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Dedication

To my wife Angela for making this possible.

Abstract

This dissertation consists of three essays.

In the first essay, Enoch Hill and I present a general equilibrium model where heterogeneous consumers endogenously choose whether to become workers, consumers or entrepreneurs in order to analyze how limits on the leverage of banks affect real output. In our model tighter limits on the leverage of banks cause an increase in the spread between the interest rate that banks charge for loans and the interest rate that banks pay for deposits. A higher spread results in two types of distortions: First, firms with the same productivity will have different size. Second, productive firms will cease to exist, while nonproductive ones will enter. These distortions result in lower production.

In the second essay, Enoch Hill and I develop a general equilibrium model of theft, private security and public law enforcement (PLE) which matches both macro and micro empirical evidence. We find a non-monotonic relation between PLE and aggregate production. In particular, for countries with relatively small amounts of PLE, increasing the level can result in a reduction of aggregate production and welfare primarily due to an increase in the incarceration rate. However, for countries with higher levels of PLE, an increase in the level improves production and welfare. We also find the private security causes a negative externality in economies with low levels of PLE.

In the third essay, Enoch Hill, Michael Maio and I propose an original model of firm hierarchy which suggests that firm structure is important for understanding the wage structure. In our model, more productive firms choose to employ more levels of management, which requires a higher average level of skill in workers and consequently a higher average skill premium. This is consistent with what we document in the Chilean data and also agrees with the firm size to skill premium relationship commonly documented in the literature. Additionally, our model predicts that skill premium is increasing in the ratio of workers to managers, a fact we also observe in the Chilean data.

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Chapter 1

Leverage Away Your Wedge: An Analysis of Banks' Impact on Output

1.1 Introduction

The banking sector doesn't produce a tangible product but it is clear that it has a tangible effect on the real economy. In this paper we develop a model which allows us to analyze what these effects are and the channel through which these effects are transmitted. Using our model we find that the leverage of banks has both direct and indirect effects on occupational choice, and also indirectly effects the distribution of firm sizes, and real output. The primary channel through which these effects are transmitted is through the spread between the interest rate that banks charge for loans and the interest rate that banks pay for deposits which we will henceforth refer to as the margin of intermediation. We will consider a model without risk so we will only focus on the downside of having limits on the leverage of banks.

In our model firms need to pay for their workers before they produce, in the spirit of Arellano, Bai, and Kehoe (2012). To do this, they can either use their own assets or can take out loans from the banking sector. The primary result from our model is that as banks become less leveraged (i.e. the ratio between deposits over equity decreases), the resulting margin of intermediation in the general equilibrium increases. This margin is responsible for two types of distortion relative to a model with an unconstrained banking sector: First, firms with the same productivity will hire different amounts of workers depending on the assets of the firm. Second, skilled unwealthy consumers will choose to work rather than become entrepreneurs, while nonskilled wealthy consumers will choose to manage firms. In addition, lower leverage in the banking sector will require more bankers to satisfy the demand for loans and deposits. Each of these factors results in a reduction in real output.

An appealing feature of our model is its clear and intuitive characterization of occupational choice among consumers. We allow heterogeneity in consumers along two dimensions; namely, wealth and skill. Rich unskilled consumers choose to become bankers while unwealthy unskilled consumers become workers and skilled consumers choose to manage firms as entrepreneurs. In the parameterization where banks are infinitely leveraged, real allocations are not dependant upon wealth and the model collapses to the model in Lucas (1978) where wealth only affects consumption but has no effect on occupational choice or real output. In this case skilled consumers choose to become entrepreneurs while unskilled consumers choose to work; and the marginal productivity across firms is constant and firms are perfectly assortative in size along the skill of entrepreneurs.

As the leverage of the banking sector decreases, the margin of intermediation increases. As a consequence, the wealth of consumers begin to have real effects on occupational choices of consumers and the hiring decisions of firms, since this causes the cost of the marginal worker to differ based on the wealth of the entrepreneur. Essentially, wealthy entrepreneurs face a lower marginal cost per worker than unwealthy entrepreneurs, which causes firm size to vary across the wealth of entrepreneurs. For two entrepreneurs with the same skill level, the wealthier entrepreneur will hire more workers than the less wealthy entrepreneur. If the skill of the entrepreneurs is sufficiently small, it is possible that the unwealthy consumer will prefer to become a worker rather than manage a firm and face the higher interest rate on loans required to hire workers. Additionally, it is possible that wealthy consumers, who would have worked in the scenario where banks are infinitely leveraged, are incented to become entrepreneurs due to the reduction in the return of their assets. Even though their skill at managing workers is low relative to the rest of the entrepreneurs, they can obtain a higher return from using their wealth to hire workers rather than investing their assets with banks.

We relate our model to the misallocation literature. Banerjee and Moll (2010), Buera, Kaboski, and Shin (2011) and Midrigan and Xu (2014), among others, analyze the role of misallocation on the productivity of an economy. In these models, misallocation arises mostly due to financial constraints. In our case, it is the leverage of the financial intermediaries that causes the misallocation.

Our model is also related to the literature relating financial development and efficiency with production. Levine (2005) offers a comprehensive literature review of this field. In Greenwood, Sanchez, and Wang (2010) and Greenwood, Sanchez, and Wang (2013) costly state verification causes a difference in the marginal product of capital and its user cost. In our model it is the leverage of the banking sector that causes different consumers to face different interest rates.

Erosa (2001) is probably the most similar model to ours. In his model intermediation costs cause an exogenous margin of intermediation. In our case this margin of intermediation arises endogenously from the leverage of the banking sector. Erosa (2001) also analyzes occupational choice, although in his setup the only heterogeneity of consumers is in age, as his model is dynamic. We explore an additional occupational choice; namely, becoming a banker.

This paper is organized as follows: Section 1.2 presents the model; Section 1.3 characterizes the solution to the model; Section 1.4 presents a benchmark model without limits on the leverage of banks; Section 1.5 shows the main results; Section 1.6 highlights preliminary relations we observe in data. Finally, Section 1.7 concludes.

1.2 Model

We consider a two period model. Consumers are heterogeneously endowed with skill and wealth. At the beginning of the first period they choose whether to become workers, entrepreneurs or bankers. Workers receive their wages in the first period and save to consume in the second period. Entrepreneurs manage firms and need to pay their workers in the first period. Nonetheless, the firms they manage produce in the second period. Therefore they might need to borrow from banks in order to pay their wage bills. Bankers set up a bank in the first period and receive the profits from the bank in the second period. Banks take deposits from consumers and lend to firms. They face an exogenous limit on their leverage.

We assume that there is a unit measure of consumers who maximize utility by choosing to be an entrepreneur, a worker or a banker. Each consumer is endowed with a skill level z and some wealth a. We assume that z and a are drawn from a distribution with positive support that we will denote by $G(z \times a)$. A consumer's decision is characterized by z and a, so we will denote consumers by the realizations of these random variables. Consider consumer (z, a). He solves the following problem

$$u(z,a) = \max_{\mathcal{O}} \left\{ u^{\mathcal{W}}(a), u^{\mathcal{E}}(z,a), u^{\mathcal{B}}(a) \right\}, \tag{1.1}$$

where $u^{\mathcal{W}}(a)$ denotes the utility derived from becoming a worker, $u^{\mathcal{E}}(z,a)$ is the utility from becoming an entrepreneur, and $u^{\mathcal{B}}(a)$ is the utility from becoming a banker.

1.2.1 Workers

Denote the set of workers by $\mathcal{W}$. In period 1 workers use their wages w and wealth to consume and to save, s. In the second period the worker's income is given by the return on savings, r^D. The utility of being a worker with wealth a is given by (1.2).

$$\begin{aligned} u^{\mathcal{W}}(a) &= \max_{s} \ln c_1 + \beta \ln c_2 \\ c_1 &= w + a - s \\ c_2 &= (1 + r^D)s. \end{aligned} \tag{1.2}$$

1.2.2 Entrepreneurs

Denote the set of entrepreneurs by $\mathcal{E}$. In the first period entrepreneurs use their wealth to consume, to pay for the workers l they hire, and they can save or borrow. In the second period entrepreneurs consume the production of the firm. If they borrowed in the first period, they repay their debt at an interest rate of r^L. If they saved, they get a return of r^D on their savings. The utility of being an entrepreneur with skill z and wealth a is given by (1.3).

$$\begin{aligned} u^{\mathcal{E}}(z,a) = \max_{s,l} \ln c_1 + \beta \ln c_2 \\ c_1 = a - wl - s \\ c_2 = zl^{\alpha} + (1+r^L)s\mathbf{1}_{\{s<0\}} \\ + (1+r^D)s\mathbf{1}_{\{s\geq 0\}}. \end{aligned} \tag{1.3}$$

Each entrepreneurs belongs to one of three types: Entrepreneurs that borrow to pay for their workers, $s < 0$, entrepreneurs that have enough wealth to pay for workers and deposit the difference, $s > 0$, and entrepreneurs that spend all their available wealth to hire workers, $s = 0$. We will denote by $\mathcal{E}_L$ the set of entrepreneurs that borrow, by $\mathcal{E}_D$ the set of entrepreneurs that save and by $\mathcal{E}_O$ the rest of entrepreneurs.

1.2.3 Bankers

Denote the set of bankers by $\mathcal{B}$. In the first period bankers consume part of their wealth. The rest of their wealth, s, is used as equity for the bank they manage. In the second period bankers consume the profits from that bank. The utility of a banker with wealth a is given by (1.4).

$$\begin{aligned} u^{\mathcal{B}}(a) = \max_{s} \ln c_1 + \beta \ln c_2 \\ c_1 = a - s \\ c_2 = \pi^B(s). \end{aligned} \tag{1.4}$$

The profits of a bank with equity s are given by (1.5)

$$\begin{aligned} \pi^B(s) \equiv \max_{L,D} \ (1+r^L)L - (1+r^D)D \\ \text{s. t. } D + s = L \\ \frac{D}{s} \leq \lambda. \end{aligned} \tag{1.5}$$

The first constraint in (1.5) is the balance sheet constraint of the bank: The bank lends its equity and the deposits it takes. The second constraint implies that there is a limit on how many resources a bank can intermediate. Specifically the limit is on how many deposits a bank can take per unit of equity. This limit is exogenous and we denote it by λ.

We now define an equilibrium for this economy:

Definition 1. *An equilibrium for this economy is allocations* $x^{\mathcal{W}}(a) \equiv \{s^{\mathcal{W}}(a)\}$, $x^{\mathcal{E}}(z,a) \equiv \{l(z,a), s^{\mathcal{E}}(z,a)\}$, $x^{\mathcal{B}}(a) \equiv \{s^{\mathcal{B}}(a)\}$ *and* $x^{B}(s) \equiv \left\{L^{B}(s), D^{B}(s)\right\}$, *prices* $p \equiv \{w, r^{L}, r^{D}\}$ *and sets* $\mathcal{W}$, $\mathcal{B}$ *and* $\mathcal{E}$ *such that*

1. $\mathcal{W}$, $\mathcal{B}$ *and* $\mathcal{E}$ *are such that* $\mathcal{O}(z,a)$ *is a solution to* (1.1) *for all* (z,a);
2. *given* p, $x^{\mathcal{W}}(a)$ *is a solution to* (1.2);
3. *given* p, $x^{\mathcal{E}}(z,a)$ *is a solution to* (1.3);
4. *given* p, $x^{\mathcal{B}}(a)$ *is a solution to* (1.4);
5. *given* p, $x^{B}(s)$ *is a solution to* (1.5);
6. *and markets clear:*

   (a) *Deposits:*

$$\int_{\mathcal{W}} s^{\mathcal{W}}(a)dG(z\times a) + \int_{\mathcal{E}_D} s^{\mathcal{E}}(z,a)dG(z\times a) = \int_{\mathcal{B}} D^{B}(s^{\mathcal{B}}(a))dG(z\times a);$$

   (b) *loans:*

$$\int_{\mathcal{E}_L} s^{\mathcal{E}}(z,a)dG(z\times a) + \int_{\mathcal{B}} L^{B}(s^{\mathcal{B}}(a))dG(z\times a) = 0;$$

   (c) *labor:*

$$\int_{\mathcal{W}} dG(z\times a) = \int_{\mathcal{E}} l(z,a)dG(z\times a);$$

   (d) *goods:*

$$\int c_1(z,a)dG(z\times a) = \int a dG(z\times a)$$
$$\int c_2(z,a)dG(z\times a) = \int_{\mathcal{E}} z l(z,a)^{\alpha} dG(z\times a).$$

1.3 Characterizing the model

We will first prove that in equilibrium the interest rate of loans is greater than the interest rate on deposits. This implies that there is an incentive to manage a bank, rather to deposit in one. Additionally, the profits from banks are linear in the wealth that is used to run them.

Lemma 1. *In equilibrium $r^L \geq r^D > -1$ and $r^L = r^D$ only as $\lambda \to \infty$. Furthermore, the profits of a bank with equity s can be written as $\pi^B(s) = (1 + r^B)s$ with*

$$r^B = r^L + \lambda(r^L - r^D);$$

$r^B \geq r^D$ and $r^B = r^D$ only as $\lambda \to \infty$. Additionally, loan supply and deposit demand are given by

$$\begin{aligned} D^d(s) &= \lambda s \\ L^s(s) &= (1 + \lambda)s. \end{aligned} \tag{1.6}$$

Proof. First notice that if $r^D \leq -1$, then banks will demand an infinite amount of deposits. If $r^L < r^D$ then banks will not supply loans since the cost of deposits is higher than the revenue they can get from lending, so it must be the case that $r^L \geq r^D$. Now, the bank is risk neutral. Therefore the second constraint in (1.5) binds. The supply of loans follows from the balance sheet constraint $(D(s) + s = L(s))$. This proves (1.6).

Plugging (1.6) into the objective function of (1.5) yields $\pi^B(s) = (1 + r^B)s$, where $r^B \equiv r^L + \lambda(r^L - r^D)$. Now, notice that the only way to have a finite r^B as $\lambda \to \infty$ is if $r^L = r^D$. Additionally, r^B can also be written as $r^B = r^D + (1+\lambda)\left(r^L - r^D\right)$. $r^L \geq r^D$ implies that $r^B \geq r^D$, with equality only as $\lambda \to \infty$. □

Now we will prove that both workers and bankers save in the first period. The reason for this is that these consumers have no source of income in the second period. Lemma 2 characterizes the solution of (1.2) and (1.4).

Lemma 2. *The solution of* (1.2) *is*

$$s^{\mathcal{W}}(a) = \frac{\beta}{1+\beta}(w + a). \tag{1.7}$$

The solution of (1.4) *is*

$$s^{\mathcal{B}}(a) = \frac{\beta}{1+\beta}a. \tag{1.8}$$

Proof. The first order condition of (1.2) is

$$\frac{1}{w+a-s} = \frac{\beta}{s}.$$

Lemma 1 implies that the first order condition of (1.4) can be written as

$$\frac{1}{a-s} = \frac{\beta}{s}.$$

Solving for s in (1.7) and (1.8) yields the result. □

Finally we will characterize the solution of (1.3). Since entrepreneurs that borrow and entrepreneurs that save face a different interest rate, there will be misallocation: Firms with the same productivity have different sizes depending on the wealth of the entrepreneur that manages them. The misallocation depends on the margin of intermediation.

Lemma 3. *The solution of* (1.3) *is*

$$l(z,a) = \begin{cases} \left(\frac{\alpha z}{(1+r^L)w}\right)^{\frac{1}{1-\alpha}} & \text{if } a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \\ \frac{\alpha\beta}{1+\alpha\beta}\frac{a}{w} & \text{if } \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \le a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \\ \left(\frac{\alpha z}{(1+r^D)w}\right)^{\frac{1}{1-\alpha}} & \text{if } a \ge \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \end{cases}$$

$$s^{\mathcal{E}}(z,a) = \begin{cases} \frac{\beta}{1+\beta}a - \frac{1+\alpha\beta}{1+\beta}\left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} & \text{if } a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \\ 0 & \text{if } \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \le a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \\ \frac{\beta}{1+\beta}a - \frac{1+\alpha\beta}{1+\beta}\left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} & \text{if } a \ge \delta_{\mathcal{E}_D,\mathcal{E}_O}(z), \end{cases}$$

where

$$\delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \equiv \frac{1+\alpha\beta}{\beta}\left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}$$
$$\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \equiv \frac{1+\alpha\beta}{\beta}\left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

Entrepreneurs with $a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$ *will borrow, entrepreneurs with* $a \geq \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ *will save and entrepreneurs with* $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ *will spend all their available wealth in paying for workers.*

Proof. If $s \neq 0$ the first order conditions of (1.3) can be written as

$$\frac{w}{a-wl-s} = \frac{\beta\alpha z l^{\alpha-1}}{zl^\alpha+(1+r)s}$$
$$\frac{1}{a-wl-s} = \frac{\beta(1+r)}{zl^\alpha+(1+r)s},$$

where $r = r^D$ if $s > 0$ and $r = r^L$ if $s < 0$. If $s = 0$ first order conditions of (1.3) can be written as

$$\frac{w}{a-wl} = \frac{\alpha\beta}{l}.$$

The proof of the Lemma follows from solving for s and l. The expressions for $\delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ and $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$ follow from analyzing when $s > 0$ or $s < 0$. □

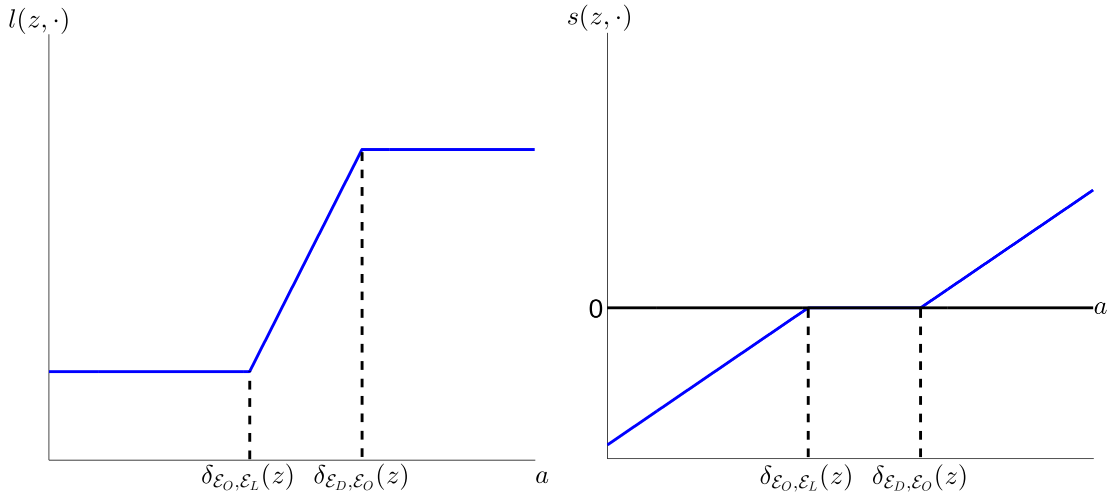
For the moment fix w. Then there are two effects on entrepreneurs of having a positive margin of intermediation: The larger this margin is, the bigger the range in firm sizes for consumers with the same skill z across the spectrum of wealth a.

Wealthy entrepreneurs will have enough wealth to pay for their workers and save the difference. Due to this, the marginal cost of an employee will depend on r^D. On the other hand, unwealthy entrepreneurs need to borrow to pay for their workers, so the marginal cost of an employee will depend on r^L. Figure 1.1a highlights this point.

Additionally, a higher margin of intermediation implies that the difference in wealth between an entrepreneur that is able to save and an entrepreneur that borrows is higher. In other words, holding w constant, a higher margin of intermediation implies a larger difference between $\delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ and $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$. As a consequence less entrepreneurs use banks as financial intermediaries, since more entrepreneurs use all their available wealth to hire workers. See Figure1.1b for a graphical representation of this point.

Figure 1.1: Entrepreneur with skill z

(a) $l(z,a)$ **(b)** $s^{\mathcal{E}}(z,a)$



Corollary 1 shows misallocation in a slightly different way. Let $r(z,a)$ be the marginal return of hiring $l(z,a)$ workers. This return will be decreasing in the wealth of the entrepreneur. $r(z,a)$ is the opportunity cost of using wealth for hiring workers. If an unwealthy entrepreneur uses one extra dollar to hire workers, he is borrowing more and therefore is spending r^L. On the other hand, an extra unit of wealth that a wealthy entrepreneur spends on hiring workers could be used to get a return of r^D if it was used instead to save in a bank.

Corollary 1. *Let* $r(z,a) \equiv \frac{z\alpha l(z,a)^{\alpha-1}}{w} - 1$. *Then*

$$r(z,a) = \begin{cases} r^L & \text{if } a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \\ z\left(\frac{\alpha}{w}\right)^{\alpha}\left(\frac{1+\alpha\beta}{\beta a}\right)^{1-\alpha} - 1 & \text{if } \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \\ r^D & \text{if } a \geq \delta_{\mathcal{E}_D,\mathcal{E}_O}(z). \end{cases}$$

$r(z,a)$ *is continuous and decreasing in* a.

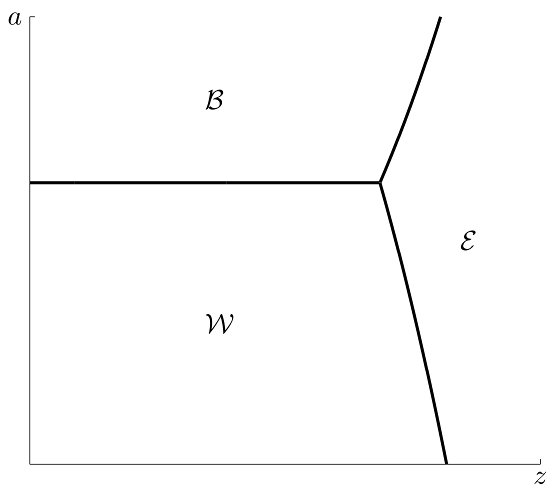
With the results shown in Lemmas 2 and 3 we are able to determine the occupational choice of consumers. In A.1 we determine explicitly the boundaries in skill and wealth that determine the occupational choice of consumers as a function of the prices in this economy. That allows us to fully characterize each consumer, taking prices as given.

From Lemma 3 we conclude that within the set of entrepreneurs, wealthy entrepreneurs (high a) will be able to save and unwealthy entrepreneurs (low a) will need to borrow. In general we find that consumers with low wealth a and low skill z will choose to become workers. The fact that these consumers have low skill makes it better for them to work than to set up a firm. Additionally, their low wealth makes it optimal for them to get an extra source of income in the first period. The only way to do this is by becoming a worker.

On the other hand, consumers with low skill and high wealth will become bankers. Similar to workers, having a low skill level is a deterrent from becoming an entrepreneur. Nonetheless, the high level of wealth makes it better for these consumers to set up a bank, rather than to become workers, since $r^B > r^D$ in equilibrium. Finally, entrepreneurs will be consumers with high skill. As shown in Lemma 3, the level of wealth will affect the size of the firm that they manage.

It is worth mentioning that the occupational choice of consumers depends on their wealth. Figure 1.2 shows graphically the different occupation choices of consumers in (z, a) space.

Figure 1.2: Types of consumers depending on skill and wealth



$\mathcal{B}$: Bankers. $\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

1.4 Model with perfectly efficient banking sector

Contrast the model characterized in Section 1.3 with a model where the margin of intermediation is 0. In this model there will only be an interest rate r. As mentioned in Lemma 1, this can be achieved in the limit as λ approaches infinity. Recall from Lemma 1 that in this case $r^B = r^L = r^D$, so only a consumer with infinite wealth will be willing to be a banker and this bank will be infinitely leveraged and have 0 profits. We can interpret this case as a model where consumers do not need a financial intermediary to get wealth from consumers that are willing to save to borrowing entrepreneurs. In this case consumers choose whether to become entrepreneurs or workers. The utility of a consumer endowed with skill level z and wealth a is given by

$$u(z,a) = \max_{\mathcal{O}} \left\{ u^{\mathcal{E}}(z,a), u^{\mathcal{W}}(a) \right\}, \tag{1.9}$$

where

$$\begin{aligned} u^{\mathcal{W}}(a) &= \max_{s} \ln c_1 + \beta \ln c_2 \\ c_1 &= w + a - s \\ c_2 &= (1+r)s. \end{aligned} \tag{1.10}$$

and

$$\begin{aligned} u^{\mathcal{E}}(z,a) &= \max_{s,l} \ln c_1 + \beta \ln c_2 \\ c_1 &= a - wl - s \\ c_2 &= zl^{\alpha} + (1+r)s. \end{aligned} \tag{1.11}$$

In this case, entrepreneurs face the same interest rate, regardless if they borrow or save. Workers, as before, will decide to save since they don't have any source of income in the second period. An equilibrium for this economy is defined as follows.

Definition 2. *An equilibrium for this economy is allocations* $x^{\mathcal{W}}(a) \equiv \{s^{\mathcal{W}}(a)\}$ *and* $x^{\mathcal{E}}(z,a) \equiv \{l(z), s^{\mathcal{E}}(z,a)\}$*, prices* $p \equiv \{w, r\}$ *and sets* $\mathcal{W}$ *and* $\mathcal{E}$ *such that*

1. *$\mathcal{W}$ and $\mathcal{E}$ are such that $\mathcal{O}(z,a)$ is a solution to* (1.9) *for all* (z,a)*;*

2. *given p, $x^{\mathcal{W}}(a)$ is a solution to* (1.10)*;*

3. *given p, $x^{\mathcal{E}}(z,a)$ is a solution to* (1.11)*;*

4. *and markets clear:*

   (a) *Savings:*
$$\int_{\mathcal{W}} s^{\mathcal{W}}(a)dG(z \times a) + \int_{\mathcal{E}} s^{\mathcal{E}}(z,a)dG(z \times a) = 0;$$

   (b) *labor:*
$$\int_{\mathcal{W}} dG(z \times a) = \int_{\mathcal{E}} l(z)dG(z \times a);$$

   (c) *goods:*
$$\int c_1(z,a)dG(z \times a) = \int adG(z \times a))$$
$$\int c_2(z,a)dG(z \times a) = \int_{\mathcal{E}} zl(z)^{\alpha}dG(z \times a).$$

Lemmas 4 and 5 characterize the solution to (1.10) and (1.11).

Lemma 4. *The solution of* (1.10) *is*

$$s^{\mathcal{W}}(a) = \frac{\beta}{1+\beta}(w+a).$$

Proof. The first order condition of (1.10) is

$$\frac{1}{w+a-s} = \frac{\beta}{s}.$$

Solving for s yields the result. □

Lemma 5. *The solution of* (1.11) *is*

$$l(z) = \left(\frac{\alpha z}{(1+r)w}\right)^{\frac{1}{1-\alpha}}.$$

$$s^{\mathcal{E}}(z,a) = \frac{\beta}{1+\beta}a - \frac{1+\alpha\beta}{1+\beta}\left(\frac{z}{1+r}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

Let

$$\delta_{\mathcal{E}_D,\mathcal{E}_L}(z) \equiv \frac{1+\alpha\beta}{\beta}\left(\frac{z}{1+r}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

Entrepreneurs will save if $a \geq \delta_{\mathcal{E}_D,\mathcal{E}_L}(z)$ *and borrow otherwise.*

Proof. The first order conditions of (1.11) can be written as

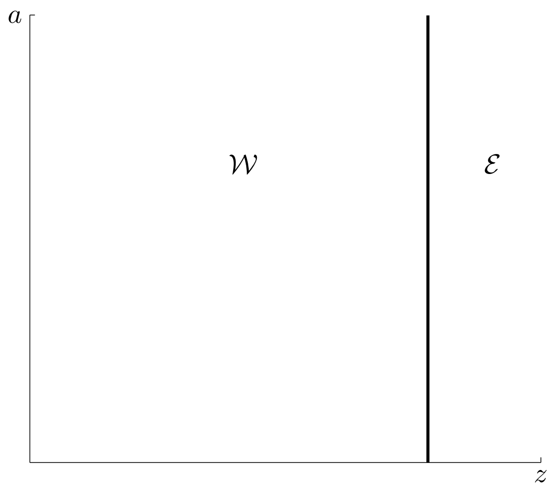
$$\begin{aligned}\frac{w}{a-wl-s} &= \frac{\beta\alpha z l^{\alpha-1}}{zl^{\alpha}+(1+r)s}\\ \frac{1}{a-wl-s} &= \frac{\beta(1+r)}{zl^{\alpha}+(1+r)s},\end{aligned}$$

The proof of the Lemma follows from solving for s and l. The expression for $\delta_{\mathcal{E}_D,\mathcal{E}_L}(z)$ follows from analyzing when $s > 0$ or $s < 0$. □

Notice that in this case the size of the firms does not depend on the wealth of the entrepreneur. With the results shown in Lemmas 4 and 5 we are able to determine the occupational choice of consumers. In A.2 we determine the explicit boundaries between the two occupational choices of consumers as functions of w and r.

Since every consumer faces the same interest rate, the boundary that determines the occupational choice between workers and entrepreneurs will not depend on the level of wealth. Similar to Lucas (1978), the occupation choice depends exclusively on skill level. Less skilled consumers will become workers since the consumption they get from setting up a firm would be lower than consumption from working. Figure 1.3 shows a graphical characterization of consumers in the (z, a) space.

Figure 1.3: Types of consumers depending on skill and wealth



$\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

We solve this model by stating and solving an equivalent Social Planner Problem (See A.3).

1.5 Results

We first prove a lemma that allows us to characterize labor remuneration in the model. We then provide an example which allows us to get a closed form solution. Finally we show some numerical results to highlight the main results of our model.

1.5.1 Labor remuneration is constant

Lemma 6 proves that labor remuneration in the model is constant.

Lemma 6. *Let A be the total amount of wealth in the economy and denote the mass of workers by $M_{\mathcal{W}}$. Then*

$$wM_{\mathcal{W}} = \alpha\beta A.$$

Proof. See A.4. □

We give an overview of the proof of Lemma 6. Consider first the case where wages are paid in the same period as when production takes place. Given the production function of the firms in our model, it holds that $wM_{\mathcal{W}} = \alpha Y$, where Y denotes total production. Now, in our model wages are paid in period 1, while production takes place in period 2. Therefore the marginal cost of labor depends on interest rates. that is,

$$(1+\tilde{r})wM_{\mathcal{W}} = \alpha Y, \tag{1.12}$$

where $\tilde{r}$ is an average interest rate of the economy.[1] Additionally, in our model total consumption in the first period is given by the total amount of wealth in the economy, A, while total consumption in the second period equals total production, Y. Finally, the fact that consumers have log utility implies the following aggregate Euler equation

$$Y = \beta(1+\tilde{r})A. \tag{1.13}$$

Plugging in (1.12) into (1.13) yields the result in the Lemma. Lemma 6 implies that the effect of productivity affects wages indirectly through the measure of workers. In other words, more productive economies will have higher wages since the measure of workers will be smaller.

1.5.2 Example

We consider a particular distribution that allows us to find a close form solution to the model. Given constraints on parameters, we are able to abstract from changes in occupational choice to focus on the main source of distortion; namely, the difference in size by firms with the same productivity. Finally we are able to derive an analytical solution for total output and show that it is increasing in λ since output decreases with the margin of intermediation. The distribution we consider is specified in Definition 3.

Definition 3. *Let $\widetilde{G}(\cdot)$ be the following distribution on z and a:*

1. *z takes values z_1 and z_2, with weights δ_z and $1-\delta_z$, respectively. We assume $z_1 < z_2$.*

2. *For $z = z_1$: a takes values a_1 and a_2, with weights δ_a and $1-\delta_a$, respectively. We assume $a_1 < a_2$.*

[1] Recall from the corollary to Lemma 3 that each type of entrepreneur faces a different interest rate in this economy.

3. *For $z = z_2$: Continuum of values of a distributed according to a uniform distribution between 0 and $\overline{a}$.*

Proposition 1 states the equilibrium prices and occupational choices in this economy. The specific set of assumptions on parameters, as well as the proof of the proposition, can be found in A.4.

Proposition 1. *Given $\widetilde{G}(\cdot)$,*

$$
\begin{aligned}
w &= \alpha\beta \frac{\delta_a\delta_z a_1 + (1-\delta_a)\delta_z a_2 + (1-\delta_z)\frac{\overline{a}}{2}}{\delta_a\delta_z} \\
1 + r^L &= \frac{z_2}{\beta}(1+\alpha\beta)^{1-\alpha}\left(\frac{\delta_a\delta_z}{\delta_a\delta_z a_1 + (1-\delta_a)\delta_z a_2 + (1-\delta_z)\frac{\overline{a}}{2}}\right)^{\alpha}\left(\frac{1-\delta_z}{2\overline{a}(1+\lambda)(1-\delta_a)\delta_z a_2}\right)^{\frac{1-\alpha}{2}} \\
1 + r^D &= \frac{z_2}{\beta}(1+\alpha\beta)^{1-\alpha}\left(\frac{\delta_a\delta_z}{\delta_a\delta_z a_1 + (1-\delta_a)\delta_z a_2 + (1-\delta_z)\frac{\overline{a}}{2}}\right)^{\alpha} \\
&\times\left(\frac{1}{1-\left(\frac{2}{\overline{a}(1-\delta_z)}\right)^{\frac{1}{2}}\left((\lambda-\alpha\beta)(1-\delta_a)\delta_z a_2 - \alpha\beta(1-\delta_z)\frac{\overline{a}}{2} - (1+\alpha\beta)\delta_a\delta_z a_1\right)^{\frac{1}{2}}}\right)^{1-\alpha}
\end{aligned}
$$

and occupational choices

$$
\begin{aligned}
\mathcal{W} &= (z_1, a_1) \\
\mathcal{B} &= (z_1, a_2) \\
\mathcal{E}_D &= \{(z_2, a) : \delta_{\mathcal{E}_D,\mathcal{E}_O}(z_2) \leq a \leq \overline{a}\} \\
\mathcal{E}_O &= \{(z_2, a) : \delta_{\mathcal{E}_O,\mathcal{E}_L}(z_2) \leq a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z_2)\} \\
\mathcal{E}_L &= \{(z_2, a) : 0 \leq a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z_2)\}
\end{aligned}
$$

are an equilibrium in this economy, where $\delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$, and $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$ are as defined in Lemma 3.

Proof. See A.4. □

The fact that the margin of intermediation is decreasing in λ is a corollary to Proposition 1.

Corollary 2. *$r^L - r^D$ is decreasing in λ.*

Additionally, we prove that the differences in sizes for firms decreases as λ increases.

Corollary 3. *The dispersion in firm size is decreasing in λ.*

Proof. The dispersion in sizes of firms is increasing in $r^L - r^D$. □

Furthermore, from Proposition 1 we are able to derive an expression for total output in this economy. Proposition 2 proves that total production is increasing in λ, since it is decreasing in the margin of intermediation.

Proposition 2. *Let*

$$\mathcal{C}_L \equiv \frac{1-\delta_z}{2\overline{a}(1+\lambda)(1-\delta_a)\delta_z a_2}$$

$$\mathcal{C}_D \equiv \frac{1}{1-\left(\frac{2}{\overline{a}(1-\delta_z)}\right)^{\frac{1}{2}}\left((\lambda-\alpha\beta)(1-\delta_a)\delta_z a_2 - \alpha\beta(1-\delta_z)\frac{\overline{a}}{2} - (1+\alpha\beta)\delta_a\delta_z a_1\right)^{\frac{1}{2}}}.$$

Then total output in this economy is equal to

$$Y = z_2\frac{1-\delta_z}{\overline{a}}\left(\frac{1}{1+\alpha\beta}\right)^{\alpha}\left(\frac{M_{\mathcal{W}}}{A}\right)^{\alpha}\left[\overline{a}\left(\frac{1}{\mathcal{C}_D}\right)^{\alpha} - \frac{\alpha}{1+\alpha}\left[\left(\frac{1}{\mathcal{C}_D}\right)^{1+\alpha} - \left(\frac{1}{\mathcal{C}_L}\right)^{\frac{1+\alpha}{2}}\right]\right]$$

Furthermore, an increase in λ decreases the last term in Y, which is a function of $r^L - r^D$.

Proof. See A.4. □

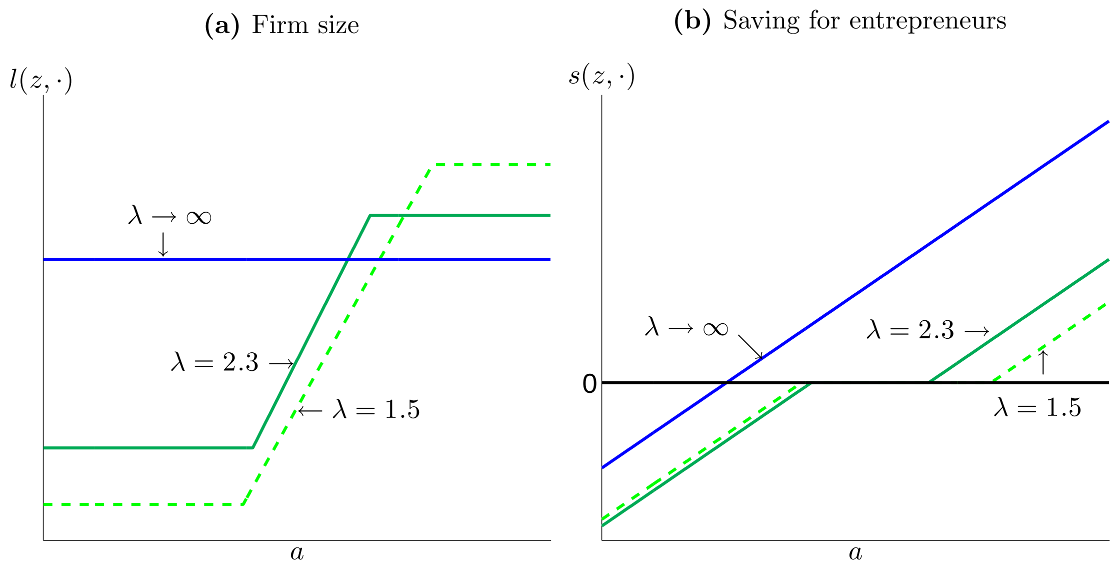
1.5.3 Numerical solutions

We now solve the model numerically by assuming that z and a are drawn from independent uniform distributions. We also set $\alpha = 0.7$ and $\beta = 0.96$.[2] We analyze what happens as λ increases. We find that $r^L - r^D$ is decreasing in λ. Additional to the effect this has on the dispersion on the sizes of firms with the same productivity, we also find that there is an effect on occupational choice; namely, we observe that unskilled consumers who choose to become entrepreneurs in economies with low levels of λ, will choose to become workers as λ increases. Also, skilled consumers who became workers for low levels of λ, choose to manage firms for higher limits on the leverage of banks.

[2]The results we show hold qualitatively for various parameterizations.

As λ increases, the margin of intermediation goes down, which implies lower misallocation: The heterogeneity in size among firms with same productivity decreases. Figure 1.4 shows what happens to misallocation as λ increases. Figure 1.4a shows the effect on firm size for an entrepreneur under different values of λ. As $\lambda \to \infty$ misallocation disappears. Firms with the same productivity will have the same size, as the continuous line shows. For finite values of λ, there will be misallocation.

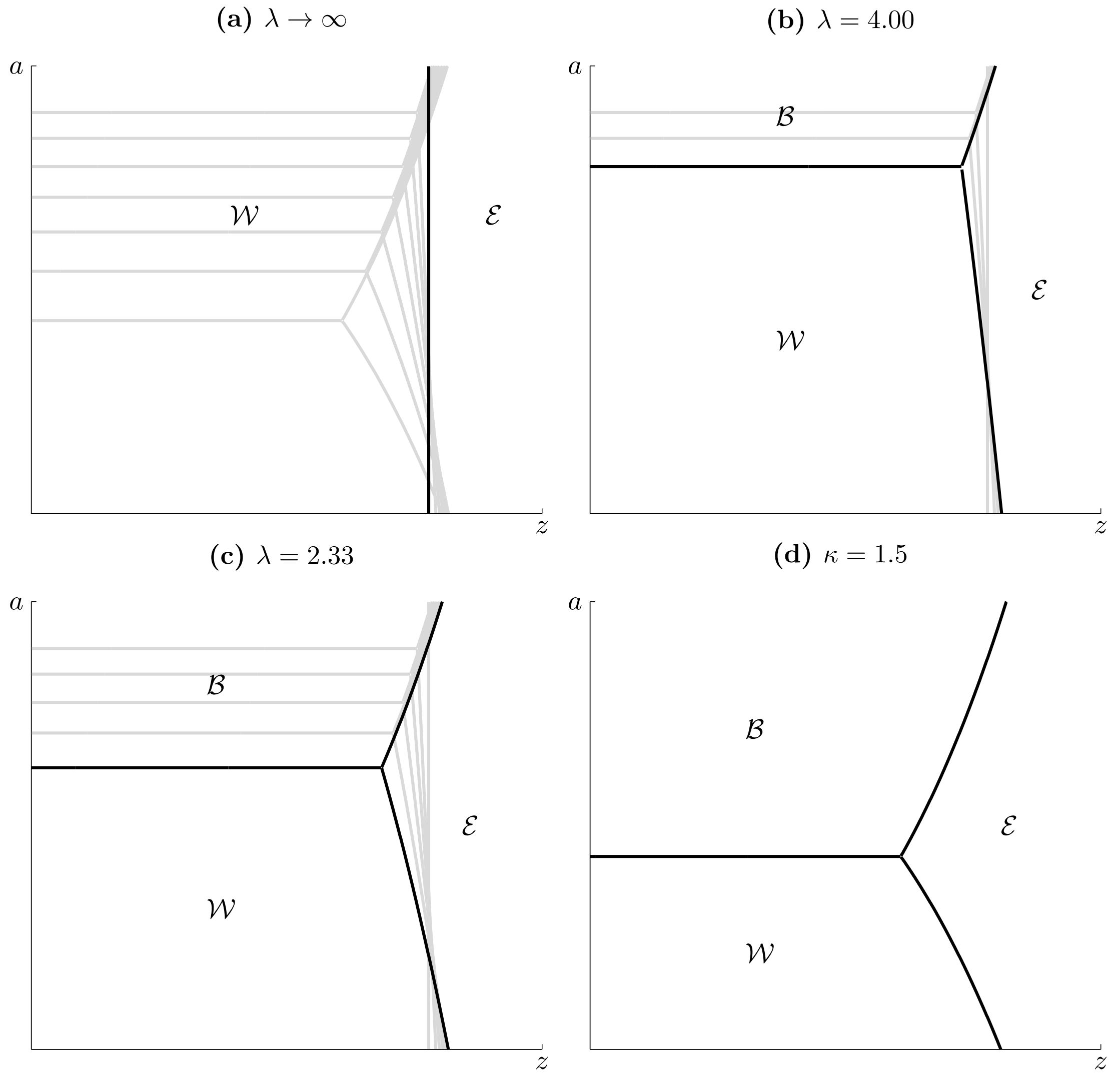
Figure 1.4: Entrepreneur with skill z



Now, recall that the kinks in firm size correspond to values of a such that $a = \delta_{\mathcal{E}_O,\mathcal{E}_L}$ and $a = \delta_{\mathcal{E}_D,\mathcal{E}_O}$. A higher value of λ implies a smaller difference between $\delta_{\mathcal{E}_O,\mathcal{E}_L}$ and $\delta_{\mathcal{E}_D,\mathcal{E}_O}$. A consequence of a higher λ is that more entrepreneurs use banks as financial intermediaries, since less entrepreneurs use all their available wealth to hire workers. Figure 1.4b highlights this point: higher levels of λ imply less entrepreneurs that neither borrow nor save.

Furthermore, as λ increases another distortion diminishes: Skilled consumers choose to manage firms, while unskilled consumers become workers. That is, consumers that choose to become entrepreneurs when $\lambda < \infty$, choose to become workers when $\lambda \to \infty$. Similarly, consumers that choose to become workers when $\lambda < \infty$, decide to become entrepreneurs when $\lambda \to \infty$.

Figure 1.5: Boundaries for different values of λ

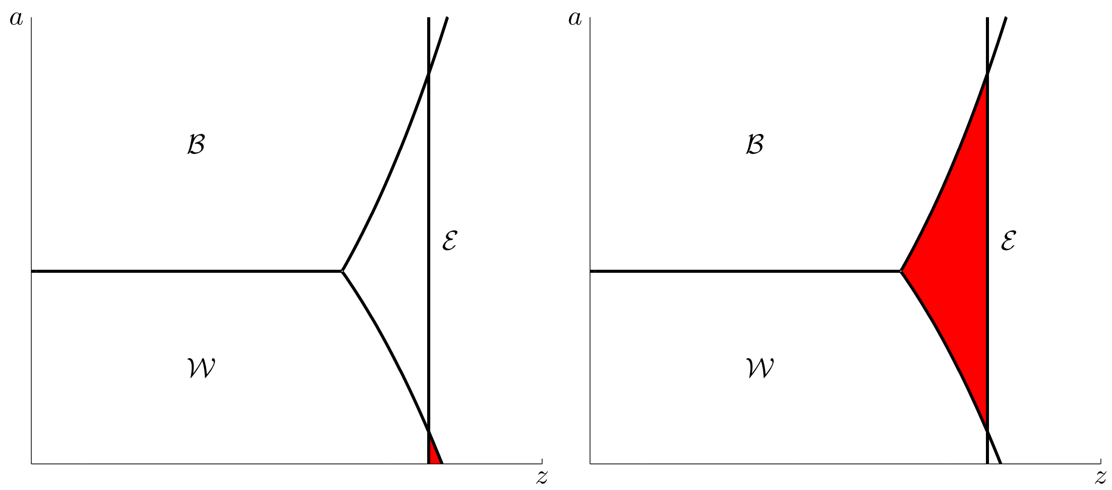


$\mathcal{B}$: Bankers. $\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

To understand the source of this distortion, Figure 1.6 shows the occupational choices of consumers in the (z, a) space for different values of λ. As λ increases the threshold in wealth above which consumers prefer to become bankers over workers goes up. That is, less workers become bankers. The main reason for why this occurs is that the spread between r^B and r^D decreases, which is largely a consequence of the decrease in the margin of intermediation.

Figure 1.6: Changes in occupational choice

(a) Skilled consumers become entrepreneurs... **(b)** ... and unskilled consumers become workers



$\mathcal{B}$: Bankers. $\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

More importantly, as λ increases, the slope of the the boundary that determines the occupation choice between workers and entrepreneurs increases. That is, the decision between becoming a worker or an entrepreneur becomes less dependent on the level of wealth of the consumer than on its skill. Given that the margin of intermediation is decreasing in λ, Lemma 7 shows this by proving that in an economy with $\lambda < \infty$ an unwealthy consumer needs to have a higher skill than a wealthy consumer in order to become an entrepreneur. Furthermore, this difference is increasing in the margin of intermediation. We prove the lemma for a particular case in which the minimum value that a attains is 0. Nonetheless the result holds for any general distribution with positive support.

Lemma 7. *A consumer with wealth above* $a_1 = \frac{1+\alpha\beta}{\beta(1-\alpha)}w$ *will become an entrepreneur as long as* $z \geq z_1$ *where*

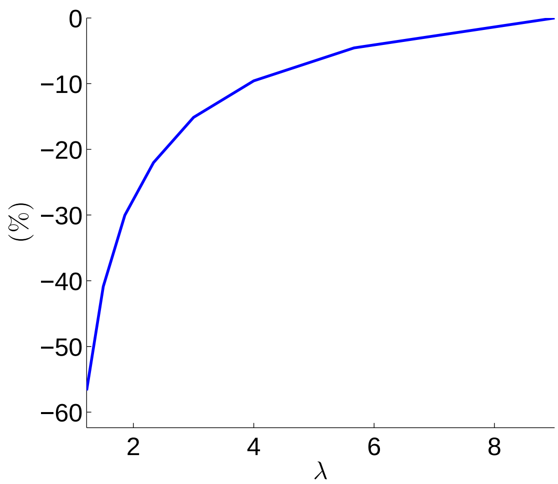
$$z_1 \equiv \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} w(1+r^D).$$

A consumer with no wealth ($a = 0$) will become an entrepreneur as long as $z \geq z_2$ where

$$z_2 \equiv \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} w(1+r^L) \left(\frac{1+r^D}{1+r^L}\right)^{\frac{\beta(1-\alpha)}{1+\beta}}.$$

$z_2 > z_1$ as long as $r^L - r^D > 0$. Furthermore, $z_2 - z_1$ is decreasing in λ.

Figure 1.7: Production relative to economy with $\lambda = 9$



Proof. See A.4. □

Figure 1.6 shows the effect of the distortion on occupation choice. Figure 1.6a shows consumers who choose to become entrepreneurs in an economy without limits on leverage ($\lambda \to \infty$) but in the model with $\lambda < \infty$ choose to become workers. That is, without limits on leverage these consumers choose to manage firms since they are skilled consumers. However, these consumers have low wealth. If they chose to become entrepreneurs when $\lambda < \infty$, they would need to finance a large portion of their wage bill by borrowing. Therefore, in an economy with $\lambda < \infty$ they are better off by becoming workers.

On the other hand, Figure 1.6b shows the consumers that would choose to become entrepreneurs in an economy with limits on leverage ($\lambda < \infty$). These consumers are wealthy enough to be able to finance a significant share of their wage bill without needing to borrow. As λ increases, these consumers are better off becoming workers, since they are not skilled enough.

The increased distortions mentioned before cause production to be lower for low levels of λ. Figure 1.7 shows how production is increasing in λ. In this figure we compare the production in an economy with a given level of λ relative to the production of an economy with $\lambda = 9$. With the current parametrization we are able to explain up to 56% differences in production.

We also analyze what happens as we change other parameters in the model. In particular, we analyze what happens as we change the ratio of wealth to skill in the model. That is, let $\overline{a}$ be the supremum of the support of the distribution for wealth and let $\overline{z}$ be the equivalent for skill. We analyze changes in $\frac{\overline{a}}{\overline{z}}$ for a given value of λ and holding other parameters constant. We find that distortions of having limits on leverage decrease: As this ratio increases, consumers are wealthier in the first period, relative to the second period. This implies that interest rates will be lower in equilibrium since there is more wealth that is going to be saved and less will be borrowed. Furthermore, banks become bigger, which causes the margin of intermediation to be lower.

1.6 Data

The objective of this section is to highlight that data supports the mechanism we mention in the model. That is, there is a negative correlation between the margin of intermediation and production. Additionally, we document that the margin of intermediation is negatively correlated with financial inclusion, where financial inclusion is defined as the percentage of firms that rely on banks to finance their working capital. This is consistent with the results in the model since a lower margin of intermediation implies that the measure of firms that borrow from banks increases.

Table 1.1: Production vs margin of intermediation

Dependent Variable: log GDP per capita	
Margin of intermediation	-0.880^{***}
	(0.072)
Number of observations	2,886

OLS estimation. Standard errors are in parenthesis.
***: Significant at 1%.
Country fixed effects.

We use data from the Global Financial Development Database (GFDD). This is an extensive database that includes measures of financial development for over 200 countries from 1960 to 2010.[3] The measures are divided in metrics of depth, access, efficiency and stability of the financial markets in order to analyze the different roles that financial systems play in an economy. See Cihak, Demirguc-Kunt, Feyen, and Levine (2012) for a further description of the database.

Table 1.2: Financial inclusion vs margin of intermediation

Dependent Variable: Financial inclusion	
Margin of intermediation	-1.117^{*}
	(0.599)
Number of observations	134

OLS estimation. Standard errors are in parenthesis.
*: Significant at 10%.
Country fixed effects.

We only consider countries whose population has been above 1,000,000 at some point in time and we estimate the following econometric model:

$$GDP_{i,t} = \beta_1 + \beta_2 MI_{i,t} + \mu_i + \varepsilon_{i,t}.$$

We define $GDP_{i,t}$ as the natural logarithm of GDP per capita in 2000 US dollars. MI is the margin of intermediation, defined in the database as the bank-lending deposit spread. We include country fixed effects to control for other country specific characteristics.

[3] Some variables are available for a shorter span of time.

Table 1.1 shows the results of the estimation. We observe that there is a negative correlation between the margin of intermediation and the GDP per capita. An increase of a percentage point in the margin of intermediation results in a decrease of 0.9% in GDP per capita. This is consistent with the results shown in Erosa (2001) and Greenwood, Sanchez, and Wang (2013).

We then test other implications of the model; namely, as the margin of intermediation decreases more firms rely on the banking sector to finance their wage bill. For this we estimate (1.14):

$$FI_{i,t} = \alpha_1 + \alpha_2 MI_{i,t} + \mu_i + \varepsilon_{i,t}. \tag{1.14}$$

We define FI as the percentage of firms that use banks to finance working capital, which we denote as financial inclusion. Cihak, Demirguc-Kunt, Feyen, and Levine (2012) compiled this measure from the Enterprise Surveys, which are surveys conducted by the World Bank to emerging countries (See The World Bank (2012) for further details). As mentioned in Cihak, Demirguc-Kunt, Feyen, and Levine (2012), financial development is positively correlated with income. Therefore, it is safe to assume that firms that don't rely on banks to finance their working capital are most likely not able to use financial markets to save either. Table 1.2 shows the result of the estimation of (1.14). We document a negative correlation between financial inclusion and the margin of intermediation: a one percent increase in the margin of intermediation reduces the percentage of firms using banks to finance their working capital by 1.1%. This is consistent with the results of our model: as the margin of intermediation decreases, the measure of firms that borrow from banks increases.

1.7 Conclusion

This paper analyzes the relationship between the leverage of banks and real allocations. Economies with banks that have a lower leverage experience a higher margin of intermediation, which affects both the occupational choices of consumers and the distribution of firm sizes across the wealth of individuals. When the margin of intermediation is large, wealthy entrepreneurs can hire workers for a significantly lower cost relative to unwealthy entrepreneurs. This variation in marginal costs of employees translates into heterogeneity in firm sizes across the spectrum of wealth for otherwise identical firms.

Occupational choice is also distorted: consumers with substantial skill but sufficiently small wealth may be dissuaded from managing firms due to the large costs of taking out loans, whereas wealthy but less skilled consumers may manage firms due to their relative advantage in inexpensive financing and the low opportunity cost of using those funds to hire workers rather than depositing. Future work will focus on analyzing the tradeoff of having limits on the leverage of banks in an economy with risk.

Chapter 2

Public Law Enforcement: More Is not Always Better

2.1 Introduction

Property rights, or namely, the ability of firms and consumers to own capital and other resources are essential to almost every economic model. However, for the most part these rights are taken as given. A walk through the streets in an urban area of virtually any developing country reveals that this is not the case. Private security guards, metal bars and large locks are commonplace to counteract high levels of theft. Economists have remained largely silent on the discussion of theft and how to counteract it even though it is important for policymakers. In this paper we propose a general equilibrium model that includes theft, private security and public law enforcement and is motivated by qualitative patterns observed in the World Bank's Enterprise Surveys and Government Indicators. In particular, the model we propose highlights the direct and general equilibrium responses by agents in their private security and theft decisions to changes in the level of public law enforcement.

The level of public law enforcement is drastically different across countries, with police force and incarceration rates varying by a factor of 100.[1] We provide a theory of why this is the case. Namely, we find that the marginal effects of changes in public law

[1] Source: http://www.prisonstudies.org/info/worldbrief/

enforcement are very different depending on the current level of public law enforcement. Further, there are a number of non-monotonicities in the relationship between public law enforcement and variables of interest. Finally, the combined direct and indirect effects of the level of public law enforcement can affect the aggregate production in a country by as much as 6%.

According to our model, in economies with relatively low levels of public law enforcement, a marginal rise in the level can actually lower aggregate production. This happens since an increase in public law enforcement increases the incarceration rate, which removes agents from the labor force. Additionally, we observe a high level of substitution between public law enforcement and expenditures on private security which dampens the effect public law enforcement has on the overall level of theft. Finally, there is a general equilibrium effect which lowers the relative income of the non-incarcerated agents through the increased burden of supporting those who are incarcerated. This also pushes some agents to become thieves. To put this in perspective, using our benchmark model, if the level of public law enforcement in Guatemala improved to the level in Mexico, we would predict a decrease in aggregate production of 0.33%.

We also observe that private security exerts a negative externality in economies with low levels of publics law enforcement. If we consider equilibria where firms can only hire a fraction of the private security that they would otherwise find optimal, then production is hiring than if firms were able to hire their optimal level of private security. The reason for this is that by restricting how much firms can spend on private security, firms end up hiring more workers.

For high enough levels of public law enforcement, we find that marginal increases in the level of public law enforcement provide large gains to aggregate production and increase the labor force. As the probability of getting caught rises, agents are deterred from stealing and at some point the drop in theft becomes larger than the increase in those thieves who are caught. The reduction in incarceration rates augments the total labor force which increases total production. Additionally, reduction in theft from firms lowers the distortionary wedge between the marginal product of labor and the wage rate which rises both the efficiency and average size of firms. Finally, the increase in the wage rate and the reduction in the burden of the incarcerated on the non-incarcerated increases the actual cost of getting caught and puts further downward pressure on the theft rate. Again using our benchmark model, if the level of public law enforcement in Mexico improved to the level of that in the United States, we would predict an increase in aggregate production of 2.58%.

Data which exists for private security consistently reveals that the correlation between private security expenditures and theft is positive. We match this fact. In our model this relation is caused by public law enforcement which both deters theft and serves as a substitute for private security. In this sense, we make the same empirical observation as North (1968) in that economies where firms have lower private security expenditures are also economies with less theft and often higher production.

In order to direct and validate the way we model the decision of thieves in when and how much to steal, and the way we model private security, we adopt two strategies. First we incorporate existing findings on theft in the psychology and sociology literature. Second we allow agents to vary across two dimensions: by aversion to theft and by varying levels of ability as in the Lucas (1978) span of control model. Granting variation across these dimensions gives insight into micro decisions of agents and across firms. We validate our modeling of theft and private security by matching these patterns to the data.

Heterogenous modeling of agents also gives further insights. Specifically, agents with lower ability earn less which decreases their cost of being caught and increases the likelihood they engage in theft. Second, the distortion from theft across firms is not uniform. Firms managed by entrepreneurs with higher ability afford larger amounts of private security which reduce the wedge between the marginal productivity of labor and the wage rate. This mechanism causes the dispersion across firm size to be increasing in the rate of theft.

As far as we are aware, we present the first general equilibrium model incorporating theft, private security and public law enforcement. However, our work contributes and builds upon a vast theoretical and empirical literature.

Our paper continues in the spirit of the seminal work by Becker (1968). In our model consumers analyze the costs and benefits of committing a crime and make a rational decision of whether to engage in criminal activity. Perhaps the model most similar to ours is the one in Fender (1999) which includes many of the same elements and some of the same results. In that model, corruptible agents choose between work and theft and there is consideration of the level of enforcement which is very similar to our notion of public law enforcement. In line with this paper, we observe similar relationships between the level of enforcement, the number of criminals and the number punished. We also observe the possibility of multiple equilibria which is consistent with both Fender (1999) and Burdett, Lagos, and Wright (2003). That is, suppose there are lots of high wage jobs available. Then workers are less inclined to criminal activity which makes the higher wage sustainable. If no such jobs are available, workers are more inclined to criminal activity which keeps wages low. In contrast to Fender (1999), our model allows thieves to both work and steal, we include a notion of firms, agents are heterogenous in ability and we incorporate general equilibrium effects. This allows us to match micro data in order to validate our macro results.

Our findings are consistent with the findings in the empirical paper by Buonanno, Drago, Galbiati, and Zanella (2011). Their work suggests that increases in the incarceration rate deters crime. In our model we support that this effect holds, but the general equilibrium effects can cause pressure on crime (specifically theft) in both directions. Due to the current absence of dynamics in our model, we are unable to address the (largely empirical) literature on the deterrents of the effects of prison on recidivism.[2]

Our paper is also related to the existing literature relating trust, extortion, distortion and firm size. We observe a similar pattern in distribution of firm size due to increases in theft as Ranasinghe (2012) observes from increases in extortion in the sense that higher levels increase dispersion of firm size. Our effects differ slightly in that all firms are smaller than they would be in the absence of theft but the distortion is greatest for the smallest firms. We also incorporate from Grobovšek (2013) the finding that increased levels of theft among workers constrains firm size through ineffective contract enforcement.

The rest of the paper is organized as follows. Section 2.2 presents an empirical motivation for our model and the main mechanism in it. Section 2.3 outlines the model. Section 2.4 presents the primary results. Section 2.5 concludes.

[2]See Drago, Galbiati, and Vertova (2009) for an example.

2.2 Data and Empirical Motivation

In order to validate the mechanisms used and the implied predictions of our model, we employ data from the World Bank Enterprise Surveys and Worldwide Governance Indicators. The surveys are conducted at the firm level using a representative sample of an economy's private sector. The World Bank selected firms for the Enterprise Surveys using stratified random sampling. All members of the population have the same probability of being sampled and no weighting of the observations is necessary. However, only firms with 5 or more employees are targeted for an interview and organizations with 100% government ownership are ineligible to participate.[3] Surveys occur face-to-face with business owners and top managers.

Surveys have been conducted every year from 2006 to 2011. Nonetheless, in any single country there have been a maximum of two surveys and the vast majority of countries have been surveyed a single time. The final dataset used in this paper includes 130 country-years and averages 373 firms interviewed per country-year combination for a total of 48,436 observations. There are 111 unique countries where surveys have been conducted. Questions are both qualitative and quantitative in nature. Qualitative questions ask perception of certain business obstacles (e.g. "Do you perceive crime, theft and disorder as a major constraint?"). Quantitative questions of particular relevance request the number of employees, the annual revenue, the amount of annual losses due to theft as well as annual private security expenditures.

Summary statistics are included in Table 2.1. Crime is identified as a "major" constraint by over a quarter of all firms interviewed. Additionally, even though only roughly a quarter of firms directly experienced theft in the year of interview, almost two thirds of firms have positive expenditures on private security. The average security expenditures for firms purchasing private security was 2.6% of total revenues. Firms which experienced theft had an average loss equivalent to 3.8% of their total revenues.

[3] The sample targets firms they believe to have 5 or more employees, however some firms are observed to have less than 5 upon conducting the interview.

Table 2.1: Summary Statistics

Share of firms that perceive theft as a major constraint	27.8%
Share of firms that had positive expenditures on private security	63.9%
Average private security expenditures*	$59,931
Average private security expenditures over revenue**	2.6%
Firms that experienced theft	24.7%
Average level of theft*	$18,786
Average theft over revenue***	3.8%

*: Levels were converted to 2000 US dollars.

**: Conditional on having positive private security expenditures.

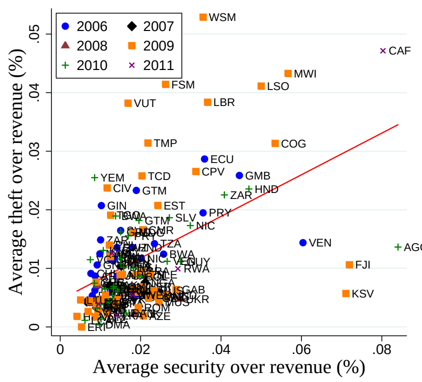
***: Conditional on having experienced theft.

Source: The World Bank (2012). Authors' calculations.

We now make a number of motivating observations where we highlight how the level of public law enforcement is important in determining theft and private security choices in equilibrium. Figure 2.1 shows average experienced theft to average private security expenditures at the country level. We observe that average theft is positively correlated with average private security expenditures and the relationship is significant at the 1% level. We assume that all else constant, theft should decrease in security expenditures. However, both theft and security decisions are endogenous to the environment. Therefore the observed positive correlation is not causal but is indicative of some tertiary effect. We posit that one of the key drivers of this relationship is public law enforcement. First, as seen in Figure 2.2a, security is decreasing in the country's Rule of Law index[4] which we use as a proxy for public law enforcement. Second, theft is also decreasing in public law enforcement as seen in Figure 2.2b. This additional information seems to support private security being an imperfect substitute for public law enforcement and that a firm's equilibrium private security decision does not fully compensate for the lack of a strong public law enforcement presence.

[4]The World Bank establishes six governance indicators, one of which is a Rule of Law Index which orders countries according to the overall rule of law. See Kaufmann, Kraay, and Mastruzzi (2010) for a description of the methodology used to calculate these indicators. According to the World Bank definition, "The Rule of Law index captures perceptions of the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, property rights, the police, and the courts, as well as the likelihood of crime and violence". We use the percentile of this index as a proxy for the level of public law enforcement in a given country.

Figure 2.1: Theft over Revenue vs Security Expenditures over Revenue

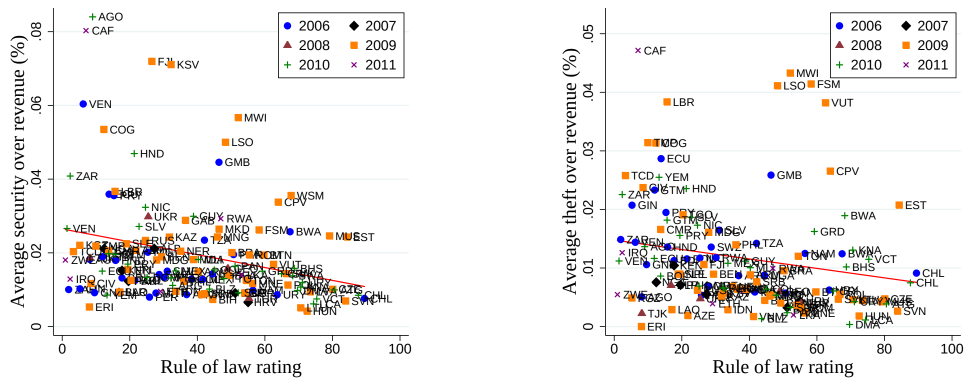


Source: The World Bank (2012). Authors' calculations.

Figure 2.2: Security Expenditures over Revenue (Sec to Rev) and Theft over Revenue (Theft to Rev) vs Rule of Law

(a) Sec to Rev vs Rule of Law

(b) Theft to Rev vs Rule of Law



Source: The World Bank (2012). Authors' calculations.

We observe a hump-shaped relationship between rule of law and incarceration rates. For low levels of public law enforcement, we observe that increases in this level are accompanied by an increase in the incarceration rate. However, once the level of public law enforcement reaches a certain point, further increases are actually related to lower incarceration rates. Table 2.2 shows both a linear and quadratic fit when regressing the incarceration rate on the Rule of Law. We observe that incarceration rates appear to be increasing in the Rule of Law. Nonetheless, when we add a quadratic Rule of Law term the coefficient on the first term becomes five times larger and increases in significance. Additionally the goodness of fit more than doubles. Both of these support the use of a quadratic term and that the hump shape in incarceration rates matches data better.

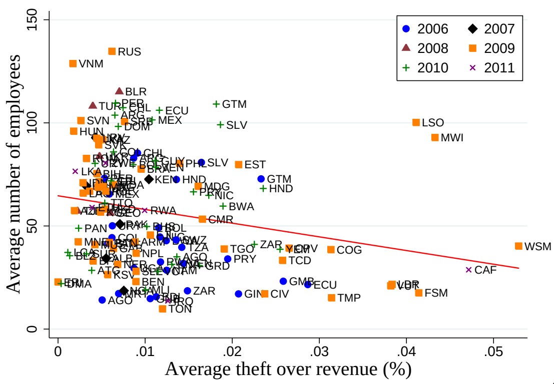
Table 2.2: Incarceration as a function of Rule of Law (RoL)

	Dependant variable is Incarceration Rate	
	(1)	(2)
Constant	112.807***	54.051***
	(13.991)	(20.537)
RoL	0.802***	4.250***
	(0.239)	(0.936)
RoL^2		-0.0340***
		(0.009)
Observations	199	199
R^2	0.054	0.119

OLS estimation. Standard errors are in parenthesis.
***: Significant at 1%.

Our final motivating observation is that we observe average firm size to be inversely correlated with average theft, and this relation is also significant at the 1% level. Figure 2.3 shows this relation. A similar observation was made by Grobovšek (2013).

Figure 2.3: Average Size of Firm vs Average Theft over Revenue



Source: The World Bank (2012). Authors' calculations.

To conclude, we find suggestive evidence that the level of public law enforcement is important in the determination of equilibrium theft and private security choices. Perhaps more importantly, effects on incarceration from marginal changes in the level of public law enforcement will vary based on the current level of public law enforcement.

2.3 Environment

Our model is constructed in the spirit of Lucas (1978). Each consumer makes two choices: whether to become a thief or not, and whether to become an entrepreneur or a worker. Consider a particular consumer. If she chooses to become a thief, she optimally chooses how much to steal from firms, taking as given how much security is hired by each firm. However, she faces an exogenous probability of getting caught and losing what she stole as well as the ability to work or manage a firm. If the consumer decides to become an entrepreneur, she takes into account how much theft she faces and chooses how much security to hire, in addition to choosing the optimal size of her firm. If she becomes a worker, then she works in firms in exchange for a wage.

2.3.1 Consumers

In this economy there is a unit mass continuum of risk neutral consumers, each endowed with a skill level and an aversion to steal. Consider a consumer with skill level z and aversion to steal θ. She maximizes her utility, given by (2.1).

$$u(z,\theta) = \max\left\{u_T(z,\theta), u_{NT}(z)\right\}, \tag{2.1}$$

where

$$\begin{aligned} u_T(z,\theta) &= (1-\lambda)\left[\max\left\{w,\pi(z)\right\} + \Pi_T - \Upsilon\right] + \lambda\underline{c} - \theta \\ u_{NT}(z) &= \max\left\{w,\pi(z)\right\} - \Upsilon. \end{aligned}$$

That is, she decides whether to become a thief and get utility $u_T(z,\theta)$ or not become a thief and get $u_{NT}(z)$. In the former case, the consumer steals from firms to get an extra income of Π_T. She gets away with stealing with an exogenous probability $1-\lambda$. With probability λ the consumer gets caught and loses all sources of income. Instead she receives consumption $\underline{c}$. We interpret $\lambda \in [0,1]$ as the level of public law enforcement and we interpret a thief being caught as implying that she goes to jail. In this way, if a consumer is caught, she neither works nor becomes an entrepreneur. Finally, Υ is a lump sum tax on consumers that do not go to jail which finances $\underline{c}$ for those in jail. That is,

$$\Upsilon = \frac{\underline{c}\lambda M_T}{1 - \lambda M_T}. \tag{2.2}$$

Regardless of the decision to become a thief, the consumer also decides whether to work for a wage w or become an entrepreneur and receive the profits $\pi(z)$ from the firm she manages. If she becomes an entrepreneur her income will depend on her skill z. If the consumer decides not to become a thief, she receives the income either from working or from being an entrepreneur, minus the lump sum tax.

We assume that θ and z are drawn from independent distributions. We will denote by $F(\cdot)$ and $G(\cdot)$ the cumulative distribution functions of θ and z, respectively. A consumer's decision is characterized by z and θ, so we will denote each agent by the realizations of these random variables.

2.3.2 Firms, theft and private security

Consider an entrepreneur with skill level z. She maximizes the profits from the firm she manages, which we will denote as firm z, by hiring workers l_y to maximize its profits. The firms produces using a decreasing returns to scale function, zl_y^{α}, where $\alpha \in (0, 1)$. From what firm z produces $(1 - \lambda)M_T\tau$ gets stolen, where M_T denotes the measure of consumers that become thieves and τ is how much each thief decides to steal from firm z.[5] Finally, firm z can hire security guards l_s to diminish theft. All firms produce the same final good and we normalize the price of this good to 1. To summarize, firm z solves problem (2.3).

$$\pi(z) \equiv \max_{l_y \geq 0, l_s \geq 0} zl_y^{\alpha} - wl_y - wl_s - (1 - \lambda)M_T\tau(z). \tag{2.3}$$

In order to determine how many security guards are hired, we assume firm z understands the thieves' problem. Consider the problem of a consumer that becomes a thief

[5] Since agents are risk averse we are able to abstract from which firms are stolen from and only care about the expected level of theft.

and decides to steal from firm z. The income derived in stealing from firm z is given by

$$\pi_T(z) \equiv \max_{\tau \geq 0} \tau - C_\tau(z), \tag{2.4}$$

where $C_\tau(z)$ denotes the cost born by those who steal from firm z. We make three assumptions regarding this cost. First, $C_\tau(z)$ is increasing and convex in the amount stolen. The more that is stolen, the greater the transportation costs, storage costs, etc. Additionally, without convexity, thieves would always attempt to steal everything possible or steal nothing at all which does not hold true empirically. Moreover, the thieves' problem does not solely consider the financial costs but also the utility costs of time and anguish involved in planning and carrying out an operation. It is reasonable that the cost of theft in terms of planning, stress, and time grows exponentially from stealing a pack of gum to stealing everything in a store.

Our second assumption is that security affects the choice of theft by making it more costly to steal. The presence of a security guard in the firm causes more planning and time in order to be able to steal. This is consistent with what is found by Kraut (1976) where the risk associated with stealing is perceived as a deterrent.

Finally, we assume that the cost of stealing is decreasing in the amount produced by the firm. This accounts for the fact that if a firm is bigger, then there are more things to steal, and so stealing the same amount as from a smaller firm is easier. This is consistent with the results reported by Smigel (1956), who finds that people are more likely to steal from big firms than from small firms.

We assume that $C_\tau(z) \equiv \frac{\phi(l_s(z))}{l_y(z)}\frac{\tau^2}{2}$, where $\phi(l_s) \equiv \left(\frac{\alpha}{1-\alpha} l_s\right)^{\frac{1-\alpha}{\alpha}}$ denotes how much security l_s guards provide for a given level of M_T and is strictly increasing and concave in l_s. The solution to (2.4) is

$$\tau(z) = \frac{l_y(z)}{\phi(l_s(z))}. \tag{2.5}$$

Then $\pi_T(z) = \frac{1}{2}\tau(z)$ and the aggregate income received from stealing Π_T is given by

$$\Pi_T \equiv \int_{(z,\theta)\in E} \pi_T(z) dF(\theta) dG(z) - \lambda \int_{(z,\theta)\in E \bigcap T} \pi_T(z) dF(\theta) dG(z), \tag{2.6}$$

where E and T denote the set of consumers that become entrepreneurs and the set of consumers that become thieves, respectively. That is,

$$E \equiv \{(z, \theta) : \pi(z) \geq w\} \tag{2.7}$$

$$T \equiv \{(z, \theta) : u_T(z, \theta) > u_{NT}(z)\}. \tag{2.8}$$

We abuse notation and also refer to E as the set of z for which consumers become entrepreneurs. The use of E will be clear from context.

The second term in (2.6) is due to the fact that thieves do not get income when stealing from the firms managed by entrepreneurs that are thieves and get caught. Recall that a fraction λ of the total entrepreneurs that become thieves go to jail and unable to manage firms.

2.3.3 Micro Support for Modeling Theft and Private Security

We use the existing literature as well as qualitative patterns in micro-data to motivate our modeling methods. Specifically we make four observations using data from the Enterprise Surveys (See Table 2.3). First, both the absolute level of theft and private security expenditures are increasing across firm size. This is consistent with Smigel (1956).

Table 2.3: Results in Theft and Security Across Firms

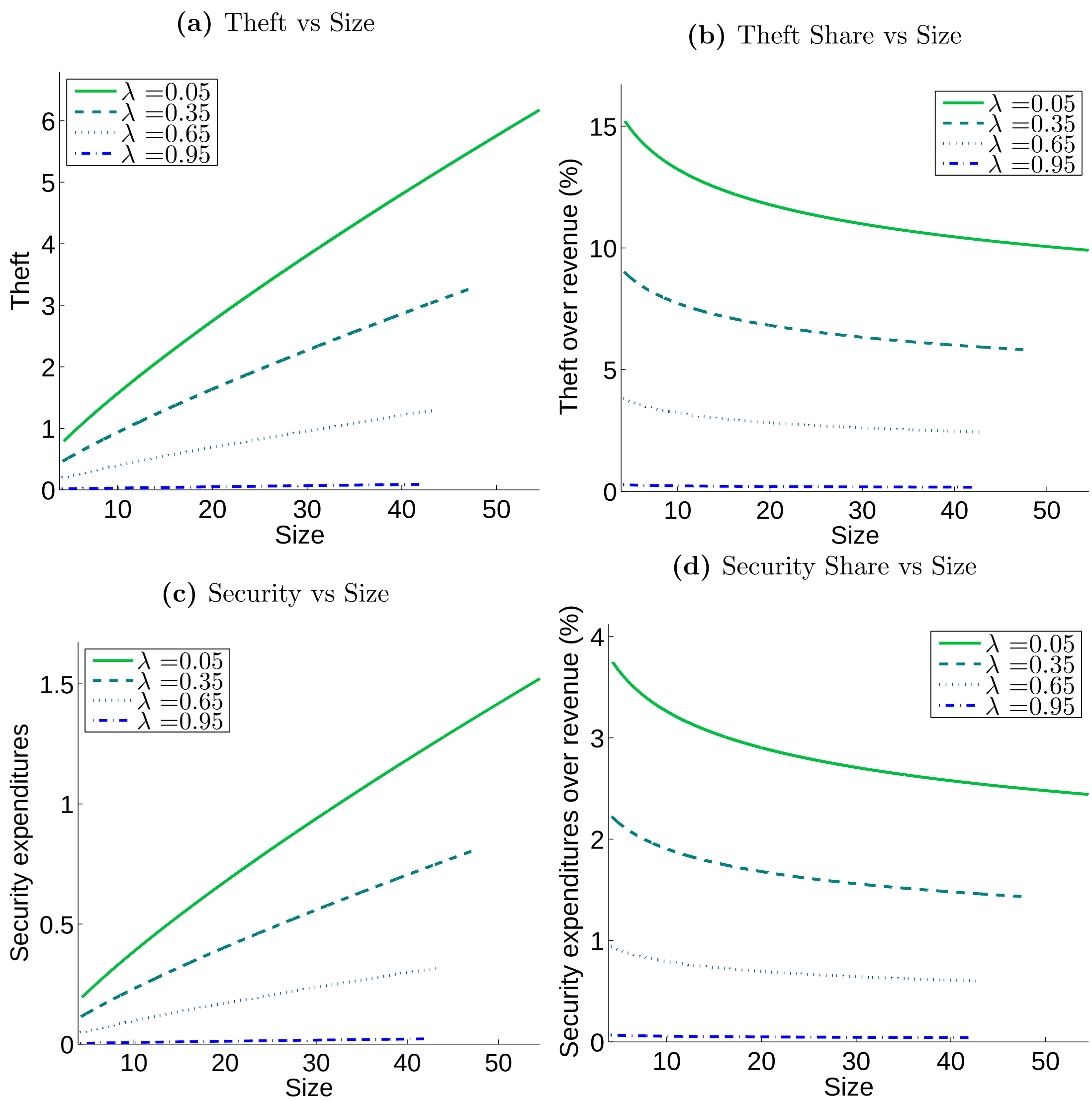
Dependent Variable:	Theft	$\frac{\text{Theft}}{\text{Revenue}}$	Security	$\frac{\text{Security}}{\text{Revenue}}$	$\frac{\text{Security}}{\text{Revenue}}$	$\frac{\text{Security}>0}{\text{Revenue}}$
Independent Variable	(1)	(2)	(3)	(4)	(5)	(6)
Size (Labor)	224.14^{***}		613.99^{***}			
	(12.45)		(42.86)			
Size ($\log$ Labor)		-0.002^{***}		0.001^{***}	0.004^{***}	-0.003^{***}
		(0.000)		(0.000)	(0.001)	(0.000)
Size ($\log \text{Labor}^2$)					-0.000^{**}	
					(0.000)	
Observations	48,299	48,299	48,299	48,299	48,299	30,838
Country-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes

OLS estimation. Standard errors are in parenthesis.
***: Significant at 1%. **: Significant at 5%. *: Significant at 10%.

Next we analyze these same variables as a share of revenue. While theft is increasing in the size of firm, theft as a percentage of revenue is decreasing in the size of firm. The relation with private security expenditures is slightly more complicated. When we regress private security expenditures as a share of revenues we find that the share is increasing slowly in the size of firm. However, when we add a quadratic term on size to the regression we find a hump shape, with private security share increasing for small firms and decreasing for larger firms. Another level of analysis reveals the cause for this hump. The probability that a firm purchases private security is increasing in size. However, given a firm purchases private security (column 6 of Table 2.3), the share of revenue spent on private security is decreasing in the size of firm.

These micro patterns were used to guide our modeling of private security and theft. To the extent possible, given the level of heterogeneity used in our model, we match these patterns for a large range of λ. Figure 2.4d matches the data in column 6 of Table 2.3. Due to the level of heterogeneity it is not within the scope of our model to exactly match the hump shape found in the data.

Figure 2.4: Matching Micro Patterns



2.3.4 Equilibrium

An equilibrium in this economy is allocations $\{\tau(z), l_y(z), l_s(z)\}_{z\in E}$, wages w, and sets E and T such that

1. $\tau(z)$ satisfies (2.5) for all $z \in E$; and $l_y(z)$ and $l_s(z)$ solve (2.10);

2. E and T satisfy (2.7) and (2.8);

3. the labor market clears:

$$\int_{(z,\theta)\in E} \left(l_y(z) + l_s(z)\right) dF(\theta)dG(z) - \lambda \int_{(z,\theta)\in E\bigcap T} \left(l_y(z) + l_s(z)\right) dF(\theta)dG(z)$$
$$= \int_{(z,\theta)\in E^c} dF(\theta)dG(z) - \lambda \int_{(z,\theta)\in E^c\bigcap T} dF(\theta)dG(z);$$

4. and the good market clears:

$$\begin{aligned} Y \equiv & \int_{(z,\theta)\in E} z l_y(z)^\alpha dF(\theta)dG(z) - \lambda \int_{(z,\theta)\in E\bigcap T} z l_y(z)^\alpha dF(\theta)dG(z) \\ = & \int_{(z,\theta)\in E} \left[w\left(l_y(z) + l_s(z)\right) + \pi(z) + (1-\lambda)\tau(z)M_T\right] dF(\theta)dG(z) \\ & -\lambda \int_{(z,\theta)\in E\bigcap T} \left[w\left(l_y(z) + l_s(z)\right) + \pi(z) + (1-\lambda)\tau(z)M_T\right] dF(\theta)dG(z), \end{aligned}$$

where

$$M_T \equiv \int_{(z,\theta)\in T} dF(\theta)dG(z).$$

2.3.5 Characterizing of the equilibrium

Lemma 8 characterizes E and T.

Lemma 8.

$$\begin{aligned} E &= \left\{(z,\theta) : z \geq z^E\right\} \\ T &= \left\{(z,\theta) : z < z^E \text{ and } \theta < \theta^W\right\} \bigcup \left\{(z,\theta) : z \geq z^E \text{ and } \theta < \theta^E(z)\right\}, \end{aligned}$$

where z^E is the unique value of z such that $\pi(z^E) = w$ and

$$\begin{aligned} \theta^W &\equiv \frac{\lambda \underline{c}}{1-\lambda M_T} + (1-\lambda)\Pi_T - \lambda w \\ \theta^E(z) &\equiv \frac{\lambda \underline{c}}{1-\lambda M_T} + (1-\lambda)\Pi_T - \lambda \pi(z). \end{aligned}$$

Proof. The production function of every firm satisfies Inada conditions, so $l_y(z) > 0$ for all $z \in E$. Now, the Envelope Theorem, (2.10) and the assumptions on ϕ imply $\pi'(z) = l_y(z)^\alpha \left(1 - \frac{(1-\lambda)M_T}{a\phi(l_s(z)|M_T)}\right) > 0$. Additionally, $\lim_{z\to 0} \pi(z) \leq 0$. Also, $\pi(z) < w$

for all z cannot be an equilibrium since in this case there would be no entrepreneurs.

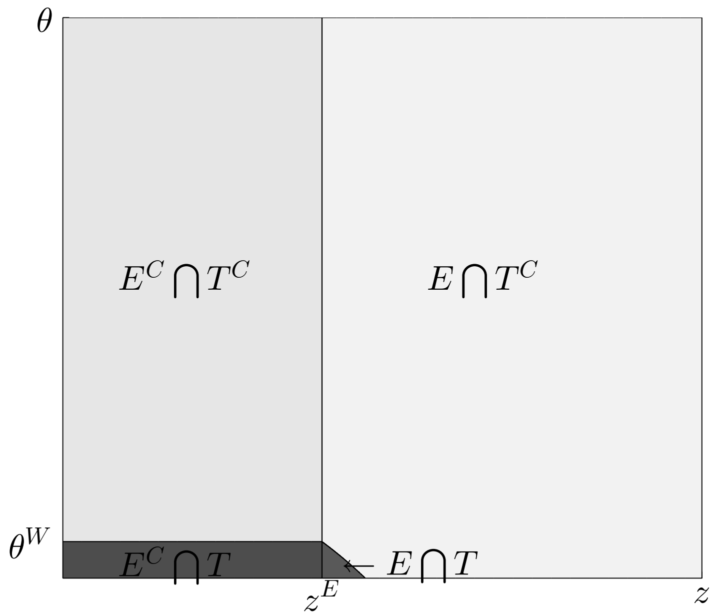
On the other hand, from (2.8), $(z,\theta) \in T$ if and only if $u_T(z,\theta) > u_{NT}(z)$. From (2.1) and (2.2) we have that $(z,\theta) \in T$ if and only if

$$\frac{\lambda \underline{c}}{1-\lambda M_T} + (1-\lambda)\Pi_T - \lambda \max\{w, \pi(z)\} > \theta. \tag{2.9}$$

The definition of z^E and (2.9) imply the result. □

From Lemma 8 we see that thieves are those agents who have the lowest levels of skill and the lowest aversion to steal. Since income is increasing in skill, we also observe that those with the smallest incomes are the most likely to become thieves. We note that these results are consistent with both the theoretical and empirical existing literature.[6] Figure 2.5 shows E and T across skill (x-axis) and aversion to steal (y-axis).

Figure 2.5: E and T across Skill and Aversion to Steal



As mentioned, firm z knows (2.5). Therefore the firm's problem can be written as

[6]For example see Freeman (1999).

stated in (2.10).

$$\pi(z) = \max_{l_y \geq 0, l_s \geq 0} z l_y^{\alpha} - \left(\frac{(1-\lambda) M_T}{\phi\left(l_s\right)} + w \right) l_y - w l_s. \tag{2.10}$$

Lemma 9 characterizes the demand for labor and security given wages w, as well as firms profits and how much is stolen from each firm.

Lemma 9. *Assume* $\alpha > 0.5$*. Then*

$$l_y(z) = \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda) M_T}{w} \right)^{\alpha} \right)^{\frac{1}{1-\alpha}}$$

$$l_s(z) = \frac{1-\alpha}{\alpha} \left(\frac{(1-\lambda) M_T}{w} \right)^{\alpha} \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda) M_T}{w} \right)^{\alpha} \right)^{\frac{\alpha}{1-\alpha}}$$

$$\pi(z) = \frac{1-\alpha}{\alpha} w \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda) M_T}{w} \right)^{\alpha} \right)^{\frac{1}{1-\alpha}}$$

$$\tau(z) = \left(\frac{w}{(1-\lambda) M_T} \right)^{1-\alpha} \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda) M_T}{w} \right)^{\alpha} \right)^{\frac{\alpha}{1-\alpha}}.$$

Proof. See B.1. □

Now, notice from (2.10) that the production function satisfies Inada conditions, so $l_y(z) > 0$ for every firm. We assume that $\alpha > 0.5$ so that $\phi(\cdot)$ is strictly concave and first order conditions with respect to l_s are also sufficient. Using the fact that the solution is always interior, taking first order conditions of the right hand side of (2.10) with respect to l_y yields

$$w + \frac{(1-\lambda) M_T}{\phi\left(l_s(z)\right)} = \alpha z l_y^{\alpha-1}. \tag{2.11}$$

In the absence of theft (i.e. $M_T = 0$) (2.11) reduces to $w = \alpha z l_y^{\alpha-1}$, or $l_y = \left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}}$. We observe that theft creates a wedge which causes the marginal productivity of labor to be greater than w by a factor of $\frac{(1-\lambda) M_T}{\phi(l_s(z))}$. Observe that the wedge is increasing in the measure of thieves and decreasing in both a higher level of public law enforcement and private security. As a consequence of theft, firms are smaller in equilibrium than in the absence of theft:

$$\left(\frac{\alpha z}{w}-\left(\frac{(1-\lambda)M_T}{w}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}<\left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}}.$$

Corollary 4 shows that the ratio of theft experienced by a firm to private security expenditures is constant and greater than 1.

Corollary 4. *The ratio of theft experienced by a firm,* $(1-\lambda)M_T\tau(z)$*, and private security expenditures,* $wl_s(z)$*, is constant and equal to* $\frac{\alpha}{1-\alpha}$*.*

Proposition 3 shows every firm finds it optimal to hire security.

Proposition 3. *Assume* $\alpha > 0.5$*. Then* $l_s(z) > 0$ *for all* $z \geq z^E$*.*

Proof. By definition z^E is such that $\pi(z^E) = w$. From the expression for $\pi(z)$ in Lemma (9),

$$z^E = \frac{w^{1-\alpha}}{\alpha}((1-\lambda)M_T)^{\alpha} + \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} w$$

and $l_s(z) > 0$ if and only if $z > \frac{w^{1-\alpha}}{\alpha}((1-\lambda)M_T)^{\alpha}$. Since $z^E > \frac{w^{1-\alpha}}{\alpha}((1-\lambda)M_T)^{\alpha}$, the result follows. □

2.4 Results

2.4.1 Parameterization

Our current parametrization is chosen such that reasonable parameter values are able to give results which qualitatively match the micro and macro patterns we observe in the data. Table 2.4 shows the values of the parameters that we use and the moments we target.

We calibrate preference and technology parameters to match key aspects of the US economy. Our model economy consists of eight parameters. In accordance with Buera, Kaboski, and Shin (2011) we assume that entrepreneurial ability is Pareto distributed with shape parameter ν and scale parameter $\underline{z}$. Since Buera, Kaboski, and Shin (2011) also fit their model to the US economy, we adopt $\nu = 4.84$ from their paper. We set the nominal Pareto scale parameter $\underline{z}$ at 1 for simplicity. The distribution for preference on theft is assumed to be uniformly distributed and is characterized by $\underline{\theta}$ and $\overline{\theta}$. We calibrate these parameters to fall within a reasonable range given annual reported property crimes and the percentage of US citizens with a criminal record.[7]

We calibrate $\underline{c}$, public expenditure on the incarcerated, by matching the costs per prisoner relative to average income.[8] Parameter α is the returns to scale of the production function. We choose α to target an effective return to scale α of 0.85, which is commonly used in the literature.[9]

Finally, we choose λ, the level of public law enforcement, and the extra degree of freedom we have from the distribution function on θ to match inventory shrinkage and loss prevention expenditures as a percentage of revenue from retail firms, as reported by the 2011 National Retail Security Survey.

Table 2.4: Calibration - Parametrization

Moment	Data	Model	Parameter
Consumption Expenditure per Criminal	0.37	0.37	$\underline{c} = 0.36$
Loss Prevention Expenditures	0.35%	0.35%	$\lambda = 0.82$
Criminal Record	3.1%-27.8%	5.00%	$\overline{\theta} = 4.00$
Inventory Shrinkage	1.42%	1.42%	$\underline{\theta} = -0.69$
Returns to Scale	0.85	0.80	$\alpha = 0.80$
Pareto Shape Parameter	4.84	4.84	$\nu = 4.84$

2.4.2 Macro Results

The primary result of our paper is that changes to the level of public law enforcement have different effects depending on the current level of public law enforcement. In Figure 2.6a we observe that for low levels of public law enforcement, increases to this level can actually decrease the amount of total production. When we compare the model to the data the pattern is quite similar; however, we only account for a relatively small portion of the differences in GDP across countries due to differences in public law enforcement.

[7]The National Employment Law Project estimates that 27.8% of US adults have a criminal record. On the other hand the FBIs UCR Program reports a property crime rate of 3.1% in 2009.

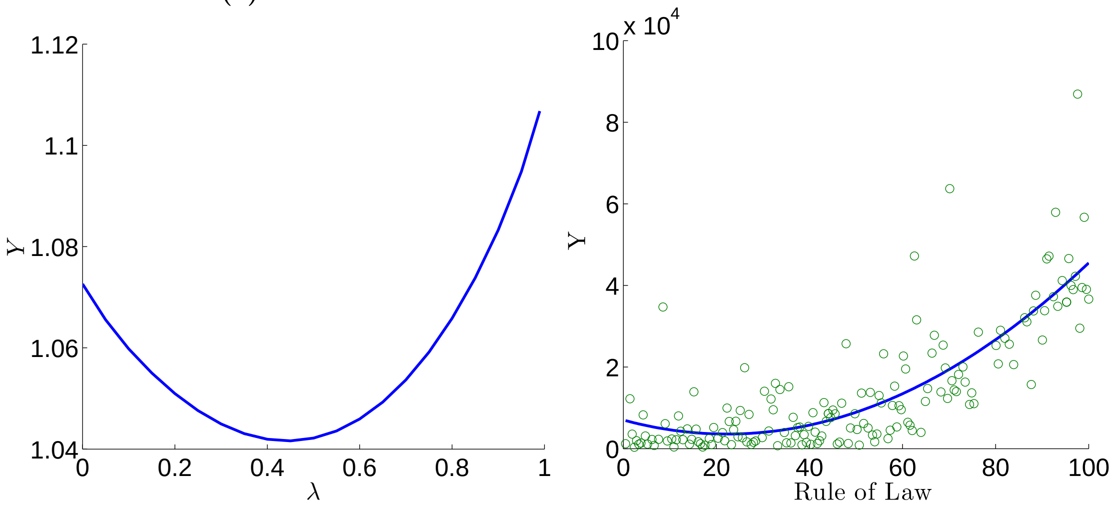
[8]According to the Bureau of Justice Statistics, as citetd in the report "Public Safety, Public Spending" prepared by the Public Safety Performance Project, the marginal cost per prisoner was $13,797 in 2005. On the other hand the Social Security Administration reports an Average Wage Index in 2005 of $36,953. We choose $\underline{c}$ so that $\frac{c}{w} = \frac{\$13,797}{\$36,953} = .37$.

[9]See Khan and Thomas (2013) or Ranasinghe (2012) for other papers using a similar number.

Small increases in the level of public law enforcement cause a decrease in GDP for those countries with low levels of public law enforcement. Countries with higher levels of public law enforcement demonstrate a positive correlation between per capita GDP and the level of public law enforcement. Finally, we note that the effect of public law enforcement on aggregate production can be as large as 6.02%.

Figure 2.6: Total Production vs Public LE

(a) Model **(b)** Data



In Figure 2.6b Y refers to the GDP per capita (PPP) of 2010. Source: World Bank. Authors' calculations.

We explain the primary mechanism for this result with Figure 2.7a. Recall from Table 2.2 the non-monotonicity in the incarceration rate. Our model produces the same pattern as we vary the level of public law enforcement holding all other parameters from the benchmark model fixed as shown in Figure 2.7a. The result can be explained rather intuitively. If we think of the incarceration rate as a rectangle with the vertical axis representing the level of public law enforcement λ, which in our model also represents the percentage of thieves who are caught, and the horizontal axis as the measure of people who steal M_T, then the incarceration rate is simply the area of this rectangle. In the benchmark model we observe that $\frac{\partial M_T}{\partial \lambda} < 0$. At some point the decrease in the measure of thieves outweighs the increase in the percentage of thieves who are caught.

This concept is visually represented in Figure 2.7b.[10]

Figure 2.7: Incarceration

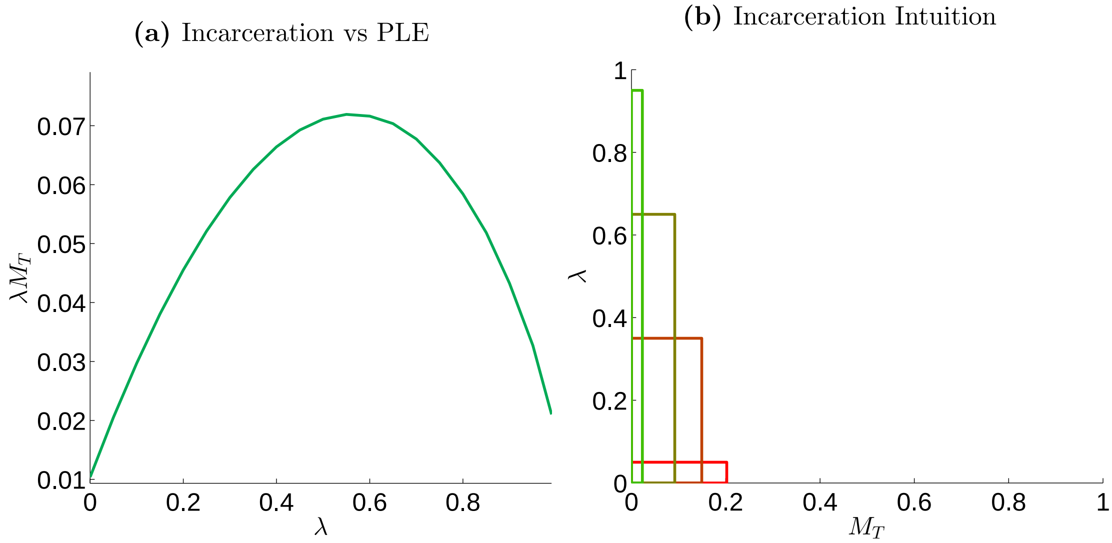
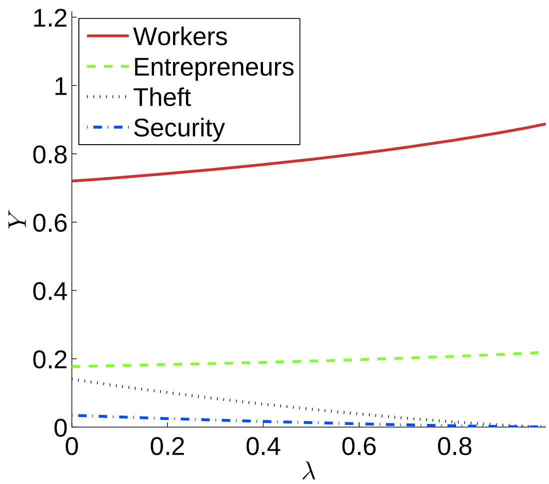


Figure 2.8 splits production into four categories, two of which are security and theft. The security line represents the total cost in final good paid to private security workers. The theft line represents the total value of goods stolen. When we look at these two variables across the level of public law enforcement we see that they match Figures 2.2a and 2.2b. While private security expenditures directly reduce theft, in equilibrium firms hire more private security and more agents choose to engage in theft when there is less public law enforcement. In this sense, public law enforcement directly reduces theft, but it also acts as a substitute for private security expenditures, which indirectly puts an upward pressure on theft. If policymakers fail to consider this indirect effect, they

[10]While the levels shown in this figure are quite large relative to incarceration rates observed in the United States, the idea is that increasing public law enforcement causes workers to be removed (or possibly misallocated) from the labor force. Multiple studies have been conducted to review the measure of people in the United States who have a criminal record. This number consistently comes out between one-quarter and one-third of the population. A recent survey from the Society of Human Resources Management shows that 92% of their members perform criminal background checks on some or all job candidates (The Society of Human Resources Management is the largest association of human resources personnel. The survey can be found in Background Checking: Conducting Criminal Background Checks (Jan. 22, 2010)). A number of articles including 65 Million "Need Not Apply" put out by the National Employment Law Project argue that having a criminal background can severely limit job opportunity. While our model is binary in whether an agent is able to work or not, we believe that the actual effect of public law enforcement observed in our model is consistent with what is observed in data.

are likely to overestimate the benefits from public law enforcement.

Figure 2.8: Uses of Production vs PLE



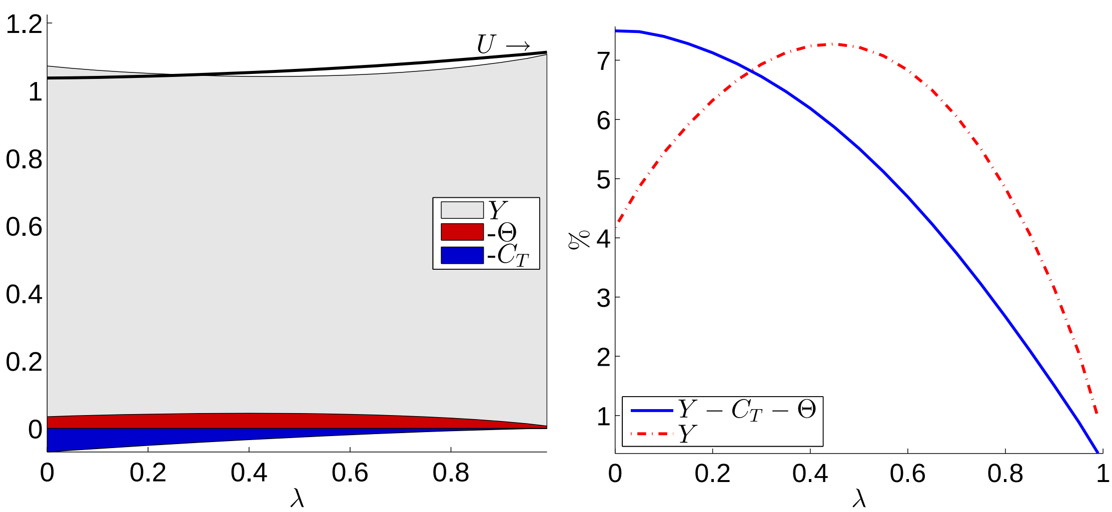
2.4.3 Implications on Welfare

Turning to welfare, we observe a similar pattern to that of production relative to the level of public law enforcement. For smaller levels of public law enforcement, increases in the level actually reduce total welfare as seen in Figure 2.9a. Nonetheless, the range of values for which welfare is decreasing is smaller than for production.

Figure 2.9: Welfare

(a) Welfare

(b) Increase in Consumption to Have the Same Utility as Without Theft



Explaining in detail this result requires analyzing the expression for welfare. Equation (2.12) shows that welfare is given by the production of the economy minus the cost incurred by thieves when stealing, C_T, and the aversion to steal, Θ. These two extra terms explain why welfare and production are not the same.

$$U \equiv \int_{(z,\theta)} u(z,\theta) dF(\theta) dG(z) = Y - C_T - \Theta, \tag{2.12}$$

where

$$C_T \equiv (1-\lambda) M_T \int_{z \geq z^E} (1 - \lambda F(\theta^E(z))) \frac{a\phi\left(l_s(z)\,|M_T\right)}{z l_y(z)^{\alpha_T}} \frac{\tau(z)^2}{2} dG(z)$$

$$\Theta \equiv \left(\int_{z<z^E} \int_{\underline{\theta}}^{\theta^W} + \int_{z \geq z^E} \int_{\underline{\theta}}^{\theta^E(z)} \right) \theta dF(\theta) dG(z).$$

Next, we analyze the effect of theft on welfare by calculating the extra consumption that consumers in our model require in order to be indifferent to an economy without theft. The economy without theft that we consider is characterized in (B.11) of B.2.

Since welfare includes non-pecuniary costs in utility due to theft and aversion to steal, we analyze how much production needs to increase both including and excluding C_T and Θ. That is, let Y^{NT} denote an economy where there is no theft. The solid line in Figure 2.9b shows $\frac{Y^{NT}+C_T+\Theta}{Y} - 1$ and the dotted line shows $\frac{Y^{NT}}{Y} - 1$.

From Figure 2.9b we can conclude that the effect of theft on welfare is considerable. For some values of λ consumption has to increase by over 7% in order to have the same utility as in an economy without theft. Notice that considering the costs of theft and aversion to steal lowers the amount by which consumption has to be increased for most values of λ since the parametrization shown in Section 2.4.1 implies that it is mostly consumers with negative values of θ who become thieves in equilibrium.

2.4.4 Negative Externality of Private Security

We now calculate the negative externality that is caused by hiring private security. For this, we consider an alternative equilibrium where firms are not allowed to hire as much private security as they find optimal. That is, let $l^*(z)$ be the optimal private security hired by firm z; i.e. the value of l_s that is a solution to (2.10). We consider an equilibrium where firm z can only hire $\widehat{l_s}(z) \equiv Sec \times l^*(z)$, for $Sec \in (0, 1)$; that is, an equilibrium where firms can only hire a fraction of the security that they find optimal. We keep all parameters of the model as in Table 2.4.

Figure 2.10: Production as a function of λ

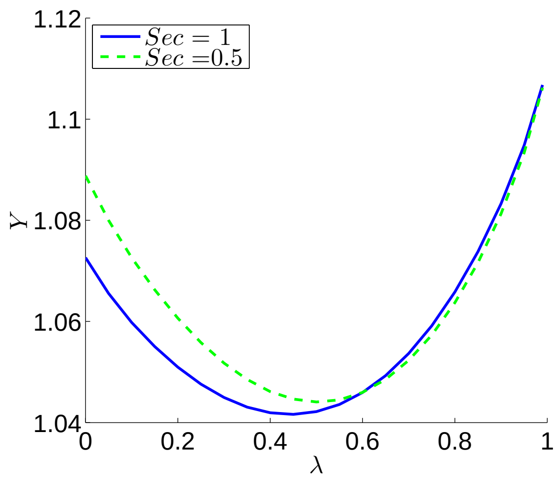


Figure 2.10 contrasts the production in equilibrium for $Sec = 1$ (i.e. the benchmark

equilibrium) and for $Sec = 0.5$ (an equilibrium where firms can only hire half as much security as they otherwise find optimal). For low values of λ, restricting to hire private security can increase production up to 1.5%. Private security helps diminish the wedge caused by theft, as seen in (2.11). Nonetheless, when $Sec = 1$, workers that could be hired to produce are hired as private security guards. When λ is low, private security causes a negative externality: workers that are hired as security guards could be hired to produce the final good. Since lower λ implies a higher percentage of revenue spent on security, the effect of reducing security is much higher for lower λ.

2.4.5 Sensitivity Analysis

We conduct a sensitivity analysis on the remaining variables to better understand how they affect the model. An important parameter in our model is $\underline{c}$. This parameter represents the consumption level received by agents who engage in theft and are caught. As $\underline{c}$ increases, the possibility of getting caught becomes less of a deterrent. Additionally, the burden borne by those who are not caught increases, reducing the value of not engaging in theft and adding incentives towards becoming a thief. As $\underline{c}$ increases, production, the average size of firms and utility all monotonically decrease, and M_T, the measure of people who become thieves, monotonically increases in $\underline{c}$. The implication is that, if you do not care strongly about very negative outcomes for those who are caught stealing, the best policy is to implement very harsh penalties. A potential reason to avoid harsh penalties is concern for the innocent and the costly as well as potentially inaccurate verification of guilt. This is currently outside the scope of this model.

The distribution of θ represents the distribution of the moral fibre of the agents in our model. Apart from matching moments in data, it is difficult to know a proper strategy for determining what this distribution should look like. However, we are able to see how changing the distribution affects the results. Conceptually there are two important components of the distribution of θ which affect theft in our model. First, the measure of people who steal is determined by the measure of people below the cutoff in the distribution of θ. Second, the sensitivity of the model to changes in various other parameters depends on the density of the distribution over θ at the cutoff.

We make the following observations. First, the model is more sensitive to changes in $\underline{\theta}$ than changes in $\overline{\theta}$. This is because changes in $\underline{\theta}$ directly increase the measure of people who prefer to steal whereas changes in $\overline{\theta}$ affect the density of people in the range of those who prefer to steal. Lowering $\overline{\theta}$ increases the density of people in the range of those who prefer to steal and vice versa. Second, for the most part the effects of θ on equilibrium moments are rather intuitive. The only unusual result is that total welfare is not monotonic in $\underline{\theta}$, but this is easily explained. In all real measures, lowering $\underline{\theta}$ makes the economy worse off, however, recall that θ is the measure of aversion to theft which factors directly into utility. Negative θ's can be interpreted as a rush or pleasure from stealing. As we increase the pleasure from stealing two things happen: The measure of people who steal increases and the extra utility those agents receive from stealing increases. If we make the aversion to theft negative enough, the overall utility can actually begin to decrease in $\underline{\theta}$.

2.4.6 Extensions

We now consider two extensions to the model: First we consider a model where theft causes a destruction of goods. That is, for every unit of good that is stolen, only a fraction δ can be consumed by thieves. In particular we replace thieves' problem (2.4) by (2.13).

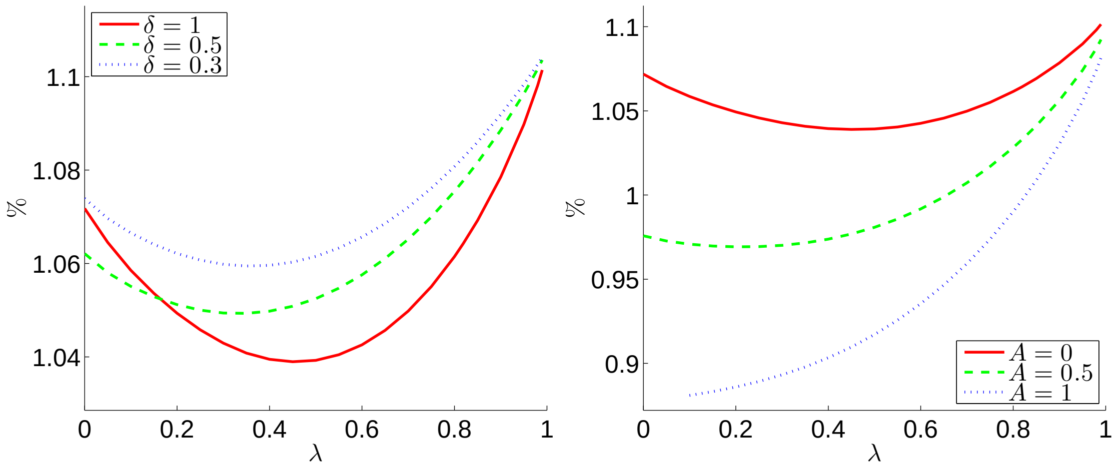
$$\pi_T(z) \equiv \max_{\tau \geq 0} \delta\tau - C_\tau(z). \tag{2.13}$$

Our benchmark model is given by $\delta = 1$ and we consider economies where we change the value of δ. Values such that $\delta < 1$ might act as a deterrent for thieves, since their return for stealing is decreased. However, it might also be the case that they might attempt to steal even more in order to achieve the same consumption as they would otherwise get when $\delta = 1$. In equilibrium we observe that for low levels of δ it is the first effect that dominates. For intermediate levels of δ, it depends on the level of public law enforcement: In economies with low λ theft increases, reducing total production in equilibrium.

In general, $\delta < 1$ causes production to be less sensitive to the level of public law enforcement. Moreover, for low values of δ production is higher across all levels of public law enforcement, relative to the case when $\delta = 1$. See Figure 2.11a.

Figure 2.11: Production for different values of δ and A

(a) Production for different values of δ **(b)** Production for different values of A



We also consider a model where theft causes labor to be less productive. We can rationalize this as entrepreneurs knowing that their employees might be stealing from the firm they are working at, or as employees working less time, since they devote some time on their job to steal. We model this feature by replacing (2.3) with (2.14).

$$\pi(z) \equiv \max_{l_y \geq 0, l_s \geq 0} z\left((1 - AM_T)l_y\right)^\alpha - wl_y - wl_s - (1-\lambda)M_T\tau(z), \qquad (2.14)$$

for $A \in [0,1]$. Our benchmark model is given by $A = 0$ and we consider economies such that $A > 0$. The fact that workers are less productive when there is theft causes production to be lower when $A > 0$ than in our benchmark model for all levels of public law enforcement (See Figure 2.11a). Additionally, production becomes more sensitive to the level of λ. In particular, for lower levels of public law enforcement, and increase in the level causes higher labor productivity. For values of A that are high enough, this increment in labor productivity counteracts the removal of agents from the labor force, thus making production increasing in λ for all levels of λ.

2.5 Conclusion

In this paper we propose a model of theft, private security and public law enforcement which matches a number of patterns in the micro data. Theft lowers total production directly and indirectly. First, theft acts as a wedge similar to a tax for firms which causes firms to be inefficiently small since the marginal product of labor is greater than the wage rate in equilibriums with positive amounts of theft. Private law enforcement helps decrease this wedge, but in order to do so, some of the labor force is taken away from producing the consumption good and used to provide security.

Perhaps the most surprising result of our model is that total production and welfare are not monotonic in levels of public law enforcement. The interaction of theft and public law enforcement is the source of the second mechanism which affects the total level of production in the economy. Public law enforcement can reduce total production and welfare because incarcerated agents are removed from the labor force. However, it also increases the disincentives of theft which causes a reduction in the measure of agents who elect to steal. This in turn reduces the measure of agents who are incarcerated. The interaction of these two forces can cause non-monotonic effects on the total level of production and welfare which might explain why we observe such vastly different levels of public law enforcement. Specifically, countries with low levels of public law enforcement do not have immediate benefits from small increases to the level of public law enforcement.

Chapter 3

Linking Firm Structure and Skill Premium

3.1 Introduction

In this paper we bring new data and new theory to bear on a longstanding question: Why do large firms pay higher wages than smaller firms? While many explanations focus on the fact that large firms employ disproportionately more labor-augmenting technology than smaller firms (e.g. Dunne and Schmitz (1995), Dunne, Foster, Haltiwanger, and Troske (2002)), we propose a different story: Larger firms choose organizational structures with more levels of management, and the wages paid to managers increase as the number of levels grows. We develop a model in which managers near the top of the management hierarchy in large firms have a larger marginal contribution to output than their counterparts at smaller firms since they oversee a larger quantity of firm activities. As a result, they receive higher wages.

Our model is consistent with empirical evidence that the wage premium offered by large firms is greater among skilled employees than among unskilled employees (e.g. Brown and Medoff (1989), Davis and Haltiwanger (1991), and Idson and Oi (1999)). Equivalently, the skill premium (the average wage a firm pays to skilled employees divided by the average wage it pays to unskilled employees) is increasing in firm size.

We are able to go a step further, using recent data from Chile's manufacturing sector to show that managers in particular receive a large wage premium at large firms - much more so than other categories of skilled workers. The empirical literature typically uses broad definitions of skilled and unskilled labor to study the skill premium. For example, some studies (e.g. Dunne and Schmitz (1995) and Pavcnik (2003)) treat blue-collar workers as skilled and white-collar workers as unskilled even though there are substantial skill differences within each group. (For example, both a CEO and a data entry employee would be classified as white-collar.) By using detailed plant-level data on employment and wages from Chile's Manufacturing Industry National Survey (ENIA) during the period 1995-2007, we are able to obtain a more granular picture of the wage premia offered by large firms within different labor categories. We construct and study several alternative measures of the skill premium at the firm level. One measure is the ratio of the average wage a firm's managers receive to the ratio of wages a firm's unskilled production workers receive. Another is the ratio of average wages received by skilled production workers ("technicians") to average wages received by unskilled production workers. The latter ought to most directly capture any skill-bias in the technology hired by large firms.

What we find is striking. The effect of firm size on the manager skill premium is close to 20 times greater than the effect of size on the production worker skill premium. Our results suggest that managerial inputs and firm hierarchies are an important reason why large firms pay a higher skill premium than small firms.

To investigate the relationship between firm size and the skill premium theoretically, we develop a hierarchical model of firms that borrows elements from Caliendo and Rossi-Hansberg (2012) in which large firms optimally choose organizational structures with more levels of management than small firms. A manager receives direct reports from employees on the next-lowest level (either production workers or lower-ranked managers) and is able to augment their output multiplicatively using her managerial labor. High-level managers at large firms, who have the scope to augment large amounts of output, receive high wages. Small firms, producing relatively little output, have no such high-productivity managers and therefore pay their managers lower wages on average. Additionally, since wages for unskilled workers are constant across firms in our model, the skill premium is increasing in firm size.

An immediate implication of the model is that managers with more direct reports receive higher wages. We find evidence in the Chilean data consistent with this prediction: The skill premium a firm pays to its managers is increasing in the firm's ratio of managers to unskilled workers.

One novel feature of our model is that the number of levels of management a firm chooses is a continuous, rather than discrete variable. Doing so gives the model substantial flexibility and allows us to map the firm's production function into a class of familiar production functions in which workers differ only in the quality units they supply. The advantage of our model, relative to those with just human capital, is that it allows us in addition to evaluate firm hierarchical choices. While the notion of having, say, 2.5 levels of management is not immediately intuitive, we will attempt to provide some clarity later when we present the model.

This paper is connected to an extensive empirical literature on the relationship between firm size and wages. Brown and Medoff (1989) provided an early contribution by showing that large firms tend to pay their workers higher wages than smaller firms. The same phenomenon has been documented by many other researchers. For instance, Davis and Haltiwanger (1991) study U.S. Census manufacturing data from 1963-86 and find a steep monotonic relationship between a firm's size (in terms of number of employees) and the average wage paid to its workers. Idson and Oi (1999) corroborate their results using more recent Census data.

Most empirical work, including the Davis and Haltiwanger (1991) study cited above, also shows that the wage gap between large and small firms is the largest among skilled workers, indicating that the within-firm skill premium increases, on average, with firm size. Haskel (1998) provides a counterpoint by showing that, among UK manufacturing firms in the 1980s, small firms had a higher skill premium than larger firms.

Many explanations have been advanced for the wage premium paid by large firms (for a summary, see Katz and Summers (1989)). One explanation for why large firms pay higher wages than small firms that has garnered substantial empirical support is that large firms demand more skilled labor than small firms. For example, Abowd, Kramarz, and Margolis (1999) provide evidence from France, and Zweimüller and Winter-Ebmer (2003) provide evidence from Switzerland. One reason why large firms demand more skilled workers is that they need skilled labor to operate more advanced production technology (See Dunne and Schmitz (1995) or Dunne, Foster, Haltiwanger, and Troske

(2002)). While differences in technology appear to contribute meaningfully to the wage gap between large and small firms, a substantial portion of the variation remains unexplained by technological differences.

The technological explanation for the firm-size wage gap has been adopted by researchers seeking to understand the effects of international trade on labor markets. Bernard and Bradford Jensen (1999) document that exporting firms, which tend to be the largest firms, raise the demand for skilled labor relative to unskilled labor, leading to increases in the skill premium. Yeaple (2005) and Bustos (2011) present models explaining exporters' high demand for skilled labor. By serving a large international market, exporters have a stronger incentive to invest in cost-reducing technologies than firms that only serve a small local market. To the extent that the new, labor-saving technologies require skilled labor to operate, large exporting firms will hire more skilled labor than smaller firms. Helpman, Itskhoki, and Redding (2010) further develop this argument theoretically.

Our theoretical approach to studying the firm-size wage gap differs from existing theories in that we focus on the effect of firm size on organizational structure. While our model is consistent with the fact that larger firms demand higher-skilled employees than smaller firms, we develop explicitly the idea that larger firms choose a hierarchical structure with more levels than smaller firms. To do so, we draw on the hierarchical model of Garicano (2000) in which firms optimally choose the number of levels of management by trading off gains from having more managers solve problems against the costs of communication between many levels of management. The basic structure of the model has been adapted to study the effects of international trade on organizational structure (Caliendo and Rossi-Hansberg (2012)) as well as aggregate productivity differences between countries (Grobovšek (2013)).

Relative to their models, our main theoretical contribution is to model the number of management levels that a firm chooses as a continuous, rather than discrete, choice. Doing so demonstrates that our model is essentially a standard, simple human capital model with a restriction on firms which require them to hire specific ratios of workers across levels of education. This gives the ability to make specific predictions about firm organizational structure and skill premia. Instead of being perfectly substitutable in production, imposing a structural hierarchy upon firms turns employees with varying levels of education into complements. Two workers cannot simply replace a manager but must work in conjunction with managers in order to accomplish additional tasks. Larger firms require more levels and a higher average level of human capital, and consequently they pay a higher average wage.

Finally, our paper relates broadly to the extensive literature on the increase in the skill premium over time observed in many countries. Skill-biased technical change is the most prominent (and perhaps the most debated) hypothesis for why the skill premium has risen. While many studies support the skill-biased technical change hypothesis, the evidence is mixed. Card and DiNardo (2002) provide a review of the debate and find the hypothesis lacking. Pavcnik (2003) studies plant-level employment data from Chile - though from an earlier time period than we do - and finds that skill-biased technical change cannot explain the increases in the skill premium observed in her sample. Doms, Dunne, and Troske (1997) come to a similar conclusion when studying U.S. manufacturing plant data. In each case, controlling for unobserved plant characteristics eliminates the effect of a plant's technology on the premium it pays to its skilled workers. While we do not focus on the behavior of the skill premium over time, our results resonate with theirs in that technology does not appear to be a key driver of our empirical results concerning the skill premium.

The paper proceeds as follows: In Section 3.2 we present our data findings. In Section 3.3 we introduce a simple, discrete firm structure. Section 3.4 extends the firm structure to allow for a flexible selection of both levels and measure of employees. Section 3.5 presents and characterizes a general equilibrium model with the firm structure introduced in the previous section. Finally, Section 3.6 concludes.

3.2 Data

In this section we use data from the Manufacturing Industry National Survey (ENIA) compiled by the Chilean National Statistics Institute (INE) to analyze the relationship between size and skill premium. This database allows us to differentiate between expenditures on different types of skilled workers. In this way we are able to explain more precisely how size affects skill premium.

ENIA compiles economic and accounting information for registered firms that are established in Chile and have 10 or more employees.[1] We have annual information from 1995 to 2007 for around 4,200 firms per year, although information differentiating types of skilled workers is only available beginning in 2000.

The available information allows us to distinguish between skilled and unskilled workers, as well as how much was spent on employees of each type. Unskilled workers are non specialized workers in charge of executing tasks, mainly manual, that are directly related to the production process ("Blue Collar"). We categorize all other employees as skilled workers ("White Collar").[2]

We now analyze the relation between size and skill premium. We define the skill premium of a firm as the ratio of the average wage per skilled worker to the average wage per unskilled worker. We are able to identify different types of skilled workers in the data. In particular skilled workers can be further divided into owners, managers, technicians and administrative personnel. For each firm we estimate five measures of skill premium: a skill premium for each of the types of skilled workers and a skill premium aggregating across all skilled workers. Table 3.1 shows the resulting skill premium by type of skilled worker. Note that the average manager gets paid over five and a half times the wage of an unskilled worker. This number is much higher than the skill premium for other types of skilled workers.

Table 3.1: Skill Premium by Type of Skilled Worker

Managers	5.56
Technicians	1.70
Owners	2.62
Administrative Personnel	1.41
All skilled workers	1.66
Mean number of workers	48.41
Std. deviation:	75.99

Source: INE. Authors' calculations.

Next, we analyze the impact of the size of a firm on the skill premium. To do this

[1] INE targets firms with 10 or more employees. If a firm has more than one establishment, each establishment is reported separately. In this case, there can be reports by establishments with under 10 employees.

[2] Only full-time workers are included in the analysis.

we run the following regression:

$$SP_{i,t} = \beta_0 + \beta_1 size_{it} + \alpha_t + \gamma_{ind} + \varepsilon_{it}, \tag{3.1}$$

where $size_{it}$ is the natural logarithm of the total number of workers at firm i in period t, α_t is a time fixed effect, and γ_{ind} is an industry fixed effect. Table 3.2 shows that the skill premium is increasing in the size of a firm. This result is robust across all types of skilled workers. Moreover, size seems to affect the skill premium of managers more than it affects the skill premium of other skilled workers. In particular, the effect of size on the skill premium of technicians is very small: Holding all other variables constant, an increase of 1% in the size of a firm raises the skill premium of technicians by 0.0008. That is, even though there is a positive relation, the size of a firm does not seem to be economically significant in determining the skill premium of technicians.

The regression results suggest that bigger firms pay higher wages for skilled workers due to a higher marginal productivity of managers, not of technicians. In particular, the data casts doubt on whether larger firms acquiring skill biased capital can explain why larger firms offer a higher skill premium, as has been proposed in the literature (e.g. Dunne and Schmitz (1995) and Dunne, Foster, Haltiwanger, and Troske (2002)).[3] Rather, our data analysis supports the importance of considering the organizational structure of a firm when analyzing the relation between size and skill premium.

[3] We get similar results if we calculate the skill premium using average hourly wage instead of average annual wage.

Table 3.2: Skill Premium vs Size of Firm

	Managers	Technicians	Owners	Administrative Personnel	All skilled workers
Size	1.767***	0.083***	1.641***	0.130***	0.201***
	(0.031)	(0.006)	(0.033)	(0.005)	(0.004)
Year fixed effects	yes	yes	yes	yes	yes
Industry fixed effects	yes	yes	yes	yes	yes
Number of Observations	$13,549$	$20,381$	$13,623$	$22,842$	$34,291$
Adjusted R^2	0.264	0.082	0.204	0.089	0.192

OLS estimation. Standard errors are in parenthesis.
***: Significant at 1%.
Source: INE. Authors' calculations.

We next analyze the relationship between the organizational structure of a firm and the skill premium. For this we extend regression (3.1) to control for the ratio of unskilled workers to managers. We label this ratio as span of control, SC. The extended regression is as follows:

$$SP_{i,t} = \beta_0 + \beta_1 size_{it} + \beta_2 SC_{it} + \alpha_t + \gamma_{ind} + \varepsilon_{it},$$

where α_t and γ_{ind} are defined as in (3.1). Table 3.3 shows the results. We find that the higher the number of unskilled workers per manager, the larger the difference in wage. This supports the idea that firms where managers have a wider span of control offer a higher compensation to managers.

Table 3.3: Skill Premium for Managers vs Composition of Firm

Size	1.619***
	(0.034)
Span of control	0.010***
	(0.001)
Year fixed effects	yes
Industry fixed effects	yes
Number of Observations	$13,549$
Adjusted R^2	0.267

OLS estimation. Standard errors are in parenthesis.
***: Significant at 1%.
Source: INE. Authors' calculations.

These data findings motivate the structure of firms used in our model. We will focus on the role of managers, abstracting from other types of skilled workers, since it is the wages of managers that drive the relation between size and skill premium. Additionally, the firm structure should explain why managers with more direct reports get paid more.

3.3 Discrete Firm Structure

To develop insight into the relationship between firm size and the skill premium paid to managers, we develop a hierarchical model of firms that draws elements from Garicano (2000) and Caliendo and Rossi-Hansberg (2012). Briefly, a firm consists of several levels of employees, with employees on the bottom level representing production workers and employees on higher levels representing managers. Production workers complete a number of tasks, but a fraction of their potential output is lost before their tasks are converted into the firm's final good. Managers on the first level above workers receive the uncompleted tasks from the workers who report directly to them and are able to complete a portion of them. Tasks that even a manager on the first level cannot complete are handed to the next level of management, and so on. A firm chooses the number of levels of management optimally to balance the benefits of being able to solve additional tasks against the costs of compensating the additional managers.

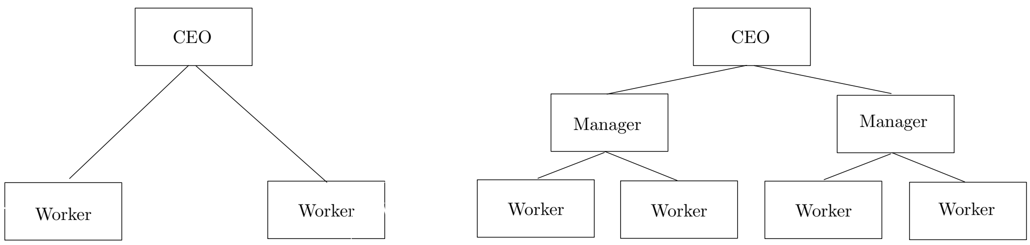
In our full general equilibrium model, the number of management levels chosen by a firm is a continuous variable. We first develop intuition by presenting a simple partial-equilibrium version of the model in which the number of management levels is discrete.

Consider a firm with $X \in \mathbb{N}$ levels. The structure of the firm is rigid in the sense that specific quantities of managers at each level are required. Each worker at level $x = 0$ receives a measure 1 of tasks. Of those tasks, he is able to solve a share equal to $1-\lambda$, and the uncompleted fraction $\lambda \in (0, 1)$ of the tasks is passed to the level immediately above. No two employees work on the same task. Each manager in level $x > 0$ is assigned R direct reports from the level immediately below and so they receive a measure $R(1 - \lambda)$ of unsolved tasks of which they are able to solve a share $1 - \lambda$. The unsolved fraction λ of the tasks are passed in turn to the next-highest level of management. The quantity R is constant within a firm so that a manager above the first management level receives direct reports from R managers on the level immediately below. A firm's total output is the number of tasks its workers and managers are collectively able to solve. (In the next section, a firm's output will be a function of the number of tasks completed.)

We refer to R as the span of control of the firm. We denote employees at the lowest level ($x = 0$), who receive no reports, as workers, while employees on higher levels are managers. We will denote managers at the highest level as CEO's and we assume that every firm has one CEO. Figure 3.1 displays the case where $R = 2$ for firms with $X = 1$ and $X = 2$.

Figure 3.1: Firm with one or two levels of management

(a) Firm with one level of management **(b)** Firm with two levels of management



Each firm chooses the optimal number of levels of management X. Hiring additional levels of management allows more tasks to be solved in exchange for a larger wage bill. Specifically, a firm with X levels hires R^{X-x} managers at level $x > 0$ and R^X workers. Furthermore, it is able to solve $R^X(1 - \lambda^{X+1})$ tasks. Lemma 10 states and proves this result.

Lemma 10. *Consider a firm with X levels and span of control R. Each manager in level $x \in \{1, \ldots, X\}$ solves a measure $(R\lambda)^x(1-\lambda)$ of tasks. The total measure of tasks completed by the firm is $R^X(1-\lambda^{X+1})$.*

Proof. See C.1. □

Notice that if $R\lambda > 1$ then the contribution of managers is increasing in the level of management x. Assume that wages of all employees - both workers and managers - are proportional to the number of tasks they complete (as they will be in the general equilibrium version of the model). Define the skill premium of a firm, $SP_{X,R}$, as the ratio of the average wage of managers to the average wage of workers. Then $SP_{X,R}$ can be expressed as

$$\begin{aligned} SP_{X,R} &= \frac{\sum_{x=1}^{X} R^{X-x}(R\lambda)^x(1-\lambda)}{\sum_{x=1}^{X} R^{X-x}} \times \frac{1}{1-\lambda} \\ &= \frac{\lambda}{1-\lambda}(R-1)\frac{R^X}{R^X-1}(1-\lambda^X), \end{aligned}$$

where the first term is the average contribution of managers and the second is the inverse of the average contribution of workers. Lemma 11 demonstrates that the skill premium is increasing in both levels of management and span of control.

Lemma 11. *If $R\lambda > 1$ then $SP_{X+1,R} > SP_{X,R}$ and $SP_{X,R+1} > SP_{X,R}$.*

Proof. See C.1. □

The assumption $R\lambda > 1$ implies that each manager completes more tasks than each of her direct reports individually. The lemma establishes two results central to the theme of our paper for the simple, discrete version of the model. The skill premium increases in firm size (proxied by X) because larger firms have top managers who receive many reports (both direct and indirect) and are therefore able to complete many tasks. On average, then, managers at large firms receive higher wages than managers at small firms. The second part of the lemma results from the fact that managers who receive more direct reports have the opportunity to solve more tasks that employees at lower levels were unable to complete.

3.4 Continuous Firm Structure

We now extend the firm structure to allow for a flexible (continuous) selection of both levels and measure of employees. Consider a firm with X levels and span of control R. To maintain consistency between the discrete version of the model and the continuous version, we assume a width one of CEOs. The span of control R determines the width of managers at level x to be R^{X-x}. Let manager $n \in [0, R^{X-x}-1]$ in level x be a manager who occupies a space of width one with lower edge equal to n. We define the "area of command" for manager n located on level $x > 0$ to be the set of workers and managers who either report directly to manager n or report to a manager within the area of command for manager n. Manager n is in charge of dealing with the unsolved tasks of everyone within her area of command. Formally, the area of command of manager n on level $x > 0$, $AC(x,n)$, is defined as

$$AC(x,n) \equiv \left\{(x',n') : x' \in [0,x)\, , nR^{x-x'} < n' \leq (n+1)R^{x-x'}\right\}.$$

To map the concept of area of command to the discrete case, consider a firm with 2 levels and span of control $R = 3$. The area of command of manager $n = 1$ on level $x = 1$ is all workers at the bottom level located strictly above position 3 ($= nR^x$) and below position 6 ($= (n+1)R^x$). Figure 3.2 illustrates an area of command. Lemma 12 characterizes the measure of employees in $AC(x,n)$.

Lemma 12. *There is a continuum of measure $\frac{1}{\ln R}(R^x - 1)$ of employees in area of command $AC(x,n)$.*

Proof. The measure of managers in $AC(x,n)$ is given by

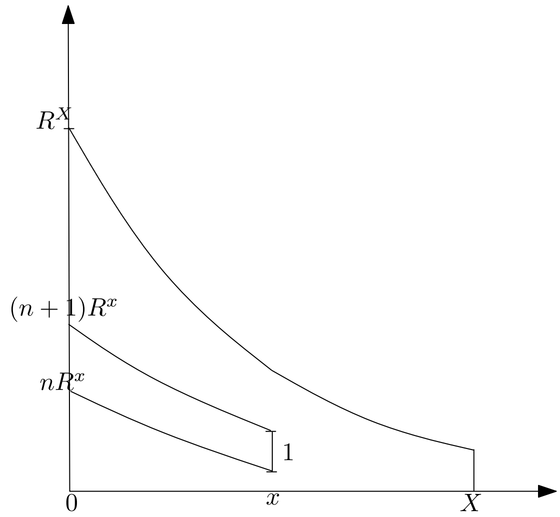
$$\int_{(x',n')\in AC(x,n)} dx'dn' = \frac{1}{\ln R}(R^x - 1)$$

□

Corollary 5. *Assume that there is a measure 1 of CEOs. Then there is a measure $\frac{R^X-1}{\ln R}$ of managers in a firm with X levels and span of control R.*

Notice that the measure of employees in an area of command doesn't depend on the location of manager n on level x. Therefore, without loss of generality, we'll denote areas of command and the tasks solved within them by the level of the manager. That is, the area of command of a manager in floor x will be denoted by $AC(x)$.

Figure 3.2: $AC(x)$



Similar to before, every employee is able to solve a measure $1 - \lambda$ of the tasks she is assigned. The remaining fraction is sent to a manager located in a higher level. As before, the measure of tasks each employee is able to solve is independent of the tasks any other employee can solve. Now, the number of reports that a manager in level x gets from employees in level $x - \Delta$ is equal to R^{Δ}. Intuitively, as we transition from the discrete to continuous case, levels come closer together and each manager gets less reports from the level directly below them, but there are more levels below them. Additionally, they are able to solve only a fraction $(1 - \lambda)\Delta$ of the tasks they receive. Proposition 4 characterizes the number of tasks that are solved in the area of command $AC(x)$.

Proposition 4. *Suppose each worker receives a measure one of tasks. Let $\mathcal{T}(x)$ be the number of tasks that are solved in the area of command $AC(x)$. Then*

$$\mathcal{T}(x) = R^x \left(1 - e^{-x(1-\lambda)}\right).$$

Proof. See C.1. □

Corollary 6. *Assume that there is a measure 1 of CEOs. Then the total number of tasks solved by a firm with X levels and R reports is $R^X(1 - e^{-X(1-\lambda)})$.*

Now, the area of command of a manager in level x, $AC(x)$, receives a measure R^x of tasks to be solved. From Proposition 4, the total measure of tasks solved within this area of command is $R^x\left(1 - e^{-x(1-\lambda)}\right)$, so there is a measure of $R^x - R^x\left(1 - e^{-x(1-\lambda)}\right) = R^x e^{-x(1-\lambda)}$ of unsolved tasks that a manager at floor x faces. Of this measure she is able to solve a fraction $1-\lambda$. Therefore the marginal contribution of a manager at level x, MC_x is given by

$$MC_x \equiv R^x e^{-x(1-\lambda)}(1-\lambda). \tag{3.2}$$

3.5 General Equilibrium Model

We now embed the structure introduced in Section 3.4 in a general equilibrium framework with consumers, firms and an education sector. Consumers supply labor inelastically and choose a quantity of education. The education a consumer receives determines the level of a firm the consumer is qualified to work at and the wage he receives. Consumers use their labor income to pay for education and the consumption good. Only by receiving education can an individual work as a manager. Firms operate a technology that converts completed tasks into the final good with heterogeneous productivities, and they choose an optimal number of levels of management, which in our model is equivalent to choosing the size of the firm. The education sector employs educators and is competitive.

3.5.1 Consumers

There is a measure L of consumers. All consumers are identical at the beginning of each period and own an equal share of all firms. Consumers choose whether to work in the education sector or the private sector by comparing the utility received from working in the education sector, V_E, with the utility received from working in the private sector, V_P.

In order to obtain competence in educating others, educators require $h_E < 1$ units of instruction from other educators. Educators supply instruction in the education sector inelastically and receive a wage of w_E. The budget constraint for an educator is

$$c_E + w_E h_E = w_E + \frac{\Pi}{L}.$$

This implies that the utility an educator receives is

$$V_E \equiv u\left(w_E(1 - h_E) + \frac{\Pi}{L}\right),$$

where Π denotes the total profits across all firms and $u(\cdot)$ is the utility function.

Consumers in the private sector choose a level of education x. The quantity x also signifies the level at which the consumer is able to manage at a firm. For example, an individual with a level of education $x = 1$ can work at the first (lowest) level of management. The amount of time required for an educator to train a consumer to work at level x is denoted by $h(x)$. Formally, the problem of a consumer in the private sector is:

$$\begin{aligned} &V_P \equiv \max_x u(c) \\ &\text{s.t. } c + w_E h(x) = w(x) + \frac{\Pi}{L} \end{aligned} \tag{3.3}$$

Since all consumers are ex-ante identical, it must be the case that consumers are indifferent between all choices in their optimal set. If we assume that there exists an x_E such that the level of education required to work at level x_E is equivalent to the level of education required to become an educator (i.e. $h(x_E) = h_E$), then the indifference requirement immediately implies that $w_E = w(x_E)$. This allows us to simplify the consumer's problem to be (3.3) since V_E is given by the utility obtained from choosing x_E.

Notice that in order to educate a manager to level x we require $h(x)$ units of educator time. Further, each of those educators requires h_E units of educator time, and those educators also need h_E units of educators time, and so on. Therefore the total time required in order to train a manager to level x is

$$h(x) + h(x)h_E + (h(x)h_E)h_E + \cdots = h(x)\sum_{n=0}^{\infty} h_E^n = \frac{h(x)}{1-h_E}.$$

3.5.2 Firms

The homogeneous final good is produced by a continuum of firms. A firm whose employees complete $\mathcal{T}$ total tasks produces the final good using technology $Y(\mathcal{T}, z)$. We make basic regularity assumptions on Y; namely, $Y_{\mathcal{T}} > 0$, $Y_{\mathcal{T}\mathcal{T}} < 0$, $Y_z > 0$, and $Y(\mathcal{T}, z)$ satisfies Inada conditions on $\mathcal{T}$. Firms have idiosyncratic technology parameters z distributed according to the density function $f(\cdot)$. Given z, the firm chooses a number of employee levels X in order to maximize profits. The structure of the firm is rigid in the sense that a firm with X levels must employ R^{X-x} employees at each level $x \in [0, X]$ at the firm. In this sense, the firm faces a modified Leontief production function for which the proportions of employees hired at each level are fixed. Workers and managers are able to complete tasks as outlined in Section 3.4. Let $\mathcal{T}(X)$ denote the number of tasks completed by a firm with X levels.[4] Profits for firms are given by

$$\begin{aligned} \pi(z) = & \max_X Y(\mathcal{T}(X), z) - \int_x w(x) l(x, X) dx \\ & \text{s.t. } l(x, X) = R^{X-x}. \end{aligned} \tag{3.4}$$

Total profits across all firms are then $\Pi = \int_z \pi(z) f(z) dz$.

3.5.3 Relation to the Standard Model with Human Capital

At its core, our model is a model with human capital or education with the added restriction that firms hire specific ratios of workers across levels of education. Instead of being perfectly substitutable in production, imposing a structural hierarchy upon firms turns employees with varying levels of education into complements. Two workers cannot simply replace a manager but must work in conjunction with managers in order to accomplish additional tasks. Larger firms require more levels and ultimately a higher average level of human capital. Consequently, they pay a higher average wage.

[4] Recall that $\mathcal{T}(X) = R^X\left(1 - e^{-X(1-\lambda)}\right)$; see the Corollary to Proposition 4.

3.5.4 Equilibrium

An equilibrium is a function which maps education levels to wages, $w : \mathcal{X} \to \mathbb{R}_+$, and a function which maps firm technology endowments to an optimal number of levels for the firm, $X^* : \mathcal{Z} \to \mathbb{R}_+$, such that:

1. X^* solves the firm's problem (3.4);

2. consumers are indifferent between any education level which is employed with positive measure;

3. the total labor market clears

$$\int_z \int_x l(x, X^*(z)) \left(1 + \frac{h(x)}{1 - h_E}\right) f(z) dx dz = L;$$

4. the goods market clears

$$\int_z Y(\mathcal{T}(X^*(z)), z) f(z) dz = \int_{x \neq x_E} c(x) g(x) dx + M_E c(x_E),$$

   where

$$M_E \equiv \int_z \int_x l(x, X^*(z)) \frac{h(x)}{1 - h_E} f(z) dx dz$$
$$g(x) \equiv \frac{1}{L} \int_z l(x, X^*(z)) f(z) dz.$$

3.5.5 Solving the Equilibrium

We make assumptions about the functional form of $h(x)$ and the firm structure in order to obtain an analytical solution to the model.

Assumption 1.

$$h(x) = (R^x e^{-x(1-\lambda)} - 1)(1 - h_E)$$

Assumption 2. *The span of control is sufficiently wide, specifically:*

$$\ln R > 1 - \lambda$$

We denote by w_0 the wage rate for an unskilled worker on the bottom level of a firm, who has received no education. By equilibrium condition 2 above we know that consumers must be indifferent between obtaining any education level which is employed with positive measure. This implies that $w(x) = w_E h(x) + w_0$. Assumption 1, the marginal productivity of a worker at level x from equation (3.2) and the fact that $w_E = \frac{w_0}{1-h_E}$ imply that the marginal wage bill for completing an additional task is constant and equal to $\frac{w_0}{1-\lambda}$. Therefore w_0 is sufficient for characterizing the entire wage schedule.

Now, due to the rigid nature of the firm's structure and Assumption 2 on the span of control R of the firm, the mapping from X to $\mathcal{T}$ is monotonic and we can simplify the firm's problem to choosing the optimal number of tasks to be solved. Proposition 5 formally states and proves this.

Proposition 5. *Under Assumptions 1 and 2 the firm's problem becomes:*

$$\max_{\mathcal{T}} Y(\mathcal{T}, z) - w_0 \mathcal{T}. \tag{3.5}$$

Proof. See Appendix. □

This immediately implies that a firm with a higher technology parameter z will solve more tasks. Since tasks are monotonically increasing in the number of levels, it is also straightforward that the number of levels in a firm is increasing in z. The continuous case offers an analytical advantage over the discrete case since solutions do not depend on cutoff values for z. The continuity in selection of tasks along z allows us to demonstrate the existence of the equilibrium which we formalize with Proposition 6.

Proposition 6. *Suppose Assumptions 1 and 2 hold. Then an equilibrium exists.*

Proof. See C.1. □

Corollary 7. *If $Y(\mathcal{T}, z) = z\mathcal{T}^{\alpha}$, then*

$$\mathcal{T}^*(z) = \left(\frac{\alpha z}{w_0}\right)^{\frac{1}{1-\alpha}}$$

for each z and

$$w_0 = \alpha \left(\frac{\mathbf{E}\left[z^{\frac{1}{1-\alpha}}\right]}{L(1-\lambda)} \right)^{1-\alpha}$$

clears the labor market.

3.5.6 Skill Premium

The model can help us understand the effects of firm size and structure on skill premium. We first develop the notion of a skill premium within the model. Similar to section 3.3, we define the skill premium for a firm with X levels and span of control R to be the ratio of the average wage for managers to the wage for workers; that is,

$$SP(X,R) \equiv \frac{\int_{x>0} w(x)l(x,X)dx / \int_{x>0} l(x,X)dx}{w_0}.$$

This aligns well with the definition we use in the data where skill premium is defined as the ratio of average wages for managers to average wages of blue collar workers. Using this definition of skill premium, we are able to show that skill premium is increasing in the size of the firm.

Lemma 13. *Under Assumptions 1 and 2,* $\frac{\partial SP}{\partial X} > 0$.

Proof. See C.1. □

Next, we show that the skill premium is increasing in the span of control of the firm.

Lemma 14. *Under Assumption 1,* $\frac{\partial SP}{\partial R} > 0$.

Proof. See C.1. □

In our model, there is a monotonic relationship between the size of firms and the number of levels the firm employs. Therefore, Lemma 13 aligns with our first empirical observation, namely, that skill premium is increasing with size. This is a direct result of the fact that managers at higher levels have a higher marginal productivity than managers at lower levels and workers. As more levels are added, the average productivity of managers increases and since the productivity per worker is constant in size, the result is obtained.

The intuition for our second result is similar. The number of tasks passing through a manager in a given level is increasing in the number of his direct reports. As the manager's span of control increases so does his marginal contribution to the firm. In our model this directly correlates to the manager's compensation due to our educational structure, thus our second result 14. This result can be generalized by relaxing our assumption on education and adding consumer heterogeneity. In any model where marginal contribution and wages are positively related a similar result will hold. This is also consistent with the second fact we observe in the data, namely, that skill premium is increasing in the ratio of workers per manager.

3.6 Conclusion

In this paper we use recent data from the Chilean Manufacturing Survey to document that, consistent with previous findings, skill premium is positively related with the size of firms. We exploit the rich information we have on different types of skilled workers to estimate the relationship between skill premia by worker type and size. We find that the effect of size on skill premium is much greater for managers than for technicians. From this we conclude that the organizational structure of the firm is important in explaining the positive relationship between size and skill premium. Further findings on the positive correlation between the ratio of workers to managers and skill premium suggest that span of control plays a key role in a firm's organizational structure.

We build on the discrete structural models in the literature à la Garicano (2000) and Caliendo and Rossi-Hansberg (2012) by developing a simple continuous version of the model which maps higher levels of management to higher levels of human capital. The model allows for a closed form analytical solution and provides comparative statics which make specific predictions about the affects of organizational structure on its wage structure; namely, the two facts in the data outlined above.

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Appendix A

Appendix to Chapter 1

A.1 Characterizing the boundaries that determine the occupational choices

In this section we will characterize the boundaries that determine the occupational choice of consumers given values of w, r^D and r^L. This characterization allows us to analyze how changes in λ affect production. Consumers will make an occupational choice depending on the utility they can achieve from that occupation (see (1.1)). Lemma 15 characterizes the utility that consumers get depending on their occupational choice.

Lemma 15. *The utility of consumer (z, a) is*

$$u(z,a) = \begin{cases} (1+\beta)\ln\frac{a+w}{1+\beta} + \beta\ln\beta(1+r^D) & \textit{if } (z,a)\in\mathcal{W} \\ (1+\beta)\ln\frac{a}{1+\beta} + \beta\ln\beta(1+r^B) & \textit{if } (z,a)\in\mathcal{B} \\ (1+\beta)\ln\left(\frac{a}{1+\beta} + \frac{1-\alpha}{1+\beta}\left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}\right) + \beta\ln\beta(1+r^L) & \textit{if } (z,a)\in\mathcal{E}_L \\ (1+\beta)\ln\left(\frac{a}{1+\beta} + \frac{1-\alpha}{1+\beta}\left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}\right) + \beta\ln\beta(1+r^D) & \textit{if } (z,a)\in\mathcal{E}_D \\ \ln\frac{a}{1+\alpha\beta} + \beta\ln z\left(\frac{\alpha\beta}{1+\alpha\beta}\frac{a}{w}\right)^{\alpha} & \textit{if } (z,a)\in\mathcal{E}_O. \end{cases}$$

Proof. In this model $c_2(z,a) = \beta(1 + r(z,a))c_1(z,a)$ for all consumers (z,a), where $r(z,a) = r^D$ for $(z,a) \in \mathcal{W}$, $r(z,a) = r^B$ for $(z,a) \in \mathcal{B}$ and it is defined in Corollary 1 of Lemma 3 for $(z, a) \in \mathcal{E}$. Therefore

$$u(z, a) = (1 + \beta) \ln c_1(z, a) + \beta \ln \beta(1 + r(z, a)).$$

The proof of the lemma follows then from Lemmas 2 and 3. $\square$

We now compare the utility derived from two occupations at a time. Lemmas 16 to 22 show the boundaries that arise from these comparisons. The proof of these lemmas follows from Lemma 15. It is worth noticing that some boundaries only depend on the level of wealth (the choice between being a worker and a banker), or exclusively on the skill level (the choice between being a worker and an entrepreneur that saves). The rest of the boundaries depend on a combination of both wealth and skill level.

Lemma 16. *Let*

$$a_{\mathcal{W},\mathcal{B}} \equiv \frac{w}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1}.$$

If $a \geq a_{\mathcal{W},\mathcal{B}}$ then $(z, a) \notin \mathcal{W}$ and if $a < a_{\mathcal{W},\mathcal{B}}$ then $(z, a) \notin \mathcal{B}$.

Lemma 17. *Let*

$$z_{\mathcal{W},\mathcal{E}_D} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} w(1 + r^D).$$

If $z < z_{\mathcal{W},\mathcal{E}_D}$ then $(z, a) \notin \mathcal{E}_D$ and if $z \geq z_{\mathcal{W},\mathcal{E}_D}$ then $(z, a) \notin \mathcal{W}$.

Lemma 18. *Let*

$$\delta_{\mathcal{W},\mathcal{E}_O}(a) = \beta^{1-\alpha}(1 + r^D) \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{a + w}{1 + \beta}\right)^{\frac{1+\beta}{\beta}} \left(\frac{1 + \alpha\beta}{a}\right)^{\frac{1+\alpha\beta}{\beta}}.$$

If $z < \delta_{\mathcal{W},\mathcal{E}_O}(a)$ then $(z, a) \notin \mathcal{E}_O$ and if $z \geq \delta_{\mathcal{W},\mathcal{E}_O}(a)$ then $(z, a) \notin \mathcal{W}$.

Lemma 19. *Let*

$$\delta_{\mathcal{W},\mathcal{E}_L}(z) = \frac{w - (1-\alpha)\left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}} \left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1}.$$

If $a < \delta_{\mathcal{W},\mathcal{E}_L}(z)$ then $(z, a) \notin \mathcal{E}_L$ and if $a \geq \delta_{\mathcal{W},\mathcal{E}_L}(z)$ then $(z, a) \notin \mathcal{W}$.

Lemma 20. *Let*

$$\delta_{\mathcal{B},\mathcal{E}_D}(z) = \frac{(1-\alpha)\left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}}-1}.$$

If $a \geq \delta_{\mathcal{B},\mathcal{E}_D}(z)$ *then* $(z,a) \notin \mathcal{E}_D$ *and if* $a < \delta_{\mathcal{B},\mathcal{E}_D}(z)$ *then* $(z,a) \notin \mathcal{B}$.

Lemma 21. *Let*

$$\delta_{\mathcal{B},\mathcal{E}_O}(z) = \frac{1}{\beta}\frac{(1+\beta)^{\frac{1+\beta}{\beta(1-\alpha)}}}{(1+\alpha\beta)^{\frac{1+\alpha\beta}{\beta(1-\alpha)}}}\left(\frac{z}{1+r^B}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

If $a \geq \delta_{\mathcal{B},\mathcal{E}_O}(z)$ *then* $(z,a) \notin \mathcal{E}_O$ *and if* $a < \delta_{\mathcal{B},\mathcal{E}_O}(z)$ *then* $(z,a) \notin \mathcal{B}$.

Lemma 22. *Let*

$$\delta_{\mathcal{B},\mathcal{E}_L}(z) = \frac{(1-\alpha)\left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1+r^B}{1+r^L}\right)^{\frac{\beta}{1+\beta}}-1}.$$

If $a \geq \delta_{\mathcal{B},\mathcal{E}_L}(z)$ *then* $(z,a) \notin \mathcal{E}_L$ *and if* $a < \delta_{\mathcal{B},\mathcal{E}_L}(z)$ *then* $(z,a) \notin \mathcal{B}$.

Additionally to the boundaries shown in the previous lemmas, there are two other boundaries we need to take into account: $\delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ and $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$. As mentioned in Lemma 3,

$$\begin{aligned}\delta_{\mathcal{E}_D,\mathcal{E}_O}(z) &\equiv \frac{1+\alpha\beta}{\beta}\left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}\\ \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) &\equiv \frac{1+\alpha\beta}{\beta}\left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.\end{aligned}$$

Entrepreneurs with $a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$ will borrow, entrepreneurs with $a \geq \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ will save, and entrepreneurs with $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ will spend all their available wealth in paying for workers. It is worth mentioning that these boundaries do not depend on utility, but rather on feasibility: If an entrepreneur borrows ($s^{\mathcal{E}}(z,a) < 0$) then this entrepreneur cannot choose to save.

Proposition 7 characterizes the sets of consumers. There are three cases: If the spread between the return of setting up a bank, r^B, and the interest rate on deposits, r^D is sufficiently low, then there will be three types of entrepreneurs. On the other extreme, if the spread between r^B and the interest rate on loans, r^L is sufficiently large, in equilibrium there will only be entrepreneurs that borrow.

Proposition 7. *If in equilibrium* $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^D}$*, then there will be the three types of entrepreneurs. In this case the occupational choice of consumers is characterized by the following sets:*

$$\begin{aligned}
\mathcal{B} &= [\underline{z}, z_{\mathcal{W},\mathcal{E}_D}) \times [a_{\mathcal{W},\mathcal{B}}, \overline{a}] \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}_D} \text{ and } a \geq \delta_{\mathcal{B},\mathcal{E}_D}(z)\} \\
\mathcal{W} &= [\underline{z}, z_{\mathcal{W},\mathcal{E}_D}) \times [\underline{a}, a_{\mathcal{W},\mathcal{B}}) \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}_D}, a \geq \underline{a}, z < z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L} \text{ and } z < \delta_{\mathcal{W},\mathcal{E}_O}(a)\} \\
&\bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}, a \geq \underline{a} \text{ and } a < \delta_{\mathcal{W},\mathcal{E}_L}(z)\} \\
\mathcal{E}_D &= \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}_D} \text{ and } \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \leq a < \delta_{\mathcal{B},\mathcal{E}_D}(z)\} \\
\mathcal{E}_O &= \{(z,a) : z \geq \delta_{\mathcal{W},\mathcal{E}_O}(a) \text{ and } \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)\} \\
\mathcal{E}_L &= \{(z,a) : a \geq \delta_{\mathcal{W},\mathcal{E}_L}(z) \text{ and } \underline{a} \leq a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z)\},
\end{aligned}$$

where

$$z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L} \equiv \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{\beta}{(1+\beta)\left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - (1+\alpha\beta)}\right)^{1-\alpha} w(1+r^L).$$

If in equilibrium $\frac{1+r^B}{1+r^L} \leq \left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} < \frac{1+r^B}{1+r^D}$*, then there will be no entrepreneurs that deposit. In this case the occupational choice of consumers is characterized by the following sets:*

$$\begin{aligned}
\mathcal{B} &= [\underline{z}, z_{\mathcal{W},\mathcal{B},\mathcal{E}_O}) \times [a_{\mathcal{W},\mathcal{B}}, \overline{a}] \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{B},\mathcal{E}_O} \text{ and } a \geq \delta_{\mathcal{B},\mathcal{E}_O}(z)\} \\
\mathcal{W} &= [\underline{z}, z_{\mathcal{W},\mathcal{B},\mathcal{E}_O}) \times [\underline{a}, a_{\mathcal{W},\mathcal{B}}) \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{B},\mathcal{E}_O}, a \geq \underline{a}, z < z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L} \text{ and } z < \delta_{\mathcal{W},\mathcal{E}_O}(a)\} \\
&\bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}, a \geq \underline{a} \text{ and } a < \delta_{\mathcal{W},\mathcal{E}_L}(z)\} \\
\mathcal{E}_O &= \{(z,a) : z \geq \delta_{\mathcal{W},\mathcal{E}_O}(a) \text{ and } \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \leq a < \delta_{\mathcal{B},\mathcal{E}_O}(z)\} \\
\mathcal{E}_L &= \{(z,a) : a \geq \delta_{\mathcal{W},\mathcal{E}_L}(z) \text{ and } \underline{a} \leq a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z)\},
\end{aligned}$$

where

$$z_{\mathcal{W},\mathcal{B},\mathcal{E}_O} \equiv \frac{(1+\alpha\beta)^{\frac{1+\alpha\beta}{\beta}}}{(1+\beta)^{\frac{1+\beta}{\beta}}}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{\beta}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}}-1}\right)^{1-\alpha} w(1+r^B).$$

If in equilibrium $\frac{1+r^B}{1+r^L} > \left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}}$*, then there will only be entrepreneurs that borrow. In this case the occupational choice of consumers is characterized by the following sets:*

$$\begin{aligned}
\mathcal{B} &= [\underline{z}, z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}) \times [a_{\mathcal{W},\mathcal{B}}, \overline{a}] \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{B},\mathcal{E}_L} \textit{ and } a \geq \delta_{\mathcal{B},\mathcal{E}_L}(z)\} \\
\mathcal{W} &= [\underline{z}, z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}) \times [\underline{a}, a_{\mathcal{W},\mathcal{B}}) \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}, a \geq \underline{a} \textit{ and } a < \delta_{\mathcal{W},\mathcal{E}_L}(z)\} \\
\mathcal{E}_L &= \{(z,a) : a \geq \delta_{\mathcal{W},\mathcal{E}_L}(z) \textit{ and } \underline{a} \leq a < \delta_{\mathcal{B},\mathcal{E}_L}(z)\}.
\end{aligned}$$

where

$$z_{\mathcal{W},\mathcal{B},\mathcal{E}_L} \equiv \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{\left(\frac{1+r^B}{1+r^L}\right)^{\frac{\beta}{1+\beta}}-1}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}}-1}\right)^{1-\alpha} w(1+r^L).$$

Proof. The strategy to prove the proposition relies on the fact that, depending on the values of $\frac{1+r^B}{1+r^D}$ and $\frac{1+r^B}{1+r^L}$, some types of entrepreneurs will not exist. This will be a consequence of the slope of some boundaries.

First, recall from Lemma 1 that $r^B > r^L > r^D$ in equilibrium as long as $\lambda < \infty$. Let

$$a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O} = \frac{1+\alpha\beta}{\beta(1-\alpha)}w.$$

Notice that

$$\begin{aligned}
\delta_{\mathcal{B},\mathcal{E}_D}\left(z_{\mathcal{W},\mathcal{E}_D}\right) &= a_{\mathcal{W},\mathcal{B}} \\
\delta_{\mathcal{W},\mathcal{E}_O}\left(a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}\right) &= z_{\mathcal{W},\mathcal{E}_D} \\
\delta_{\mathcal{E}_D,\mathcal{E}_O}\left(z_{\mathcal{W},\mathcal{E}_D}\right) &= a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}.
\end{aligned}$$

As long as $a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}$ there will exist entrepreneurs that save in equilibrium. $a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}$ and $\delta_{\mathcal{B},\mathcal{E}_D}(z) \geq \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ if and only if $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^D}$. Finally,

$\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) > \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ since $r^L > r^D$, so if $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^D}$ there will be entrepreneurs that neither borrow nor deposit.

Now consider the case where there are no entrepreneurs that save in equilibrium. Notice that

$$\delta_{\mathcal{W},\mathcal{E}_O}\left(a_{\mathcal{W},\mathcal{B}}\right) = z_{\mathcal{W},\mathcal{B},\mathcal{E}_O}$$
$$\delta_{\mathcal{B},\mathcal{E}_O}\left(z_{\mathcal{W},\mathcal{B},\mathcal{E}_O}\right) = a_{\mathcal{W},\mathcal{B}}.$$

Let

$$a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L} \equiv \frac{1+\alpha\beta}{(1+\beta)\left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - (1+\alpha\beta)} w.$$

Notice that

$$\delta_{\mathcal{W},\mathcal{E}_O}\left(a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}\right) = z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}$$
$$\delta_{\mathcal{E}_O,\mathcal{E}_L}\left(z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}\right) = a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}$$
$$\delta_{\mathcal{W},\mathcal{E}_L}\left(z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}\right) = a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L},$$

so as long as $a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}$ there will exist entrepreneurs that neither borrow nor save. $a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}$ and $\delta_{\mathcal{B},\mathcal{E}_O}(z) \geq \delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$ if and only if $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^L}$.

Finally notice that

$$\delta_{\mathcal{W},\mathcal{E}_L}\left(z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}\right) = a_{\mathcal{W},\mathcal{B}}$$
$$\delta_{\mathcal{B},\mathcal{E}_L}\left(z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}\right) = a_{\mathcal{W},\mathcal{B}}.$$

If $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} < \frac{1+r^B}{1+r^L}$ there will only be entrepreneurs that borrow in equilibrium. □

Lemma 23 determines the skill level $z^0_{\mathcal{W},\mathcal{E}_L}$ above which there will be no workers.

Lemma 23. *Let*

$$z^{\underline{a}}_{\mathcal{W},\mathcal{E}_L} = \left(\frac{w - \underline{a}\left(\left(\frac{1+r^L}{1+r^D} \right)^{\frac{\beta}{1+\beta}} - 1 \right)}{1-\alpha} \right)^{1-\alpha} \left(\frac{w}{\alpha} \right)^{\alpha} (1+r^L) \left(\frac{1+r^D}{1+r^L} \right)^{\frac{\beta(1-\alpha)}{1+\beta}}$$

Then

$$\delta_{\mathcal{W},\mathcal{E}_L}\left(z^{\underline{a}}_{\mathcal{W},\mathcal{E}_L} \right) = \underline{a}.$$

A.2 Characterizing the boundaries that determine the occupational choice as $\lambda \to \infty$

In this section we will characterize the boundaries that determine the occupational choice of consumers given values of w and r. Consumers will choose their occupation depending on the utility they can derive from it (see (1.9)). Lemma 24 characterizes the utility derived from each occupation.

Lemma 24. *The utility of consumer* (z, a) *is*

$$u(z,a) = \begin{cases} (1+\beta)\ln \frac{a+w}{1+\beta} + \beta \ln \beta(1+r) & \text{if } (z,a) \in \mathcal{W} \\ (1+\beta)\ln \left(\frac{a}{1+\beta} + \frac{1-\alpha}{1+\beta} \left(\frac{z}{1+r} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} \right) + \beta \ln \beta(1+r) & \text{if } (z,a) \in \mathcal{E}. \end{cases}$$

Proof. In this model $c_2(z,a) = \beta(1+r)c_1(z,a)$ for all consumers (z,a). Therefore

$$u(z,a) = (1+\beta)\ln c_1 + \beta \ln \beta(1+r).$$

The proof of the lemma follows from Lemmas 4 and 5. □

We now compare the utility derived from the two occupations. Lemma 25 shows the boundary that arises from this comparison. The proof of this lemma follows from Lemma 24. It is worth noticing that this boundary only depends on the skill level.

Lemma 25. *Let*

$$z_{\mathcal{W},\mathcal{E}} \equiv \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} w(1+r).$$

If $z < z_{\mathcal{W},\mathcal{E}}$ *then* $(z,a) \in \mathcal{W}$ *and if* $z \geq z_{\mathcal{W},\mathcal{E}}$ *then* $(z,a) \in \mathcal{E}$.

Proposition 8 characterizes the sets of consumers.

Proposition 8. *Consumer occupations are characterized by*

$$\mathcal{W} = \{(z,a) : z < z_{\mathcal{W},\mathcal{E}}\}$$
$$\mathcal{E} = \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}}\}.$$

A.2.1 Solving the model

Let

$$A \equiv \int a dG(z \times a)$$

Since for each individual, consumption in the two periods is related by

$$c_2(z,a) = \beta(1+r)c_1(z,a).$$

This must also hold in summation, which implies:

$$\int c_2(z,a)dG(z \times a) = \int \beta(1+r)c_1(z,a)dG(z \times a). \tag{A.1}$$

We use Lemmas 5 and 25 and the market clearing condition for labor to solve for $C_{w,r} \equiv w(1+r)$. This allows us to have an expression for total production. Then we use A and total production to solve for r using (A.1). Finally we solve for w.

A.3 Social Planner Problem

The Social Planner Problem is

$$\max_{l(z),c_1(z,a),c_2(z,a),o} \int \left(v(c_1(z,a) + \beta v(c_2(z,a))\right) dG(z \times a)$$

$$\int c_1(z,a) dG(z \times a) = \int a dG(z \times a)$$

$$\int c_2(z,a) dG(z \times a) = \int_{\mathcal{E}} z l(z)^{\alpha} dG(z \times a)$$

$$\int_{\mathcal{E}} l(z) dG(z \times a) = \int_{\mathcal{W}} dG(z \times a).$$

Notice that aggregate consumption in period 1 is exogenous, so maximizing social welfare implies maximizing production in the second period. Now, the marginal productivity of labor is constant across all firms, but adding an extra entrepreneur implies increasing the marginal productivity of labor, since the average size of a firm decreases. So solving the Social Planner Problem reduces to finding a boundary above which consumers will become entrepreneurs. We know that

$$l(z) = \left(\frac{\alpha z}{MPL}\right)^{\frac{1}{1-\alpha}}.$$

From the labor market clearing condition we have

$$\left(\frac{\alpha}{MPL}\right)^{\frac{1}{1-\alpha}} \int_{z \geq z_{\mathcal{W},\mathcal{E}}} z^{\frac{1}{1-\alpha}} dG(z \times a) = \hat{G}(z_{\mathcal{W},\mathcal{E}}),$$

where

$$\hat{G}(z') \equiv \int_{z<z'} dG(z \times a).$$

Then

$$MPL = \alpha \left(\frac{(1-\hat{G}(z_{\mathcal{W},\mathcal{E}}))\mathbf{E}\left[z^{\frac{1}{1-\alpha}} | z \geq z_{\mathcal{W},\mathcal{E}}\right]}{\hat{G}(z_{\mathcal{W},\mathcal{E}})} \right)^{1-\alpha}.$$

The Social Planner Problem is equivalent to

$$\begin{aligned}\max_{z_{\mathcal{W},\mathcal{E}}} H(z_{\mathcal{W},\mathcal{E}}) &\equiv \max_{z_{\mathcal{W},\mathcal{E}}} \int_{\mathcal{E}} z f(l(z)) dG(z\times a)\\ &= \max_{z_{\mathcal{W},\mathcal{E}}} \left(\frac{\alpha}{MPL(z_{\mathcal{W},\mathcal{E}})}\right)^{\frac{\alpha}{1-\alpha}} \int_{z\geq z_{\mathcal{W},\mathcal{E}}} z^{\frac{1}{1-\alpha}} dG(z\times a)\\ &= \max_{z_{\mathcal{W},\mathcal{E}}} \left(1-\hat{G}(z_{\mathcal{W},\mathcal{E}})\right)^{1-\alpha} \mathbf{E}\left[z^{\frac{1}{1-\alpha}}|z\geq z_{\mathcal{W},\mathcal{E}}\right]^{1-\alpha} \hat{G}(z_{\mathcal{W},\mathcal{E}})^{\alpha}.\end{aligned}$$

A.4 Select Proofs

A.4.1 Proof of Lemma 6

To prove this Lemma it is sufficient to consider aggregate wealth across the different occupational choices. That is, let $A_{\mathcal{O}}$ denote the aggregate wealth endowed to consumers who choose occupation $\mathcal{O}$, where $\mathcal{O}\in\{\mathcal{W},\mathcal{B},\mathcal{E}_L,\mathcal{E}_O,\mathcal{E}_D\}$. Additionally, let

$$\begin{aligned}Z_{\mathcal{E}_D} &\equiv \left[\int_{\mathcal{E}_D} z^{\frac{1}{1-\alpha}} dG(z\times a)\right]^{1-\alpha}\\ Z_{\mathcal{E}_L} &\equiv \left[\int_{\mathcal{E}_L} z^{\frac{1}{1-\alpha}} dG(z\times a)\right]^{1-\alpha}.\end{aligned}$$

That is, $Z_{\mathcal{E}_D}^{\frac{1}{1-\alpha}}$ $(Z_{\mathcal{E}_L}^{\frac{1}{1-\alpha}})$ is the $\frac{1}{1-\alpha}$-th moment of the skill endowed to the entrepreneurs that are depositors (borrowers). Finally let $M_{\mathcal{W}}$ be the mass of workers:

$$M_{\mathcal{W}} \equiv \int_{\mathcal{W}} dG(z\times a).$$

Then the market clearing condition for deposits can be written as

$$\begin{aligned}&\frac{\beta}{1+\beta}(wM_{\mathcal{W}}+A_{\mathcal{W}})+\frac{\beta}{1+\beta}A_{\mathcal{E}_D}-\frac{1+\alpha\beta}{1+\beta}\left(\frac{Z_{\mathcal{E}_D}}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}\\ &=\lambda\frac{\beta}{1+\beta}A_{\mathcal{B}};\end{aligned} \tag{A.2}$$

for loans as

$$\frac{\beta}{1+\beta}A_{\mathcal{E}_L} - \frac{1+\alpha\beta}{1+\beta}\left(\frac{Z_{\mathcal{E}_L}}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} + (1+\lambda)\frac{\beta}{1+\beta}A_{\mathcal{B}} = 0; \tag{A.3}$$

and for labor as

$$M_{\mathcal{W}} = \left(\frac{\alpha Z_{\mathcal{E}_L}}{(1+r^L)w}\right)^{\frac{1}{1-\alpha}} + \frac{\alpha\beta}{1+\alpha\beta}\frac{A_{\mathcal{E}_O}}{w} + \left(\frac{\alpha Z_{\mathcal{E}_D}}{(1+r^D)w}\right)^{\frac{1}{1-\alpha}}. \tag{A.4}$$

(A.2) and (A.3) can rewritten as (A.5) and (A.6), respectively.

$$\left(\frac{\alpha Z_{\mathcal{E}_L}}{(1+r^L)w}\right)^{\frac{1}{1-\alpha}} = \frac{\beta}{1+\alpha\beta}\left(A_{\mathcal{E}_L} + (1+\lambda)A_{\mathcal{B}}\right)\frac{\alpha}{w} \tag{A.5}$$

$$\left(\frac{\alpha Z_{\mathcal{E}_D}}{(1+r^D)w}\right)^{\frac{1}{1-\alpha}} = \frac{\beta}{1+\alpha\beta}\left(wM_{\mathcal{W}} + A_{\mathcal{W}} + A_{\mathcal{E}_D} - \lambda A_{\mathcal{B}}\right)\frac{\alpha}{w}. \tag{A.6}$$

Plugging (A.5) and (A.6) into the labor market clearing condition we get

$$M_{\mathcal{W}} = \frac{\beta}{1+\alpha\beta}\left(A_{\mathcal{E}_L} + (1+\lambda)A_{\mathcal{B}}\right)\frac{\alpha}{w} + \frac{\alpha\beta}{1+\alpha\beta}\frac{A_{\mathcal{E}_O}}{w} + \frac{\beta}{1+\alpha\beta}\left(wM_{\mathcal{W}} + A_{\mathcal{W}} + A_{\mathcal{E}_D} - \lambda A_{\mathcal{B}}\right)\frac{\alpha}{w}. \tag{A.7}$$

The result follows from reorganizing (A.7).

A.4.2 Assumptions on parameters in Section 1.5.2

The following assumptions on parameters guarantee that the prices stated in Proposition 1 are an equilibrium:

$$z_2 > \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} w(1+r^L) > z_1.$$

This assumption guarantees that consumers with $z = z_2$ choose to become entrepreneurs are entrepreneurs while the other consumers don't.

$$a_2 > \frac{w}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1} > a_1.$$

This assumption guarantees that consumers with $a = a_2$ choose to become bankers and consumers with $a = a_1$ choose to become workers.

$$\overline{a} > \frac{1+\alpha\beta}{\beta}\left(\frac{z_2}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

This assumption guarantees that there are entrepreneurs that deposit.

$$0 < (\lambda-\alpha\beta)(1-\delta_a)\delta_z a_2 - \alpha\beta(1-\delta_z)\frac{\overline{a}}{2} - (1+\alpha\beta)\delta_a\delta_z a_1 < (1-\delta_z)\frac{\overline{a}}{2}.$$

The first inequality is consistent with the existence of consumers other than workers that save; namely, entrepreneurs that deposit. The second inequality is consistent with the existence of consumers that do not save; namely. entrepreneurs that borrow and entrepreneurs that neither borrow nor deposit.

$$\begin{aligned}&(1+\lambda)(1-\delta_a)\delta_z a_2\\&<\left(\left(\frac{1-\delta_z}{2\overline{a}}\right)^{\frac{1}{2}} - \left((\lambda-\alpha\beta)(1-\delta_a)\delta_z a_2 - \alpha\beta(1-\delta_z)\frac{\overline{a}}{2} - (1+\alpha\beta)\delta_a\delta_z a_1\right)^{\frac{1}{2}}\right)^2.\end{aligned}$$

This final assumption guarantees that $r^L > r^D$, which is necessary to have an equilibrium.

A.4.3 Proof of Proposition 1

The strategy to prove this proposition will be to assume that parameter values are such that an equilibrium exist. Then we show that the conditions on this parameters satisfy what is stated in A.4.2. We will also build on the proof of Lemma 6. Given the definitions in A.4.1, under $\widetilde{G}(\cdot)$ we have

$$\begin{aligned}
M_{\mathcal{W}} &= \delta_a \delta_z \\
A_{\mathcal{W}} &= a_1 \delta_a \delta_z \\
A_{\mathcal{B}} &= a_2 (1-\delta_a)\delta_z \\
A_{\mathcal{E}_D} &= \frac{1-\delta_z}{\overline{a}} \int_{\delta_{\mathcal{E}_D,\mathcal{E}_O}(z)}^{\overline{a}} a da \\
A_{\mathcal{E}_L} &= \frac{1-\delta_z}{\overline{a}} \int_{0}^{\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)} a da \\
Z_{\mathcal{E}_D} &= z_2 \left(\frac{1-\lambda_z}{\overline{a}}\right)^{1-\alpha} \left(\overline{a} - \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)\right)^{1-\alpha} \\
Z_{\mathcal{E}_L} &= z_2 \left(\frac{1-\lambda_z}{\overline{a}}\right)^{1-\alpha} \left(\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)\right)^{1-\alpha}.
\end{aligned} \tag{A.8}$$

Reorganizing (A.5) and (A.6) we get

$$\begin{aligned}
1 + r^L &= Z_{\mathcal{E}_L} \left(\frac{\alpha}{w}\right)^{\alpha} \left(\frac{1+\alpha\beta}{\beta\left(A_{\mathcal{E}_L} + (1+\lambda)A_{\mathcal{B}}\right)}\right)^{1-\alpha} \\
1 + r^D &= Z_{\mathcal{E}_D} \left(\frac{\alpha}{w}\right)^{\alpha} \left(\frac{1+\alpha\beta}{\beta\left(wM_{\mathcal{W}} + A_{\mathcal{W}} + A_{\mathcal{E}_D} - \lambda A_{\mathcal{B}}\right)}\right)^{1-\alpha}.
\end{aligned} \tag{A.9}$$

The result follows form plugging (A.8) into (A.9), and using the result from Lemma 6. The proof that the assumptions stated in A.4.2 are sufficient restrictions on the parameters arises from guaranteeing that there are no negative roots in the resulting r^L and r^D and by using the characterization of the boundaries that determine the occupational choices in the model (See A.1).

A.4.4 Proof of Proposition 2

The proof of this proposition relies on the result stated in Proposition 1. Notice that $M_{\mathcal{W}}$ and A are constant in the model. Then we can write r^L and r^D as

$$\begin{aligned}
1 + r^L &= \frac{z_2}{\beta}(1+\alpha\beta)^{1-\alpha} \left(\frac{M_{\mathcal{W}}}{A}\right)^{\alpha} \mathcal{C}_L^{\frac{1-\alpha}{2}} \\
1 + r^D &= \frac{z_2}{\beta}(1+\alpha\beta)^{1-\alpha} \left(\frac{M_{\mathcal{W}}}{A}\right)^{\alpha} \mathcal{C}_D^{1-\alpha}.
\end{aligned}$$

Notice that $r^L - r^D$ is a function of $\mathcal{C}_L - \mathcal{C}_D$. Furthermore, from the corollary to Proposition 1 this difference is decreasing in λ.

Now, total production in this economy is given by the sum of what is produced by each type of entrepreneur. Let $Y_{\mathcal{T}}$ be total production by entrepreneur of type $\mathcal{T}$, where $\mathcal{T} \in \{L, O, D\}$. Taking into account the results in Proposition 1 and Lemma 3 we have

$$Y_L = z_2 \frac{1-\delta_z}{\bar{a}} \left(\frac{1}{1+\alpha\beta}\right)^{\alpha} \left(\frac{M_{\mathcal{W}}}{A}\right)^{\alpha} \left(\frac{1}{\mathcal{C}_L}\right)^{\frac{1+\alpha}{2}}$$

$$Y_O = z_2 \frac{1-\delta_z}{\bar{a}} \left(\frac{1}{1+\alpha\beta}\right)^{\alpha} \left(\frac{M_{\mathcal{W}}}{A}\right)^{\alpha} \frac{1}{1+\alpha} \left[\left(\frac{1}{\mathcal{C}_D}\right)^{1+\alpha} - \left(\frac{1}{\mathcal{C}_L}\right)^{\frac{1+\alpha}{2}}\right]$$

$$Y_D = z_2 \frac{1-\delta_z}{\bar{a}} \left(\frac{1}{1+\alpha\beta}\right)^{\alpha} \left(\frac{M_{\mathcal{W}}}{A}\right)^{\alpha} \left(\bar{a} - \frac{1}{\mathcal{C}_D}\right) \left(\frac{1}{\mathcal{C}_D}\right)^{\alpha}.$$

The result follows from adding $Y_{\mathcal{T}}$ for $\mathcal{T} \in \{L, O, D\}$.

A.4.5 Proof of Lemma 7

The proof of this lemma relies on what is proven in A.1. First, from Proposition 7 we see that a consumer with skill z_1 and wealth a_1 choose to be an entrepreneur that is able to use its wealth to pay for workers. Additionally, if $\underline{a} = 0$ a consumer with skill z_2 and wealth $a = 0$ chooses to become an entrepreneur that needs to borrow to pay for workers according to Lemma 17 and Proposition 7. Finally,

$$z_2 - z_1 = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} (1+r^D)^{\frac{\beta(1-\alpha)}{1+\beta}} \left[(1+r^L)^{\frac{1+\alpha\beta}{1+\beta}} - (1+r^D)^{\frac{1+\alpha\beta}{1+\beta}}\right],$$

which is increasing in $r^L - r^D$.

Appendix B

Appendix to Chapter 2

B.1 Proof of Lemma 9

Since $\alpha > 0.5$, the following first order conditions of (2.10) characterize the solution to this problem:

$$w + \frac{(1-\lambda)M_T}{\phi(l_s)} = \alpha z l_y^{\alpha-1} \tag{B.1}$$

$$w = (1-\lambda)M_T l_y \frac{\phi'(l_s)}{\phi(l_s)^2}. \tag{B.2}$$

Solving for l_y in (B.1) yields

$$l_y = \left(\frac{\alpha z}{w + \frac{(1-\lambda)M_T}{\phi(l_s)}}\right)^{\frac{1}{1-\alpha}}. \tag{B.3}$$

Plugging (B.3) in (B.2) yields

$$w = \left(\frac{\alpha z}{w\phi(l_s) + (1-\lambda)M_T}\right)^{\frac{1}{1-\alpha}} (1-\lambda)M_T\phi'(l_s)\phi(l_s)^{\frac{1}{1-\alpha}-2}.$$

Our assumption that $\phi(l_s) \equiv \left(\frac{\alpha}{1-\alpha}l_s\right)^{\frac{1-\alpha}{\alpha}}$ satisfies

$$\phi'(l_s)\phi(l_s)^{\frac{1}{1-\alpha}-2} = 1,$$

so in equilibrium

$$l_s(z) = \frac{1-\alpha}{\alpha}\left(\frac{(1-\lambda)M_T}{w}\right)^{\alpha}\left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda)M_T}{w}\right)^{\alpha}\right)^{\frac{\alpha}{1-\alpha}}. \tag{B.4}$$

Plugging (B.4) into (B.3) yields

$$l_y(z) = \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda)M_T}{w}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{B.5}$$

Plugging (B.4) and (B.5) into (2.5) yields

$$\tau(z) = \left(\frac{w}{(1-\lambda)M_T}\right)^{1-\alpha}\left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda)M_T}{w}\right)^{\alpha}\right)^{\frac{\alpha}{1-\alpha}}. \tag{B.6}$$

Finally, $\pi(z)$ results from plugging (B.4) to (B.6) into the objective function of (2.3):

$$\pi(z) = \frac{1-\alpha}{\alpha} w \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda)M_T}{w}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}.$$

B.2 Model $\lambda = 1$

Assume $\lambda = 1$. Depending on parameter values, in equilibrium there could be theft. That is, even if thieves cannot consume what they steal, their aversion to becoming thieves, θ, and the consumption they get when they get caught, $\underline{c}$, can be such that some households are better off stealing. If $\lambda = 1$ then Lemma 8 implies

$$\begin{aligned} \theta^W &= \frac{\underline{c}}{1-M_T} - w \\ \theta^E(z) &= \frac{\underline{c}}{1-M_T} - \pi(z). \end{aligned} \tag{B.7}$$

There will be theft in an equilibrium with $\lambda = 1$ as long as $\theta^W \geq \inf_\theta \operatorname{supp}\{F(\theta)\}$. In this case firm z's problem is

$$\pi(z) \equiv \max_{l_y \geq 0, l_s \geq 0} z l_y^{\alpha} - w l_y - w l_s. \tag{B.8}$$

The solution of (B.8) is

$$\begin{aligned} l_y(z) &= \left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}} \\ l_s(z) &= 0. \end{aligned} \tag{B.9}$$

Plugging (B.9) into (B.8) we have

$$\pi(z) = (1-\alpha) z^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

Notice that $\pi(z)$ is strictly increasing in z, so there exists a cutoff z^E such that $\pi(z^E) = w$, which implies consumers choose to be workers for $z < z^E$ and decide to be entrepreneurs for $z \geq z^E$ and

$$z^E = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} w.$$

Then the equilibrium in this case is characterized by w and M_T such that

$$\begin{aligned} M_T &= F(\theta^W) G(z^E) + \int_{z \geq z^E} F(\theta^E(z)) dG(z) \\ \int_{z \geq z^E} l_y(z)(1 - F(\theta^E(z))) dG(z) &= (1 - F(\theta^W)) G(z^E) \\ \theta^W &\geq \inf_{\theta} \operatorname{supp} \{F(\theta)\}. \end{aligned} \tag{B.10}$$

If the first two equations of (B.10) are satisfied, but the third one is not, then we have an economy as in Lucas (1978). That is, there is no theft in equilibrium and consumers choose between being workers or entrepreneurs. Firms' profits are given by

(B.8) with $M_T = 0$, so the equilibrium of this economy is characterized by (B.11).

$$\begin{aligned} M_T &= 0 \\ l_y(z) &= \left(\frac{z\alpha}{w}\right)^{\frac{1}{1-\alpha}} \\ z^E &= \left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha} w \\ \int_{z \geq z^E} l_y(z) dG(z) &= G(z^E). \end{aligned} \tag{B.11}$$

Appendix C

Appendix to Chapter 3

C.1 Select Proofs

C.1.1 Proof of Lemma 10

We will first prove by induction on x that each manager at level $x > 0$ is able to solve $(R\lambda)^x(1-\lambda)$ tasks. Consider first the measure of tasks that workers solve: by assumption each worker receives a measure one of tasks of which he is unable to solve a fraction λ which he passes on to managers in level $x = 1$. Now, each manager at level $x = 1$ receives R direct reports. Therefore each manager at level $x = 1$ receives a measure $R\lambda$ of tasks, out of which they solve a fraction $1-\lambda$ and passes on the remaining to managers at level $x = 2$.

Consider a manager at level x and assume that he solves a measure $(R\lambda)^x(1-\lambda)$ of tasks. A manager at level $x+1$ receives R direct reports, each of which passes on a measure $\lambda(R\lambda)^x$ of unsolved tasks. Of this measure he is able to solve a fraction $1-\lambda$. This implies that $(R\lambda)^{x+1}(1-\lambda)$.

Since there are R^{X-x} managers in level x, the total number of tasks $\mathcal{T}_X$ is given by

$$\begin{aligned}\mathcal{T}_X &= \sum_{x=0}^{X} R^{X-x}(R\lambda)^x(1-\lambda)\\ &= R^X(1-\lambda^{X+1}).\end{aligned}$$

C.1.2 Proof of Lemma 11

First we will prove that $SP_{X+1,R} > SP_{X,R}$. For this it is sufficient to prove that

$$\frac{R^{X+1}}{R^{X+1}-1}(1-\lambda^{X+1}) > \frac{R^X}{R^X-1}(1-\lambda^X),$$

which is equivalent to proving that

$$R\frac{\sum_{x=0}^{X}\lambda^x}{\sum_{x=0}^{X-1}\lambda^x} > \frac{\sum_{x=0}^{X}R^x}{\sum_{x=0}^{X-1}R^x}. \tag{C.1}$$

(C.1) holds if and only if

$$\begin{aligned} R\left(\sum_{x=0}^{X-1}R^x\right)\left(\sum_{x=0}^{X}\lambda^x\right) &> \left(\sum_{x=0}^{X}R^x\right)\left(\sum_{x=0}^{X-1}\lambda^x\right) \\ \iff \lambda^X\sum_{x=1}^{X-1}R^x &> \sum_{x=0}^{X-1}\lambda^x \\ \iff \sum_{x=0}^{X-1}\lambda^x\,(R\lambda)^{X-x} &> \sum_{x=0}^{X-1}\lambda^x. \end{aligned} \tag{C.2}$$

Since $R\lambda > 1$, λ^x on the left hand side of (C.2) is multiplied by a term greater than one for all $x \in \{0,\ldots,X-1\}$. Therefore the left hand side is strictly greater than the right hand side and the proof follows.

Now we prove that $SP_{X,R+1} > SP_{X,R}$. For this it is sufficient to prove that

$$R\frac{(R+1)^X}{(R+1)^X-1} > (R-1)\frac{R^X}{R^X-1},$$

which is equivalent to proving that

$$R(R^X-1)(R+1)^X > (R-1)R^X((R+1)^X-1). \tag{C.3}$$

(C.3) holds if and only if

$$(R^X-R)(R+1)^X + (R^{X+1}-R^X) > 0,$$

which holds if $R > 1$. Notice that $R\lambda > 1$ and $\lambda \in (0,1)$ imply $R > 1$, which completes the proof.

C.1.3 Proof of Proposition 4

We will prove the result by first considering levels of width Δ and calculating by induction the total number of tasks solved in $AC(x)$ as $\Delta \to 0$. In each level $x' \in [0,x]$ of area of command $AC(x)$ there are $R^{x-x'}$ employees. Workers in $AC(x,n)$ receive a measure 1 of tasks and are able to solve a fraction $\Delta(1-\lambda)$ of them. Each manager in level Δ receives $R^{\Delta}(1-\Delta(1-\lambda))$ tasks and is able to solve a fraction $\Delta(1-\lambda)$. Since there is a measure $R^{x-\Delta}$ of managers at level Δ, the total measure of tasks that managers at level Δ solve is $R^x(1-\Delta(1-\lambda))\Delta(1-\lambda)$. Each employee in level 2Δ manages a width of R^{Δ} direct reports, each of whom passes on $R^{\Delta}\left(1-\Delta(1-\lambda)\right)^2$ unsolved tasks. Each manager in level 2Δ is able to solve a fraction $\Delta(1-\lambda)$ of those tasks. There is a measure $R^{x-2\Delta}$ of managers at level 2Δ, so the total measure of tasks that these managers solve is $R^x(1-\Delta(1-\lambda))^2\Delta(1-\lambda)$.

Now assume that managers at level $i\Delta$ solve a measure $R^{i\Delta}(1-\Delta(1-\lambda))^i\Delta(1-\lambda)$ of tasks. There are $R^{x-i\Delta}$ managers at this level. So the total measure of tasks solved by managers at this level is $R^x(1-\Delta(1-\lambda))^i\Delta(1-\lambda)$. Each employee at level $(i+1)\Delta$ manages a width of R^{Δ} direct reports, each of whom passes on $R^{i\Delta}\left(1-\Delta(1-\lambda)\right)^{i+1}$ unsolved tasks. Each manager in level $(i+1)\Delta$ is able to solve a fraction $\Delta(1-\lambda)$ of those tasks. Since there are $R^{x-(i+1)\Delta}$ managers at level $i+1$, the total measure of tasks solved by these managers is $R^x(1-\Delta(1-\lambda))^{i+1}\Delta(1-\lambda)$. Therefore the total measure of tasks solved in area of command $AC(x)$ is

$$
\begin{aligned}
\mathcal{T}(x) &\equiv \lim_{\Delta\to 0}(1-\lambda)R^x\Delta\sum_{i=0}^{\lfloor \frac{x}{\Delta}\rfloor}\left(1-\Delta(1-\lambda)\right)^i \\
&= \lim_{\Delta\to 0} R^x(1-(1-\Delta(1-\lambda))^{\lfloor \frac{x}{\Delta}\rfloor+1}) \\
&= R^x\left(1-e^{-x(1-\lambda)}\right),
\end{aligned}
$$

where the last line is due to the fact that

$$\lim_{\Delta\to 0}\frac{x}{\Delta}\ln\left(1-\Delta(1-\lambda)\right) = -x(1-\lambda).$$

C.1.4 Proof of Proposition 5

Recall from Proposition 4 that the number of tasks a firm solves is given by $\mathcal{T}(X) = R^X\left(1-e^{-X(1-\lambda)}\right)$. Equilibrium condition 2 requires that consumers be indifferent between obtaining any education levels which are employed with positive measure. This implies that $w(x) = w_E h(x) + w$. Now, for an educator it must hold that $w_E = w_E h_E + w$, so $w_E = \frac{w}{1-h_E}$. Therefore

$$w(x) = \frac{w}{1-h_E}h(x) + w. \tag{C.4}$$

Plugging (C.4) and the labor requirements at each level into the firm's problem we get:

$$\begin{aligned}
&\max_X Y(\mathcal{T}(X),z) - \int_x R^{X-x}\left(\frac{w}{1-h_E}h(x)+w\right)dx \\
=&\max_X Y(\mathcal{T}(X),z) - R^X\int_x we^{-x(1-\lambda)}dx \\
=&\max_X Y(\mathcal{T}(X),z) - w\mathcal{T}(X),
\end{aligned} \tag{C.5}$$

where the second line is a consequence of Assumption 1.

Assumption 2 on the span of control implies that $\mathcal{T}(X)$ is strictly increasing in X.[1] Therefore (C.5) is equivalent to

$$\max_{\mathcal{T}} Y(\mathcal{T},z) - w\mathcal{T}.$$

C.1.5 Proof of Proposition 6

Proposition 5 implies that the solution to (3.5) is sufficient to derive a function $X^* : \mathcal{Z} \to \mathbb{R}_+$ that satisfies the equilibrium conditions given wages.

[1] $\mathcal{T}'(X) = R^X\left(\ln R - e^{-X(1-\lambda)}\left(\ln R - (1-\lambda)\right)\right)$. Since $\ln R > 1-\lambda$, $1-\lambda > 0$ and $e^{-X(1-\lambda)} < 1$, then $\mathcal{T}'(X) > 0$.

Denote by $\mathcal{T}^*(z,w)$ the solution to (3.5). The regularity conditions on $Y(\mathcal{T},w)$ imply that $\mathcal{T}^*(z,w)$ is strictly decreasing in w, $\lim_{w\to\infty}\mathcal{T}^*(z,w)=0$ and $\lim_{w\to 0}\mathcal{T}^*(z,w)=\infty$ since $\mathcal{T}^*(z,w)$ satisfies

$$Y_{\mathcal{T}}(\mathcal{T}^*(z,w),z)=w.$$

Denote by $X^*(z,w)$ the value of X that achieves $\mathcal{T}^*(z,w)$. Notice that $X^*(z,w)$ is also strictly decreasing in w. Let

$$\begin{aligned}
H(w) &\equiv \int_z\int_x l(x,X^*(z,w))\left(1+\frac{h(x)}{1-h_E}\right)f(z)dxdz \\
&= \int_z\int_x R^{X^*(z,w)-x}R^x e^{-x(1-\lambda)}f(z)dxdz \\
&= \frac{1}{1-\lambda}\int_z \mathcal{T}^*(z,w)f(z)dz.
\end{aligned}$$

Then the regularity conditions on $Y(\mathcal{T},w)$ imply that $H(w)$ is strictly decreasing in w, $\lim_{w\to\infty}H(w)=0$ and $\lim_{w\to 0}H(w)=\infty$. Since the labor market clearing condition implies $L=H(w)$, the result follows.

C.1.6 Proof of Lemma 13

Plugging in $h(x)$ from Assumption 1 and the definition of $l(x,X)$ we get

$$SP(X)=\ln R\frac{1}{1-\lambda}\frac{R^X}{R^X-1}(1-e^{-X(1-\lambda)}).$$

Consider

$$g(X)\equiv\frac{R^X}{R^X-1}(1-e^{-X(1-\lambda)}).$$

It is sufficient to prove that $g'(X)>0$. Now

$$g'(X)=\frac{R^X}{e^{(1-\lambda)X}(R^X-1)^2}\left((1-\lambda)(R^X-1)-\ln R\left(e^{X(1-\lambda)}-1\right)\right).$$

It is sufficient to prove that $\widehat{g}(X)\equiv(1-\lambda)(R^X-1)-\ln R\left(e^{X(1-\lambda)}-1\right)>0$ for all $X>0$. First notice that $\widehat{g}(0)=0$. Now,

$$\widehat{g}'(X)=(1-\lambda)\ln R\left(R^X-e^{X(1-\lambda)}\right),$$

so $\ln R > 1 - \lambda$ implies that $\widehat{g}'(X) > 0$.

C.1.7 Proof of Lemma 14

Plugging in $h(x)$ from Assumption 1 and the definition of $l(x, X)$ we get

$$SP(X) = \ln R \frac{1}{1-\lambda} \frac{R^X}{R^X - 1}(1 - e^{-X(1-\lambda)}).$$

Let $g(R) = \ln R \frac{R^X}{R^X - 1}$. It is sufficient to prove that $g'(R) > 0$. Now

$$g'(R) = \frac{R^{X-1}}{(R^X - 1)^2} \left(R^X - 1 - \ln R^X\right).$$

Since $x - 1 > \ln x$ for any $x > 0$, the result follows.