LIMITATIONS FOR DAYTIME DETECTION OF STARS USING THE INTENSIFIER IMAGE ORTHICON

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This paper presents an analysis of the theoretical limitation for daytime detection of stars. The paper deals first with the fundamental limits in detection as valid for an ideal device and then with the limitations for a device having a quantum conversion efficiency of less than one, as valid for the intensifier image orthicon. This type of camera tube was suggested by the author in 1952 for overcoming the noise in the scanning section of conventional camera tubes in order to reach the limitations in detection as determined by the noise of the photocathode, when using a sequential light amplifier system (U. S. Patent 2,955,158: Light amplifier, assigned to the U. S. government, inventor Radames K. H. Gebel, filed Nov. 1958, patented Oct. 1960). The development of the intensifier image orthicon was sponsored and monitored by the Aeronautical Research Laboratory at Wright Air Development Center, and most effectively carried out by the Radio Corporation of America Laboratories at Princeton under the guidance of Dr. George A. Morton (Morton and Ruedy, 1960).

For mathematical simplicity, some of the equations derived are approximations. However, they suffice for determining a first approximation of an "optimistic detectable apparent magnitude," which is the purpose of this paper. A rectangular distribution of the quantum density is assumed rather than the actual graded distribution, where the two have the same total value. Technical details concerning the intensifier image orthicon and associated circuitry are not explained since this has been presented in an earlier paper (Gebel, 1961).

FUNDAMENTAL LIMIT OF DETECTION BY AN IDEAL DEVICE

Accomplishing optimum conditions for detection, using photoelectric devices, requires that the focal length of the optical system be related to the resolution of the sensor, so that the scintillating image of a star or the main portion of the central disk of the diffraction pattern caused by radiation from the star is about the same size as the smallest resolvable area at the photosensor. If the focal length chosen is longer than required for this condition, the quantum density for the individual resolution element becomes unnecessarily small. If the focal length is too short, the individual sensor element receives more stray background radiation than necessary.

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According to accepted standard definitions, the limiting theoretical resolution angle α_{min} of a telescope is reached when the central maximum of the image from one point source falls onto the center of the first dark ring of the image of another point source. It is

$$\alpha_{\min} = \frac{1.22 \times 10^{-6} \lambda}{D} \tag{1}$$

where α_{min} is in radians, λ is the effective wave length of light in μ (microns), and D is the diameter of the lens in meters. The diameter of a circle that lies inside and concentric to the perimeter of the resolution disk in the focal plane, where the intensity is ≈ 0.37 of the peak, will be arbitrarily considered in this paper as the theoretical linear resolution d_r of the lens; then

$$\mathbf{d_r} = \alpha_{\min} \mathbf{f} \tag{2}$$

where f is the focal length of the telescope. To make d_r equal to the smallest resolvable element of the photosensor, which will be called matching the telescope to the photosensor, Eq. (1) can be substituted into Eq. (2) and the resulting equation solved for f, yielding

$$f \approx \frac{8.2 \times 10^5 d_r D}{\lambda_{max}}$$
 (3)

where λ_{max} is the longest effective wavelength in μ .

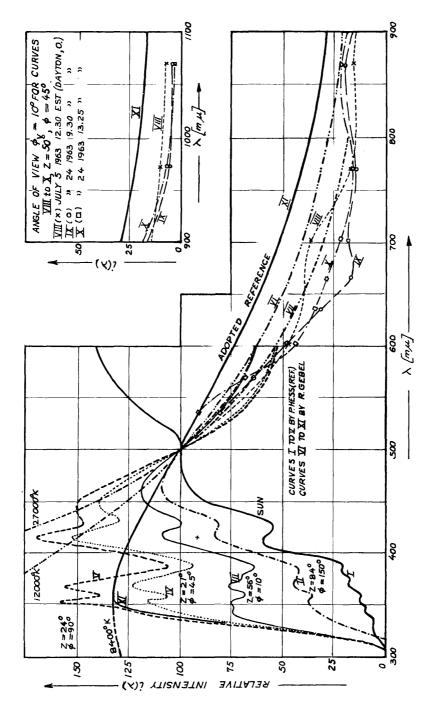
For convenient calculation of a first approximation of the theoretical limit for daytime detection, a conversion from apparent magnitude into quantum flux density, per unit of receptor area and per unit of time, has to be established. All the following calculations will use the radiation from the sun as standard and therefore can be applied to all celestial bodies having the same spectral distribution as the sun; for celestial bodies having other color temperatures, corrections are necessary. Values of the solar energy per cm² per sec, and also the quanta per mm² per sec, reaching the earth's atmosphere from our sun were derived by the author and presented in an earlier paper (Gebel, Hayslett, and Wylie, 1962). There the number of quanta Q counting from λ 0.30 μ to 0.80 μ received by a telescope for the exposure time t in seconds from any celestial body with apparent visual magnitude, M, and having the same spectral distribution as our sun was determined to be

$$Q \approx 3.5 \times 10^{(10-0.4M)} D^2 \eta_T t$$
 (4)

where η_T is the transmission efficiency of the telescope and atmosphere.

For computing the faintest apparent magnitude detectable during the daytime, the ratio of the quanta of light coming from a celestial body to the quanta of light coming from the background must be determined. For this, the amount of stray light from the sky and also the spectral distribution of that light must be known. The spectral distribution of the daytime sky radiation is shown in figure 1. Curves II to V show the distribution between λ 300 $m\mu$ and 600 $m\mu$ as a function of the angular distance Z from the sun (P. Hess, 1939). For 350 $m\mu$ light the intensity decreases by a factor of about four as Z varies from 30° to 150°.

An extensive review of work in this field can be found in the "Handbuch der Physik," by Fritz Möller. The "American Cinematographer Handbook and Reference" by Jackson J. Rose gives for the color temperature of a blue sky 12,000 to 27,000 K, of a hazy or smoggy sky 7500 to 8400 K, and of an overcast sky, 6800 K. In figure 1 black body radiation curves with color temperatures of 12,000 K and 27,000 K (curves VI and VII) are superimposed on Hess curves II to V; none of these seem to represent a suitable average value for use in this paper. A color temperature of 8400 K seems to give a more fitting average value below $500 \ m\mu$. Obviously it is impossible to select any value as a true standard



FIGURET. SPECTRAL ENERGY DISTRIBUTION OF DAYTIME SKY RADIATION

for either the brightness or the color temperature. Nevertheless, it is necessary, to adopt some reasonable average value for the flux of quanta illuminating the Earth's surface, which may be used in the following computations for obtaining first approximations for the limiting detectable apparent magnitude. To supplement the Hess data beyond $600~m\mu$, I have made measurements between $500~m\mu$ and $1100~m\mu$, (curves VIII to X). These measurements were made with a supersensitive portable lightmeter which I designed (Gebel 1954, 1955), sensitive to the visible and near infrared. Eleven calibrated Spectracoat interference filters made by Optics Technology were used. Hess found the cutoff of the daytime stray light at about $310~m\mu$. Considering the large fluctuations in Hess' curves, to me it seems reasonable for the computations in this paper to adopt curve XI. This is essentially a black body curve with a color temperature of 8400° K, modified at the short wavelength end to approximate curve V found by Hess. Curve XI lies above the measured values of curves VIII to X but the 8400° K long wave tail was retained to make some allowance in the adopted value for less favorable conditions. Table 2, computed from curve XI which is the adopted reference

Table 1
Photocathode response to direct sunlight

Spectral band Δλ mμ	Quanta flux by bands Δλ Q' _c quanta mm ⁻² sec ⁻¹	Typical average photocathode efficiency by bands $\Delta\lambda$ $\eta'(S-10)$ $\eta'(S-20)$ electrons/quantum		Photocathode output by bands $\Delta\lambda$ E'c(S-10) E'c(S-20) electron mm ⁻² sec ⁻¹	
300-310 310-320 320-330 330-340 340-350 350-360 360-370 370-380 380-490 400-420 420-440 440-480 480-500 500-550 550-600 600-650 650-700	8.266×10^{12} 1.069×10^{13} 1.247 1.393 1.539 1.691 1.858 2.047 2.251 2.492 6.741 7.880 9.190 1.002×10^{14} 1.036 2.571 2.645 2.628 2.532 4.688	0.016 0.029 0.043 0.049 0.052 0.055 0.057 0.061 0.061 0.058 0.054 0.048 0.037 0.025 0.014 0.005	0.060 0.095 0.123 0.143 0.161 0.177 0.185 0.192 0.197 0.200 0.200 0.171 0.153 0.135 0.110 0.079 0.052	2.00×10^{11} 4.04 6.62 8.29 9.66 1.13×10^{12} 1.28 1.47 4.11 4.81 5.33 5.41 4.97 9.51 6.61 3.68 1.27 4.69×10^{11}	$\begin{array}{c} 4.96\times10^{11}\\ 1.02\times10^{12}\\ 1.53\\ 1.99\\ 2.48\\ 2.99\\ 3.44\\ 3.93\\ 4.43\\ 4.98\\ 1.35\times10^{13}\\ 1.50\\ 1.57\\ 1.53\\ 1.40\\ 2.83\\ 2.09\\ 1.37\\ 7.60\times10^{12}\\ 4.22\\ \end{array}$
300-800	$Q_c = 2.12 \times 10^{15}$	$\eta_{\rm c} = 0.025$	$\eta_{\rm c} = 0.083$	$E_c = 5.31 \times 10^{13}$	$E_c = 1.76 \times 10^{14}$

for the stray light, shows by bands the number of quanta Q_b per mm² per sec received from the sky by a horizontal plane at sea level. This assumes a homogeneous spectral distribution over the entire sky and an illumination of the horizontal plane of 15×10^3 Lux (Dorno, 1919; Leistner, 1952).

For the small angle α_{min} , the number of quanta of stray light O_{str} per second in the corresponding area of the focal plane with the diameter d_r of Eq. (2) may be assumed to be

$$Q_{\rm str} \approx \frac{Q_{\rm b} \pi D^2 \times 10^6}{4} \left(\sin^2 \frac{\alpha_{\rm min}}{2} \right) \eta_{\rm o} \approx \frac{Q_{\rm b} \pi D^2 \times 10^6}{4} \frac{d_{\rm r}^2}{4 f^2} \eta_{\rm o} \approx \frac{2 \times 10^5 Q_{\rm b} D^2 d_{\rm r}^2 \eta_{\rm o}}{f^2}$$
(5)

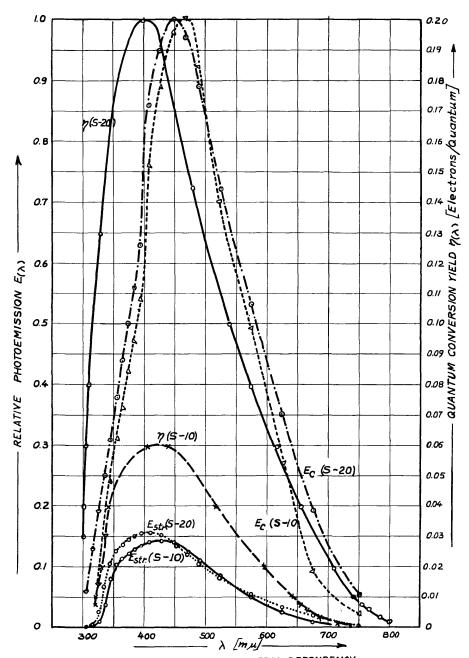


FIGURE II . TYPICAL AVERAGE SPECTRAL DEPENDENCY
OF S -10 & S-20 PHOTOCATHODE CHARACTERISTICS

where η_0 is the transmission efficiency of the telescope and Q_b is the total flux of quanta as adopted for the stray light counting from 300 $m\mu$ to 800 $m\mu$ as shown in table 2. Brightness measurements of the daytime sky made with a 15-m focal length telescope showed that the intensity increases very rapidly when viewed closer to the sun than $Z=30^{\circ}$. Obviously then, Table 2 should be restricted to $Z>30^{\circ}$ and then as an approximate guide only. Then, using the value of f from Eq. (3) in Eq. (5) gives the smallest number

of quanta of stray light Q_{str} per sec for a matched system. Thus

$$Q'_{\rm str} \approx \frac{2 \times 10^5 Q_b D^2 d_r^2 \eta_o}{\left(\frac{8.2 \times 10^5 d_r D}{\lambda_{\rm max}}\right)^2} \approx 3 \times 10^{-7} Q_b \eta_o \lambda_{\rm max}^2$$
(6)

Taking the value of Q_b from Table 2, using $\lambda_{max} = 0.80 \mu$ and substituting these in Eq. (6) will produce the smallest number of quanta of stray light per sec, that needs to be considered for practical purposes for a resolution element at the focal plane of a matched system. Thus

$$Q'_{str} \approx 3 \times 10^{-7} \times 2.48 \times 10^{14} \times 0.8^{2} \eta_{o} \approx 4.8 \times 10^{7} \eta_{o}$$
 (7)

Obviously, if the star to be detected has a different color temperature than the daytime sky, the use of appropriate spectral and/or polarization filters is possible to increase the detectable apparent magnitude. However, since the objective of this study is to obtain a first approximation of the limiting detectable magnitude, this has not been taken into consideration here.

The probability of detection is a function of the ratio r of the number of quanta of light received by a given area from a star to the square root of the average number of quanta of stray light or background light received by an equal area in the image plane (Gebel, 1961). It is commonly assumed that detection is theoretically possible if $r \ge 1$ (Rose, 1948). Hence, setting r = 1 and using a matched system the smallest number of quanta Q from a point source which can theoretically be detected is

$$Q = (Q'_{str}t)^{1/2}$$
 (8)

where t is the time of exposure in seconds. Setting Eq. (8) equal to Eq. (4) and neglecting the small differences between values η_0 and η_T , which is the atmospheric absorption and is small under optimum conditions, yields

$$(Q'_{str}t)^{1/2} = 3.5 \times 10^{(10-3.4M)} D^2 \eta_T t$$
 (9)

Solving for the most optimistic value of the apparent magnitude M_{th} that is theoretically detectable during daytime for any system gives:

$$M_{\rm th} \approx 2.5 \log \frac{3.5 \times 10^{10} D^2 \eta_{\rm T} t^{1/2}}{O_{\rm str}^{1/2}}$$
 (10)

Example 1.—For a 10 inch telescope and for $\eta_T = 0.5$; t = 1 sec; $Q_{str} = 4.8 \times 10^7 \times 0.5$ from Eq. (7) and Eq. (10) the theoretical limiting apparent magnitude for daytime detection is found to be

$$M_{\rm th}\!\approx\!2.5\,\log\!\frac{3.5\times10^{10}\times\!0.25^2\times\!0.5\times1^{1\!/\!2}}{(4.8\times10^7\times\!0.5)^{1\!/\!2}}\!\approx\!13.4.$$

THE LIMITING STAR MAGNITUDE DETECTABLE BY THE INTENSIFIER IMAGE ORTHICON DURING DAYTIME HOURS

The spectral conversion yields, η^{\dagger} in electrons per quantum, of the commercially available S-10 (Ag-Bi-O-Cs) and S-20 multialkali ([Cs](NaK)₃ Sb) photocathodes are shown by spectral bands in table 1 and figure 2, and the number of electrons E_{sir} produced per mm² per sec by these photocathodes are shown in table 2, also

by spectral bands, for the adopted reference for daytime stray light (curve XI). Counting from λ 0.30 μ to 0.80 μ and comparing the number of electrons E_{str} per sec caused by the stray radiation N (table 2) with the number of quanta Q_b comprising that radiation, the average quantum efficiency η_b in electrons per quantum of these photocathodes may be obtained for stray light. Thus

$$\eta_{\rm b} = \frac{E_{\rm str}}{O_{\rm b}} \tag{11}$$

which, for the S-20 photocathode yields

$$\eta_{\rm b} = \frac{2.55 \times 10^{13}}{2.48 \times 10^{14}} \approx 0.103.$$

By considering the smallest practical value of Q'_{str} , as in Eq. (7), the number of electrons E_b released per sec by a resolution element of the photocathode under matched conditions is, according to Eqs. (7) and (11),

Table 2

Adopted sky background reference¹

Spectral band	Energy flux by bands Δλ	Quanta flux by bands		Electron flux by bands $\Delta\lambda$	
Δλ	N'	Q' _b	E' _{str} . (S-10)	E' _{str} . (S-20)	
$m\mu$	erg mm ⁻² sec ⁻¹	quanta mm ⁻² sec ⁻¹		mm ⁻² sec ⁻¹	
300-310	0.20	3.08×10 ¹⁰	· · · · · · · · · · · · · · · · · · ·	1.85×10 ⁹	
	$\frac{0.20}{2.78}$				
310-320		4.42×10^{11}	0.00 \ (1010	4.20×10^{10}	
320-330	$\frac{10.72}{20.64}$	1.76×10^{12}	2.82×10^{10}	2.16×10^{11}	
330-340	$\frac{22.64}{20.72}$	3.83	1.11×10^{11}	5.48	
340-350	29.79	5.19	2.23	8.36	
350-360	31.38	5.63	2.76	9.97	
360-370	31.78	5.86	3.05	1.08×10^{12}	
370-380	31.38	5.94	3.27	1.14	
380-390	30.98	6.02	3.43	1.19	
390-400	30.58	6.10	3.60	1.22	
400-420	59.58	1 . $23 imes 10^{13}$	7.50	2 . 46	
420 - 440	57 . 20	1.24	7.56	2.36	
440-460	54.82	1.25	7.25	2.14	
460-480	52.04	1.24	6.70	1.90	
480-500	49.26	1.22	5.86	1.65	
500-550	111.20	$\frac{1}{2}.\overline{95}$	1.09×10^{12}	3.25	
550-600	93.35	$\frac{2.70}{2.71}$	6.78×10^{11}	2.14	
600-650	80.45	$\frac{2.54}{2.54}$	3.56	1.32	
650-700	67.50	$\frac{2.31}{2.30}$	1.15	6.90×10^{11}	
700-800	105.26	3.99	3.99×10^{10}	3.59	
300-800	$N = 9.53 \times 10^2$	$Q_b = 2.48 \times 10^{14}$	$E_{\rm str.} = 7.7 \times 10^{12}$	$E_{\rm str.} = 2.55 \times 10$	

¹Ground illumination and photoemission for a spectral distribution as in Curve XI, Fig. I.

$$E_b = Q_{str}' \eta_b \approx 4.8 \times 10^7 \eta_o \ 0.103 \approx 5 \times 10^6 \eta_o.$$
 (12)

Using the spectral conversion yields η' of table 1 and taking the number of quanta per mm² per sec as presented by spectral bands by Gebel *et al.* (1962), the number of electrons E_c released per mm² per sec by an S-10 or S-20 photocathode may be determined for direct sunlight, also shown in table 1 by bands. Using table 1, the average conversion efficiency η_c in electrons per quantum is, for light similar to sun light,

$$\eta_c = \frac{E_c}{Q_c} \approx \frac{1.76 \times 10^{14}}{2 \times 10^{15}} \approx 0.083$$
(13)

where Q_c is the total number of quanta of sunlight between 0.30 μ and 0.80 μ and E_c is the total number of electrons per mm² per sec caused by Q_c .

It is commonly accepted that the threshold of a detecting device is reached when the internal noise of the device becomes equal to the signal produced by the information. Since in the intensifier image orthicons the quantum to electron conversion is accomplished by using a photocathode with an efficiency of less than 1 as shown above, η_c and the fluctuations in the dark current produced by the photocathode and the fluctuations in the scanning beam represent the limiting factors which determine the deviation from the previously determined theoretical

magnitude number (Gebel, 1961).

The number of electrons per second comprising the dark current at 25 C for the multialkali photocathode is less than 1000 per mm². Since, in accordance with Eq. (12), in a matched system the smallest number of electrons caused by the stray light and emitted by a resolution element is $5 \times 10^6 \,\eta_o$ electrons per sec, the dark current and its fluctuations may be neglected in this case. The amplification of the intensifier section is sufficiently high that any other fluctuations in the system are small in comparison with the fluctuations in the number of electrons caused by the daytime stray light and may also be neglected (Gebel, 1961).

In Eq. (10) the theoretical limit was determined for an ideal system having a conversion efficiency of one. However, for the intensifier image orthicon the fact that the quantum efficiency of the photocathode is less than one has to be taken into consideration. Therefore

$$(Q_{\rm str}^{\prime} \eta_{\rm b} t)^{\frac{1}{2}} = Q \eta_{\rm c}. \tag{14}$$

Substituting Q from Eq. (4) into (14), again neglecting the losses in the atmosphere by setting $\eta_o = \eta_T$ and then solving for the detectable apparent magnitude M_p yields, for devices such as the intensifier image orithcon under the assumed conditions.

$$M_{P} = 2.5 \log \frac{3.5 \times 10^{10} D^{2} t^{1/2} \eta_{T} \eta_{c}}{(Q_{ctr}^{1})^{1/2} \eta_{b}^{1/2}}.$$
 (15)

Here it becomes of interest to find the difference M_D between the theoretically detectable magnitude M_{th} and that of M_P as in Eq. (15). Then using Eqs. (10) and (15) for substitution gives:

$$M_D = M_{th} - M_P = 2.5 \log \frac{\eta_b^{1/2}}{\eta_c}$$
 (16)

Obviously $M_{th} \ge M_P$, therefore Eq. (16) is valid only if $\eta_b^{1/2} \ge \eta_c$ which is the case if the assumptions are observed.

Example 2.—Using the conversion efficiency of the S-20 multialkali photocathode as previously determined, and assuming the most optimistic operating conditions, yields

$$M_D \approx 2.5 \log \frac{0.103^{\frac{1}{2}}}{0.083} \approx 1.5$$

and using M_{th} from Example 1

$$M_P = 13.4 - 1.5 = 11.9$$

which is approximately six apparent magnitudes (250 times) fainter than the unaided human eye can see at night.

In the spring of 1957, I suggested the use of the intensifier image orthicon in connection with a suitable sequential light amplifier chain for fast daytime detection and image recording of celestial bodies utilizing the enormous contrast enhancement capability of such a system (Gebel, 1961). One of the Cat Eye arrangements constructed by the Aeronautical Research Laboratories was installed in the Weaver Observatory of Wittenberg University in Springfield, Ohio, for this purpose, and a number of photographs of celestial bodies were taken at noon time, which could not have been obtained employing conventional photography (Gebel, Devol, and Wylie, 1960).

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