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On the Accuracy of Representing Heartbeats with Hermite Basis Functions

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Resumen

Desde que en 1959 Pipberger digitalizó por primera vez el electrocardiograma (ECG) y diseñó los primeros programas de ordenador para su análisis, la atención que ha despertado desde múltiples disciplinas científicas ha sido extraordinaria. El ECG pronto se convirtió en una prueba sencilla y de bajo coste recomendada para el estudio de cardiopatías, acaparando un gran interés debido a la mortalidad de las enfermedades cardiovasculares, que se han situado como la primera causa de muerte por enfermedad en el mundo.

Pero más allá de servir de instrumento al servicio del estudio de la patología cardiaca, el ECG se ha mostrado como una fuente todavía inagotable de investigación médica al poner de manifiesto la compleja interacción entre distintos procesos fisiológicos que concurren en las alteraciones del impulso eléctrico en el miocardio. En este sentido, la aparición de nuevas aplicaciones médicas del análisis del ECG es continua: encontramos algunos ejemplos en la estimación de la salud fetal en obstetricia, el seguimiento de pacientes crónicos como en el caso de la diabetes , la enfermedad pulmonar obstructiva crónica, o la apnea-hipopnea del sueño, entre otras, o incluso en el diseño de nuevos fármacos.

El análisis automático de ECG requiere en primer lugar la elección de una forma de representación del latido cardiaco. Una de las opciones más habituales es utilizar una base de funciones, expresando cada latido como una combinación lineal de estas funciones. Los coeficientes de la combinación lineal son utilizados para representar el latido, consiguiendo una representación muy compacta. Una de las base de funciones más utilizada por su calidad en la representación es la compuesta por los polinomios de Hermite. La cantidad de polinomios utilizados para representar cada latido cambia bastante entre los distintos autores, algunos utilizan tan solo 3 polinomios por latido mientras que otros llegan a utilizar hasta 20. Usualmente los autores justifican poco o nada la elección del número de polinomios.

Este artículo pretende analizar el impacto de elegir un cierto número de polinomios de Hermite en la exactitud de la representación del latido. Para ello se ejecutó un cojunto de tests sobre la base de datos MIT-BIH Arrhythmia Database variando el número de polinomios utilizados entre 2 y 20. Se utilizaron tres diferentes estrategias para determinar la posición del latido y se aportan los datos de error para cada uno de los test. Basándose en los resultados obtenidos se proporcionan ciertas indicaciones acerca de cómo elegir un número de polinomios adecuado para representar el latido según la aplicación.

On the Accuracy of Representing Heartbeats with Hermite Basis Functions
Extended Version

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Abstract: Automatic ECG analysis requires choosing a representation for heartbeats. A common approach is using some basis of functions to represent the heartbeat as a linear combination of these functions. The coefficients of the linear combination are used as the features that represent the heartbeat, providing a very compact representation. The most used basis of functions is the one made up of the Hermite functions. Some authors have used as few as 3 Hermite polynomials to represent each heartbeat, while others have used as many as 20. Often little or no justification for the choice of the number of polynomials is given. This paper aims to analyze the impact of using a certain number Hermite polynomials on the accuracy of heartbeat representation. Tests were run fitting the heartbeats of the MIT-BIH arrhythmia database with a number of polynomials ranging from 2 to 20. Three different strategies to determine the heartbeat's position were used. The fitting errors are reported here. Based on these results, some guidelines to choose a suitable number of Hermite polynomials for different applications are given.

1 INTRODUCTION

The electrocardiogram is a simple and inexpensive test for the diagnosis of multiple cardiovascular diseases. Its main disadvantage is probably the large amount of information that it generates; e.g., a 24-hour Holter recording can contain up to 100,000 heartbeats. Thus, visual inspection of the recording can be a tedious and time-consuming task. This is the reason why the biomedical engineering community has attempted to provide tools for the automatic analysis of ECG recordings.

Automatic ECG analysis starts with the detection and characterization of heartbeats. Errors in this task can invalidate the rest of the analysis. The first step in the characterization is to choose the features that will represent the heartbeat. Then these features are usually fed to an automatic classifier capable of recognizing the different morphological families of beats (Braccini and Edenbrandt, 1997) (de Chazal et al., 2004) (de Chazal and Reilly, 2006) (Osowski and Stodolski, 2003) (Park et al., 2008) .

In the literature there are three main approaches to represent beats: using the digitized signal (Hu et al., 1993), extracting heartbeat interval features (de Chazal and Reilly, 2006) and using some basis of functions (Jane et al., 1993) . Using the digitized signal prevents any loss of information, but this representation is difficult to work with due to its large size, and it is very sensitive to noise. Using heartbeat interval features, such as QRS height and width, QT segment, etc., is the closest representation to the clinicians' modus operandi when they interpret beats. However, good interval features need to be selected in order to achieve good classification results, and it is difficult to obtain a robust extraction of these features. The basis of functions have a good performance under noisy conditions and can provide a very compact representation of the beat (a number as low as 3-4 features per beat may be enough). The main disadvantage of this approach is the loss of interpretability of the features.

The basis of functions most commonly used is the one made up of the Hermite functions. This basis exploits the similarity of the shapes of these polynomials with the QRS complexes (Sörnmo et al., 1981) (Lagerholm et al., 2000). Hermite functions are orthonomal; thus each feature has independent information and the signal can be accurately represented as a linear combination of a low number of Hermite functions. The coefficients of the linear combination are used as the features that characterize the shape of the beat.

When using this approach, a choice must be made about the number of Hermite polynomials to be used in the representation of the beats. As a general rule, the more polynomials are used, the more accuracy is achieved in the representation of the morphology of the beat. But a high number of polynomials (features) means a high dimensionality feature space, which can cause problems when training the automatic beat classifier. Furthermore, the higher order Hermite functions have high frequency components which could model high frequency artifacts present in the signal, rather than the beat. There are some authors that use as few as 3 polynomials (Braccini and Edenbrandt, 1997), and others use as many as 20 (Park et al., 2008). Other authors have used, for example, 6 (Lagerholm et al., 2000), 11 (Haraldsson et al., 2004), 15 (Xu and Wunsch, 2005). Usually the different authors provide no good justification for the number of polynomials used in their work

This paper aims to analyze the impact of using a certain number of Hermite polynomials in the representation of a heartbeat. Section 2 describes the database used in our analysis, the preprocessing applied to the ECG signal, and how the error between the representation obtained from the Hermite basis functions and the original signal was calculated. Section 3 describes the results obtained when fitting the beats with different numbers of Hermite polynomials, and Section 4 discusses these results, providing some guidelines to choose a suitable number of Hermite polynomials.

2 MATERIAL AND METHOD

2.1 ECG Database

The database most commonly used in the papers dealing with automatic beat classification is the MIT-BIH arrhythmia database (Osowski and Stodolski, 2003) (de Chazal et al., 2004) (Braccini and Edenbrandt, 1997) (de Chazal and Reilly, 2006) (Park et al., 2008) (Lagerholm et al., 2000) . Therefore, this will also be the database we shall use in our study. The MIT-BIH arrhythmia database (Moody and Mark, 2001) is made up of 48 ECG recordings of two channels among the modified limb lead II (MLII) and the modified leads V1,V2,V3,V4 and V5. The recordings are digitized at 360 Hz sampling rate. All beats in the database were annotated by two or more cardiologist.

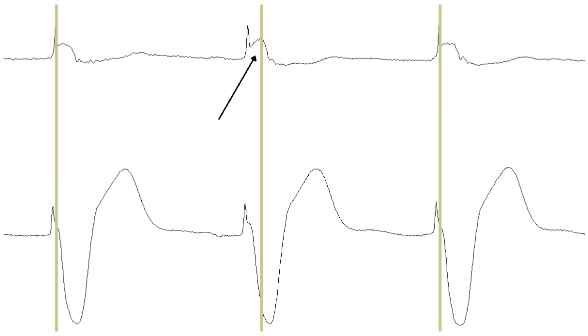

Figure 1: Instability of the handmade beat annotations made by the cardiologists in the MIT-BIH arrhythmia database

2.2 Preprocessing

To eliminate the baseline drift a wavelet based filter was used. The filter was a low pass filter that passes only the low frequencies corresponding with the baseline drift. Then from the coefficients of the Discrete Wavelet Transform (DWT) the baseline drift was reconstructed. This reconstruction was subtracted from the original signal, thus removing the baseline drift (Blanco-Velasco et al., 2008). To remove the high frequency noise a low-pass 4 order Butterworth filter with a cutoff frequency of 40 Hz was used. One of the theoretical advantages of representing beats with the Hermite polynomials is the robustness of the representation in the presence of noise. To empirically test this, we shall run our tests both directly on the recordings, and over a filtered version of the recordings.

Theoretically, Hermite polynomials will provide a better characterization of the beat if the point of maximum symmetry is selected as the center of the window of signal to be fitted. This point is usually the peak of the QRS complex, the R wave. Furthermore, setting the beat location in a stable position within the QRS complex will lead to more reproducible results, and therefore to features that will be more easily recognized by an automatic classifier. As it can be seen in Figure 1, the annotations handmade by the cardiologists have inaccuracies due to the imprecision introduced by the user interface.

To try to achieve a more stable beat's position within the QRS complex, and to get as close as possible to the point of maximum symmetry, an algorithm to improve the beats' location provided in the MIT-BIH arrhythmia database was used. The algorithm calculates the mean in a 200 ms window around the annotation provided in the database (the annotation handmade by cardiologists). Usually, the R wave peak is the farthest point from the mean value. This point is selected and a new window of 200 ms around it is extracted from the signal.

The correction to the beat's position can be applied only to one channel or to both channels independently. If it is only applied to one channel, the position of the R wave peak is assumed to be equal for both channels (this is not necessarily true in practice). Otherwise, the location of the R wave peak may be slightly different for each channel.

We have run one test using the beat's positions provided by the MIT-BIH arrhythmia database, the solution most commonly used in the literature. A second test was performed applying the beat location correction algorithm over the first channel and using the same beat location in the second channel. Finally, a third test was run applying the beat location correction algorithm over both channels independently. Each of the three strategies was applied directly over the MIT-BIH arrhythmia database signal recordings, and over the filtered version of the recordings, yielding a total of six different tests.

2.3 Hermite Functions

We will extract each heartbeat's QRS by taking a 200 ms window of sampled ECG centered on the beat's position, being the beat's position calculated by one of the three strategies presented in the previous section. This window is wide enough to encompass the entire QRS complex of a normal beat, but narrow enough not to include the P and T waves. The width of this window is the one normally used in the literature (Lagerholm et al., 2000) (Mugler and Clary, 2002). All the Hermite functions converge to zero both in ∞ and in $-\infty$. Thus, we shall add 100 ms zeros on each side of the 200 ms window containing the QRS. Let us denote by $x(t)$ the resulting 400 ms window. $x(t)$ can be represented as:

$$x(t) = \sum_{n=0}^{N-1} c_n(\sigma)\phi_n(t,\sigma) + e(t) \tag{1}$$

where N is the number of Hermite polynomials used in the representation of the beat, $\phi_n(t,\sigma)$ is the n Hermite function, c_n are the coefficients of the linear combination, σ is a parameter that controls the width of the polynomial, and $e(t)$ is the error between $x(t)$ and the Hermite approximation. The Hermite functions $\phi_n(t,\sigma)$, $0 \leq n < N$, are defined as:

$$\phi_n(t,\sigma) = \frac{1}{\sqrt{\sigma 2^n n! \sqrt{\pi}}} e^{-t^2/2\sigma^2} H_n(t/\sigma) \tag{2}$$

where σ is a parameter that controls the width of the polynomial. The Hermite polynomial $H_n(t/\sigma)$ can be obtained recursively:

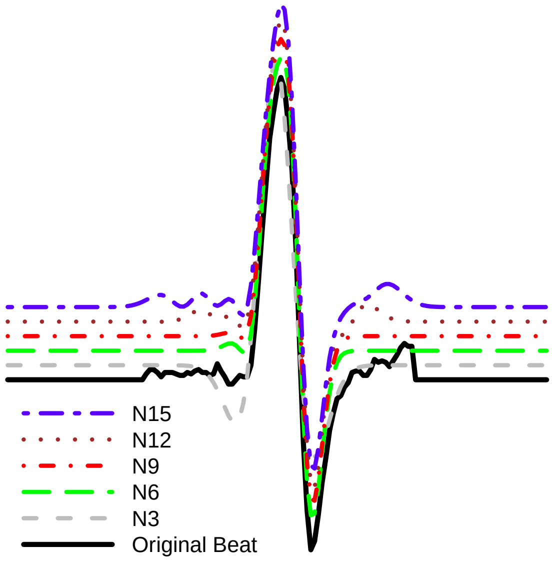

Figure 2: Original beat and hermite appoximations with N=3,6,9,12 and 15 for a fixed σ

$$H_n(x) = 2xH_{n-1}(x) - 2(n-1)H_{n-2}(x) \tag{3}$$

where $H_0(x) = 1$ and $H_1(x) = 2x$. For example $H_2(x) = 4x^2 - 2$, $H_3(x) = 8x^3 - 12x$, and so on.

To adjust the width of the Hermite function to each QRS complex, the σ value is used. Hence, each QRS complex is represented by the N coefficients of the linear combination $c_n(\sigma)$, $0 \leq n < N$, and by σ. Figure 2 illustrates how the higher the order of the Hermite functions used, the more accurate the approximation of the beat is. However, using high degree polynomials has the risk of modeling noise in the signal, and not the actual shape of the QRS complex (see the wavy behavior just before the start of the QRS complex in the approximation N=15 in Figure 2).

For a given value of σ, the hermite functions form an orthonormal basis:

$$\sum_{t=-\infty}^{\infty} \phi_n(t,\sigma)\phi_m(t,\sigma) = \delta_{mn}. \tag{4}$$

This permits an efficient calculation of $c_n(\sigma)$ in Equation 1. Without an infinite window size, Equation 4 does not hold. However if $\phi_n(\sigma)$ is close to zero on the edges of the window, Equation 4 is still a good approximation. For a given σ the coefficients $c_n(\sigma)$ are calculated by minimizing the summed square error

$$\sum_t |e(t)|^2 = \sum_t |x(t) - \sum_n c_n(\sigma)\phi_n(t,\sigma)|^2 \tag{5}$$

The minimum of the square error is easily calculated thanks to the orthogonality property:

$$c_n(\sigma) = \vec{x} \cdot \vec{\phi_n}(\sigma) \tag{6}$$

where the vectors are defined as $\vec{x} = \{x(t)\}$ and $\vec{\phi}_n = \{\phi_n(t,\sigma)\}$.

An iterative stepwise increment of σ was done by recomputing Equation 6 and Equation 5 for each step and selecting the σ that minimizes the error. Defining $\phi_n(\sigma)$ as being close enough to zero outside the window

$$|\phi_n(-t_0,\sigma)| = |\phi_n(t_0,\sigma)| < \frac{1}{10} \max_{t\in[-t_0,t_0]} |\phi_n(t,\sigma)| \tag{7}$$

and

$$|\phi_n(t,\sigma)| \leq |\phi_n(t_0,\sigma)| \ \ \forall |t| > t_0 \tag{8}$$

we can obtain the maximum values for σ. The value of the increment in each step was $\frac{frecuency}{1000}$ from 0 to the maximum.

2.4 Error Measurement

(Lagerholm et al., 2000) used the following measure to quantify the error of the approximation:

$$\in = \frac{\sum_t |e(t)|^2}{\sum_t |x(t)|^2} \tag{9}$$

This measure will be calculated in our test, to be able to compare our results with the ones of Lagerholm et al. We shall also calculate another measure that we believe is more easy to interpret: the normalized root-mean-square error (NRMSE) between the Hermite reconstruction and the sampled signal:

$$NRMSE = \frac{RMSE}{x_{max} - x_{min}} = \frac{\sqrt{\frac{\sum_t |e(t)|^2}{N}}}{x_{max} - x_{min}} \tag{10}$$

where N is the size of the window in samples. The NRMSE can be interpreted as the average error expressed as a percentage of the range of values in the signal fragment ($x_{max} - x_{min}$).

3 RESULTS

The algorithms described in the previous section were implemented by the authors in the Java programming language, with the exception of the wavelet-based filter and the high frequency filter. The filters were implemented in Matlab. From Matlab we generated filtered versions of the recordings of the MIT-BIH arrhythmia database that were fed to the algorithms implemented in Java. Tests were run for all the filtered and the unfiltered recordings of the MIT-BIH arrhythmia database. In each case three different runs were performed: a first one uses the beat annotations provided in the database as center for the window of the Hermite interpolation; a second one searching for the point of maximum symmetry of the beat on the first ECG channel and using this position also on the second channel; and third one searching independently in each channel for the point of maximum symmetry of the beats.

The results of the average NRMSE (see Equation 10) through all recordings are shown in Figure 3. The errors of each channel and the average error of the two channels are shown. The results corresponding with the beat's positions provided in the database, the beat's position correction applied to the first ECG channel, and the beat's position correction applied to both channels are marked with triangles, squares and circles, respectively. The bar shows the standard deviation of each error. The graphs on the left are the results for the unfiltered signal and the graphs on the right are the results for the filtered signal.

Figure 4 shows Lagerholm's error measure (see Equation 9) when using the beat positions provided in the database, and when the correction is applied to both channels. Results are shown both for the filtered and unfiltered signal.

During the tests our software measured the time required for fitting each beat with the Hermite polynomials. The average time required to fit each beat with the Hermite approximation of degree N is shown Figure 5. These test were executed in an Intel Core i5 CPU at 3.1 GHz with 4Gb of RAM running on Linux (CentOS-6.1).

4 DISCUSSION

The results in the previous section show that even with a small number of Hermite functions, beats can be represented acceptably. This is not surprising at all; there are authors in the literature that use as few as 3 functions to represent the beats (Braccini and Edenbrandt, 1997). 7 polynomials may be a sweet spot; between 6 and 7 we can still appreciate a significant improvement in Figure 3 and Figure 4; but after 7 the improvements are smaller. At least when the final goal is to obtain a beat classification, it is questionable whether it is worth using a number as high as 20 polynomials (Park et al., 2008), since the benefits obtained from a slightly more accurate representation of the beats may be overtaken by the disadvantages of training classifiers in a higher dimension space: going from 12 functions to 20 produces a decrease of approximately 0.005 in the total NRMSE both over the filtered and the unfiltered signal (see Figure 3).

The beat's position correction algorithm, especially when applied to both channels, provides noticeable improvements of the results. These improve-

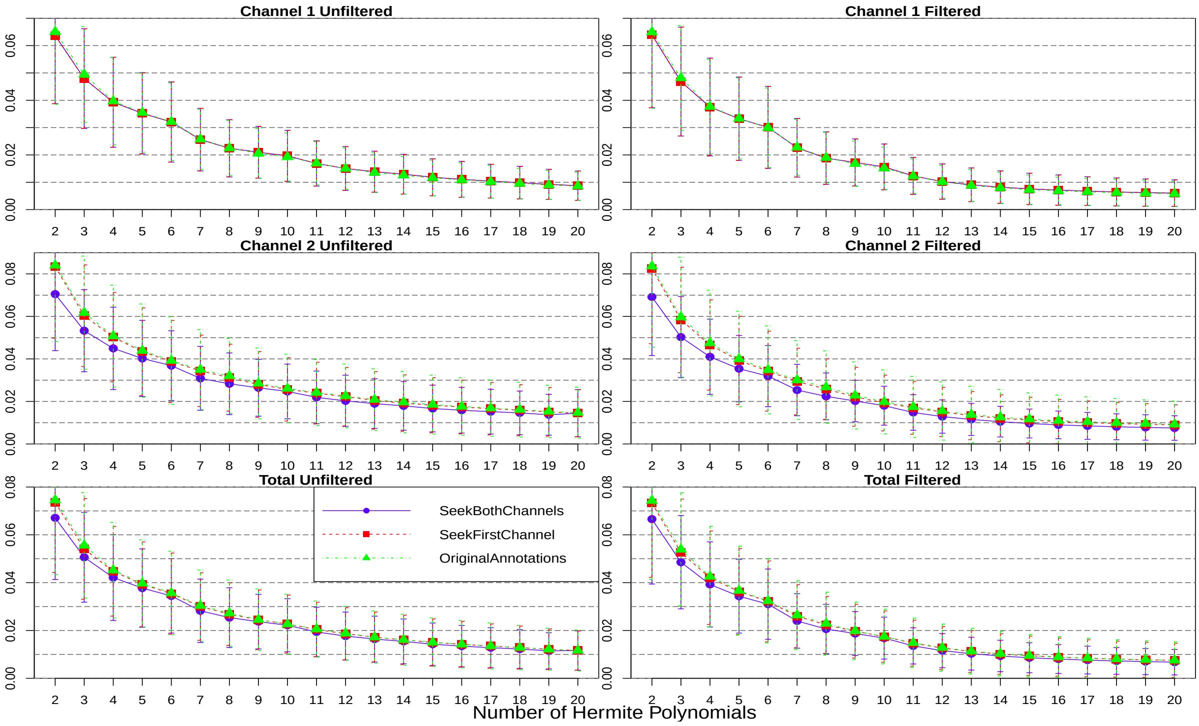

Figure 3: NRMSE results for the unfiltered and filtered signal for the three different strategies to determine the beat's position

ments are more marked in the second channel, especially when using low numbers of Hermite functions. The reason why the correction provides better results on the second channel is probably because the MIT-BIH arrhythmia database has been annotated over the first channel (Moody and Mark, 2001). The reason why more improvement is obtained for a low number of polynomials is because when using a high number of polynomials it is posible to represent the beat accurately even if the point chosen as the center of the fitting window is not the point of maximum symmetry (see Figure 3).

Filtering provides significant improvements in the results (see Figure 3 and Figure 4). We have performed independent tests using only high frequency filtering and only baseline drift removal. The removal of baseline drift alone produced virtually identical results to working directly with the unfiltered signal; almost all the improvements that can be seen in Figures 3 and 4 when using the filtered signal arise from the high frequency filtering. This suggests that Hermite approximation is more affected by high frequency noise than by baseline drift. For example, a 2% of NRMSE can be achieved without filtering with 11 polynomials but with filtering only 8 are required; and we cannot reach a 1% of NRMSE without filtering, not even with 20 polynomials, while with filtering is possible to reach this error with 13 (see Figure 3). It should be noted that many authors that have used Hermite polynomials to represent beats do not apply high-frequency filtering before the interpolation (Hu et al., 1993) (Park et al., 2008) (Hu et al., 1997) (Braccini and Edenbrandt, 1997) (Osowski et al., 2004) (Osowski and Stodolski, 2003) (Lagerholm et al., 2000).

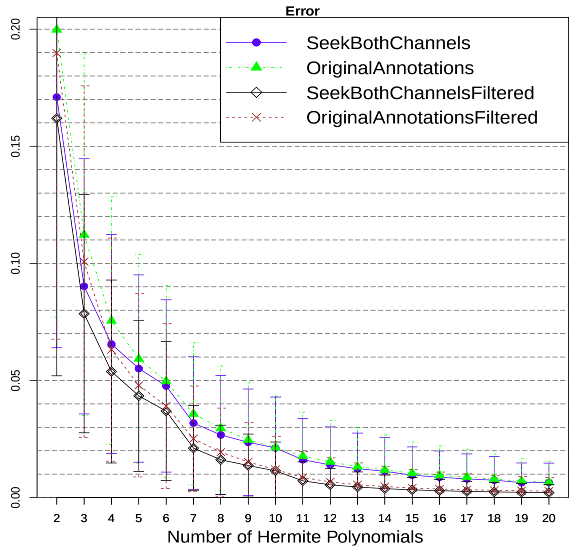

Figure 4: Lagerholm et al. error measure

Among the papers we have reviewed only (Lagerholm et al., 2000) reports error results for the Hermite approximation. Lagerholm et al. calculated the error

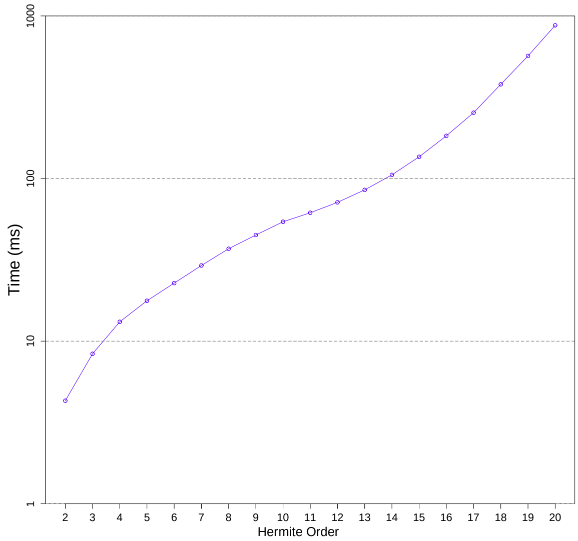

Figure 5: Average CPU time, in logarithmic scale, required to represent one beat with the Hermite basis functions

with Equation 9. They only report the values for 3, 4, 5 and 6 Hermite polynomials; the errors are 9.7%, 6.8% , 5.5% and 4.5%, respectively. These results are slightly lower than the results we obtained with our beat correction algorithm applied over both channels with the unfiltered signal. However, when using the filtered signal the errors we obtain are lower than the results of Lagerholm et al., both when using the original beat annotations from the database, and when using the beat position correction over both channels. It should be noted that Lagerholm et al. applied no high-frequency filtering.

CPU time used when calculating the Hermite representation increases very fast with the number of polynomials (see Figure 5). Our implementation of the algorithms for order 20 cannot process an electrocardiogram in real time on a modern computer (a Intel Core i5 CPU at 3.1 GHz). If the algorithms are going to be implemented in a device with low computing power, such a cell phone of a microcontroller, using a low order representation over the high-frequency filtered signal and applying beat position correction would probably yield a good compromise between accuracy in the representation and computing power requirements. Both filtering and beat's position correction consume relatively little CPU time, but provide significant improvements, especially for low order representations. For example, a characterization with the annotations of the database and the unfiltered signal and orders $N = 3, 7, 11$ have errors NRMSD of 0.0556 ,0.0303 and 0.0206, with average execution time per beat of 9.94ms, 34.5ms and 73.4ms respectively. If the signal is filtered, and beat position correction is applied over both channels, the execution times are almost identical but the NRMSD fall to 0.0486, 0.0240, 0.0136, respectively.

5 CONCLUSIONS

We have analyzed the impact of using a certain number of Hermite polynomials on the accuracy of heartbeat representation. Tests were run over the MIT-BIH arrhythmia database with a number of polynomials ranging from 2 to 20. Three different strategies to determine the heartbeat's position were used. Runs were performed over the original signal, over the signal after removing baseline drift, and over the signal after removing baseline drift and high frequency noise.

Our results suggest that using 7 polynomials is the sweet spot that provides a better compromise between accuracy of representation and working with a low number of features. However, with a smaller number of polynomials fairly good approximations can be obtained. Especially when using a smaller number of polynomials, correcting the beats' position and filtering high frequency noise provides significant improvements in the accuracy of the representation. The removal of baseline drift appears not to have a significant impact.

In this paper we have determined the accuracy of the representation with a measure of the error between the reconstruction obtained from the Hermite polynomials and the original signal. However, if the final goal of representing beats with Hermite polynomials is to classify them in different morphological families (instead of, for example, compression of the ECG (Jane et al., 1993)), the features that minimize this error need not to be those that provide the best separation between the different classes of beats. It would be interesting to study how the features obtained when representing the beats with a different number of Hermite polynomials enable the different beat families to be separated by an automatic classifier. This will be one of our lines of future work.

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