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Equilibrium Tuition, Applications, Admissions and Enrollment in the College Market

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Abstract

I develop and structurally estimate an equilibrium model of the college market. Students, who are heterogeneous in both abilities and preferences, make college application decisions, subject to uncertainty and application costs. Colleges observe only noisy measures of student ability and set up tuition and admissions policies to compete for more able students. The model incorporates tuition, applications, admissions and enrollment as the joint outcome from a subgame perfect Nash equilibrium. I estimate the structural parameters of the model using the NLSY 97 data, via a three-step estimation procedure to deal with potential multiple equilibria. I use the estimated model to perform three counterfactual experiments. First, I explore the impacts of incomplete information on the market. A perfect measure of student ability would lead to higher enrollee ability across colleges and a $2500 increase in average student welfare. Second, I examine the equilibrium consequences of funding cuts to public colleges. All colleges, public and private, increase their tuition, and the drop in student welfare is three times as large as government savings. Finally, I study the extent to which the government can expand college access by increasing the supply of lower-ranked colleges. At most 2.1% more students could be drawn into colleges.

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EQUILIBRIUM TUITION, APPLICATIONS, ADMISSIONS AND ENROLLMENT IN THE COLLEGE MARKET

To my parents Yongbang and Shuli.

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ABSTRACT

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Kenneth I. Wolpin

I develop and structurally estimate an equilibrium model of the college market. Students, who are heterogeneous in both abilities and preferences, make college application decisions, subject to uncertainty and application costs. Colleges observe only noisy measures of student ability and set up tuition and admissions policies to compete for more able students. The model incorporates tuition, applications, admissions and enrollment as the joint outcome from a subgame perfect Nash equilibrium. I estimate the structural parameters of the model using the NLSY 97 data, via a three-step estimation procedure to deal with potential multiple equilibria. I use the estimated model to perform three counterfactual experiments. First, I explore the impacts of incomplete information on the market. A perfect measure of student ability would lead to higher enrollee ability across colleges and a $2500 increase in average student welfare. Second, I examine the equilibrium consequences of funding cuts to public colleges. All colleges, public and private, increase their tuition, and the drop in student welfare is three times as large as government savings. Finally, I study the extent to which the government can expand college access by increasing the supply of lower-ranked colleges. At most 2.1% more students could be drawn into colleges.

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Chapter 1

Introduction

Expanding college access has been a continuing policy goal in the U.S. One way used to achieve this goal is to provide government support directly for college education. For example, Figure 1.1 shows that from 1980 to 1997, local and state governments' per capita expenditure on higher education grew by about 50%.[1] Figure 1.2 shows that concurrently, among recent high school completers, the college enrollment rate increased from 49% to 67%, and the four-year college enrollment rate, in particular, increased from below 30% to 44%.[2,3] However, from 1998 to 2006, although governments' expenditure on higher education grew even faster than in earlier years, the college enrollment rates remained almost flat. This observation raises the following question: How much further could the government expand college access simply through its support for college education?

The pursuit for this long-run goal has been interrupted by the recent government budget crisis: by September 2009, at least 34 states had implemented funding cuts

[1] Measured in 2006 dollars. Data source: U.S. Census Bureau: State and Local Government Finances.

[2] "Recent high school completers" refers to individuals ages 16 to 24 who graduated from high school or completed a GED during the preceding 12 months.

[3] Public four-year colleges accommodate over 60% of four-year college attendees. (*Digest of Education Statistics*. National Center for Education Statistics.)

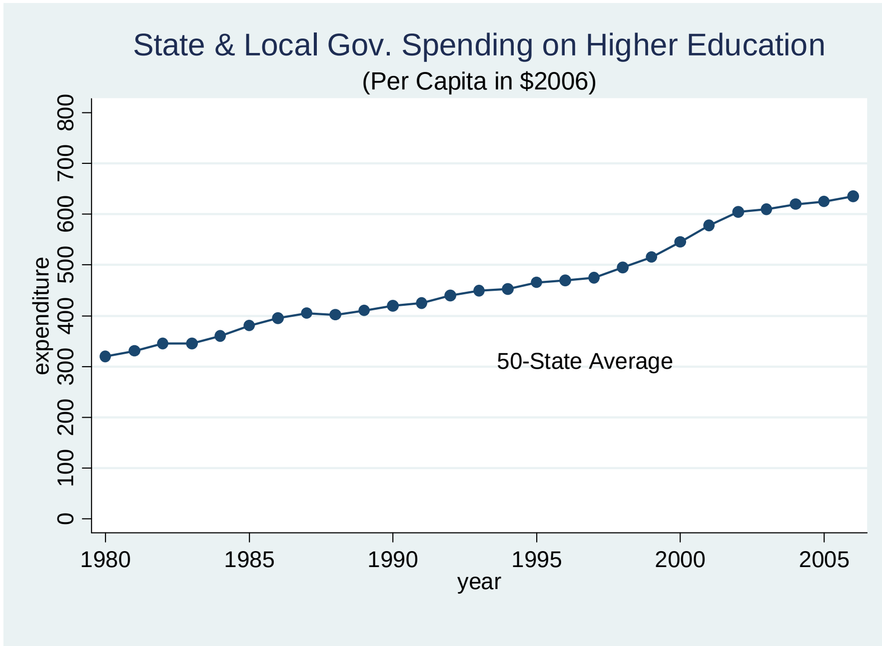

Figure 1.1: State and Local Gov. Spending on Higher Education (Per Capita in 2006 Dollar)

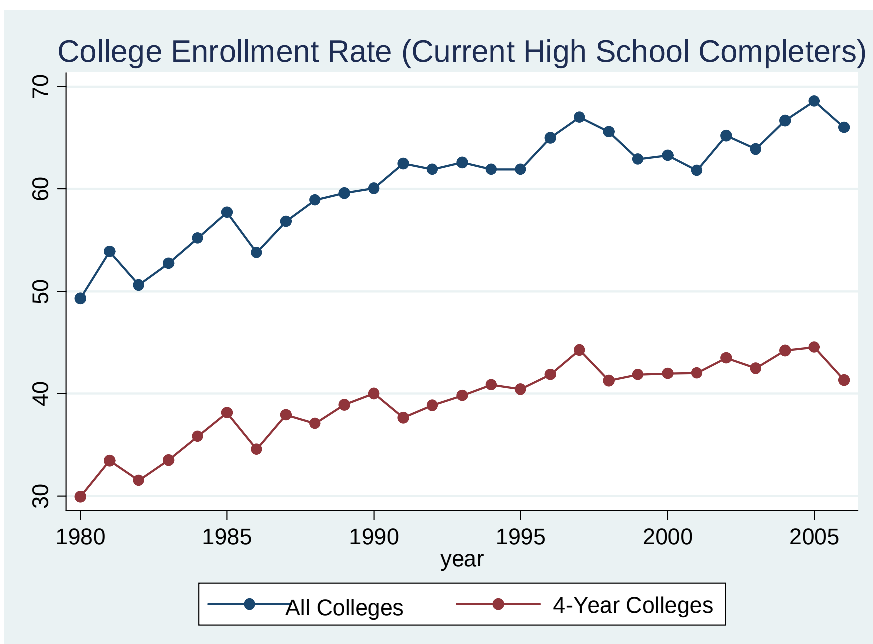

Figure 1.2: College Enrollment Rate

to public colleges and universities. For example, the University of California and California State University received 20% less state funding in 2009 than they did in 2007. How would these funding cuts affect the college market?[4] Colleges may react by changing their admissions policies and/or their tuition policies. For example, public colleges may have to increase their tuition, but by how much is unclear. Private colleges may also increase their tuition. However, not increasing tuition may enable them to attract more able students from public colleges. Which strategy is better depends on how important student ability is to colleges, relative to tuition revenue. More important, it depends on how students would respond to the changes in both the tuition policies and the admissions policies of *all* colleges in the market.

In order to predict the impacts of the recent funding cuts, as well as to assess policies aimed at achieving the long-run goal of expanding college access, we need to understand how the student and the college sides of the market interact over the process of tuition setting, applications, admissions and enrollment. This is the goal of my paper.

I develop and structurally estimate an equilibrium model of the college market. The model builds on the theoretical work of Chade, Lewis and Smith (2008), who develop an equilibrium model of college admissions process, with decentralized matching of students and two colleges.[5] Students, with heterogeneous abilities, make application decisions subject to application costs and noisy evaluations. Colleges, observing only noisy signals of student ability, compete for better students by setting admissions standards for student signals. These standards act like prices, allocating the scarce

[4] This paper focuses on the four-year college market and college refers to four-year college hereafter.

[5] Nagypál (2004) analyzes a model in which colleges know student types, but students themselves can only learn their type through normally distributed signals.

slots to qualified students.

I extend their framework in the following ways: On the student side, first, students are heterogeneous in their preferences for colleges as well as in their abilities, both unknown to the colleges. Second, I allow for two noisy measures of student ability. One measure is subjective and its assessment is known only to the college, for example, a student essay. I model this measure as a signal, following Chade, Lewis and Smith (2008). The other measure is the objective SAT score, which is known both to the student and the colleges she applies to, and may be used strategically by the student in her applications.[6] Third, in addition to the admission uncertainty caused by noisy evaluations, students are subject to post-application shocks. On the college side, I model multiple colleges (public and private), each choosing its tuition and admissions policies to maximize its payoff, which depends on expected enrollee ability and tuition revenue.

The market operates in three stages. First, colleges simultaneously announce their tuition. Second, students make their application decisions and colleges simultaneously choose their admissions policies. Finally, post-application shocks are realized and students make their enrollment decisions. The model incorporates tuition, applications, admissions and enrollment as the joint outcome from a subgame perfect Nash equilibrium (SPNE). I show that SPNE exists, but need not be unique. Multiple equilibria could arise from two sources: 1) different self-fulfilling expectations held by the student about admissions policies, and 2) the strategic interplay among colleges.

To estimate the model with potentially multiple equilibria, I extend the two-step estimation strategy proposed by Moro (2003), who estimates a statistical discrim-

[6] For example, a low-ability student with a high SAT score may apply to top colleges to which she would not otherwise apply; a high-ability student with a low SAT score may apply less aggressively than she would otherwise.

ination model in which only one side of the market is strategic. I show how the extended strategy can be used to estimate a model in which both sides of the market are strategic, and hence, the additional second source of multiple equilibria arises. I estimate the model in three steps; the first two steps recover all the fundamental parameters involved in the application-admission subgame without having to impose any equilibrium selection rule. In particular, each application-admission equilibrium can be uniquely summarized in the set of probabilities of admission to each college for different types of students. The first step treats these probabilities as parameters and estimates them along with fundamental student-side parameters in the student decision model, thereby identifying the equilibrium that generated the data. The second step recovers the college-side parameters by imposing each college's optimal admissions policy. I implement the first step via the simulated maximum likelihood estimation method and the second step via a simulated minimum distance estimation method. Step three recovers the rest of the parameters by matching colleges' optimal tuition with the data tuition.

To implement the empirical analysis, I use data from the National Longitudinal Survey of Youth 1997, which provides detailed information on student applications, admissions, financial aid and enrollment. I also use tuition information from the Integrated Postsecondary Education Data System.

I use the estimated structural model to conduct three counterfactual experiments. First, I consider the degree to which the market is affected by incomplete information. With a perfect measure of student ability, average student welfare would increase by \$2500, or 6%. Colleges obtain higher-ability students, although their admissions rates increase to almost 100%, which highlights the fact that the selectivity of a college need not reflect its quality in terms of enrollee ability.

In the second experiment, I examine the equilibrium impacts of a funding cut to public colleges. All colleges - public and private - increase their tuition. Although the government saves on educational expenses, the loss in student welfare is three times as large as government savings.

Finally, I investigate the extent to which the government can expand college access by increasing the supply of lower-ranked colleges, private or public. At most 2.1% more students can be drawn into colleges, although the enlarged colleges lower their tuition dramatically and adopt an open admissions policy. Therefore, neither the tuition cost nor the number of available slots is a major obstacle to college access. A large group of students, mainly low-ability students, prefer the outside option over any of the college options.

Although this paper is the first to estimate a market equilibrium model that incorporates college tuition setting, applications, admissions and enrollment, it builds on various studies on similar topics. For example, Manski and Wise (1983) use reduced-form approaches to study various stages of the college admissions problem separately in a partial equilibrium framework. Most relevant to my paper, they find that applicants do not necessarily prefer the highest quality school.[7] Arcidiacono (2005) develops and estimates a structural model to address the effects of college admissions and financial aid rules on future earnings. Taking admissions probabilities as exogenous, he models student's application, enrollment and choice of college major and links education decisions to future earnings.

The study by Epple, Romano and Sieg (2006) is most related to my paper to the extent that both papers build and empirically implement a market equilibrium

[7] Light and Strayer (2002), Bowen and Bok (1998) and Brewer, Eide and Goldhaber (1999) are examples of reduced-form studies that focus on the role of race in college education.

model for college education. Abstracting from college applications, Epple, Romano and Sieg (2006) focus on admissions, net tuition and enrollment, in an environment with complete information and no uncertainty.[8] Students make enrollment decision, and colleges choose financial aid and admissions policies to maximize the quality of education provided to their students. They base their empirical analysis on a sample of incoming freshmen and detailed college-level data. By comparison, I build a college market model with incomplete information and uncertainty. I model application decisions as well as (gross) tuition policies, admissions policies and enrollment decisions, taking financial aid policies as given. I estimate my model on a sample of potential college applicants and college-level tuition data. The two papers complement each other in understanding the college market. Epple, Romano and Sieg (2006) provide a more comprehensive view of colleges' pricing strategies. My model incorporates students' application behavior and therefore contributes to the assessment of policies that would affect application decisions.

The rest of the paper is organized as follows: Chapter 2 lays out the model, defines the equilibrium and proves existence. Chapter 3 explains the estimation strategy, followed by a brief discussion of identification. Chapter 4 describes the data. Chapter 5 presents empirical results, including parameter estimates and model fit. Chapter 6 conducts the counterfactual experiments. The last Chapter concludes the paper. The appendix contains some details and additional tables.

[8] In their paper, (1) it is implicitly assumed that either application is not necessary for admission, or all students apply to all colleges. (2) Student ability is equivalent to their SAT scores, and hence is observed by all economic agents as well as the econometrician.

Chapter 2

Model

2.1 Primitives

2.1.1 Players

There are J colleges, indexed by $j = 1, 2, ...J$. In the following, J will also denote the set of colleges. A college's payoff depends on the total expected ability of its enrollees and its tuition revenue. To maximize its payoff, each college has the latitude to choose its tuition and admissions policies, subject to its fixed capacity constraint κ_j, where $\kappa_j > 0$ and $\sum\limits_{j \in J} \kappa_j < 1$.

There is a continuum of students, making college application and enrollment decisions. Students differ in their SAT scores and family backgrounds, and they are of different types. In addition, each student also has her own idiosyncratic (permanent) tastes for colleges. In particular, SAT scores ($SAT \in \{1, 2, ...\overline{SAT}\}$) and family backgrounds (B) are jointly distributed according to $H(SAT, B)$.[1] A student type T is defined as $T \equiv (A, Z) \in \{1, 2, ..., \overline{A}\} \times \{1, 2, ..., \overline{Z}\}$, with A denoting ability, and Z representing the non-ability dimension. Type T is correlated with (SAT, B) and distributed according to $P(T|SAT, B) = \Lambda(A|SAT, B)P(Z|A)$, where $\Lambda(A|SAT, B)$

[1] The distribution of $H(SAT, B)$ is nonparametric and comes directly from the data.

is an ordered logistic distribution and $P(Z|A)$ is non-parametric.[2] Student's idiosyncratic (permanent) tastes for colleges are captured by a J-dimensional random vector ϵ. ϵ is i.i.d. $N(0, \Omega_\epsilon)$, where Ω_ϵ is restricted to be a diagonal matrix with $\sigma^2_{\epsilon_j}$ denoting the variance of ϵ_j.

2.1.2 Application Cost

Applications are costly to the student. The cost of application is restricted to be a function of only the number of applications sent, regardless of where they are sent. The cost function, denoted as $C(\cdot)$, satisfies the following: $C : \{1, ..., J\} \rightarrow \mathbb{R}_{++}$, with $C(n+1) \geq C(n)$. I treat $C(\cdot)$ non-parametrically.

2.1.3 Financial Aid

A student may obtain financial aid that helps to fund her attendance in any college, and she may also obtain college-specific financial aid. The amounts of various financial aid depend on the student's family background and SAT, via the exogenous financial aid functions $f_j(B, SAT)$, for $j = 0, 1...J$, with 0 denoting the general aid and j denoting college j-specific aid.[3] The final realizations are subject to post-application shocks $\eta \in \mathbb{R}^{J+1}$. I assume that η is i.i.d. $N(0, \Omega_\eta)$, where Ω_η is a diagonal matrix with $\sigma^2_{\eta_j}$ denoting the variance of shock η_j. The realized financial aid for student i is given by

$$f_{ji} = \max\{f_j(B_i, SAT_i) + \eta_{ji}, 0\} \text{ for } j = 0, 1, ..J.$$

[2] In implementation, only one element of B, family income, enters $P(A|SAT, B)$.

[3] See the appendix for functional forms of financial aid.

2.1.4 Student Preference

Given tuition profile $t \equiv \{t_j\}_{j=1}^{J}$, the ex-post value of attending college j for student i is given by

$$u_{ji}(t) = (-t_j + f_{0i} + f_{ji})(1 + \delta + \delta^2 + \delta^3) + \overline{u}_{jT_i} + \epsilon_{ji}, \tag{2.1}$$

where t_j is tuition for attending college j, f_{0i} and f_{ji} refer to the realized general aid and college j-specific aid for student i, and δ is the discount factor. The first term of (2.1) summarizes student i's net monetary cost of attending college j for four years. The non-pecuniary value of attending college j for student i is captured by: $\overline{u}_{jT_i}$, the average utility from attending college j among type T_i students, and by ϵ_{ji}, i's idiosyncratic taste for college j.[4]

An outside option is always available to the student and its net expected value is normalized to zero. After application, the outside option is subject to a random shock ζ, which is i.i.d. $N(0, \sigma_\zeta^2)$, and the ex-post value of the outside option is $u_{0i} = \zeta_i$.

2.1.5 College Payoff

The payoff for college $j \in J$ is given by:

$$\pi_j = \sum_{a=1}^{\overline{A}} \omega_a n_{ja} + (m_{j1} t_j + m_{j2} t_j^2) \times \sum_{a=1}^{\overline{A}} n_{ja}, \tag{2.2}$$

where ω_a is the value of ability $A = a$, with $\omega_{a+1} > \omega_a$ and ω_1 normalized to 1. n_{ja} is the measure of j's enrollees with $A = a$. The first term in (2.2) is college j's total enrollee ability. The second term in (2.2) is college j's payoff from its tuition revenue, where m_j is college j's valuation of tuition relative to that of enrollee ability. In particular, if college j uses tuition only as a tool to maximize enrollee ability, m_j would be 0.

[4] $\overline{u}_{jT}$'s are treated non-parametrically.

2.1.6 Timing

First, colleges simultaneously announce their tuition levels, to which they commit. In the second stage, students make their application decisions, and all colleges simultaneously choose their admissions policies. Finally, students learn about admission results and post-application shocks to the outside option and financial aid, and then make their enrollment decisions.[5]

2.1.7 Information Structure

Upon student i's application, each college she applies to receives a signal $s \in \{1, 2, ..\overline{s}\}$ drawn from the distribution $P(s|A_i)$, the realization of which is known only to the college. I assume that for $A < A'$, the distribution $P(s|A')$ first order stochastically dominates $P(s|A)$.[6] Conditional on the student's ability, signals are i.i.d. across the colleges she applies to.

$P(s|A)$, the distributions of characteristics, preferences, payoff functions and financial aid functions are public information. Individual student's SAT score is known both to her and the colleges she applies to. Throughout all stages, a student has private information about her type T, her idiosyncratic taste ϵ and her family background B. To ease notation, let $X \equiv (T, B, \epsilon)$. After application, the student observes her post-application shocks. For any individual applicant, college j has information only about her SAT and the signal she sends to j. In particular, it does not observe whether the student also applies to other colleges.

[5] This paper excludes early admissions, which accounts for only a small fraction of the total applications. For example, in 2003, 17.7% of all four-year colleges offered early decision. In these colleges, the mean percentage of all applications received through early decision was 7.6%. Data source: *Admission Trends Survey* (2004), National Association for College Admission Counseling.

[6] That is, if $A < A'$, then for any $s \in \{1, 2, ..\overline{s}\}$, $\Pr(s' \leq s|A) \geq \Pr(s' \leq s|A')$.

In the following, I will first specify and solve the student problem and the college problem for a given tuition profile and define an application-admission equilibrium. Then, I will specify the first stage tuition game and define the equilibrium of the whole game.

2.2 Applications, Admissions and Enrollment

2.2.1 Enrollment Decision

Knowing her post-application shocks and admission results, student i chooses the best among the outside option and admissions on hand, i.e., $\max\{u_{0i,}\{u_{ji}(t)\}_{j\in O_i}\}$, where O_i denotes the set of colleges that have admitted student i. For students not admitted anywhere, $\max\{u_{ji}(t)\}_{j\in\emptyset} = -\infty$. Let

$$v(O_i, X_i, SAT_i, \eta_i, \zeta_i|t) \equiv \max\{u_{0i,}\{u_{ji}(t)\}_{j\in O_i}\} \tag{2.3}$$

be the optimal ex-post value for student i, given admission set O_i. Denote the associated optimal enrollment strategy $d(O_i, X_i, SAT_i, \eta_i, \zeta_i|t)$.

2.2.2 Application Decision

Given her admissions probability $p_j(A_i, SAT_i|t)$ to each college j, which depends on her ability and SAT, the value of application portfolio $Y \subseteq J$ for student i is given by:

$$V(Y, X_i, SAT_i|t) \equiv \sum_{O\subseteq Y} \Pr(O|A_{i,}SAT_i, t)E_{(\eta,\zeta)}[v(O, X_i, SAT_i|t)] - C(|Y|), \tag{2.4}$$

where $|Y|$ is the size of portfolio Y, and

$$\Pr(O|A_{i,}SAT_i, t) = \prod_{j\in O} p_j(A_i, SAT_i|t) \prod_{k\in Y\setminus O} (1 - p_k(A_i, SAT_i|t))$$

is the probability that the subset of colleges $O \subseteq Y$ admit student i. The student's application problem is

$$\max_{Y \subseteq J} \{V(Y, X_i, SAT_i | t)\}. \tag{2.5}$$

Let the optimal application strategy be $Y(X_i, SAT_i|t)$.

2.2.3 Admissions Policy

Given tuition, a college chooses its admissions policy in order to maximize the total expected ability of its enrollees, subject to its capacity constraint.[7] Its optimal admissions policy must be a best response to other colleges' admissions policies while accounting for students' strategic behavior. In particular, observing only signals and SAT scores of its applicants, the college has to infer: first, the probability that a certain applicant would accept its admission, and second, the expected ability of this applicant conditional on her accepting the admission, both of which depend on the strategies of all the other players. For example, the conditional expected applicant ability need not be monotone in SAT because students know their SAT and could use this information strategically in their applications.

Formally, given tuition profile t, students' strategies $Y(\cdot), d(\cdot)$ and other colleges'

[7] It is implicitly assumed that the tuition weights m are such that $\gamma_j(s, SAT|t, e_{-j}, Y, d) + m_{j1}t_j + m_{j2}t_j^2 \geq 0$ for any (s, SAT) under reasonable tuition levels. Otherwise, the college may rather be under capacity than admit certain (s, SAT) students. I estimate m without imposing this restriction, but the estimated $\widehat{m}$ (and a wide range of m around $\widehat{m}$) satisfies this restriction. For example, even if $\gamma_j(s, SAT|t, e_{-j}, Y, d) = 1$ (the lowest ability value), in order to get $1 + \widehat{m}_{j1}t_j + \widehat{m}_{j2}t_j^2 < 0$ with $t_j \geq 0$, t_j needs to be higher than \$64,895 for private colleges, and higher than \$46,290 for public colleges, which are far beyond the tuition levels in the data.

admissions policies e_{-j}, college j solves the following problem:

$$\max_{e_j(\cdot|t)} \sum_{s,SAT} e_j(s,SAT|t)\alpha_j(s,SAT|t,e_{-j},Y,d)\gamma_j(s,SAT|t,e_{-j},Y,d)\mu_j(s,SAT|\cdot)$$

$$s.t. \sum_{s,SAT} e_j(s,SAT|t)\alpha_j(s,SAT|t,e_{-j},Y,d)\mu_j(s,SAT|t,e_{-j},Y,d) \leq \kappa_j \qquad (2.6)$$

$$e_j(s,SAT|t) \in [0,1],$$

where $e_j(s,SAT|t)$ is college j's admissions policy for its applicants with (s,SAT), $\alpha_j(s,SAT|t,e_{-j},Y,d)$ is the probability that such an applicant would accept college j's admission, $\gamma_j(s,SAT|t,e_{-j},Y,d)$ is the expected ability of such an applicant conditional on her accepting j's admission, and $\mu_j(s,SAT|t,e_{-j},Y,d)$ is the measure of j's applicants with (s,SAT). The first order condition for problem (2.6) is

$$\gamma_j(s,SAT|t,e_{-j},Y,d) - \lambda_j + \nu_0 - \nu_1 = 0, \qquad (2.7)$$

where λ_j is the multiplier associated with capacity constraint, i.e., the shadow price of a slot in college j. ν_0 and ν_1 are adjusted multipliers associated with the constraint that $e_j(s,SAT) \in [0,1]$.[8]

If it admits an applicant with (s,SAT) and the applicant accepts the admission, college j has to give up a slot from its limited capacity, which induces the marginal cost λ_j. The marginal benefit, on the other hand, is the expected ability of such an applicant conditional on her accepting j's admission. Conditioning on acceptance is necessary for correct inference about the student's ability because of the potential "winner's curse": the student might accept college j's admission because she is of low ability and is rejected by other colleges. Balancing between the marginal benefit and

[8] ν_0, ν_1 are the multiplier associated with $\alpha_j(s,SAT|t,e_{-j},Y,d)\mu_j(s,SAT|t,e_{-j},Y,d)e_j(s,SAT) \in [0,1]$.

the marginal cost, the solution to college j's admissions problem is characterized by:

$$e_j(s,SAT|t)\begin{cases} =1 & \text{if } \gamma_j(s,SAT|t,e_{-j},Y,d)>\lambda_j \\ =0 & \text{if } \gamma_j(s,SAT|t,e_{-j},Y,d)<\lambda_j \\ \in[0,1] & \text{if } \gamma_j(s,SAT|t,e_{-j},Y,d)=\lambda_j \end{cases}, \tag{2.8}$$

$$\sum_{s,SAT} e_j(s,SAT|t)\alpha_j(s,SAT|t,e_{-j},Y,d)\mu_j(s,SAT|t,e_{-j},Y,d)\leq\kappa_j, \tag{2.9}$$

and

$$\lambda_j\begin{cases} \geq 0 \text{ if } (2.9) \text{ is binding} \\ =0 \text{ if } (2.9) \text{ is not binding} \end{cases}.$$

Let the $W_j(t,e_{-j},Y,d)$ denote the maximized value of (2.6) for college j.

To implement its admissions policy, college j will rank its applicants with different (s,SAT) by their $\gamma_j(s,SAT|t,e_{-j},Y,d)$, then admit from the top group to the bottom group until its capacity is filled in expectation.[9] All applicants with the same (s,SAT) are identical to the college and hence are treated equally. In particular, everyone in an (s,SAT) group will be admitted if two conditions hold: 1) this (s,SAT) group is ranked highest among the groups whose admissions are still to be decided, and 2) the expected enrollment of this group, $\alpha_j(s,SAT|t,e_{-j},Y,d)\mu_j(s,SAT|t,e_{-j},Y,d)$, is no larger than college j's remaining capacity, where j's remaining capacity equals κ_j minus the sum of expected enrollment of groups ranked above, i.e., groups that college j has already decided to admit. A random fraction of an (s,SAT) group is admitted if condition 1) holds but 2) does not, where the fraction equals the remaining capacity divided by the expected enrollment of this group. As a result, a typical set of admissions policies for the ranked (s,SAT) groups would be $\{1,...,1,q,0,...,0\}$, with $q\in(0,1)$ if the capacity constraint is binding, and $\{1,...,1\}$ if the capacity constraint is not binding or just binding. Appendix A.1 provides details on how to calculate $\alpha_j(\cdot)$ and $\gamma_j(\cdot)$.

[9] However, colleges sent out their admission decisions for *all* applicants simultaneously.

2.2.4 Link Among Various Players

The probability of admission to each college for different (A, SAT) groups of students, $\{p_j(A, SAT|t)\}$, summarizes the link among various players. The knowledge of p makes the information about admissions policies $\{e_j(s, SAT|t)\}$ redundant. In particular, students' application decisions are based on p, and college j can make inferences about $\alpha_j(\cdot)$ and $\gamma_j(\cdot)$, and therefore choose its admissions policy, based on p_{-j}. The relationship between p and e is given by:

$$p_j(A, SAT|t) = \sum_s P(s|A)e_j(s, SAT|t). \tag{2.10}$$

The role of p as the link among players and the mapping (2.10) are of great importance in the estimation strategy to be specified later.

2.2.5 Application-Admission Equilibrium

Definition 2.1. *Given tuition profile t, an application-admission equilibrium, denoted as $AE(t)$, is*

$(d(\cdot|t), Y(\cdot|t), e(\cdot|t))$*, such that*

(a) $d(O, X, SAT, \eta, \zeta|t)$ is an optimal enrollment decision for every (O, X, SAT, η, ζ);

(b) Given $e(\cdot|t)$, $Y(X, SAT|t)$ is an optimal college application portfolio for every (X, SAT), i.e., solves problem (2.5) *;*

(c) For every j, given $(d(\cdot|t), Y(\cdot|t), e_{-j}(\cdot|t))$, $e_j(\cdot|t)$ is optimal admissions policy for college j, i.e., solves problem (2.6) *.*

Proposition 2.1. *For any given tuition profile t, an application-admission equilibrium exists.*

Proof. (see the appendix). $\square$

2.3 Tuition Policy

Before the application season begins, colleges simultaneously announce their tuition levels, understanding that their announcements are binding and would affect the following application-admission subgame. Although the subsequent game could admit multiple equilibria from the econometrician's point of view, I assume that the players agree on the equilibrium selection rule and only this selected set of equilibrium profiles $AE(\cdot)$ would be considered when colleges choose tuition policies.[10] Given t_{-j} and the equilibrium profiles $AE(\cdot)$ in the following subgame, college j's problem is to maximize $E(\pi_j)$, which is equivalent to

$$\max_{t'_j}\{W_j(t'_j, t_{-j}, e_{-j}, Y, d) \tag{2.11}$$

$$+\sum_{s,SAT} e_j(s, SAT|t'_j, t_{-j})\alpha_j(s, SAT|t'_j, t_{-j}, e_{-j}, Y, d)$$

$$\mu_j(s, SAT|t'_j, t_{-j}, e_{-j}, Y, d)(m_{j1}t'_j + m_{j2}t'^2_j)\}. \tag{2.12}$$

The first term in (2.11) is college j's payoff from the total expected ability of its enrollees resulting from $AE(t'_j, t_{-j})$. The second term is college j's monetary payoff from its expected tuition revenue, where the expected enrollment, $\sum_{s,SAT} e_j(\cdot|\cdot)\alpha_j(\cdot|\cdot)\mu_j(\cdot|\cdot)$, also results from $AE(t'_j, t_{-j})$.

Independent of m, the college has to consider the strategic role of its tuition in the subsequent $AE(t'_j, t_{-j})$. On the one hand, low tuition makes the college more attractive to students and more competitive in the market. On the other hand, high tuition serves as a screening tool and leads to a better pool of applicants if high-ability students are less sensitive to tuition than low-ability students. Together with the monetary incentives for tuition revenue, such trade-offs determine the optimal

[10] The way in which the equilibrium selection rule is reached is beyond the scope of this paper. But as an example, it may result from repeated interactions between players.

tuition level for the college.

2.4 Subgame Perfect Nash Equilibrium for the College Market

Definition 2.2. *A subgame perfect Nash equilibrium for the college market is* $(t^*, d(\cdot|\cdot), Y(\cdot|\cdot), e(\cdot|\cdot))$ *such that:*

(a) *For every* t, $(d(\cdot|t), Y(\cdot|t), e(\cdot|t))$ *constitutes an* $AE(t)$, *according to Definition 1*;

(b) *For every* j, *given* t^*_{-j}, t^*_j *is optimal tuition for college* j, *i.e., solves problem* (2.11) .

Proposition 2.2. *Under usual regularity conditions, a subgame perfect Nash equilibrium exists for the college market.*

Proof. (see appendix). □

Chapter 3

Estimation Strategy and Identification

3.1 Estimating Application-Admission Subgame

First, I fix the tuition profile at its equilibrium (data) level and estimate the parameters that govern the application-admission subgame. To save notation, I suppress the dependence of endogenous objects on tuition.

The estimation is complicated by the fact that the application-admission subgame may have multiple equilibria and econometricians do not observe the equilibrium selection rule used by the players.[1] One way to deal with this complication is to impose some equilibrium selection rule assumed to have been used by the players and consider only the selected equilibrium. However, for models like the one in this paper, there is not a single compelling selection rule.[2] I use a two-step strategy to estimate the

[1] Models with multiple equilibria do not have a unique reduced form and this indeterminacy poses practical estimation problems. An important issue associated with maximum likelihood estimation of such models is that one should maximize the likelihood not only with respect to the structural parameters but also with respect to the types of equilibria that may have generated the data. The latter can be a very complicated task: first, computing all the equilibria associated with each trial value of the parameters can be computationally very demanding; and second, the number of possible combinations of equilibria increases exponentially with sample size.

[2] See, for example, Mailath, Okuno-Fujiwara and Postlewaite (1993).

application-admission subgame without having to impose any equilibrium selection rule.

Each application-admission equilibrium is uniquely summarized in the admissions probabilities $\{p_j(A, SAT)\}$, which provide sufficient information for players to make their unique optimal decisions. In the student decision model, these admissions probabilities are taken as given. Step one treats the equilibrium admissions probabilities p as parameters and estimates them along with structural student-side parameters, thereby identifying the equilibrium that generated the data. Step two imposes each college's optimal admissions policy. The resulting admissions policies $\{e_j(s, SAT)\}$ yield a new set of admissions probabilities, which, under the true college-side parameters, should match the reduced-form equilibrium admissions probabilities estimated in the first step.

3.1.1 Step One: Estimate Fundamental Student-Side Parameters and Equilibrium Admissions Probabilities

I use the method of simulated maximum likelihood estimation (SMLE) in the first step: together with estimates of the fundamental student-side parameters $\left(\widehat{\Theta}_0\right)$, the estimated equilibrium admissions probabilities $\widehat{p}$ should maximize the probability of the observed outcomes of applications, admissions, financial aid and enrollment conditional on observable student characteristics, i.e., $\{(Y_i, O_i, f_i, d_i | SAT_i, B_i)\}_i$. Θ_0 is composed of 1) type-specific preference parameters and idiosyncratic taste distribution parameters $\Theta_{0u} = [\{\overline{u}_j(T)\}, \{\sigma_{\epsilon_j}\}]'$, 2) application cost parameters $\Theta_{0C} = \{C(n)\}'$, 3) financial aid parameters Θ_{0f}, 4) the standard deviation of the shock to the outside option $\Theta_{0\zeta} = \sigma_\zeta$ and 5) the parameters involved in the distribution of types Θ_{0T}.

Suppose student i is of type T. Her contribution to the likelihood, denoted by $L_{iT}(\Theta_{0u}, \Theta_{0C}, \Theta_{0f}, \Theta_{0\zeta}, p)$, is composed of the following parts:

$L_{iT}^{Y}(\Theta_{0u}, \Theta_{0C}, \Theta_{0f}, \Theta_{0\zeta}, p)-$ the contribution of Y_i,

$L_{iT}^{O}(p)-$ the contribution of $O_i|Y_i$,

$L_{iT}^{f}(\Theta_{0f})-$ the contribution of $f_i|O_i$, and

$L_{iT}^{d}(\Theta_{0u}, \Theta_{0f}, \Theta_{0\zeta})-$ the contribution of $d_i|(O_i, f_i)$.

Hence,

$$L_{iT}(\cdot) = L_{iT}^{Y}(\cdot)L_{iT}^{O}(\cdot)L_{iT}^{f}(\cdot)L_{iT}^{d}(\cdot).$$

Now, I will specify each part in detail. Conditional on (T, SAT_i, B_i), there are no unobservables involved in the probabilities of $O_i|Y_i$ and $f_i|O_i$. The probability of $O_i|Y_i$ depends only on ability and SAT, and is given by

$$L_{iT}^{O}(p) \equiv \Pr(O_i|Y_i, A, SAT_i) = \prod_{j \in O_i} p_j(A, SAT_i) \prod_{k \in Y_i \setminus O_i} [1 - p_k(A, SAT_i)].$$

Let $J_i^f \subseteq \{0, O_i\}$ be the sources of observed financial aid for student i, where 0 denotes general aid.[3] The probability of the observed financial aid depends only on SAT and family background:

$$\begin{aligned} L_{iT}^{f}(\Theta_{0f}) &\equiv \Pr(f_i|O_i, SAT_i, B_i) \\ &= \begin{cases} \prod\limits_{j \in J_i^f} \phi(\frac{f_{ji} - f_j(SAT_i, B_i)}{\sigma_{\eta_j}})^{I(f_{ji}>0)} \Phi(\frac{-f_j(SAT_i, B_i)}{\sigma_{\eta_j}})^{I(f_{ji}=0)} & \text{if } J_i^f \neq \emptyset \\ 1 & \text{otherwise} \end{cases}, \end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal density and cumulative distribution, respectively, and $I(\cdot)$ is the indicator function. The choices of Y_i and $d_i|(O_i, f_i)$ both depend on the unobserved idiosyncratic tastes ϵ, therefore the two probabilities have to be calculated jointly. Let $G(\epsilon, \zeta, \{\eta_j\}_{j \in \{0, O_i\} \setminus J_i^f})$ be the joint distribution of

[3]By definition, a student can get admitted only to the colleges she applies to and can obtain financial aid from a college only if she is admitted to the college. That is, $O_i \subseteq Y_i$ and $J_i^f \subseteq \{0, O_i\}$.

idiosyncratic taste, outside option shock and unobserved financial aid shocks,

$$\begin{aligned}&L_{iT}^{Y}(\Theta_{0u},\Theta_{0C},\Theta_{0f},\Theta_{0\zeta},p)L_{iT}^{d}(\Theta_{0u},\Theta_{0f},\Theta_{0\zeta})\equiv\\&\int I(Y_i|T,SAT_i,B_i,\epsilon)I(d_i|O_i,T,SAT_i,B_i,\epsilon,\zeta,\{\eta_j\}_{j\in\{0,O_i\}\backslash J_i^f},\{f_{ji}\}_{j\in J_i^f})\\&dG(\epsilon,\zeta,\{\eta_j\}_{j\in\{0,O_i\}\backslash J_i^f}),\end{aligned}$$

where the indicator function $I(\cdot)$ is used because the two conditional student decisions are unique.[4] The multi-dimensional integration has no closed-form solution and is approximated by a kernel smoothed frequency simulator.[5] This involves assigning to each student, characterized by (T, SAT_i, B_i), R sets of random draws of taste and ex-post shocks $\{\left(\epsilon,\zeta,\{\eta_j\}_{j\in\{0,O_i\}\backslash J_i^f}\right)_r\}_{r=1}^{R}$, solving the optimization problem for each of these R cases, and then integrating over these R cases.[6]

To obtain the likelihood contribution of a student as observed in the data, I integrate over the unobserved type T:

$$L_i(\Theta_0,p)=\sum_T P(T|SAT_i,B_i;\Theta_{0T})L_{iT}(\Theta_{0u},\Theta_{0C},\Theta_{0f},\Theta_{0\zeta},p). \tag{3.1}$$

Finally, the log likelihood for the whole random sample is

$$L(\Theta_0,p)=\sum_i \ln(L_i(\Theta_0,p)). \tag{3.2}$$

3.1.2 Step Two: Estimate College-Side Parameters

The college-side parameters to be estimated at step two, denoted Θ_2, are the distributional parameters of signals conditional on ability $P(s|A)$, the capacity constraint

[4]When the realization of financial aid is observed by the econometrician, there is no need to simulate ex-post shocks for it. However, simulation is needed in cases where information on financial aid is missing.

[5]See McFadden (1989) for the properties of such simulators.

[6]See Appendix D.1 for details.

parameters κ and the value of different abilities ω. They are estimated via simulated minimum distance estimation (SMDE). Based on $\widehat{\Theta}_0$, I simulate a population of students and obtain their optimal application and enrollment strategies under $\widehat{p}$, which also yields the equilibrium enrollment in each college group. Under the assumption that the expected capacity constraint is binding and that the law of large numbers applies, the total enrollment in j, as a fraction of the student population, should equal j's expected capacity.[7] These equilibrium capacities, together with the estimated equilibrium admissions probabilities $\widehat{p}$, serve as the targets to be matched in the second-step estimation.

The estimation explores each college's optimal admissions policy: taking student strategies and $\widehat{p}_{-j}$ as given, college j chooses its admissions policy $\{e_j(s, SAT)\}_{s,SAT}$. This leads to the admissions probability to college j for each (A, SAT) type, according to equation (2.10). Ideally, the admissions probabilities derived from step two should match the equilibrium admissions probabilities from step one, and the capacity parameters in step two should match the equilibrium capacities. The estimates of the college-side parameters minimize the weighted sum of the discrepancies. Let $\widehat{\Theta}_1 = [\widehat{\Theta}_0', \widehat{p}']'$; the objective function in the second step is

$$\min_{\Theta_2}\{q(\widehat{\Theta}_1, \Theta_2)'\widehat{W}q(\widehat{\Theta}_1, \Theta_2)\}, \tag{3.3}$$

where $q(\cdot)$ is the vector of the discrepancies mentioned above, and $\widehat{W}$ is an estimate of the optimal weighting matrix. The choice of W takes into account that $q(\cdot)$ is a function of $\widehat{\Theta}_1$, which are point estimates with variances and covariances. Appendix D.2 gives more details about the second-step estimation.

[7] The assumption of a binding capacity constraint is justified by the fact that the admission rate is lower than 100% for any college group in the data.

3.2 Tuition Weights

Given other colleges' equilibrium (data) tuition t^*_{-j}, I solve college j's tuition problem (2.11). Under the true tuition weight parameters m, the optimal solution should match data tuition. The objective in step three is

$$\min_m \{(t^* - t(\widehat{\Theta}, m))'(t^* - t(\widehat{\Theta}, m))\},$$

where t^* is the data tuition profile, $t(\cdot)$ consists of each college's optimal tuition, $\widehat{\Theta} \equiv [\widehat{\Theta}_0, \widehat{\Theta}_2]$ is the vector of fundamental parameter estimates from the previous two steps. I obtain the variance-covariance of $\widehat{m}$ using the Delta method, which exploits the variance-covariance structure of $\widehat{\Theta}$.

3.2.1 Solving the Optimal Tuition Problem

Given $\widehat{\Theta}$, t^*_{-j} and some m, I examine college j's expected payoff at each trial tuition level t'_j and obtain the optimal tuition associated with this m. This procedure requires computing the series of application-admission equilibria $AE\left(., t^*_{-j}\right)$, which can only be achieved through simulation. To do so, I use an algorithm motivated by the rule of "continuity of equilibria," which requires, intuitively, that $AE(t'_j, t^*_{-j})$ should be close to $AE(t_j, t^*_{-j})$ when t'_j is close to t_j. To be specific, I start from the equilibrium at the data tuition level $\left(t^*_j, t^*_{-j}\right)$, which is numerically unique for nontrivial initial beliefs.[8] Then, I gradually deviate from t^*_j: for $\left(t^{"}_j, t^*_{-j}\right)$, I start the search for new equilibrium, i.e., the fixed point of admissions policies $e\left(.|\left(t^{"}_j, t^*_{-j}\right)\right)$, using, as the initial guess, the $e\left(.|\left(t'_j, t^*_{-j}\right)\right)$ that is associated with the most adjacent $\left(t'_j, t^*_{-j}\right)$. The resulting series of $AE\left(., t^*_{-j}\right)$ is used to solve college j's tuition problem.

[8] "Nontrivial initial beliefs" requires that the student's initial belief about admissions probability is not 0 at any college. $AE(t^*)$ is found to be unique numerically in my search for equilibrium starting from 500 different combinations of nontrivial initial beliefs.

3.3 Identification

3.3.1 Student-Side Parameters

In my model, students are of unobserved (discrete) types and have normally distributed idiosyncratic tastes. Econometricians observe students' discrete choices without observing their latent utilities. Therefore, the student-side model can be viewed as a finite mixture of multinomial probits. In the appendix, I prove the identification of a mixed probit model with two types.[9] The identification in the more general case of mixed multinomial probits with multiple types would require more complicated algebraic analysis but would nevertheless follow the same logic. The observed variation in students' behavior arises from their heterogeneity both across and within types. In order to disentangle these two sources of heterogeneity, I need additional within-type variation that is driven by some observable. I assume that only SAT and family income (a 5-year average) enter the type distribution, i.e., SAT and family (permanent) income summarize all information that is correlated with ability. By contrast, financial aid depends on SAT and all family-background variables. For example, conditional on family permanent income, family assets vary with factors, such as housing prices and stock prices, that are not correlated with ability. Variations in financial aid have different impacts on students across types, which helps to identify the type distribution and type-specific utilities.[10]

Given the type distribution identified from the mixture of probits, I now discuss

[9] The proof builds on Meijer and Ypma (2008), who show the identification for a mixture of two univariate normal distributions.

[10] This exclusion restriction is sufficient but not necessary for identification. For example, I could allow family asset to enter type distribution as a categorical variable, and to enter financial aid function as a continuous variable. The within-category variation in asset would be enough for identification.

the major sources for the identification of other student-side parameters. First, the probabilities of admissions $\{p_j(A, SAT)\}$ are identified mainly from the observed variation in admissions across students with the same SAT but different family income, due to the exclusion restriction that family income affects admissions rates only via ability. Second, the vector σ_ϵ for the i.i.d. idiosyncratic tastes is identified from the variation in expected financial aid across students within a college, given that student utility is measured in monetary units and that the coefficient on net tuition is normalized to one. Third, application costs are identified mainly from the variation in the numbers of applications, using the restriction that $C(\cdot)$ is the same across students.[11] Finally, the fraction of admitted students who chose not to attend any college serves as the major identification source for σ_ζ, the standard deviation of the outside option shock.

3.3.2 College-Side Parameters

The identification of capacity parameters κ follows directly from the equilibrium college capacities calculated based on $\widehat{\Theta}_1$, and the identification of $\left\{\widehat{P}(s|A)\right\}$ is facilitated by the restriction that signal distribution is the same across colleges. However, the vector of ability values ω is not point identified, even after normalizing ω_1. The reason is as follows: each college j faces discrete (s, SAT) groups of applicants and its admissions policy depends on the rankings of these groups in terms of their conditional expected abilities $\gamma_j(\cdot)$. These relative rankings remain unchanged for a range of ω's, implying that ω cannot be point identified. Consequently, I set up a grid of ω's and implement the second step estimation given each of these ω's. The best fit

[11] For example, by comparing $V(\{j\})$ and $V(\emptyset)$, I can identify $\left(\overline{u}_j(A, Z) - \frac{C(1)}{p_j(SAT, A)}\right)$. Then I can separately identify $\overline{u}_j$ and $C(1)$ because application cost is independent of SAT.

occurs with ω's around $[1, 2, 3]'$, therefore, I fix $\widehat{\omega} = [1, 2, 3]'$. At other values of ω around $[1, 2, 3]'$, the estimates for the other parameters in steps two and three will change accordingly. However, the counterfactual experiment results are robust to these changes.[12]

[12] The appendix shows counterfactual experiment results with alternative ω's around $[1, 2, 3]'$.

Chapter 4

Data

4.1 NLSY Data and Sample Selection

The National Longitudinal Survey of Youth 1997 consists of a sample of 8984 youths who were 12 to 16 years old as of December 31, 1996. There is a core nationally representative random sample and a supplemental sample of blacks and Hispanics. Annual surveys have been completed with most of these respondents since 1997. A college choice series was administered in years 2003-2005 to respondents from the 1983 and 1984 birth cohorts who had completed either the 12th grade or a GED at the time of interview. Respondents provided information about each college they applied to, including name and location; any general financial aid they may have received; whether each college to which they applied had accepted them for admission, along with financial aid offered. Information was asked about each application cycle.[1] In every survey year, the respondents also reported the college(s), if any, they attended during the previous year.[2] Other available information relevant to this paper includes SAT/ACT score and financial-aid-relevant family information (family income, family

[1] An application cycle includes applications submitted for the same start date, such as fall session 2004.

[2] The NLSY97 geocode (restricted-use) data provide information on the names and locations of the colleges related to the student.

assets, race and number of siblings in college at the time of application). The sample I use is from the 2303 students within the representative random sample who were eligible for the college choice survey in at least one of the years 2003-2005. To focus on first-time college application behavior, I define applicants as students whose first-time college application occurred within 12 months after they became eligible. Under this definition, 1756 students are either applicants or non-applicants.[3] I exclude applications for early admission (21 obs.).[4] I also drop observations where some critical information, such as the identity of the college applied to, is missing (89 obs.). The final sample size is 1646.

4.2 Aggregation of Colleges

Two major constraints make it necessary to aggregate colleges. One is computational feasibility: with a large number of colleges, solving the student optimal portfolio problem and/or computing the equilibrium poses major computational challenges.[5] Another major constraint is sample size: without some aggregation, the number of observations for each option would be too small to obtain precise parameter estimates. Consequently, I aggregate colleges into groups by observed characteristics and treat each group as one college in the estimation. By doing so, I abstract from some idiosyncratic factors such as regional preferences that may be important at a disaggregate level but are less likely to be important at a more aggregate level.

[3] I exclude students who were already in college before their first reported application. If a student is observed in more than one cycle, I use only her/his first-time application/non-application information.

[4] There is no direct information on early admission; I identify early admission according to the rules specified in the appendix.

[5] The choice set for the student application problem grows exponentially with the number of colleges. Moreover, a fixed point has to be found for each college's admissions policy in order to solve for the equilibrium.

The aggregation goes as follows: first, I divide all four-year colleges into private and public categories, and then I use the within-category rankings from U.S. News and World Report 2003-2006 for further division. Since 1983, U.S. News and World Report has been publishing annual rankings of U.S. colleges and is the most widely quoted of its kind in the U.S.[6] Each year, seven indicators are used to evaluate the academic quality of colleges for the previous academic year.[7] The report years I use correspond to the years when most of the students in my sample applied to colleges, and the rankings had been very stable during that period. Table 4.1 shows the detailed grouping: I group the top 30 private universities and top 20 liberal arts colleges into Group 1, the top 30 public universities into Group 2, and all other four-year private (public) colleges into Group 3 (Group 4).

To accommodate the aggregation of colleges, the empirical definitions of application, admission and enrollment in this paper are as follows: a student is said to have applied to group j if she applied to any college within group j; is said to have been admitted to group j if she was admitted to any college in group j; and is said to have enrolled in group j if she enrolled in any college in group j. With these definitions, this paper is meant to capture the behavior of the majority of students: 60% of applicants in the sample applied to no more than one college within a group; on the other hand, cross-group application is a significant phenomenon in the data. Table 4.2 shows, conditional on applying to the college group in the row, the fraction

[6] With the exception of 1984, when the report was interrupted.

[7] These indicators include: assessment by administrators at peer institutions, retention of students, faculty resources, student selectivity, financial resources, alumni giving, and (for national universities and liberal arts colleges) "graduation rate performance", the difference between the proportion of students expected to graduate and the proportion who actually do. The indicators include input measures that reflect a school's student body, its faculty, and its financial resources, along with outcome measures that signal how well the institution does its job of educating students.

Table 4.1: Aggregation of Colleges

Variable	Group 1	Group 2	Group 3	Group 4
Num. of colleges (Potential[a])	51	32	1921	619
Num. of colleges (Applied[b])	37	32	312	292
Capacity[c] (%)	1.0	4.6	11.2	24.4

Group 1: Top private colleges; Group 2: Top public colleges;
Group 3: Other private colleges; Group 4: Other public colleges.
a. Total number of colleges in each group (IPEDS).
b. Number of colleges applied to by some students in the sample.
c. Capacity = Num. of students in the sample enrolled in group j/sample size.

Table 4.2: Applications|Applied to a Certain Group

%	Group 1	Group 2	Group 3	Group 4
Group 1	100.0	32.7	70.9	40.0
Group 2	12.2	100.0	39.9	52.7
Group 3	13.0	19.6	100.0	47.2
Group 4	4.1	14.5	26.4	100.0

Conditional on applying to the group in the row,
the fraction that applied to each group in the column.

of applicants that applied to each of the college groups in the column. For example, 32.7% of Group 1 applicants also applied to Group 2. Moreover, among the applicants who applied to both groups within the public/private category, very few applied to cross-group colleges that are close in ranking.[8]

I also adjust the definitions of tuition and financial aid to college aggregation. I use the within-group average tuition as the group tuition, based on the tuition information from the Integrated Postsecondary Education Data System (IPEDS). If a student got financial aid offers from more than one college within the group she enrolled in, the financial aid from the attended college is viewed as the aid she got from this group; if she was offered aid from more than one college within a group she

[8] Among the applicants who applied to both groups within the public/private category, I define a student as a "close applicant" if the ranking distance is less than 10 between the best lower-ranked college and the worst top college she applied to. For Group 1-and-Group 3 applicants, 10% are close applicants. For Group 2-and-Group 4 applicants, none are close applicants.

Table 4.3: Student Characteristics

Variable	Non-Applicants	Applicants	Attendees
Female	43.2%	53.0%	54.1%
Black	17.7%	13.3%	12.1%
Family Income[a]	39835.5	68481.1	70605.61
	(32361.0)	(51337.0)	(51279.3)
$Inc^b = 1$	34.5%	13.7%	12.7%
$Inc = 2$	50.8%	48.9%	47.6%
$Inc = 3$	14.7%	37.5%	39.7%
$SAT^c = 1$	79.8%	16.5%	13.7%
$SAT = 2$	17.0%	59.7%	60.6%
$SAT = 3$	3.2%	23.8%	25.7%
Observations	899	747	678

a. in 2003 dollars

b. Inc=1 if family income is below 25th percentile (group mean $10,017)
Inc=2 if family income is in 25-75th percentile (group mean $45,611)
Inc=3 if family income is above 75th percentile (group mean $110,068)

c. SAT=1 if SAT or ACT equivalent is lower than 800.[10]
SAT=2 if SAT or ACT equivalent is between 800 and 1200.
SAT=3 if SAT or ACT equivalent is above 1200.
Score conversion follows SAT-ACT Concordance Tables (College Board).

did not enroll in, the highest financial aid from that group is used.[9]

4.2.1 Summary Statistics

Table 4.3 summarizes characteristics among non-applicants, applicants and attendees. There are clear differences between non-applicants and applicants: the latter are much more likely to be female, white, with higher SAT scores and with higher family income. Conditional on applying, attendees and non-attendees are not significantly different. Similar patterns have been found in other studies using different data.[11]

[9] Given the assumption that all colleges are identical within a group, the highest financial aid from the group together with the non-pecuniary utility from that group is the highest bid for the student from that group.

[11] For example, Howell (2005), using data from National Education Longitudinal Study of 1988, and Arcidiacono (2005), using data from the National Longitudinal Study of the Class of 1972, report similar patterns.

Table 4.4: Distribution of Portfolio Sizes

%	Size= 0	Size= 1	Size= 2	Size= 3	Size= 4
All	54.6	31.0	11.2	2.9	0.3
White	53.3	31.6	11.7	3.1	0.3
Black	61.6	27.9	8.1	1.9	0.3
$SAT = 1$	85.4	12.7	1.4	0.5	0.0
$SAT = 2$	25.5	50.0	19.0	5.0	0.3
$SAT = 3$	14.0	49.8	28.0	6.8	1.4
$Inc = 1$	75.2	19.7	3.6	1.0	0.5
$Inc = 2$	55.6	32.4	10.3	1.7	0.0
$Inc = 3$	32.0	39.6	20.4	7.3	0.7

Table 4.4 summarizes the distribution of application portfolio size. Fifty-five percent of students did not apply to any four-year college. Among applicants, 67% applied to only one group, and only 7% of applicants applied to three groups or more. Relating portfolio size to student characteristics: whites, students with higher SAT and students with higher family income are not only more likely to apply but also more likely to apply to more groups.

Table 4.5 shows group-specific application rates and admissions rates. The application rate, defined as the fraction of applicants that apply to a certain group, increases as one goes from Group 1 to Group 4.[12] But relative to their capacities as shown in Table 4.1, top colleges still receive disproportionately higher fractions of applications than lower-ranked colleges. For example, Group 4 is almost 25 times as big as Group 1, but the application rate for Group 4 is only 10 times as high as that for Group 1. Consistently, the admissions rate increases monotonically from 58% in Group 1 to 96% in Group 4. The appendix includes further summary statistics by race, by SAT and by family income.

[12] Application rates across groups do not need to add up to 100%, since some students applied to multiple college groups.

Table 4.5: Application and Admission: All Applicants

%	Group 1	Group 2	Group 3	Group 4
Application Rate	7.4	19.8	40.3	72.0
Admission Rate	58.2	76.4	91.7	95.7

Num of all applicants: 747

Application rate=num. of group j applicants/num. of all applicants

Admission rate=num. of students admitted to group j/num. of group j applicants

Table 4.6: Final Allocation of Admitted Students (in percentage)

Group 1	Group 2	Group 3	Group 4	Outside
2.2	10.6	25.6	55.7	6.0

Num. of students with at least one admission: 720.

Table 4.7: Tuition and Financial Aid

	Group 1	Group 2	Group 3	Group 4	General aid
Tuition[a]	27009	5347	17201	3912	N/A
Fraction of Aid Recipients[b]	42.4%	32.8%	67.1%	46.6%	39.9%
Mean Aid for Recipients	12836.1	8967.9	11346.6	5344.8	4325.6

a. Tuition and aid are measured in 2003 dollars.

b. Fraction of aid recipients=$\frac{\text{Num. of aid recipients in the sample}}{\text{Num. of admitted students in the sample}}$

Table 4.6 shows the final distribution of students who obtained at least one admission. Over 80% of them attended lower-ranked colleges, with Group 4 accommodating 56%. By contrast, only 2% attended colleges in the top-ranked private Group 1. Six percent of admitted students rejected all admissions and chose the outside option, suggesting the existence of post-application shocks.

Table 4.7 summarizes tuition and financial aid. Private colleges are about four to five times as costly as public colleges of similar ranking. Within the public/private category, the higher-ranked colleges are more costly. Financial aid information is shown in the last two rows. The first four columns show the fraction of financial aid recipients among those admitted to each group and the average amount of financial aid obtained. Relative to students admitted to top groups, a higher fraction of students admitted to lower-ranked groups receive college financial aid. Conditional on obtaining some aid, the amount of aid is monotone in the tuition cost.[13] As shown in the last column, 40% of admitted students receive some outside financial aid that helps to fund college attendance in general, but the average amount of general aid is lower than that of any college-specific aid.

[13] Financial aid can exceed tuition, since it may also cover other expenditures necessary for college attendance.

Chapter 5

Empirical Results

This section presents structural parameter estimates (with standard deviation in parenthesis) and model fits. I allow for six types of students, with $(A, Z) \in \{1, 2, 3\} \times \{1, 2\}$, three SAT levels, $SAT \in \{1, 2, 3\}$, and three signal levels, $s \in \{1, 2, 3\}$.[1] I allow the tuition weight vector m to differ across public and private categories, but restrict it to be the same within the public/private category. The discount factor δ is fixed at 0.95.

5.1 Student-Side Parameter Estimates

5.1.1 Preference Parameter Estimates

Table 5.1 reports the estimates of preference parameters. Rows 1 to 3 show the mean values attached to colleges by type $Z = 1$ students with $A = 1$ to $A = 3$, respectively. $\overline{\psi}_j(A)$'s shown in the next two rows are the additional values attached to each college group by type $Z = 2$ students relative to type $Z = 1$ students, conditional on ability. That is, $\overline{u}_j(A, Z = 2) = \overline{u}_j(A, Z = 1) + \overline{\psi}_j(A)$.[2] The next three rows report $\overline{u}_j(A, Z = 2)$. Within the same Z type, students of different ability

[1] A, SAT and s go from low to high as the levels go from 1 to 3.

[2] I restrict $\overline{\psi}_j(1) = \overline{\psi}_j(2)$.

levels have very different valuations for colleges. An average student of the lowest ability ($A = 1$) derives large negative utility from any college; i.e., the outside option is preferable to any college option for her, and her college utility levels are universally much lower than those of higher-ability students. For students of the two higher ability levels, some college options are better, while some are worse than the outside option. Moreover, their valuations of colleges are not universally monotone in ability: on average, $A = 3$ students have higher valuations for top colleges (Groups 1,2) and lower valuations for lower-ranked colleges (Groups 3,4) than $A = 2$ students do. Holding ability constant, $Z = 2$ type value private colleges much more and public colleges much less than $Z = 1$ type.

The next row of Table 5.1 shows the standard deviations of idiosyncratic tastes: even within T type, students are still very different in their tastes for colleges. For example, although college 1 is worth only \$124, 188 for an average student in $(A = 3, Z = 2)$ type, this value becomes \$271, 618 at the 90^{th} percentile.[3] The last row shows the estimate of the standard deviation of the ex post shock to the outside option. Relative to the variation in permanent tastes, the variation in the ex post shocks is smaller: the major driving force in a student's decision is her permanent taste. However, together with the ex post financial aid shocks, the ex post shock to the outside option introduces non-trivial uncertainty into a student's application problem. For example, ex post, a student might opt out even with some admissions in hand.

[3] Table 7.2 in the appendix illustrates the importance of within-type taste dispersion by showing the mean evaluations of colleges among all students, applicants and attendees, from a simulated example.

Table 5.1: Preference Parameter Estimates

	Group 1	Group 2	Group 3	Group 4
$\overline{u}_j(A=1,Z=1)$	-233937.51	-287044.58	-216980.82	-119861.20
	(79801.99)	(18949.38)	(8143.23)	(4483.13)
$\overline{u}_j(A=2,Z=1)$	-222379.95	-97657.36	-20898.31	81493.20
	(43606.00)	(9341.44)	(3203.74)	(1096.97)
$\overline{u}_j(A=3,Z=1)$	-57506.06	59719.85	-52037.77	11025.54
	(3536.88)	(6397.90)	(6164.44)	(4629.37)
$\overline{\psi}_j(A=1,2)$	159977.95	-22777.16	155710.58	-124851.87
	(40945.39)	(10115.50)	(4188.71)	(6790.05)
$\overline{\psi}_j(A=3)$	181694.72	-66612.01	89018.65	-115946.48
	(26718.71)	(7528.33)	(9597.78)	(21069.53)
$\overline{u}_j(A=1,Z=2)$	-73959.56	-309821.73	-61270.24	-244713.07
$\overline{u}_j(A=2,Z=2)$	-62402.00	-120434.52	134812.27	-43358.67
$\overline{u}_j(A=3,Z=2)$	124188.65	-6892.15	36980.88	-104920.94
σ_{ϵ_j}	115039.64	91646.92	77914.55	43567.67
	(1164.68)	(3807.92)	(1964.19)	(1506.58)
σ_ζ		10433.37		
		(2916.06)		

$\overline{u}_j(A,Z=2)=\overline{u}_j(A,Z=1)+\overline{\psi}_j(A)$

In sum, there is significant heterogeneity in students' preferences for colleges, both across types and within each type. Not only do students attach different values to the same college, but they also rank colleges differently.[4] It would be misleading to assume that all students value colleges in the same way or that students' benefits from attending colleges are monotone in ability. Rather, the preference parameter estimates suggest that the college market is highly horizontally differentiated, with each option (including the outside option) best catering to some groups of students.

5.1.2 Type Distribution Parameter Estimates

[4]In line with the finding from this paper, Dale and Krueger (2002) also find that "a more selective school is not the income-maxmizing choice for all students".

Table 5.2: Ordered Logit Ability Distribution

cut_1^a	cut_2^a	Family Income	$SAT = 2$	$SAT = 3$
2.4782	5.4100	0.00001	2.8052	3.6927
(0.1555)	(0.2220)	(0.000002)	(0.16147)	(0.2297)

a. cut_1, cut_2 are the cutoff parameters for the ordered logit.

Table 5.3: Z Type Distribution

	$A = 1$	$A = 2$	$A = 3$
$\Pr(Z = 1 \mid A)$	0.8347	0.7359	0.6313
	(0.0695)	(0.0265)	(0.0814)
$\Pr(Z = 2 \mid A)$	0.1636	0.2641	0.3687

Table 5.2 shows the parameter estimates for the ordered logit distribution of ability conditional on family income and SAT. Students with higher SAT scores and those with higher family income are more likely to be of higher ability. Table 5.3 shows the distribution of Z types by ability: at all ability levels, most students are of type $Z = 1$ (78% of all students), but the fraction goes down as ability goes up. In other words, higher-ability students are more likely to be of the type that values private colleges over public colleges.

Based on the estimates in Tables 5.2 and 5.3, I simulate a population of students and report their type distribution in Table 5.4. Of all students, 57% are of ability 1, and only 9% are of ability 3. Conditional on being a type $Z = 2$, the ability distribution first order stochastically dominates that conditional on being a type $Z = 1$. The ability distribution among SAT 1 students is distinctively different from that among higher-SAT students: over 88% of SAT 1's are of ability 1 and fewer than 1% of them are of ability 3. Between SAT 2 and SAT 3 students, the ability difference is less obvious: the majority of students of both SAT levels are of ability 2, but SAT 3 students are less likely to be of ability 1 and more likely to be of ability 3 than SAT 2 students. SAT, as a noisy measure of student ability, is more powerful in

Table 5.4: Ability Distribution: Simulation

%	$A = 1$	$A = 2$	$A = 3$
All	57.2	33.9	8.9
$Z = 1$	60.9	31.9	7.2
$Z = 2$	43.5	41.3	15.2
$SAT = 1$	88.6	10.7	0.7
$SAT = 2$	28.3	58.3	13.4
$SAT = 3$	12.7	57.8	29.5
$Inc = 1$	76.7	20.5	2.8
$Inc = 2$	59.7	33.4	6.9
$Inc = 3$	32.4	48.3	19.3

distinguishing ability 1 students from the others, but less so in distinguishing between ability 2 and ability 3 students. Finally, the last three rows of Table 5.4 illustrate the relationship between family income and ability. The majority of students from both low- and middle-income families are of ability 1, with the fraction being 77% for those from low-income families. By contrast, only 32% of students from high-income families are of ability 1. Although ability 3 students are in the minority at all family income levels, their fraction goes up steeply with family income levels.[5]

5.1.3 Application Costs and Financial Aid

One of the major features of this model is that applications are costly for the student, which is confirmed by Table 5.5. The cost for the first application is $\$6,477$, which is higher than the annual tuition of public colleges. But as the number of applications goes up, the marginal cost goes down very fast, suggesting the existence of some economies of scale.

[5] As stated earlier, ability in this paper refers to college preparedness rather than innate ability. These estimates suggest that students from higher-income families are better prepared.

Table 5.5: Application Costs

	$n=1$	$n=2$	$n=3$	$n=4$
$C(n)$	6477.40	7977.17	8335.54	8589.00
	(323.92)	(188.42)	(202.62)	(213.22)

Table 5.6: Financial Aid

	General aid		College-Specific Aid	
	Coefficient	Std. Error	Coefficient	Std. Error
Constant	−4907.82	(817.06)	−13664.32	(1756.28)
Black	1490.72	(915.24)	3277.25	(1033.22)
Family Income	−0.0253	(0.0107)	−0.0461	(0.0092)
Family Asset	−0.0041	(0.0027)	−0.0045	(0.0024)
$SAT=2$	3993.10	(854.49)	8141.64	(1837.64)
$SAT=3$	6081.56	(1079.32)	15227.48	(1843.60)
Sibling in College[a]			4336.62	(897.90)
$(SAT=2)\times public$			−4068.05	(2487.06)
$(SAT=3)\times public$			−7821.93	(2563.67)
Group 2			3993.83	(2870.72)
Group 3			9511.52	(1811.92)
Group 4			6854.97	(2278.03)
σ_η	8034.08	(169.34)	9758.76	(285.86)

a. Whether the student has some siblings in college at the time of application.

Table 5.6 displays the estimated parameters for the Tobit specifications of financial aid.[6] The left panel reports parameter estimates for general aid. Being black and having higher SAT scores increases one's expected financial aid, while higher family income and/or assets reduces it. These patterns also hold for college-specific aid, as shown in the right panel. However, the effect of SAT is greater in private colleges than in public colleges. Having siblings in college at the time of application also increases college-specific financial aid. Top colleges are less generous in giving out financial aid, especially Group 1, although it charges the highest tuition. By contrast, Group 3 is most generous in giving out financial aid. The last row shows the standard deviations of financial aid: there is a significant amount of uncertainty involved in the final realization of financial aid, adding to the total uncertainty faced by the student upon application.

5.2 College-Side Parameter Estimates

The overall chi square goodness of fit statistic is 41.06 for the second-step SMDE.[7] Table 5.7 reports parameter estimates for signal distribution conditional on ability. By sending out signals, the highest ability students can successfully distinguish themselves from the others: they are much more likely to send the highest signal, and almost never send out the lowest signal. Ability 2 students are most likely to send the medium signal, and they distinguish themselves from ability 1 students mainly because of their lower probability of sending out the lowest signal. However, their chance of obtaining the highest signal is almost the same as ability 1 students. As a

[6] The explanatory variables are chosen based on published financial aid policies and on Tobit regressions using only financial aid data: insignificant regressors are omitted. The results reported in Table 10 are estimated jointly with other student-side parameters via SMLE.

[7] $\chi^2_{27,0.05} = 40.11$.

Table 5.7: Signal Distribution

	$P(s=1\|A)$	$P(s=2\|A)$	$P(s=3\|A)$
$A=1$	0.2210	0.3851	0.3939
	(0.0769)		(0.0954)
$A=2$	0.0253	0.5807	0.3940
	(0.0047)		(0.0810)
$A=3$	0.000001	0.2876	0.7124
	(0.0577)		(0.0575)

Table 5.8: Capacities

κ_1	κ_2	κ_3	κ_4
0.0096	0.0459	0.1082	0.2456
(0.0015)	(0.0013)	(0.0009)	(0.0021)

result, it is hard to distinguish between the two lower-ability types by signals.

The estimated expected capacity of each college group is given in Table 5.8: the more selective colleges and private colleges are smaller than their counterparts. These capacity estimates closely match the capacities observed in the data shown in Table 4.1.

Finally, Table 5.9 shows the results for tuition weights, where tuition is measured in $\$1,000$. Besides using their tuition to compete for better students, colleges have positive but bounded incentives to raise tuition. If we do not consider the effect of tuition on enrollee ability, the "monetarily optimal" tuition is around $\$26,400$ for private colleges, and $\$5,700$ for public colleges.

Table 5.9: Tuition Weights

$j \in \{1,3\}$ private		$j \in \{2,4\}$ public	
m_{j1}	m_{j2}	m_{j1}	m_{j2}
0.0674	-0.0013	0.0073	-0.00063
(0.002)	(0.0004)	(0.0034)	(0.00015)

Tuition is measured in thousands of dollars.

Table 5.10: Distribution of Portfolio Sizes (in percentage)

Size	Data	PE	AE	SPNE
0	54.6	54.9	55.1	55.7
1	30.9	29.6	30.9	31.5
2	11.2	11.8	10.7	9.6
3	2.9	3.3	3.0	2.9
4	0.3	0.2	0.3	0.2
χ^2 Stat		2.95	0.47	5.64

PE: Partial Equilibrium Model
AE: Application-Admission Equilibrium
ME: Market Equilibrium Model
$\chi^2_{4,0.05}$= 9.49

5.3 Model Fit

Given the parameter estimates, I first fix tuition profile at the data level and simulate the student-side partial equilibrium model (PE) and the application-admission equilibrium model (AE). Then I endogenize tuition and simulate the whole subgame perfect Nash equilibrium model (SPNE). This section shows model fits for the whole sample; the appendix reports the fits by race, by SAT and by family income.

Table 5.10 shows the fit for the distribution of portfolio sizes: all three models fit the data well, with SPNE slightly understating the fraction of multiple applications. Table 5.11 displays the fit of application and admissions rates among applicants. The first set of rows show that all three models closely match application rates, except that the SPNE model under-predicts the application rate for Group 4. The fit for admissions rates is shown in the second set of rows: PE closely matches the admissions rates for all groups. AE and SPNE under-predict the admissions rate for Group 1 and over-predict that for Group 3. Table 5.12 displays the fits of student allocation. The first set of columns shows the allocation for all students, and the second set of columns shows that for students with at least one admission: all models closely fit

Table 5.11: Application and Admission: All Applicants (in percentage)

Application Rate	Data	PE	AE	SPNE
Group 1	7.4	7.6	7.1	7.4
Group 2	19.8	21.1	19.9	20.2
Group 3	40.3	41.4	41.2	41.9
Group 4	72.0	72.5	70.8	67.0*
Admission Rate				
Group 1	58.2	54.2	44.1*	43.6*
Group 2	76.4	80.2	81.9	82.0
Group 3	91.7	90.9	95.3*	98.6*
Group 4	95.7	95.0	95.0	97.1

* $\chi^2 > \chi^2_{1,0.05}$

the allocation patterns, with SPNE fit being the best.

Finally, Table 5.13 contrasts SPNE predicted tuition levels with the data. The model fits Group 4's tuition almost perfectly, but it under-predicts College 2's tuition and over-predicts College 3's tuition by about 10%. The deviation of the SPNE tuition from data tuition comes mainly from the SPNE structure. Table 7.13 in the appendix shows each college's tuition as the best response to others' equilibrium (data) tuition (i.e., the fit for the third-step estimation), which closely matches the data. We can also compare the SPNE tuition with the monetarily optimal tuition ($26,400 for private colleges and $5,700 for public colleges). All groups charge less than they would if tuition policies do not play a strategic role in the subsequent application-admission game. Moreover, the under-pricing is more significant for lower-ranked groups than for top groups. Although within the public/private category colleges share the same monetary incentives for tuition, lower-ranked colleges have to resort to much lower prices than top colleges so as to attract enough qualified applicants.

Table 5.12: Final Allocation of Students (in percentage)

	All Students				Students With Some Admission			
	Data	PE	AE	SPNE	Data	PE	AE	SPNE
College 1	1.0	1.1	1.0	1.0	2.2	2.7	2.2	2.2
College 2	4.6	4.5	4.3	4.5	10.6	10.6	10.1	10.5
College 3	11.2	10.7	11.3	11.1	25.6	24.9	26.4	25.8
College 4	24.4	23.5	24.0	24.3	55.7	54.8	55.9	56.3
Outside	58.8	60.2	59.4	59.1	6.0	7.0	5.3	5.1
χ^2 Stat.		2.11	0.54	0.12		1.93	1.54	1.45

$\chi^2_{4,0.05} = 9.49$

Table 5.13: Tuition

	Group 1	Group 2	Group 3	Group 4
Data	27009	5347	17201	3912
SPNE	26162	4555	19173	3925

Chapter 6

Counterfactual Experiments

With the estimated model, which fits the data reasonably well, I conduct three counterfactual experiments. Comparisons are made between the baseline SPNE and the new SPNE, simulated using the same set of random draws.

6.1 Perfect Signals

To quantify the impact of incomplete information on the equilibrium, I conduct a counterfactual experiment where signals measure student ability perfectly, i.e., for all A, $P(s = A|A) = 1$.[1]

Table 6.1 contrasts the distributions of portfolio sizes. Perfect signaling eliminates the admission uncertainty in most cases and enables students to target their applications better.[2] Students without a chance of getting admitted are discouraged from applying at all; hence fewer students apply. Moreover, when admission is certain, multiple applications remain meaningful only as a way to guard against ex post

[1] With perfect signals, SAT no longer affects any decision.

[2] With perfect signals, students would face admission probabilities of either 1 or 0 in most cases. But a student is still subject to rationing if a college's remaining capacity cannot accommodate all applicants in her ability group.

Table 6.1: Perfect Signals: Distribution of Portfolio Sizes

%	$Size = 0$	$Size = 1$	$Size = 2$	$Size = 3$	$Size = 4$
Base SPNE	55.7	31.5	9.6	2.9	0.2
New SPNE	57.0	34.3	7.7	0.9	0.1

Table 6.2: Perfect Signals: Admission Rates

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	43.6	82.0	98.6	97.1
New SPNE	99.3	97.4	97.4	98.2

shocks, which leads to fewer applications sent by applicants.

Table 6.2 shows the changes in admissions rates: as student applications get better targeted, colleges face only well-qualified applicants and all admissions rates increase to near 100%, with Group 1's admissions rate being the highest. Obviously, in this case, "selectivity" as reflected by admissions rate bears no indication about a college's quality, as measured by the ability of its students, which is shown in Table 6.3. The perfect ability measure enables the top groups to fill their capacities with (almost) only the highest-ability students. The lower-ranked groups, although losing some of the highest-ability students, are (almost) free of the lowest-ability students. As a result, the average student ability increases in all college groups.

Table 6.3: Perfect Signals: Ability Distribution Within Each Destination

%	Group 1	Group 2	Group 3	Group 4	Outside
Base SPNE					
$A = 1$	3.7	0.2	7.4	0.9	94.9
$A = 2$	6.5	15.7	80.9	91.9	3.0
$A = 3$	89.9	84.1	11.7	7.2	2.1
New SPNE					
$A = 1$	0.0	0.0	1.8	0.0	96.4
$A = 2$	0.0	0.2	86.6	94.0	2.4
$A = 3$	100.0	99.8	11.6	6.0	1.2

Table 6.4: Perfect Signals: Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26162	4555	19173	3925
New SPNE	26409	3307	16579	2956

Table 6.5: Perfect Signals: Mean Student Welfare

	Base SPNE ($)	New SPNE($)	Change ($)	Change (%)
All	41402	43860	2458	5.94
$A = 1$	677	162	-515	-76.07
$A = 2$	98630	103373	4743	4.81
$A = 3$	84673	97453	12780	15.09

Table 6.4 shows the changes in tuition under the new SPNE. Given perfect signals, colleges no longer need to use tuition as a screening tool. All colleges but Group 1 lower their tuition. Relative to the number of students with highest ability and a strong preference for Group 1, the slots in Group 1 are still scarce. When the signal is perfect, not only does Group 1 admit only the highest-ability students, but it also charges its monetarily optimal tuition. Other colleges do not enjoy the same preferable market position: the perfect signal makes their competition for better students more severe, which drives down their tuition.

Finally, changes in student welfare are reported in Table 6.5. The lowest ability students lose significantly, since they are denied admission to almost any college. However, all other students gain, and the highest-ability students benefit the most. On average, student welfare increases by 6%, 24% of which comes directly from the changes in tuition.

Table 6.6: Funding Cuts: Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26162	4555	19173	3925
New SPNE	26401	5034	19355	4801

6.2 State Budget Crisis: Funding Cuts

Now I use the model to address the concern about funding cuts: what would happen to the college market if the government cuts funding for public colleges? I fix all the other parameters at their original levels and increase m_1 for public colleges by 10%. As a result, the monetarily optimal tuition for public colleges is increased by 10% from $\$5,700$ to $\$6,270$.

The new equilibrium tuition levels are shown in Table 6.6. Top public colleges (Group2) increase their tuition by about 10%, but the response from lower-ranked public colleges (Group 4) is more dramatic: they increase their tuition by 22%. The overall increase in public tuition is 20%. In response, private colleges also increase their tuition, but only by less than 1%.

Higher tuition lowers the payoff from attending college, and students react by reducing their applications, as shown in Table 6.7. Table 6.8 reports that both groups of public colleges increase their admissions rates after the big increases in their tuition. On the one hand, higher admissions rates compensate students for the higher tuition to keep public colleges attractive. On the other hand, lower-ability students, whose payoffs from colleges are lower, self-select out when tuition is increased, which also contributes to the higher admissions rates in public colleges.

Table 6.9 examines the changes in the composition of students within each destination. All colleges except Group 4 end up with an unambiguously higher-quality pool of enrollees, due to the screening effect of higher tuition. Group 4, although it

Table 6.7: Funding Cuts: Distribution of Portfolio Size

%	$Size = 0$	$Size = 1$	$Size = 2$	$Size = 3$	$Size = 4$
Base SPNE	55.7	31.5	9.6	2.9	0.2
New SPNE	55.9	31.8	9.2	2.9	0.2

Table 6.8: Funding Cuts: Admission Rates

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	43.6	82.0	98.6	97.1
New SPNE	43.5	83.2	98.6	98.5

gets rid of some of the lowest-ability students, also loses some of the highest-ability students, as a result of its tuition increase of over 20%.

Last but not least, Table 6.10 reports changes in student welfare: all students lose, and the mean welfare decreases by \$700. If the government uses the increased public tuition revenue on a one-for-one basis to save on its education expenses, it could save \$234 per student, which is only 1/3 of the welfare loss suffered by students. Over all colleges, the increase in tuition revenue is \$257 per student. As a result, most of the student welfare loss is due to the indirect non-tuition distortions imposed by the funding cut.

Table 6.9: Funding Cuts: Ability Distribution Within Each Destination

%	Group 1	Group 2	Group 3	Group 4	Outside
Base SPNE					
$A = 1$	3.7	0.2	7.4	0.9	94.9
$A = 2$	6.5	15.7	80.9	91.9	3.0
$A = 3$	89.9	84.1	11.7	7.2	2.1
New SPNE					
$A = 1$	3.6	0.2	7.0	0.7	95.1
$A = 2$	6.4	15.5	81.2	92.3	2.8
$A = 3$	90.0	84.3	11.8	7.0	2.1

Table 6.10: Funding Cuts: Mean Student Welfare

	Base SPNE	New SPNE($)	Change ($)
All	41402	40703	−699
$A = 1$	677	649	−28
$A = 2$	98630	97018	−1612
$A = 3$	84673	83149	−1524

6.3 Creating More Opportunities

Finally, I use the model to answer the long-run policy question: to what extent can the government further expand college access simply by increasing the supply of colleges? I conduct two series of experiments: in one series, I increase the capacity of the lower-ranked private colleges (Group 3) by growing magnitudes while keeping the capacities in other groups fixed; in the other, I do the analogous experiments with the capacity of the lower-ranked public colleges (Group 4). The response of college enrollment to the increase in supply is the same for either series and is shown in Figure 6.1. At the beginning, there is a one-to-one response of college enrollment to the increase in supply. Then, enrollment reaches a satiation point where there is neither excess demand nor excess supply of colleges in the enlarged group and the equilibrium outcomes remain the same thereafter. The following tables report the case when Group 3's (Group 4's) supply is at the satiation point, labeled SPNE_3 (SPNE_4).

Table 6.11 reports changes in tuition. To attract enough students, the enlarged Group (Group 3 in SPNE_3, Group 4 in SPNE_4) dramatically lowers its tuition, and its closest competitor, i.e., the other lower-ranked group (Group 4 in SPNE_3, Group 3 in SPNE_4), also lowers its tuition.[3] For example, in SPNE_4, Group 4 cuts its tuition

[3] Colleges do not have to fill their capacities, and they can charge high tuition and leave some slots vacant. However, under the current situation, it is not optimal for them to do so.

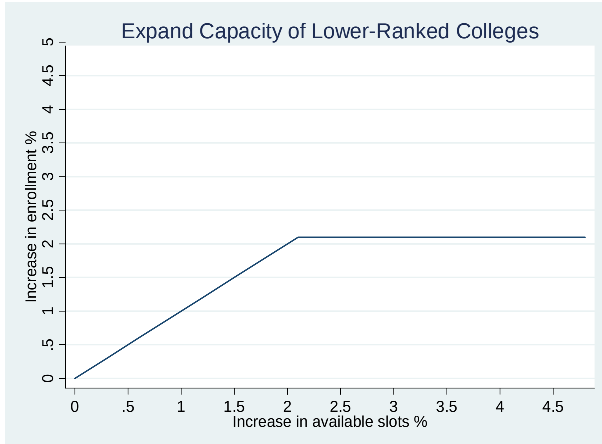

Figure 6.1: State and Local Gov. Spending on Higher Education (Per Capita in 2006 Dollar)

Table 6.11: Increasing Supply: Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26162	4555	19173	3925
SPNE_3	26306	7245	13426	3162
SPNE_4	27549	6473	17394	136

from $3, 925 to an almost negligible level of $136. Its private counterpart, Group 3, also lowers its tuition by about 9%. However, in both SPNE_3 and SPNE_4, the two top groups increase their tuition.

Table 6.12 indicates that in both SPNE_3 and SPNE_4, admissions rates increase in all colleges and reach (almost) 100% except for Group 1. The major driving forces for the increased admissions rates are likely to differ across college groups. For lower-ranked groups, higher admissions rates and lower tuition reflect their efforts to enroll enough students. Top groups increase their admissions rates mainly because they are faced with a better self-selected applicant pool: the increased tuition in top groups push, and the tuition and admissions policies in lower-ranked colleges pull lower-ability applicants toward lower-ranked groups.

Table 6.12: Increasing Supply: Admission Rates

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	43.6	82.0	98.6	97.1
SPNE_3	47.3	99.0	100.0	99.7
SPNE_4	47.7	99.0	99.1	100.0

Table 6.13: Increasing Supply: Attendance

%	Base SPNE	SPNE_3	SPNE_4	All Open&Free
All	40.9	43.0	43.0	51.1
$A = 1$	1.9	3.4	3.2	14.9
$A = 2$	94.7	97.5	97.7	99.4
$A = 3$	86.4	90.1	90.3	98.6

Table 6.13 shows the allocation effect. The first row displays the attendance rate over all students: regardless of the 100% admissions rate and the dramatically lowered tuition in the enlarged group, in either SPNE_3 or SPNE_4, only 2.1% more students can be drawn into colleges. Since there will be an excess supply of colleges in the enlarged group if its capacity is further increased, this 2.1% increase represents the upper limit to which the government can increase college attendance by supplying more of the lower-ranked colleges, private or public. To further understand these equilibrium results, I conduct a partial equilibrium experiment where all colleges are open and free, and the attendance rate is reported in the last column of Table 6.13: only 51%, or 10% more students, would attend colleges. Therefore, neither college capacity nor tuition is a major barrier to college access. A vast majority of students who do not attend colleges under the base SPNE prefer the outside option over any college option. Among them, most are of low ability. In fact, as indicated in the last three rows of Table 6.13, only 2% of the lowest-ability students attend college in the base SPNE, and fewer than 15% of them would attend college even if colleges were

free and open. By contrast, the majority of students of higher ability attend college in the base SPNE, and almost all of them would attend college if colleges become free and open. The major limit to college access, therefore, is ability and the associated preferences, which is in line with findings from earlier research.[4]

[4] For example, Cameron and Heckman (1998) and Keane and Wolpin (2001).

Chapter 7

Conclusion

In this paper, I have developed and structurally estimated an equilibrium model of the college market that incorporates tuition setting, applications, admissions and enrollment. In the model, students are heterogeneous in their abilities and preferences. They face uncertainty and application costs when making their application decisions. Colleges, observing only noisy measures of student ability, compete for more able students via tuition and admissions policies. I have shown that a subgame perfect Nash equilibrium exists for the college market. I have estimated the structural model via a three-step estimation procedure to cope with the complications caused by potential multiple equilibria. The empirical results suggest that the model is able to replicate most of the patterns in the data well.

The estimated structural model has been used to conduct three counterfactual experiments that examine, respectively, the distortion imposed on the market by incomplete information, the equilibrium impacts of funding cuts to public colleges, and the extent to which the government can further expand college access by increasing the supply of lower-ranked colleges. The results suggest that (1) neither tuition cost nor college capacity is a major obstacle to college access, (2) a large fraction of students, mainly low-ability students, prefer the outside option over any college option, and (3) expanding the supply of colleges only draws at most 2.1% more students into colleges.

Several extensions of this model would be interesting to pursue. First is to control for additional sources of observed heterogeneity, such as minority status. There are different ways to incorporate such heterogeneity into the model. For example, affirmative action, in terms of more preferable admissions rates for minority groups, may result from colleges' pursuit of racial diversity, or race-specific ability distributions, or some combination of both. All of these conjectures would lead to different equilibrium results, which could be brought to confront the data.

The second extension is to endogenize capacity constraints and to study the long-run equilibrium. One approach is to introduce a cost function for college education, assuming free entry to the market. Equilibrium of the model would then depend on the form of the cost function. Estimation of such a model may require additional data on college expenses and non-tuition revenues.

The third extension is to endogenize financial aid. Treating financial aid as an equilibrium object together with applications and admissions is a great challenge for future research.

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Appendices

Appendix A: Model Details

A.1 College Admission Problem:

Calculation of $\alpha_j(s, SAT|t, e_{-j}, Y, d)$ and $\gamma_j(s, SAT|t, e_{-j}, Y, d)$:

All objects defined in A.1 depend on $\{t, e_{-j}, Y, d\}$, unless explicitly specified otherwise, but to save notation, the dependence is suppressed. Given $\{t, e_{-j}, Y, d\}$, let $\Pr(accept|X, SAT, \eta, \zeta, j)$ be the probability of acceptance by a student with characteristics (X, SAT, η, ζ) that applies to j. Let $F(X, \eta, \zeta|s, SAT, j)$ be the distribution of (X, η, ζ) conditional on (s, SAT) and application to j. The probability that an applicant with (s, SAT) accepts j's admission is:

$$\alpha_j(s, SAT) = \int \Pr(accept|X, SAT, \eta, \zeta, j) dF(X, \eta, \zeta|s, SAT, j).$$

Let $\Pr(O_{-j}|A, SAT) \equiv \prod_{l \in O \setminus j} p_l(A, SAT) \prod_{k \in Y \setminus O} (1 - p_k(A, SAT))$ be the probability of admission set O for a student with (A, SAT), with college j admitting her for sure, let $I(\cdot)$ be the indicator function, then

$$\Pr(accept|X, SAT, \eta, \zeta, j) =$$
$$\sum_{O_{-j} \subseteq Y(X,SAT)\setminus\{j\}} \Pr_{(A,SAT)}(O_{-j}|A, SAT) I(j = d(X, SAT, \eta, \zeta, O)),$$

that is, the student will accept j's admission if j is the best post-application choice for her. The distribution $F(X, \eta, \zeta | s, SAT, j)$ is given by

$$dF(X, \eta, \zeta | s, SAT, j) = \frac{P(s|A)I(j \in Y(X, SAT))dF(X, \eta, \zeta | SAT)}{\int P(s|A)I(j \in Y(X, SAT))dF(X, \eta, \zeta | SAT)},$$

where $F(X, \eta, \zeta | SAT) = P(T|SAT, B)G(\epsilon, \eta, \zeta)H(B|SAT)$ is exogenous. Finally, the expected ability of applicant (s, SAT) conditional on acceptance is

$$\gamma_j(s, SAT) = \frac{\int A \times \Pr(accept|X, SAT, \eta, \zeta, j)dF(X, \eta, \zeta | s, SAT, j)}{\alpha_j(s, SAT)}.$$

A.2 Proof of Proposition 1 (Existence of Application-Admission Equilibrium)

To ease illustration, I will prove existence of equilibrium in a simpler model, but the idea applies to the more complicated model. Assume there are two colleges $j \in \{1, 2\}$, a continuum of students divided into two types defined by ability: $A \in \{1, 2\}$. The utility of outside option is normalized to 0. The utility of attending college 1 is $u_1(A)$ for everyone with ability A, and the utility of attending college 2 is $u_2(A) + \epsilon$, where ϵ is i.i.d. idiosyncratic taste. There are two SAT levels:$\{1, 2\}$ and two signal levels $s \in \{1, 2\}$, with conditional distribution $P(s|A)$. There is no ex-post shock.

Proof. Some notations to be used: for an (A, SAT) group, let the fraction of students that do not apply to any college be $\theta^0_{A,SAT}$, the fraction of those applying to college j only be $\theta^j_{A,SAT}$ and let $\theta^{12}_{A,SAT}$ be the fraction applying to both. For each (A, SAT) group, $\theta_{A,SAT} \in \Delta$, a 3-simplex; and for all four (A, SAT) groups, $\theta \in \Theta \equiv \Delta^4$. On the college side, each college chooses admissions policy $e_j \in [0, 1]^4$, where 4 is the number of (s, SAT) groups faced by the college.

Step 1: The application-admission model can be decomposed into the following sub-mappings:

Taking the distribution of applicants, and the admissions policy of the other college as given, college j's problem (2.6) can be viewed as the sub-mapping

$$M_j : \Theta \times [0,1]^4 \rightrightarrows [0,1]^4,$$

for $j = 1, 2$. Taking college admissions policies as given, the distribution of students is obtained via the sub-mapping

$$M_3 : [0,1]^4 \times [0,1]^4 \rightrightarrows \Theta.$$

An equilibrium is a fixed point of the mapping:

$$M : \Theta \times [0,1]^4 \times [0,1]^4 \rightrightarrows \Theta \times [0,1]^4 \times [0,1]^4$$

$$s.t.(\theta, e_1, e_2) \in M(\theta, e_1, e_2),$$

where

$$\theta \in M_3(e_{1,}e_2)$$

$$e_j \in M_j(\theta, e_k) \; j, k \in \{1, 2\}, j \neq k.$$

Step 2: Show that Kakutani's Fixed Point Theorem applies in this mapping and hence an equilibrium exists.

1) The domain of the mapping, being the product of simplexes, is compact and non empty.

2) It can be shown that the correspondence $M_j(\cdot, \cdot)$ is compact-valued, convex-valued and upper-hemi-continuous, for $j = 1, 2$. In particular, the $(s, SAT)'$th component of $M_j(\theta, e_k)$ is characterized by (2.8) and (2.9), where $\gamma_j(s, SAT) - \lambda_j$ is continuous in (θ, e_k).

3)Aggregate individual optimization into distribution of students with different portfolios.

It is easy to show that, generically, each student has a unique optimal application portfolio as the solution to (2.5). Moreover, for given (A, SAT), there exist $\epsilon^*(e) \geq \epsilon^{**}(e)$, both continuous in e, such that:

For $\epsilon \geq \epsilon^*(e)$, $Y(A, SAT, \epsilon) = \begin{cases} \{2\} \text{ if } C(2) - C(1) > k_1(e) \\ \{1, 2\} \text{ otherwise} \end{cases}$;

for $\epsilon \in [\epsilon^{**}(e), \epsilon^*(e)), Y(A, SAT, \epsilon) = \{1, 2\}$; and

for $\epsilon < \epsilon^{**}(e)$, $Y(A, SAT, \epsilon) = \begin{cases} \{1\} \text{ if } C(1) \leq k_2(e) \\ \emptyset \text{ otherwise} \end{cases}$,

where $k_1(e)$ and $k_2(e)$ are continuous in e. Therefore, given e, the (A, SAT) population can be mapped into a distribution $\theta_{A,SAT} \in \Delta$, and this mapping is continuous in e. Because the mapping from $[0, 1]^4 \times [0, 1]^4$ into the individual optimal portfolio is compact-valued, convex-valued and upper-hemi-continuous, and the mapping from the individual optimization to the distribution Θ is continuous, the composite of these two mappings, M_3, is compact-valued, convex-valued and upper-hemi-continuous.

Given 1)-3), Kakutani's Fixed Point Theorem applies. □

A.3 Proof of Proposition 2 (Existence of SPNE in the College Market)

Proof. Since for every t, $AE(t)$ exists in the subsequent game, a SPNE exists if a Nash equilibrium exists in the tuition setting game. Let $\bar{t}_j$ denote some large positive number, such that for any t_{-j}, the optimal $t_j < \bar{t}_j$. It is easy to find such a $\bar{t}_j$, since the expected enrollment, hence college j's payoff, would go to 0, when t_j becomes too high. Now define the strategy space for college j as $[0, \bar{t}_j]$, which is nonempty, compact and convex. The objective function of college j is continuous in t, since the distribution of applicants, and hence the total expected ability, is continuous in

t. Given certain regularity conditions, the objective function is also quasi-concave in t_j.[1] The general existence proof for Nash equilibrium applies. $\square$

Appendix B: Data Details

B.1 Empirical Definition of Early Admission:

1) Applications were sent earlier than Nov. 30th, for attendance in the next fall semester and

2) The intended college has early admissions/ early decision/ rolling admissions/ priority admissions policy,[2] and

3) a. Either one application was sent early and yielded admission or

b. some application(s) was (were) sent early but rejected, and other application(s) was (were) sent later.

Appendix C: Detailed Functional Forms

C.1 Conditional Ability Distribution

For $a = 1, 2, 3$

$$\Pr(A_i = a) = \frac{1}{1 + e^{-cut_a + \alpha_1 y_i + \alpha_2 I(SAT_i = 2) + \alpha_3 I(SAT_i = 3)}} - \frac{1}{1 + e^{-cut_{a-1} + \alpha_1 y_i + \alpha_2 I(SAT_i = 2) + \alpha_3 I(SAT_i = 3)}}$$

where y_i denotes family income of i, $I(\cdot)$ is the indicator function, $cut_0 = -\infty$ and $cut_3 = +\infty$.

[1] One counter example occurs when m_{j1} and m_{j2} are both positive and large, which is not supported by the estimation result.

[2] Data source for college early admission programs: 1) Christopher et. al. (2003), and 2) web information posted by individual colleges.

C.2 Financial Aid

(1) General Aid:

$$f_0(SAT_i, B_i) = \beta_0^0 + \beta_1^0 I(race_i = black) + \beta_2^0 I(SAT_i = 2) + \beta_3^0 I(SAT_i = 3) + \beta_4^0 y_i + \beta_5^0 asset_i$$

$$f_{0i} = \max\{f_0(SAT_i, B_i) + \eta_{0i}, 0\},$$

where $\eta_{0i}\tilde{\ } i.i.d.N(0, \sigma_{f_0}^2)$.

(2) College-Specific Financial Aid:

$$f_j(SAT_i, B_i) = \beta_0^1 + \beta_1^1 I(race_i = black) + \beta_2^1 I(SAT_i = 2) + \beta_3^1 I(SAT_i = 3) + \beta_4^1 y_i + \beta_5^1 asset_i + \beta_6^1 I(nsib > 0) + \beta_7^1 I(SAT_i = 2) I(j \in public) + \beta_8^1 I(SAT_i = 3) I(j \in public) + \beta_9^1 I(j = 2) + \beta_{10}^1 I(j = 3) + \beta_{11}^1 I(j = 4)$$

$$f_{ji} = \max\{f_j(SAT_i, B_i) + \eta_{ji}, 0\}$$

where $nsib$ denotes the number of siblings in college at the time of i's application and $\eta_{ji}\tilde{\ } i.i.d.N(0, \sigma_{f_1}^2)$.

C.3 Preferences

Preferences for colleges $\overline{u}_j(A, Z = 1)$ are fully non-parametric and $\overline{u}_j(A, Z = 2) = \overline{u}_j(A, Z = 1) + \overline{\psi}_j(A)$, with the restriction that $\overline{\psi}_j(1) = \overline{\psi}_j(2)$.[3]

Appendix D: Estimation and Identification

D.1 Details on MLE

[3]This restriction is imposed to save the number of parameters. The restricted model cannot be rejected at 10% significance level.

(1) To form the individual likelihood, I need to calculate the following integration:[4]

$$\int I(Y_i|T, SAT_i, B_i, \epsilon)I(d_i|O_i, T, SAT_i, B_i, \epsilon, \zeta, \eta)dG(\epsilon, \zeta, \eta). \tag{7.1}$$

One alternative is to use the frequency simulator, which converges to the true probability when the number of simulation draws gets to infinity. However, this simulator is not smooth and will lead to poor performance of the optimization routine. In this paper, the integration is approximated via a kernel smoothed frequency simulator.

For each student (SAT_i, B_i), I draw shocks $\{(\epsilon_{ir}, \zeta_{ir}, \eta_{ir})\}_{r=1}^{R}$ from their joint distribution $G(\cdot)$.[5] These shocks are the same across T for the same student i, but are i.i.d. across students. All shocks are kept fixed throughout the estimation. Let $v_ir(j)$ be the ex-post value of college j for student$_ir$ with $(T, SAT_i, B_i, \epsilon_ir, \zeta_ir, \eta_ir)$, let $v_ir^* = \max\{0, \{v_ir(j)\}_j \in O_i\}$, let $V_ir(Y)$ be the ex-ante value of portfolio Y for this student, and $V_ir^* = \max_Y \subseteq J\{V_ir(Y)\}$, (see the next subsection for details of this calculation). (7.1) is then approximated by:

$$\frac{1}{R}\sum_{r=1}^{R}\frac{\exp[(V_{ir}(Y_i) - V_{ir}^*)/\tau_1]}{\sum_{Y\subseteq J}\exp[(V_{ir}(Y) - V_{ir}^*)/\tau_1]}\frac{\exp[(v_{ir}(d_i) - v_{ir}^*)/\tau_2]}{\sum_{j\in O_i}\exp[(v_{ir}(j) - v_{ir}^*)/\tau_2]},$$

where τ_1, τ_2 are smoothing parameters, and when $\tau \to 0$, the approximation converges to frequency simulator.

(2) Solving the optimal application problem for student $(T, SAT_i, B_i, \epsilon_{ir})$: Calculating the Emax

With rational expectation, the student's application decision is based on her ex-

[4] I describe the situation where I do not observe any information about the student's financial aid. For students with some financial aid information, the observed financial aid replaces the random draw of the corresponding financial aid shock.

[5] Since shocks are assumed to be independent, each shock is drawn from its marginal distribution.

pectation of post-application values:

$$\begin{aligned} V_i(Y) &= \sum_{O\subseteq Y} \Pr_i(O) E v_{ir}(O) - C(|Y|) \\ &= \sum_{O\subseteq Y} \Pr_i(O) E_{(\eta,\zeta)} \max\{u_{0ir,}\{u_{jir}\}_{j\in O}\} - C(|Y|). \end{aligned}$$

The Emax function has no closed-form expression and is approximated via simulation. For each $(T, SAT_i, B_i, \epsilon_{ir})$, draw M sets of shocks $\{(\eta_m, \zeta_m)\}_{m=1}^{M}$. For each of the M sets of $(T, SAT_i, B_i, \epsilon_{ir}, \eta_m, \zeta_m)$, calculate $\max\{u_{0irm,}\{u_{jirm}\}_{j\in O}\}$, where u_{jirm} denotes u_{jir} evaluated at the shock (η_m, ζ_m). The Emax is the average of these M maximum values.

D.2 Details on Second-Step Estimation

(1) Targets to be matched: for each of the Groups $2, 3$ and 4, there are nine admissions probabilities to be matched $\{(A, SAT)\}_{(A,SAT)\in\{1,2,3\}\times\{1,2,3\}}$. For Group 1, there are six admissions probabilities to be matched: since no one in $SAT = 1$ group applied to Group 1, the admissions probabilities to Group 1 for $\{(A, SAT = 1)\}_{A\in\{1,2,3\}}$ are fixed at 0, and are not included in the target set. The other four targets are the capacities of the four groups simulated from the first step. In all, there are 37 targets to be matched using college-side parameters: $\{P(s|A)\}, \{\kappa_j\}_j$, 10 of which are free.

(2) Optimal Weighting Matrix:

Let Θ^* to denote the true parameter values. The first-step estimates $\widehat{\Theta}_1$, being MLE, are asymptotically distributed as $N(0, \Omega_1)$. It can be shown that the optimal weighting matrix for the second-step objective function (3.3) is $W = Q_1 \Omega_1 Q_1'$, where Q_1 is the derivative of $q(\cdot)$ with respect to $\widehat{\Theta}_1$, evaluated at $\left(\widehat{\Theta}_1, \Theta_2^*\right)$. The estimation of W involves the following steps:

1) Estimate the variance-covariance matrix $\widehat{\Omega}_1$: in the case of MLE, this is minus the outer product of the score functions evaluated at $\widehat{\Theta}_1$. The score functions are

obtained via numerically taking partial derivatives of the likelihood function with respect to each of the first step parameters evaluated at $\widehat{\Theta}_1$.

2) Obtain preliminary estimates $\widetilde{\Theta}_2 \equiv \arg\min_{\Theta_2}\{q(\widehat{\Theta}_1, \Theta_2)'\widetilde{W}q(\widehat{\Theta}_1, \Theta_2)\}$, where $\widetilde{W}$ is any positive-definite matrix. The resulting $\widetilde{\Theta}_2$ is a consistent estimator of Θ_2^*.

3) Estimate Q_1 by numerically taking derivative of $q(\cdot)$ with respect to $\widehat{\Theta}_1$, evaluated at $\left(\widehat{\Theta}_1, \widetilde{\Theta}_2\right)$. In particular, let Δ_m denote a vector with zeros everywhere but the m'th entry, which equals a small number ε_m. At each $\left(\widehat{\Theta}_1 + \Delta_m, \widetilde{\Theta}_2\right)$, I simulate the student decision model and calculate the targets for the second-step estimation; then holding student applications fixed, I solve for college optimal admissions and calculate the distance vector $q\left(\widehat{\Theta}_1 + \Delta_m, \widetilde{\Theta}_2\right)$. The m'th partial component of Q_1 is approximated as $[q\left(\widehat{\Theta}_1 + \Delta_m, \widetilde{\Theta}_2\right) - q\left(\widehat{\Theta}_1, \widetilde{\Theta}_2\right)]/\varepsilon_m$.

D.3 Identification of A Mixture of Two Probits

Assume there are two unobserved types of individuals $A \in \{1, 2\}$, and $\Pr(A = 1) = \lambda$. Let the continuous variable $z \in Z \subseteq R$ be an observed individual characteristics and $f(\cdot)$ be a differentiable function of z. Let $y \in \{0, 1\}$ be the observed discrete choice, which relates to the latent variable y^* in the following way:

$$y(z) = 1 \text{ if only if } y^*(z) \equiv f(z) + u_1 I(A = 1) + u_2 I(A = 2) + \epsilon > 0$$

where $\epsilon \tilde{} i.i.d.N(0, 1)$. The model implies that

$$P(z) \equiv \Pr(y(z) = 1) = \lambda\Phi(f(z) + u_1) + (1 - \lambda)\Phi(f(z) + u_2) \tag{7.2}$$

Theorem 7.1. *Assume that 1) $\lambda \in (0, 1)$, 2) there exists an open set $Z^* \subseteq Z$ such that for $z \in Z^*$, $f'(z) \neq 0$. Then the parameters $\theta = (\lambda, u_1, u_2)'$ in (7.2) are locally identified.*

Proof. The proof draws on the well-known equivalence of local identification with positive definiteness of the information matrix. In the following, I will show the positive definiteness of the information matrix for model (7.2).

Step 1. Claim: *The information matrix $I(\theta)$ is positive definite if and only if there exist no $w \neq 0$, such that $w' \frac{\partial P(z)}{\partial \theta} = 0$ for all z.*

The log likelihood of an observation (y, z) is

$$L(\theta) = y \ln(P(z)) + (1-y)\ln(1 - P(z)).$$

The score function is given by

$$\frac{\partial L}{\partial \theta} = \frac{y - P(z)}{P(z)(1-P(z))} \frac{\partial P(z)}{\partial \theta}.$$

Hence, the information matrix is

$$I(z) = E\left[\frac{\partial L}{\partial \theta}\frac{\partial L}{\partial \theta'}|z\right] = \frac{1}{P(z)(1-P(z))}\frac{\partial P(z)}{\partial \theta}\frac{\partial P(z)}{\partial \theta'}.$$

Given $P(z) \in (0,1)$, it is easy to show that the claim holds.

Step 2. Show $w' \frac{\partial P(z)}{\partial \theta} = 0$ for all $z \Longrightarrow w = 0$.

$\frac{\partial P(z)}{\partial \theta}$ is given by:

$$\begin{aligned}
\frac{\partial P(z)}{\partial \lambda} &= \Phi(f(z) + u_1) - \Phi(f(z) + u_2) \\
\frac{\partial P(z)}{\partial u_1} &= \lambda \phi(f(z) + u_1) \\
\frac{\partial P(z)}{\partial u_2} &= (1-\lambda)\phi(f(z) + u_2)
\end{aligned}$$

Suppose for some w, $w' \frac{\partial P(z)}{\partial \theta} = 0$ for all z :

$$w_1[\Phi(f(z) + u_1) - \Phi(f(z) + u_2)] + w_2 \lambda \phi(f(z) + u_1) + w_3(1-\lambda)\phi(f(z) + u_2) = 0$$

Take derivative with respect to z evaluated at some $z \in Z^*$

$$\begin{aligned}
& w_1[\phi(f(z) + u_1) - \phi(f(z) + u_2)]f'(z) + w_2 \lambda \phi'(f(z) + u_1) f'(z) \\
& + w_3(1-\lambda)\phi'(f(z) + u_2) f'(z) = 0
\end{aligned} \tag{7.3}$$

Define $\gamma(z) = \frac{\phi(f(z)+u_1)}{\phi(f(z)+u_2)}$, divide Equation (7.3) by $\phi(f(z) + u_2)$:

$$w_1[\gamma(z) - 1] - w_2\lambda(f(z) + u_1)\gamma(z) - w_3(1-\lambda)(f(z) + u_2) = 0$$

$$\gamma(z)[w_1 - w_2\lambda(f(z) + u_1)] - [w_1 + w_3(1-\lambda)(f(z) + u_2)] = 0 \tag{7.4}$$

Since $\gamma(z)$ is a nontrivial exponential function of z, (7.4) hold for all $z \in Z^*$ only if both terms in brackets are zero for each $z \in Z^*$, i.e.

$$w_1 - w_2\lambda(f(z) + u_1) = 0 \tag{7.5}$$

$$w_1 + w_3(1-\lambda)(f(z) + u_2) = 0.$$

Take derivative of (7.5) again with respect to z, evaluated at $z \in Z^*$:

$$w_2\lambda f'(z) = 0$$

$$w_3(1-\lambda)f'(z) = 0,$$

which implies

$$w_2\lambda = 0$$

$$w_3(1-\lambda) = 0$$

Since $\lambda \in (0, 1)$, $w = 0$. $\square$

Appendix E: Additional Tables

Table 7.1: Conditional Distributions of SAT and Income

%	$SAT = 1$	$SAT = 2$	$SAT = 3$	$Inc = 1$	$Inc = 2$	$Inc = 3$
$SAT = 1$				36.0	52.1	11.9
$SAT = 2$				15.2	47.8	37.1
$SAT = 3$				9.2	47.3	43.5
White	47.2	38.6	14.2	19.6	52.3	28.1
Black	71.7	24.4	3.9	54.3	37.2	8.5

Table 7.2: Application, Admission: Applicants by Race

(%)	Group 1	Group 2	Group 3	Group 4
White Applicants				
Application Rate	8.2	21.3	40.3	70.5
Admission Rate	58.5	77.5	92.7	95.6
Black Applicants				
Application Rate	2.0	10.1	40.4	81.8
Admission Rate	50.0	60.0	85.0	96.3

Num of applicants: whites (648), blacks (99)

Application rate=num. of applicants to group j/num. of all applicants

Admission rate=num. of admissions to group j/num. of applicants to j

Table 7.3: Application, Admission: Applicants by Race

%	Group 1	Group 2	Group 3	Group 4
$SAT = 1$ Applicants				
Application Rate	0	7.3	42.3	66.7
Admission Rate	N/A	33.3	82.6	87.8
$SAT = 2$ Applicants				
Application Rate	4.5	16.6	37.9	79.1
Admission Rate	45.0	72.6	95.5	96.0
$SAT = 3$ Applicants				
Application Rate	19.7	30.9	44.9	57.9
Admission Rate	65.7	89.1	93.8	100.0

Num of applicants: $SAT1$ (123), $SAT2$ (446), $SAT3$ (178)

Table 7.4: Application, Admission: Applicants by Income

%	Group 1	Group 2	Group 3	Group 4
$Inc = 1$ Applicants				
Application Rate	3.9	14.7	37.2	72.5
Admission Rate	25.0	73.3	92.1	93.4
$Inc = 2$ Applicants				
Application Rate	6.6	13.7	38.6	72.0
Admission Rate	66.7	76.0	90.1	95.1
$Inc = 3$ Applicants				
Application Rate	9.6	29.6	43.6	71.8
Admission Rate	55.6	77.1	93.4	97.5

Num of applicants: Inc1 (102), Inc2 (365), Inc3 (280)

Table 7.5: Non Pecuniary College Value: A Simulated Example

	All	Applicants	Attendees
$\widetilde{u}_1(A = 1, Z = 1)$	−234068.4	85945.8	99942.4
	(115117.4)	(30811.5)	(34730.2)
$\widetilde{u}_1(A = 2, Z = 1)$	−222834.1	117083.6	157089.1
	(115493.2)	(43664.9)	(44945.0)
$\widetilde{u}_1(A = 3, Z = 1)$	−57699.4	134435.5	159033.6
	(115636.8)	(58596.4)	(58609.5)
$\widetilde{u}_1(A = 1, Z = 2)$	−74090.3	108956.4	126051.4
	(115117.4)	(50605.7)	(50773.6)
$\widetilde{u}_1(A = 2, Z = 2)$	−62856.0	133911.5	158914.7
	(115493.2)	(57014.3)	(58820.5)
$\widetilde{u}_1(A = 3, Z = 2)$	123994.5	187099.1	211289.7
	(115637.1)	(82266.7)	(79100.2)

Table 7.6: Model Fit: Distribution of Portfolio Size (by Race)

%	White		Black	
	Data	SPNE	Data	SPNE
0	53.3	53.0	61.6	70.8
1	31.6	33.3	27.9	21.5
2	11.7	10.3	8.1	6.0
3	3.1	3.2	1.9	1.6
4	0.3	0.2	0.3	0.1
χ^2 Stat.		3.36		10.5

PE: Partial Equilibrium Model;

χ^2: Size 2-4 are combined due to small cell size in the sample.
$\chi^2_{2,0.05}$= 5.99

Table 7.7: Model Fit: Distribution of Portfolio Size (by SAT)

%	$SAT = 1$		$SAT = 2$		$SAT = 3$	
	Data	SPNE	Data	SPNE	Data	SPNE
0	85.4	86.4	25.5	27.6	14.0	12.7
1	12.7	12.1	50.0	49.4	49.7	58.1
2	1.4	1.5	19.0	17.0	28.0	21.4
3	0.5	0.0	5.0	5.5	6.8	7.2
4	0.0	0.0	0.3	0.4	1.4	0.5
χ^2 Stat.		1.31		1.54		6.37

χ^2 : Size 2-4 are combined due to small cell size in the sample.
$\chi^2_{2,0.05}$= 5.99

Table 7.8: Model Fit: Distribution of Portfolio Size (by Income)

%	$Inc = 1$		$Inc = 2$		$Inc = 3$	
	Data	SPNE	Data	SPNE	Data	SPNE
0	75.2	74.0	55.6	58.2	32.0	32.7
1	19.7	19.5	32.4	30.1	39.6	46.1
2	3.6	5.1	10.3	8.9	20.4	15.5
3	1.0	1.3	1.7	2.6	7.3	5.3
4	0.5	0.1	0.0	0.2	0.7	0.4
χ^2 Stat.		1.34		2.43		13.9

χ^2 : Size 2-4 are combined due to small cell size in the sample.
$\chi^2_{2,0.05}$= 5.99

Table 7.9: Model Fit: Application and Admission Among Applicants (by Race)

(%)	Whites		Blacks	
Application Rate	Data	SPNE	Data	SPNE
Group 1	8.2	7.7	2.0	5.3
Group 2	21.3	20.9	10.1	14.0
Group 3	40.3	41.5	40.4	45.6
Group 4	70.5	66.8*	81.8	67.9*
Admission Rate				
Group 1	58.5	44.2	50.0	35.7
Group 2	77.5	83.4	60.0	78.2
Group 3	92.7	98.8*	85.0	97.0*
Group 4	95.6	97.2*	96.3	96.9

* $\chi^2 > \chi^2_{1,0.05}$

Table 7.10: Model Fit: Application and Admission Among Applicants (by SAT)

%	$SAT = 1$		$SAT = 2$		$SAT = 3$	
Application	Data	SPNE	Data	SPNE	Data	SPNE
Group 1	0.0	0.0	4.5	7.4*	19.7	12.0*
Group 2	7.3	0.0*	16.6	20.8	30.9	31.5
Group 3	42.3	44.7	37.9	41.5	44.9	41.0
Group 4	66.7	66.2	79.1	70.7*	57.9	58.5
Admission						
Group 1	N/A	N/A	45.0	28.2	65.7	66.7
Group 2	33.3	N/A	72.6	72.8	89.1	99.4*
Group 3	82.6	91.7*	95.5	100*	92.8	100.0*
Group 4	87.8	96.2*	96.0	97.7	100.0	97.8

* $\chi^2 > \chi^2_{1,0.05}$

Table 7.11: Model Fit: Application and Admission Among Applicants (by Income)

(%)	$Inc = 1$		$Inc = 2$		$Inc = 3$	
Application	Data	SPNE	Data	SPNE	Data	SPNE
Group 1	3.9	4.6	6.6	6.4	9.6	9.8
Group 2	14.7	12.9	13.7	17.6	29.6	26.1
Group 3	37.2	45.8	38.6	42.6	43.6	39.4
Group 4	72.5	67.3	72.0	68.2	71.8	65.3*
Admission						
Group 1	25.0	35.6	66.7	42.6*	55.6	45.7
Group 2	73.3	78.8	76.0	81.7	77.1	84.8
Group 3	92.1	96.7	90.1	98.5*	93.4	99.5*
Group 4	93.4	96.9	95.1	97.4*	97.5	97.8

* $\chi^2 > \chi^2_{1,0.05}$

Table 7.12: Tuition Fit When m=0

	Group 1	Group 2	Group 3	Group 4
Data	27009	5347	17201	3912
SPNE($m = 0$)	36933	819	2935	69

Table 7.13: Tuition Fit in Step-3

	Top Priv.	Top Pub.	Low Priv.	Low Pub.
Data	27009	5347	17201	3912
Best Response	27579	4954	18010	3921

Table 7.14: Robustness Check: Perfect Signal-Tuition

	Group 1	Group 2	Group 3	Group 4	
Base SPNE	26940	4773	19907		4392
New SPNE	27004	3825	17251		3718

All the tables showing robustness check assume $\omega = [1, 1.4, 2]'$.
For other ω's around $[1, 2, 3]$, the results are similarly robust.

Table 7.15: Robustness Check: Perfect Signal-Distribution of Portfolio Sizes

%	$Size = 0$	$Size = 1$	$Size = 2$	$Size = 3$	$Size = 4$
Base SPNE	55.9	31.8	9.2	2.9	0.2
New SPNE	57.1	34.5	7.4	1.0	0.1

Table 7.16: Robustness Check: Perfect Signal-Admission Rates

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	44.1	82.7	99.0	98.2
New SPNE	93.8	97.7	97.5	99.2

Table 7.17: Robustness Check: Perfect Signal-Ability Distribution Within Each Destination

%	Group 1	Group 2	Group 3	Group 4	Outside
Base SPNE					
$A = 1$	3.4	0.2	7.4	0.8	95.0
$A = 2$	6.2	15.3	81.1	92.1	2.9
$A = 3$	90.4	84.5	11.5	7.1	2.1
New SPNE					
$A = 1$	0.0	0.0	1.8	0.0	96.4
$A = 2$	0.0	0.1	86.6	94.2	2.3
$A = 3$	100.0	99.9	11.5	5.8	1.3

Table 7.18: Robustness Check: Perfect Signal-Mean Student Welfare

	Base SPNE ($)	New SPNE($)	Change ($)	Change (%)
All	41396	43575	2179	5.3
$A = 1$	670	161	-509	-76.0
$A = 2$	98248	102550	4302	4.4
$A = 3$	84550	95740	11190	13.2

Table 7.19: Robustness Check: Funding Cuts-Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26940	4773	19907	4392
New SPNE	27103	5394	20095	4821

Table 7.20: Robustness Check: Funding Cuts- Mean Student Welfare

	Base SPNE	New SPNE($)	Change ($)
All	41396	40639	-757
$A = 1$	670	647	-23
$A = 2$	98248	96497	-1751
$A = 3$	84550	82903	-1647

Table 7.21: Robustness Check: Increasing Supply-Tuition

	Group 1	Group 2	Group 3	Group 4
Base SPNE	26940	4773	19907	4392
$SPNE_3$	27386	7397	13862	3103
$SPNE_4$	27534	6890	18176	98

Table 7.22: Increasing Supply: Admission Rates

%	Group 1	Group 2	Group 3	Group 4
Base SPNE	44.1	82.7	99.0	98.2
$SPNE_3$	48.7	95.3	100.0	99.4
$SPNE_4$	47.3	95.3	99.8	100.0

Table 7.23: Robustness Check: Increasing Supply- Attendance Rate

Base SPNE	$SPNE_3$	$SPNE_4$
40.9	43.0	43.0