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NOTE: At the time of publication, author Marek W. Kowarz was affiliated with the Institute of Optics, University of Rochester, Rochester, NY 14627

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# Antenna radiation in the presence of a chiral sphere

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The radiation emitted by electromagnetic sources placed both inside and outside of a homogeneous sphere of chiral media is studied using an exact formulation. For both cases, dyadic Green's functions are found in terms of spherical vector wave functions. The radiated fields and radiation resistance are examined for a dipole located at the center of the chiral sphere. For this case, it is shown that by choosing the sphere's size and material parameters properly, purely circular polarization can be achieved in the far field. It is also demonstrated that the radiation resistance of the dipole can be increased.

## I. INTRODUCTION

*Chirality*, which refers to the handedness of an object or a medium, has played an important role in a variety of fields including chemistry,<sup>1</sup> optics,<sup>2</sup> particle physics,<sup>3</sup> and mathematics.<sup>4</sup> *Electromagnetic chirality* represents the role of chirality in electromagnetics and is exhibited in a class of materials called chiral materials. Owing to the handed nature of their constituents, these materials themselves possess an intrinsic handedness. For the time harmonic ( $e^{-i\omega t}$ ) and isotropic case, they are characterized by the following set of constitutive relations:

$$\mathbf{D} = \epsilon_c \mathbf{E} + i\xi_c \mathbf{B}, \quad (1)$$

$$\mathbf{H} = i\xi_c \mathbf{E} + \mathbf{B}/\mu_c, \quad (2)$$

where  $\epsilon_c$ ,  $\mu_c$ , and  $\xi_c$  are, respectively, the permittivity, permeability, and chirality admittance of such media.<sup>5</sup> It has been shown that electromagnetic waves in these media display two unequal characteristic wave numbers for the right- and left-circularly polarized (RCP, LCP) eigenmodes.<sup>6</sup> These unequal wave numbers give rise to a circular birefringence which results in both optical activity and circular dichroism. It is worth noting that, in isotropic chiral media, this birefringence is independent of the direction of wave propagation, whereas in anisotropic materials it does depend on the wave's direction of propagation. The set of chiral constitutive relations given in (1) and (2) is actually a subset of the more general constitutive relations used to describe bianisotropic media. These generalized relations have been studied extensively by Kong.<sup>7-9</sup>

Recently, there has been renewed attention brought to the area of wave propagation and radiation in chiral media due to the possibility of fabricating such materials for microwaves and millimeter waves. In the past few years, several fundamental problems of electromagnetic wave interaction with chiral materials have been investigated and reported in the literature. Among these, one should mention dyadic Green's functions in chiral media,<sup>6,10</sup> waveguiding structures filled with chiral materials,<sup>11-13</sup> transition radiation caused by a chiral slab,<sup>14</sup> Doppler effects in chiral media,<sup>15</sup> wave propagation in periodic chiral structures,<sup>16</sup> point and distributed radiators embedded in chiral media,<sup>17</sup> and

reflection and refraction at a chiral-nonchiral interface.<sup>18,19,20</sup>

The present paper is aimed at investigating the interaction of radiation emitted by an electromagnetic source with a sphere made from an isotropic lossless chiral material. The motivation behind the present investigation, besides its theoretical and academic importance, is provided by the potential applications of chiral materials to the designs of novel antenna radomes and superstrates, chiral lenses for microwave and optical regimes, and array polarization control. The antenna radomes made from chiral materials, which we name *chirodomes*, are of particular interest since the polarization birefringence present in chiral materials may give rise to new radiation characteristics. Furthermore, for both the optical and microwave regimes, lenses made of chiral or optically active materials may yield novel and interesting properties such as bifocal lengths.

## II. PROBLEM FORMULATION

In order to study the radiation characteristics of an electromagnetic source in the presence of a homogeneous chiral sphere, it is necessary to derive dyadic Green's functions for such a problem. However, before proceeding, we must first consider the case of an unbounded chiral medium. The dyadic Green's function for this case will provide the framework from which to devise those for the chiral sphere. Bassiri *et al.* have already obtained<sup>6</sup> one form from a coordinate free approach. Here, we will use an eigenfunction expansion in spherical coordinates, which is similar to that employed by Tai for nonchiral spheres,<sup>21</sup> and express the dyadic Green's function in terms of spherical vector wave functions. This expansion will later greatly facilitate the matching of boundary conditions at the interface between the chiral sphere and the nonchiral dielectric surrounding it.

The Helmholtz equation for the electric field in a chiral medium is<sup>6</sup>

$$\nabla \times \nabla \times \mathbf{E} - 2\omega\mu_c\xi_c \nabla \times \mathbf{E} - \omega^2\mu_c\epsilon_c \mathbf{E} = i\omega\mu_c \mathbf{J}. \quad (3)$$

Since the equation is linear, the electric field can be found from a three-dimensional transform of current density  $\mathbf{J}$  with the dyadic Green's function  $\Gamma_c(\mathbf{r}, \mathbf{r}')$ :

$$\mathbf{E}(\mathbf{r}) = i\omega\mu_c \int \Gamma_c(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dV'. \quad (4)$$

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A differential equation for  $\Gamma_c(\mathbf{r}, \mathbf{r}')$  is found by substituting Eq. (4) into Eq. (3), which yields

$$\nabla \times \nabla \times \Gamma_c(\mathbf{r}, \mathbf{r}') - 2\omega\mu_c \xi_c \nabla \times \Gamma_c(\mathbf{r}, \mathbf{r}') - \omega^2 \mu_c \epsilon_c \Gamma_c(\mathbf{r}, \mathbf{r}') = \mathbf{I} \delta(\mathbf{r} - \mathbf{r}'), \quad (5)$$

with unit dyadic  $\mathbf{I}$  and Dirac delta function  $\delta(\mathbf{r} - \mathbf{r}')$ .

Equation (5) will now be solved in terms of spherical vector wave functions. The two orthogonal functions  $\mathbf{M}_{\sigma mn}(\kappa)$  and  $\mathbf{N}_{\sigma mn}(\kappa)$  are usually defined as follows:

$$\mathbf{M}_{\sigma mn}(\kappa) = \nabla \times [\psi_{\sigma mn}(\kappa) \mathbf{r}] = (1/\kappa) \nabla \times \mathbf{N}_{\sigma mn}(\kappa), \quad (6a)$$

$$\begin{aligned} \mathbf{N}_{\sigma mn}(\kappa) &= (1/\kappa) \nabla \times \nabla \times [\psi_{\sigma mn}(\kappa) \mathbf{r}] \\ &= (1/\kappa) \nabla \times \mathbf{M}_{\sigma mn}(\kappa), \end{aligned} \quad (6b)$$

where  $\kappa$  is a yet undetermined wave number,  $\mathbf{r}$  is the piloting radial vector, and  $\psi_{\sigma mn}(\kappa)$  is the generating function given by

$$\psi_{\sigma mn}(\kappa) = j_n(\kappa r) P_n^m(\cos \theta) \frac{\cos(m\phi)}{\sin \theta}. \quad (7)$$

Here,  $j_n(\kappa r)$  is a spherical Bessel function with order  $n$  and  $P_n^m(\cos \theta)$  is an associated Legendre function of the first kind with order  $(n, m)$ . Only integer values of  $n$  and  $m$  will be used in the remainder of this paper. It should also be noted that the subscripts  $e$  and  $o$  do not refer to the nature of  $\mathbf{M}_{\sigma mn}(\kappa)$  or  $\mathbf{N}_{\sigma mn}(\kappa)$ , but rather to the even or odd character of the generating function. The coordinates  $(r, \theta, \phi)$ , with their

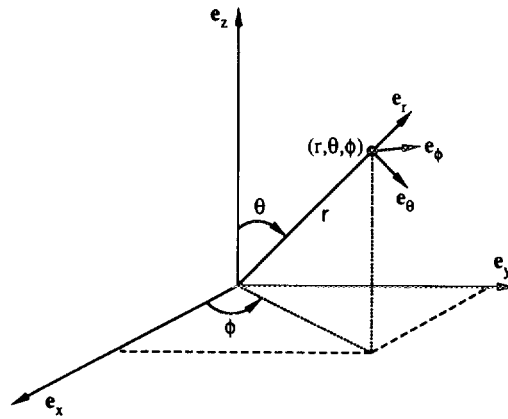


FIG. 1. Spherical coordinate system.

corresponding unit vectors, are defined in Fig. 1. When unprimed (primed), these coordinates represent the location of the observation (source) point. It is apparent that the functions  $\mathbf{M}_{\sigma mn}(\kappa)$  and  $\mathbf{N}_{\sigma mn}(\kappa)$  are not themselves solutions to the source-free form of Eq. (5), since the curl term converts one function into the other. However, as has previously been noted by Bohren,<sup>22</sup> it is possible to find linear combinations of  $\mathbf{M}_{\sigma mn}(\kappa)$  and  $\mathbf{N}_{\sigma mn}(\kappa)$  which do satisfy the equation. These combinations,  $\mathbf{V}_{\sigma mn}(\kappa)$  and  $\mathbf{W}_{\sigma mn}(\kappa)$ , are normalized such that

$$\begin{aligned} \mathbf{V}_{\sigma mn}(\kappa) &= \frac{\mathbf{M}_{\sigma mn}(\kappa) + \mathbf{N}_{\sigma mn}(\kappa)}{\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \left[ \mp \frac{m}{\sin \theta} P_n^m(\cos \theta) \frac{\sin(m\phi)}{\cos(m\phi)} \left( j_n(\kappa r) \mathbf{e}_\theta + \frac{1}{\kappa r} \frac{\partial}{\partial r} [r j_n(\kappa r)] \mathbf{e}_\phi \right) \right. \\ &\quad \left. + \frac{\partial P_n^m(\cos \theta)}{\partial \theta} \frac{\cos(m\phi)}{\sin(m\phi)} \left( \frac{1}{\kappa r} \frac{\partial}{\partial r} [r j_n(\kappa r)] \mathbf{e}_\theta - j_n(\kappa r) \mathbf{e}_\phi \right) + n(n+1) P_n^m(\cos \theta) \frac{\cos(m\phi)}{\sin(m\phi)} \frac{j_n(\kappa r)}{\kappa r} \mathbf{e}_r \right] \end{aligned} \quad (8a)$$

and

$$\begin{aligned} \mathbf{W}_{\sigma mn}(\kappa) &= \frac{\mathbf{M}_{\sigma mn}(\kappa) - \mathbf{N}_{\sigma mn}(\kappa)}{\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \left[ \mp \frac{m}{\sin \theta} P_n^m(\cos \theta) \frac{\sin(m\phi)}{\cos(m\phi)} \left( j_n(\kappa r) \mathbf{e}_\theta - \frac{1}{\kappa r} \frac{\partial}{\partial r} [r j_n(\kappa r)] \mathbf{e}_\phi \right) \right. \\ &\quad \left. + \frac{\partial P_n^m(\cos \theta)}{\partial \theta} \frac{\cos(m\phi)}{\sin(m\phi)} \left( -\frac{1}{\kappa r} \frac{\partial}{\partial r} [r j_n(\kappa r)] \mathbf{e}_\theta - j_n(\kappa r) \mathbf{e}_\phi \right) - n(n+1) P_n^m(\cos \theta) \frac{\cos(m\phi)}{\sin(m\phi)} \frac{j_n(\kappa r)}{\kappa r} \mathbf{e}_r \right]. \end{aligned} \quad (8b)$$

Thus,  $\mathbf{V}_{\sigma mn}(\kappa)$  and  $\mathbf{W}_{\sigma mn}(\kappa)$  now satisfy

$$\nabla \times \mathbf{V}_{\sigma mn}(\kappa) = (1/\kappa) \nabla \times \mathbf{V}_{\sigma mn}(\kappa), \quad (9a)$$

$$\nabla \times \mathbf{W}_{\sigma mn}(\kappa) = -(1/\kappa) \nabla \times \mathbf{W}_{\sigma mn}(\kappa). \quad (9b)$$

With the normalization chosen in (8a) and (8b), the orthogonality relations for  $\mathbf{V}_{\sigma mn}(\kappa)$  and  $\mathbf{W}_{\sigma mn}(\kappa)$  remain the same as those for  $\mathbf{M}_{\sigma mn}(\kappa)$  and  $\mathbf{N}_{\sigma mn}(\kappa)$ , that is

$$\int \mathbf{V}_{\sigma mn}(\kappa) \mathbf{W}_{\sigma m'n'}(\kappa') d^3 r = 0 \quad (10)$$

and

$$\begin{aligned} &\int \left\{ \begin{array}{l} \mathbf{V}_{\sigma mn}(\kappa) \mathbf{V}_{\sigma m'n'}(\kappa') \\ \mathbf{W}_{\sigma mn}(\kappa) \mathbf{W}_{\sigma m'n'}(\kappa') \end{array} \right\} d^3 r \\ &= \frac{(2 - \delta_{m0}) \pi^2 n(n+1)(n+m)!}{\kappa^2 (2n+1)(n-m)!} \delta_{mm'} \delta_{nn'} \delta(\kappa - \kappa'), \end{aligned} \quad (11)$$

where  $\delta_{m0}$ ,  $\delta_{mm'}$ , and  $\delta_{nn'}$  are Kronecker delta functions. As can be seen in Eq. (11),  $\mathbf{V}_{\sigma mn}(\kappa)$  and  $\mathbf{W}_{\sigma mn}(\kappa)$  are orthogo-

nal functions, but they do not satisfy the standard orthonormal conditions, i.e., inner product equal to one, however, the definitions used for these functions are completely adequate for our purposes.

With the help of the new spherical vector wave functions, the right-hand side of Eq. (5) may be expressed as follows by using a technique similar to that of Tai<sup>21</sup>:

$$\delta(\mathbf{r} - \mathbf{r}') = \int_0^\infty d\kappa \sum_{n=1}^\infty \sum_{m=0}^n [\mathbf{V}_{\sigma mn}(\kappa) \mathbf{A}_{\sigma mn}(\kappa) + \mathbf{W}_{\sigma mn}(\kappa) \mathbf{B}_{\sigma mn}(\kappa)], \quad (12)$$

where  $\mathbf{A}_{\sigma mn}(\kappa)$  and  $\mathbf{B}_{\sigma mn}(\kappa)$  can easily be obtained with the help of the orthogonality relations:

$$\begin{Bmatrix} \mathbf{A}_{\sigma mn}(\kappa) \\ \mathbf{B}_{\sigma mn}(\kappa) \end{Bmatrix} = \frac{2 - \delta_{m0}}{2\pi^2} \frac{(2n+1)(n-m)!}{n(n+1)(n+m)!} \kappa^2 \begin{Bmatrix} \mathbf{V}'_{\sigma mn}(\kappa) \\ \mathbf{W}'_{\sigma mn}(\kappa) \end{Bmatrix}. \quad (13)$$

The prime on  $\mathbf{V}_{\sigma mn}(\kappa)$  and  $\mathbf{W}_{\sigma mn}(\kappa)$  represents the dependence of the functions on the source point  $\mathbf{r}'$ . Considering the expansion in (12), it is reasonable to choose

$$\Gamma_c(\mathbf{r}, \mathbf{r}') = \int_0^\infty d\kappa \sum_{n=1}^\infty \sum_{m=0}^n [\alpha(\kappa) \mathbf{V}_{\sigma mn}(\kappa) \mathbf{A}_{\sigma mn}(\kappa) + \beta(\kappa) \mathbf{W}_{\sigma mn}(\kappa) \mathbf{B}_{\sigma mn}(\kappa)] \quad (14)$$

for the form of the dyadic Green's function. Substituting (12) and (14) into Eq. (5), we find

$$\alpha(\kappa) = \frac{1}{\kappa^2 - 2\omega\mu_c \xi_c \kappa - \omega^2 \mu_c \epsilon_c} = \frac{1}{(\kappa - k_+)(\kappa + k_-)}, \quad (15a)$$

$$\beta(\kappa) = \frac{1}{\kappa^2 + 2\omega\mu_c \xi_c \kappa - \omega^2 \mu_c \epsilon_c} = \frac{1}{(\kappa + k_+)(\kappa - k_-)}, \quad (15b)$$

where the wave numbers  $k_+$  and  $k_-$  are given by

$$k_+ = \omega\mu_c \xi_c + \sqrt{\omega^2 \mu_c^2 \xi_c^2 + \omega^2 \mu_c \epsilon_c}, \quad (16a)$$

$$k_- = -\omega\mu_c \xi_c + \sqrt{\omega^2 \mu_c^2 \xi_c^2 + \omega^2 \mu_c \epsilon_c}. \quad (16b)$$

These two wave numbers correspond to the two circularly polarized modes present in the unbounded chiral medium.<sup>6</sup>

$$\Gamma_c(\mathbf{r}, \mathbf{r}') = \frac{i}{2\pi(k_+ + k_-)} \sum_{n=1}^\infty \sum_{m=0}^n (2 - \delta_{m0}) \frac{(2n+1)(n-m)!}{n(n+1)(n+m)!} \begin{cases} k_+^2 \mathbf{V}_{\sigma mn}^{(1)}(k_+) \mathbf{V}'_{\sigma mn}(k_+) + k_-^2 \mathbf{W}_{\sigma mn}^{(1)}(k_-) \mathbf{W}'_{\sigma mn}(k_-) & r > r' \\ k_+^2 \mathbf{V}_{\sigma mn}(k_+) \mathbf{V}_{\sigma mn}^{(1)'}(k_+) + k_-^2 \mathbf{W}_{\sigma mn}(k_-) \mathbf{W}_{\sigma mn}^{(1)'}(k_-) & r < r'. \end{cases} \quad (22)$$

The superscript (1) in  $\mathbf{V}_{\sigma mn}^{(1)}(k_+)$ ,  $\mathbf{V}_{\sigma mn}^{(1)'}(k_+)$ ,  $\mathbf{W}_{\sigma mn}^{(1)}(k_-)$ , and  $\mathbf{W}_{\sigma mn}^{(1)'}(k_-)$  is present to indicate the substitution of  $h_n^{(1)}$  for  $j_n$  in the generating function  $\psi_{\sigma mn}(\kappa)$ . This result, although different in form, is physically equivalent to that ob-

The RCP mode propagates with wave number  $k_+$  and the LCP mode with  $k_-$ .

In order to obtain a closed form for  $\Gamma_c(\mathbf{r}, \mathbf{r}')$ , the integral in Eq. (14) must be evaluated using contour integration. Since the differentiation of  $\psi_{\sigma mn}(\kappa)$ , with respect to spatial coordinates, and its integration, with respect to  $\kappa$ , may be interchanged, it is only necessary to examine the following integral:

$$I = \int_0^\infty d\kappa \frac{F(\kappa)}{(\kappa - k_+)(\kappa + k_-)} j_n(\kappa r) j_n(\kappa r'), \quad (17)$$

where  $F(\kappa)$  is a polynomial of  $\kappa$ . Equation (17) may be rewritten, in terms of regular Bessel functions of order  $n + 1/2$ , as

$$I = \frac{\pi}{2\sqrt{rr'}} \int_0^\infty d\kappa \frac{F(\kappa)/\kappa}{(\kappa - k_+)(\kappa + k_-)} \times J_{n+1/2}(\kappa r) J_{n+1/2}(\kappa r'). \quad (18)$$

This equation may be written in two equivalent forms when expressed in terms of Hankel functions<sup>23</sup>:

$$I = \frac{\pi}{4\sqrt{rr'}} \int_{-\infty}^\infty d\kappa \frac{F(\kappa)/\kappa}{(\kappa - k_+)(\kappa + k_-)} \times H_{n+1/2}^{(1)}(\kappa r) J_{n+1/2}(\kappa r') \quad (19)$$

and

$$I = \frac{\pi}{4\sqrt{rr'}} \int_{-\infty}^\infty d\kappa \frac{F(\kappa)/\kappa}{(\kappa - k_+)(\kappa + k_-)} \times J_{n+1/2}(\kappa r) H_{n+1/2}^{(1)}(\kappa r'). \quad (20)$$

Because of the radiation condition, which ensures that, in the far zone, waves propagate away from the source and not towards it, only the residues corresponding to positive poles are chosen for the contour. It is important to note that the singularity of the Hankel function at the origin dictates the regions over which (19) and (20) are to be used. Therefore, the integral reduces to

$$I = \frac{i\pi F(k_\pm)}{k_+ + k_-} \begin{cases} h_n^{(1)}(k_\pm r) j_n(k_\pm r') & r > r' \\ j_n(k_\pm r) h_n^{(1)}(k_\pm r') & r < r', \end{cases} \quad (21)$$

where  $h_n^{(1)}(k_\pm r')$  is a spherical Hankel function.

We now have the desired dyadic Green's function expressed in terms of the new spherical vector wave functions:

tained by Bassiri *et al.*<sup>6</sup> The association of  $\mathbf{V}_{\sigma mn}$  with  $k_+$  ( $\mathbf{W}_{\sigma mn}$  with  $k_-$ ) in Eq. (22) suggests that the  $\mathbf{V}_{\sigma mn}$  ( $\mathbf{W}_{\sigma mn}$ ) functions yield RCP (LCP) waves. However, as is seen in Eq. (8a) and (8b), these functions do contain a rapidly de-

caying radial component and, therefore, are not truly circularly polarized waves until one reaches the far zone of the source. It is also interesting to note that both  $V_{jmn}$  and  $W_{jmn}$  contain this radial component, whereas in the original set of spherical vector wave functions only  $N_{jmn}$  possessed it. Thus, using the original set for the nonchiral case,  $M_{jmn}$  could be used to represent the transverse waves, such as TE or TM, emitted by the source. In the chiral case, only hybrid waves can be excited and, hence, neither  $V_{jmn}$  nor  $W_{jmn}$  can be purely transverse.

Our solution for  $\Gamma_c(\mathbf{r}, \mathbf{r}')$  can easily be verified by selecting a point dipole excitation located at the origin and of the form<sup>24</sup>

$$\mathbf{J}(\mathbf{r}') = \mathbf{e}_r \frac{I_0 \delta(r') \delta(\theta') \delta(\phi')}{r'^2 \sin \theta'} \quad (23)$$

Substituting Eq. (23) into Eq. (4) and noting that

$$P_n^m(\cos \theta')|_{\theta'=0} = \begin{cases} 0 & \text{if } m \neq 0 \\ 1 & \text{if } m = 0 \end{cases} \quad (24)$$

and

$$\frac{j_n(k_{\pm} r')}{k_{\pm} r'}|_{r'=0} = \begin{cases} 0 & \text{if } n \geq 2 \\ 1/3 & \text{if } n = 1 \end{cases} \quad (25)$$

gives the radiation pattern

$$\mathbf{E}(\mathbf{r}) = -\frac{\omega \mu_c I_0}{2\sqrt{2}\pi(k_+ + k_-)} \times [k_+^2 \mathbf{V}_{\omega 01}^{(1)}(k_+) - k_-^2 \mathbf{W}_{\omega 01}^{(1)}(k_-)]. \quad (26)$$

It should be mentioned that this compact expression is valid for all observation points in both the near and far zones. In the far zone of the source, the Hankel functions present in the equation become

$$\frac{h_1^{(1)}(k_{\pm} r)}{k_{\pm} r} \xrightarrow{k_{\pm} r \gg 1} -\frac{e^{ik_{\pm} r}}{k_{\pm} r}, \quad (27)$$

$$\frac{1}{k_{\pm} r} \frac{\partial}{\partial r} [r h_n^{(1)}(k_{\pm} r)] \xrightarrow{k_{\pm} r \gg 1} -i \frac{e^{ik_{\pm} r}}{k_{\pm} r}. \quad (28)$$

Thus, the far-field radiation pattern is found to be the same as that obtained by Bassiri *et al.*<sup>6</sup>:

$$\mathbf{E}(\mathbf{r}) = -\frac{i\omega \mu_c I_0 \sin \theta}{4\pi(k_+ + k_-)} \left( \frac{k_+ e^{ik_+ r}}{r} (\mathbf{e}_\theta + i\mathbf{e}_\phi) + \frac{k_- e^{ik_- r}}{r} (\mathbf{e}_\theta - i\mathbf{e}_\phi) \right). \quad (29)$$

The sums  $(\mathbf{e}_\theta + i\mathbf{e}_\phi)$  and  $(\mathbf{e}_\theta - i\mathbf{e}_\phi)$  indicate RCP and LCP modes, respectively.

### III. INTERIOR DYADIC GREEN'S FUNCTIONS FOR A CHIRAL SPHERE SURROUNDED BY A NONCHIRAL DIELECTRIC

Having expressed the dyadic Green's function for an unbounded chiral medium in terms of the new spherical vector wave functions, we may now use that result to construct one for a chiral sphere. The geometry of interest is shown in Fig. 2. It consists of a chiral sphere of radius  $a$  located at the origin and embedded in a nonchiral dielectric of infinite ex-

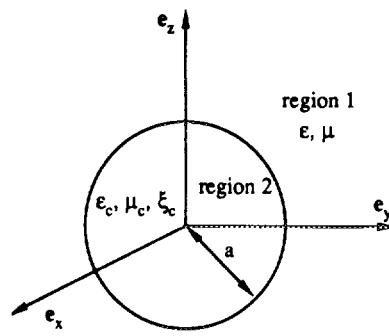


FIG. 2. Geometry of the chiral sphere. The interior of the sphere is characterized by  $\epsilon_c, \mu_c$ , and  $\xi_c$ , while the exterior is described by only  $\epsilon$  and  $\mu$ .

tent. As depicted in Fig. 2, the exterior and interior of the chiral sphere are denoted regions 1 and 2, respectively. Since the dyadic Green's function for sources in region 1 is different from that for sources in region 2, the two cases are considered separately. Here, we examine the case where the source is at the interior of the sphere. The exterior case may be found in the Appendix.

The boundary conditions require that the total tangential components of  $\mathbf{E}$  and  $\mathbf{H}$  be continuous across the interface:

$$\mathbf{e}_r \times \mathbf{E}_1 = \mathbf{e}_r \times \mathbf{E}_2, \quad (30)$$

$$\mathbf{e}_r \times \mathbf{H}_1 = \mathbf{e}_r \times \mathbf{H}_2, \quad (31)$$

where  $\mathbf{E}_1$  and  $\mathbf{H}_1$  are the fields in region 1 and  $\mathbf{E}_2$  and  $\mathbf{H}_2$  are those in region 2. As before, the electric field can be written as

$$\mathbf{E}_1(\mathbf{r}) = i\omega \mu_c \int \Gamma_{\text{tot}}^{(12)}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_2(\mathbf{r}') dV' \quad (32a)$$

and

$$\mathbf{E}_2(\mathbf{r}) = i\omega \mu_c \int \Gamma_{\text{tot}}^{(22)}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_2(\mathbf{r}') dV'. \quad (32b)$$

The first superscript of the dyadic indicates the location of the observation point, while the second gives that of the source. With the electric field representation in Eq. (32a) and (32b), boundary condition (30) at  $r = a$  becomes

$$\mathbf{e}_r \times \Gamma_{\text{tot}}^{(12)}(\mathbf{r}, \mathbf{r}') = \mathbf{e}_r \times \Gamma_{\text{tot}}^{(22)}(\mathbf{r}, \mathbf{r}'). \quad (33)$$

In order to express boundary condition (31) in terms of these same functions, we replace  $\mathbf{H}$  by  $\mathbf{E}$  with the help of the appropriate constitutive relation and Maxwell's equation  $\nabla \times \mathbf{E} = i\omega \mathbf{B}$ , which yields

$$\begin{aligned} (1/\mu) [\mathbf{e}_r \times \nabla \times \Gamma_{\text{tot}}^{(12)}(\mathbf{r}, \mathbf{r}')] \\ = -\omega \xi_c [\mathbf{e}_r \times \Gamma_{\text{tot}}^{(22)}(\mathbf{r}, \mathbf{r}')] \\ + (1/\mu_c) [\mathbf{e}_r \times \nabla \times \Gamma_{\text{tot}}^{(22)}(\mathbf{r}, \mathbf{r}')]. \end{aligned} \quad (34)$$

From scattering superposition, the total dyadic Green's functions in (32a) and (32b) may be written

$$\Gamma_{\text{tot}}^{(12)}(\mathbf{r}, \mathbf{r}') = \Gamma_s^{(12)}(\mathbf{r}, \mathbf{r}'), \quad r \gg a, \quad (35a)$$

$$\Gamma_{\text{tot}}^{(22)}(\mathbf{r}, \mathbf{r}') = \Gamma_c(\mathbf{r}, \mathbf{r}') + \Gamma_s^{(22)}(\mathbf{r}, \mathbf{r}'), \quad r \leq a, \quad (35b)$$

where good choices for the forms of  $\Gamma_s^{(12)}(\mathbf{r}, \mathbf{r}')$  and  $\Gamma_s^{(22)}(\mathbf{r}, \mathbf{r}')$  are

$$\Gamma_s^{(12)}(\mathbf{r}, \mathbf{r}') = \frac{i}{2\pi(k_+ + k_-)} \sum_{n=1}^{\infty} \sum_{m=0}^n (2 - \delta_{m0}) \frac{(2n+1)(n-m)!}{n(n+1)(n+m)!} \\ \times \left\{ \left[ a_n^v \mathbf{V}_{\rho mn}^{(1)}(k) + a_n^w \mathbf{W}_{\rho mn}^{(1)}(k) \right] \mathbf{V}'_{\rho mn}(k_+) + \left[ b_n^v \mathbf{V}_{\rho mn}^{(1)}(k) + b_n^w \mathbf{W}_{\rho mn}^{(1)}(k) \right] \mathbf{W}'_{\rho mn}(k_-) \right\} \quad (36a)$$

and

$$\Gamma_s^{(22)}(\mathbf{r}, \mathbf{r}') = \frac{i}{2\pi(k_+ + k_-)} \sum_{n=1}^{\infty} \sum_{m=0}^n (2 - \delta_{m0}) \frac{(2n+1)(n-m)!}{n(n+1)(n+m)!} \\ \times \left\{ \left[ c_n^v \mathbf{V}_{\rho mn}(k_+) + c_n^w \mathbf{W}_{\rho mn}(k_-) \right] \mathbf{V}'_{\rho mn}(k_+) + \left[ d_n^v \mathbf{V}_{\rho mn}(k_+) + d_n^w \mathbf{W}_{\rho mn}(k_-) \right] \mathbf{W}'_{\rho mn}(k_-) \right\}, \quad (36b)$$

since these already satisfy their respective homogeneous Helmholtz equations.<sup>25</sup> Furthermore, as is required to match  $\Gamma_s^{(12)}(\mathbf{r}, \mathbf{r}')$  and  $\Gamma_s^{(22)}(\mathbf{r}, \mathbf{r}')$  with  $\Gamma_c(\mathbf{r}, \mathbf{r}')$  at the boundary, the arguments of the primed spherical vector wave functions agree with those for  $\Gamma_c(\mathbf{r}, \mathbf{r}')$ . Also, here, four unknowns are present in each of (36a) and (36b), whereas only two were needed in the nonchiral version of the problem.<sup>26</sup> The difference arises in the matching of boundary conditions. As has been observed elsewhere,<sup>18</sup> it is not possible to satisfy these conditions separately with each eigenmode, which explains why different coefficients are needed for  $\mathbf{V}_{\rho mn}$  and  $\mathbf{W}_{\rho mn}$ .

Now, all that remains to be done is to solve for the unknown coefficients  $a_n^v, a_n^w, b_n^v, b_n^w, c_n^v, c_n^w, d_n^v,$  and  $d_n^w$ . We introduce the following simplifying notation for this task:

$$j = j_n(ka) \quad \text{and} \quad \partial j = \frac{1}{ka} \frac{\partial}{\partial r} [r j_n(kr)] \Big|_a, \\ j_{\pm} = j_n(k_{\pm} a) \quad \text{and} \quad \partial j_{\pm} = \frac{1}{k_{\pm} a} \frac{\partial}{\partial r} [r j_n(k_{\pm} r)] \Big|_a, \\ h = h_n^{(1)}(ka) \quad \text{and} \quad \partial h = \frac{1}{ka} \frac{\partial}{\partial r} [r h_n^{(1)}(kr)] \Big|_a, \\ h_{\pm} = h_n^{(1)}(k_{\pm} a) \quad \text{and} \quad \partial h_{\pm} = \frac{1}{k_{\pm} a} \\ \times \frac{\partial}{\partial r} [r h_n^{(1)}(k_{\pm} r)] \Big|_a.$$

Substituting (36a) and (36b) into Eqs. (33) and (34) yields the following sets of linear relations:

$$\mathbf{T} \begin{bmatrix} a_n^v \\ a_n^w \\ c_n^v \\ c_n^w \end{bmatrix} = \begin{bmatrix} k_+^2 \partial h_+ \\ k_+^2 h_+ \\ k_+^2 \partial h_+ \\ k_+^2 h_+ \end{bmatrix} \quad (37a)$$

and

$$\mathbf{T} \begin{bmatrix} b_n^v \\ b_n^w \\ d_n^v \\ d_n^w \end{bmatrix} = \begin{bmatrix} -k_-^2 \partial h_- \\ k_-^2 h_- \\ k_-^2 \partial h_- \\ -k_-^2 h_- \end{bmatrix}, \quad (37b)$$

with the matrix  $\mathbf{T}$  given by

$$\mathbf{T} = \begin{bmatrix} \partial h & -\partial h & -\partial j_+ & \partial j_- \\ h & h & -j_+ & -j_- \\ l \partial h & l \partial h & -\partial j_+ & -\partial j_- \\ l h & -l h & -j_+ & j_- \end{bmatrix} \quad (38)$$

and where  $l$ , the impedance ratio between the sphere's interior and its exterior, is

$$l = \sqrt{\epsilon/\mu} / \sqrt{\epsilon_c^2 + (\epsilon_c/\mu_c)}. \quad (39)$$

The inverse of  $\mathbf{T}$  can be found by Gaussian-Jordan elimination. However, because of the tediousness of the task, we used the Mathematic<sup>TM</sup> computer mathematics system instead, which yielded the result:

$$\mathbf{T}^{-1} = \frac{1}{2D} \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ t_{41} & t_{42} & t_{43} & t_{44} \end{bmatrix}, \quad (40)$$

with

$$t_{11} = (l-1)j_- \partial j_+ + h - (l+1)j_+ \partial j_- - h \\ + 2lj_+ j_- \partial h, \\ t_{12} = -(l+1)j_- \partial j_+ \partial h + (l-1)j_+ \partial j_- \partial h \\ + 2l \partial j_+ \partial j_- h, \\ t_{13} = -(l-1)j_- \partial j_+ h - (l+1)j_+ \partial j_- h \\ + 2j_+ j_- \partial h, \\ t_{14} = -(l+1)j_- \partial j_+ \partial h - (l-1)j_+ \partial j_- \partial h \\ + 2 \partial j_+ \partial j_- h, \\ t_{21} = (l+1)j_- \partial j_+ h - (l-1)j_+ \partial j_- h \\ - 2lj_+ j_- \partial h, \\ t_{22} = (l-1)j_- \partial j_+ \partial h - (l+1)j_+ \partial j_- \partial h \\ + 2l \partial j_+ \partial j_- h, \\ t_{23} = -(l+1)j_- \partial j_+ h - (l-1)j_+ \partial j_- h \\ + 2j_+ j_- \partial h, \\ t_{24} = (l-1)j_- \partial j_+ \partial h + (l+1)j_+ \partial j_- \partial h \\ - 2 \partial j_+ \partial j_- h, \\ t_{31} = -2l \partial j_- h^2 + 2l^2 j_- h \partial h, \\ t_{32} = -2lj_- \partial h^2 + 2l^2 \partial j_- h \partial h, \\ t_{33} = -2l \partial j_- h^2 + 2j_- h \partial h,$$

$$\begin{aligned}
t_{34} &= -2lj_- \partial h^2 + 2 \partial j_- h \partial h, \\
t_{41} &= 2l \partial j_+ h^2 - 2l^2 j_+ h \partial h, \\
t_{42} &= -2lj_+ \partial h^2 + 2l^2 \partial j_+ h \partial h, \\
t_{43} &= -2l \partial j_+ h^2 + 2j_+ h \partial h, \\
t_{44} &= 2lj_+ \partial h^2 - 2 \partial j_+ h \partial h,
\end{aligned}$$

and<sup>29</sup>

$$\begin{aligned}
D &= 2l (h^2 \partial j_+ \partial j_- + \partial h^2 j_+ j_-) \\
&\quad - h \partial h (l^2 + 1) (j_+ \partial j_- + j_- \partial j_+). \quad (41)
\end{aligned}$$

Hence, the coefficients of the dyadic Green's functions for sources at the interior of the chiral sphere are

$$\begin{aligned}
a_n^v &= k_+^2 (l+1) (j_- \partial h - \partial j_- h) \\
&\quad \times (j_+ \partial h_+ - \partial j_+ h_+) / D, \quad (42a)
\end{aligned}$$

$$\begin{aligned}
a_n^w &= -k_+^2 (l-1) (j_- \partial h + \partial j_- h) \\
&\quad \times (j_+ \partial h_+ - \partial j_+ h_+) / D, \quad (42b)
\end{aligned}$$

$$\begin{aligned}
b_n^v &= -k_-^2 (l-1) (j_+ \partial h + \partial j_+ h) \\
&\quad \times (j_- \partial h_- - \partial j_- h_-) / D, \quad (43a)
\end{aligned}$$

$$\begin{aligned}
b_n^w &= k_-^2 (l+1) (j_+ \partial h - \partial j_+ h) \\
&\quad \times (j_- \partial h_- - \partial j_- h_-) / D, \quad (43b)
\end{aligned}$$

$$\begin{aligned}
c_n^v &= k_+^2 [(l^2 + 1) h \partial h (j_- \partial h_+ + \partial j_- h_+) \\
&\quad - 2l (j_- h_+ \partial h^2 + \partial j_- \partial h_+ h^2)] / D, \quad (44a)
\end{aligned}$$

$$c_n^w = -k_+^2 (l^2 - 1) h \partial h (j_+ \partial h_+ - \partial j_+ h_+) / D, \quad (44b)$$

$$d_n^v = -k_-^2 (l^2 - 1) h \partial h (j_- \partial h_- - \partial j_- h_-) / D, \quad (45a)$$

$$\begin{aligned}
d_n^w &= k_-^2 [(l^2 + 1) h \partial h (j_+ \partial h_- + \partial j_+ h_-) \\
&\quad - 2l (j_+ h_- \partial h^2 + \partial j_+ \partial h_- h^2)] / D. \quad (45b)
\end{aligned}$$

Substituting the coefficients (42a)–(45b) into Eqs. (36a) and (36b), we obtain the complete expression of the dyadic Green's function for electromagnetic sources at the interior of the chiral sphere.

#### IV. DIPOLE RADIATION

While our purpose here is not to examine the general use of the dyadic Green's functions we have derived, we shall examine their characteristics for one simple case. We consider the radiation pattern from an electric dipole source at the center of a chiral sphere. The current distribution  $\mathbf{J}_2(\mathbf{r}')$  is again given by Eq. (23). Inserting this distribution and Eq. (36a) into (32a) and using relations (24) and (25), we obtain

$$\begin{aligned}
\mathbf{E}_1(\mathbf{r}) &= -\frac{\omega \mu_c I_0}{2\sqrt{2}\pi(k_+ + k_-)} \\
&\quad \times [\{a_1^v - b_1^v\} \mathbf{V}_{\sigma 01}^{(1)}(k) + \{a_1^w - b_1^w\} \mathbf{W}_{\sigma 01}^{(1)}(k)]. \quad (46)
\end{aligned}$$

Thus, as one might expect, the form of the radiated field for all regions is identical to that in Eq. (26). Only the magnitude of each mode is different. For the far field, i.e.,  $kr \gg 1$ , (46) reduces to

$$\begin{aligned}
\mathbf{E}_1(\mathbf{r}) &= -\frac{i\omega \mu_c I_0 \sin \theta}{4\pi(k_+ + k_-)} \frac{e^{ikr}}{kr} \\
&\quad \times [\{a_1^v - b_1^v\} (\mathbf{e}_\theta + i\mathbf{e}_\phi) \\
&\quad + \{b_1^w - a_1^w\} (\mathbf{e}_\theta - i\mathbf{e}_\phi)]. \quad (47)
\end{aligned}$$

We limit our attention to large spheres, for which<sup>28</sup>

$$j_{\pm} \xrightarrow[k_{\pm} a \gg 1]{} -\frac{\cos(k_{\pm} a)}{k_{\pm} a}, \quad (48)$$

$$\partial j_{\pm} \xrightarrow[k_{\pm} a \gg 1]{} \frac{\sin(k_{\pm} a)}{k_{\pm} a}, \quad (49)$$

$$h_{\pm} \xrightarrow[k_{\pm} a \gg 1]{} -\frac{e^{ik_{\pm} a}}{k_{\pm} a}, \quad (50)$$

$$\partial h_{\pm} \xrightarrow[k_{\pm} a \gg 1]{} -i \frac{e^{ik_{\pm} a}}{k_{\pm} a}. \quad (51)$$

Thus, Eq. (47) becomes

$$\mathbf{E}_1(\mathbf{r}) = -\frac{i\omega \mu_c I_0 \sin \theta}{4\pi(k_+ + k_-)} \frac{e^{ik(r-a)}}{r} \frac{[\lambda_+ (\mathbf{e}_\theta + i\mathbf{e}_\phi) + \lambda_- (\mathbf{e}_\theta - i\mathbf{e}_\phi)]}{2l \cos[(k_+ + k_-)a] - i(l^2 + 1) \sin[(k_+ + k_-)a]}, \quad (52)$$

where

$$\lambda_+ = k_+ (l+1) e^{-ik_+ a} + k_- (l-1) e^{ik_+ a}, \quad (53a)$$

$$\lambda_- = k_- (l+1) e^{-ik_- a} + k_+ (l-1) e^{ik_- a}. \quad (53b)$$

It is worth noting that, due to the geometry of the problem, the angular dependence of the dipole's radiated fields are similar to those of a dipole in an unbounded chiral or nonchiral medium.

The total radiated power  $P$  is given by

$$P = \frac{1}{2} \operatorname{Re} \int \mathbf{n} \cdot (\mathbf{E}_1 \times \mathbf{H}_1^*) dS. \quad (54)$$

Since, outside the sphere, the ratio of the electric field to the magnetic field in the far zone is  $\sqrt{\mu/\epsilon}$ , this may be written in terms of the  $\theta$  and  $\phi$  components of the electric field as

$$P = \frac{1}{2\sqrt{\mu/\epsilon}} \int (|E_\theta|^2 + |E_\phi|^2) r^2 \sin \theta d\theta d\phi. \quad (55)$$

Substituting (52) into this relation and performing the integration, we obtain



$$P = \frac{\omega^2 \mu_c^2 I_0^2}{3\pi \sqrt{\mu/\epsilon} (k_+ + k_-)^2} \frac{(k_+^2 + k_-^2)(l^2 + 1) + 2k_+ k_- (l^2 - 1) \cos[(k_+ + k_-)a]}{4l^2 \cos^2[(k_+ + k_-)a] + (l^2 + 1)^2 \sin^2[(k_+ + k_-)a]} \quad (56)$$

Therefore, the radiation resistance<sup>24</sup> is

$$R = \frac{2\omega^2 \mu_c^2 \Delta^2}{3\pi \sqrt{\mu/\epsilon} (k_+ + k_-)^2} \frac{(k_+^2 + k_-^2)(l^2 + 1) + 2k_+ k_- (l^2 - 1) \cos[(k_+ + k_-)a]}{4l^2 \cos^2[(k_+ + k_-)a] + (l^2 + 1)^2 \sin^2[(k_+ + k_-)a]} \quad (57)$$

since  $P = I^2 R / 2$ . The graph of Eq. (57) as a function of  $(k_+ + k_-)a$  is shown in Fig. 3 for  $\epsilon/\epsilon_c = 0.16$ . Two notable effects are seen in the graph: the first is the strong resonance that occurs when  $(k_+ + k_-)a$  is an odd multiple of  $\pi$  and the second is the increase in radiation resistance due to increased chirality. The former effect is simply a result of constructive interference occurring with the sphere. The second one enters into the problem by changing both the impedance of the sphere and the radiation characteristics of the dipole, i.e., by exciting the RCP mode more strongly than the LCP mode. More will be said on this latter effect of chirality once we have examined the polarization of the electric field in Eq. (52).

Using the standard representation for the Poincaré sphere,<sup>29</sup> we may express the polarization of a point on the Poincaré sphere with latitude  $2\chi$  as

$$\sin 2\chi = \frac{|\lambda_-|^2 - |\lambda_+|^2}{|\lambda_-|^2 + |\lambda_+|^2} = - \frac{2(k_+^2 - k_-^2)l}{(k_+^2 + k_-^2)(l^2 + 1) + 2k_+ k_- (l^2 - 1) \cos[(k_+ + k_-)a]}, \quad (58)$$

since

$$|\lambda_{\pm}|^2 = k_{\pm}^2 (l+1)^2 + k_{\mp}^2 (l-1)^2 + 2k_+ k_- (l^2 - 1) \cos[(k_+ + k_-)a]. \quad (59)$$

For right-hand elliptically polarized waves  $-1 < \sin 2\chi < 0$ , whereas for left-hand elliptically polarized waves  $0 < \sin 2\chi < 1$ . At the extremes,  $\sin 2\chi = -1$  and  $\sin 2\chi = 1$ , the waves are RCP and LCP, respectively. Furthermore, when  $\sin 2\chi = 0$  the polarization is linear. Therefore, it follows from Eqs. (58), (16a), and (16b) that for positive (negative)  $\xi_c$  the radiated field is always of right- (left-) handed polarization. A plot of Eq. (58) is found in Fig. 4 for several values of positive  $\xi_c$ .

It is evident from Fig. 4 that, when  $(k_+ + k_-)a$  is an even multiple of  $\pi$ , one may achieve complete right circular polarization. The physical conditions which permit this phenomenon are of particular interest to radome design. To ex-

amine these conditions, we must seek the roots of Eq. (59) or, in a more intuitive form, of

$$k_{\mp}^2 R^2 - 2k_+ k_- \cos[(k_+ + k_-)a]R + k_{\pm}^2 = 0, \quad (60)$$

where the reflection coefficient  $R = -(l-1)/(l+1)$ . The solution of (60) is

$$R = (k_{\pm}/k_{\mp}) \{ \cos[(k_+ + k_-)a] \pm \sqrt{-\sin^2[(k_+ + k_-)a]} \}, \quad (61)$$

which requires that  $(k_+ + k_-)a = n\pi$  in order to have real values for  $R$ . Therefore, for  $|\lambda_+|^2 = 0$ ,

$$R = \begin{cases} k_+/k_- & n \text{ even} \\ -k_+/k_- & n \text{ odd} \end{cases} \quad (62a)$$

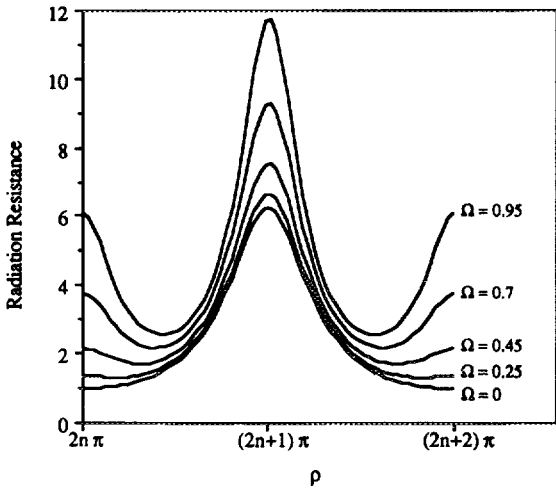


FIG. 3. A plot of the normalized radiation resistance as a function of sphere size. The normalization used is  $R/R_0$ , where  $R_0 = \omega^2 \mu_c^2 \Delta^2 / 6\pi \sqrt{\mu/\epsilon}$  is the radiation resistance in the absence of the chiral sphere. Here,  $\rho = (k_+ + k_-)a$ ,  $\Omega = \xi_c \sqrt{\mu_c/\epsilon_c}$ ,  $n$  is a large integer,  $\mu = \mu_c$ , and  $\epsilon/\epsilon_c = 0.16$ .

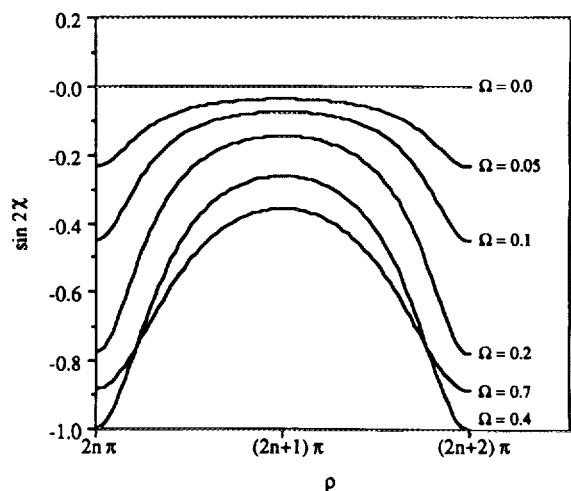


FIG. 4. A plot of ellipticity of the polarization ellipse vs the size of the chiral sphere. As before,  $\mu = \mu_c$  and  $\epsilon/\epsilon_c = 0.16$ .

and for  $|\lambda_-|^2 = 0$ ,

$$R = \begin{cases} k_- / k_+ & n \text{ even} \\ -k_- / k_+ & n \text{ odd} \end{cases} \quad (62b)$$

However, since  $\xi_c$  is fixed at a particular value, it is only possible to satisfy one of the two equations in a given medium. We can suppress the LCP wave only when  $\xi_c$  is positive and the RCP wave only when it is negative. Physically, the elimination of one of the modes, let us say the LCP mode, may be explained as follows: when condition (62b) is satisfied, a fraction  $k_- / k_+$  of the RCP wave is reflected at the sphere's boundary and becomes an LCP wave. This latter

wave now has the same magnitude as the original LCP wave since, for an unbounded chiral medium, the ratio of the amplitude of the RCP mode to that of the LCP mode is  $k_+ / k_-$ . If the latter wave is  $180^\circ$  out of phase with the original LCP wave radiated by the source, the LCP mode is completely canceled. Furthermore, as the chirality progressively increases, a smaller and smaller portion of the RCP mode is needed for the cancellation, which results in the greater radiation efficiency seen in Fig. 3.

As a final consideration, we investigate the effects of matching the sphere's impedance to that of the surrounding medium, that is, the case where  $l = 1$ . Equation (52) then simplifies to

$$\mathbf{E}_1(\mathbf{r}) = -\frac{i\omega\mu_c I_0 \sin\theta}{4\pi(k_+ + k_-)} \frac{e^{ik(r-a)}}{r} [k_+ e^{ik_+ a}(\mathbf{e}_\theta + i\mathbf{e}_\phi) + k_- e^{ik_- a}(\mathbf{e}_\theta - i\mathbf{e}_\phi)]. \quad (63)$$

Comparing this form with Eq. (29), we see that the fields are exactly the same at  $r = a$ . As expected, past this boundary, the field in (63) propagates with a single wave number  $k$ , whereas that in (29) continues with both  $k_+$  and  $k_-$ . Also, with the impedance matching condition, the radiation resistance reduces to the simple form previously found by Bassiri *et al.*<sup>6</sup> for a dipole embedded in an unbounded chiral medium:

$$R = \frac{\omega^2 \mu_c^2 \Delta^2 (k_+^2 + k_-^2)}{3\pi \sqrt{\mu/\epsilon} (k_+ + k_-)^2} = \frac{\omega^2 \mu_c^2 \Delta^2 (2\xi_c^2 + \epsilon_c/\mu_c)}{6\pi \sqrt{\xi_c^2 + \epsilon_c/\mu_c}}. \quad (64)$$

It must be noted that due to the impedance matching offered by  $l = 1$ , the above radiation resistance is independent of the sphere's radius.

## V. CONCLUSIONS

We have studied the radiation characteristics of electromagnetic sources in the presence of a sphere of isotropic, homogeneous, lossless chiral material. Two cases have been considered: (1) the source placed at the interior of the chiral sphere and (2) the source located outside the sphere. For both cases, using an exact formulation, the dyadic Green's functions have been derived and expressed in terms of the appropriately defined spherical vector wave functions. As an illustrative example, a short electric dipole found at the center of the chiral sphere has been studied in detail, and the radiated fields and radiation resistance have been examined. We have shown that, in this case, the dipole's radiated fields are, in general, elliptically polarized. Furthermore, by choosing the sphere's size and material parameters properly, circular polarization may be obtained. We have also demonstrated that the radiation resistance of the dipole depends on the sphere's size and increases monotonically with chirality parameter  $\xi_c$ .

Results of this study have potential applications to novel radome designs, spherical lenses for microwave, millimeter-wave and optical regimes, and multipolarized antennas with polarization control. In particular, spherical lenses made of homogeneous chiral materials and their characteristics are currently being examined. The results of this investigation will be reported shortly.

## ACKNOWLEDGMENTS

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## APPENDIX: EXTERIOR DYADIC GREEN'S FUNCTIONS FOR A CHIRAL SPHERE

In this appendix, we examine the case where the excitation is at the exterior of the sphere. Equations (32a) and (32b) are changed to

$$\mathbf{E}_1(\mathbf{r}) = i\omega\mu \int \Gamma_{\text{tot}}^{(11)}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_1(\mathbf{r}') dV', \quad (A1a)$$

$$\mathbf{E}_2(\mathbf{r}) = i\omega\mu \int \Gamma_{\text{tot}}^{(21)}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_1(\mathbf{r}') dV'. \quad (A1b)$$

The boundary conditions in (33) and (34) are unaltered if  $\Gamma_{\text{tot}}^{(12)}(\mathbf{r}, \mathbf{r}')$  is replaced by  $\Gamma_{\text{tot}}^{(11)}(\mathbf{r}, \mathbf{r}')$  and  $\Gamma_{\text{tot}}^{(22)}(\mathbf{r}, \mathbf{r}')$  by

$\Gamma_{\text{tot}}^{(21)}(\mathbf{r}, \mathbf{r}')$ . Using scattering superposition again:

$$\Gamma_{\text{tot}}^{(11)}(\mathbf{r}, \mathbf{r}') = \Gamma_0(\mathbf{r}, \mathbf{r}') + \Gamma_s^{(11)}(\mathbf{r}, \mathbf{r}'), \quad r \gg a, \quad (\text{A2a})$$

$$\Gamma_{\text{tot}}^{(21)}(\mathbf{r}, \mathbf{r}') = \Gamma_s^{(21)}(\mathbf{r}, \mathbf{r}'), \quad r \leq a, \quad (\text{A2b})$$

where

$$\Gamma_0(\mathbf{r}, \mathbf{r}') = \frac{i}{4\pi k} \sum_{n=1}^{\infty} \sum_{m=0}^n (2 - \delta_{m0}) \frac{(2n+1)(n-m)!}{n(n+1)(n+m)!} \begin{cases} k^2 \mathbf{V}_{\sigma mn}^{(1)}(k) \mathbf{V}'_{\sigma mn}(k) + k^2 \mathbf{W}_{\sigma mn}^{(1)}(k) \mathbf{W}'_{\sigma mn}(k) & r > r' \\ k^2 \mathbf{V}_{\sigma mn}(k) \mathbf{V}'_{\sigma mn}(k) + k^2 \mathbf{W}_{\sigma mn}(k) \mathbf{W}'_{\sigma mn}(k) & r < r' \end{cases} \quad (\text{A3})$$

$$\Gamma_s^{(11)}(\mathbf{r}, \mathbf{r}') = \frac{i}{4\pi k} \sum_{n=1}^{\infty} \sum_{m=0}^n (2 - \delta_{m0}) \frac{(2n+1)(n-m)!}{n(n+1)(n+m)!} \times \left\{ \left[ e_n^v \mathbf{V}_{\sigma mn}^{(1)}(k) + e_n^w \mathbf{W}_{\sigma mn}^{(1)}(k) \right] \mathbf{V}_{\sigma mn}^{(1)'}(k) + \left[ f_n^v \mathbf{V}_{\sigma mn}^{(1)}(k) + f_n^w \mathbf{W}_{\sigma mn}^{(1)}(k) \right] \mathbf{W}_{\sigma mn}^{(1)'}(k) \right\}, \quad (\text{A4a})$$

and

$$\Gamma_s^{(21)}(\mathbf{r}, \mathbf{r}') = \frac{i}{4\pi k} \sum_{n=1}^{\infty} \sum_{m=0}^n (2 - \delta_{m0}) \frac{(2n+1)(n-m)!}{n(n+1)(n+m)!} \times \left\{ \left[ g_n^v \mathbf{V}_{\sigma mn}(k_+) + g_n^w \mathbf{W}_{\sigma mn}(k_-) \right] \mathbf{V}_{\sigma mn}^{(1)'}(k) + \left[ h_n^v \mathbf{V}_{\sigma mn}(k_+) + h_n^w \mathbf{W}_{\sigma mn}(k_-) \right] \mathbf{W}_{\sigma mn}^{(1)'}(k) \right\}. \quad (\text{A4b})$$

Applying the boundary conditions to these equations:

$$\mathbf{T} \begin{bmatrix} e_n^v \\ e_n^w \\ g_n^v \\ g_n^w \end{bmatrix} = \begin{bmatrix} -k^2 \partial_j \\ -k^2 j \\ -lk^2 \partial_j \\ -lk^2 j \end{bmatrix} \quad (\text{A5a})$$

and

$$\mathbf{T} \begin{bmatrix} f_n^v \\ f_n^w \\ h_n^v \\ h_n^w \end{bmatrix} = \begin{bmatrix} k^2 \partial_j \\ -k^2 j \\ -lk^2 \partial_j \\ lk^2 j \end{bmatrix} \quad (\text{A5b})$$

where  $\mathbf{T}$  is still given by Eq. (38) and  $\mathbf{T}^{-1}$  by Eq. (40). Therefore, the desired coefficients are

$$e_n^v = k^2 \{ j_+ \partial_j - [(l+1)^2 \partial_j h + (l-1)^2 j \partial h] + \partial j_+ j_- [(l-1)^2 \partial_j h + (l+1)^2 j \partial h] - 4l(j_+ j_- \partial j \partial h + \partial j_+ \partial j_- j h) \} / 2D, \quad (\text{A6a})$$

$$e_n^w = -k^2 (l^2 - 1) (j_+ \partial j_- + \partial j_+ j_-) (j \partial h - \partial j h) / 2D, \quad (\text{A6b})$$

$$f_n^v = -k^2 (l^2 - 1) (j_+ \partial j_- + \partial j_+ j_-) (j \partial h - \partial j h) / 2D, \quad (\text{A7a})$$

$$f_n^w = k^2 \{ j_+ \partial j_- [(l-1)^2 \partial_j h + (l+1)^2 j \partial h] + \partial j_+ j_- [(l+1)^2 \partial_j h + (l-1)^2 j \partial h] - 4l(j_+ j_- \partial j \partial h + \partial j_+ \partial j_- j h) \} / 2D, \quad (\text{A7b})$$

$$g_n^v = k^2 l(l+1) (j_- \partial h - \partial j_- h) (j \partial h - \partial j h) / D, \quad (\text{A8a})$$

$$g_n^w = -k^2 l(l-1) (j_+ \partial h + \partial j_+ h) (j \partial h - \partial j h) / D, \quad (\text{A8b})$$

$$h_n^v = -k^2 l(l-1) (j_- \partial h + \partial j_- h) (j \partial h - \partial j h) / D, \quad (\text{A9a})$$

$$h_n^w = k^2 l(l+1) (j_+ \partial h - \partial j_+ h) (j \partial h - \partial j h) / D. \quad (\text{A9b})$$

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<sup>24</sup> $J_0$  is in reality the constant product of the current  $I$  with the dipole length  $\Delta$ .

<sup>25</sup>The homogeneous Helmholtz equation for region 1 is  $\nabla \times \nabla \times \mathbf{E} - k^2 \mathbf{E} = 0$ , where  $k = \omega \sqrt{\mu \epsilon}$ . The corresponding one for region 2 is given by Eq. (3) with the right-hand side set equal to zero.

<sup>26</sup>In the nonchiral case, half the coefficients are eliminated because  $a_n^v = a_n^w$ ,  $b_n^v = b_n^w$ ,  $c_n^v = c_n^w$ , and  $d_n^v = d_n^w$ .

<sup>27</sup>In these equations,  $\partial h^2$  should be interpreted as  $(\partial h)^2$ .

<sup>28</sup>Because of the geometry involved, all of the spherical Bessel and Hankel functions in this section are of order 1.

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