Journal of Modern Accounting and Auditing, ISSN 1548-6583
December 2013, Vol. 9, No. 12, 1622-1636

The Terminal Value (TV) Performing in Firm Valuation: The Gap of Literature and Research Agenda

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The uncertainty about the future of firms must be modeled and incorporated in the valuation of enterprises outside the explicit period of analysis, i.e., in the continuing or terminal value (TV). There is a multiplicity of factors that influence the TV of firms which are not being considered within current evaluation models. This aspect leads to the incurring of unrecoverable errors, thus leading to values of goodwill or bad will far away from the substantial value of intrinsic assets. As a consequence, the evaluation results will be presented markedly different from market values. There is no consensus in the scientific community about the method of computation of the TV as a forecast in an infinite horizon. The size of the terminal, or non-explicit period, assumed as infinite, is never called into question by scientific literature, or the probability of business bankruptcy. This paper aims to promote a study of the existing literature on the TV, to highlight the fragility of the evaluation models of companies that have been used by the academic community and by financial analysts, and to point out lines for future research to minimize these errors.

Keywords: continuing value (CV), terminal value (TV), perpetuity, life expectancy

Introduction

One of the existing problems with an enterprise's valuation models, which are broadly used by the financial community, is related to the calculation of the continuing value (CV), also known as terminal value (TV) or residual value (RV). This component is present not only in the dividend discount model (DDM) by Gordon (1959) and in the discounted cash flow (DCF) model, but also in the market value added (*MVA*) or in the refined economic value added (REVA) models, and it usually represents a large part of the valuation of the firm value as observed by Berkman, Bradbury, and Ferguson (1998), Copeland, Koller, and Murrin (2000), and Buus (2007), contributing to a great uncertainty as far as the valuation of the enterprise is concerned. According to these models, the computing or calculation of the firm value is usually divided into two stages: (1) The first one considers an explicit period (a short period with high predictability) that corresponds to a temporary horizon of 5-7 years habitually, where the forecasts are more easily feasible and where volatility is not a very relevant factor; and (2) The second focus is on the quantification of the RV or TV, which is deeply unpredictable in its estimation, using the mathematic model of a constant perpetuity or with growth of a certain attribute.

The models of valuation of firms have enormous theoretical and practical limitations. The market rate value (in the case of rated enterprises), which is the most used, may not represent the real value of the firm,

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also due to the deviation from other factors that are not based upon the internal and external premises of the enterprise growth, as for example, the asymmetric information among the different economic agents. The firms' managers are the first to look for a non-complex and easily applicable scansion. To illustrate this point, if we considered a constant perpetuity without the growth of a flow of 100 c.u., for example, and an actualization rate of 10%, the current firm value, or else, the value of their equity capital would be of 1,000 c.u. obtained by the perpetuity mathematical expression. It is necessary to emphasize that at the end of 20 years, the updated value of that annuity is 851 c.u., 978 c.u. at the end of 40 years, and 991 c.u. at the end of 50 years. In other words, the present financial modulations do not take into account the average life expectancy of an enterprise. In the previous example, the enterprise value would be 99.1% of the perpetuity only after a period of 50 years.

It is possible to determine a "mortality table" through a Coface Mope's database with about 242,661 records on insolvencies, dissolutions, and ceasing of activity in Portugal from 1900 to April 2012, and through the date of establishment and the date of dissolution or insolvency records. In the first five years, 30% of the enterprises "die". The average life expectancy at the enterprises' birth is 12 years, with a standard deviation of 11 years, reaching a maximum of 169 years. These results show the fragility of the valuation models based on the calculation of the RV, assuming it with a perpetuity or a multiple of the results.

Accordingly, the main objective of this paper is concentrated upon the revision of the literature that is more relevant to the estimation of the RV of the firm, which has been used more often by the valuation models. This paper is organized as follows. In Section 2, the traditional models for the evaluation of the firms are presented in summary. In Section 3, the main limitations are presented. Section 4 focuses upon the review of literature related to the definition of the TV, which has been used among the different models of valuation, including the main empirical conclusions led to by this literature. Finally, in Section 5, the main conclusions of this study are presented and clues for future investigation are pointed out.

The Traditional Valuation Models: A Review

The traditional models of firms valuation can be grouped into five categories: (1) models based on the discount of cash flows (CF), where DCF is included based on the free cash flows (*FCF*), discounted to the weighted average cost of capital (*WACC*), the equity cash flow (*ECF*)[1], the available flow for the equity holders, discounted to the cost of the equity capital, the capital cash flow (*CCF*), available flow for shareholders and creditors, discounted to the *WACC* before taxes and the adjusted present value (APV) or value of the unlevered enterprise plus the discounted value of the tax shield, or tax savings derived from tax deduction of interest; (2) a model of dividends which some literature considers to belong to the previous group, however, much of that literature points out reasons for its self-sufficiency; (3) models related to the value creation, such as the economic value added (EVA), the economic profit (EP), the cash value added (*CVA*)[2], and the cash flow return on investment (CFROI), are highlighted in this group; (4) models based on accounting elements, where we can find the multiples methodology, the most common, the accounting value of equity capital, the goodwill method, corresponding to the substantial value (or substitution value) plus the goodwill

[1] $ECF = FCF - [\text{interest} \times (1 - \text{tax})]$ – debt repayment + debt disbursement, and $CCF = ECF + DeCF$; $DeCF$ = interest – debt variation, with *DeCF* the debt flow.

[2] *CVA* = Net operating profit after tax (*NOPAT*) + depreciations and amortizations – (initial invested capital × *WACC*). *CVA* is a trademark of Boston Consulting Group and it is seen as an alternative for EVA.

(this last one identical to a number of times the net result or a determined percentage of businesses volume); and (5) lastly, the sustaining models in real options, where we highlight the Black and Scholes (1973) method. This study focuses on the first three groups, including a short reference to the multiples methodology due to its relevance in literature.

The DCF Method

The DCF[3] method has existed since the 1970s, it is considered one of the most used methods by the financial community and it represents the present value of the foreseen future cash flow. Pursuant to this model, the value of equity capitals of the enterprise in the present moment (*EqV*) is given by Expression (1), with *WACC* differentiated:

$$EqV_t = \sum_{\tau=1}^{T} \frac{FCF_{t+\tau}}{\prod_{j=1}^{\tau}(1+WACC_{t+j})} + \frac{FCF_{T+1}}{WACC_{T+1}-g} \frac{1}{\prod_{j=1}^{T}(1+WACC_{t+j})} - D \quad (1)$$

where *EqV* is the market value of the equity capital and *D* is the net debt of cash. Copeland et al. (2000), Koller, Goedhart, and Wessels (2010), as well as Damodaran (2002; 2006a) added the extra exploration assets to the previous expression, the value of financial participations and included the debt of operational leasings. The DCF model works in two stages: the finite horizon, explicit period, in other words, the first stage usually constitutes an estimated temporal horizon of 5-7 years and, the infinite horizon, the second stage, also named as CV or TV, which corresponds to a forecast of the activity evolution of the enterprise during an unlimited period, usually presented mathematically under the shape of a growing or a constant perpetuity of the *FCF*. The end of the first stage implies the end of any possible supernormal profit, or of a competitive advantage of the enterprise that is subjected to valuation from the competition, and therefore initiates the stable growth period. As a result, the dynamics of the *FCF* growth, in the second stage, is stated as a function of a unique stable growth rate. The TV, in other words, the so-called value after the explicit period or after the analysis horizon, is based on Gordon's (1959) model which can be observed in the second parcel of Expression (1), and it consits of only an infinite extrapolation of the *FCF* in that period. In that formulation, a constant growth of the *FCF* is assumed and, consequently, of the firm during that infinite period.

The Models Based on Residual Income

This approach of modeling has its origins within the work of Edwards and Bell (1961), who reformulated Gordon's (1959) model based on the market data (dividends) in order to turn it into a simpler valuation model, so that it could be used by the enterprises that did not distribute dividends. Therefore, with the contribution of Ohlson (1995), they redefined the model so that it could be based on measures of accounting profitability (accounting results), overcoming the high sensibility of the calculation of the TV, present in the dividends. They created the Edwards-Bell-Ohlson (EBO) model. To calculate the TV, they only projected the residual or abnormal profit (profit above the cost of equity capital). Nevertheless, this model presented a serious flaw. It was based only on accounting criteria and it was liable to various forms of recording a single patrimonial fact. According to these authors, the market value of equity capital of a firm corresponds to its present value of equity capital, to accounting values, and to what is assumed to be added in the future due to abnormal profits, i.e., superior than the remuneration of equity capital invested properly updated to the cost of equity capital. One of the examples of models based on the residual income is the EVA, which considers the fraction of the result,

[3] Williams (1997) introduced the basis for the DDM and the DCF.

which exceeds the cost of capital, as the true created wealth of the shareholder. Another example, which is very common, is the MVA, which corresponds to EVA, visible in the numerator of the last part of Expression (2), added to the invested capital:

$$VE = MVA + CI = (Do + Eo) = (Do + Ebvo) + \sum_{t=1}^{\infty} \frac{NOPAT_t - WACC_t . (D_{t-1} + Ebv_{t-1})}{\prod_{i=1}^{t}(1 + WACC_i)} \quad (2)$$

where *VE* represents the market value of the enterprise, *D* is the debt, *E* is the market value of equity capital, *Ebv* is the accounting value of equity capital, and *MVA* is the market value added. Created by Stern Stewart in 1991, EVA[4] is obtained through $NOPAT$ deducted from the product of $WACC$ by the invested capital to accounting values. EVA only reflects the incremental gains to a firm after considering the cost of capital. EVA is a superior performance measure, because only when the *NOPAT* is superior to the cost of capital applied to the value of capital invested in accounting value does the enterprise creates value. EVA is a portion of *MVA*, since this last one is obtained by the discount of future EVA to the present value, and if the capital initially invested is added to it, it will allow us to get the enterprise value.

The model of abnormal earnings growth (AEG) or capitalization model (CM) is introduced by Ohlson (1995) and appeared further inserted in the concept of residual income model (RIM). It is another model based on accounting data, but it uses the capitalized future results to replace the accounting value of equity capital. In this model, as seen in Expression (3), the firm value is determined by the combination among net future results to which the dividends as well as the resulting product of its reinvestment are added (i.e., the dividend of the increased period of the product of the reinvestment of the dividend from the previous period):

$$V_t^T = (\rho^T - 1)^{-1} . E\left[\sum_{\tau=1}^{T} \tilde{X}_{t+\tau} + \sum_{\tau=1}^{T} (\rho^{T-\tau} - 1) . \tilde{d}_{t+\tau}\right] \quad (3)$$

where V represents the market value of equity capital, *X* represents the net results and the last part of the expression, the total of dividends plus the product of reinvestment of those at the rate of cost of capital (with ρ^T = 1 + the cost of equity capital), and *d* represents the expected dividends.

The DDM

Gordon (1959) estimated the current value of a firm based on the future dividend discount to the return rate required by the shareholder. The general formulation of Gordon's (1959) model presents a capitalization of dividends constant over time. Expression (4), a particular case of Gordon's (1959) model, shows the situation in which dividends grow at a constant rate, *g*, such rate being lower than the rate of return required by shareholders, *K*:

$$V_0 = \frac{D_0(1+g)}{K-g} \quad (4)$$

where *V* represents the firm value, *D* is the dividend, *K* is the minimum return rate required in order to invest in stock, and *g* represents the constant and predictable rate of the dividends growth. Gordon's (1959) model is based on the computation of the value of the firm through the premise of future dividends discounted at the cost required by shareholders.

Models Based on Comparable Multiples of Market

The methods based on the value of the stock market, or method of comparable multiples of the market, are methods widely used by the financial community (Shaked & Kempainen, 2009) because of its easy application

[4] A trademark of Stern Stewart & Co..

and perception of the firm value in view of its comparable peers (firms in the same industry or sector). The price earnings ratio (PER) is frequently used and it corresponds to the multiple of market price compared with the net result, the price/sales (P/S), the multiple of the market price regarding the sales and services rendered, the price/earnings before interest, taxes, depreciation, and amortization (EBITDA), multiple of the rate compared with the operational profit without amortization, the price/earnings before interest and taxes (EBIT), multiple of the rate compared with the operational profit and the price book value (P/BV), and multiple of the rate related to the accounting value of the equity capital.

There is a measure analogous to this multiple on the BV, Tobin's *Q*, which corresponds to the ratio between the market value of the firm (equity and interest bearing debt) and its BV.

Limitations of Traditional Models

DCF Limitations

Despite their long existence, the techniques of DCF suffer a number of shortcomings associated with their use. The major constraints related to these models based on perpetual CF are situated in the measurement[5] of the length of the firm's life (non-explicit period, unlimited), in the computation of the update rate of that period, in the definition of the flow to perpetuate, in calculation of the growth rate of the flow to be perpetuated, and also in the fact that the mentioned techniques do not consider the management capacity of managers to change the future of the cash flows with their own decisions.

Kaplan and Ruback (1995) identified another limitation of the DCF method, showing that the *WACC* is affected by changes in the capital structure. The DCF method, by using the numerator *FCF*, presents enforcement problems in several highly leveraged operations, restructurings, project finance operations, and other cases in which changes in capital structure are common over time. It is usual in these situations that the capital structure has to be redefined and estimates are used to calculate the *WACC* that is appropriate for each period. Where the debt is foreseen by levels, rather than a percentage of the total value of the firm, the *CCF* is much easier to use, because the tax savings resulting from debt interest is computed easily as well as its consideration in CF. In respect to that, Akalu (2001) pointed out that the measure of the DCF is not applicable to business areas related to information technology, research and development (R&D), and with some real estate activities, given the high volatility of CF. This method neither takes into account the dynamic activity of managers, nor their ability and creativity in management. The assets of a firm are not held passively as they are assumed by these models.

In order to evaluate firms with losses, in early activity, or in both situations, there are additional difficulties inherent in the DCF models according to Damodaran (2001). Still in the DCF methodology, by assuming a constant and unique growth rate, we anticipate that in the horizon out of our sight, the headings of *FCF* and of the investment will have a constant behavior. The difficulties of establishing a long-term growth rate were studied by several authors that always showed complex conclusions. In this regard, Cassia, Plati, and Vismara (2007) determined that it is not possible to maintain growth rates of return of the investment above the cost of the invested capital. This competitive advantage would be difficult to maintain in the early period of steady or terminal growth.

Other authors face the difficulty of determining the value of the correct CF to perpetuate. Among these

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[5] Ikromov and Yavas (2012) argued that the firm value in the DCF depends on the magnitude, timing, risk, and volatility of *FCF*.

authors, we highlight Jennergren (2008). This author has reformulated the expression proposed by Gordon, including in his analysis the tax effect of depreciation of investment expenditure at infinity and the effect of inflation on those expenses, also considering the economic period of the life of the investment. According to the author, much of the *FCF* included in perpetuity results from the investment (non-current tangible and intangible assets) already acquired, where most of the error of the traditional models based on perpetuities is situated. Later on, the same author, Jennergren (2010) argued that the investment spending and the tax benefits inherent in the amortizations are difficult to predict and they depend not only on the level of income but also on the history of investment. He claimed that errors may arise when the level of investment is put in the perspective of fixed tangible assets with a percentage of business volume, when the enterprise expects rapid real growth in the explicit period and moderated or no real growth in the period after explicit period.

Petersen and Plenborg (2009) concluded that in the observed evaluations performed by investment banks, there were many errors in the calculation of TV and of the growth rate implied, which could entangle litigants. Martins (2011) presented a case study that ended in litigation in courts and concluded that any changes, as insignificant as they might be, in the growth rate in the computation of the RV significantly influence the value of the firm.

RIM Limitations

With respect to the limitations of the EVA model, Plenborg (2002) and Shil (2009) considered that after calculating the EVA with all the extra accounting adjustments that have to be made to the invested capital and to the result, this method presents the following limitations: (1) It is a short-term measure; (2) Future returns are estimated subjectively; (3) It is not a good measure for firms that invested heavily at an early stage and that are only going to obtain EVA from their investments after a long-term period; and (4) There are other factors, such as inflation, which are not considered. Arzac (2005) added that such methods are not aimed at the evaluation of firms but rather to assess their performance, and thus, they can be used to define compensations and rewards to their employees.

DDM Limitations

The model based on the dividend discount presents the estimation of long-term dividends in conjunction with the possibility of those not being distributed by the society but incorporated for capitalization of society as its main difficulty. In other words, it is a model that is not widely used, given the unpredictability of the value and timeliness in which there is a distribution of dividends by firms that opt for auto-financing. Foerster and Sapp (2005) indicated the DDM as presenting a good performance, explaining the observed price for an enterprise that has a long history (120 years) of distribution of dividends.

Common Limitations to All Models

Ohlson (1995) considered that the value of a company corresponds to the accumulated value of their accounting results plus reinvestment, at a risk-free interest rate, of the expected dividends in the future.

By linking this reasoning with what is defended by Fernandez (2002), in which the shareholder return over a given period corresponds to the variation in share price plus received dividends, of the share buybacks, of free attribution of new shares, of discounts against the pair in emissions of new securities (deducting capital calls), of the exercise of options or warrants related to capital, and of the conversion of liabilities into equity, we find that there is some inconsistency in the existing valuation models. Those do not provide any reinvestment by the holder of the enterprise that is subjected to analysis of the flows that the enterprise can generate and distribute to its holders in any of the forms, i.e., an increase/decrease in value that the models do not incorporate and that represents an important factor in the overall valuation of the entity object of evaluation as well as from the perspective of the holder of that entity. The process of reinvestment of earnings in the view of the holder is not incorporated in most models, providing that the rate of return is above any discount rate. It is a fact that the discount rate used may reflect this potential asset derived from the reinvestment of *FCF*, but its self-sufficiency, as suggested by Ohlson (1995), does not seem unreasonable, because the discount rates almost always refer to the specific risk of the security in question and not to others, where the investor can perhaps invest. Neither the tax nor the fruits of reinvestment are considered either in the valuation models or in discount rates.

Another limitation corresponds to the discount or liquidity premium and/or discount or control premium and they are two of the greatest impacts which determine the value of a company. Not part of the traditional models, such discounts or premiums apply *a posteriori* the outcome of the evaluation, by a multiplicative factor. According to Pratt (2009), the discount resulting from the valuation of a minority position is applied to the value that results from the evaluation of the enterprise as a whole, taking into account the premises associated to a detainment or tenure that assumes a controlling position. After subtracting the minority position, the discount implied by the lack of liquidity is applied to the resulting value. The value options for a shareholder who controls, as for a non-controlling shareholder, are different and so it is natural that there are different ways and methods of valuation for each case. The alternatives for a majority shareholder may go through: (1) perform value by placing the company in the market; (2) the settlement itself; (3) merging with another business; and (4) the sale and priority access to dividends. For a minority shareholder of a non-rated company, the value options are only possible through the dividends and the sale. In the latter case, it may not have an output option, which may not affect decisions as to receiving dividends. These value chain accesses can and often do determine the value of the enterprise.

The models do not incorporate in their traditional form these grounds, discounts and premiums of control and liquidity and do not contemplate the company perspective (entity), shareholder (owner), or both. Pratt (2009) went on to identify variables that involve discounts on firm value and, as mentioned, are not included in traditional valuation models, especially in their incorporation in TV, as: (1) the lack of a key person in the organization; (2) the taxation of capital assets or dividends in the sphere of the holder; and (3) discounts upon firms with diverse businesses (diversification) inhomogeneous (portfolio discount), among others.

The Definition of TV or CV: The State of the Art

The TV as a Perpetuity

The overwhelming majority of authors, as hereafter indicated, who study the valuation models and business performance, employ in their construction, whether they belong to the group of DCF, RIM, or DDM, a perpetual TV, with or without growth. Levin and Olsson (2000) demonstrated theoretically, by using a system of simultaneous expressions formed from the income statement and balance sheet, the conditions for a steady growth state typical of the TV. They ensured conditions so that there is a steady evolution after the horizon for profits (RIM), for *FCF* (DCF) and dividends (DDM). They demonstrated how the violation of the principles of stability implies errors in estimates of the value of firms. The steady growth state is essential so that we can have equality among the DDM, RIM, and DCF models. They assumed the basic premise that at infinity, the firm assumes a stable state of development. They indicated that all flows after the horizon will be calculated according to a growth rate applied to capital stocks that generate flows according to a growth rate of business volume. Thus, the profitability remains constant during infinity and they also ensured the equality models.

Damodaran (2002) through the H model (created by Fuller & Hsia, 1984) divided the TV into two stages: the first, with an extraordinary growth, where the growth rate decreases linearly towards stability and the second, with a stabilized growth, by applying here the traditional expression of Gordon's (1959) model.

For Fernandez (2005; 2007b), the TV is calculated according to a perpetuity (which may be without growth), where investment corresponds to the replacement of depreciation to maintain the assets at a level that can sustain constant CF. The same author also equated the TV through a perpetuity with constant growth where the investment is not restricted to the mere replacement of amortizations.

Sabal (2007) shared the same ideas of Kaplan and Ruback (1995), when he argued that for firms that have a highly unstable debt ratio, the *WACC* is not appropriate for the calculation of TV and rather, the APV method should be used, where, for the calculation of TV, the *CCF* is used as an attribute discounted to *WACC* before taxes for a given growth rate in that period.

Jennergren (2008) argued that the TV is derived from the application of Gordon's (1959) expression to a simple extrapolation of the *FCF* at the end of the explicit forecast period. This author examined the components of the TV, namely, the investment expenses and the tax savings arising from the amortization of fixed capital investment after the post-horizon period, concluding that a part of the TV arises from the CF associated with the capital assets already acquired. According to the author, the TV is the sum of eight components, which can be divided into three groups: (1) one, which has to do with the fixed capital existing prior to the start of the period after horizon; (2) another, derivative from activities associated with the replacement of assets already acquired before the end of the explicit period, in order to maintain the productive capacity (the economic life of those goods ends and they have to be replaced); and (3) finally, the third group relates to activities associated with capital expenditures inclined to the real growth of the business. Therefore, the TV is variable dependent upon investment expenditures and associated tax savings, of inflation, of the asset's economic life, and of capital intensity.

Ross, Westerfeld, and Jaffe (2007) advocated the DCF model as the most suitable for the valuation of firms. Because CF can be irregular, they make assumptions in order to adjust those CF. Annuities and perpetuities are used to compute the present value of the CF.

The TV Obtained by Different Calculation Alternatives

Hitchner and Mard (2003) considered various methods of calculation of the TV. The first is based on the output model in multiples (using a multiple of output based on the net result, EBIT, and EBITDA). This method aims to apply a multiple to the value of those attributes in the last year of the explicit period. It is used more as a second proof to the TV model based on Gordon's (1959) expression. Another method is based on the H model used by Damodaran (2002), which divides the TV into two stages, the first of extraordinary growth and the other of stabilized growth by applying the traditional expression of Gordon. Finally, the third alternative is proposed for the calculation of TV based on the named value driver formula (*VDF*) created by Koller et al. (2010).

Arzac (2005) decomposed the TV into three parts: (1) the value of reproduction; (2) the present value of the power of gains; and (3) the value of growth opportunities. The first corresponds to the potential cost that potential competitors, who are to enter the market, are willing to pay for the business. The second corresponds to the excess *FCF* generated by the firm above the value of reproduction. It is calculated by assuming that the firm has no real growth in volume (number of units sold or services rendered) in the future. This component, also called franchise value, identifies itself as the value of its competitive advantage, i.e., the value of its customers or business portfolio. Finally, the third component is calculated from the difference between the DCF valuation and its value without growth, corresponding to the valuation of growth opportunities. This component is the most difficult to quantify, because it assesses the possibility of the firm to produce a higher profitability than its cost of capital.

Damodaran (2006a; 2006b) defended the computation of the TV used in the DCF models in three ways: (1) by the expected liquidation value of each of the assets or businesses, or cash-generating units in the case of the existence of a market for this purpose; (2) by the multiples of market; and (3) by the perpetuity, stabilized growth. He assumed that the firm cannot grow larger than the growth rate of the economy wherein the company operates. He advocated a model with three stages where the third corresponds to the TV.

Fruhan (1998) suggested five alternative methods of calculating the TV: (1) TV as a growing perpetuity of *FCF*; (2) TV as a constant perpetuity of *FCF*; (3) TV as a multiple of BV of the capital invested at the start date of the terminal period; (4) TV as a multiple of profits (PER); and (5) TV as a net value of assets, calculating also the fiscal impact of the liquidation. For example, as far as this matter is concerned, Titman and Martin (2010) also advocated the DCF with a TV based on a multiple of EBITDA.

According to Pascual and Jiménez (2009), the TV can be considered as the product of the last multiple available by the value driver expected at the end of the explicit period, for example, a multiple on sales or on EBITDA. According to these authors, the TV, when calculated through CF's independent multiples, provides a better adherence to reality. They also concluded that sales, as a multiple, show better results than the EBIDTA. Shaked and Kempainen (2009) calculated the TV based on multiples, as EBITDA, or results through a range of projected growth rates for the *FCF* in the terminal period. The results obtained indicate that the analysis through the DCF uses the EBITDA as a more frequent attribute in the TV calculation, multiple, closely followed by the growing perpetuity. They also found that the growth rate in the terminal period is closely linked to inflation.

The TV as a Corrective Factor

Regarding the treatment of the finite and infinite period, Penman and Sougiannis (1998) reported that the use of the TV is to correct the fact that the DCF has no rules of addition or accounting result. For these authors, all models are particular cases of the DDM with specific TV. In the analysis of these authors, the TV works as a correction factor. Thus, for the DDM, as a foundation for the TV, they use the sum of the expected dividend per share plus the value resulting from the repurchase of shares and the amount corresponding to the free or non-free allotment of shares to the partners. For the DCF, they use the present value of dividends, including the terminal dividend because, according to the authors, this reflects the current value of the *FCF*. For the RIM, Penman and Sougiannis (1998) used the criterion of Bernard (1994) using the difference between the last available stock market value and the BV of the enterprise as attributes for TV calculation.

Penman (1998) argued that the evaluation of the three most popular methods, DCF, DDM, and RIM, works for a limited forecast horizon. He argued that these valuation techniques can be replaced by the method of dividends with an appropriate TV. To determine the value of the company, he used the ideal TV. That ideal TV is a combination based on amassed results plus a portion of the equity of the enterprise. The calculation of the TV by the ideal method of Penman (1998) corrects the errors of the DCF accumulated until the end of the explicit period, regularizing through that procedure the accounting result of non-cash effects and they are not incorporated in the traditional DCF. For the DDM, the authors of the current paper proposed the following expression:

$$P_t^T = \sum_{\tau=1}^{T} \rho^{-t} E_t\,(d_{t+\tau}) + \rho^{-t}[(\rho^S - K_S)^{-1} E_t(\sum_{\tau=1}^{S} X_{t+T+\tau}^{CS} - (K_S - 1)B_{t+T})] \quad (5)$$

where *P* represents the market value of the enterprise's equity, *E*(*d*) represents the average of the expected dividends, *ρ* = 1 + capital cost, *K* is the expected growth in the premium or in error, *X* represents the amassed profits, and *B* represents the BV of equity.

The Non-existent or Insignificant TV

Edwards and Bell (1961) and Ohlson (1995) argued that the TV includes only the present value of abnormal profits after the explicit period. Those authors considered that the return on equity (ROE) is not greater than the cost of capital after the horizon (explicit period) and due to that, the TV is zero, because the residual profit is nil.

Bernard (1994) concluded that only for the part of investment in R&D, where ROE is infinitely greater than the capital cost, the TV has relevance presenting, according to his research, a weight on average of 28% of the value of the overall evaluation of an enterprise. In the sample considered by this author, the TV in the DCF model achieved a 70% portion of the total value, i.e., large subjectivity implying that a large portion of the enterprise's value derives from a period that cannot be evaluated in as pragmatic as the one associated with an explicit period, finite, i.e., in a horizon close to the current one. Miller (2008) demonstrated that the concerns about the growth rate in the terminal period, being higher, lower, or equal to the growth rate of the economy, are irrelevant in competitive markets. The TV should be estimated as the BV of the invested capital at the end of the period of competitive advantage (explicit period), where the EVA should be null, i.e., there would be no place for the residual result, because return on invested capital (ROIC) would be identical to $WACC$.

The TV as a Perpetuity Where the Growth Rate Is Related to the Reinvestment Rate in the Terminal Period

In order to calculate the TV, Koller et al. (2010) used the *VDF*[6] expression expanded as follows:

$$VDF = \frac{NOPLAT_{t+1}(1-\frac{g}{RONIC})}{WACC-g} \quad (6)$$

The simple *VDF* (without growth in perpetuity) corresponds to net operating profit less adjusted taxes (*NOPLAT*)/*WACC*, with return on new invested capital (*RONIC*) the profitability of new invested capital, which represents the increase of *NOPLAT* (similar concept to *NOPAT*) relative to the new investment.

It must be highlighted that g/*RONIC* represents the reinvestment rate, so the numerator is also a *FCF* estimated for perpetuity. Koller et al. (2010) concluded that firm value does not change with the size of the explicit period, but decreases the weight of the TV, while the other increases. Berkman et al. (1998) argued that the calculation of the TV of the *FCF* and its growth rate are not to be calculated separately. It is defended that both depend on the retention rate of the *FCF* at the beginning of the terminal year. Berkman et al. (1998) provided empirical evidence that if the growth rate did not evidence the reinvestment rate in the TV, the evaluations would be less settled.

6 The expanded expression of the *VDF* equals the traditional TV expression in the DCF.

If the growth rate reflects the reinvestment at the beginning of the terminal period, the evaluations that use *FCF* as an attribute in TV are identical to the ones that use *NOPAT* with a growth equaling to zero. This happens, because the current growth value is zero if *RONIC* is equal to the cost of capital in the TV. Thus, potential errors in the evaluations can be avoided by setting the growth rate to zero and simply discounting the *NOPAT* of the terminal year to the *WACC*.

Hitchner and Mard (2003) defended the *VDF* approach, among several hypotheses of setting the TV. This traditional expression without growth in perpetuity does not imply that the nominal growth of *NOPLAT* has to be zero. However, it implies that growth does not add value, because the return associated with growth simply equates to the cost of capital (*WACC*). To Damodaran (2006b), this form of calculation of TV, which determines the rate of reinvestment of the enterprise, should reflect the expected growth rate and the rate of return on capital. ROIC and growth rate are twin drivers responsible for value creation, but rarely have an equal importance.

Buus (2007) defined the TV as the value of the equity of a firm at some future point in time, in which it is assumed that the enterprise has a stable growth of profits and investments or if it is converging to a growth rate towards a stable state. The process of TV computation involves the calculation of the investment in working capital and fixed capital through a proportion of the growth rate of the CF. This author assumed the equivalence in the terminal period between the return on investment and the cost of capital, assuming also that the growth rate is equal to zero, that is, the flow to perpetuate does not grow during the terminal period.

Cassia et al. (2007) conducted a sensitivity analysis of the firm value with changes in growth rates during the terminal period. The length of the explicit period would be decisive, because it is where the enterprise has abnormal profits and competitive advantages. The errors in setting the duration of this competitive advantage, which are not evidenced in the evaluation model through an appropriate duration of that period, lead to the transmission of the errors of that evaluation to the second stage.

Cassia and Vismara (2009) identified three conditions to define the period of stable or infinite growth: (1) The *RONIC* is constant in that period and equal to the one estimated for the terminal year; (2) The incremental rate is constant so that the average ROIC varies only in the second stage as a result of new investments; and (3) The reinvestment rate (net investment on operational profit) is identical to the reinvestment rate in the last year of the explicit period. As the reinvestment rate is constant in the second period, the *NOPAT* growth coincides with the growth of *FCF*. The critical point lies in predicting the finite horizon for Cassia and Vismara (2009). The criterion for verifying the transition to the second stage (terminal period, infinite or of stable growth) implies an equality between the rate of ROIC profitability and the cost of invested capital (*WACC*). In short, an ideal constant growth rate of CF in the infinite period corresponds to the product of *WACC* by the coefficient of reinvestment estimated for the last year of the explicit period.

More recently, Jennergren (2011) argued that the value driver original expression of Koller et al. (2010) is not very significant when compared with the expression of Gordon's TV. He concluded that the value driver is only relevant for new projects that present working capital needs which are different from the existing projects in the explicit period and that remain in post-forecast horizon. Even in this case, Gordon's perpetual formulation gives the same results, thus decreasing the interest in the value driver. He showed the computation of the TV (in each of the expressions reviewed by Gordon) as the sum of two expressions of Gordon instead of one, keeping, in that way, the stability situation substantially intact, which is a characteristic of the terminal period. He indicated that the value driver is just another way to write Gordon's original expression, because the numerator of the value driver is the *FCF* of period $t + 1$, equaling the traditional expression of the perpetuity of *FCF*. If *RONIC* is identical to *WACC* in the terminal period, the TV boils down to *NOPAT*/*WACC*.

The TV Is Not Perpetual, It Has an End

Tuller (2008) considered the TV concept as one of the major problems found in the evaluation of companies. He claimed that the larger and diversified a company is, the more important the calculation of TV is. He defended generically that capital intensive enterprises, with product and quality managers' diversity and a strong market presence, will theoretically produce infinite CF. Only in companies where their continuing success depends on specific qualities or skills of the managers or owners of capital, the TV may not be pertinent. Tuller (2008) indicated that we should use a series of up to 100 years, assuming that the CF of the last year would be the typical attribute for the TV and that value should be multiplied by the number of years in the terminal period. In the given example, he estimated the terminal period in 50 years. He assumed that the discount rate is the same for each period.

After being presented the main lines of theoretical orientation, the main conclusions of the theoretical/empirical studies that have focused on the quantification of the TV or continuity are summarized in Table 1.

Table 1

Main Conclusions of Empirical Studies on the Quantification of the TV

Author	Model	TV
Bernard (1994)	RIM	For this author, the TV is zero from a given time by the inability to produce residual results eternally.
Kaplan and Ruback (1995)	DCF with *CCF*	The perpetuity growth for the terminal CF, assuming the investment expenditures at least equal to amortizations and a growth rate in perpetuity that will reflect either the inflation or the real growth of that *CCF* and the tax savings derived from the tax deductibility of interest.
Bernard (1995)	RIM	Used the previsions of value line to get the premium quotation estimated over the BV on the horizon of the 5th year in order to get the TV, demonstrating that the RIM calculated accordingly explained 80% of the cross-section variation of the quotation.
Berkman et al. (1998)	DCF	Assuming the *VDF* of Koller et al. (2010), but with a null growth.
Penman and Sougiannis (1998)	RIM	The TV appears to correct errors resulting from having truncated the horizon. All models are particular cases of the DDM with a specific TV.
Francis, Olsson, and Oswald (2000)	RIM	TV with perpetuity.
Courteau, Gray, Kao, Keefe, and Richardson (2007)	DCF/RIM	Showing that DCF and RIM with TV computed by future quote calculated by value line have better performance than the DCF and RIM with TV calculated from the traditional mathematical formulations of perpetuity.
Fernandez (2007a; 2007b)	DCF (APV)	To calculate the *WACC* in TV establishes the debt objective ratio (relation of the debt and equity to total assets) over BV for those items and not for the market value.
Jennergren (2008)	DCF	This author reformulates the expression proposed by Gordon, including in his analysis the tax effect of depreciations of investment expenditure at infinity and the effect of inflation on those expenditures, considering also the useful life of the investment.
Pascual and Jiménez (2009)	DCF	Sales multiple.
Cassia et al. (2009)	DCF	Considering a growing perpetuity of *FCF*. The incremental return on the new investment (*RONIC*) is constant during stable and identical to ROIC at the beginning of the terminal period, and the average ROIC in the terminal period only varies in response to new investments. The investment rate is constant in the terminal period and identical to the rate observed at the end of the explicit period.

Conclusions

Despite the differences of opinion on methods of firm evaluation and their application, there is some convergence of ideas between academics and practitioners, according to Rogers (2009), on the following topics: (1) The core values in the evaluation of an enterprise are its expected flows, its growth, and the associated risk; (2) There is value creation if ROIC exceeds *WACC*; (3) Businesses that reach high ROIC tend to reach normal and stabilized rates of ROIC through competition; and (4) There is an equivalence among models that deduct future incomes (*FCF*, dividends, results, etc.), provided the imputations are consistently incorporated in the models. On the other hand, there is still a disagreement on issues such as the lack of attention devoted to explaining the firms' valuation by using the traditional financial models. The TV is no more than a very random and volatile way of predicting future uncertain behaviors of the firm and that, according to the models, comes down to a perpetuity of an attribute with a growth rate and an average cost of capital on the assumption that enterprise enters a steady or state of equilibrium, which unfortunately does not happen, or at least in the implicit amount of years within the TV.

Pennman (1998) indicated that it is worth studying the forecast in an infinite horizon, because there is no consensus in the scientific community about the method of computation of the TV. The size of the terminal, or non-explicit period, assumed as infinite, is never called into question by scientific literature, or the probability of business bankruptcy. There is only a reference made by Morris (2009) on the firms' life and the inclusion of a probability of bankruptcy of those in the financial models. This author made a tenuous attempt to incorporate that predicted mortality. In the same way, Adams and Thorton (2009) provided empirical evidence relating to the positive relationship between a firm's age and its value. After the explicit period, all the commonly used models present the TV to justify the permanence of the enterprise over an infinite horizon.

The vast majority of analysts and academics incorporate globally, in the specific risk of action premium, all factors that may influence the value of the enterprise. Either because of capital asset pricing model or the arbitrage price theory, analysts seek to incorporate in the calculation of cost of equity some of the main aspects that potentially affect profitability, namely, the inherent systematic or non-diversifiable risk in the market. The market risk is also incorporated in the specific risk premium and here is where literature puts all the other variables in a "bag". The academics as well as practitioners only focus on economic variables trying to anticipate behaviors based on cycles of growth, maturity, and decline, associated with the theory of a product cycle. Therefore, if evaluations all aim to evaluate an infinitive cycle of CF, how can a crisis condition the whole, infinitive, enterprise's life? Will there be no additional factors that explain the value of firms? Should those be highlighted and made autonomous from the discount rate and from the level of considered risk? There will be other attributes that explain the TV that may be identified and that will include not only financial variables inherent in the enterprise, but also non-financial variables, also defended by Laitinen (2004). Questioning the current format of calculating the TV of a firm opens a wide range of possibilities for a new mathematics and theoretical formatting, with the possibility of being empirically tested. This research effort should merit, in our opinion, a strong attention to future work.

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