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Instrument for Simultaneous Measurement of Density and Viscosity

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Instrument for Simultaneous Measurement of Density and Viscosity

Abstract

The speed of torsional stress waves transmitted in solid waveguides submerged in a liquid depends, among other things, on the liquid's density and viscosity and the waveguides' cross-sectional geometry. By measuring the speed of torsional stress waves in two waveguides of different cross-sectional geometries, one can obtain both the liquid's density and viscosity. An online, real-time sensor for the simultaneous measurement of density and viscosity is described. The article details the sensor's principles of operation and reports experimental results conducted using viscosity standard calibration liquids with well-known thermophysical properties. For fluids with density $\rho_f > 1 \times 10^3 \text{ kg/m}^3$, it is estimated that the instrument can measure density with a precision better than 0.5%. For fluids with the product shear viscosity (μ) and density, $\rho_f \mu > 100 \text{ kg}^2/(\text{m}^4\text{s})$, it can measure the shear viscosity with a precision better than 1%.

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Instrument for simultaneous measurement of density and viscosity

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The speed of torsional stress waves transmitted in solid waveguides submerged in a liquid depends, among other things, on the liquid's density and viscosity and the waveguides' cross-sectional geometry. By measuring the speed of torsional stress waves in two waveguides of different cross-sectional geometries, one can obtain both the liquid's density and viscosity. An on-line, real-time sensor for the simultaneous measurement of density and viscosity is described. The article details the sensor's principles of operation and reports experimental results conducted using viscosity standard calibration liquids with well-known thermophysical properties. For fluids with density $\rho_f > 1 \times 10^3 \text{ kg/m}^3$, it is estimated that the instrument can measure density with a precision better than 0.5%. For fluids with the product shear viscosity (μ) and density, $\rho_f \mu > 100 \text{ kg}^2/(\text{m}^4 \text{ s})$, it can measure the shear viscosity with a precision better than 1%.

INTRODUCTION

In this article, we describe an on-line, real-time instrument for the simultaneous measurement of fluid density and viscosity. When one is interested in the measurement of density alone, the instrument provides a means for correcting for viscous effects. The measurement of density and viscosity is of importance in many industrial and technological processes. Hence, it is not surprising that a considerable amount of effort has been invested in the development of instruments and measurement techniques for this purpose. For obvious reasons, it is possible to cite only a small fraction of the relevant literature pertaining to viscosity and density measurement.

Many of the devices employed for the measurement of viscosity are based on observing the period and decay (damping) constant of torsional oscillations of an axially symmetric body, such as a disk¹⁻⁴ or a sphere, suspended from an elastic strand and submerged in the measured fluid. Nieuwoudt and co-workers¹⁻³ advanced a theory which correlates the damping constant and the period of oscillations with the fluid's density and viscosity. Another technique which has been employed for precision measurements of shear viscosity involves the use of the torsional piezoelectric crystal method.⁵ In this technique, the viscosity is correlated with the resonance-curve bandwidth. However, in order to calculate the shear viscosity, independent knowledge of the fluid's density is required.

The instrument we describe operates on a different principle than the aforementioned techniques. We measure the speed of torsional stress waves transmitted in waveguides with circular and noncircular cross sections. The measured quantities are correlated with the fluid's density and viscosity. The instrument contains no moving parts, has a fast response, is relatively rugged, and can be permanently installed inside a flow conduit or a chemical reactor to continuously monitor density and viscosity in real time. These virtues may make it suitable for industrial use.

Below, we describe the principle of operation of our sen-

sor. The discussion focuses on Newtonian fluids. It is anticipated that a similar methodology could be used for non-Newtonian fluids. When torsional stress waves are transmitted in a waveguide with a circular cross section, viscous drag will induce motion in the liquid. Thus, the stress wave needs to overcome the combined inertia of the solid and the added inertia of the fluid (hereafter referred to as the "viscous inertia"). The net effect is a reduction in the wave speed. Because of the axisymmetry of the waveguide, it is not difficult to develop a theory which correlates the stress-wave speed with the fluid's shear viscosity. For example, Wang⁶ showed that, to the first approximation, this reduction in speed correlates with $(\rho_f \mu)^{1/2}$. Thus, by measuring the speed of the torsional stress wave, one can obtain information on the product $(\rho_f \mu)$. Similar information can also be obtained by measuring the wave's attenuation. The latter method was used, for example, by Roth and Rich⁷ for the measurement of the viscosity of viscoelastic materials. We prefer to measure the wave speed since it can be done with higher precision than the attenuation measurement. In principle, one could determine simultaneously shear viscosity and density by measuring both the wave speed and the attenuation. Such a measurement technique may, however, not be accurate since both the attenuation and reduction in wave speed depend primarily on the product $(\rho_f \mu)^{1/2}$, while in most circumstances, the influence of density alone is relatively weak (see Ref. 6 for further details).

When the torsional stress waves are transmitted in waveguides with noncircular cross sections, fluid motion is induced by two mechanisms. The first mechanism is the normal velocity component at the solid-fluid interface. This normal velocity component exists only when the solid's cross section is noncircular. As a result of this fluid motion, the torsional stress wave encounters added fluid inertia which exists even in the absence of viscosity. Hence, we shall refer to it as the "inviscid inertia." Bau⁸ and Kim and Bau⁹ obtained analytically and numerically the flow field and the apparent "inviscid inertias" associated with various waveguide's cross sections. The second mechanism, just as in the case of the waveguide with the circular cross section, is vis-

ous drag. As before, we shall refer to the apparent fluid inertia attributable to viscous effects as the "viscous inertia." It is more difficult to calculate analytically the viscous contribution in this instance than it is in the axisymmetric case (for an approximate evaluation of this quantity, see Davis¹⁰). However, unless the liquid viscosity is high, one can neglect the "viscous inertia" altogether.⁹ This allows one to use the waveguide with the noncircular cross section as a densimeter. Prototypes of such densimeters have been manufactured by Lynnworth,¹¹ who has obtained a patent for the device.¹² These sensors have been used in experiments to measure liquid density,¹³ liquid level,¹³⁻¹⁷ void fraction of wet steam,¹⁸ low gravity gauging of satellites' liquid propellant,¹⁹ and an aircraft's residual fuel mass.

I. THE EXPERIMENTAL APPARATUS

The apparatus consisted of either two separate waveguides with different cross-sectional geometries (Fig. 1) or a single waveguide with two sections of different cross-sectional geometries (Fig. 2). The latter arrangement has the advantage of requiring a single signal-processing unit, and therefore it may be more attractive to industrial users. Most of the results reported here were obtained using the two separate waveguides. But we tested and verified the feasibility of using a single waveguide (Fig. 2) for the same purpose. First, we shall describe the two-waveguide arrangement, and subsequently we shall provide some data on the single waveguide.

The two waveguides were made of an elastic material of density ρ_s (Fig. 1). One waveguide had a hollow circular cross section, while the other had a noncircular cross section. Experimental results for a rectangular cross section are reported later in this paper. The waveguides were submerged in a liquid of density ρ_f and shear viscosity μ . Because of the high dependence of the viscosity on temperature, the experiments were conducted in a constant-temperature bath. The temperature was occasionally varied in order to increase the range of viscosities which could be obtained from any given liquid. The liquids used were viscosity standards, typically employed in laboratory calibration of viscosimeters, for which accurate viscosity and density data are available. Subsequent to submersion in the liquid, the waveguides were subjected to torsional pulses.

The torsional stress waves were conveniently introduced utilizing magnetostrictive phenomena. One end of a delay line made of a magnetostrictive material was soldered

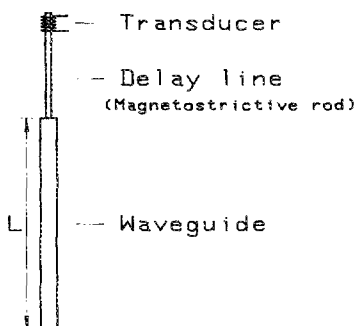


FIG. 1. Schematic description of the torsional-wave sensor consisting of a waveguide with a single cross-sectional geometry.

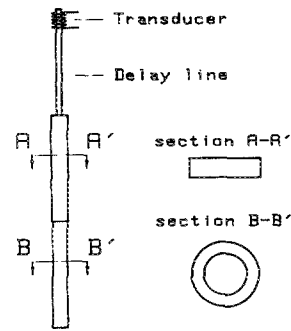


FIG. 2. Schematic description of the torsional-wave sensor consisting of a waveguide with two different sections. One section has a circular cross section, while the other has a rectangular one.

or glued to the waveguide. A coil was placed around the other end of the delay line (Fig. 1). The delay line was electrically polarized so as to develop a circumferential, permanent, magnetic field inside the magnetostrictive wire. The introduction of a current pulse into the coil caused a time-varying axial magnetic field to develop. The interaction between the two aforementioned fields led to a twisting force on the magnetostrictive wire and the generation of a torsional pulse. This is known as the Wiedemann effect.²⁰ The resulting torsional stress wave traveled in the magnetostrictive wire. Part of the wave was reflected at the magnetostrictive-wire-waveguide interface. The other part traveled through the waveguide and was reflected from its other end. It caused electromotive force in the coil which then acted as a receiver. This is known as the inverse Wiedemann effect. The signal was viewed on an oscilloscope screen. In Fig. 3, we depict typical signal traces from our experiments. The signal denoted (A) in Fig. 3 is a result of the reflection from the delay-line-waveguide interface, while the signal denoted (B) in Fig. 3 is a result of the reflection from the waveguide's end. By measuring the time (t) which elapses between the two signals, one can calculate the speed of the torsional stress wave in a waveguide of known length. The time span is measured peak to peak. As we shall show later, this time span (or the wave speed) depends, among other things, on the densities of the waveguide and the adjacent fluid, on the fluid's viscosity, and on the shape of the waveguide.

In our experiments, we typically employed waveguides made of stainless steel with length $L = 300$ mm. The sensor's sensitivity will improve if lower-density waveguides are used. The delay line is made of Remendur (Co-Fe-V) of length about 1000 mm. The typical travel time of the tor-

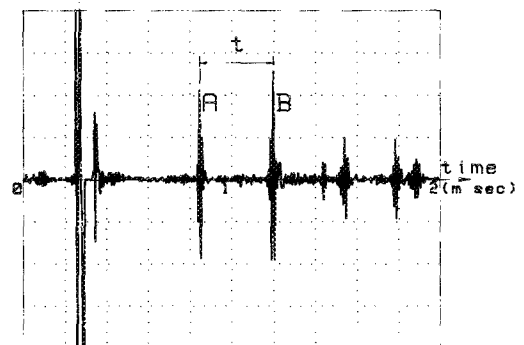


FIG. 3. The signal trace of the reflected signal of the single cross-section waveguide.

sional stress wave in a waveguide with a rectangular cross section (2 mm × 5 mm) in air is about 360 μs. The time span can typically be measured with a precision of 5 ns. The reflectivity of the delay-line-waveguide interface can be controlled by controlling the mechanical impedance mismatch at the interface. This was typically done by soldering a small ring around the waveguide. Our experiments suggested insensitivity to eccentric placement of the delay line with respect to the waveguide.

The operation of the instrument consisting of a single waveguide with two different sections is similar to what has been described above except that in this case an additional reflection of the torsional stress wave was obtained. In Fig. 4, we depict a typical signal trace from our experiments. The signals denoted *A*, *B*, and *C* are, respectively, the results of reflections from the delay-line-waveguide interface, the circular-noncircular sections' interface, and the waveguide's end. By measuring the distances between any two adjacent peaks, one can determine the travel time in the circular and noncircular sections.

II. THEORY

Consider a torsional stress wave traveling in a waveguide with a uniform cross section submerged in a liquid. As the torsional wave travels through the waveguide, the solid-liquid interface is alternately accelerated and decelerated. Consequently, the inertia which needs to be overcome by the torsional pulse is a combination of the solid waveguide's inertia (I_s) and the adjacent liquid's apparent inertia (I_f). According to Ref. 8, to the first-order approximation, the torsional wave speed (c) can be calculated from the equation

$$c = K(G/\rho_s)^{1/2}(1 + \rho_f I_f / \rho_s I_s)^{-1/2} \quad (1)$$

where G is the shear modulus of the solid, $K = \sqrt{(D^*/I_s)}$, and D^* is the torsional rigidity. In the experiments, we measured the flight time of the torsional stress wave. Hence, it is desired to derive an expression for the effect of apparent inertia on the flight time. Let t_0 and t denote, respectively, the flight time in air (which is assumed to be a good approximation for flight time in vacuum) and in liquid at the same temperature. $Dt = t - t_0$ denotes the difference in the transmission time of a wave in a waveguide submerged in liquid and one in air. Since typically $(\rho_f I_f) / (\rho_s I_s) \ll 1$, Eq. (1)

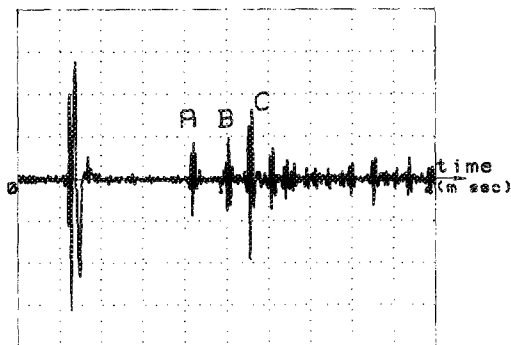


FIG. 4. The signal trace of the reflected signal for the waveguide with two different sections.

suggests, with a good approximation, that

$$Dt/t_0 \cong \rho_f I_f / 2\rho_s I_s. \quad (2)$$

As we explained in the Introduction, when the waveguide has a noncircular cross section, the fluid's motion is induced via the generation of a pressure field and a drag force. The pressure field is generated when the motion of the solid's surface induces a normal velocity component in the fluid. The drag force results from viscous effects. In short, the apparent inertia of the fluid (I_f) can be taken as resulting from a combination of these two effects. We denote the inviscid contributions to the apparent inertia as $I_{f,i}$ and $I_{f,v}$, respectively. That is, $I_f = I_{f,i} + I_{f,v}$. The inviscid inertia for rectangular and elliptical cross sections of various aspect ratios was calculated in Bau⁸ and for other cross sections in Kim and Bau.⁹ An approximate method of evaluating the viscous inertia is given in Ref. 10. Our earlier analysis^{8,9} showed that since the scale of the pressure-induced flow field is of the same order of magnitude as the size of the waveguide's cross section, $I_{f,i} \cong C_1 I_s$, and C_1 is a constant of order one. C_1 depends on the cross section's geometry and aspect ratio. For example, for a rectangular cross section of aspect ratio 3.3, $C_1 = 1.062$.⁸ The scale of the drag-induced flow field is comparable to the thickness of the viscous boundary layer. The thickness of the viscous boundary layer in a Newtonian fluid is of the order $(\nu/\omega)^{1/2}$, where $\nu = \mu/\rho_f$ is the kinematic viscosity and ω is the wave's frequency. Hence, one would expect $I_{f,v} \cong C_2 I_s (\nu/\omega a^2)^{1/2}$, where a is a characteristic dimension of the cross section and c_2 is a geometry-dependent constant which we determined empirically. For example, for a waveguide with $a = 0.005$ m, operating at a frequency $\omega = 50$ kHz at room temperature in water and glycerin, $(\nu/\omega a^2)^{1/2} \sim 2 \times 10^{-3}$ and 2×10^{-2} , respectively. We shall demonstrate that even in the latter case, viscous effects contribute less than 10% to the added inertia (I_f). In summary, the apparent inertia of the rectangular cross section can be expressed as

$$I_f/I_s = C_1 + C_2(\nu/\omega a^2)^{1/2}. \quad (3)$$

On the other hand, when the waveguide's cross section is circular, there is only drag-induced apparent inertia (i.e., $C_1 = 0$). For a waveguide with cross-sectional radius a , we have, to the first approximation,⁶ that

$$I_f/I_s = (8\nu/\omega a^2)^{1/2}. \quad (4)$$

The constants C_1 and C_2 are fixed for any given sensor, and they do not depend on the adjacent liquid. Thus, by measuring Dt/t_0 for waveguides with circular and noncircular (rectangular, say) cross sections, we can obtain both the density (ρ_f) and the viscosity (μ).

III. EXPERIMENTS

The experimental results reported in this section were conducted using stainless-steel waveguides with rectangular cross sections (2.58 mm × 0.73 mm), and an aspect ratio of 3.53, and aluminum waveguides with hollow circular cross sections of outer and inner radii of 1.22 and 0.78 mm, respectively. In our experiments, we measured the transmission times for the torsional stress wave in waveguides submerged

in liquid (t) and in air (t_0) at the same temperature. Since the interfacial velocity of the waveguide is very small (on the order of magnitude of 10^{-3} m/s or smaller), we neither expect nor observe vortex shedding from the waveguide's edges.

In Fig. 5, we depict the ratio Dt/t_0 as a function of the relative density (the ratio between the liquid's density and that of pure water) for the waveguide with a rectangular cross section. The symbols represent experimental data, while the solid line corresponds to an analytical prediction based on Eq. (2) with the viscous effects being neglected. The + signs correspond to a waveguide submerged in a solution of calcium chloride in water. The solution's density is varied by changing the salt concentration. The viscosity of the calcium chloride solution is approximately similar to that of water. The circles correspond to experiments conducted in a glycerin-water solution. Both the viscosity and density of the solution are varied by changing the relative concentration of each component. The points denoted 1, 2, 3 and 4 in Fig. 5 correspond to liquids with kinematic viscosities $\nu = 1180, 620, 310,$ and 50×10^{-6} m²/s. The scatter of the experimental data (Dt/t_0) was smaller than 0.15%. For the calcium chloride solution, an excellent agreement is observed between the inviscid theory and the experimental observations. Significant deviations between the inviscid theory and the experiment are observed for the glycerin solution for $(\nu/\omega a^2)^{1/2} > 0.01$. These deviations are attributed to viscous effects which were not included in the inviscid theory.

Next, we focus our attention on the effect of viscosity on the performance of the waveguide with the rectangular cross section. To this end, we depict in Fig. 6

$$\chi = (Dt/t_0) - (\rho_f/2\rho_s)C_1$$

as a function of $(\rho_f\mu)^{1/2}$. The standard deviation of the results depicted in Fig. 6 is smaller than 0.1%. In these experiments, the waveguide was submerged in liquids which are typically used as viscosity standards for laboratory calibration of viscosimeters (S6, S60, S600, and N100). Measurements were carried out at temperatures 20, 25, 40, and 50 °C.

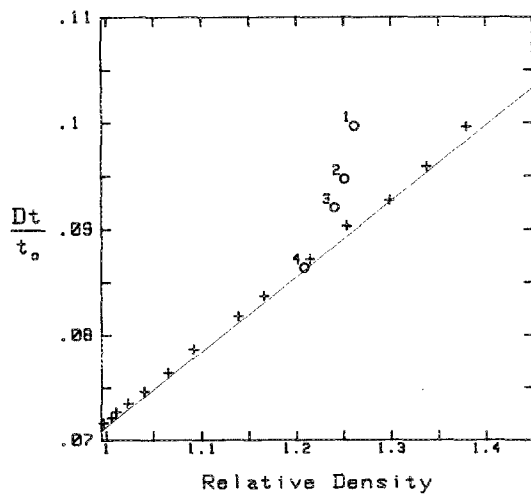


FIG. 5. The ratio Dt/t_0 is depicted as a function of the relative density for a waveguide with a rectangular cross section. The symbols and solid line represent theoretical and experimental results, respectively.

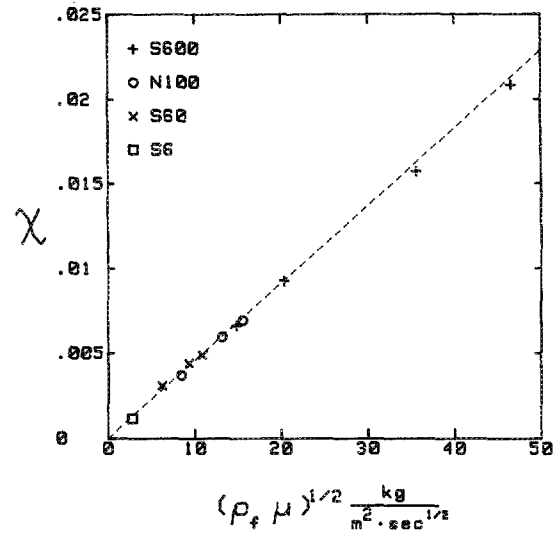


FIG. 6. χ is depicted as a function of $(\rho_f\mu)^{1/2}$ for a waveguide with a rectangular cross section submerged in various viscosity standard liquids.

The temperature in the test chamber was uniform within 0.5 °C. The observed linear relationship between χ and $(\rho_f\mu)^{1/2}$ in Fig. 6 agrees well with the assertion of expression (3). Since the wave used in our experiments was not monochromatic, we did not calculate the constant C_2 explicitly. χ , however, is a quantity which is specific to the sensor and is independent of the tested liquid. Thus, χ can be determined as a part of sensor calibration.

It is evident from the foregoing that the flight time of the torsional stress wave in a waveguide with a noncircular (rectangular, in our case) cross section is affected by both the liquid's density (ρ_f) and the product $(\rho_f\mu)$. In order to determine the density alone, we need a means of measuring the product $(\rho_f\mu)$. This is afforded by the waveguide with the circular cross section. In Fig. 7, we depict the flight time of the torsional stress wave in a hollow, circular tube as a func-

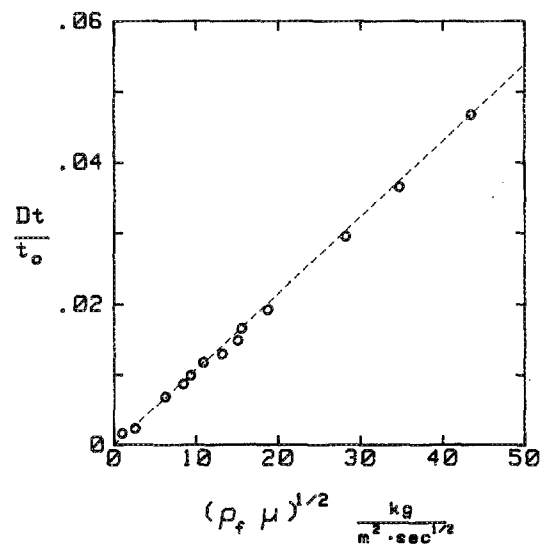


FIG. 7. The ratio Dt/t_0 is depicted as a function of $(\rho_f\mu)^{1/2}$ for a waveguide with a hollow, circular cross section submerged in various viscosity standard liquids.

tion of $(\rho_f \mu)^{1/2}$. The standard deviation of the results reported in Fig. 7 was smaller than 0.2%. In accordance with expression (4), we observe a linear relationship with the constant of proportionality depending on the sensor, but not on the tested liquid. Thus, the measurement with circular tube allows us to determine $(\rho_f \mu)^{1/2}$.

IV. DISCUSSION

We have presented in this article an instrument for the simultaneous determination of both viscosity and density. The product $(\rho_f \mu)^{1/2}$ is measured by deploying a waveguide or a section of a waveguide with a circular cross section. Subsequently, one can use formula (3) together with the reading obtained using a waveguide or a section of a waveguide with a rectangular cross section to calculate the liquid's density. The apparatus will be most effective for measurements in liquids or high-pressure gases. Its ruggedness and fast response may make it attractive for industrial users.

By comparing our measurements with the available thermophysical properties of the tested liquids, we estimate that the apparatus can resolve density within 0.5% for fluids with density $\rho_f > 1 \times 10^3 \text{ kg/m}^3$, and shear viscosity within 1% for fluids with the product shear viscosity (μ) and density (ρ_f), $(\rho_f \mu) > 100 \text{ kg}^2/(\text{m}^4 \text{ s})$.

The precision of the apparatus could conceivably be improved if one were to measure the shift in its resonance frequency under loading conditions. This could be achieved either by inducing stress waves in our waveguide at variable frequencies or by employing torsional piezoelectric crystals with noncircular cross sections.

ACKNOWLEDGMENTS

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