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## **Keywords**

Chaos, synchronization, wave propagation

## **Comments**

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# Synchronization Recovery of Chaotic Wave Through an Imperfect Channel

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**Abstract**—We present a novel idea to recover synchronization using the genetic algorithm after a chaotic wave passes through an imperfect channel with constant attenuation and offset. The compensation block, which is added before the receiver, is used to compensate the distortion of the imperfect channel. A new concept, the synchronization mismatch is defined and used as the cost function in genetic algorithm to design the compensation block. The validity of this approach is suggested by numerical simulations.

**Index Terms**—Chaos, synchronization, wave propagation.

## I. INTRODUCTION

THE presence of chaotic phenomena in physical and electrical systems is common and has been extensively demonstrated [1]. Chaos in physical systems was originally thought to be a form of noise or random disturbance. It is now well-known that chaos is not noise but rather an intricate and often repetitive pattern that arises in the behavior of some nonlinear systems that are extremely sensitive to changes in initial condition. Because of this sensitivity, the application of chaos to system design was originally thought to be difficult. However, it has been demonstrated that certain chaotic systems possess the property of synchronization and can be subject to control [2]. This has piqued interest in new, nonlinear devices and systems for secure communications [3]–[7] and other applications. Communication systems, especially wireless systems, based on the superior performance of nonlinear technology operating in the chaotic regime are expected to play an increasing role in commercial and other communications systems.

Chaotic communication systems, both digital and analog, require high-frequency circuits and systems for transmission and detection. The use of synchronized chaotic systems for communications usually relies on the robustness of the synchronization within the transmitter–receiver pair. Recently, Wu and Jaggard [8], [9] investigated chaotic wave propagation through an imperfect channel. A new concept, synchronization mismatch, was introduced and it was found that if the communication channel was imperfect, the distorted signal at the receiver input might cause a considerable synchronization mismatch between the transmitter and receiver systems.

The genetic algorithm is a subset of evolutionary algorithms that approximate some biological growth processes to optimize

highly complex cost functions. The genetic algorithm allows a population composed of many individuals to evolve under specified selection rules to a state that maximizes the “fitness” (i.e., minimizes the cost function) [10]. It has been used in many applications in communication systems, such as communication network [11], [12], multi-user detection [13], wide-band antenna design [14], and communication controller design [15].

In this paper, we propose a novel approach using genetic algorithm to recover synchronization after a chaotic wave propagates through an imperfect channel with constant attenuation and offset. To achieve this result, a compensator is added before the receiver. The genetic algorithm is applied to estimate the attenuation and offset. From the examples given here, we find that the chaotic synchronization may be recovered efficiently. Although the results are based on the Lorenz systems, they can be extended to other systems as well.

## II. CHAOTIC SYNCHRONIZATION

The concept of chaotic synchronization in a communication system is shown schematically in Fig. 1. Here, we use the Lorenz systems as examples and assume that the transmitter and receiver are identical Lorenz systems, except that for the receiver system we use the receiver input signal  $\tilde{u}_1$  to replace the receiver output signal  $u_2$  in some of the expressions. The dynamic equations for the two systems are, respectively,

$$\begin{cases} u_1' = K\alpha(v_1 - u_1) \\ v_1' = K(\rho u_1 - v_1 - u_1 w_1) \\ w_1' = K(u_1 v_1 - \beta w_1) \end{cases} \quad (1)$$

$$\text{and } \begin{cases} u_2' = K\alpha(v_2 - u_2) \\ v_2' = K(\rho \tilde{u}_1 - v_2 - \tilde{u}_1 w_2) \\ w_2' = K(\tilde{u}_1 v_2 - \beta w_2). \end{cases} \quad (2)$$

Here, the prime ' denotes derivative with respect to time  $t$ , and  $K$  is a scaling factor. It is noted that we use  $\tilde{u}_1$  as input signal (drive signal) instead of  $u_2$  on the right-hand side of the last two equations of the response system.

If the input signal  $\tilde{u}_1$  of the response system is the same as  $u_1$  of the drive system (i.e., the channel is perfect), these two systems will synchronize. This means that if these two systems start from different initial conditions, but share the same set of parameters  $\rho$ ,  $\alpha$ ,  $\beta$ , and  $K$ , the variables  $u_2$ ,  $v_2$  and  $w_2$  of the response system will soon approach the values  $u_1$ ,  $v_1$  and  $w_1$ , respectively, of the drive system [16]. Likewise, if we choose  $\tilde{u}_1$  to be a delayed version of  $u_1$  (the channel is a time delayed system),  $u_2$ ,  $v_2$  and  $w_2$  will also soon approach  $u_1$ ,  $v_1$  and  $w_1$ , respectively.

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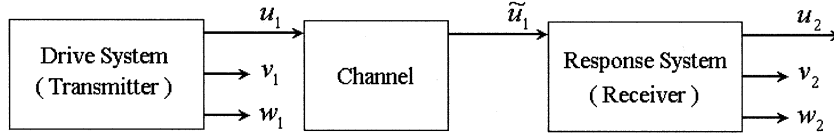


Fig. 1. A chaotic synchronization system. We assume that the transmitter and receiver are Lorenz systems sharing the same set of parameters, so that the outputs of the two systems are synchronized.

However, if  $\tilde{u}_1$  is a distorted version of  $u_1$  after passing through a channel, there will be a *synchronization mismatch* [4] between  $u_1$  and  $u_2$ . We can quantitatively define the synchronization mismatch (*SM*) as a dimensionless quantity: the maximum absolute value of the difference between  $\tilde{u}_1$  and  $u_2$  over the root mean square (rms) value of  $u_1$ , which is

$$SM = \frac{\max_{t_c \leq t \leq \infty} |u_2(t) - \tilde{u}_1(t)|}{u_{\text{rms}}} \quad (3)$$

where  $t_c$  can be chosen to be a time value after which the difference between  $\tilde{u}_1$  and  $u_2$  becomes less significant for a perfect channel.  $u_{\text{rms}}$  is the rms value of  $u_1$  and defined as

$$u_{\text{rms}} = \lim_{t \rightarrow \infty} \frac{1}{t - t_c} \sqrt{\int_{t_c}^t |u_1(t)|^2 dt}. \quad (4)$$

The synchronization mismatch defined above may be considered as a signature of the channel.

For a perfect channel, *SM* is very small. However, for an imperfect channel, the distortion from the channel may introduce a considerable *SM*, or even destroy the chaotic synchronization completely.

### III. SYNCHRONIZATION RECOVERY AND COMPENSATOR DESIGN

#### A. Imperfect Channel With Attenuation and Offset

If the channel is imperfect, the synchronization mismatch may be quite large. Let us consider an imperfect channel with attenuation and offset. In Fig. 1, the output  $\hat{u}_1$  of the channel is given by  $\hat{u}_1 = u_1/A + B$ , where  $A$  is the attenuation factor and  $B$  is the offset of the channel. The values of  $A$  and  $B$  will affect the synchronization status. We can find that when  $A$  is as small as 1.1, the synchronization mismatch can be detected, although it is not significant. However, if  $A$  goes up to 5, the mismatch can be considerable. The offset  $B$  has the similar effect on the synchronization as the attenuation factor  $A$ . When offset  $B$  increases from 1 to 5, the mismatch grows up simultaneously. As shown in Fig. 2, when the distortion from the channel is characterized as  $A = 1.5$  and  $B = 5$ , the synchronization mismatch is very significant and the synchronization can hardly hold. So far it is very clear that the mismatch increases as the attenuation or offset increases. The even worse case is that the synchronization may be totally lost.

#### B. Synchronization Recovery

In the following, we will show the idea to recover the chaotic synchronization. An intuitive way is to add a compensation

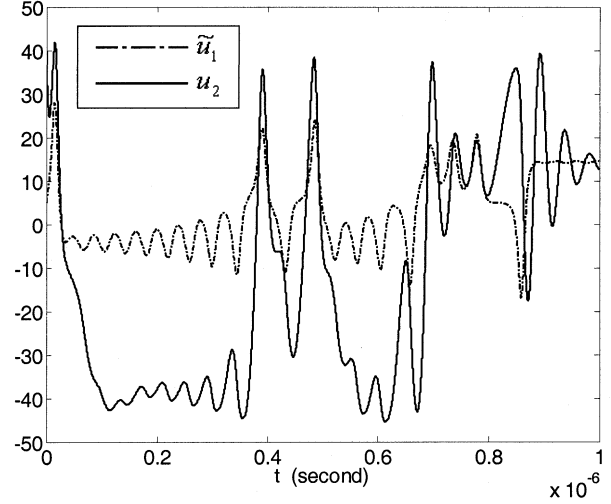


Fig. 2. Due to the distortion from the imperfect channel, attenuation factor  $A = 1.5$  and offset  $B = 5$ , the chaotic synchronization between the receiver input  $\tilde{u}_1$  (dashed line) and the receiver output  $u_2$  (solid line) cannot hold. The Lorenz system parameters are  $\alpha = 16$ ,  $\beta = 4$ ,  $\rho = 50$  and  $K = 10^7$ .

system before the receiver as shown in Fig. 3. We can design the compensator to be

$$\tilde{u}_1 = (\hat{u}_1 - \hat{B}) \cdot \hat{A}. \quad (5)$$

Therefore,

$$\tilde{u}_1 = (\hat{u}_1 - \hat{B}) \cdot \hat{A} = \frac{u_1}{A} \cdot \hat{A} + (B - \hat{B}) \cdot \hat{A}. \quad (6)$$

From (6), we can find that if  $\hat{A}$  and  $\hat{B}$  approach  $A$  and  $B$ , respectively,  $\tilde{u}_1$  will approach  $u_1$ , and the synchronization can be recovered by the response system.

Here, the objective is to find the estimation values of  $\hat{A}$  and  $\hat{B}$ . In this paper, we use the genetic algorithm to obtain these values. The cost function used in optimization is the synchronization mismatch between the receiver input and receiver output.  $\hat{A}$  and  $\hat{B}$  are the approximations of  $A$  and  $B$  when the value of the cost function approaches zero.

#### C. Numerical Example

Let us consider an example to show the robustness of our approach to recover synchronization. For the systems in (1) and (2), the Lorenz system parameters are  $\alpha = 16$ , and  $\beta = 4$ ,  $\rho = 50$  and  $K = 10^7$ . The distortion from the channel is characterized as  $A = 1.5$  and  $B = 5$ . As shown in Fig. 2, the synchronization mismatch is very significant and the synchronization can hardly hold.

Using genetic algorithm, the initial guesses are in the range  $[0,5]$  for  $\hat{A}$  and  $[-10,10]$  for  $\hat{B}$ . As shown in Fig. 4, the value of

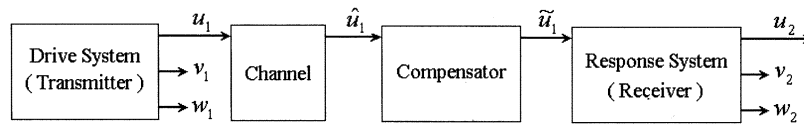


Fig. 3. A chaotic synchronization system with a compensator. We assume that the transmitter and receiver are Lorenz systems sharing the same set of parameters.

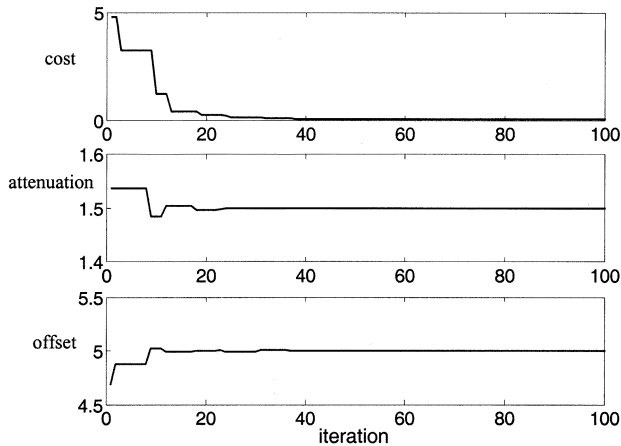


Fig. 4. Parameters of the compensation system found by genetic algorithm. An intuitive way to recover the chaotic synchronization is to add a compensator before the receiver (Fig. 3). (a) Cost versus iteration. (b) Attenuation versus iteration. (c) Offset versus iteration.

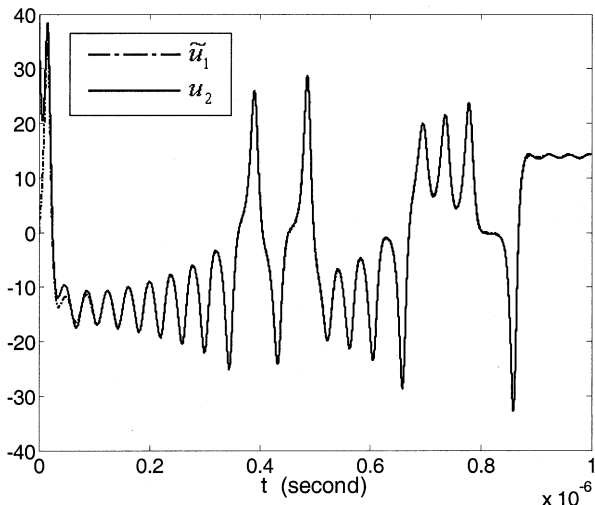


Fig. 5. The recovery of chaotic synchronization between the receiver input  $\hat{u}_1$  (dashed line) and the receiver output  $u_2$  (solid line). The Lorenz system parameters are  $\alpha = 16$ ,  $\beta = 4$ ,  $\rho = 50$  and  $K = 10^7$ .

the cost function approaches zero gradually. At the meanwhile,  $\hat{A}$  and  $\hat{B}$  are also varying toward their optimum values. After 100 iterations, the results are:  $\hat{A} = 1.4996$  and  $\hat{B} = 5.0027$ . The results are very close to the actual values of  $A$  and  $B$ , where  $A = 1.5$  and  $B = 5$ . Applying  $\hat{A}$  and  $\hat{B}$  in (4), the expression of the compensator becomes

$$\tilde{u}_1 = (\hat{u}_1 - \hat{B}) \cdot \hat{A} = (\hat{u}_1 - 5.0027) \cdot (1.4996). \quad (7)$$

And, as shown in Fig. 5, with the compensator in (7) added before the receiver, the synchronization mismatch has been reduced remarkably. The synchronization is excellently recovered by the compensator.

#### IV. CONCLUSION

In this paper, we have proposed a novel idea to use the genetic algorithm to recover synchronization after a chaotic wave passing through an imperfect channel with constant attenuation and offset. The simulation results have shown the robustness of our approach. Although this paper is based on the Lorenz systems, it can be extended to other systems as well.

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