

1992

## Comparison of Two Models for Breakdown Waves

Debra Burris

*Arkansas Tech University*

Mostafa Hemmati

*Arkansas Tech University*

Follow this and additional works at: <http://scholarworks.uark.edu/jaas>

 Part of the [Optics Commons](#)

---

### Recommended Citation

Burris, Debra and Hemmati, Mostafa (1992) "Comparison of Two Models for Breakdown Waves," *Journal of the Arkansas Academy of Science*: Vol. 46 , Article 38.

Available at: <http://scholarworks.uark.edu/jaas/vol46/iss1/38>

This article is available for use under the Creative Commons license: Attribution-NoDerivatives 4.0 International (CC BY-ND 4.0). Users are able to read, download, copy, print, distribute, search, link to the full texts of these articles, or use them for any other lawful purpose, without asking prior permission from the publisher or the author.

This Article is brought to you for free and open access by ScholarWorks@UARK. It has been accepted for inclusion in Journal of the Arkansas Academy of Science by an authorized editor of ScholarWorks@UARK. For more information, please contact [scholar@uark.edu](mailto:scholar@uark.edu).

# COMPARISON OF TWO MODELS FOR BREAKDOWN WAVES

DEBRA BURRIS and MOSTAFA HEMMATI

Physics Department  
Arkansas Tech University  
Russellville, AR 72801

## ABSTRACT

In this paper, the two theories concerning the propagation of breakdown waves are compared. The two theories are as follows:

1. The photoionization theory, in which the driving force of the propagation is the electromagnetic radiation from the hot gas generated at the electrode with the greatest potential gradient.
2. The electron fluid dynamical theory, in which the driving force of the propagation is the partial pressure of the high temperature electron gas generated in the neighborhood of the pulsed electrode. Successes in explaining the experimental data will be compared.

## INTRODUCTION

Lightning, one of nature's most awesome phenomena, has intrigued mankind for centuries. The scientific community has devoted years of research in an attempt to duplicate the lightning stroke in the laboratory and study its nature. As early as the 1700's, physicists were observing discharges similar to lightning. Hauksbee (1706/1707) saw flashes of light coming from the evacuated tube over the mercury column of a barometer when it was vibrated. Thompson (1893) began using a 15 meter long discharge tube to study ionizing waves. He was able to measure the speed of waves traveling at one half the speed of light. Beams (1930) using a Kerr cell, observed that the fast light pulse began at the high potential electrode. Technology improved after the second World War, and many new studies were performed. The 1960's mark the introduction of sound theoretical advances in the study of breakdown waves. This paper will introduce and compare the two theories (the photoionization theory and the fluid-dynamical theory) regarding the driving force of the propagation of the breakdown waves.

The photoionization theory assumes that the ionizing radiation from the hot gas formed at the electrode with the greatest potential gradient is the primary driving mechanism for the front that moves out from the hot gas. The fluid-dynamical model treats the waves as a three fluid model consisting of electrons, positive ions, and neutral particles. The primary driving mechanism in this treatment is the partial pressure of the high-temperature electron gas behind the shock front. From the beginning of the 1940's until the mid 1970's the photoionization theory received much attention in both the experimental and theoretical fields. Although this theory has been investigated fully, mathematical formulations with solutions in good agreement with the experimental results have not been achieved. The mathematical formulation of the fluid-dynamical model has shown great success in explaining the experimental data collected up to now. Hence, most of the recent works have centered around the fluid-dynamical model.

## THEORIES

Snoddy *et al.* (1937) varied different experimental parameters to study their effects on wave speeds. They found that their computed speeds were approximately 40% greater than the average speed measured over a given distance. Their work also showed an apparent increase in the speed as the negative input voltage wave traveled down the tube, while the positive input voltage wave decreased in speed. They said the speed was greatly increased by the ionization in the gas ahead of the front. Although they could not determine the exact mechanism of discharge, they speculated that it depended on the transfer of potential down the tube by ionization processes of the Townsend type.

The photoionization model consists of the following several points. The breakdown initiates at the electrode with the largest potential gradient. There, a localized region of hot ionized gas is formed. Ionizing radiation from this hot gas is thought to be the primary driving mechanism for the ionization front that moves out from the hot gas. Photons from excited atoms propagate through the neutral gas ionizing and exciting atoms in front of the wave. In turn, these newly excited atoms emit photons which carry on the process. The ionization front consists of a thin photo-absorbing region between the ionized gas behind the front and the neutral gas ahead of the front. The velocity of the front is determined by the intensity of the ionizing radiation. A single-fluid model is used since there is assumed to be no electrical current. The final form of the set of equations derived by Nelson (1964) using the photoionization theory and the analysis of his derived equations are being discussed fully in his paper.

Paxton and Fowler (1962) used the electron fluid-dynamical approach and obtained good agreement with experimental data from several different experiments. Shelton and Fowler (1968) began working with the fluid model using only one-dimensional calculations. They proposed that the one-dimensional model would be valid due to the cylindrical geometry and symmetry. In other words, if the direction of the propagation of the wave is considered to be along the x-axis, then the structure of the wave in the y and z directions remains constant and calculations need only be applied to the direction of propagation. When Shelton and Fowler (1968) applied their equations to the data of Snoddy *et al.* (1937), they found that the expected value for the acceleration was 29% instead of the 40% previously reported. They speculated that most of the acceleration of waves could be attributed to the increase in the electric field during the propagation down the tube.

Haberstich (1964) studied waves produced by impulse potentials in an un-ionized gas. He derived a one-dimensional theory for the propagation of the front. Haberstich (1964) assumed that the propagation required only one electron ahead of the front. However, his results are questionable on several counts. He never measured electron temperature. Velocities of the waves were determined by observing only a single event. Also the purity of the gas samples used was uncertain due to the type of pump used to evacuate the system.

In the electron fluid-dynamical model, a small quantity of gas near the electrode with highest potential gradient is ionized and the electrons that are produced are given kinetic energy by the electric field. This high-temperature electron gas rapidly expands, producing a shock wave of electrons which partially ionizes the neutral molecules in the ambient gas. The shock waves are of steady profile, which means if an observer were to view the wave in a reference frame traveling with the wave, there would be no time variation of the structure of the wave.

Shelton and Fowler (1968) proposed one of the most satisfying theories to date. They used a three fluid hydrodynamical model to analyze the case of a one dimensional wave traveling in the direction that an electron would be accelerated by the applied electric field. In this work, Shelton and Fowler (1968) also introduced the concept of the proforce and antiproforce waves. Proforce waves are waves for which the external electric field accelerates the electrons in the direction of wave propagation. The antiproforce waves are waves where the electric field force acting on the electrons is in the opposite direction of the wave propagation.

By introducing the dimensionless variables  $\theta$ ,  $v$ ,  $\psi$ ,  $\eta$ , and  $\xi$  as electron temperature  $T_e$ , electron concentration  $n$ , electron velocity  $v$ , electric field  $E$ , and position in the wave profile respectively, the set of equations, the equation of conservation of mass, the equation of conservation of momentum, and the equation of conservation of energy, including the Poisson's equation respectively become

$$\frac{d(v\psi)}{d\xi} = \kappa\mu v, \quad (1)$$

$$\frac{d}{d\xi} [v\psi(\psi - 1) + v\alpha\theta] = -v\eta - \kappa v(\psi - 1), \quad (2)$$

$$\frac{d}{d\xi} [v\psi(\psi^2 - 1) + 5v\psi\alpha\theta + v\psi\alpha] = -2v\psi\eta - 2\kappa v(\psi - 1), \quad (3)$$

$$\frac{d\eta}{d\xi} = \frac{v}{\alpha}(\psi - 1). \quad (4)$$

In the above equations  $\mu$  and  $\kappa$  are the ionization rate, and the elastic collision frequency respectively.  $\alpha = \frac{2e\phi}{mV^2}$  where  $\phi$  is the ionization potential of the gas.

Nelson (1964) criticized the Paxton and Fowler's (1962) fluid-dynamical model. He said that they failed to prove the validity of their zero-current assumption in the wave, because according to his calculations although they had no current flow in the wave frame, they still had current flow in the lab frame. However, if one begins with Poisson's equation

$$\frac{dE}{dx} = \frac{e}{\epsilon_0}(N_i - n), \quad (5)$$

and the equations for the production of ions and electrons

$$\frac{\partial N_i}{\partial t} + \frac{\partial(N_i V)}{\partial x} = \beta n, \quad (6)$$

$$\frac{\partial n}{\partial t} + \frac{\partial(nv)}{\partial x} = \beta n, \quad (7)$$

the above mentioned statement by Nelson (1964) can be proven invalid. In the above equations  $N_i$ ,  $V$ , and  $\beta$ , are ion density, wave velocity, and ionization frequency respectively. By subtracting equation (7) from equation (6), then multiplying both sides of the resulting equation by electron charge  $e$ , one obtains the equation

$$\frac{\partial}{\partial t} [e(N_i - n)] + \frac{\partial}{\partial x} [e(N_i V - nv)] = 0. \quad (8)$$

Then, by applying Poisson's equation, this equation becomes

$$\frac{\partial}{\partial x} [\epsilon_0 \frac{\partial E}{\partial t} + e(N_i V - nv)] = 0 \quad (9)$$

Integration of this equation gives the following equation

$$\epsilon_0 \frac{\partial E}{\partial t} + e(N_i V - nv) = i_0(t), \quad (10)$$

where  $i_0(t)$  is the current ahead of the wave. This equation shows that the total current, convection plus displacement, is independent of position. The electric field ahead of the wave  $E_0$  had been specified as a constant and the wave is said to propagate into neutral, un-ionized gas, so the right-hand side of the above equation is zero, thus satisfying the zero current condition

$$N_i V - nv = 0. \quad (11)$$

This condition holds true for any one-dimensional frame of reference. This disproves the cornerstone in Nelson's criticism of the electron fluid-dynamical model proposed by Paxton and Fowler (1962).

Using a revision of the breakdown apparatus used by Haberstich (1964), Blais and Fowler (1973) investigated Shelton's one dimensional fluid dynamical theory. They confirmed the relationship between wave speed and applied electric field as proposed by Shelton, but found the pressure dependence to be more complicated than previously believed. Blais and Fowler (1973) also established an exponential decrement rule for wave speed as a function of distance down the tube.

Finally Fowler et al. (1984) published a paper dealing with the exact numerical solutions of the set of equations pertaining to the electric breakdown waves. Their studies centered around the approximations which the equation set used by previous physicists was derived and solved. They investigated the addition of new terms to the equation of conservation of energy to try to improve agreement with experimental results. They studied the newly added terms effects on the final outcome of the integration of the set of equations in the shock region. Their most important discovery was the significant relevance of the heat conduction

term -  $\frac{5\alpha^2 v \theta}{\kappa} \frac{d\theta}{d\xi}$  and the acceptance of the temperature derivative

discontinuity at the shock front. Two other terms were found to be relevant for the integration of equations on meeting the boundary conditions at the end of the shock region. These two terms are due to the energy loss by the electrons to the heavy particles in elastic collisions and have to be added to the right hand side of the energy equation

$[-\frac{6m\kappa v \alpha \theta}{M} \text{ and } -\frac{2m\kappa v}{M}(\psi - 1)]$ .  $m$  and  $M$  are the electron and heavy

particle mass's respectively. The boundary conditions at the end of the shock region are: 1) the electrons have to come to rest relative to the ions and neutral particles, and 2) the electric field had to reduce to zero.

## CONCLUSION

Hemmati and Fowler (1985) were able to apply their modified equations to different classes of waves and found that the solutions were in good agreement with the experimental results obtained by Blais and Fowler (1973). This helped show the fluid model's application to be quite successful in theoretical explanation of the breakdown waves. In recent years, the fluid model has found more acceptance in the scientific community as opposed to the photoionization model which fails to receive much consideration due to its inability to explain a wide range of experimental results.

## LITERATURE CITED

- BEAMS, J.W. 1930. The propagation of luminosity in discharge tubes. *Phys. Rev.* 28:475-477.
- BLAIS, R., and R.G. FOWLER. 1973. Electron wave breakdown of helium. *Phys. of Fluids* 16:2149-2154.
- FOWLER, R.G., M. HEMMATI, R.P. SCOTT, and S. PARSENAJADH. 1984. Electric breakdown waves: Exact numerical solutions. *Phys. of Fluids* 27:1521-1525.
- HABERSTICH, A. 1964. Experimental and theoretical study of an ionizing potential wave in a discharge tube. Ph.D. thesis, University of Maryland.

### Comparison of Two Models for Breakdown Waves

HAUKSBEE, F. 1706/1707. An account of an experiment, confirming one lately made, touching the production, by the effluvia of one glass falling on another in motion. *Phil. Trans. Roy. Soc.* 25:2413.

HEMMATI, M. and R.G. FOWLER. 1985. Electric breakdown waves: Exact solutions. Part II. *Phys. of Fluids* 28:2744-2750.

NELSON, R.A. 1964. Theories of fast luminous front. *Phys. of Fluids* 8:23-25.

PAXTON, G.W. and R.G. FOWLER. 1962. Theory of breakdown wave propagation. *Phys. R.ev.* 128:993-997.

SHELTON, G.A. and R.G. FOWLER. 1968. Nature of electron fluid-dynamical waves. *Phys. of Fluids* 11:740-746.

SNODDY, L.B., J.R. DEITRICH, and J.W. BEAMS. 1937. Propagation of potential in discharge tubes. *Phys. Rev.* 52:739-746.

THOMPSON, J.J. 1893. *Recent researches in electricity and magnetism.* Oxford University Press, 115 pp.