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Abstract

A methodology and algorithm is presented that generates motions imitating the way humans complete a lifting task under various loading conditions. The path taken depends on "natural" parameters: the figure geometry, the given load, the final destination, and especially, the *strength model* of the agent. Additional user controllable parameters of the motion are the *comfort* of the action and the *perceived exertion* of the agent. The algorithm uses this information to incrementally compute a motion path of the end effector moving the load. It is therefore instantaneously adaptable to changing force, loading, and strength conditions. Various strategies are used to model human behavior (such as pull back, add additional joints, and jerk) that compute the driving torques as the situation changes. The strength model dictates acceptable kinematic postures. The resulting algorithm offers torque control without the tedious user expression of driving forces under a dynamics model. The algorithm runs in near-realtime and offers an agent-dependent toolkit for fast path prediction. Examples are presented for various lifting tasks, including one- and two-handed lifts, and raising the body from a seated posture.

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GRAPHICS LAB 31

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Abstract

A methodology and algorithm is presented that generates motions imitating the way humans complete a lifting task under various loading conditions. The path taken depends on "natural" parameters: the figure geometry, the given load, the final destination, and especially, the *strength model* of the agent. Additional user controllable parameters of the motion are the *comfort* of the action and the *perceived exertion* of the agent. The algorithm uses this information to incrementally compute a motion path of the end effector moving the load. It is therefore instantaneously adaptable to changing force, loading, and strength conditions. Various strategies are used to model human behavior (such as pull back, add additional joints, and jerk) that compute the driving torques as the situation changes. The strength model dictates acceptable kinematic postures. The resulting algorithm offers torque control without the tedious user expression of driving forces under a dynamics model. The algorithm runs in near-realtime and offers an agent-dependent toolkit for fast path prediction. Examples are presented for various lifting tasks, including one- and two-handed lifts, and raising the body from a seated posture.

1 Introduction

Realistic articulated figure animation is a long-sought goal of computer graphics researchers. While progress in modeling and image generation has been remarkable, many animation schemes still rely on human skill and creativity to effect natural-looking motion. As recent efforts have examined physically-based models to achieve plausibility and accuracy of synthetic models, attempts at realistic human motion have just begun to trace their origins back to biomechanical principles (Thompson et al. 1989, Bruderlin and Calvert 1989, Lee 1990). We believe that human motion models are going to be hybrids of many motion and path generation techniques. The goal is to find effective combinations that provide realistic motion behaviors while simultaneously offering the animator/user reasonable and intuitive control mechanisms. Often described as "task level animation" (Zeltzer 1985; Thalmann 1989; Esakov at al. 1989; Badler 1989), the idea is to find parametric procedures that implement various basic human activities such as grasping. reaching, lifting, walking, and so on. Presumably the list is finite: other behaviors are derived from the sequential or parallel execution of such tasks, combined with appropriate transitions for smooth action.

One of our research goals is to provide task-level control algorithms for human figures (Badler et al. 1987; Zhao and Badler 1989; Phillips et al. 1990, Badler et al. 1989). Tasks we are primarily interested in include: multiple constrained reach, view, and object manipulation by an end effector. The latter is the one we will discuss here; in particular, we examine the problem of moving a load to a specified position in space. The resulting motion is dictated by the geometry but especially by the *strength and comfort* of the agent.

Such problems have application in the ergonomic design and evaluation of workplaces for human operators and maintenance facilities for service personnel.

2 Background

Torques may be used to physically simulate motions of a figure. Typically the joint responses are conditioned by springs and dampers so that responses to external forces can be computed. Such dynamic animations have been studied by many researchers (Girard 1987; Girard 1990; Armstrong et al. 1987; Isaacs and Cohen 1987; Wilhelms 1987; Wilhelms and Moore 1988; Hahn 1988; Hoffmann and Hopcroft 1987; Otani 1989; Baraff 1989). Solving the dynamic equations, an initial value problem, is computationally expensive, especially if the joints are stiff (Armstrong et al. 1987). Moreover, such motions are annoyingly difficult to control by an animator unless free-swinging motions are in fact desired. The resulting motions appear to drive a hapless mannequin or puppet by external forces such as gravity and collision reactions. When the torques are derived from a spring or vibration model, convincing motions of worms, snakes, and other flexible objects may be simulated (Miller 1988; Pentland and Williams 1989), but this cannot be the same mechanism used for normal articulated figure motion. Kinematic and inverse kinematic approaches are easier to manipulate and may create the right "look," but suffer from potentially unrealistic motion (velocity, torque) in the body joints. These problems have been addressed as boundary value problems with constraining objective functions. The trajectories are then solved by global optimization approaches (Witkin and Kass 1988; Breen 1989) or control theory (Brotman and Netravali 1988), but their methods presume complete knowledge of the driving conditions and overall constraints. Relatively successful animation of locomotion has been achieved by hydrid approaches combining kinematic and dynamic constraints (Girard 1987; Girard 1990; Bruderlin and Calvert 1989).

In robotics, the emphasis is to accomplish a motion within constraints and optimize with respect to some criteria, i.e. time, torque, energy, and obstacles (Shiller and Dubowsky 1985; Dubowsky et al. 1986; Hollerbach and Suh 1985; Kazerounian and Nedungadi 1987; Khatib 1987; Maciejewski and Klein 1985; Singh and Leu 1987; Luh and Lin 1981; Rajan 1985). Bioengineers are curious to determine if human motion conforms to some optimality criterion, such as energy (Beckett and Chang 1968; Chao and Jacobson 1971; Yen and Nagurka 1987; Yeo 1976). They have found that human motion is not optimal.

Despite these diverse approaches to the human motion problem, none has been successful at specifing a task by describing a load and a placement goal, and then completing the task in a realistic (though possibly suboptimal) manner. There is much work on generating a path between two endpoints (Sahar and Hollerbach 86; Kahn and Roth 1979; Schmitt et al. 1985), but the usual solution incorporates constraints and a single objective function that is optimized.

3 Our Approach

We offer a solution which blends kinematic, dynamic and biomechanical information when planning and executing a path. The task is described by the starting position, the load (weight) that needs to be transported, and a goal position for the load. Some simple additional parameters help select from the wide range of possible paths by invoking biomechanical and performance constraints in a natural fashion. *Thus a path is determined from* *a general model rather than provided by the animator.* In addition, the algorithm has the ability to adapt to changing forces that are required to complete a task. The basic premise of the method is that a person tends to operate within a *comfort* region which is defined by *available strength.* This is even more probable when the person has to move a heavy object.

We support the general principle that constraints predict motion. We considered applying the constraints which would be similar to those that a human encounters when moving. Because human motion is diverse, a model of the current state of the body is required so that a predicted path is determined by compromising the task to be accomplished and the body's resources. Also, because of the diversity in motion, no single type of path or strategy is applicable for all cases. To address this, we developed a toolkit of motion generators which produce characteristic motions and a corresponding toolkit of transition identifiers which points to the appropriate motion generator for any situation. The toolkits are the foundation of our human simulation system which is fast enough to allow interactive animation of the completion of a task. The constraints for this system should be intuitive so that manipulation of constraint parameters would generate the expected results. Finally, the constraint parameters should be easy to modify, allowing the animator some flexibilty and creativity in producing a desired motion.

We assume that a person tends to operate within a comfort region dictated by muscular strength, especially when moving a heavy object. When a person has to accomplish a task he starts from some initial posture and then plans the direction for his hand to move. This planning is based on the person's perception of his strength, comfort, and the importance of staying along a particular path. After a direction is determined, he tries to move in that direction for a short distance with joint rates that maintain the body's discomfort level below a particular threshold. Once the distance is reached another direction is selected by balancing the need to finish the task as directly as possible with restrictions derived from the body's limitations. Again, joint rates can be determined once a new direction is established.

4 Problem Specification

The problem is to find the trajectories, both joint and end-effector, that a human-like linkage would traverse to complete a task (Figure 1) . The task can be specified in a simple manner since the joints have to either overcome a force or impart a force. At every instance in a trajectory a downward force (assuming the task involves working against gravity) acts on a hand or end-effector. The task specification can be generalized to describe a complicated task by letting the force be a function of body position, hand position, time or other factors. In general, task specification can be represented by a *force trajectory.* The force direction may not be collinear with the direction to the final position (goal position). As a result, path planning is based on a combination of a force trajectory and the direction to the final position which we call *tracking trajectory.* In addition to task specification by a force and tracking trajectory, human motion is guided by many constraints that limit the joint and end-effector trajectories. Constraints that will guide this work are *comfort level*, *perceived exertion*, and *strength.*

Comfort level is defined in a mechanical sense. It is found by calculating over the entire body the maximum torque ratio: current torque divided by the maximum torque at each individual joint for the current joint position and velocity. In general when humans move they try to maintain their effort below a particular discomfort level. Therefore, it is

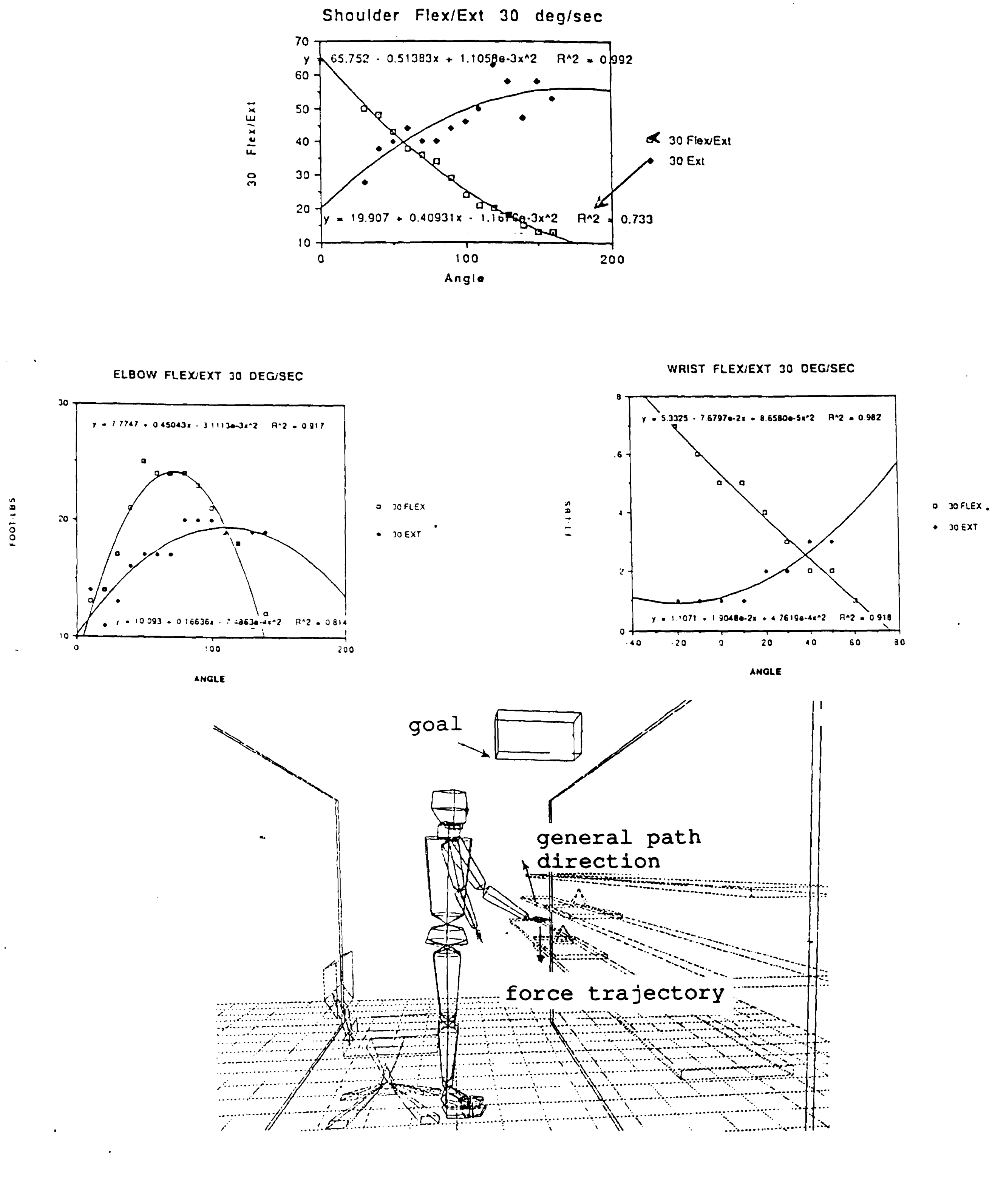

Figure 1: Definitions used in describing the lifting task and its parameters. Sample strength curves shown at the top.

desirable to dictate a motion that minimizes the maximum torque ratio of a body in order to maximize the comfort level.

Perceived exertion is a variable used to indicate the expected level of difficulty in completing a task. It depends on the perception of the amount of strength required (an implicit function of the force trajectory) and the amount of strength available. If perceived exertion is low then the comfort level is not expected to be exceeded for the paths 'likely' to be taken to satisfy a task, especially for a path that travels a straight line between the initial body position and the goal. However, if the perceived exertion is high, then the joint and end-effector paths need to be deviated from a straight path in order to abide by the comfort constraint. Perceived exertion is represented by a cone which is defined by the maximum deviation angle of a path from its current position.

5 Strength Model

Ultimately, the shape of an end effector's motion is derived from the body's resource strength: the maximum achievable joint torque. Strength information (maximum torques) is defined as muscle group strengths and is stored on a joint degree of freedom (DOF) basis. Modeling strength in terms of muscle group strength allows different people to possess different strength capacities in different muscle groups. Thus, the difference between two people such as a dancer and a pianist can be readily modeled and illustrated. Each DOF of a joint has two joint movements which are associated with two different muscle groups. For example, an elbow is modeled to have one DOF. It can only rotate around one axis; its rotational movements are extension and flexion which correspond to muscle groups extensor and flexor. Therefore, strength information of extension and flexion are stored for an elbow joint. Each muscle group strength is modeled as a function of body position, anthropometry, gender, handedness, fatigue, and other strength parameters (Asmussen and Heeboll-Nielson 1962; Laubach 1976; NASA 1978; Ayoub et al. 1981; Imrhan 1983; Chaffin and Anderson 1984; Heyward et al. 1986). In terms of body position, we chose a more generalized model that takes effects of adjacent joint angles into consideration (Schanne 1972). For example, the muscle group strengths of a shoulder are modeled to be functions not only of the shoulder angles but also of the elbow angle.

Strength information is maintained in *SASS* (Spreadsheet Anthropometry Scaling System) (Grosso et al. 1989) and is available through the Peabody description (Phillips and Badler 1988) of the figure. *SASS* is a spreadsheet-like system which allows flexible interactive access to all anthropometric variables (segment dimensions, mass, joint limits, strength, etc.) needed to size a human figure described structually by a Peabody body file. A sample *SASS* spreadsheet screen showing a portion of the right upper limb strength data is shown in Figure 2. *SASS* provides a user with easy and effective access to strength information either by a table lookup from empirical observations or an interpolation function through a prediction or scaling equation. Variations due to anthropometry, gender, handedness, fatigue, etc. may be included. Strength information of specific individuals or percentiles within a population may be obtained through *SASS*.

Strength curves for each joint can be developed for its entire range of motion. They generally fall into one of three categories: (1) ascending, (2) descending, and (3) a combination of ascending and descending (Clarke 1966; Kulig et al. 1984). In our strength model, there are two strength curves for each DOF of a joint because each DOF is associated with two movements (in opposite directions) which correspond to two muscle group strengths.

POPULATION : Strength Test Data		■Next Page	■Query
FIGURE TYPE : Skinny Body	STRENGTH TYPE: Isometric	■Prev. Page	■Disp. Indv.
GENDER : Male	MOTION SPEED : 0	■deg -> rad	■void
MASS : 82.20kg (50.00%)	HANDEDNESS : Right	■Input Data	■void
STATURE : 182.20cm (50.00%)	TRAINING : 0	■Create Fig.	■ft-lb -> N-m
GROUP %TILE : 50.00%	FATIGUE LEVEL: 0	■Quit	■right -> left

Motion Impetus (right)		Body Configuration / Joint Angles (deg)							Resultant Strength	
		Shoulder			Elbow	Wrist			Value (ft-lb)	%
		x	y	z	y	x	y	z		
Shoulder	Abduction	150.00	45.00	-	90.00	-	-	-	21.18	50
	Adduction	150.00	45.00	-	90.00	-	-	-	19.64	50
	Flexion	45.00	150.00	45.00	90.00	-	-	-	17.80	50
	Extension	45.00	150.00	45.00	90.00	-	-	-	12.06	50
	Lateral Rotation	45.00	-	150.00	-	-	-	-	15.77	50
	Medial Rotation	45.00	-	150.00	-	-	-	-	15.71	50
Elbow	Flexion	-	90.00	-	145.00	-	-	-	14.57	50
	Extension	-	90.00	-	145.00	-	-	-	11.88	50
Wrist	Abduction	-	-	-	-	25.00	-	45.00	3.62	50
	Adduction	-	-	-	-	25.00	-	45.00	2.69	50
	Flexion	-	-	-	90.00	20.00	75.00	45.00	2.10	50
	Extension	-	-	-	90.00	20.00	75.00	45.00	2.86	50
	Supination	-	-	-	-	-	45.00	90.00	1.92	50
	Pronation	-	-	-	-	-	45.00	90.00	4.60	50

Press LEFTMOUSE to select items. [page: 1]

Figure 2: Sample display page from *SASS* showing right upper limb strength data.

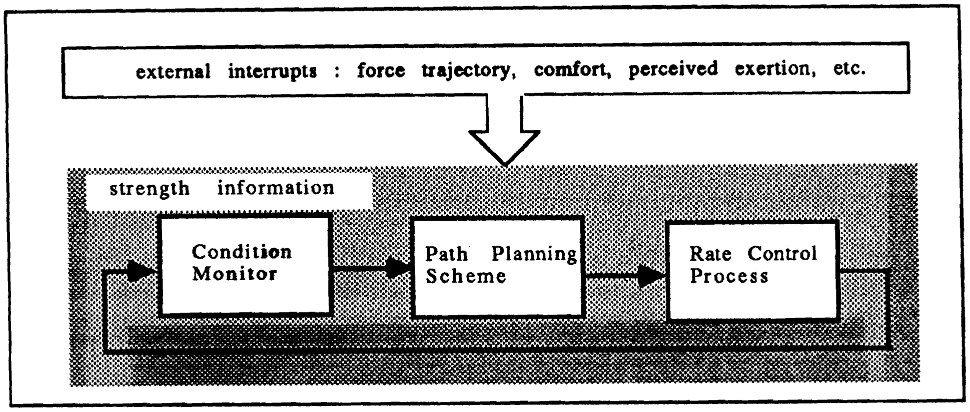

Figure 3: Overall system architecture.

Figure 1 shows extension and flexion strength curves for shoulder, elbow, and wrist, respectively (Greenisen 1989). The maximum strength value used for the comfort computation is called the *effective maximum strength.*

6 System Architecture

The generation and control of motion consist of three components (Figure 3):

1. Condition Monitor which monitors the state of a body and suggests motion strategies.
2. Path Planning Scheme (PPS) which plans the direction that an end-effector will move.
3. Rate Control Process (RCP) which determines the joint rates for motion.

The *condition monitor* reports on the current state of a body: the current position, maximum strength for a current position, current joint torques, etc. It then suggests motion strategies to the *path planning scheme* which determines a direction of travel from a body position. The length of travel in the suggested direction of travel can be arbitrarily set. The rate of travel, constrained by torque, for a path interval can then be computed by the *rate control process.* After the joint rates are resolved and new joint positions are found, these procedures are repeated until the entire joint path is mapped out in a manner that satisfies the specified task. This system architecture is an iterative process which allows changes to the parameters at any time through other external process. Possible situations to alter any of the parameters are dropping or changing the mass of a load, redirecting the goal, or encountering an obstacle. This is different from the global nature of optimal control-based algorithms. We handle similar global considerations through an external process (Lee 1990).

6.1 Condition Monitor

The condition monitor gathers information about the current state of a body, assembles the information, and suggests a motion strategy for the next procedure to process. The motion strategies are a function of the constraint parameters: comfort, perceived exertion, and strength. Each motion strategy, based on the constraints, concentrates on a separate fundamental aspect of motion. The strategies can be catagorized by whether an end-effector's need to reach a goal is more important than joint considerations or whether joint considerations are more important than reaching a goal. We can also interpret the strategies as particular optimization problems. The condition monitor is the highest level of the three procedures in predicting a path.

6.2 Path Planning Scheme

The path planning scheme, guided by the condition monitor, determines the direction to move. In general, the output of any system is bounded by its headroom[1]. In the case when there is much strength in a system, the headroom can be used to suggest incremental joint displacements, $d\theta$. A larger headroom allows a larger displacement. The mapping between the cartesian displacement and the joint displacement is

$$d\mathbf{x} = \mathbf{J}d\theta \tag{1}$$

where $\mathbf{J}$ is a $3 \times n$ matrix and n is the number of joint displacements. If the headroom for each joint, which is represented by a weighting vector $\mathbf{w}$, is proportional to $d\theta$, then

$$d\hat{\mathbf{x}} = \mathbf{J}\mathbf{w} \tag{2}$$

where $d\hat{\mathbf{x}}$ is a normalized direction of reach. $d\hat{\mathbf{x}}$ is then compared to a cone, which represents a set of feasible directions to travel and is derived from perceived exertion. If $d\hat{\mathbf{x}}$ is within the cone then the direction of motion should be $d\hat{\mathbf{x}}$, otherwise the direction can be $d\hat{\mathbf{x}}$ projected onto the cone.

When there is not much strength in the system, the suggested direction of motion must not violate the strength constraints. The decision process should shift from one where the desirability to reach a goal is a major component of determining a suggested motion to one where avoiding positions where the joints that are strained becomes more important. As a result, determining directions to move should demand greater consideration in joint space than in task space. This will be more apparent when the methods for PPS are discussed in the Appendix.

6.3 Rate Control Process

The rate control process, the most basic of the three procedures, resolves the speed with which a body moves along a prescribed end-effector path. This requires the use of dynamics, especially when the motion is fast. Previously, where dynamics was considered in solving the problem, solutions consisted of specifying a set of force functions for the end-effector or each joint, with the expectation that the computed positions would appear as desired. This solution is appropriate for forward dynamics simulation but not in situations where motion control is needed.

Dynamics equations can be interpreted as constraint equations solving for joint trajectories if they satisfy the conditions imposed by specific end-effector path and torque limits. The dynamics equations can be reformulated so that they provide a mapping between an

[1]The available range of a variable within a constraint.

end-effector path and a binding torque constraint. A binding torque constraint is the maximum torque allowed to drive a body with maximum end-effector acceleration without the end-effector deviating from the prescribed path. A greater torque would cause excessive inertial force and therefore, undesirable path deviation. It is evident from the derivation of the reformulated dynamics equations (see Appendix), which were originally derived to solve for path completion in minimum time (Shiller and Dubowsky 1985), that joint trajectories can be found from the acceleration of an end-effector. As a result, the reformulated dynamic equations implicitly determine the force functions (joint torques) to guide an end-effector along a specified path.

Torque limits are established by a current comfort constraint. The comfort level variable, cl, determines the torque limit at each joint by a simple relation:

$$cl = \frac{\tau_{c,i}}{\tau(\theta)_{max,i}} \tag{3}$$

where $\tau_{c,i}$ is the torque limit for a particular joint, i. $\tau(\theta)_{max,i}$ contains the maximum torque for the joint's current position. $\tau(\theta)_{max,i}$ is obtained by examining the strength curves or querying the *SASS* database. When the value of cl becomes greater than one, there is no more strength to accomplish a task and therefore the attempt to complete a task should cease. Comfort level, which can be adjusted to achieve a desired motion, influences not only the direction of travel but also the rate of task completion.

7 Motion Strategies

This is a catalogue of human motion strategies and the conditions that trigger their use. The strategies are given in the order of increasing discomfort. The modeling of these strategies is discussed in the Appendix.

Available Torque

When a person moves, the tendency is to move the stronger joint. This is much like the forces due to a spring or other types of potential forces. A stronger spring, based on the spring's stiffness coefficient, would yield a larger displacement per unit time than a weaker spring. Similarly, for a human, the amount of displacement for a joint depends not only on the strength that a joint is capable of but also on the amount of strength that is currently available. The amount of strength available, which is based on the difference between the current required torque to support a particular position and the effective maximum strength, is called *torque availability.* If torque availability is low, motion would not be encouraged. Conversely, if the torque availability is high, the joint will do more of the work. Torque availability is the driving factor for a joint to move and to redistribute the joint torques so that the comfort level is more uniform. This assumes that a body has enough effective strength to allow its joints to move to positions where the overall stress level of the body is smaller than if the joints were only guided by kinematic demands.

Reducing Moment

In following the Available Torque Strategy, a joint would be discouraged from moving when its torque value approaches its effective maximum value. We will define this condition as *stress lock.*

As a joint approaches its effective maximum strength, usually a region where the strength curves (for humans) are relatively flat, the joint tries to avoid stress lock while trying to reach a goal. A path towards the goal is still possible as long as the maximum strength is not surpassed for any of the joints. When all the joints reach stress lock, the body attempts to reduce the moment caused by a force trajectory and the distance to the force trajectory's point of application. In addition, a reduction in moment increases the torque availability of (at least) the joint that is approaching stress lock. The reduction in the total moment involves trying to reduce moment on a joint by joint basis. At each joint a virtual displacement is given to determine if that displacement provides sufficient moment reduction to continue moving in that direction.

Pull Back

The two previous strategies depend on the current torque to be less than the maximum strength. In these cases, maneuverability in torque space is high and therefore, an end-effector can still consider moving toward a goal without exceeding any joint's maximum strength.

However, when a particular joint reaches its maximum strength, then that joint can no longer advance toward a goal from the current configuration. The Pull Back strategy proposes that the end-effector approaches the goal from another configuration. In an effort to determine another approach to the goal, the constraint of moving toward a goal within a restricted path deviation can be relaxed. The emphasis of the strategy is one where the joints dictate an improved path in terms of torques. This can be accomplished by increasing the *ultimate available torque* – the difference of maximum strength to current torque – for a set of *weak joints* – joints that are between the joint which has no ultimate available strength and an end-effector.

In general, the joint with the least amount of ultimate available torque will reverse direction and cause the end-effector to pull back (move away from its goal). A result of this strategy is that the overall comfort level is increased. When the joints form a configuration that has a greater level of comfort, there might be enough strength to complete the task. If this occurs, the governing strategy could return to Reducing Moment.

Added Joints, Recoil, and Jerk

When the three modes, Available Torque, Reducing Moment, and Pull Back have been exhausted and an agent still cannot complete a task, it is obvious that the active joints (the joints that were initially assigned to the task) cannot supply sufficient strength. When this occurs it should be determined if the task should be aborted or if there are other means of acquiring additional strength. Acquiring more strength involves activating one or more additional joints.

When additional joints are activated, a *stable configuration* should be formed by the active joints because they cannot get to configurations where there is enough strength to complete a task. When the active joints form a stable configuration, they can either move very quickly or get jerked by the added joint. The added joint must be much stronger than any of the active joints. Jerking reduces the forces that are necessary for the active joints to move. Once the added joint has jerked, the active joints can try to reach their goal since the torques that they must generate have decreased. The added joint, at its current position, may not be able to supply enough force to effect the necessary force reduction for the active joints. In such a case it needs to move to a position where it can supply enough force. This is usually done by recoiling.

A stable configuration is a posture that a set of joints should form so that it can withstand large forces such as those caused by jerking. Finding a stable body configuration is necessary because during jerking discomfort is intensified from impact caused by going from a static situation to one that is dynamic. The Pull Back Strategy leads to a posture of stable configuration.

8 Comfort, PE, and Motion Strategies

Comfort and perceived exertion are two constraints whose influences on motion are coupled. It is extremely difficult to synthesize the effect of each constraint on a motion. However, in developing motion strategies, it was required to understand the bearing they have in the determination of a motion. In the most primitive process of the system, which involves dynamics, the two constraining parameters predicted a motion precisely.

In a higher level of control (such as PPS) their relationship may not be as exact (despite our efforts). However, we could arrive at an impression of the role they play in motion. We organize this in terms of increasing discomfort levels (Figure 4).

- High comfort (or lazy comfort). Perceived exertion constraint is not active but comfort constraint is because any changes in acceleration (not necessarily large) may cause a joint to exceed the comfort constraint. In general, any force trajectory associated with a motion of high comfort is negligible. Also, dynamics is important because of the relatively large inertial effects of the body. This group is bounded by motions that are catagorized by *zero jerk* condition (see efforts by (Girard 1990)) and Available Torque.

- Regular comfort. The end-effector can advance toward the goal. Perceived exertion and comfort are not constraining and dynamics should be evaluated. Available Torque and Reducing Moment bounds this comfort level.

- Discomfort. At this point the comfort level for one or more joints are surpassed. Perceived exertion is relaxed (larger path deviation is allowed) because required forces are greater than expected. Motion should have slowed down considerably and therefore, dynamics is not important and (most likely) is not meaningful. This group is formed by Reducing Moment and Pull Back.

- Intolerable discomfort. Many of the joints' comfort levels have been exceeded and there may be other joints which could be approaching their ultimate available torque. In such a situation, stategies can be combined. The pool of strategies are Pull Back, Recoil and Jerk. Nevertheless, perceived exertion is relaxed and depending on the combination of the strategies, dynamics might be important.

9 Results

The strategies of Available Torque, Reducing Moment, and Pull Back are implemented. Figures used for this discussion, which show paths resulting from the algorithm, follow the Appendix. The task is to pick an increasingly heavy load from three separate initial

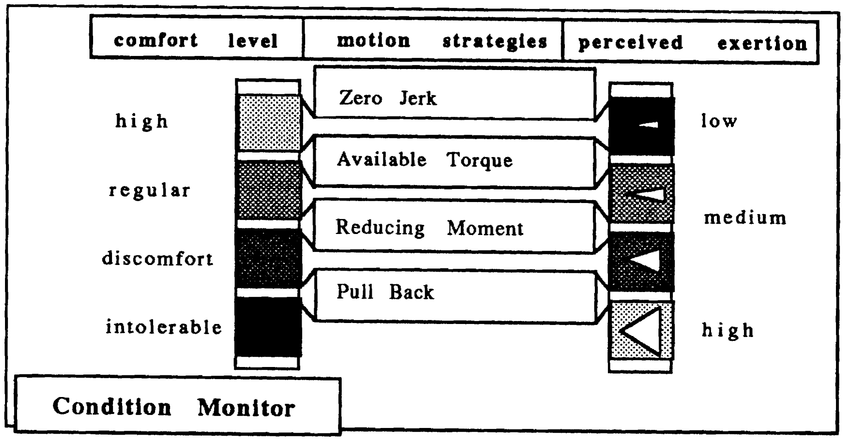

Figure 4: Relation of comfort level. motion strategies, and perceived exertion in the condition monitor. Note that as perceived exertion increases, the cone of allowable displacement enlarges.

positions to a goal (located on the corner of the box above the body's head). The comfort level for these examples is fixed at (0.5). Perceived exertion (pe) may also be altered.

In Figure A1 the body ($pe = 0.3$), shown in its initial position, is instructed to pick an object, which weighs 10N, from the lowest shelf. The actual handling of the object is omitted since it would block the drawing of the end-effector's path. In Figure A2, because the object is light, a fast motion is predicted and the solution resembles a minimum time path. Figure A3 shows the hand reaching the target goal.

In Figure A4, the same body is lifting a 20N object from the same starting position as in Figure A1. The heavier weight draws the hand closer to the body. The paths are rough because they are at the boundary of a solution determined by Available Torque and Reducing Moment. Figure A5 shows the complete behavior for lifting the 20N weight. Also the path from the 10N lift is included for comparison.

In Figure A6 the body ($pe = 0.3$) picks an object (20N) from the middle shelf. It started in a more comfortable position and as a result, the motion is less jagged than the one in Figure A4 even though the task involves the same load. Since it is more comfortable, it does not have to decide between strategies as often (the ability to anticipate motion, which would smooth out most instantances of when the jagged motion would occur, has not been incorporated).

Figure A7 shows the path (left curve, curve for the task described in Figure A6 is on the right) for the same task as the one depicted in Figure A6 but with a heavier load (30N, $pe = 0.15$). The body immediately executes Pull Back. In this case, the body pulled back to a region of high comfort and so the approach to the goal is smooth.

In Figure A8 the body ($pe = 0.15$) lifts from the top shelf (hand moved away from initial viewing of the curves). The curve on the right is for a 30N object. The left curve, representing a task of moving a 45N object, the body has to pull back more because the object is much heavier. However, in this instance the pull back is only to a region that is at the edge of the body's discomfort level and therefore, there is much shifting between strategies. The larger external force requires greater path deviation along this boundary. The greater path deviation allows more space to find a more comfortable posture. This can be studied by comparing with Figure A6 where the path deviation for a lighter object is less.

The algorithm can be applied to any types of task, as long as it is force based. Figure A9 to A11 shows a body rising from a chair. A force trajectory represents its body weight. Figure A10 shows the body leaning forward to balance his weight (a consequence of Reducing Moment) and to reach a goal, the position of his upper torso when standing.

These paths can be used to create keyframes which can be interpolated for animation purposes. The average time of a path generation is under 10 seconds. Since our examples mainly involved heavy loads, static torque computations were used. The joint position are determined by inverse kinematics computation (Zhao and Badler 1989).

10 Discussion

In this paper a structure to solve the problem of path generation for animation to complete a task in 'real' articulated figure motion modeling is presented. The task, represented by a force trajectory, can depend on position, time, etc. The method maps out an entire path automatically and incrementally for a force trajectory over a set of constraints which consist of: comfort level, perceived exertion, and strength. Because the body is constantly updated, these constraints can also be a function of an external model, such as fatigue.

Motion is generated by integrating a condition monitor which suggests basic motion strategies, a path planning scheme which locally plans the end-effector's path and a rate control process which controls the joint rates. The condition monitor offers strategies to pursue by balancing the goal of the task and the resources that are currently available. The path planning scheme proposes a direction of travel by executing the basic strategies. The elusive force funtion that previous investigators have sought can be found by changing the role of the dynamic equations to a constraint equation which is established with a dynamics model. By selecting the most binding constraint from the constraint equations, the maximum joint rates can be computed.

Altering the constraints used in this problem gives an animator the ability to create a motion that conforms to physical laws. The fast computations in this system permit the animator to generate and achieve the desired motion quickly. We see this capabilty as a inherently model-driven, flexible, and crucial component in the on-going search for task-oriented human movement animation.

A Appendix

A.1 Rate Control Process

The standard dynamic equations can be written as

$$\tau = \mathbf{M}(\theta)\ddot{\theta} + \mathbf{H}(\theta, \dot{\theta}) + \mathbf{F}(\theta) \tag{4}$$

where τ is the torque vector, $\mathbf{M}$ is the inertia matrix, $\mathbf{H}$ is the matrix which contains the coriolis and centripetal terms, and $\mathbf{F}$ is from the force trajectory. Let s be a parameterization of the suggested path from PPS and let $\mathbf{r}$ and $\mathbf{R}$ be vectors denoting the end-effector's degrees-of-freedom. The path may then be expressed in terms of s as:

$$\mathbf{r}(\theta) = \mathbf{R}(s). \tag{5}$$

This equation equates two descriptions of the same point which can be used to obtain $\dot{\theta}$ and $\ddot{\theta}$ as functions of $\dot{s}$ and $\ddot{s}$. Differentiating the above equation with respect to time

$$\mathbf{r}_\theta \dot{\theta} = \mathbf{R}_s \dot{s}. \tag{6}$$

Here $\mathbf{r}_\theta$ and $\mathbf{R}_s$ are $3 \times n$ and 3×1 jacobian matrices respectively. Solving for the joint velocity vector, we obtain:

$$\dot{\theta} = \mathbf{r}_\theta^\dagger \mathbf{R}_s \dot{s} \tag{7}$$

where $\mathbf{r}_\theta^\dagger$ is a generalized inverse. If this generalized inverse is related to minimizing the joint velocities then $\mathbf{r}_\theta^\dagger = \mathbf{J}^\dagger = (\mathbf{J}^T\mathbf{J})^{-1}\mathbf{J}^T$. The jacobians can also be weighted by a mass matrix or a torque matrix which can then be related to minimizing energy or power respectively. For example, in a lifting task, this would depend on the weight of the object relative to the weight of the segments.

Differentiating the previous equation the joint accelerations can be solved:

$$\ddot{\theta} = \mathbf{r}_\theta^\dagger(\mathbf{R_s}\ddot{\mathbf{s}} + \mathbf{R}_{ss}\dot{s}^2 - \dot{\mathbf{r}}_\theta\dot{\theta}) \tag{8}$$

And the dynamics equation becomes

$$\tau = \mathbf{K}_1(s)\ddot{s} + \mathbf{K}_2(s, \dot{s}) + \mathbf{F} \tag{9}$$

$$\mathbf{K}_1(s) = \mathbf{M}(\theta)[\mathbf{r}_\theta]^\dagger \mathbf{R}_s \tag{10}$$

$$\mathbf{K}_2(s) = \mathbf{M}(\theta)[\mathbf{r}_\theta]^\dagger(\mathbf{R}_{ss}\dot{s}^2 - [\dot{\mathbf{r}}_\theta]\dot{\theta}) + \mathbf{H}(\theta, \dot{\theta}). \tag{11}$$

By imposing the torque limit vector, τ_c, we obtain:

$$\mathbf{K}_1(s)\ddot{s} + \mathbf{K}_2(s, \dot{s}) + \mathbf{F} \leq \tau_c. \tag{12}$$

Solving the above equation for the acceleration term gives:

$$\mathbf{K}_1(s)\ddot{s} \leq \tau_c - \mathbf{K}_2(s, \dot{s}) - \mathbf{F}. \tag{13}$$

The above set of n equations solved for $\ddot{s}$ forms a set constraint conditions

$$\ddot{s} \leq \mathbf{A}(s, \dot{s}) \tag{14}$$

where $\mathbf{A}$ is n-element column vector whose elements are the maximum acceleration resulting from the torque limits of each joint. They form n-constraints of eligible acceleration. Obviously, $\ddot{s}$ can be choosen from the constraint

$$\ddot{s} \leq min[\mathbf{A}(s, \dot{s})] \tag{15}$$

where min is a function which returns the minimum element of a column vector. This equation translates the torque limits into cartesian acceleration limits. In order for the end-effector to maintain its path this relation must be satisfied.

This equation offers the advantange of resolving the joint trajectories by integrating a single constraint equation instead of a system of dynamic equations. Because the parameterized path can be determined by an integration of a single equation and the joint trajectories can be determined by multiplication, the required dynamic computations should be relatively fast.

Dynamics are important when a task involves a fast motion such as moving a light object or jerking because the body's inertial forces have a major impact on the forces of the system. However, when the body's inertial forces are relatively small compared to a force trajectory and there is no jerk in the motion, the motion is slow and static approximation probably would yield reasonable results. The speed of the motion is probably determined by other factors. Because of this slow speed, the resolution of the joints does not require dynamics. The joint resolution of a redundant figure can be approximated by inverse kinematics (Phillips et al. 1990).

A.2 W Vectors

These $\mathbf{w}$ vectors can be applied to the the equation $d\hat{\mathbf{x}} = \mathbf{J}\mathbf{w}$.

A.2.1 W Based on Available Torque

The elements of $\mathbf{w}$, a $n \times 1$ vector (n is the number of degrees-of-freedom) which is based on available torque is created by using strength curves. The joint that has the larger amount of available torque should move more. For example, consider two joints, A and B, which have the same value for their maximum torque, τ_{max}. If the current torque, τ_{cur}(which can be computed very quickly from forward dynamics) for joint A is lower than the current torque for joint B, the torque availability, τ_{max} - τ_{cur}, in A would then be greater than the torque availability in B. This would encourage moving joint A (assuming that we are dealing with fully contracted muscles) more than joint B and thereby decreasing more joint A's available torque than joint B's. Based on the need to weight the motion to the joint that has the potential to be more active, the value of each element of $\mathbf{w}$ can be

$$w_i = \frac{\tau_{max,i} - \tau_{cur,i}}{max\left[\tau_{max,n}\right]} \tag{16}$$

where i is enumerated for each of the joints and max is a function that returns the value of the maximum torque of all the joints. This ratio follows the property that when a joint's torque value is near its maximum torque, movement is discouraged, and when it is far from the maximum, movement is encouraged towards the position where the maximum torque occurs.

Let's define the $\mathbf{w}$ that was just presented as $\mathbf{w}_1$.

A.2.2 Stress Lock

$\mathbf{w}_1$ assumes that torque maneuverability is high and the current comfort level can be exceeded, but not excessively. However, stress locked joints should be exempted from deciding where to move. This can be done by setting $\mathbf{w}_{1i} = 0$ where i represents the stress locked joints.

A.2.3 W Based on Reducing Moment

When several joint are in the neighborhood of stress lock, the joint rate of most of the joints is close to zero so the joint torques can be appoximated with the static force equation

$$\tau_{static} = \mathbf{J}(\theta)^T \mathbf{F}(\theta) \tag{17}$$

where τ_{static} is the static force vector and $\mathbf{F}$ is the force trajectory at the current position. The static force equation can be used to determine which joints will benefit, with respect to moment reduction, with a change in joint angle. The static torque vector computed for a small change in joint i's position is

$$\tau_{d\theta_i} = \mathbf{J}(\theta + \delta\theta_i)^T \mathbf{F}(\theta). \tag{18}$$

The ith component of $\tau_{d\theta_i}$, $\tau_{d\theta_i,i}$, can be compared with the ith component of τ_{static}, $\tau_{static,i}$, to determine the effect, on each joint torque, of altering each joint angle. Let's define a $\tau_{dif,i}$ to be equal to $\tau_{static,i} - \tau_{d\theta_i,i}$. Then for the joints that are proximal to the joint that is stress locked, both a positive and a negative $d\theta$ should be investigated. The one that yields the larger difference should be used to define $\mathbf{w}$. Finally,

$$w_i = \pm \frac{\tau_{dif,i}}{max[\tau_{dif,n}]} \tag{19}$$

where the denominator is the maximum τ_{dif} from all the joints. w_i would be negative if the $\tau_{d\theta}$ used was based on a negative $d\theta$. It can be shown that $\mathbf{w}_2$ is similar to $\frac{\partial \tau_i}{\partial \theta_i}$ (Lee 1990). This weighting vector for moment reduction is labelled $\mathbf{w}_2$.

A.3 Direct Joint Control

A.3.1 Pull Back

A set of weak joints participate in Pull Back. The objective is to acquire more strength. The ultimate available torque is evaluated beginning from the weakest joint to the joint nearest an end-effector. The equations for this is

$$\tau_{u(plus),i} = \tau_{s,i}(\theta + \delta\theta_i) - \tau_{d\theta_i,i} \tag{20}$$

$$\tau_{u(minus),i} = \tau_{s,i}(\theta - \delta\theta_i) - \tau_{d\theta_i,i} \tag{21}$$

where $\tau_{s,i}(\theta + \delta\theta_i)$ and $\tau_{s,i}(\theta - \delta\theta_i)$ are the strength values (with the appropriate strength curve) for small changes in joint i's position. Then the most stressed joint moves in the direction of improving $\tau_{u,i}$. The next topologically closer joint to the end-effector performs the same calculation, incorporating the displaced joint values. This process is repeated until the joint nearest the end-effector is reached. The amount of joint displacements can be an arbitrary joint displacement weighted by its discomfort level which causes the most stressed joint to move the most.

A.3.2 Singular Configuration and Recoil

Pull Back brings the body to a stable configuration in terms of torque. To model a stable configuration completely, the structural capacity of a figure should be included in the model. The Pull Back strategy can also be used to find postures involving motions of high exertion (throwing a baseball, football, and shotputting). This involves using sensitivity analaysis (Lee 1990).

Recoiling is similar to Pull Back – acquiring additional strength – except that it involves only the added joints. The amount of recoil must satisfy the static force condition

$$\tau(\theta)_{add} \geq \mathbf{J}^T \mathbf{F}(\theta)_{recoil} \tag{22}$$

where $\tau(\theta)_{add}$ is the maximum strength of the added joints. Finally,

$$\mathbf{F}_{recoil} = \mathbf{F}_t + (\mathbf{F}_t - (\mathbf{J}^T)^{\dagger}\tau_s) \tag{23}$$

where $\mathbf{F}_t$ is a force trajectory and $(\mathbf{J}^T)^{\dagger}\tau_s$ is the amount of force the active joints can supply. Since the joints are starting from rest, the static force condition is applicable.

Acknowledgments

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Figure A1

Initial Position
bottom shelf

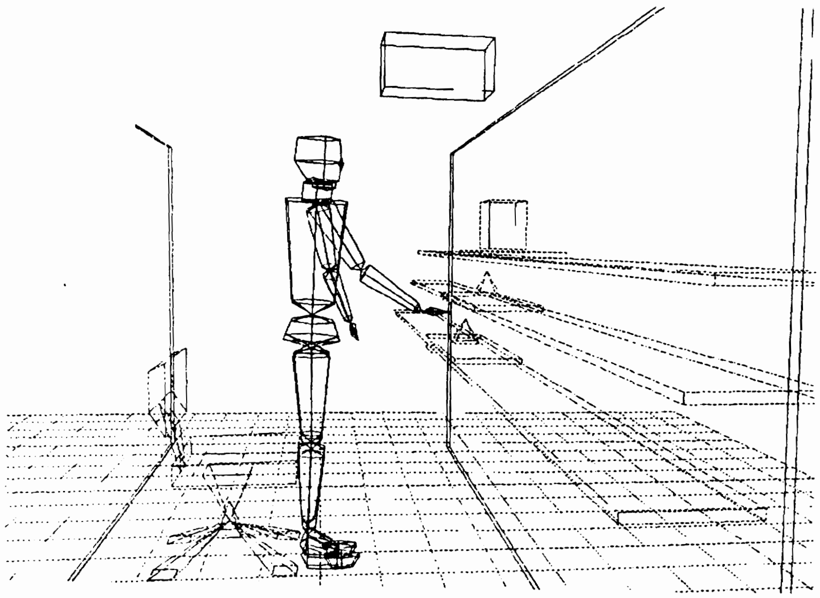

Figure A2

bottom shelf:
weight of object 10N
perceived exertion 0.3

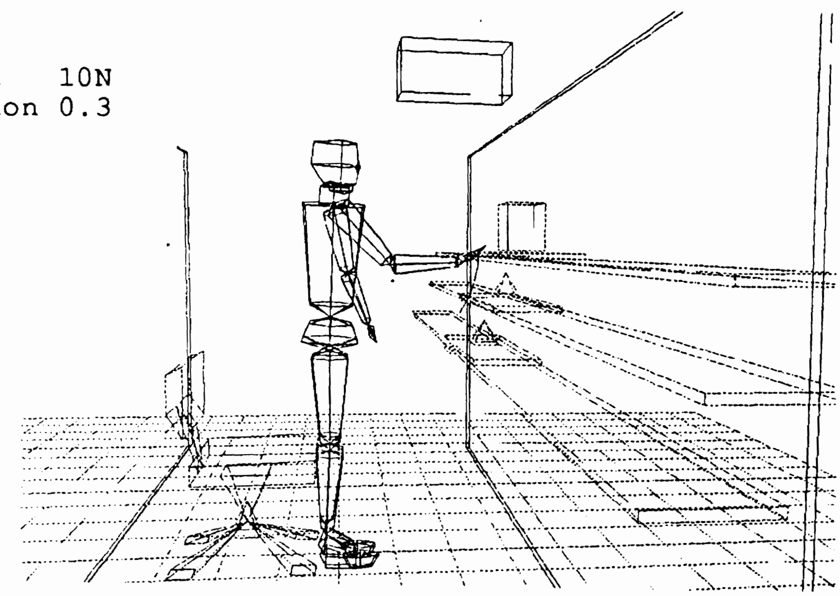

Figure A3

bottom shelf:
weight of object 10N
perceived exertion 0.3

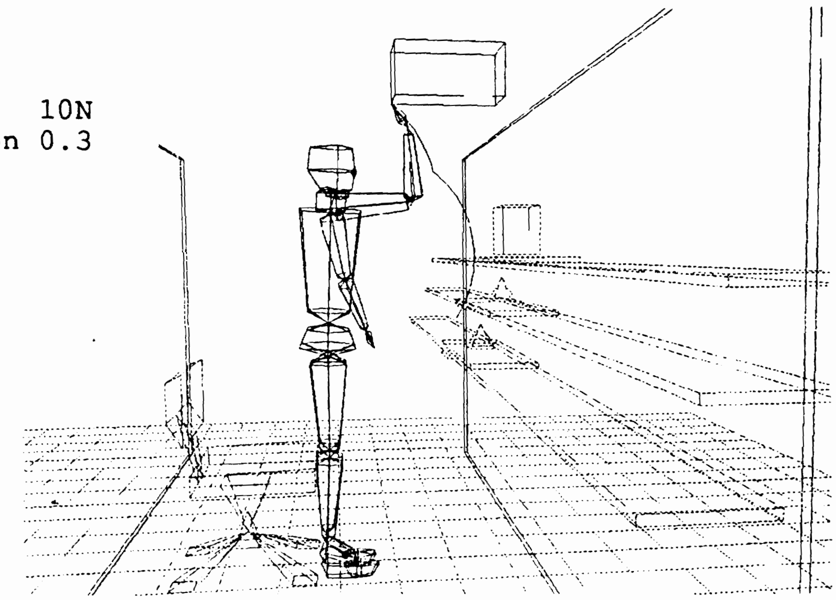

Figure A4

bottom shelf:
weight of object 20N
perceived exertion 0.3

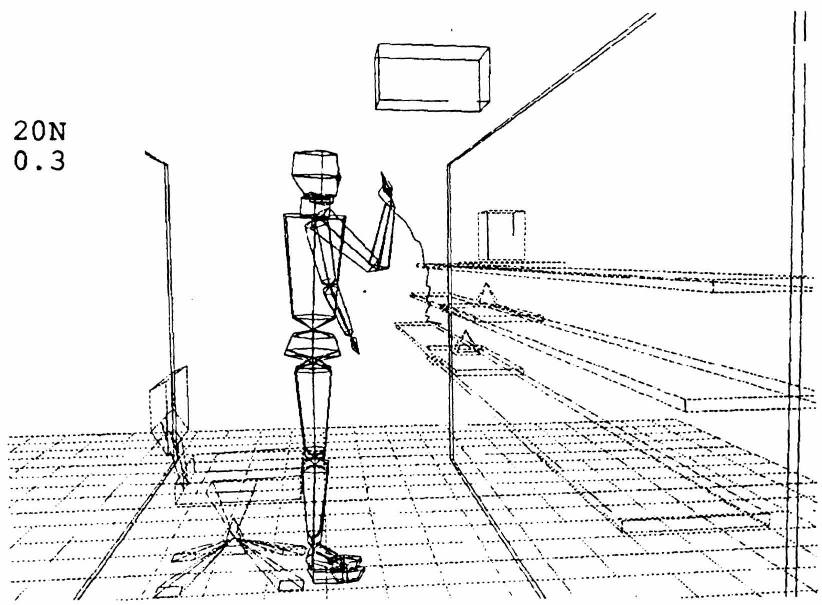

Figure A5

bottom shelf:

(left curve)
weight of object 20N
perceived exertion 0.3

(right curve)
weight of object 10N
perceived exertion 0.3

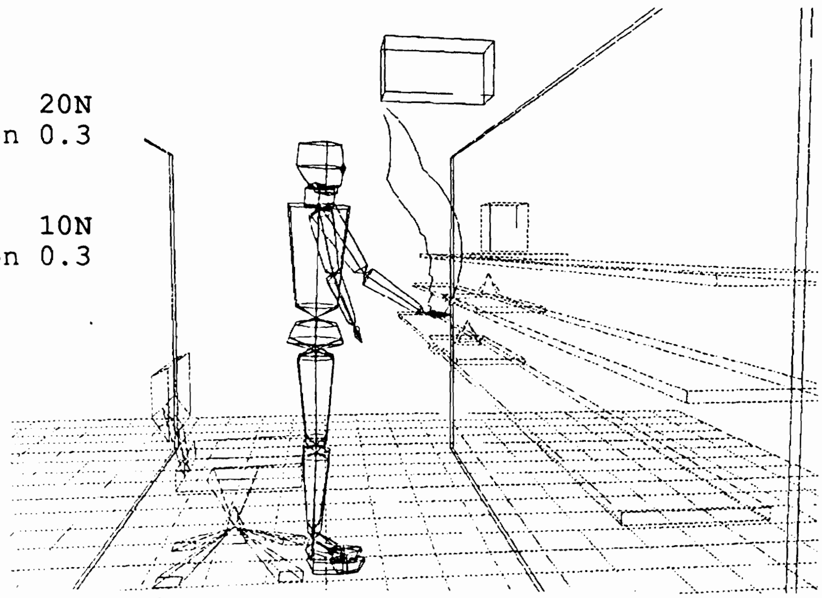

Figure A6

middle shelf:
weight of object 20N
perceived exertion 0.3

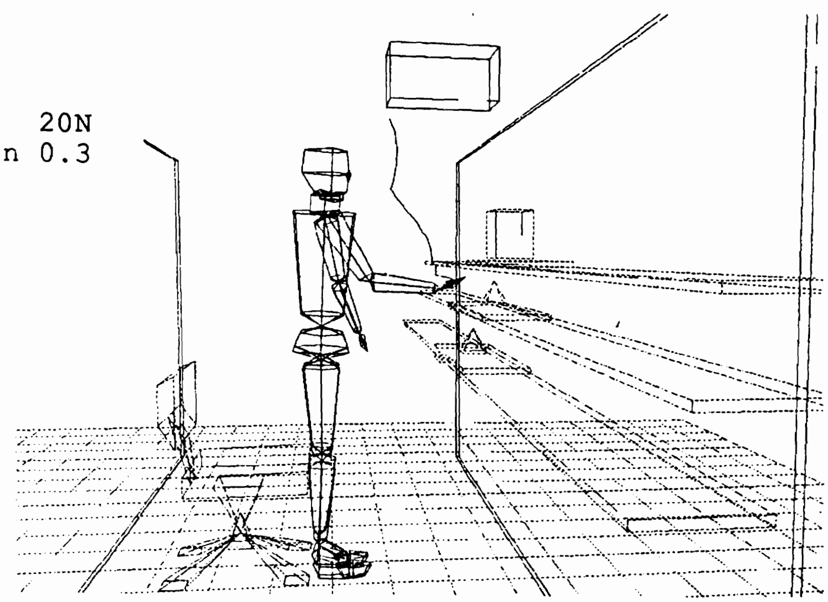

Figure A7

middle shelf:

(left curve)
weight of object 30N
perceived exertion 0.15

(right curve)
weight of object 20N
perceived exertion 0.3

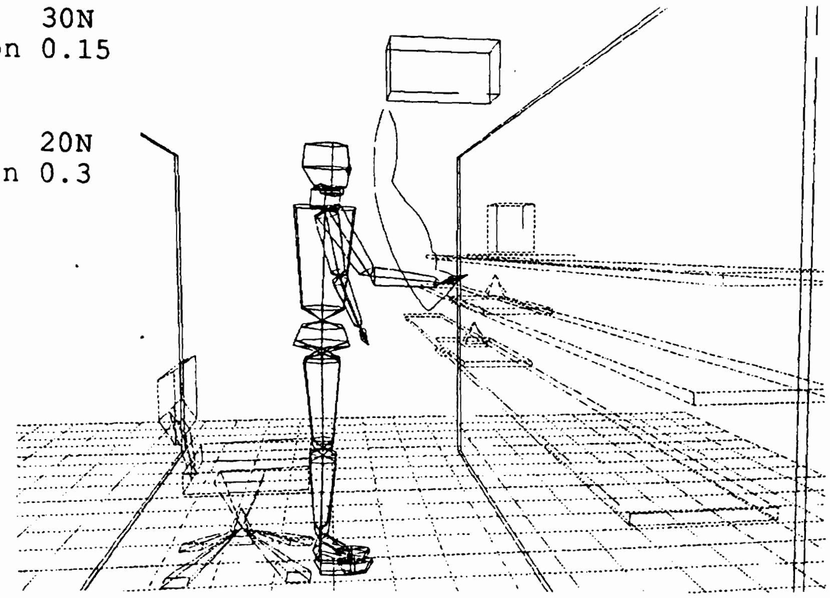

Figure A8

top shelf:

(left curve)
weight of object 45N
perceived exertion 0.15

(right curve)
weight of object 30N
perceived exertion 0.15

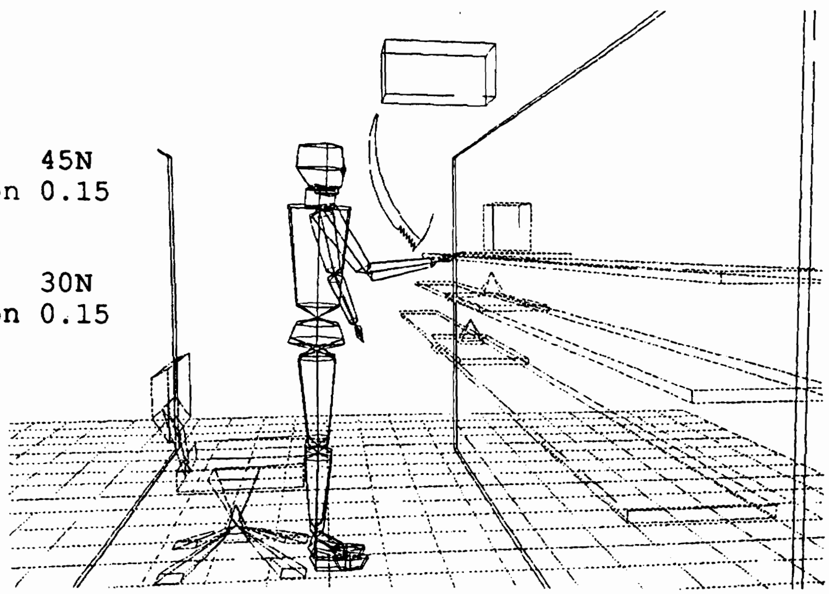

Figure A9

sitting position

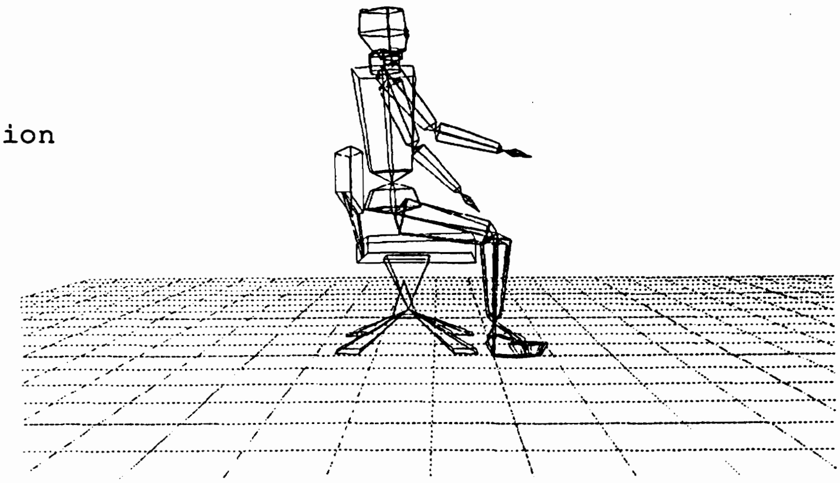

Figure A10

getting up

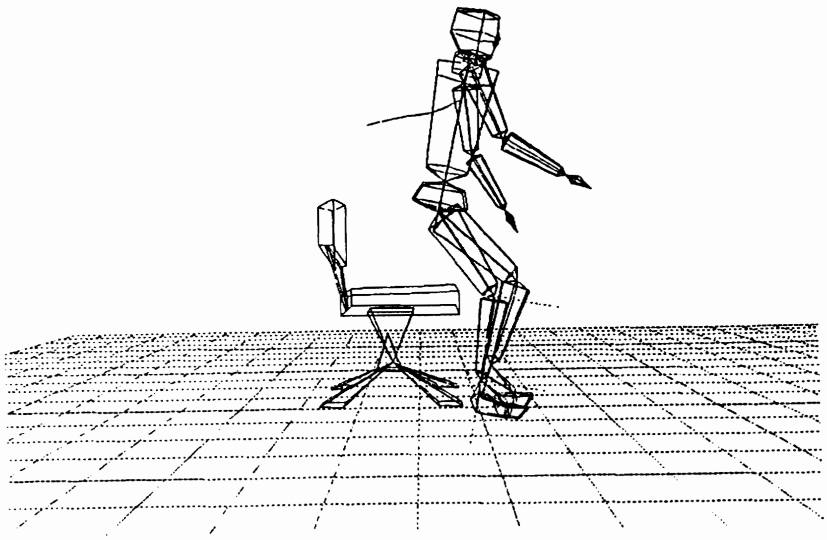

Figure A11

standing up