3-4-1991

Controlling a Chaotic System

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Suggested Citation:
Singer, Jonathan, Y-Z Wang, and Haim H. Bau. (1991) *Controlling a Chaotic System*. Physical Review Letters. Vol. 66(9).

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Abstract
Using both experimental and theoretical results, this Letter describes how low-energy, feedback control signals can be successfully utilized to suppress (laminarize) chaotic flow in a thermal convection loop.

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(Received 22 October 1990)

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PACS numbers: 05.45.+b

Chaotic behavior is abundant both in nature and in man-made devices. On occasion, chaos is a beneficial feature as it enhances mixing and chemical reactions and provides a vigorous mechanism for transporting heat and/or mass. However, in many other situations, chaos is an undesirable phenomenon which may lead to vibrations, irregular operation, and fatigue failure in mechanical systems, temperature oscillations which may exceed safe operational conditions in thermal systems, and increased drag in flow systems. Also, since chaotic behavior cannot be predicted in detail, it may be detrimental to the operation of various devices. Clearly, the ability to control chaos (i.e., promote or eliminate it) is of much practical importance. Although the topic of enhancing chaos has attracted some attention in the scientific literature,[1] there are, indeed, very few theoretical publications[2,3] and even fewer experimental works which address the probably more difficult topic of chaos suppression.

In the first part of this Letter, we describe an experiment conducted with a thermal convection loop, in which for heating rates exceeding a certain threshold value the flow exhibited chaotic behavior. By making small adjustments to the heating rate in response to events detected inside the loop (feedback control), we succeeded in suppressing the chaotic behavior and "laminarizing" the flow. In order to achieve a better understanding as to how our controller operates, we applied a similar control strategy to a simplified mathematical model capable of qualitatively describing the flow in the loop. This theoretical investigation is described in the second part of the Letter. The success of our effort gives hope that it may be possible to suppress chaos in more complicated systems.

The experimental apparatus consists of a pipe of diameter d ($=0.030$ m) bent into a torus of diameter D ($=0.760$ m) containing liquid (i.e., water). The apparatus stands in the vertical plane. The lower half of the apparatus is heated with a uniform-heat-flux resistance heater while the upper half is submerged in a jacket containing flowing coolant so as to approximate a uniform wall temperature (Fig. 1). The apparatus is similar to the one employed by Creveling *et al.*[4] and Gorman, Widmann, and Robins,[5] who have described it in detail. We measured the temperature differences between positions 3 and 9 o'clock and between 6 and 12 o'clock around the loop as functions of time. The heating (cooling) of the lower (upper) half causes temperature gradients within the liquid which under certain conditions may cause fluid motion inside the loop.

For low heating rates ($Q < 190$ W), the flow inside the loop is steady and unidirectional. That is, depending on initial conditions, the fluid flows in either the counterclockwise or the clockwise direction. Above a certain critical heating rate (about 190 W in our experiment), the steady motion loses its stability and the flow becomes chaotic. The chaotic flow appears as irregular oscillations in the flow rate and occasional reversals in the direction of the flow. For example, in Fig. 2, we depict the experimentally obtained temperature difference between positions 3 and 9 o'clock as a function of time for the heating rate of 600 W. The corresponding Rayleigh number is 3.16 times its value at the onset of chaos. Positive (negative) values of the temperature difference in Fig. 2 indicate flow in the counterclockwise (clockwise) direction. Witness the relatively high temperature oscillations associated with the chaotic flow.

Our objective is to suppress the oscillations so as to make the flow approximately steady. That is, we wish to retain the steady unidirectional flow as it existed before the onset of chaos (albeit with higher cross-sectionally

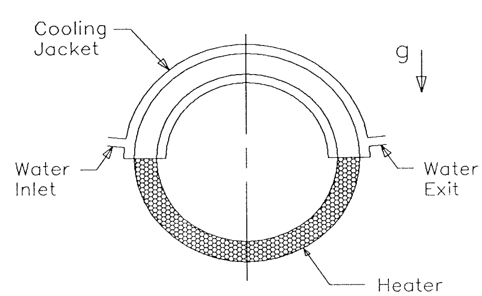

FIG. 1. Schematic description of the thermal convection loop. The lower half of the loop is heated with a uniform-flux resistance heater. The upper half is cooled by passing water through the jacket.

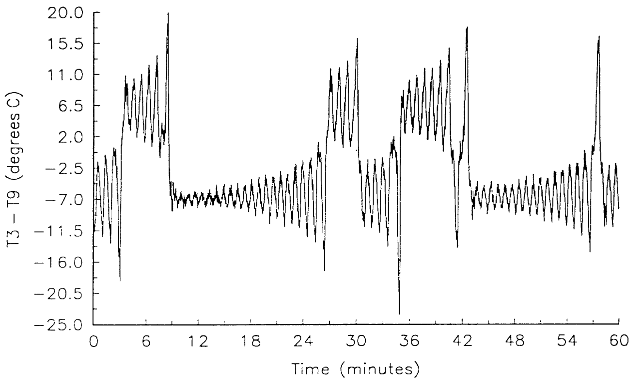

FIG. 2. The experimentally obtained temperature difference between positions 3 and 9 o'clock around the loop as a function of time for a heating rate of 600 W without a controller. The change in sign indicates a change in the direction of the flow.

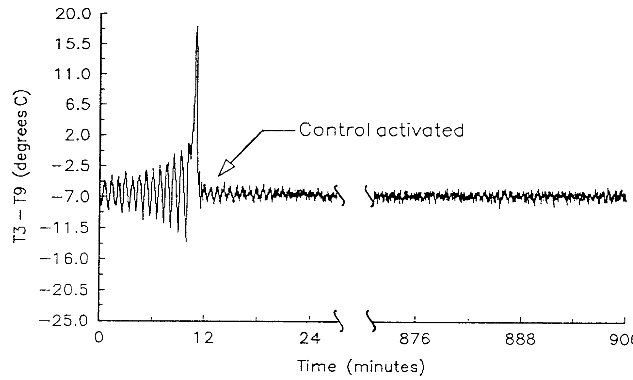

FIG. 3. The experimentally obtained temperature difference between positions 3 and 9 o'clock around the loop as a function of time for a heating rate of 600 W. The controller is turned on 12.5 min into the run, "laminarizing" the flow.

averaged velocity, reflecting the higher heating rate). To accomplish this objective, we adopted a relatively simple control strategy. We change the heating rate by a relatively small increment as a function of the low-pass-filtered temperature difference between positions 6 and 12 o'clock around the loop. When the above temperature difference exceeds or drops below some average value, the heating rate is increased or decreased by a preset increment (i.e., 25 W in Fig. 3) after a time delay of a few seconds. The results of this strategy are depicted in Fig. 3 where we show the temperature difference between positions 3 and 9 o'clock depicted as a function of time. Initially, the flow was uncontrolled and we observed similar oscillations to the ones depicted in Fig. 2. The controller was activated 12.5 min into the run in Fig. 3. The transition from the chaotic flow into a relatively steady, laminar flow is self-evident. We ran the experiment for over 15 h maintaining the type of steady flow shown in Fig. 3. The controller also succeeded in overcoming finite-amplitude disturbances purposely introduced into the loop. It is likely that the magnitude of the control signal could be further reduced by adopting a more sophisticated control strategy than the one reported here.

In order to gain physical insight into how the controller operates, it is useful to briefly describe the mechanism responsible for the chaotic, oscillatory behavior of the flow in the loop.[6] To this end, imagine that a small disturbance causes the flow to slow down below the steady-state flow rate. As a result, the fluid spends more time in the heater (cooler) section, gains (loses) more (less) heat than usual, and emerges from the heater (cooler) with a temperature higher (lower) than usual. This, in turn, causes an increase in the buoyancy force with a corresponding increase in the fluid's velocity. Once the fluid velocity increases, the reverse effect occurs with a subsequent reduction in the fluid velocity. Under appropriate conditions, in the absence of a control mechanism, these oscillations amplify and eventually lead to the chaotic behavior depicted in Fig. 2. The controller detects the appearance of disturbances by monitoring deviations in the temperature difference between top and bottom from the corresponding steady-state value $(z-z_0)$. Once such a deviation is detected, the controller takes action to counteract the effect of this deviation. For instance, if the deviation tends to accelerate (decelerate) the flow, the heating rate is increased (decreased) to counteract this effect. As the controller applies only relatively small perturbations to the input power, it will be able to counteract only small oscillations. Consequently, when the controller is applied to a chaotic flow, it may take some time before the temperature oscillations become small enough for the controller to take effect. This amount of time will decrease as the magnitude of the control signal increases. Ott, Grebogi, and Yorke[3] argue that this length of time is proportional, on the average, to a negative power of the control signal. Once the controller succeeds in laminarizing the flow, it will prevent the oscillations from increasing beyond the controllable magnitude. It should be noted that, due to the presence of noise in the system, it is necessary to maintain the control signal above some minimal value.

To attain further insight into how the controller operates, we examined a simple mathematical model based on the Lorenz equations.[7,8] The solutions of the Lorenz equations provide a good qualitative resemblance to the observed flow in the loop.[9,10] The solutions of these equations, depending on the magnitude of the Rayleigh number (which in our case is proportional to the heating rate), include a no-motion state, two steady flow states (consisting of flows in the counterclockwise and clockwise directions), and chaotic flow of the type depicted in Fig. 2. The model also predicts periodic windows within the chaotic regime, but these have not yet been observed in experiments. The variables (x,y,z) in the equations below correspond, respectively, to the cross-sectionally averaged velocity in the loop, the temperature difference between positions 3 and 9 o'clock, and the temperature difference between positions 12 and 6 o'clock. The Lorenz equations with the on-off controller are

$$\frac{dx}{dt}=p(y-x),\quad \frac{dy}{dt}=-xz-y\ ,$$
$$\frac{dz}{dt}=xy-z-[R+\varepsilon\,\mathrm{sgn}(z-z_0)]\,. \tag{1}$$

As in the experiment, the controller reacts to deviations of z from some preset, average value and modifies the magnitude of the Rayleigh number (R) which is proportional to the heating rate in the experiment. In the above, p is the Prandtl number, ε represents the magnitude of the control signal (in the classical Lorenz equations $\varepsilon=0$), and sgn corresponds to the sign of $z-z_0$. We carried out numerical experiments to observe the effect of the controller on the behavior of the flow. The results of our numerical experiments are depicted in Fig. 4, where we show the controlled (uncontrolled) signals with thick (light) lines. In Fig. 4, the controller has been switched on at a nondimensional time $t=9$. Comparing Figs. 4 and 3, we observe that the physical and simulated controller cause a similar effect.

In order to analyze the controller's action, it is convenient to construct the Lyapunov functional for the controlled system. To this end, we define a new set of dependent variables $\{X,Y,Z\}=\{x-\sqrt{R-1},y-\sqrt{R-1},z+1\}$, where the fixed point $\{X,Y,Z\}=\{0,0,0\}$ corresponds to a steady-state-motion solution of the Lorenz system (1). We focus on R values for which this solution is nonstable. The Lyapunov functional

$$E=\tfrac{1}{2}(X^2+pY^2+pZ^2)\geq 0 \tag{2}$$

satisfies

$$\frac{1}{p}\frac{dE}{dt}=-(X-Y)^2-Z^2+[\sqrt{R-1}X-\varepsilon\,\mathrm{sgn}(Z)]Z\,. \tag{3}$$

For stability, we require that (3) be negative. This can be satisfied provided that X is sufficiently small. Thus, there is a domain of attraction in phase space $\{X,Y,Z\}$ in the vicinity of the fixed point $\{0,0,0\}$. In other words, once the system enters into this domain of attraction (and eventually it would), it will stay in it as long as externally imposed perturbations are not too large. The Lyapunov functional presented above, although not optimal, demonstrates the effect of the controller on the stability characteristics of the loop.

We note in passing that the chaotic attractor includes, in addition to the nonstable time-independent flow, also an assortment of nonstable periodic and quasiperiodic orbits. Ott, Grebogi, and Yorke[2,3] argue that it might be possible to stabilize any of the aforementioned orbits. In numerical experiments, using a controller, we indeed succeeded in obtaining a stable periodic flow in the nominally chaotic regime.

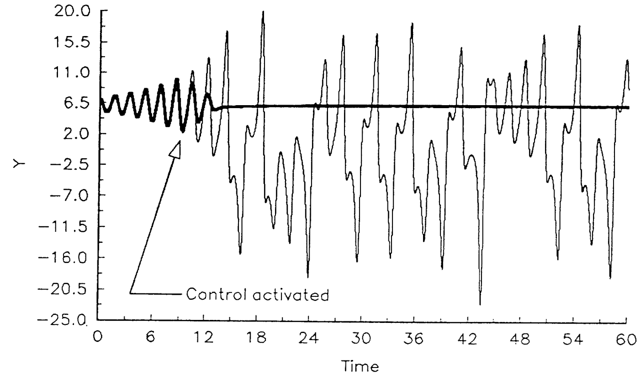

FIG. 4. The numerically generated temperature difference between positions 3 and 9 o'clock around the loop as a function of time. The uncontrolled and controlled signals are shown in light and thick lines, respectively.

In conclusion, we have demonstrated experimentally and theoretically that a simple control strategy can be effectively used to suppress chaos in a simple dynamical system. It is our hope that similar control strategies can be successfully implemented for more complicated situations.

This work was supported, in part, by the National Science Foundation through Grant No. CBT 83-51658.

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