FLEXIBILITY IN MENTAL CALCULATION

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PROJECT - NUMERICAL THINKING AND FLEXIBLE CALCULATION: CRITICAL ISSUES

OBJECTIVES

To identify and describe students' conceptual knowledge associated with the different levels of understanding of numbers and multiplication / division and analyze if and how this knowledge facilitates adaptive thinking and flexible calculation

METHODOLOGY

Methodological

Options: Design research (Gravemeijer & Cobb, 2013)

Participants: Primary grade students and their teachers

Data collection: Participant observation (videotaped classes), clinical interviews and students' written work

TEACHING EXPERIMENT

To prepare teaching experiments the project team:

- designs adequate tasks to develop students' flexible and adaptive mental calculation (Threlfall, 2002, 2009)
- analyzes how children use their knowledge about numbers and operations to solve the tasks (interviews)

THE TASK *WALKING*

The approach and real calculation used by Miguel (11 years old) to solve the task *Walking (*Threlfall´s model, 2002; 2009)

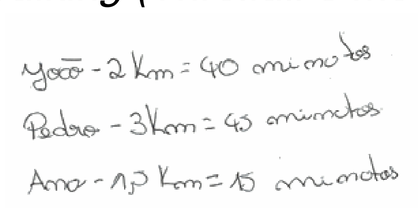

Fig. 1. Data recording used by Miguel

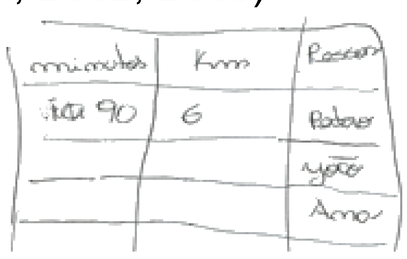

Fig. 2.Table with the distance that Pedro walked in 90 minutes

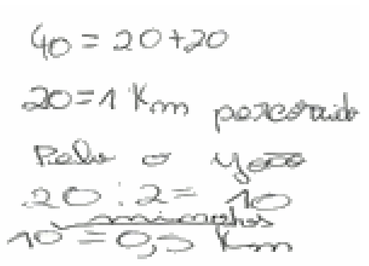

Fig. 3. Distance that João walks in 10 minutes

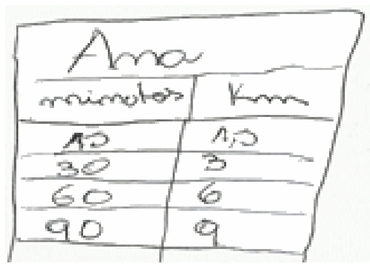

Fig. 4. Distance that Ana walks in 90 minutes

MIGUEL'S PERSONAL KNOWLEDGE

NOTICING AND TRANSIENT THOUGHTS

Additive and multiplicative facts

40 =20+20; 80 =2×40; 90 =2×45;

20 : 2=10; 2×30= 60

Proportional relationship

- Miguel's own symbolization (Fig1).
- Miguel's own verbalization:

First approach thought:

"Now I'm going to see, for instance, in 80 mins how much it is … to see a number that is easier for these two" (João and Pedro).

Second approach thought:

"I'm going to do, for instance, in 90 mins, how much each one walks".

APPLIED RELATIONSHIPS

Proportional relationship

To calculate the distance walked by Pedro, informal use of the proportional relationship underlying the scalar method (fig 2).

Miguel knows that he can use this reasoning with both, natural numbers and decimals (fig 3).

He uses similar reasoning for the case of Ana (fig. 4)

He answers "Who walks faster is Ana"- multiplicative comparison in the context of proportional relationships in two fields of measure.

- The previous analysis suggests that Miguel has developed certain flexibility.
- Data suggests that his ways of symbolizing allows him to think and calculate flexibly, adaptively and efficiently (Hatano, 2003; Threlfall, 2002).

MIGUEL'S CALCULATION PRACTICE

EXPLORATIVE PARTIAL CALCULATION

Use of personal symbolic notation of proportional relationship (2 Km-40 min) to construct a table that allows him to envision the three relations between time and distance (90 as the total time).

He connects 90 with 6, doubling (mentally) both numbers of the initial relation: 3 – 45 ↷ 6 - 90

APPLIED CALCULATION METHOD/PROCEDURE

Application of the same scalar method to find, in the other situation of the task, the number (quantity of km) that has to be associated with 90 (quantity of time)

Uses, in both cases, the same stepwise strategy, proportionally increasing both quantities (or decreasing).

Unique, local adaptations

João's walk (2 - 40): Use of doubling both numbers in combination with breaking the total distance.

Ana´s walk (1,5 - 40) – Use of repeated doubling

Comparison reasoning

Derives the quicker child from the biggest number of km walked in the same time (90).

- From the point of view of the used scheme of thinking and calculating, it looks as if Miguel is less flexible than the "ideal" expert (Hatano, 2003).
- However, from the point of view of the unique personal calculation, it is evident that Miguel has already developed important building stones for flexible mental calculation.

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