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Recommended Citation

J. Kovářík, D. Levin, and Tao Wang. (2016). "Ellsberg Paradox: Ambiguity And Complexity Aversions Compared". *Journal Of Risk And Uncertainty*. Volume 52, Issue 1. 47-64. DOI: 10.1007/s11166-016-9232-0
https://works.swarthmore.edu/fac-economics/353

Ellsberg Paradox: Ambiguity and Complexity Aversions Compared*

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January 16, 2016

Abstract

We present a simple model where preferences with complexity aversion, rather than ambiguity aversion, resolve the Ellsberg paradox. We test our theory using laboratory experiments where subjects choose among lotteries that "range" from a simple risky lottery, through risky but more complex lotteries, to one similar to Ellsberg's ambiguity urn. Our model ranks lotteries according to their complexity and makes different–at times contrasting–predictions than most models of ambiguity in response to manipulations of prizes. The results support that complexity aversion preferences play an important and separate role from beliefs with ambiguity aversion in explaining behavior under uncertainty.

Keywords: Ambiguity, Complexity, Compound Risk, Ellsberg paradox, Risk, Uncertainty.
JEL Classification: C91, D01, D81.

*Our paper greatly benefited from comments and suggestions of a referee and the Editor. We also thank Itzhak Gilboa, Yoram Halevy, Brian Hill, Dimitry Mezhvinsky, Jim Peck, and participants of the DT conference 2009 in HCE Paris and SEET 2013. Jaromír Kovářík greatly acknowledges support from Spanish Ministry of Science and Innovation (ECO 2012-31626, ECO 2012-35820), the Basque Government (IT-783-13), and GACR (14-22044S).

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1 Introduction

In his thought experiment, Ellsberg (1961) offered subjects (some of whom were renowned economists) a bet on red (black) that paid a high prize if the color drawn from the chosen urn was red (black). Subjects had to choose between two urns. Urn I contained 100 red and black balls in an unknown ratio and urn II contained exactly 50 red and 50 black balls. A significant percentage of subjects chose urn II in both the bet on red and the bet on black, violating principles implied by Savage's (1954) expected utility theory (EUT) as these choices could not be rationalized by any subjective prior distribution of the states of the world envisioned by EUT. Ellsberg's paradox highlighted that Savage's axioms, the sure-thing principle in particular, are quite restrictive and that there is a need for a revised or even new theory of decision making under uncertainty. A variety of models have been developed to generalize EUT and resolve Ellsberg's paradox so far (see Gilboa and Marinacci (2013) for an excellent review).[1] This literature mostly focuses on the principles governing the formulation of beliefs, with little attention to the possible direct impact of uncertainty/ambiguity on utilities.

In contrast, we recognize that, say, a prize of $10 from an ambiguous urn may not be viewed as equivalent to a prize of $10 from a simpler decision problem (risky urn) and provide a simple alternative explanation to the Ellsberg's paradox. We argue that choices that seem to contradict the existence of a prior distribution could have been made by subjects who *do* have a prior distribution of the different states of nature, but who are averse to complexity and regard it as a cost that taxes the utility of the lottery.[2] Having a diffuse prior over the (101) different states of nature in Ellsberg's urn transforms it into a risky but compound lottery.[3] In addition, our complexity-averse decision makers (DMs) do not regard compound lotteries as equivalent to their reduced form counterparts. As a result, the Ellsberg paradox can be explained by complexity aversion that is applied to utilities and *not* beliefs as in (most of) the literature on ambiguity aversion.

Recently, Halevy (2007) has documented a strong association between attitudes to ambiguity and the failure to reduce compound (objective) lotteries (see also Dean and Ortoleva (2011) and Abdellaoui et al. (2011)), suggesting that the failure to reduce compound lotteries might be one contributing factor of the Ellsberg paradox. However, this work was not designed to disentangle whether

[1]Leading theories include Schmeidler (1989), Gilboa and Schmeidler (1989), Segal (1987, 1990), and more recently Klibanoff et al (2005) and Maccheroni et al. (2006). Section 2 and Online Appendix A provide a more detailed exposition of these theories in the context of our experiment.

[2]Modeling complexity aversion as taxing utilities due to the extra (cognitive) costly effort to compute reduced-form equivalents would result in a circular argument. The reason is that, once such effort is already taken, it becomes sunk and should not affect choices. However, we do not assume that DMs reduce the lotteries, and our simple model and treatment manipulations enable us to distinguish complexity from uncertainty aversion. See below.

[3]We model the DM as having a particular prior on the Ellsberg urn lottery only to illustrate our claim: There *do* exist priors with complexity aversion that resolve the Ellsberg paradox.

and/or to what extent uncertainty/ambiguity affects DMs through beliefs or utility.

Our theory generates several predictions, some of which contrast with previous literature. First, it induces a ranking of alternatives that exhibit the same degree of riskiness but differ in their complexity or compoundness. Second, the model generates predictions of DMs' response to prize manipulations that differ from the literature on ambiguity and we design an experiment that enables us to distinguish between complexity aversion and the leading theories of uncertainty/ambiguity aversion. Some of our findings are statistically weak, yet overall they support the idea that complexity aversion should be viewed as an important and a separate force from ambiguity aversion.[4]

We do not provide a formal definition of complexity or a way to measure it. Instead, we relate complexity in our experiment to the compoundness (the number of layers) and the number of alternatives (branches) of the lotteries. Note that Ellsberg did not provide a formal measure of ambiguity or its aversion. Instead, he called attention to what seems like a paradox under EUT that the literature, for over 50 years, had tried to resolve by focusing on new belief formation. In contrast, we offer an alternative (additional) explanation via complexity aversion that affects the utility but not beliefs.

We neither dispute the significance of Ellsberg's observation nor the valuable contribution of the research on uncertainty/ambiguity, nor argue that our theory replaces approaches that are based on belief formations. The argument we envision here is that complexity aversion may play a relevant, distinct, and important role *alongside* uncertainty/ambiguity aversion and thus deserves attention. Since we find evidence of complexity aversion, disentangling the individual principles behind the Ellsberg paradox (e.g. Ahn *et al.*, 2014) or evaluating the performance of different models of ambiguity aversion (such as Halevy, 2007) becomes more challenging as the impact of ambiguity on beliefs might be confounded with the effects of complexity on utilities.

Our model can explain a phenomenon reported recently by Chew et al. (2013). Consider an Ellsberg-like urn with 100 balls where there are only either $n \geq 50$ or $100 - n$ red balls with unknown probabilities. This spans an unambiguous risky urn if $n = 50$, any ambiguous urn with two possible amounts of red balls if $n \in (50, 100)$, and the extreme situation with either all balls red or all black ($n = 100$). As n increases from $n = 50$ (the risk case) to 100, Chew et al (2013) find that the willingness to pay for those lotteries declines with n in line with some ambiguity aversion theories. However, for $n = 100$ the willingness to pay sharply increases and this lottery is indistinguishable from the risky $n = 50$ one. Note that all the uncertainty resolves in one stage in the $n = 100$ lottery, making this ambiguous option basically non-compound.[5] Thus, since complexity scales with the compoundness, complexity aversion pre-

[4] The results of Halevy (2007) and Abdellaoui et al. (2011) are generally in line with the idea that people rank prospects according to their complexity. See also Yates and Zukowsky (1976).

[5] In contrast, the urns with $n \in (50, 100)$ can be viewed as twice compound and thus more complex lotteries.

dicts the $n = 50$ and $n = 100$ urns to be similar and both to be preferred to the $n \in (50, 100)$ variations.

In Section 2, we describe complexity aversion and derive the experimental hypotheses. We present our experimental design in Section 3 and the results in Section 4. Section 5 concludes. Some additional material can be found in the Online Appendix.

2 Hypotheses

2.1 Complexity Aversion

We argue that people dislike complexity. Complexity-averse individuals tax the outcomes of complex lotteries by a constant $\delta > 0$. Consequently, when a DM faces a complex lottery with two final prizes H and L, $H > L \geq 0$, she views the lottery as having prizes $H - \delta$ and $L - \delta$. We assume that the parameter δ scales up with the complexity of the lottery. Hence, if a complexity-averse DM is presented with a choice between two lotteries with identical prizes and probabilities with one lottery being a compound variation of the second, she might prefer the latter *only* because of the increased complexity of the former.[6]

Throughout the paper, we assume that subjective probabilities coincide with objective probabilities if they are known. In the present theory, ambiguous lotteries are viewed by DMs as compound and more complex versions of risky lotteries, for which the corresponding probabilities can be calculated.[7] For instance, the ambiguous Ellsberg urn can be viewed as a risky lottery in which there is a uniform probability of each possible number of red/black balls in the urn. This way, the Ellsberg's urn represents a relatively complex risky lottery and complexity-averse DMs might choose risky lotteries over ambiguous urns because of the complexity involved in the computation of expectations corresponding to Ellsberg's urn.

Since complexity is understood as an ordinal measure here, the present theory allows for a preference ranking over lotteries that are identical in both the final outcomes and their corresponding probabilities, but differ in the way they are compounded and one can analyze such preference ranking in a population using experimental techniques.

We focus on a simple version of complexity aversion. We assume that people dislike the cognitive effort associated with computation of probabilities in complex lotteries and avoid performing them. Instead, they estimate (or guess) the probabilities and discount the final prizes of these lotteries.

Consider a choice between a pure risky lottery A and its more complex counterpart B, which both give high prize H with probability one half and L otherwise.[8] The DM does not make the calculation of probabilities in the

[6] We use the linear, additive structure for expositional simplicity. However, qualitative results will not change with any utility that is declining in δ.

[7] This approach is also taken by e.g. Segal (1987) or Klibanoff et al. (2005).

[8] We use one half for expositional simplicity. The arguments extend for different probability levels.

complex lottery, but forms an estimate θ about the probability of drawing H in lottery B. Then, $B \succ A$ if[9]

$$\theta(H-\delta)+(1-\theta)(L-\delta) > \frac{1}{2}(H+L)$$

or

$$\theta > \theta^* = \frac{1}{2} + \frac{\delta}{H-L}.$$

We assume that there is a distribution of θ in the population (as in Figure 1a) and that the mode of the distribution is equal to the true probability (one half in our example).[10] People whose estimated θ lies below the threshold θ^* prefer the simpler lottery A, but a possibly non-negligible amount of people, whose beliefs about the probability of the high prize in the complex lottery B are large enough, might prefer the more complex B to A. In addition, since θ^* is increasing in δ and decreasing in $(H-L)$, the fraction of people who prefer B to A decreases with the complexity of B and increases with the difference between the high and low prizes. These observations generate two questions.

Question 1 (complexity aversion): *Do most people prefer the risky lottery to its compound or ambiguous variations and does a non-negligible fraction of subjects still prefer the latter?*

Note that, if one allows for subjective priors over the ambiguous events in the existing theories of ambiguity aversion, any theory allows ambiguity-averse individuals to prefer the ambiguous urn if they believe that the high prize is likely enough (see Online Appendix A for formal analysis). Hence, even if our theory predicts a non-negligible amount of people to prefer B, *Question 1* does not allow us to distinguish our complexity aversion from other models.

Question 2 (degree of complexity): *Does the number of people preferring the risky lottery A to B increase with the complexity of B?*

It is the second question that allows the testing our theory *vis-à-vis* other models explaining Ellsberg's paradox. An important implication of complexity aversion is that the ranking of lotteries will depend on their complexity even if they are payoff-equivalent and even if only objective risk is involved. Any theory in which people reduce compound lotteries predicts that people should be indifferent between reduced lotteries and their compound variations, thus predicting the same number of people preferring A to B independently of their complexity. The differentiation of payoff-equivalent non-ambiguous lotteries requires a two-stage approach toward risk and uncertainty as in rank-dependent

[9] Note that δ in fact measures the difference in complexity between A and B, rather than being an absolute measure of complexity of lottery B.

[10] Obviously, the complexity of the lottery itself may shape this distribution through the variance of the estimates in the population. This, however, does not affect any of the predictions below.

utility (RDU; Segal, 1987, 1990). Under RDU, people evaluate each lottery layer and each lottery within each layer separately providing an argument for why people do not reduce compound lotteries. Ambiguous lotteries are viewed similarly in this theory. In particular, the RDU model allows for the ranking in *Question 1* if the probability weighting function $f(.)$ in Segal (1987,1990) satisfies several properties.[11] The alternative two-stage approach of Klibanoff et al. (2005) transforms the expected utilities (conditional on the state) using a function $\phi(.)$, which reflects attitudes toward ambiguity.[12] However, this function does not apply for compound lotteries. As a result, they may generate similar predictions as complexity aversion under ambiguity but differ starkly from complexity aversion if only objective risk is involved. Consequently, RDU is the only theory that cannot be distinguished from complexity aversion in terms of *Question 2*.

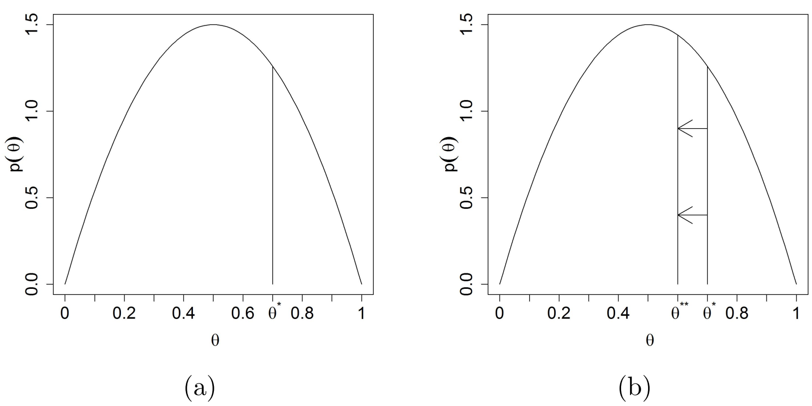

Figure 1. *Left*: The distribution of estimates of probability of high prize in the complex lottery B and threshold θ^*; *Right*: The effect of stake increase on the threshold probability.

2.2 Multiplying prizes and complexity aversion

Since δ only changes with the degree of complexity, then if, for example, we double the prizes without altering any probability the threshold θ becomes

$$\theta^{**} = \frac{1}{2} + \frac{\delta}{2(H-L)} = \theta^* - \frac{\delta}{2(H-L)} < \theta^*. \tag{1}$$

[11] See Appendix A for more details.

[12] Ambiguity aversion requires the concavity of ϕ, whereas complexity aversion is modeled as a linear transformation of payoffs which depends on the complexity of alternative prospects. This model is related to source-dependent approaches to uncertainty (Ergin and Gul, 2009; Grant et al., 2009). See Appendix A for more details.

In words, if we double all of the lottery prizes, the threshold θ that makes people indifferent between choosing the simple risky lottery over its complex (e.g. compound) "cousin" is lower and, hence, the fraction of the population selecting the complex lottery ought to increase. Figure 1b illustrates this effect graphically. If in particular B is an ambiguous urn, such prediction contrasts starkly with most of the belief-based explanations of the Ellsberg paradox (see below).

Notice that the above prediction abstracts from the possibility that, if more is at stake (i.e. prizes are multiplied by a factor larger than one), DMs may exert more effort in forming their probability estimates in more complex lotteries. That is, larger stakes provide an incentive to "think harder" and may result in a distribution of estimates that is more concentrated around the true probabilities (see Figure 2a).

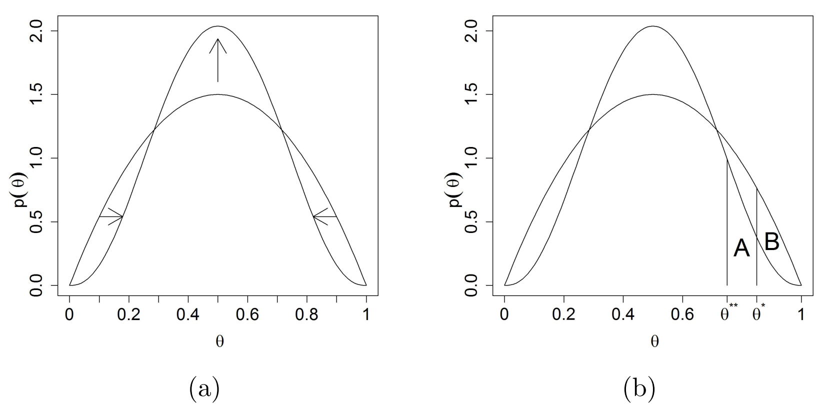

Figure 2. *Left*: Effect of increased effort while estimating the probability of the high prize in complex lotteries. *Right*: If stakes are multiplied and effort increased, the difference between the mass of people that prefer B to A is given by (i) people who switch from A to B due to the lowering of θ^* to θ^{**} (area A) and (ii) people who switch from B to A due to a better estimate of the true probability (area B).

Figure 2 reveals that complexity aversion and the increased effort estimating the probability of the high prize work one against the other. The reduction of the threshold θ (from θ^* to θ^{**}) increases the mass of people who prefer the complex/ambiguous lottery (area A in Figure 2b), while the shrinkage of tails of the distribution due to higher effort decreases the number of B-preferring individuals (area B in Figure 2b). Since the potential higher effort works against the above prediction of more people preferring B after the prize multiplication, any evidence in favour of complexity aversion would be even stronger.

Although the effect of higher stakes on precision of probability beliefs prevents us from making general predictions concerning prize increase, the threshold θ^{**} (and the increase of B-preferring individuals in Fig. 2b) depends on the parameter δ and thus on the complexity of lottery B. In particular, the higher the general complexity of the lotteries the more likely we are to observe more people preferring the complex lottery B after the payoffs are increased. Such interplay between the stakes and the overall complexity plays an important role in the interpretation of the results.

This discussion generates the following question:

Question 3 (prize multiplication): *Does an increase in prizes while keeping the same complexity make more (fewer) people prefer the ambiguous urn B to the risky A?*

What are the predictions of the theories of ambiguity regarding the change of DMs' choices that result from (pure) multiplications (e.g. doubling) of prizes? Note that since probabilities are the same before and after the prize multiplication the priors ought not to be affected. As a result, one group of models predicts no change in ranking due to such prize manipulation. This group includes subjective expected utility (SEU; Savage, 1957; Anscombe and Aumann, 1963), Choquet expected utility (CEU; Schmeidler, 1989), maximin expected utility (MMEU; Gilboa and Schmeidler, 1989), $\alpha-$maximin model ($\alpha-$MEU, Ghirardato et al., 2004), and rank-dependent utility (RDU; Segal, 1978, 1990). Hence, even though *Question 2* does not allow us to separate RDU from complexity aversion, these theories generate different predictions for such prize manipulations: it should not affect the ranking under RDU while complexity aversion predicts it would.

As mentioned above, the smooth model of Klibanoff et al. (2005), labeled as REU hereafter, transforms the expected utilities (conditional on the state) using a function ϕ which reflects attitudes toward ambiguity. This model provides no general prediction about the effect of prize multiplication as it depends on the distribution of preferences (shapes of ϕ) in the population, not the complexity. REU permits more people to prefer the ambiguous lottery B after the prize multiplication.[13] Apart from aversion to complexity, this is the only theory that allows for such a prediction, but it suggests that the effect (whatever it is) should be the same, independently of the complexity of the lotteries.

Finally, variational preferences (VP) due to Maccheroni et al. (2006) take into account the possibility that DMs are well aware that their beliefs might not be correct. They axiomatize a model such that the expected utility from lotteries and actions with unknown probabilities accounts for such uncertainty, using a function $c(.)$. Since there is no uncertainty in risky prospects and $c(.)$ is independent of the stakes, the role of this function only influences the utilities from the ambiguous urns and loses importance as prizes increase. As a result, if someone changes her choice after prizes increase, she can switch from the

[13] We prefer this more conservative prediction. However, if most of the subjects in the experiment are averse to ambiguity (concave ϕ) fewer people should prefer B over A after our prize manipulation. See Online Appendix A for more details.

ambiguous B to A but not vice versa and VP unambiguously predict that more people will prefer the risky urn after such prize multiplication.

Another question is what the above models of ambiguity aversion predict concerning the ranking of risky and ambiguous lotteries if we again take into account the possibility that people exert more effort estimating probabilities in the ambiguous lotteries for larger stakes, as in Figure 2a. Observe that those theories, for which the threshold for accepting the ambiguous lottery is unaffected, will only experience a decrease of the B-selecting DMs after the prize multiplication.[14] The effect of multiplication and larger effort estimating the probability of H reinforce each other in VP. This should also hold for REU if most of the subjects are averse to ambiguity; if, in contrast, most people are ambiguity-loving, both effects go against each other as in the complexity aversion, but the final effect should again be the same independently of the general complexity of the lotteries.

In sum, REU is the only theory that allows for more people preferring the ambiguous lottery after the payoff manipulation, but only if most people are ambiguity-loving. REU predicts the effect to be the same independently of other features of the experimental design though.[15]

3 Experimental Design

A total of 717 undergraduate students enrolled in a number of economics courses at a large Midwestern university participated in our experiments. Each student was only allowed to participate in one of the sessions. We conducted multiple sessions of 15-40 students and each session lasted approximately 15 minutes. Upon permission by the instructor, the experimenter would enter the classroom toward the end of a class and explain to the students the procedure and compensation in the experiment. Students would stay if they chose to participate. A detailed description of the four lotteries and a questionnaire were distributed. The experimenter explained the lotteries, answered any questions regarding the lotteries, and participants were asked to rank the lotteries from the most to the least preferred. Indifference was allowed.[16]

Participants were informed that one out of ten students would be randomly chosen and invited to participate in an actual lottery at the end of the experiment. For each invited student, the experimenter would randomly pick two out of the four ranked lotteries and implement the preferred one by the student in the experiment. The students were compensated according to the result of the chosen lottery. We used such a procedure to create the incentive to rank all the lotteries, rather than concentrating only on the most preferred one. This was explained to the students.

[14] To see this, note that the area A in Figure 2b does not exist and these theories only suffer the loss of the B-preferring individuals corresponding to area B.

[15] Appendix A.3 contains the formal analysis of these arguments.

[16] The experimental instructions can be found in the Online Appendix.

3.1 Lotteries

The experiment is an extension of Ellsberg's original design. In each lottery below, the participant chooses a color. If the chip drawn is of the chosen color, a high prize H is awarded; otherwise, a small prize of L is awarded. We conducted two waves of the experiment, differing in the general complexity of lotteries (number of balls), and two treatment variations with respect to the stakes in both waves. In the low-stake treatment $H = 50$ and in the high-stake treatment $H = 100$, but in all cases $L = 5$.[17]

The participants were presented with four options: A - risky, B - ambiguous, C - once compound, and D - twice compound lotteries in Wave 1. In Wave 2, the twice compound lottery D was replaced by a once compound but more complex urn.[18] The particular lotteries were as follows:[19]

Wave 1:

- *Lottery A* (risk) contains 5 white and 3 black balls/chips.
- *Lottery B* (Ellsberg) contains 6 chips of unknown color and 2 chips of a color chosen by the subject.
- *Lottery C* (once compound): A coin is tossed to select one bag. Bag 1 contains 2 white and 2 black chips, while Bag 2 contains 3 white and 1 black chip.
- *Lottery D* (twice compound): The content of the bag is decided through two coin flips. The first flip decides between left and right. On the left, another flip selects between two bags: Bag 1 with 1 white and 3 black chips and Bag 2 with 4 white chips; on the right, another flip determines whether Bag 3 with 3 white and 1 black chips or Bag 2 with 2 white and 2 black chips is used.

Wave 2:

- *Lottery A* (risk) contains 3 white and 1 black chips.
- *Lottery B* (Ellsberg) contains 1 chip of unknown color and 1 chip of a color chosen by the subject.
- *Lottery C* (once compound): A six-sided die is rolled to select one out of two bags: If 1 or 2 turns up (probability $\frac{1}{3}$), Bag 1 with 1 white and 3 black chips is chosen; if 3 to 6 (probability $\frac{2}{3}$) Bag 2 containing 4 white and 0 black chips is chosen.

[17] Note that we did not multiply the low prize in the experiment. In Appendix A, we show formally that it neither affects the derived theoretical predictions of complexity aversion nor alters the predictions of the ambiguity literature.

[18] Naturally, the terms risky, ambiguous, and compound were never used in the actual experiment.

[19] As mentioned, we use $\frac{1}{2}$ as the probability of high prize to simplify the exposition of complexity aversion, whereas in the actual experiments the probabilities are $\frac{3}{8}$ in Wave 1 and $\frac{3}{4}$ in Wave 2. The reason is to prevent subjects from focusing on the symmetric $\frac{1}{2}$ while evaluating or estimating probabilities of prizes during the experiment. The theoretical predictions are robust to considering $\frac{1}{2}$, $\frac{3}{8}$ or $\frac{3}{4}$.

- *Lottery D* (more complex compound): A six-sided die is rolled to select one out of three bags: If 1 or 2 (probability $\frac{1}{3}$), Bag 1 with 2 white and 2 black chips is chosen; if 3 or 4 (probability $\frac{1}{3}$) Bag 2 containing 3 white and 1 black chip(s) is chosen; if 5 or 6 (probability $\frac{1}{3}$) Bag 3 with 4 white and 0 black chips is chosen for the draw.

First, note that the design allows for a paradox a là Ellsberg. For instance, in Wave 1 we have a risky urn A with eight balls, five black and three white. The Ellsberg-like urn B contains six black and white balls in unknown proportion. A DM is asked to add two balls of a chosen color, i.e. two black, two white or one black and one white, to the Ellsberg urn (so that the total number of balls in the urn is eight) and to bet on his/her chosen color in either the risky or the Ellsberg's urn. Let x denote the number of black balls among the original six balls in the Ellsberg urn and y the total number of black balls in the Ellsberg urn after the addition of the two chosen balls. Suppose that the DM believes that $x \in \{4, 5, 6\}$. She can add two black balls and bet on black from the Ellsberg urn. Since $y \geq 6$ in this case, the probability of drawing a black ball and winning is at least $2/3 > 5/8$ (the probability of winning by betting on black from the risky lottery). If conversely $x \in \{0, 1, 2\}$ the DM can add two white balls and bet on white from Ellsberg's B, leading to $y = x \leq 2$ and the probability of drawing white from B (and winning) is at least $6/8 > 5/8$ (the probability of winning on black from the risky lottery). In both case, an ambiguity neutral DM should strictly prefer to bet on the Ellsberg urn or be indifferent if $x = 3$. Thus, in the spirit of Ellsberg there exist no prior (probability) beliefs that can rationalize a DM who strictly prefers to bet on the risky urn when in reality most of subjects strictly prefer to bet on the risky (rather than Ellsberg's) urn in our experiment.

3.2 Experimental Lotteries and Complexity

Although we do not provide a formal definition and/or measurement of complexity, we associate it with the compoundness (or layers) and the number of alternatives (branches) of the lotteries. Note though that the most complex lottery–the ambiguous B–was presented as second. The reason is to break the relation between the order of presentation and order of complexity.

In Wave 1, we can rank the four lotteries according to two elements of complexity: compoundness and the number of possible urn compositions. The composition is known in lottery A (that is, no compoundness and one composition); C is once compound and can result in two alternative urns; D is twice compound and may lead to four different urns. The ambiguous B is ranked last. It can be viewed as once compound but it generates seven differing urn compositions. Moreover, B requires the formation of beliefs about each composition, leading to additional complexity. Complexity aversion may thus predict $A \succ C \succ D \succ B$ in Wave 1.

Our Wave 2 is motivated by sharpening further the distinction between ambiguity and complexity to the extent possible.[20] First, Wave 2 is overall less complex than Wave 1 with respect to the number of balls in the individual urns. Furthermore, the differences in complexity across individual lotteries within Wave 2 are smaller than in Wave 1. A is again a simple risky lottery; lotteries C and D are both once compound but D can lead to three different urn compositions. Hence, we expect the following ranking under complexity aversion: $A \succ C \succ D$.

As for B in Wave 2, it only contains one chip of unknown color. Suppose, for instance, that a subject chooses a white chip to introduce into the bag, then the only possible distributions of the chips in the bag are: (1) two white and zero black chips or (2) one white and one black. This makes the evaluation of probabilities over potential outcomes considerably simpler and less complex in Wave 2 than in Wave 1 as there only are two branches here (compared to seven in B in Wave 1). It represents the least complex ambiguous lottery possible. In fact, B can be viewed as a compound lottery with two alternative urn compositions, contrasting with C only in that the likelihood of each composition is unknown. Hence, we expect $A \succ C \succ B$.

The last question is the comparison in terms of complexity between the once compound D and the ambiguous B. They both exhibit the same degree of compoundness, but D can lead to three (as opposed to two) possible urn compositions with known probabilities unlike B, where they have to be estimated. The question is whether more alternatives are viewed as more complex than unknown probabilities. We form no particular hypothesis, but our data in both Waves suggest that unknown probabilities may be viewed as more complex than the number of alternatives.

3.3 Wave 1 vs. Wave 2

As mentioned above, Wave 2 is overall less complex than Wave 1. Furthermore, the differences in complexity across individual lotteries within Wave 2 are smaller than in Wave 1.

Existing models do not provide a unanimous ranking of ambiguity in Wave 2 relative to Wave 1 as each model uses a different argument for why people may exhibit aversion to ambiguity. Ambiguity can be modeled as mean-preserving spreads in the induced distribution of expected utilities in REU of Klibanoff et al. (2005). Hence, lottery B in Wave 2 maximizes their definition of ambiguity. As a result, complexity aversion should play a marginal role in Wave 2, while ambiguity aversion should be more salient under REU. However, mean-preserving spreads are only unambiguously less preferred in REU. In contrast to REU, they can be more preferred in RDU. Other literature is generally mute about how the priors about the ambiguous states will change in such cases. This prevents us from providing a general analysis of the interaction of complexity and ambiguity aversion across the two waves that would hold for all the considered models of ambiguity aversion. These considerations notwithstanding, as

[20] We thank the participants in the DT conference in HCE, Paris, 2009 for suggesting this design.

mentioned above both waves can be compared in terms of their absolute and relative complexity.

The last difference between the two waves allows us to explore whether subjects view “vertical” (more layers, Wave 1) and “horizontal” (more branches, Wave 2) complexity differently.

		Wave 1	Wave 2
Stakes	$H = 50$; $L = 5$	181	227
	$H = 100$; $L = 5$	159	149

Table 1: Treatments and number of observations

Table 1 summarizes the experimental design and the number of observations in each treatment. See the Online Appendix for a more detailed description of the experiment and a visual representation of the lotteries in Wave 1 as presented to the subjects.

4 Results

Tables 2 and 3 summarize the experimental results in Wave 1 and 2, respectively.[21]

4.1 Question 1: Complexity Aversion

First, note from Tables 2 and 3 that in 22 out of 24 cells above the main diagonal the percentages are larger than 50%.[22] That is, most subjects almost always prefer the less complex variation of the lottery in pair-wise comparisons. To test this claim (say) for lotteries A and B, we construct a variable that takes values of $\{1, 0, -1\}$ if $\{A \succ_i B, A \sim_i B, B \succ_i A\}$, respectively, indicating that individual i prefers A to B, is indifferent between A and B, or prefers B to A.

We then use the Wilcoxon signed-rank to test whether the median of this variable is equal to zero, and we reject this hypothesis in all ($p < 0.005$) but three cases. The first exception involves the comparison of B and D in Wave 1 for $H = 100$, where 54.7% prefer the compound complex D to the ambiguous B. This percentage goes in the correct direction, but is not significantly different from 50% ($p = 0.207$). The other two exceptions involve the compound and complex compound lotteries C and D in Wave 2. In both waves, in contrast to the prediction, less than half of the subjects prefer the ambiguous to the complex compound lottery, but both numbers are not statistically different from 50% ($p = 0.352$ and 0.381 for low and high prizes, respectively). Given that it is robust to prize manipulation, it suggests that complexity in branches is not

[21]Note that, in contrast to the experiment, the lotteries are ordered according to their complexity here. Hence, the Ellsberg lottery B appears as last in Tables 2 and 3.

[22]The two exceptions, both in Wave 2, are still close to 50%.

perceived as a sufficient complication to induce a ranking between these two options.

$H = 50$ ($N = 181$)

	A: Risk	C: Once Compound	D: Twice Compound	B: Ellsberg
A: Risk		70.4% (127.5)	72.7% (131.5)	80.7% (146)
C: Once Compound	29.6% (53.5)		61.6% (111.5)	71.3% (129)
D: Twice Compound	27.3% (49.5)	38.4% (69.5)		60.8% (110)
B: Ellsberg	19.3% (35)	28.7% (52)	39.2% (71)	

$H = 100$ ($N = 159$)

	A: Risk	C: Once Compound	D: Twice Compound	B: Ellsberg
A: Risk		71.4% (113.5)	70.8% (112.5)	81.4% (129.5)
C: Once Compound	28.6% (45.5)		64.8% (103)	67.9% (108)
D: Twice Compound	29.2% (46.5)	35.2% (56)		54.7% (87)
B: Ellsberg	18.6% (29.5)	32.1% (51)	45.3% (72)	

Note: If $A \sim B$, then we count 0.5 observations for each, A and B, not to lose these data. Hence, the decimals in paretheses.

Table 2. Lottery pairwise ranking: Wave 1. The percentage (number) in each cell is the percentage (number) of subjects who prefer the row over the column lottery.

In addition and consistent with our model, there always exists an important fraction of individuals that still prefer the more complex–be it compound or ambiguous–variation of each lottery.[23] The exact numbers vary across treatments. In both waves, around 20% prefer the ambiguous to the risky lotteries.

[23]Wilcoxon signed-rank test rejects that the percentages are significantly different from zero in all cases ($p = 0$ in all cases).

4.2 Question 2: Degree of Complexity

The fraction of people preferring the lottery in each row increases with the complexity of the alternative in each column (as we move in the tables in each row from the left to the right). For instance, in Wave 1 the risky lottery A is preferred by 70.4% to the once compound C and by 72.7% to the twice compound D. This fraction increases to 80.7% in case of the ambiguous B. With few exceptions, this is the general picture in Tables 2 and 3.[24]

There is one systematic and one non-systematic exception to the above finding. As for the former, the fraction of population that prefers the risky lottery A to either C or D is always very similar, independently of the treatment variation. At first sight, this finding suggests that people view the two compound variations as similar. Such an explanation would in turn imply that people are on average indifferent between C and D. This is confirmed in Wave 2 but it contrasts with Wave 1, in which 61.6% and 64.8% of subjects prefer C over D. These figures are significantly different from 50% ($p = 0$). This discrepancy between Waves 1 and 2 (together with the unique exception in *Question 1* above) suggests that the additional branch in lottery D in Wave 2 is not a sufficient complication for subjects to perceive D as more complicated than C and rather points to the difference between "horizontal" and "vertical" complexity. Since the non-systematic violation of complexity mentioned above involves C and D (with respect to Ellsberg's B), we also attribute this observation to subjects' perception of "horizontal" complexity.

In sum, the lotteries are generally ranked in the experiment according to their complexity. This contrasts with any theory that assumes that people reduce compound lotteries and are thus indifferent among them and their reduced counterparts, whereas this is in line with predictions of RDU and complexity aversion.

4.3 Questions 3: Prize Multiplication

Previous sections derive several possible predictions due to multiplication of prizes by a factor larger than one. Two of them are derived from the idea of complexity aversion, while the others are predictions of the ambiguity literature. Complexity aversion predicts that, at least in the more complex Wave 1, more people will prefer the complex urns for high stakes. This is not an obvious prediction and no other theory permits such an effect, with the exception of REU of Klibanoff et al. (2005) if most DMs are ambiguity-loving.

[24] Pairwise Wilcoxon signed-rank tests confirm this finding with some exceptions. First, the first percentage in each row is always lower than the third percentage (e.g. A over C vs. A over B in the first row etc.) in both tables ($p < 0.002$ in all cases but Wave 1 $L = 50$ with $p < 0.045$). The comparison between the first and second percentage is almost always significant at 5%, except the A row ($p > 0.46$ in Wave 1 and $p > 0.11$ in Wave 2), the D row in Wave 1 for $H = 100$ ($p = 0.132$), and the C row for $H = 50$ ($p = 0.063$). The last comparison, the second vs. third percentage, is again almost always significant at 5%, the exceptions being row A for $H = 50$ ($p = 0.098$), row C for ($p = 0.065, 0.698$ for $H = 50, 100$, resp.), and row B ($p = 0.99, 0.114$ for low and high stakes, resp.). Note that most of these exceptions are at least marginally ($p \leq 0.1$) or close to marginally significant.

A quick look at the results of the treatment effects in Tables 2 and 3 suggests the following tendency: more people prefer more complex lotteries in Wave 1 with higher stakes, while the contrary occurs in Wave 2 where complexity is minimized. This observation is consistent with complexity aversion and is at odds with theories that predict either no effect on the ranking or fewer people always preferring the ambiguous lottery B after the prize manipulation. The evidence here is weaker in the former case than in the latter though. However, most of the violations of our model again involve the lotteries C or D or both.

The general picture notwithstanding, with one exception, none of the differences across the stake variations in any cell are statistically significant at 5% using the non-parametric Wilcoxon-Mann-Whitney rank sum test. In Wave 1, the fraction of people preferring Ellsberg's lottery B for $H = 100$ increases by 3.4% ($p = 0.504$) in comparison with C and 6.1% in comparison with D ($p = 0.236$). The preference is roughly the same in the risky vs. Ellsberg comparison ($-0.7\%; p = 0.82$). The comparison of the risky and twice compound lotteries goes in the predicted direction ($+1.9\%; p = 0.7$), while the C vs. D comparison does not (-3.2%; $p = 0.5$). The two remaining cases provide roughly the same ranking in both stake variations ($p > 0.82$). The picture is clearer in Wave 2, but still statistically weak: fewer people always prefer the more complex lottery after the prize multiplication. Such a treatment effect is only significant in the comparison of the risky and Ellsberg urn ($p = 0.019$), and very marginally significant in the A vs. C ($p = 0.148$) and B vs. C comparisons ($p = 0.128$). The differences are small in the C vs. D (0.2%) and B vs. D (1%) cases.

To move beyond these pairwise comparisons, we test the overall tendency to prefer more complex lotteries if stakes are high in Wave 1 and the contrary effect in Wave 2. To this aim, we pool the data from all the pairwise lottery comparisons and all the treatments, and regress the choice of the simpler lottery on a dummy for high stakes and Wave 2 and their interaction.[25] This approach allows the testing of whether the treatment effects are significant but also serves to test whether the treatment effect is different across the two waves. This is important because ambiguity aversion predicts that it should not change from one wave to the other, whereas complexity aversion predicts different reactions to the stake manipulations across the two waves (see *Question 3*). We report two regressions in Table S3 in the Online Appendix, one that contains all the pairwise lottery comparisons and one that only considers the comparisons that involve the ambiguous lotteries B.

[25] Since the dependent variable takes values -1, 0, and 1 if, respectively, the individual prefers the more complex lottery, she is indifferent between them, or she prefers the simpler option, we estimate the ordered-logit model. The results are generally robust to alternative specifications of the dependent variable, different estimation techniques, or panel data regressions, in which subjects' different choices play the role of the time-series variable.

$H = 50$ $(N = 227)$

	A: Risk	C: Compound	D: Complex Compound	B: Ellsberg
A: Risk		59.5% (135)	61.7% (140)	79.7% (181)
C: Compound	40.5% (92)		46.9% (106.5)	73.1% (166)
D: Complex Compound	38.3% (87)	53.5% (120.5)		71.6% (162.5)
B: Ellsberg	20.3% (46)	26.9% (61)	28.4% (64.5)	

$H = 100$ $(N = 149)$

	A: Risk	C: Compound	D: Complex Compound	B: Ellsberg
A: Risk		66.1% (98.5)	61.1% (91)	88.6% (132)
C: Compound	33.9% (50.5)		46.6% (69.5)	79.9% (119)
D: Complex Compound	38.9% (58)	53.4% (79.5)		74.5% (111)
B: Ellsberg	11.4% (17)	20.1% (30)	25.5% (38)	

Note: If $A \sim B$, then we count 0.5 observations for each, A and B, not to lose these data. Hence, the decimals in parentheses.

Table 3. Lottery pairwise ranking: Wave 2. The percentage (number) in each cell is the percentage (number) of subjects who prefer the row over the column lottery.

The regressions corroborate that people increase their preference for more complex lotteries after the prize multiplication in Wave 1, whereas the contrary occurs in Wave 2. Once again, this effect is statistically weak in Wave 1 but the overall effect is significant in Wave 2 ($p = 0.035$ and 0.018 if all comparisons or only B-involving comparisons are considered, respectively).

Most importantly, the treatment effects differ across the two waves ($p = 0.064$ and 0.015, respectively). This points to a non-trivial interaction between complexity and ambiguity, contrasting the belief-models of ambiguity aversion and supporting our complexity aversion. The support for our theory is generally stronger if only B-involving comparisons are considered (compared to the regression considering all the comparisons; see Table S3 in Online Appendix C). We suspect that this may be due to the problematic comparisons between C and D.

To conclude, complexity aversion that takes into account the effect of prize manipulation on the distribution of estimated probabilities of lottery prizes is the best predictor of the qualitative shift of preferences observed in the lab. Even though statistically weak, the treatment effects move in the predicted direction. Moreover, we find strong evidence for different impact of our payoff manipulation across the two waves, pointing to an interaction between ambiguity and complexity.[26]

5 Conclusions

We propose a simple alternative (and/or additional) resolution to the Ellsberg paradox. Ours focuses on aversion to complexity, presented here as compoundness, affecting utility but otherwise consistent with EUT, rather than the typical resolutions based on new structures of beliefs. Complexity-averse decision-makers regard complexity as disutility and thus less valued than a similar but less complex lottery. Our simple model allows the existence of *a priori* in Ellsberg's ambiguous urn, making a choice under risk but with compound lotteries. We demonstrate that the Ellsberg paradox can be explained by complexity aversion even if we abstract from ambiguity aversion. Our experimental results are in most respects consistent with complexity aversion and with previous evidence (e.g. Halevy (2007)). In particular, subjects exhibit a preference ranking over lotteries that reflects their degree of complexity and the impact of payoff manipulation seems to interact with the general complexity of our experimental lotteries.

Recognizing possible interactions between ambiguity and complexity, we design our experiment to illustrate that complexity aversion plays a separate role from ambiguity aversion. When we move to an overall more simple scenario (from Wave 1 to Wave 2), we do not see an increase in the number of subjects preferring the risky A over the ambiguous B. When we multiply prizes but keep complexity the same, the observed effects can be explained by complexity aversion theory, whereas ambiguity aversion is mostly silent on that.

[26] In his experiment, Halevy (2007) provides a robustness test of his main treatment, in which he multiplies the stakes by 10. We used his data and found that 22.2% prefer the Ellsberg-like lottery to the risky one for low stakes, but this fraction increases to 25% if stakes are higher. The difference is not statistically significant, though. There are three reasons why this additional finding may support complexity aversion: (i) any increase already contradicts most of the theories of ambiguity aversion, (ii) there are only 38 observations in the high-stake treatment, (iii) multiplying the stakes by 10 should increase subjects' estimation effort considerably and thus work against the prediction of complexity aversion. In contrast, his ten-ball lotteries are more complex than ours. Hence, we consider his data stimulating for further exploration of complexity aversion but prefer to be cautious using this data to make general conclusions concerning our theory.

This evidence points out that the future literature should take into account that people are likely to be averse to complexity *vis-à-vis* utilities and not just to ambiguity due to unknown probabilities, and that complexity aversion (e.g. as modeled here) can affect decisions and behavior in a different way. Complexity aversion may play a complementary role alongside risk and ambiguity aversion. Obviously, complexity (as separate from ambiguity) aversion should be further studied in the laboratory and in the field to understand it better.

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Appendix A

Let there be a strictly increasing (and concave) von Neuman-Morgenstern utility function $u(.)$, with $u(0) = 0$, and assume that subjective probabilities are replaced by the objective ones if the former are known.

A.1 Question 1: Complexity aversion

Most of the models focus on how DMs form their beliefs about the probabilities of individual outcomes. With two outcomes, H and L in our example, all these theories say that $B \succeq A$ by a DM if

$$\gamma u(H) + (1-\gamma)u(L) \geq \tfrac{1}{2}u(H) + \tfrac{1}{2}u(L) \Leftrightarrow [u(H) - u(L)](\gamma - \tfrac{1}{2}) \geq 0. \quad (2)$$

In words, if $\gamma \geq \frac{1}{2}$, DMs prefer B to A. That is, all these models allow people to choose the ambiguous B if γ is high enough. This holds for subjective expected utility (SEU), Choquet expected utility (CEU), maximin expected utility (MMEU), and $\alpha-$maximin model ($\alpha-$MEU).

Under SEU (e.g. Savage, 1957, or Anscombe and Aumann, 1963), DMs choose the complex lottery B over A if their belief about the probability of high prize in B is higher than one half. With two options, the CEU model of Schmeidler (1989) makes the same prediction. The subjective probability is replaced by a "capacity" measure $v(.)$ that does not have to add up to one to reflect the degree of beliefs in the probabilities of the prizes of a lottery. Again, if $v(H)$ is high enough, the subject might prefer lottery B. In MMEU, Gilboa and Schmeidler (1989) propose to replace the capacities by the most pessimistic priors (in terms of expected utilities), while the $\alpha-$MMEU (Ghirardato et al., 2004) argues in favour of a convex combination of the most pessimistic and the most optimistic beliefs. If these priors are biased toward high probabilities of H, B can be preferred to A.

The rank-dependent utility or RDU (Segal, 1987, 1990) incorporates the rank-dependent utility model of Quiggin (1982). In this theory, it is assumed that DMs view ambiguous lotteries as compounded variations of reduced lotteries for which corresponding probabilities can be computed, but they do not reduce them correctly. More precisely, DMs see the Ellsberg urn as a two stage lottery. The first stage corresponds to the probabilities of chosen ball selection given the urn composition, while the second stage refers to the probabilities over the possible states of nature (e.g. the possible 101 compositions of the original Ellsberg urn). Which lottery is then selected depends on the beliefs about the likelihood of the high prize in both the ambiguous and risky lotteries.

To simplify matters, consider a ten-ball variations of the original Ellsberg urns. In RDU, outcomes are ranked and probabilities are weighted by an increasing function $f : [0,1] \to [0,1]$, satisfying $f(0) = 0$ and $f(1) = 1$. Let $\beta_i \in [0,1]$ denote the DM's belief that the urn contains i balls of the color the DM bets on; $i = 0, 1, 2, ..., 10$ and $\sum_{i=0}^{10} \beta_i = 1$. DMs first compute the expected RDU for each possible situation in the second stage and the corresponding certainty equivalents of each alternative are used to compute the final weights in the first-stage lottery. Consequently, the DM prefers the ambiguous to the risky lottery if

$$\gamma u(H) + (1-\gamma)u(L) \geq f(\tfrac{1}{2})u(H) + [1 - f(\tfrac{1}{2})]u(L), \tag{3}$$

where $\gamma = \left\{ \sum_{i=1}^{10} f(\sum_{j=i}^{10} \beta_i) \left[f(\sum_{j=0}^{i} \frac{j}{10}) - f(\sum_{j=0}^{i-1} \frac{j}{10}) \right] \right\}$. We can see that, depending on the priors about the composition of the ambiguous urn and the shape of the weighting function $f(.)$, the RDU DMs will prefer one or the other option. For example, if the DM believes that the ambiguous urn contains 10 red balls with probability one (i.e. $\beta_{10} = 1$ and $\beta_i = 0$ for $i = 0, 1, ..., 9$), the left-hand side of (3) reduces to $u(H)$ and she bets on red and selects the ambiguous urn. If, instead, the DM has a uniform prior about the second-stage lottery, Theorem 4.2 in Segal (1987) establishes several conditions on $f(.)$ such that the DM behaves as ambiguity averse, i.e. she is indifferent between the two colors but selects the risky urn.

As in RDU, Klibanoff et al. (2005) assume that people see ambiguous lotteries as two-stage probability draws and propose a theory labeled as recursive expected utility (REU) by Halevy (2007). The REU from choosing the ten-ball variation of Ellsberg's lottery (labeled B here) has the following form:

$$U_{REU}(B) = \sum_{j=i}^{10} \beta_i \phi \left[\frac{i}{11} u(H) + \frac{11-i}{11} u(L) \right].$$

The von Neuman-Morgenstern $u(.)$ determines the risk aversion of DMs, while the attitudes toward ambiguity are captured by the function ϕ. The choice of a DM between the risky and ambiguous lotteries will be driven by the shape of ϕ: if ϕ is concave (convex) the DM is ambiguity averse (loving).[27] In our simple example with two options, $A \succ B$ if and only if

$$\tfrac{1}{2}u(H) + \tfrac{1}{2}u(L) \geq \beta\phi[u(H)] + (1-\beta)\phi[u(L)]. \tag{4}$$

Hence, people may reject the ambiguous B for two reasons. Either they are ambiguity-averse (reflected by ϕ) and/or their belief about H is low enough.

Last, variational preferences (VP) of Maccheroni et al. (2006) take into account the possibility that DMs are well aware that their beliefs might not be correct. They axiomatize a model such that the expected utility from lotteries and actions with unknown probabilities accounts for such uncertainty, using a function $c(p)$. This function reflects the degree of uncertainty about subjective probabilities described by p and can be viewed as a measure of distance between the priors considered, p, and the best guess of the DM. In risky lotteries, we can set $c(p) = \infty$ for any p different from the objective probabilities and these

[27] There exist a class of models known as source dependent or second-order expected utility which can be thought of as a special case of REU (e.g. Ergin and Gul (2009), Grant et al. (2009)). In this approach, people reduce compound lotteries but view risk and uncertainty differently. For these reasons, we do not discuss this approach in more detail here and focus on REU.

possible priors are never relevant for the decision. Formally,[28]

$$U_{VP}(A) = \tfrac{1}{2}u(H) + \tfrac{1}{2}u(L) \geq \min_{p \in \triangle(S)} [pu(H) + (1-p)u(L) + c(p)] = U_{VP}(B). \quad (5)$$

Hence, people who believe that balls of a certain color are highly present in the ambiguous urn and put high enough confidence in these beliefs might choose this color and select B over the risky A.

A.2 Question 2: Degree of complexity

Since all the one-stage theories of ambiguity aversion assume that people reduce compound lotteries when information is provided objectively, they predict that subjects should be indifferent among the compound variations of the same risky lottery.[29] This also holds for the two-stage REU, since if probabilities are objective $\phi(.)$ does not apply. Nevertheless, we observe a systematic ranking according to the "degree of complexity" in the experiment even if the probabilities are objective.

The only exception is the two-stage RDU, since both the compound and ambiguous options are viewed as compound lotteries independently of whether the probabilities are objective or subjective. In particular, if the conditions on $f(.)$ corresponding to ambiguity aversion hold, people may rank a risky lottery over its once compound payoff-equivalent variation, the latter over the twice compound variation and so on. Let us illustrate this on the lotteries from Wave 1.[30] Denote Wj the bet on white in lottery $j \in \{A, B, C, D\}$. Then,

$$U_{RDU}(WA) = u(L) + [u(H) - u(L)]f(0.5).$$

In the once compound lottery C, a RDU DM first evaluates the second-stage sublotteries $SL^2_{left} = (H, 0.5; L, 0.5)$, and $SL^2_{right} = (H, 0.75; L, 0.25)$ as follows: $U_{RDU}(WSL^2_{left}) = u(L) + [u(H) - u(L)]f(0.5)$ and $U_{RDU}(WSL^2_{left}) = u(L) + [u(H) - u(L)]f(0.75)$.

Since $U_{RDU}(WSL^2_{right}) > U_{RDU}(WSL^2_{left})$,

$$\begin{aligned} U_{RDU}(WC) &= u\left\{u^{-1}\left[U_{RDU}(WSL^2_{left})\right]\right\} + \\ &\quad + \left(u\left\{u^{-1}\left[U_{RDU}(WSL^2_{right})\right]\right\} - u\left\{u^{-1}\left[U_{RDU}(WSL^2_{left})\right]\right\}\right) f(0.5) \\ &= u(L) + [u(H) - u(L)]\, f(0.5)\, [1 + f(0.75) - f(0.5)] \end{aligned}$$

Then, $U_{RDU}(WA) > U_{RDU}(WC)$ if and only if

$$\frac{f(5/8)}{f(0.5)} > 1 + f(0.75) - f(0.5).$$

[28] S can be considered the set of subjective beliefs about the possible states of the world.

[29] In VP, $c(p) = \infty$ for any p different from the objective probabilities in VP. The function $c(.)$ thus plays no role in risky and compound lotteries.

[30] The intuitions extend to Wave 2.

In a similar vein, we can compute $U_{RDU}(WD)$ and show that, under certain conditions on $f(.)$ characterized in Theorem 4.2 in Segal (1987), RDU DM can exhibit the following ranking: $U_{RDU}(WA) > U_{RDU}(WC) > U_{RDU}(WD) > U_{RDU}(WB)$ if she holds, for instance, uniform priors in B. Hence, this is the only model that can generate the same prediction as our complexity aversion, while all other theories of aversion to ambiguity predict indifference between risky and their payoff-equivalent compound variations.

A.3 Question 3: Prize Multiplication and Ambiguity Literature

What do the belief-based explanations of the Ellsberg paradox predict when lottery prizes are multiplied by two? To answer this question, note first that (i) if the probabilities are the same before and after the prize multiplication the priors should not be affected by such payoff manipulation, and (ii) the expression (2) is satisfied if and only if

$$\gamma u(2H) + (1-\gamma)u(2L) \geq \tfrac{1}{2}u(2H) + \tfrac{1}{2}u(2L) \Leftrightarrow [u(2H) - u(2L)](\gamma - \tfrac{1}{2}) \geq 0. \quad (6)$$

That is, as well as in (2), DMs prefer B to A if and only if $\gamma \geq \frac{1}{2}$. As a result, the theories that can be characterized by (2) may differ in the weight γ and, thus, generate different predictions about the number of people preferring one option to the other. However, they all agree that the number of people preferring each option should not change before and after the prize multiplication, since the weights γ from (2) and (6) are the same and compare to $\frac{1}{2}$ in these theories. This is true for SEU, CEU[31], MMEU, and $\alpha-$MEU.

This intuition also extends to RDU. Hence, even though RDU and complexity aversion cannot be separated on basis of Question 2, they generate a different hypothesis regarding the prize multiplication.

The REU model of Klibanoff et al. (2005) provides no general prediction about the effect of prize multiplication. If we focus on ambiguity-averse REU (that is, a concave ϕ), the concavity implies that if (4) holds

$$\tfrac{1}{2}u(2H) + \tfrac{1}{2}u(2L) \geq \beta\phi[u(2H)] + (1-\beta)\phi[u(2L)].$$

In words, ambiguity-averse subjects who choose the risky option for low stakes should also do so in the high-stakes treatment. Only B-choosing ambiguity-averse individuals may switch their choice to A for high stakes. However, the contrary holds for risk seeking individuals. Consequently, the effect depends on the distribution of shapes of ϕ in the population. If we expect the majority of our subjects to be ambiguity-averse as typically observed in experiments, REU would predict more ambiguity aversion-like behavior after the prize manipulation.

One more conservative implication of REU is that we can observe more, fewer or the same number of people preferring the ambiguous lotteries over

[31] Formally, $[u(H) - u(L)](v(H) - \frac{1}{2}) \geq 0$ if and only if $[u(2H) - u(2L)](v(H) - \frac{1}{2}) \geq 0$.

risky ones after the prize multiplication. Apart from aversion to complexity, this is the only theory that permits more people preferring B after the prizes increase. However, this prediction does not depend on any other feature of the experiment; the effect only depends on the distribution of preferences in the population. As a result, REU predicts the same effect (whatever it is), independently of the general complexity of experimental lotteries in each wave. In contrast, complexity aversion predicts an interplay between complexity and ambiguity across the two waves.

Finally, VP of Maccheroni et al. (2006) make the same prediction as REU under convex ϕ. Formally, since $c(.)$ is independent of the stakes, it generates an interplay between stakes and the impact of the confidence in own subjective probabilities and (5) thus implies

$$U_{VP}(A) = \tfrac{1}{2}u(2H)+\tfrac{1}{2}u(2L) \geq \min_{p \in \triangle(S)} [pu(2H) + (1-p)u(2L) + c(p)] = U_{VP}(B).$$

Under VP, given that the function $c(p)$ loses relative importance for higher stakes people that prefer the risky A before the multiplication will also prefer it afterwards. Hence, if someone changes her choice, she switches from the ambiguous B to A, but not vice versa. Consequently, VP predict that unambiguously more people will prefer the risky urn after our prize manipulation.[32]

A.4 Holding L fixed in prize manipulation

In the main exposition of complexity aversion, we multiply both prizes by the same factor, but in the actual experiment we only double H, holding L fixed. Here, we show that all the results derived above extend to such a scenario. The threshold before the multiplication remains the same. If we double H (and hold L), $B \succ A$ if and only if

$$\theta(2H-\delta) + (1-\theta)(L-\delta) > \frac{1}{2}(2H+L) \Leftrightarrow \theta > \widetilde{\theta} = \frac{1}{2} + \frac{\delta}{2H-L}.$$

It is easy to see that $\widetilde{\theta} < \theta^*$. Hence, the threshold also falls in this case.

The contrast of complexity aversion and the ambiguity literature also holds under the current specification. Note that

$$\gamma u(H) + (1-\gamma)u(L) \geq \tfrac{1}{2}u(H) + \tfrac{1}{2}u(L) \Leftrightarrow [u(H)-u(L)](\gamma - \tfrac{1}{2}) \geq 0 \qquad (7)$$

if and only if

$$\gamma u(2H) + (1-\gamma)u(L) \geq \tfrac{1}{2}u(2H) + \tfrac{1}{2}u(L) \Leftrightarrow [u(2H)-u(L)](\gamma - \tfrac{1}{2}) \geq 0. \quad (8)$$

[32] Last, the confidence model of Hill (2011) is directly based on the idea that stakes should interact with how much confidence in their subjective beliefs DMs require in order to bet on options with unknown probabilities. In particular, he argues that the higher the stakes the more confident DMs should be. As a consequence, the model predicts that, *ceteris paribus*, fewer people should choose the ambiguous lotteries if prizes are multiplied.

As a consequence, SEU, CEU, MMEU and $\alpha-$MEU predict no change after such prize manipulation, and similar considerations apply to RDU. The predictions for REU are independent of the stakes in question, while the sensitivity of VP and Hill's (2011) confidence model on stakes remains as long as at least one of the prizes increases.

In sum, all the predictions stated in the main text extend to the context presented to experimental subjects.

Appendix B: Experimental Instructions (Wave 1, low stakes)

Experiment of Lottery Preferences

Welcome to our study. This experiment is being conducted as a part of a research project on people's preference. The whole session will last approximately 15 minutes.

Instructions:

1. After we read the instructions and you understand the task, if you are willing to participate, please print your name on the last page of this questionnaire (consent form), sign and date it.
2. Read the descriptions of four lotteries in the following pages. Try to understand how they operate and rank/order them according to your preference. Put down your preferences where asked in the questionnaire.
3. Hand in the questionnaire.
4. Wait for our announcement of those who are invited to participate in our real lotteries.

Rewards:

After we have collected all the questionnaires, we will randomly pick several out them (roughly 1/10 of the class size). Those whose questionnaires are drawn will be invited to try their luck in an actual realization of the lotteries we have devised for this study which are explained below.

We will pick two of the four lotteries and run the lottery, which you prefer according to your answers in this questionnaire. You will be able to observe the result instantly. You can then collect the prize money after just a little paper work.

If you are one of the lucky ones, but prefer to do the actual lottery privately; please let us know and we can set up an appointment.

Caution:

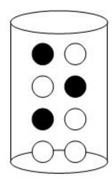
This is a serious experiment. Please avoid discussing, looking at others' questionnaire or exclaiming. Should you have any questions please raise your hand and we will be happy to assist you.

The lotteries:

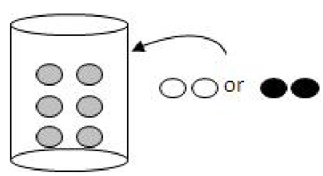
We have created four lotteries labeled A, B, C and D. Please follow our description of each of them below and pay attention to our explanations. If you have any questions about any of them, please do not hesitate to ask us.

All the information provided here is accurate to our knowledge and is intended to help you understand the mechanism of the lotteries. Any student who is invited to participate in our actual realization of the lotteries is welcome to verify that the provided information is true.

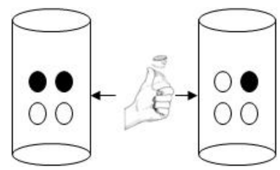
Lottery A: There is one non-transparent brown bag, with 8 chips in it. 5 of these chips are white and the other 3 are black, as in figure A. You are asked to announce a color, white or black, at your own choice. Then a chip is drawn from the bag. You will be awarded \$50 if the color of the chip matches the color of your choice, and \$5 otherwise.



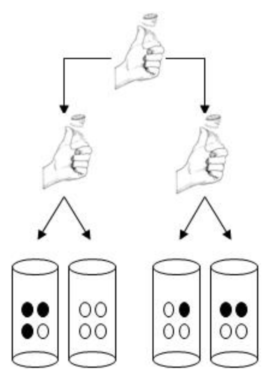
Lottery B: There is one non-transparent brown bag with 6 chips in it, as in figure B. Each chip is either white or black, but you do not know the exact number of chips of either color. You are asked to announce a color, white or black, at your own choice. 2 chips of that color are put in to the brown bag to make it 8 chips in total. Then a chip is drawn from the brown bag. You will be awarded \$50 if the color of the chip matches the color of your choice, and \$5 otherwise.



Lottery C: There are two non-transparent brown bags each with 4 chips in it, as in figure C. The first bag contains 2 white chips and 2 black chips. The second bag contains 3 white chips and 1 black chip. You are asked to announce a color, white or black, at your own choice. After that, we flip a fair coin. If it is heads we use the first bag, if tails, we use the second bag. Therefore, each bag has equal chance to be selected. Then a chip is drawn from the chosen bag. You will be awarded \$50 if the color of the chip matches the color of your choice, and \$5 otherwise.



Lottery D: This is illustrated in Figure D. Like the previous lotteries, you are asked to announce a color, black or white, at your choice. Then we flip a fair coin. If it is heads then we proceed with the left branch. We flip a fair coin one more time. If it is heads, we use a brown bag with 1 white chip and 3 black ones. If it is tails, we use a bag with 4 chips all white. If it is tails we proceed with the right branch. We flip a fair coin one more time. If it is heads, we use a brown bag with 3 white chips and 1 black one. If it is tails, we use a bag with 2 white chips and 2 black ones. After a bag is chosen this way, a chip is drawn out of the bag. You will be awarded $50 if the color of the chip matches the color of your choice, and $5 otherwise.



Lottery preferences:

If you are sure about how these lotteries operate, please take a moment to evaluate them and then rank them according to your preference. Suppose you are given a chance to enter for one of the lotteries, which one would you like most? If that one is not available, which will come in as second choice and so forth?

Please note that your answer will help decide which lottery you will get if you are chosen to take part in a real one, and hence it may affect your potential payoff. So, please take your time, try to understand how each lottery works before you put down your preference.

If you have made your choices, please print the letters associated with each lottery (A, B, C or D) in one of the boxes in your questionnaire: most preferred (rank 1); somewhat preferred (rank 2); less preferred (rank 3) or least preferred (rank 4).

Please put only one letter in each box. If you feel you cannot decide between two or more lotteries, you may put them in adjacent boxes and put an equal sign “=” in between. However, if you are to be chosen to play in our realized lottery and the two lotteries available deemed “equal” by you, we will pick one for you to participate.

Most Preferred (Rank 1)	Somewhat Preferred (Rank 2)	Less Preferred (Rank 3)	Least Preferred (Rank 4)

Thanks for your participation. Please do not forget to sign the next page.

Appendix C: Additional Tables and Regressions

Pref.	Freq. $H = 50$	Freq. $H = 100$	Pref.	Freq. $H = 50$	Freq. $H = 100$
$A \succ C \succ D \succ B$	31	25	$C \succ A \succ B \succ D$	8	6
$A \succ C \succ B \succ D$	19	19	$C \succ A \succ D \succ B$	8	9
$A \succ B \succ C \succ D$	10	18	$C \succ B \succ A \succ D$	1	1
$A \succ D \succ C \succ B$	25	11	$C \succ D \succ A \succ B$	9	4
$A \succ D \succ B \succ C$	4	2	$D \succ A \succ B \succ C$	4	2
$A \succ B \succ D \succ C$	3	3	$D \succ A \succ C \succ B$	8	14
$B \succ A \succ C \succ D$	10	8	$D \succ B \succ A \succ C$	1	1
$B \succ \{A, C, D\}$	9	8	$D \succ C \succ A \succ B$	9	10
$\{B, C, D\} \succ A$	6	5	$\{A, B, C, D\}$	16	13
Total	$H = 50$: **181** obs. & $H = 100$: **159** obs.				

Note: A≻B = A preferred to B; {A,B} = A indifferent to B.

Table S1. Preference ranking in Wave 1.

Pref.	Freq. $H=50$	$H=100$	Pref.	Freq. $H=50$	$H=100$
$A \succ B \succ C \succ D$	5	5	$B \succ D \succ C \succ A$	5	-
$A \succ B \succ D \succ C$	-	2	$B \succ A \succ \{C,D\}$	1	1
$A \succ C \succ B \succ D$	10	6	$B \succ \{A,C,D\}$	1	1
$A \succ C \succ D \succ B$	15	8	$\{B,D\} \succ \{A,C\}$	-	1
$A \succ D \succ B \succ C$	6	5	$C \succ A \succ B \succ D$	3	3
$A \succ D \succ C \succ B$	19	19	$C \succ A \succ D \succ B$	15	10
$\{A,B\} \succ C \succ D$	1	2	$C \succ B \succ A \succ D$	4	-
$\{A,B\} \succ D \succ C$	1	1	$C \succ B \succ D \succ A$	-	3
$\{A,B\} \succ \{C,D\}$	2	-	$C \succ D \succ A \succ B$	10	11
$\{A,C\} \succ B \succ D$	3	2	$\{C,D\} \succ A \succ B$	5	2
$\{A,C\} \succ D \succ B$	4	-	$\{C,D\} \succ B \succ A$	2	-
$\{A,D\} \succ B \succ C$	2	-	$C \succ \{A,B\} \succ D$	1	1
$\{A,D\} \succ C \succ B$	6	10	$C \succ \{A,D\} \succ B$	2	2
$A \succ \{B,C\} \succ D$	1	-	$C \succ \{B,D\} \succ A$	-	1
$A \succ \{B,D\} \succ C$	2	1	$C \succ A \succ \{B,D\}$	1	2
$A \succ \{C,D\} \succ B$	5	7	$C \succ D \succ \{A,B\}$	3	-
$A \succ B \succ \{C,D\}$	-	1	$D \succ A \succ B \succ C$	5	2
$A \succ C \succ \{B,D\}$	-	3	$D \succ A \succ C \succ B$	15	12
$A \succ D \succ \{B,C\}$	5	-	$D \succ B \succ A \succ C$	-	1
$\{A,B,D\} \succ C$	3	-	$D \succ C \succ A \succ B$	18	8
$\{A,C,D\} \succ B$	13	1	$D \succ C \succ B \succ A$	3	1
$\{A,B,C,D\}$	3	4	$D \succ \{A,C\} \succ B$	3	4
$B \succ A \succ C \succ D$	8	2	$D \succ A \succ \{B,C\}$	-	2
$B \succ A \succ D \succ C$	8	1	$D \succ C \succ \{A,B\}$	1	-
$B \succ C \succ A \succ D$	5	-	$D \succ \{A,B,C\}$	1	-
$B \succ D \succ A \succ C$	1	1	**Total**	**227**	**149**

Note: A$\succ$B = A preferred to B; {A,B} = A indifferent to B.

Table S2. Preference ranking in Wave 2.

Dependent variable: Choice of the simpler lottery (-1,0,1)		
	All comparisons	B-involving comparisons
High stake	-.042	-.124
	(.087)	(.126)
Wave 2	-.156*	.254**
	(.081)	(.122)
High stake × Wave 2	.232*	.464**
	(.125)	(.191)
Obs.	4295	2148
Pseudo R^2	0.001	0.010
p-value (model)	0.123	0
Effect stake in W2 (p)	.035**	.018**

St. errors in parentheses; **,* significant at 5%, 10%, resp.

Table S3. Ordered logit regressions of choice, pooled data.

Appdendix D: Wave 2 Instructions

Experiment of Lottery Preferences

Welcome to our study. This experiment conducted as a part of a research project on people's preference. The whole session will last approximately 15 minutes.

Instructions:

1. After we read the instructions and you understand the task, if you are willing to participate, please print you name on the last page of this questionnaire (consent form), sign and date it.
2. Read the descriptions of four lotteries in the following pages. Try to understand how they operate and rank order them according to your preference. Put down your preferences where asked in the questionnaire.
4. Hand in the questionnaire.
5. Wait for our announcement of those who are invited to participate in our real lotteries.

Rewards:

After we have collected all the questionnaires, we will randomly pick several out them (roughly 1/10 of the class size). Those whose questionnaires are drawn will be invited to try their luck in an actual realization of the lotteries we have devised for this study which are explain below.

We will pick two of the four lotteries and run the lottery, which you prefer according to your answers in this questionnaire. You will be able observe the result instantly. You can then collect the prize money after just a little paper work.

If you are one of the lucky ones, but prefer to do the actual lottery privately; please let us know and we can set up an appointment.

Caution:

This is a serious experiment. Please avoid discussing, looking at others' questionnaire or exclaiming. Should you have any questions please raise your hand and we will be happy to assist you.

The lotteries:

We have created four lotteries labeled A, B, C and D. Please follow our description of each of them below and pay attention to our explanations. If you have any questions about any of them, please do not hesitate to ask us.

All the information provided here is accurate to our knowledge and is intended to help you understand the mechanism of the lotteries. Any student who is invited to participate in our actual realization of the lotteries is welcome to verify that the provided information is true.

Lottery A: There is one non-transparent brownbag, with four (4) chips in it. Three (3) of these chips are white and the other one (1) is black, as in figure A. You are asked to announce a color, white or black, at your own choice. Then one (1) chip is drawn from the bag. You will be awarded $100 if the color of the chip matches the color of your choice, and $5 otherwise.

Figure A

Lottery B: There is one non-transparent brown bag with only one (1) chip in it, as in figure B. The chip is either white or black. You are asked to announce a color, white or black, at your own choice. One (1) chip of that color is put in to the brownbag to make it two (2) chips in total. Then one (1) chip is drawn from the brownbag. You will be awarded $100 if the color of the chip matches the color of your choice, and $5 otherwise.

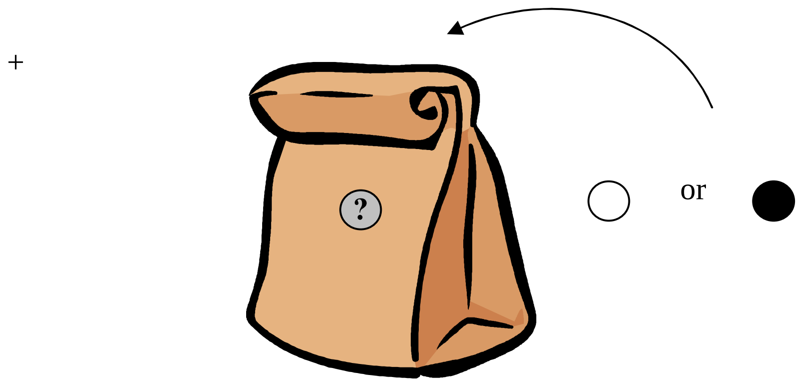


Figure B

Lottery C: There are two (2) non-transparent brown bags each with four (4) chips in it, as in figure C. The first bag contains one (1) white chip and three (3) black chips. The second bag contains four (4) white chips. You are asked to announce a color, white or black, at your own choice. After your announcement, we will roll a fair die in front of you. If it ends up with 1 or 2 on the top, we will use the first bag; otherwise, we use the second bag. Then one (1) chip is drawn from the chosen bag. You will be awarded $100 if the color of the chip matches the color of your choice, and $5 otherwise.

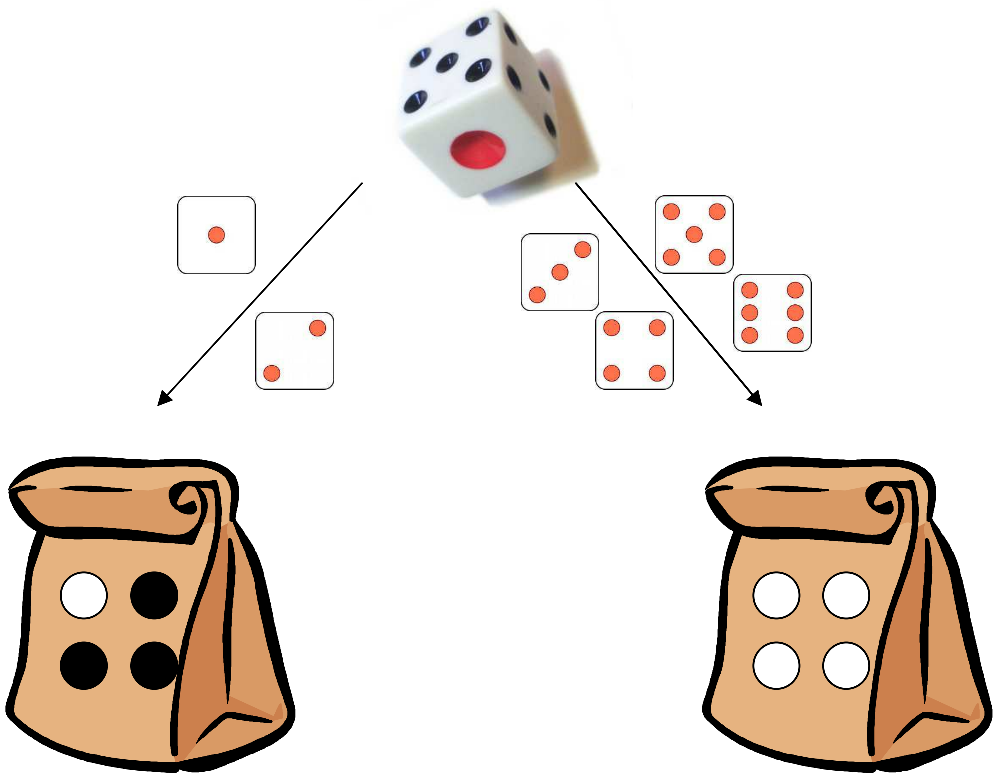


Figure C

Lottery D: There are three (3) non-transparent brown bags each with four (4) chips in it, as in figure D. The first bag contains two (2) white chip and two (2) black chips. The second bag contains three (3) white chips and one (1) black chip. The third bag contains four (4) white chips. You are asked to announce a color, white or black, at your own choice. After your announcement, we will roll a fair die in front of you. If it ends up with 1 or 2 on the top, we will use the first bag; 3 or 4, the second bag; 5 or 6, the third bag. Then one (1) chip is drawn from the chosen bag. You will be awarded $100 if the color of the chip matches the color of your choice, and $5 otherwise.

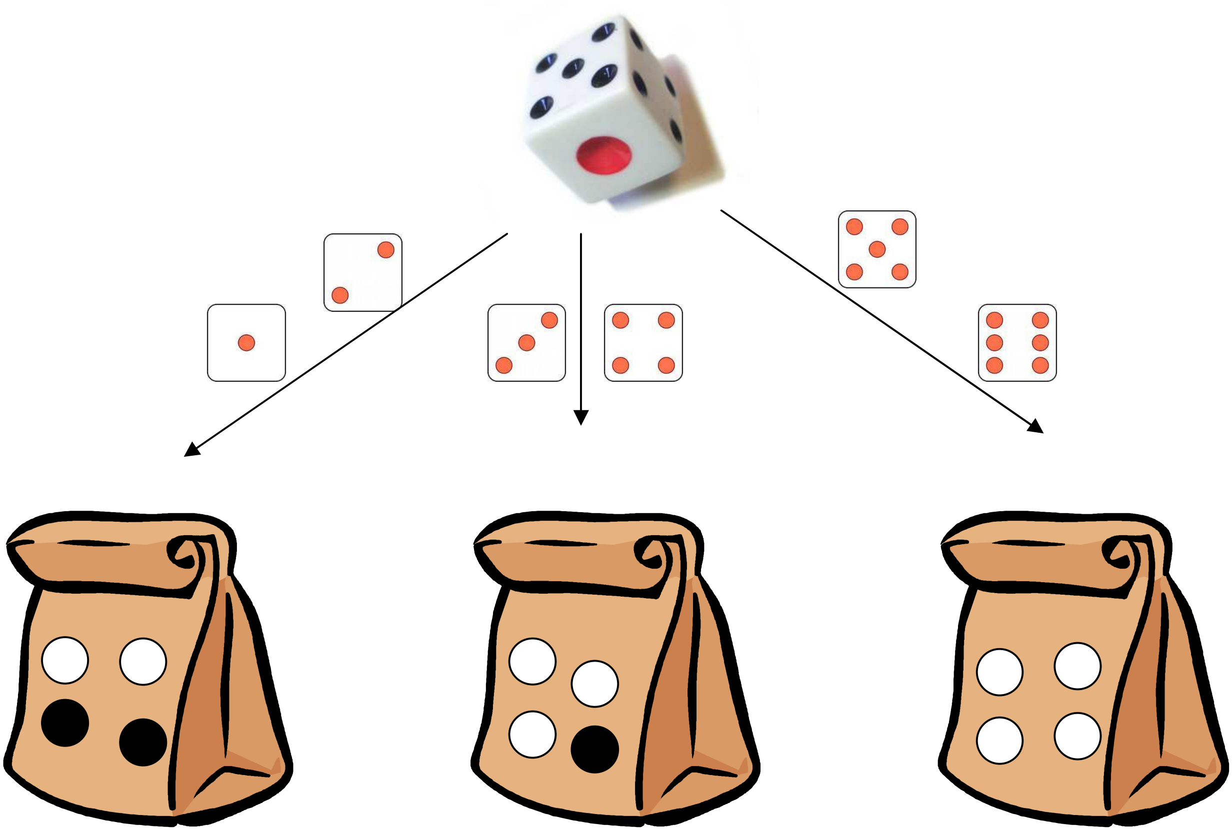


Figure D

Lottery preferences:

If you are sure about how these lotteries operate, please take a moment to evaluate them and then rank them according to your preference. Suppose you are given a chance to enter for one of the lotteries, which one would you like most? If that one is not available, which will come in as second choice and so forth?

Please note that your answer will help decide which lottery you will get if you are chosen to take part in a real one, and hence it may affect your potential payoff. So, please take your time, try to understand how each lottery works before you put down your preference.

If you have made your choices, please print the letters associated with each lottery (A, B, C or D) in one of the boxes in your questionnaire: most preferred (rank 1); somewhat preferred (rank 2); less preferred (rank 3) or least preferred (rank 4).

Please put only one letter in each box. If you feel you cannot decide between two or more lotteries, you may put them in adjacent boxes and put an equal sign "=" in between. However, if you are to be chosen to play in our realized lottery and the two lotteries available deemed "equal" by you, we will pick one for you to participate.

Most Preferred (Rank 1)	Somewhat Preferred (Rank 2)	Less Preferred (Rank 3)	Least Preferred (Rank 4)

Thanks for your participation. Please do not forget to sign the next page.