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Weather Variability and Economic Analysis

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INTRODUCTION

Weather affects the profitability of many agricultural enterprises. Its effects might be immediate, as demonstrated by a rain shower that halts harvest operations, or long run, such as a long drouth that dries up shallow wells and small streams. In either case, most weather variables affecting agricultural production are random and unpredictable in nature, thereby surrounding the farmer with an abundance of uncertainty.

Consideration of weather in analyses of agricultural or other types of problems is long overdue. A recent report of a National Research Council Committee (22) states:

> Nor is the atmosphere the setting for physical processes only. It is also the milieu for every biological process on the face of the earth. It is, indeed, the environment into which we are born; it is the environment in which we live and carry on our multifarious human activities; and it is the environment which each year claims some of our kind before their normal life expectancy has been reached. From such a point of view, a proper understanding of this milieu takes an enormous economic importance. Do we wish to fly an airplane? Do we wish to grow a crop? Do we wish to ship perishable goods in an unheated or unrefrigerated conveyance? Do we wish to sail a ship? Do we wish to live securely by the ocean's edge or on the rich flood plain? Do we wish to regulate our supply of natural gas at the source so that homes will be heated and complicated gas transmission systems will be efficient? Do we wish to enjoy outdoor recreation? Do we need to fight a battle? Do we—but the list is endless. The value of applying our still admittedly incomplete knowledge of the atmosphere to climatic analysis and weather forecasting has been placed at more than a billion dollars a year.
>
> While weather forecasting is but one aspect of meteorology, it is the cruicible in which our understanding of the atmosphere and its mode of operation is tested. To the extent that we cannot observe and cannot understand and cannot explain atmospheric processes, our forecasts of the future consequences of these processes will be imperfect—and the poorer our understanding, the less perfect the forecasts. By the same token, however, increments of knowledge will mean increaments of accuracy and, superimposed on the by-no-means negligible level of current accuracy, each increment of improvement in weather prediction will have increasingly greater economic significance. Ultimately, it may be expected that business, industry, and agriculture will adjust their activities to utilize dependence on weather rather than seek to avoid dependence on weather. Agricultural planning is only one example of such an activity. One is led to the inescapable conclusion that our nation's best economic interests require a more active attention to the meteorological problem. Indeed, the question might well be raised: "Can we afford not to?"

The manager of a farm or business is responsible for many long range strategic and many short range operational decisions, each involving some degree of uncertainty concerning the eventual profit or loss resulting from the decisions. Management, in fact, is necessary only in the presence of uncertainty. When the outcome of a business operation is certain, all that is required is for the owners and employees to follow a set of instructions leading to the goals of the business, such as profit maximization or maximization of the value of the physical plant. In the real world, however, decisions must be made in the presence of uncertainty. While a manager might prefer to delay a decision until the outcome is more predictable, he evenutally must make the decision if business operations are to continue. Thus, the basic function of management is to make strategic and operational decisions in such a manner that the business is able to survive and, if possible, prosper in the presence of uncertainty.

Historically, most farmers or managers of other business enterprises faced with decisions affected by random weather inputs have developed techniques or "rules of thumb" that represent a type of applied meteorological analysis. With few exceptions, managers of these enterprises still use these same rules when making weather-related decisions. In recent decades, the science of meteorology has advanced rapidly, making a wealth of weather information available to managers. The purpose of this study is to develop a method of analysis that will allow a manager to use meteorological information to arrive at a "best" decision for a particular enterprise. The method presented is general—managerial decisions that are most effective for one enterprise are not necessarily best for another enterprise, or for the same enterprise in a different climate.

Year's Fuel Supply in One Day—Free

To illustrate the importance of weather inputs in agriculture, consider a farm which has 100 acres of corn land. To produce a crop of corn, the manager of this farm would have to provide energy, in the form of fuel for tractors, electricity, and manual labor. Energy from the sun is also required to heat the soil, evaporate water from the soil, heat the air, and activate the complex photosynthesis process in the growing leaves of the corn plants.

On a sunny day in June in Central Missouri, 100 acres of land would receive about 8,820,900,000 B.T.U.'s of heat from the sun. If one gallon of gasoline equals 124,000 B.T.U.'s, this would be equal to 71,136 gallons. It has been estimated that the photosynthesis process makes use of about 1 percent of the energy from the sun. If this figure is accepted, the photosynthesis process in 100 acres of corn would take the equivalent of 711 gallons of gasoline in one day. This would almost equal a year's fuel supply for the tractors used on 100 acres of corn!

An even larger percentage of the energy from the sun would be involved in maintaining soil temperatures, evaporating water, and heating the atmosphere. Thus, it is apparent that a farmer growing 100 acres of corn has control of only a very small fraction of the total energy required to produce a crop.

The Specific Problem

It is relatively easy to set up a theoretical model, such as the one that is presented later in this bulletin. It is more difficult to apply such a model in the analysis of the factors involved in a specific decision. There are few enterprises where careful records have been kept in a form that permits analysis. Where such records have been kept, they rarely are available for more than a few years. When more than just a few years of records are available, management practices have often changed several times, making the analysis of specific decisions more difficult. Further, weather data have often not been recorded at the location of the enterprise in question. Weather data are available from several of the agricultural experiment stations in Missouri, as in most other states, but are rarely recorded on privately operated commercial farms.

Before a specific weather-related decision was selected for analysis, a list was composed of probable decisions facing the manager of a farm which includes corn as one of its enterprises. This list was developed following discussion with personnel at the University of Missouri.

1. Should corn be grown on this farm?
2. What rotation of crops, including corn, would be best on this farm?
3. What soil conservation practices should be used on this farm?
4. What machinery and buildings are needed for a corn enterprise on this farm?
5. Should irrigation of corn be attempted on this farm?
6. How many acres of corn should be grown on this farm during the coming season?
7. What fields should the corn acreage include?
8. What variety of seed should be used?
9. What general fertilizer program should be used this season?
10. What changes in machinery and labor use will be necessary in this corn enterprise this year?
11. When should the cornland be plowed?
12. When should the final seed-bed preparation be done?
13. How should the final seed-bed preparation be done?
14. Should the fertilizer program be modified at the time plowing is being done?
15. When should planting be done this year?
16. What rate of planting should be used?
17. Should the use of starter fertilizer be modified at planting time?
18. Should pre-emergence spray be applied at planting time?
19. Should pre-emergence irrigation be used?
20. When should the corn be cultivated?
21. How many times should the corn be cultivated?
22. What change in machinery should be made during cultivation?
23. Should plans for the use of side-dress fertilizer be modified during cultivation?

24. Should chemical weed and insect control be used during cultivation?
25. Should the corn be irrigated during the tasselling-fruiting phase?
26. Should chemical insect control be applied during the tasselling-fruiting phase?
27. When should the corn be harvested?
28. What harvesting machinery should be used?
29. Should the crop be sold immediately or stored on the farm?
30. Should grain drying equipment be used?

The first five are long-range decisions involving more than one season. Several of the remaining decisions are both long-range and seasonal, and some involve only one season. Some of them, once made, are irrevocable for a long period of time; others can be revised any time before the actual occurrence of the event. Each of these decisions had to be made by considering some aspect of the weather.

Decision number 30, "Should grain drying equipment be used?" was chosen as an example for several reasons. Brooker and McQuigg (2) had developed a method of relating weather effects on the in-storage drying process that was suitable for synthesizing a long period of "experience." Costs and returns in terms of present day prices could be associated with this experience in a form that would permit economic analysis.

Grain drying equipment is becoming more common in corn producing areas. Many corn farms now involve large quantities of capital investment in seed, fertilizer, land, and machinery. Many managers would prefer not to leave the last stages of maturity of the crop to the whims of fall weather.

The installation of grain drying equipment and the accompanying loading and unloading facilities requires some time. A manager would expect to use such equipment during several crop seasons. Therefore, the decision to use or not use grain drying equipment is a long-range decision. Decisions related to the operation of equipment during a specific year are short term operational decisions not considered in the analysis that follows. The essential question to be analyzed is: "From a long range viewpoint, will it pay me to buy, install, and operate grain drying equipment on my farm?" The method of analysis, however, is applicable to many other problems.

REVIEW OF LITERATURE

An extensive body of literature in the field of decision-making and operations research has been published since World War II. Wasserman and Silander (29) have collected a review of the literature in this field. Most of the work on decision-making theory apparently has analyzed problems connected with industrial, engineering, political, and personnel management. None of the references quoted in Wasserman and Silanders' bibliography specifically discussed a weather-related decision.

Analysis of weather-crop and weather-livestock, weather-industry, and weather-commerce relations is needed if economic analyses are to be meaningful. A bibliography compiled by G. L. Barger (1) includes much of the work that has been done in this field.

Research in production economics is usually directed toward the maximization of profits, or the minimization of costs. The usual treatment in analyses that involve a weather input is to assume that weather conditions will be "average." An article of this type is the one in which Peterson and Swanson (21) analyzed estimated annual yield and fertilizer and labor requirements for eight crop rotations and two farm sizes, using linear programming methods and a modern electronic computer. One of the assumptions of their analysis is that weather and growing conditions are normal.

Munson and Doll (18) discuss several methods of analyzing crop yield data resulting from various fertilizer combinations. They point out that in addition to the usual analysis of fertilizer applications and yields, other factors, such as cropping sequence, weather, and residual fertility, must be considered. Some other recent work has considered weather as a source of yield variability. Brown and Oveson (3) present ten years of wheat response to various amounts of nitrogen per acre. These data show considerable variation from year to year in response to nitrogen treatment. They mention that about 40 percent of the variance is due to nitrogen applications and the remaining 60 percent is due to "unexplained causes, such as differences in available moisture and climate from year to year."

Orazem and Herring (19) analyzed six years of experimental data, relating grain sorghum yields in southwest Kansas to soil moisture at planting time, rainfall during the growing season, and various levels of nitrogen application. This analysis shows that all three factors are important in determining final yields, but that the strongest relationship exists between soil moisture at seeding time and yields. They point out that recommendations of the best level of nitrogen application can be made only when the soil moisture at seeding time is considered.

Knetsch (15) analyzed the effects of nitrogen applications on corn yields at the Tennessee Agricultural Experiment Station, Jackson, Tenn. He related corn yields, nitrogen application, and the number of drouth days during the various critical stages of development of the corn crop. Drouth days were defined by a method developed by van Bavel (28). The highest correlation with yields and the number of drouth days occurred during the 30 days following tasselling. An analysis of 30 years of weather records by Knetsch and Smallshaw (16) provided probabilities of occurrence of various drouth intensities. Expected returns from several levels of nitrogen application were computed and recommended levels presented. This analysis was based on three years crop data and applied to 30 years of weather records.

Stallings (24) analyzed the variation in yields due to weather. He did not base his analysis on weather data but attempted to remove all sources of variation in yield except weather and attribute the remaining variation to weather.

Indexes of the influence of weather on national production of major crops in the United States were developed for use in aggregate model building.

Parks and Knetsch (20) included a measure of effects of drouth intensity on the response of corn yields to various rates of nitrogen. These effects are computed by using a set of equations which the authors developed. Their equations were based on three years of crop data. This analysis did not take into account costs or returns.

Halter and Bradford (11) used weather and crop yield data for the period 1927 through 1954 to develop a regression of yield on the number of drouth days. This regression was used in an analysis of the changes that had occurred in certain management practices from 1953 to 1957. To make farm production data comparable, corrections were made for change in price and for variations in weather.

Williamson and Riley (30) analyzed the effects of weather, defoliation, and mechanical picking upon effects cotton quality. Their analysis does not include cost and return data, but it does point out some economically useful weather-production patterns.

Thompson and Brier (26) developed a method of relating the economic factors involved in the use of weather forecasts, and a way to determine the economic utility of weather forecasts. This method was based on a comparison of the ratio of the cost of protection to the loss that would occur if the event took place and no protection were provided, with the forecast probability that the adverse weather would occur. This method dealt with short-range decisions and costs, but not economic returns.

Gringorten (10) has commented on the problem of evaluating weather forecasts in an economic sense. He presents a model in the form of an income matrix which relates the profit or utility of each possible course of action to the various states of weather that can occur. He computed an excepted operational gain for each course of action. This analysis is concerned with profits, or utility, and with various methods of estimating, or predicting, the probabilities of occurrence of weather events.

Gleeson (9) has developed a method which also relates economic gains and losses to weather events. He presents two techniques, one of which is to be used when the probabilities of weather events can be forecast or computed from weather records, and the other, based on game-theory, to be used when the probabilities of weather events are not known. Gleeson suggests using the method which yields the highest expected return. His techniques are not limited only to short range decisions, but can be used for both short-range and long-range decisions and can be used in connection with either meteorological forecasts or climatologically determined probability estimates.

The major conclusion obtained from the survey of literature is that only a beginning has been made in the analysis of weather-economic decision problems. The purpose of the remainder of this study is to develop further some of the basic ideas found in this review and apply them to a specific decision.

DEVELOPMENT OF THE ANALYTICAL MODEL

A production function representing the yield forthcoming from a production process can be written

(1) $Y = f_1 (X_1, X_2, X_3, \ldots, X_n)$

where Y is the resulting yield and the X_i ($i = 1, 2, 3, \ldots, n$) represents the inputs. Some inputs are fixed and some are variable for a given time period; over time, the manager of the firm must make appropriate decisions regarding each input if the firm is to survive.

Some of the inputs used in the production process may be controlled in amount by the manager. In this case, input use is governed by the well-known marginal principles leading to minimum cost and maximum profit combinations. Economic literature is replete with logical and empirical analyses of this type.

Other inputs involved in the production process, such as weather inputs in agriculture, may be random in nature.[1] These inputs may have significant effects upon yields and costs even though they cannot be controlled by the manager. He must take what comes. Because of this, he is unable to equate the marginal value product of the random inputs to the cost of the random input. In fact, in many cases, such as rainfall, which usually has a positive marginal product, or hail, which usually has a negative marginal product, the random weather input has no cost to the manager.

The basic logic underlying revenue and costs for a production process involving controlled inputs is well developed. The next step would be to introduce the random weather input into the existing analysis and examine the results. For this purpose, assume the production process involves one input that may be controlled, X_c, and one random weather input, X_r. The results may be generalized to more variables. The random input, X_r, may have an effect on costs, returns, or both.

The production function to be considered now is

(2) $Y = f_2 (X_c, X_r)$

where X_c and X_r are both variable for the time period under consideration. While the following analysis deals primarily with variable inputs that are completely utilized in one production period, X_c could be a durable input lasting over several seasons. In either case, the same general principles will apply. Also, fixed inputs may or may not be relevant for a particular analysis, but usually will need to be considered for most empirical applications in agriculture.

No Interaction Between Controlled and Random Inputs

If the first derivative of production function (2) taken with respect to X_c, that is, the marginal product equation for X_c, is of the general form

(3) $\frac{\partial Y}{\partial X_c} = f_3 (X_c)$

[1]While day-to-day weather is undoubtedly correlated, many meteorologists believe that monthly or yearly weather variations are random. For further discussion of this, see Friedman (8) and Thom (25).

then the inputs X_c and X_r do not interact. The absence of interaction between the input variables insures that the profit maximizing amount of X_c will be the same regardless of the magnitude of X_r. Thus, the profit maximizing amount of X_c is determined by equating the marginal value product of X_c to the cost per unit of X_c as follows

$$(4) \qquad (P_y) \frac{\partial Y}{\partial X_c} = P_c$$

where P_y is the price per unit of output and P_c is the cost per unit of X_c. The appropriate stability conditions must be present, of course, to insure a maximum rather than a minimum of profit.[2]

Thus, conventional profit maximizing procedures can be applied to X_c when X_c and X_r do not interact. In this case, the manager would be irrational not to apply the marginal principles of resource allocation to X_c. However, the random weather variable, X_r, does affect costs, revenue, and eventual profit.

When X_c is held constant at the profit maximizing level, production function (2) may be expressed

$$(5) \qquad Y = f_4 (X_r)$$

and revenue, R, can be written

$$(6) \qquad R = P_y f_4 (X_r).$$

Revenue may increase, decrease, or remain constant as X_r increases. R could be total revenue for the firm or, in a partial budgeting situation, the revenue added by the enterprise being considered. Various possibilities are shown in Figure 1.

In a similar manner, a total cost function, C, for X_r could be written

$$(7) \qquad C = g_1 (X_r).$$

C could be the sum of all fixed and variable costs involved in the production process plus any costs associated with X_r. Or, in partial budgeting, C would be the costs associated with X_r. Because X_r is not controlled by the manager, changes in C associated with changes in X_r are not necessarily the costs of procuring X_r. Such changes could represent costs resulting from the occurrence of a given quantity of X_r, such as increased harvesting costs incurred as rainfall increases yield; they could represent the cost of protecting against the occurrence of X_r, as in the case of levees or hail insurance. X_r might have no effect on costs. To affect profits, it is necessary only that X_r affect either revenues or costs, not both. Various effects of X_r on costs could be shown by the curves in Figure 1.

Cost and revenue functions for X_r are combined in Figure 2. Obviously, functions differing in form from the ones presented are possible. Profit, P, is

[2]This problem can be presented in a game theory context by regarding the manager, controlling X_c, as the maximizing player with the weather, controlling X_r, as the minimizing player. When X_r and X_c do not interact a situation exists wherein one strategy, the economic optimum, dominates all other strategies available to the manager. Game theory is not used in this paper because (1) game theory assumes no knowledge of the probability of a competitor's actions—an assumption that does not hold true in weather problems when frequency distributions can be determined, (2) we wish to integrate our analysis with the theory of the firm analysis and consider alternative solutions. There are many references on game theory, for example see Dorfman, Samuelson and Solow (6).

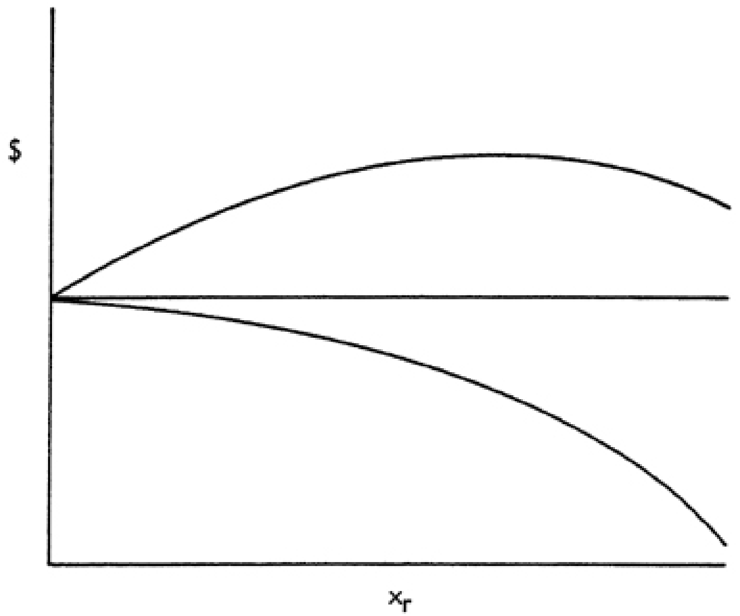

Figure 1

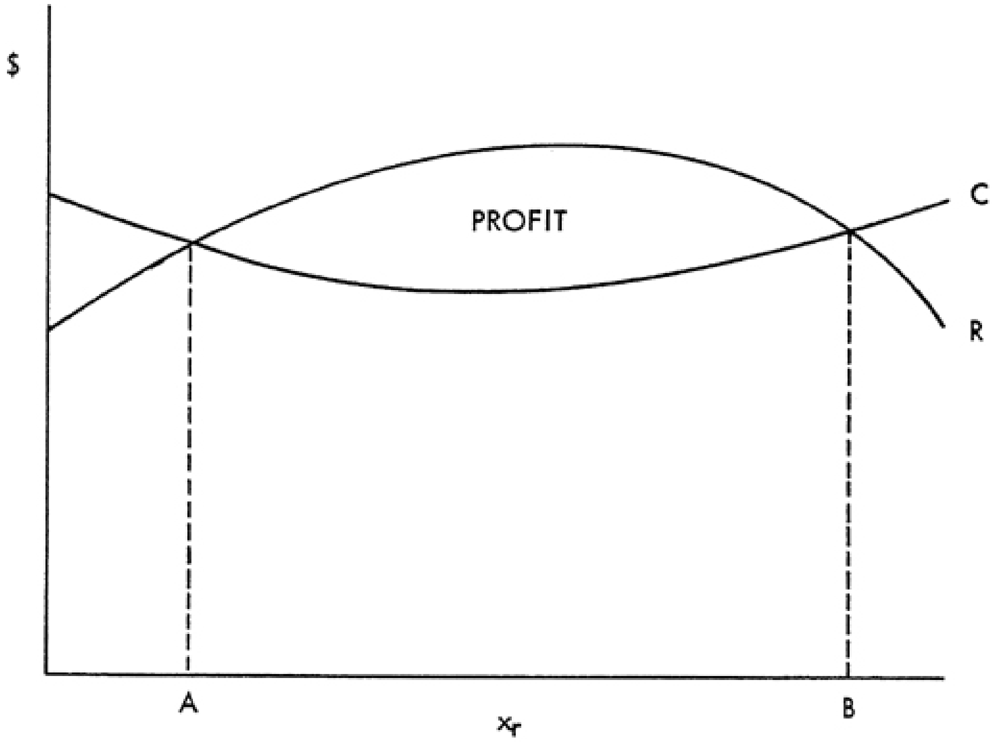

Figure 2

equal to R minus C. In Figure 2, A and B represent limits of X_r within which a profit is made. A and B are break-even points, where R - C = O. If a minimum allowable profit, K, were desired, perhaps because of an opportunity cost, the limits could be shortened so that for each limit, R - C = K.

Thus far, to effectively analyze the effects of a random weather variable upon a production process, a need for the following information has been shown: First, the specific weather input affecting the process must be known. This suggestion is not as naive as it seems. Weather inputs have been subjected to so little rigorous study that managers are often uncertain or unaware of the specific weather input that affects their enterprise. Second, the cost and revenue functions associated with the weather input should be known. Third, the amounts or limits of the weather input quantities within which a profit can be earned must be determined.

A manager has two types of meteorological knowledge to use in combination with the three needs outlined above and presented in Figure 2. They are weather forecasts and historical climatological data. A weather forecast can be categorical, stating that X_r will have a definite value. If the forecast is accurate, an analysis as in Figure 2 will reveal the effect upon profits.

However, managerial decisions must often be made weeks or months in advance. When the time interval between the decision and the occurrence of the event is beyond the capabilities of the forecast, or when durable assets are being purchased, historical climatological data must be used.

Thus far, even if cost and revenue functions were known, the manager would still not be informed about the expected level of profit. To determine expected profit, the probability distribution of the random weather input, X_r, must be known. While the true population distribution of X_r will never be known, it can often be estimated from historical climatological records. Therefore, if cost and revenue functions for X_r are known as in Figure 3a and X_r is distributed as in Figure 3b, the manager could expect a loss almost 100 percent of the time, because almost 100 percent of the values of X_r fall outside the limits A and B. If X_r were distributed as in Figure 3c, the manager could expect a profit almost 100 percent of the time. Thus, the area of the probability distribution occurring between A and B is extremely important. In most practical situations, the relationship of the probability distribution to the cost and revenue curves for random input variables undoubtedly falls between the two extremes presented in Figure 3. It is worth noting, however, that little research in agricultural technology has been devoted to determining either the limits for an enterprise or the probability of the random input occurring within these limits. Usually, farmers or applied farm management men have been left to determine such facts for themselves.

When revenue and cost functions, along with the probability distribution, for X_r are known, the expected profit, E (P), resulting from the production

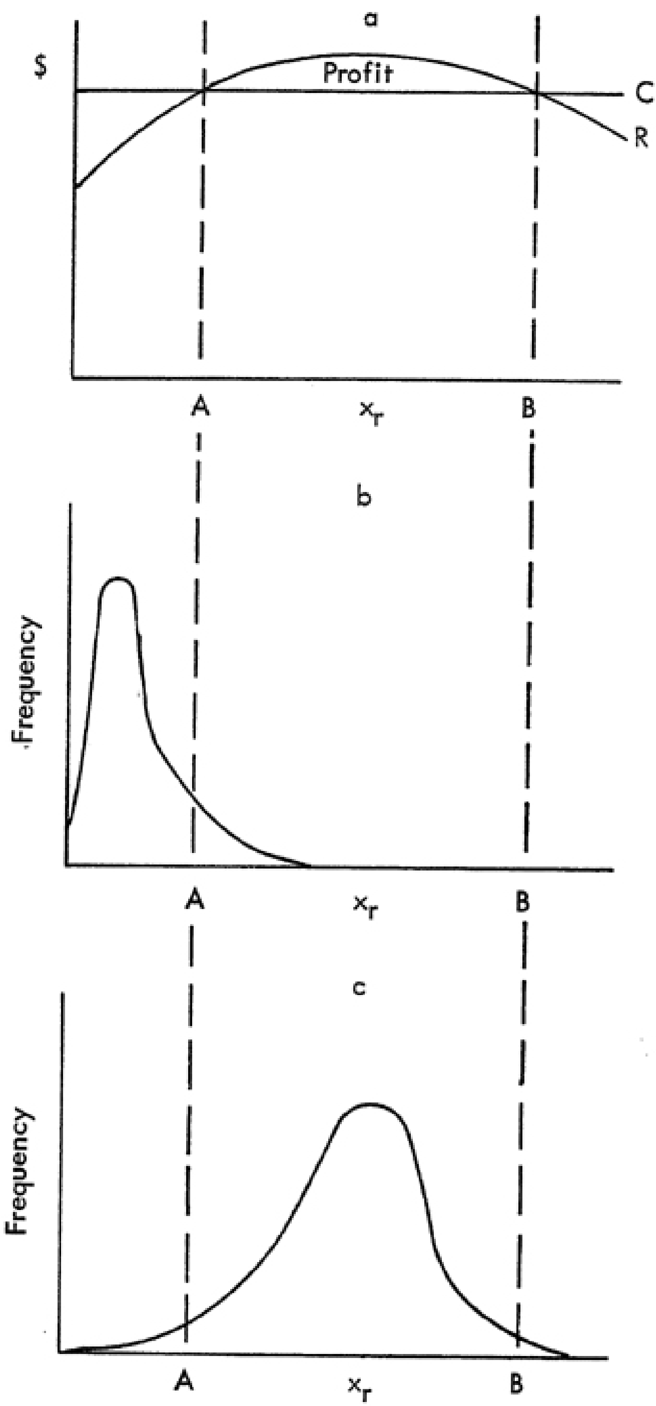

Figure 3

process can be computed using usual methods of computing mathematical expectations.[3] If E (P) > K, where K is defined as above and may be zero, then the production process would be profitable over time. When several alternative technologies result in E (P) > K, the most profitable alternative will be selected.

When E (P) ≤ K, the production process is unprofitable over time. In this situation, the manager has two general alternatives. First, he may adopt different technologies, where technology means all of the production practices used. The most readily apparent solution would be the adoption of practices that would increase the profit area by shifting or changing the shape of the revenue and cost functions; this might involve increasing production, decreasing costs, increasing production more than costs, etc. The effects of these changes would be to increase the probability of making a profit (increase the distance between A and B) or to increase the magnitude of profits when earned. In the latter case, notice that E (P) could be increased even when the area of the probability distribution between A and B—the probability of making any profit—is reduced. Another technological change would involve a shift in the timing of the operation to take advantage of different climatological probabilities. Still another would be to abandon the procedure involving X_r and substitute another input; thus, a farmer in a low rainfall area may decide to purchase irrigation equipment. Other changes in technology may be dictated, depending upon the nature of the enterprise. As an alternative to a change in technology, the manager may decide to completely abandon the enterprise.

The discussion above indicates that the manager is faced with many alternative goals when dealing with random weather inputs; he may act to maximize expected profits, maximize the probability of making a profit, etc.[4] These may not always be mutually exclusive categories. In the short run, the manager possessing limited operating capital for reserves may prefer to maximize the probability of making a profit in order to meet family living expenses, fixed mortgage payments, or other expenses needed to insure the short run survival of the firm and the family. In the long run, these goals would appear to be most readily accomplished by maximizing E (P).

Practical interpretations for analyses similar to that in Figure 3 are numerous. Managers with high cost or low revenue functions not only decrease profit

[3]Expected profits, E (P), is equal to the sum obtained by multiplying the amount of profit resulting from each amount of X_r by the probability of occurrence of that amount of X_r and summing over all possible values of X_r. For example:

Amount of X_{ri}	Probability	Profit
X_{r1}	2/6	$12
X_{r2}	3/6	24
X_{r3}	1/6	42

E (P) = 2/6 (12) + 3/6 (24) + 1/6 (42) = $23. Expectations for continuous functions can be derived using the probability density function and integral calculus.

[4]Other alternatives may also be possible and rational. In conventional game theory, for example, the manager is pictured as selecting the alternate that insures a maximum-minimum return. Or, he may wish to maintain a certain balance in his savings account, etc.

but could also decrease the probability of a profit; thus, efficiency in farming can benefit the farmer two ways. Also, high opportunity costs decrease the probability of a profit, that is, of $E(P) > K$. High product prices or yield increasing technologies increase both profit and the probability of a profit. Readers will undoubtedly develop other examples. The shapes of the relevant functions must always be specified.

Interaction Between Controlled and Random Inputs

If the first derivative of the production function taken with respect to X_c is of the general form

$$(8) \quad \frac{\partial Y}{\partial X_c} = f_5 (X_c, X_r)$$

then the inputs X_c and X_r interact.

In this case, the optimum amount of X_c will vary depending on the magnitude of the random input. Thus, the manager is faced with the dilemma of utilizing a small amount of X_c and foregoing a profit many years or of utilizing a larger amount of X_c and incurring a loss many years. This problem can be solved by determining A, B and E (P) for each quantity of X_c, assuming a finite number of possibilities. Such a solution is pictured in Figure 4 for three levels of X_c. In Figure 4a, X_c is utilized at the rate of two units, where "unit" is purposely ambiguous and could be any measure, and a loss is incurred regardless of the occurrence of X_r. Figure 4b and 4c are drawn so that the probability of a profit is greater when X_c is four than when X_c is six, but E (P) is larger for X_c equals six. In this case, the manager should choose four units of X_c if he wants to maximize the probability of a profit and six units of X_c if he wants to maximize E (P), long run profits. Again, other examples could be chosen so that the maximum E (P) and probability of a profit could occur at the same level of X_c. When the inputs interact, the manager makes his decision by comparing expected profits and the probability of a profit for various levels of X_c[5]. Thus, the solution is not appreciably different from that used in selecting technologies when inputs do not interact. The assumption here is that the best technology is selected before X_c is determined. The reader may wish to relax this assumption; the best level of technology could vary with differing amounts of X_c.

The profit maximizing amount of X_c referred to in this section means the amount of X_c resulting in the largest E (P) and not the amount of X_c at which the marginal product of X_c equals the unit cost of X_c. In fact, the latter amount

[5]In game theory terminology, the choice of an amount of X_c, whether to maximize E (P) or the probability of a profit, implies the use of a pure strategy. The alternative of using two or more amounts of X_c in a given ratio, the rate for any one year selected at random, has not been discussed. While game theory and thus the use of mixed strategies is applicable only when competitors' probable actions are not known, the problem is complicated when probability estimates are sample estimates rather than population values. See Gleeson (9).

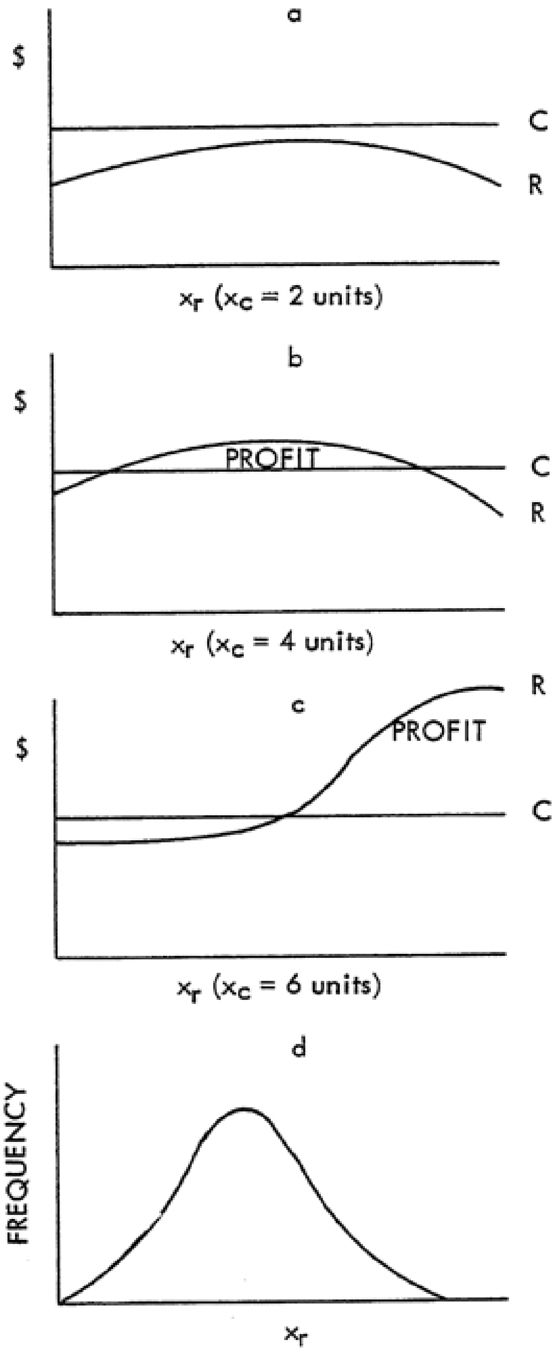
a
$
C
R
xr (xc = 2 units)
b
$
PROFIT
C
R
xr (xc = 4 units)
R
c
PROFIT
$
C
xr (xc = 6 units)
d
FREQUENCY
xr

Figure 4

of X_c can not be determined unless the magnitude of the random weather input, X_r, is specified. When the weather variable interacts with the controlled input, there seems to be little logic in referring to the profit maximizing amount of X_c, where the term "profit maximizing" is used in the traditional sense.

APPLICATION OF MODEL TO A GRAIN DRYING PROBLEM

The decision, "Should grain drying equipment be used?" was chosen for detailed analysis for reasons mentioned above. A farm manager faced with making this decision would know or be able to determine some of the physical or cost figures which will be assumed in the following analysis. He would know, for example, the number of acres of corn he has and he would have a notion of the variability of his corn yields. Also, he can determine, by shopping the local markets, the costs of purchasing and operating drying equipment. The main consideration in the following presentation is to demonstrate a method which could be used to analyze such a problem, even though specific acreages, yields, costs, and prices differ from those used here. A partial budgeting approach will be used; only costs and returns added by the grain drying equipment will be considered.

Ideally, the decision to use or not to use grain drying equipment on a farm or in a given farming area would be based on experience gained from years of experimental operations of such equipment in an area with similar climatic conditions. This experimental operation would include costs and returns, as well as physical data. It would include data comparing the drying of corn under natural conditions in the field to the use of grain drying equipment.

The weather affects profits from grain drying two ways. First, weather conditions during the growing season affect yields and consequently the number of grain drying bins needed. Second, weather conditions after harvest affect the cost of drying. As mentioned above, an analysis of the effects of these weather variables would be facilitated if experimental data of the type needed were available. In the absence of such data, methods of synthesizing the data were used. The level and variability of corn yields for Central Missouri were estimated using 21 years of data from a fertilized, continuous corn plot on an experimental field.[6] The effects of weather conditions after harvest on the length of time required for drying corn in the bin was estimated using a mathematical model developed by Brooker and McQuigg (2). Their model, based on physical and engineering concepts, considers the temperature and moisture content of the air that is forced through the grain bin and the moisture content of the grain. Daily values of the drying capacity of the air were computed for Columbia, Mo., for September, based on 21 years of weather records. These daily values can be used to obtain the frequency distribution of the number of days required to dry grain from an initial moisture content to any desired moisture level.

[6]Corn yields are from plot 18, continuous corn, on Sanborn Field of the Missouri Agricultural Experiment Station.

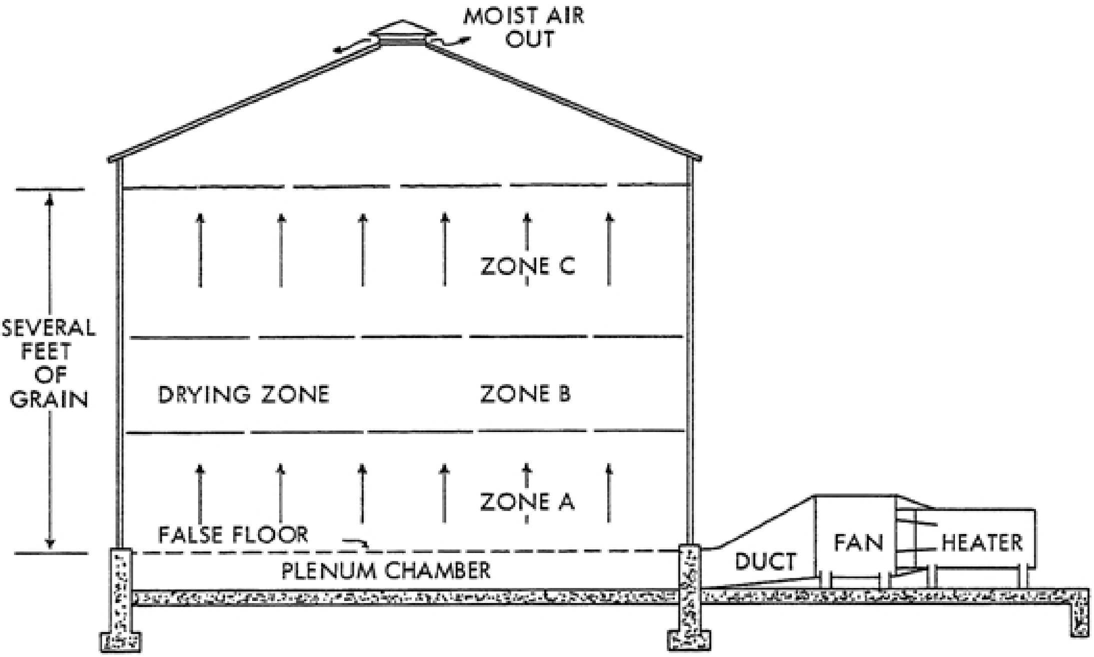

Figure 5—A typical farm grain drying system. Heated or unheated air, from the fan is forced into a plenum chamber, through a perforated metal false floor, and up through the grain. Zone B is a drying zone that is formed early in the drying period and "moves" from the bottom to the top of the bin. Grain in Zone A is dry and grain in Zone C has not yet begun to dry.

Assumptions

For purpose of illustration, two types of grain drying systems will be considered. One, a natural air drying system, forces outside air through the grain with a motor-driven fan while the other, a supplemental heat drying system, forces heated air through the bin. Both are the small, stationary type commonly found on farms; details of their physical layout are shown in Figure 5.

The analysis will be conducted for a farm with 120 acres of corn land that yields from zero to 100 bushels per acre, wet basis; that is, computed before harvest while the corn is still in the field at 25 percent moisture content. Each grain drying bin holds 2000 bushels of shelled corn.

The assumed cost of the bin, exclusive of drying equipment, is $700; depreciation is 5 percent of the original cost each year. The cost of a 5 h.p. electric motor, fan, duct work, and false floor in each bin is $900; depreciation is 10 percent of the original cost per year. The cost of a unit that will add 15° F to the temperature of the air going into the bin is $150 per bin, also depreciated at 10 percent per year.

The natural air drying bins will be loaded only once per season. Supplemental heat drying bins may be loaded more than once a season. Labor for operating the equipment is estimated at one hour per day at $1.25 per hour. Labor for emptying the supplemental heat drying bins is estimated to be six hours per bin unloaded at $1.25 per hour. No labor charge is included for filling the bins.

The cost of operating the 5 h.p. electric motor and fan is estimated to be $2.40 per day of operation of the bin, with electricity at $0.02 per KWH. The additional cost per day of operation for fuel for each supplemental heat drying bin, with LP gas priced at $0.15 per gallon, is estimated to be $4.80.

Total field loss using a picker-sheller is assumed to be 10 percent, when corn moisture is 25 percent. Loss increases to 15 percent when corn moisture is 17 percent (13).

Alternative Management Possibilities

There are many alternative methods the manager could utilize to harvest, sell, or store his corn crop. To analyze all of them would be impossible. In order to limit the problem to manageable proportions, the following two alternatives have been selected:

1. Corn is harvested with a picker-sheller at 25 percent moisture content, with a field loss of 10 percent. The shelled corn is dried to 14 percent moisture using either natural air or supplemental heat equipment. The dried corn is then stored under a government commodity loan at $1.14 per bushel. Harvest begins in September.

2. Corn is left in the field to dry to 17 percent moisture content and is harvested with a picker-sheller with a field loss of 15 percent. It is sold on the open market immediately with a discount to 15.5 percent moisture. The price received in this alternative will be left variable.

Under these alternatives grain drying could increase total revenue from corn production in two ways. First, field losses from harvesting are reduced, thus making available added amounts of corn for sale. Second, alternative 1, the grain drying alternative, permits the farmer to take advantage of the government storage program. If the price of corn on the open market is below the loan price, total revenue will be increased by using alternative 1. In actual practice, a farm manager using either alternative would be able to decide after harvest whether to sell on the open market, store in anticipation of increased prices later in the year, or take a government loan. Also, the use of grain drying equipment does not necessitate placing the grain under government loan. Obviously, the selection of these options would affect the profits accruing from grain drying. However, the purpose of this section is to apply the model developed above. For this reason only the two alternatives are presented.

Analysis of Costs and Returns

Natural Air Drying. The costs of using the natural air grain drying equipment described above can be expressed as

$$C_n = DN_i + (L + E) N_d n$$

where

C_n = Annual cost of owning and operating natural air grain drying equipment.

D = Annual depreciation of the bin and the drying unit.

N_i = Number of bins with drying units installed.
L = Labor cost per bin per day of operation.
E = Cost of fuel per day per bin.
N_d = Number of bin loads dried.
n = Number of operating days or days drying time.

Using the cost figures presented above, the cost function for natural air drying is

(9) $C_n = \$125\ N_i + \$3.65\ N_d\ n$

This equation presents cost as a function of inputs under the control of the operator and random or uncontrollable inputs. N_i, the number of installed units, can be determined by the farm manager, while N_d and n are dependent upon the weather during the growing season and the drying period, respectively.

Total revenue from alternative 2 is equal to the price per bushel of corn at harvest time multiplied by the number of bushels of corn remaining after the corn is field dried to 17 percent moisture and a 15 percent field loss is sustained, and after the resulting amount is discounted to 15.5 percent moisture at the time of sale.[7] The number of bushels of corn for sale in alternative 2, Y_2, is

$$Y_2 = \left(\frac{100 - 25}{100 - 17}\right)\left(\frac{100 - 17}{100 - 15.5}\right) 0.85\ Y_o = 0.754\ Y_o$$

where Y_o is the number of bushels of corn in the field at 25 percent moisture. For the second alternative, 2222 bushels of corn in the field will finally result in 1675 bushels available for sale. Total revenue from alternative 2 is

(10) $R_2 = P_2\ Y_2 = P_2\ (0.754)\ Y_o$

where P_2 is the price per bushel of corn on the open market at harvest time.

Total revenue from the use of drying equipment in alternative 1 is equal to the bushels of corn available after the drying process is completed multiplied by the loan price per bushel of corn. To fill a bin, 2222 bushels of corn standing in the field at a 25 percent moisture content are needed. After sustaining a 10 percent field loss, 2000 bushels or an even bin load remain; after drying to 14 percent moisture, only 1744 bushels remain in the bin. When the total amount of corn grown exceeds the drying capacity of the number of drying bins installed, the excess corn is assumed to be left in the field to dry and to be harvested and marketed as in alternative 2.

The number of bushels stored or sold under alternative 1, Y_n, is

$$Y_n = 2222\ (0.90)\left(\frac{100 - 25}{100 - 14}\right) N_d + 0.754\ (Y_o - 2222\ N_d)$$

or

$$Y_n = 1744\ N_d + 0.754\ (Y_o - 2222\ N_d)$$

where the symbols are as previously defined. Total revenue from alternative 1 is

(11) $R_n = (1.14)\ 1744\ N_d + P_2\ (0.754)\ (Y_o - 2222\ N_d).$

[7]The amount of corn left after drying from an initial moisture content to a final moisture content is calculated as follows: Final amount of corn = $\frac{100 - \text{initial moisture content}}{100 - \text{final moisture content}}$ x initial amount of corn

Under the assumptions of the analysis, each grain drying bin saves approximately 69 bushels of corn. In addition to this increase in amount of corn, drying provides the opportunity to increase price through use of a government loan on the 1675 bushels available for sale in alternative 2. If grain drying is to be profitable, the value of the 69 bushels plus the increased returns on the 1675 bushels must more than cover the costs of grain drying. Thus, if the corn price on the open market at harvest time is high enough, grain drying will not be profitable.

The break-even harvest price, that is, the price of corn that must prevail on the open market at harvest time if there is to be no advantage from the use of either alternative, can be determined by substituting the above numerical values into the expression

$$R_n - C_n = R_2$$

and solving for P_2. Break-even prices for several combinations of N_i, N_d, and n, determined in the manner just described, are presented in Table 1. Thus, if a manager installs one grain drying bin, has a total yield that requires the operation of only one bin for six days, and the price of corn turns out to be $1.10 per bushel at harvest time, he will break even on his grain drying investment. If the market price is above $1.10, he will lose money; if it is less than $1.10, he

TABLE 1-OPEN MARKET CORN PRICES THAT MUST PREVAIL AT HARVEST TIME IF RETURNS FROM ALTERNATIVE 2 ARE TO EQUAL RETURNS FROM ALTERNATIVE 1, NATURAL AIR DRYING.

Number of Bins Installed, N_i	Number of Days Required to Dry Bin Load of Grain from 25 to 14 Percent Moisture							
	6	8	10	12	14	16	18	20
	Number of bins used, N_d = 1 (Y_o = 2222 bu.)							
1	1.10	1.10	1.10	1.09	1.09	1.08	1.08	1.07
2	1.03	1.02	1.02	1.01	1.01	1.00	1.00	0.99
3	0.95	0.94	0.94	0.93	0.93	0.92	0.92	0.91
4	0.88	0.87	0.87	0.87	0.86	0.85	0.85	0.85
5	0.80	0.79	0.79	0.78	0.78	0.78	0.77	0.77
	N_d = 2 (Y_o = 4444 bu.)							
2	1.10	1.09	1.09	1.09	1.08	1.08	1.07	1.07
3	1.06	1.05	1.05	1.04	1.04	1.03	1.03	1.03
4	1.02	1.02	1.02	1.01	1.00	1.00	1.00	0.99
5	0.99	0.99	0.98	0.97	0.97	0.97	0.96	0.96
	N_d = 3 (Y_o = 6666 bu.)							
3	1.10	1.09	1.09	1.09	1.08	1.08	1.07	1.07
4	1.07	1.07	1.06	1.06	1.06	1.05	1.05	1.04
5	1.05	1.04	1.04	1.03	1.03	1.03	1.02	1.02
	N_d = 4 (Y_o = 8888 bu.)							
4	1.10	1.09	1.09	1.09	1.08	1.08	1.07	1.07
5	1.08	1.07	1.06	1.06	1.06	1.05	1.05	1.04
	N_d = 5 (Y_o = 11110 bu.)							
5	1.10	1.10	1.09	1.09	1.09	1.08	1.07	1.07

will make a profit. If he installs five bins and yields are such that he only needs to operate one bin, the break-even price is only \$0.77 per bushel when 20 days drying time are required. The number of bins operated is dependent upon corn yields. As long as the capacity used is equal to installed capacity, $N_i = N_d$, the break-even harvest price remains about the same for each drying period. Unused capacity lowers the break-even price more than increases in drying time.

Two questions become relevant at this point in the analysis. First, if yield is sufficient to fill one or more natural air drying bins, can a profit be made by drying? Second, how many drying bins should be installed to make the most profit? The answer to the first question depends upon number of days required to dry a load of grain and upon the open market price of corn at harvest time; the answer to the second depends upon the amount of corn to be dried.

The number of days required to dry a load of grain is determined by random weather fluctuations during the drying period. When other inputs are held constant, yield levels are a function of random weather conditions during the growing season. The price of corn, however, is not a random variable. That is, a farm manager could undoubtedly obtain better price information by relying on current economic information, such as farm outlook or announced government price support levels, than by calculating a mean or expected price based on a distribution of past prices. Thus, two frequency distributions must be used to determine the most profitable number of grain bins: one for the number of days drying time and one for corn yields. The open market price of corn at harvest time is not random and must be specified.

Table 2 gives a frequency distribution of the number of days required to dry a bin of corn from 25 to 14 percent moisture, beginning on September 10. This distribution, determined as described earlier, is based on 21 years of weather records. E (n) for the natural air drying equipment described herein is 11 days.

Expected profit from drying a load of grain can be computed by using the frequency distribution in Table 2 and equations (9), (10), and (11). Table 3 includes expected profits for several combinations of units installed, units operated, and two levels of P_2, price per bushel of corn on the open market at harvest time. For example, if two natural air drying bins are installed and corn yields are such that only one bin is used, the expected profit from grain drying is \$50 when $P_2 = \$0.98$ and \$122 when $P_2 = \$0.94$. Or, if five bins are installed and five are used, expected profit is \$899 when $P_2 = \$0.98$ and \$1234 when $P_2 = \$0.94$. These expected profits represent average profits in the long run, based upon the expected number of drying days. When the actual number of drying days is less than 11, actual profit from drying will be larger than expected profits; when actual drying days are more than expected, actual profit will be smaller than expected profit. The data in Table 3 again suggest the importance of fully utilizing grain drying equipment.

Cost and return functions for n, the number of days of drying time, can be depicted graphically. When $Y_o = 4444$, $N_i = 4$ $N_d = 2$, and $P_2 = \$0.98$, the appropriate revenue and cost curves, along with a frequency distribution for n,

TABLE 2-FREQUENCY DISTRIBUTION OF DAYS REQUIRED TO DRY A BIN LOAD OF CORN FROM 25 TO 14 PERCENT MOISTURE CONTENT USING NATURAL AIR DRYING EQUIPMENT, BASED ON 21 YEARS WEATHER RECORDS FOR COLUMBIA, MISSOURI.[a]

	Required Number of Days															
	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Frequency	$\frac{1}{21}$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{1}{21}$	$\frac{1}{21}$	$\frac{4}{21}$	$\frac{1}{21}$	$\frac{1}{21}$	$\frac{4}{21}$	$\frac{1}{21}$	$\frac{2}{21}$	--	--	$\frac{1}{21}$	--	$\frac{1}{21}$

[a]Beginning drying date is September 10.

TABLE 3-DOLLARS OF PROFIT EXPECTED FROM DRYING ONE TO FIVE BIN LOADS OF CORN FOR ALTERNATIVE 1, USING NATURAL AIR GRAIN DRYING EQUIPMENT.[a]

Number of Bins Installed N_i	Number of Bins Used, N_d 1	2	3	4	5
			P_2 = \$0.98		
1	180				
2	50	359			
3	- 70	234	539		
4	-195	109	414	719	
5	-320	- 16	389	594	899
			P_2 = \$0.94		
1	247				
2	122	493			
3	- 3	368	740		
4	-128	243	615	987	
5	-253	118	490	862	1234

[a]Based on a support price of \$1.14 per bushel of corn.

are shown in Figure 6. The revenue function is actually $R_1 - R_2$, the increase in returns from grain drying. The cost function, C, represents the increase in costs due to grain drying. Profits decrease with drying time because costs increase and returns remain constant.

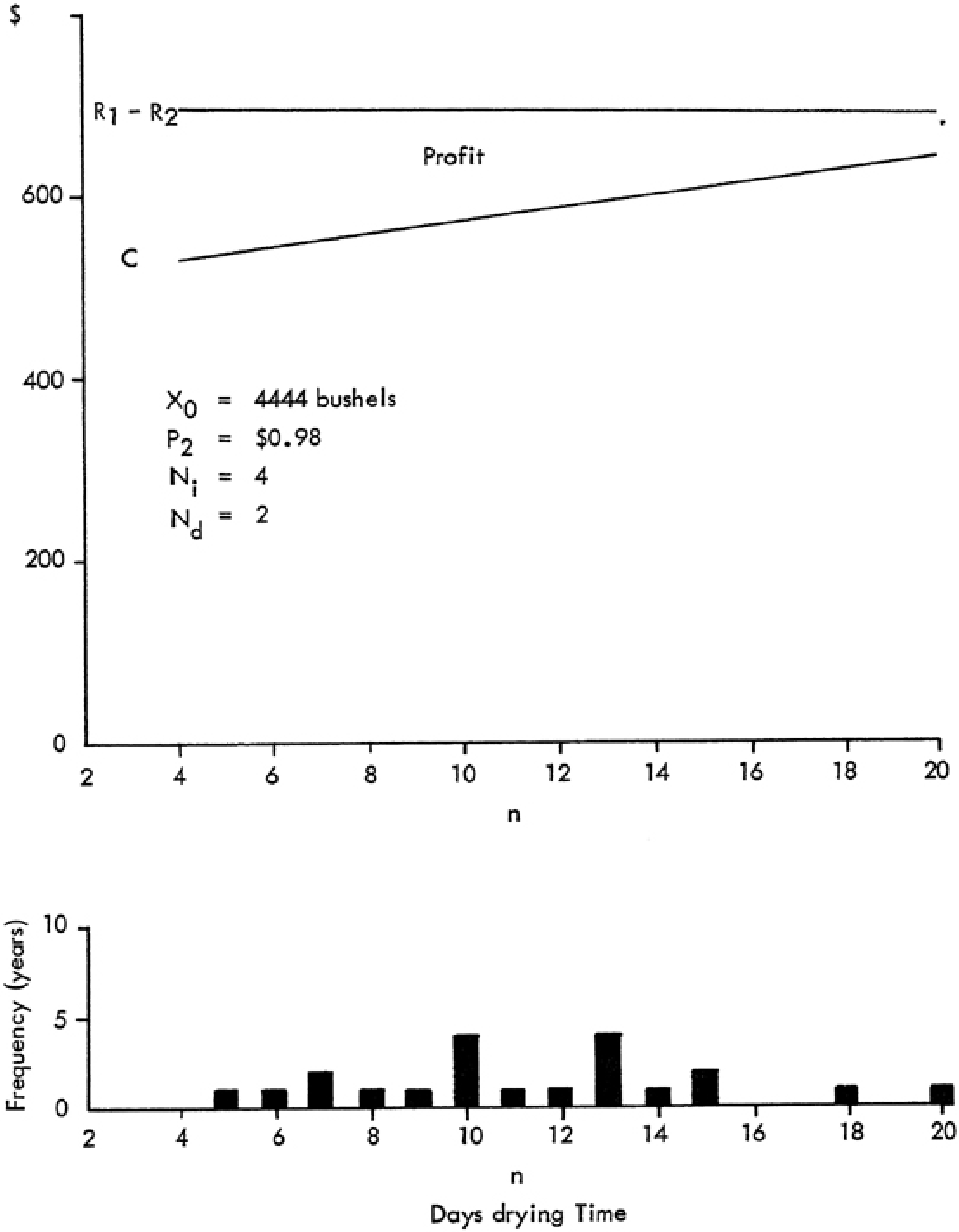

Figure 6

A frequency distribution of corn yields for Columbia, Mo., is presented in Table 4. As mentioned earlier, these data were collected over a 21-year period from an experimental plot. They are undoubtedly more variable than actual farm yields would be over the same time period but probably approximate yield variations on individual farms better than would county average yields. Similar management and fertilization practices are used on the experimental plots each year. The plots are harvested by hand at a high moisture content. In the absence of more refined measures of the effects of yearly weather conditions upon yield, the frequency distribution in Table 4 is used to approximate the distribution of Y_o. The class limits of the yields in Table 4 have been delimited to represent the required number of grain drying bins. If the yield on the 120 acres is less than 18.5 bushels per acre, no bins are needed. If the yield is between 18.5 and 34.9 bushels, one bin would be required and any excess would be left in the field and marketed as in alternative 2.

TABLE 4-FREQUENCY DISTRIBUTION OF CORN YIELDS FOR COLUMBIA, MISSOURI

Bushels per acre	Frequency
0.0 - 18.4	$\frac{1}{21}$
18.5 - 36.9	$\frac{7}{21}$
37.0 - 55.4	$\frac{9}{21}$
55.5 - 73.9	$\frac{1}{21}$
74.0 - 92.4	$\frac{3}{21}$

[a]Corn yields are from plot 18, continuous corn, on Sanborn Field of the Missouri Agricultural Experiment Station.

By making the realistic assumption that weather conditions affecting yield are independent of those affecting drying time, the probabilities in Table 4 may be used along with the expected profits in Table 3 to determine expected profits for grain bins installed. Expected profits for two open market harvest price levels are presented in Table 5. Profits in Table 3 were based only upon the dis-

TABLE 5-EXPECTED PROFIT IN ALTERNATIVE 1 FROM THE USE OF NATURAL AIR GRAIN DRYING EQUIPMENT.[a]

Number of Bins Installed N_i	Open Market Price of Corn per Bushel	
	$0.98	$0.94
1	$165	$229
2	229	334
3	162	280
4	80	208
5	-45	83

[a]Based on a support price of $1.14 per bushel.

tribution ot days drying time; profits in Table 5 are based both upon the distribution of drying time and the distribution of yields. For example, Table 3 presents expected profits for five installed bins when one, two, etc., are used. The profits in Table 5 are computed using the probabilities that zero, one, two, etc., bins will be used. Thus, in addition to the cost of drying, profits in Table 5 consider the losses incurred from installation of too many bins. When the market price is $0.98 per bushel, two bins result in the highest expected profit for 120 acres of corn, $229; when the price drops to $0.94, two bins still result in the highest profit, $334. These are expected or average profits. Actual profits from two bins may be either larger or smaller.

Because of space limitations, no attempt will be made to compare expected profits and the probability of a profit for various combinations of installed and used grain drying bins. However, because the revenue and cost functions for the number of days drying time are linear, any change that increases expected profit from this variable will also increase the probability of a profit and any change that decreases expected profit will decrease probability of a profit. For example, adding a fifth, unused drying bin to the analysis in Figure 6 would increase costs and decrease both expected profit and the probability of a profit. With respect to yield variability, the probability of a profit increases as the number of bins purchased decreases. Actual losses can be incurred regardless of yield level, depending upon corn prices and the required number of days of drying time.

The number of drying bins and the number of drying days do not interact. Profits are always largest or losses smallest when the number of drying bins purchased equals the number utilized, regardless of the length of the drying time. The weather "variable" affecting yields does interact with the number of drying bins; as yields increase, the number of bins required increases.

Supplemental Heat Drying. The cost of supplemental heat drying can be expressed as

$$C_s = DN_i + (L + E)\,N_d\,n + AU + KB$$

where, in addition to terms defined earlier

C_s = annual cost of owning and operating supplemental heat drying equipment.

A = Cost of unloading one bin.

U = Number of bins unloaded.

B = Number of auxiliary bins.

K = Depreciation cost of auxilary bins.

Using the cost figures presented above for a 15° F increase in the natural air temperature, the cost function becomes

$$(12) \qquad C_s = \$140\,N_i + \$8.45\,N_d\,n + \$7.50\,U + \$35\,B.$$

Again, this equation expresses costs of drying grain as a function of a controlled input, N_i, and random inputs, N_d and n.

Total revenue from alternative 2, R_2, is again computed using equation

TABLE 6 - OPEN MARKET CORN PRICES THAT MUST PREVAIL AT HARVEST TIME IF RETURNS FROM ALTERNATIVE 2 ARE TO EQUAL RETURNS FROM ALTERNATIVE 1; SUPPLEMENTAL HEAT DRYING EQUIPMENT USED TO DRY ONE BIN LOAD OF GRAIN

Number of Bins Installed, N_i	Number of Days Required to Dry Bin Load of Grain from 25 to 14 Percent Moisture			
	2	4	6	8
	Number of bins used, N_d = 1 (Y_o = 2222 bu.)			
1	1.04	1.08	1.07	1.06
2	1.01	1.00	0.99	0.98
3	0.93	0.92	0.91	0.90
4	0.84	0.83	0.82	0.81
5	0.76	0.75	0.74	0.73
	N_d = 2 (Y_o = 4444 bu.)			
2	1.09	1.08	1.07	1.06
3	1.05	1.04	1.03	1.02
4	1.01	1.00	0.99	0.98
5	0.97	0.96	0.95	0.94
	N_d = 3 (Y_o = 6666 bu.)			
3	1.09	1.08	1.07	1.06
4	1.07	1.06	1.05	1.04
5	1.04	1.03	1.02	1.01
	N_d = 4 (Y_o = 8888 bu.)			
4	1.09	1.08	1.07	1.06
5	1.07	1.06	1.05	1.04
	N_d = 5 (Y_o = 11111 bu.)			
5	1.09	1.08	1.07	1.06

(10). The number of bushels of corn resulting from the use of the supplemental heat drying units, Y_s, can be expressed as

$$Y_s = \left(\frac{100 - 25}{100 - 14}\right) (0.90)\ Y_o = 0.785\ Y_o$$

where Y_o is the amount of corn in the field at 25 percent moisture. As before, a 10 percent field loss is sustained and the remaining corn is dried to 14 percent moisture. However, using supplemental heat, bins may be reloaded at least twice, thus allowing partial bin loads to be dried. Total revenue from the use of supplemental heat drying is

(13) $R_s = \$1.14\ (0.785)\ Y_o = \$0.895\ Y_o$.

The break-even harvest prices are again determined by solving the expression

$$R_s - C_s = R_2$$

for P_2. Break-even prices are presented in Table 6 for supplemental drying equipment used once; that is, to dry one bin load. These prices are only slightly smaller than break-even prices for natural air drying, the difference being due to the extra cost of the supplemental heating equipment. Again, break-even prices are the highest when all installed equipment is utilized. Unused equipment lowers the break-even price more than an increase in drying time.

Table 7 gives a frequency distribution of the number of days drying time required to dry a bin of corn from 25 to 14 percent moisture using supplemental heating equipment. E (n) for this distribution is six days. Thus, the additional heat causes the expected drying time to be reduced to approximately half that for natural air equipment. Also, as a comparison of Tables 2 and 7 shows, the variance in drying time is much smaller for supplemental heat drying.

TABLE 7-FREQUENCY DISTRIBUTION OF DAYS REQUIRED TO DRY A BIN LOAD OF CORN FROM 25 TO 14 PERCENT MOISTURE CONTENT USING SUPPLEMENTAL HEAT DRYING EQUIPMENT; BASED ON 21 YEARS OF WEATHER RECORDS FOR COLUMBIA, MISSOURI.[a]

	Required Number of Days				
	4	5	6	7	8
Frequency	$\frac{5}{21}$	$\frac{7}{21}$	$\frac{7}{21}$	$\frac{1}{21}$	$\frac{1}{21}$

[a]Based on a 15°F increase in the air temperature and a starting date of September 10.

Expected profits from the use of supplemental heat drying equipment can be computed by using the cost and revenue functions (10), (12), and (13) and the frequency distribution of drying days given in Table 7. Table 8 includes expected profits computed for two open market price levels and various combinations of installed and used bins. Any number of bins may be installed, but the number used will depend upon corn yields. When the open market price of corn is $0.94 per bushel, the expected profit when one bin is installed and used is $162. Two bins installed and used return $323, or twice the return of one bin, and so on. Because the largest costs in grain drying are fixed costs determined by depreciation on the original investment, installation of more bins

TABLE 8-EXPECTED DOLLARS OF PROFIT FROM DRYING ONE TO FIVE BIN LOADS OF CORN FOR ALTERNATIVE 1; EACH SUPPLEMENTAL HEAT DRYING EQUIPMENT USED TO DRY ONE BIN LOAD.[a]

Number of Bins Installed, N_i	Number of Bins Used, N_d 1	2	3	4	5
			P_2 = $0.98		
1	162				
2	22	323			
3	-118	183	485		
4	-258	43	345	647	
5	-398	-97	205	507	809
			P_2 = $0.94		
1	229				
2	89	457			
3	- 51	317	686		
4	-191	177	546	915	
5	-331	37	406	775	1144

[a]Based on a support price of $1.14 per bushel of corn.

than are used reduces profits. Thus, if five bins are installed and only one used, the manager will lose about $400 when the market price of corn is $0.98 per bushel. The reduction in the market price of corn by $0.04 increases profit from alternative 1 by $67.

Table 8 contains expected profits from the use of supplemental heat equipment in alternative 1 for various combinations of installed and used equipment. That is, the data in Table 8 are computed for selected yield levels—those that will fill one to five bins. The only random weather variable used to compute expected profits in Table 8 is the number of days drying time. Next, the probability of needing different numbers of bins should be considered.

Expected profits from the installation of different numbers of bins can be computed by using the yield distribution in Table 4 and assuming that growing season weather is independent of drying weather. For supplemental heat drying, expected profits for two open market prices are presented in Table 9.

TABLE 9-EXPECTED PROFITS IN ALTERNATIVE 1 FROM SUPPLEMENTAL HEAT GRAIN DRYING EQUIPMENT USED TO DRY ONE BIN LOAD.[a]

Number of Bins Installed, N_i	Open Market Price per Bushel of Corn	
	$0.98	$0.94
1	$ 147	$211
2	193	299
3	111	230
4	15	142
5	-125	2

[a]Based on a support price of $1.14 per bushel.

These expected profits are based upon the frequency distribution of days of drying time and the frequency distribution of corn yields. Over time, the manager of the hypothetical farm setup in this example would make the most profit from supplemental heat grain drying in alternative 1 if he installed two bins. Given the prices and costs used here, the manager would average $299 a year from installation of two bins when the market price of corn in $0.94. He would not make $299 each year. Some years he would make more; other years, if no yield occurred, he could lose his fixed costs, $280.

One of the major advantages of supplemental heat drying not yet discussed is that more than one bin load of grain can be dried. The farm manager can load the drying bin, dry the grain, and then unload the dried grain, placing it in ordinary storage. Using the costs presented above and assuming a $35 yearly depreciation on bins used to store dried grain, open market corn prices needed to make alternative 2 as profitable as alternative 1 are presented in Table 10. These break-even prices are larger than those in Table 6, indicating the increased profitability in multiple drying. In fact, the break-even prices almost reach the assumed price support level.

TABLE 10-OPEN MARKET CORN PRICES THAT MUST PREVAIL AT HARVEST TIME IF RETURNS FROM ALTERNATIVE 2 ARE TO EQUAL RETURNS FROM ALTERNATIVE 1; SUPPLEMENTAL HEAT DRYING EQUIPMENT USED TO DRY MORE THAN ONE BIN LOAD.

One Bin	Number of Days Required to Dry Grain from 25 to 14 Percent Moisture Content							
Used to Dry:	6	8	10	12	14	16	18	20
Two Loads During the Season.	1.12	1.12	1.11	1.11	1.10	1.10	1.09	1.09
Three Loads During the Season.	---	---	1.13	1.12	1.12	1.12	1.12	1.11

TABLE 11-FREQUENCY DISTRIBUTION OF DAYS REQUIRED TO DRY CORN FROM 25 TO 14 PERCENT MOISTURE USING SUPPLEMENTAL HEAT DRYING EQUIPMENT; BASED ON 21 YEARS WEATHER RECORDS FOR COLUMBIA, MISSOURI.[a]

Number of Loads Dried per Drying Unit	Required Number of Days													
	9	10	11	12	13	14	15	16	17	18	19	20	21	22
2	$\frac{4}{21}$	$\frac{2}{21}$	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{3}{21}$	$\frac{1}{21}$	$\frac{1}{21}$	--	$\frac{1}{21}$	--	--	--	--	--
3	--	--	--	--	--	$\frac{3}{21}$	$\frac{2}{21}$	--	$\frac{4}{21}$	$\frac{3}{21}$	$\frac{2}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{2}{21}$

[a]Based on a 15°F increase in air temperature and a starting date of September 10.

Frequency distributions were derived for the number of days required to dry two and three loads of grain in one drying unit, allowing one day for unloading between each two loads. These distributions are presented in Table 11. E (n) for two loads is 12 days and E (n) for three loads is 18 days. The probability that the actual drying time will exceed the expected time is greater for three loads than for two.

If yields are such that two loads of grain can be dried in one season using one supplemental heat unit, the expected profit of alternative 1 over alternative 2 is $421 when the open market price of corn is $0.98 per bushel and $555 when the open market price is $0.94. If three loads are dried, expected profit for the $0.98 price is $680 and $880 for the $0.94 price. Increased profits result from the use of the drying facilities more than once and spreading the fixed costs of grain drying over a larger volume of yield. The profit figures just presented consider variability in drying time.

Incorporating yield variability into the analysis through the use of the yield distribution in Table 4, expected profit when the market price is $0.98 would be $316 when one drying bin is installed. Expected profit would be $222 when two bins are installed. When the market price is $0.94, expected profit is $434 for one bin and $350 for two bins. Thus, when multiple dryings are allowed, one supplemental heat drying bin per 120 acres of corn land becomes the most profitable. Also, expected profits are substantially larger than when one load is dried in each bin.

Summary of Grain Drying Analysis

Under the assumptions and data used in this analysis, grain drying is profitable. When only one load is dried in a season and the open market price of corn is $0.98 per bushel, expected profit from each bin load of grain is $180 for natural air drying equipment and $162 for supplemental heat drying equipment. If the open market price of corn drops to $0.94, profit from each system increases $67.

The profit figures presented in the above paragraph were for each bin load dried. A more important question is: How many grain drying bins should be installed for each 120 acres of corn? When one load per season is dried in each installation, two natural air drying bins are the most profitable, resulting in an expected profit of $229 when the open market corn price is $0.98. For the same market price, two supplemental heat drying bins result in a slightly lower expected profit of $193.

Supplemental heat bins may be used to dry more than one load per drying season. When multiple dryings are allowed, one supplemental heat drying bin per 120 acres of corn is the most profiable, resulting in an expected profit of $316 for a market price of $0.98 when two loads per bin are dried. Expected profits increase as the market price of corn drops.

A consideration as important as expected profits is the expected drying time. Even when one bin load of grain is dried, the difference in expected profits from the methods of drying is not large. The drying time required for natural air drying is highly variable, ranging from five to 20 days. For all 21 years, the supplemental heat unit completed the drying process in a four-to eight-day period. Thus, even when only one bin load is dried, the farm manager may prefer to sacrifice some expected profit to add some certainty to the drying process. He need not do this blindly or by guess; the analysis indicates that a sacrifice of $18 expected profit will reduce the expected drying time from 15 days to 5 days. Also, the supplemental heat unit provides more flexibility. The heat may be increased if warranted.

The grain drying analysis is an applied example of the theoretical model presented earlier. It is not meant to be a complete development of grain drying problems. The analysis has been limited to two production alternatives—the question examined was grain drying and storage at the government loan price versus no grain drying or government loan. Other alternatives are possible and should be considered in overall farm planning. If the manager specifies his alternatives, they can be analyzed by the method presented.

The expected profits depend entirely upon the price and cost data and the frequency distributions of the relevant weather variables. Costs, returns, and the weather vary. Also, the weather probabilities used are estimated rather than known or population values. Thus, all final figures are necessarily estimates.

Timeliness of operation has not been considered. An early corn harvest may permit the manager to sow his land to a fall crop or may provide for more timely labor requirements. Also, the probability of losing all or part of a crop because of unfavorable harvesting conditions in the late fall was not included in this analysis because adequate information was not available. Finally, investment in grain drying equipment should be considered along with other possible capital outlays.

SUMMARY AND CONCLUSIONS

This study presents and applies a model that can be used to analyze a production situation which involves one or more random weather inputs. To apply the model, the random inputs affecting the production process must be defined. For grain drying, the random weather inputs are the weather affecting corn yields and the temperature and humidity of the air during the September drying period. Once identified, cost and return functions must be determined for the random variables. Finally, frequency distributions of the variables must be determined and used along with the cost and return functions to estimate expected profits. If expected profit is negative, the production process is unprofitable over time. If positive, the process is profitable.

The presence of interaction among random and controlled inputs used in the production process determines the relevance of using conventional profit maximizing procedures to allocate controlled resources. When random and controlled inputs do not interact, the optimum amount of the controlled input—

determined by equating the marginal value product of the input to the price of the input—is not affected by variations in the random input. When the two types of inputs do interact, the optimum amount of the controlled input cannot be determined without specifying an amount of the random input. Thus, the "optimum" amount of the controlled input has little meaning when the two types of inputs interact. In this case, it seems more rational to refer to the amount of the controlled input that maximizes expected profit, maximizes the probability of a profit, minimizes the probability of a loss, etc., depending on the manager's goals. In the short run, the manager may select any of these goals; in the long run, maximization of expected profits would seem most likely to insure survival of the firm.

Identifying weather variables and deriving their frequency distributions changes uncertainties into risks. That is, the manager gains knowledge of the probable occurrences of weather events about which he either has limited knowledge or no knowledge. Over time, the distribution can be used to determine expected profits or losses. Within a single growing season or production period, the manager does not know what will occur and hence still faces an uncertain situation. In this case, he can use the frequency distribution to evaluate the odds for or against a particular alternative. Thus, while he does not know what will occur, he knows what is likely to occur or likely not to occur. Once he knows the probability of success for a managerial decision, he can evaluate for himself whether or not to undertake action.

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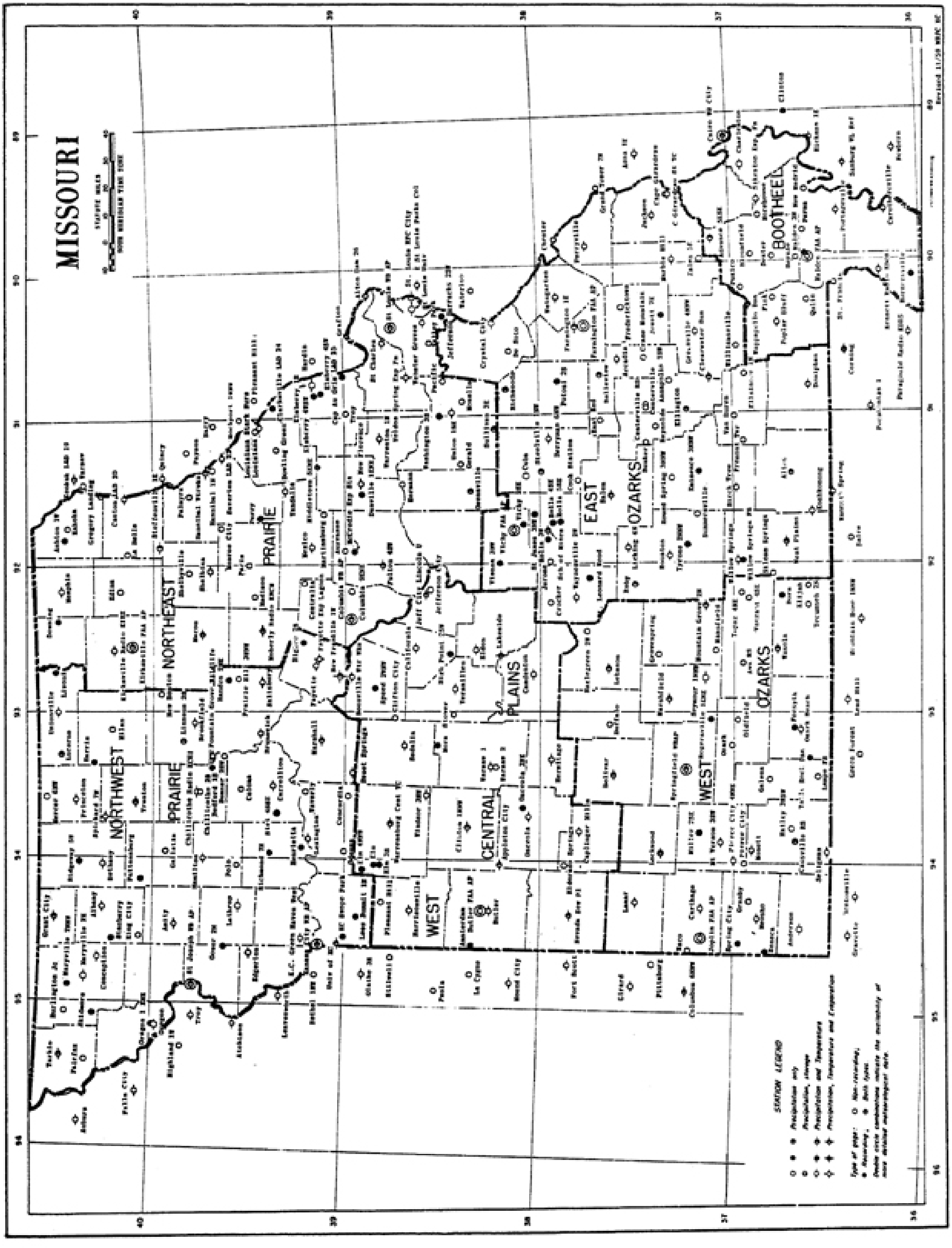
APPENDIX

The development and application of the theoretical model showed that some measure of the frequency with which the weather parameter falls between some critical limits is necessary. There are three ways this frequency can be obtained:

1. When short-term decisions are involved, a forecast of meteorological events may sometimes be used.
2. When a sufficiently long period of record of weather events is available, a frequency distribution of these events can be prepared by counting the times the events occurred within certain ranges.
3. When no observations have been made of a weather parameter at a particular place or where the observations cover only a limited number of years of record, adequate estimates of the liklihood of occurrence of the weather parameter can be made by using observed data from nearby weather stations, or by using information from one of several mathematically defined meteorological frequency distributions.

Short-term forecasts, covering periods up to 5 days are issued in Missouri by U. S. Weather Bureau offices in Kansas City, St. Joseph, Columbia, Springfield and St. Louis. The forecasts from the Weather Bureau cover weather elements that are of general interest to the citizens of the state. There are a growing number of privately operated meteorological consulting firms which provide special forecasts and do research for individuals and companies. A list of the names and addresses of the private meteorological consultants in the United States may be found in recent issues of the Bulletin of the American Meteorological Society, on file in many libraries and most


MISSOURI
NORTHWEST
PRAIRIE
NORTHEAST
PRAIRIE
CENTRAL
PLAINS
WEST
EAST
OZARKS
WEST
OZARKS
BOOTHEEL
STATION LEGEND

Weather Bureau stations. This list can also be obtained by writing to the Secretary, American Meteorological Society, 45 Beacon Street, Boston 8, Mass.

Weather data are recorded at a variety of locations in each state. Figure 7 shows weather stations in operation in Missouri. A similar network exists in each of the states in this country. The network in Missouri includes professionally staffed Weather Bureau stations at Kansas City, St. Joseph, Columbia, Springfield, and St. Louis. These stations record hourly observations on a larger number of weather parameters. The data are placed on IBM punched cards and microfilm and are also published monthly in a series called "Local Climatological Data."

A group of stations operated by the Federal Aviation Agency also record hourly data for a variety of weather parameters. These stations were located, as of December, 1960, at Kirksville, Vichy, Cape Girardeau, and Joplin. Data from this group of FAA stations are forwarded to the Weather Bureau each month, and are microfilmed and published in less detailed form in the series called "Climatological Data."

A large group of stations, operated by volunteer weather observers who use official Weather Bureau equipment, record the highest and lowest temperature each day, and the amount and kind of precipitation that occurs. Some of these stations are also equipped with recording rain gauges, from which hourly data can be extracted. These hourly data are published in a series called "Hourly Precipitation Data."

Many of these meteorological records have been placed on IBM punched cards and are available at the National Weather Records Center in Asheville, N. C. Also, several summaries of Missouri weather data have been prepared by the Weather Bureau and the University of Missouri. These summaries are listed in a supplemental bibliography at the end of this appendix.

There are several theoretical frequency distributions which provide useful estimates of the probability of occurrence of weather parameters. Several of these theoretical distributions have been used for many years. Others did not come into use until the development of electronic data processing and computer equipment made it possible to handle large numbers of mathematical operations and complex mathematical models.

The normal, or Gaussian, distribution has been widely used in many statistical analyses. It has been found appropriate for estimating probabilities of occurrence of weather elements which are unbounded, such as temperature and pressure. For example, the date of occurrence of the first temperature in the fall and of the last temperature in the spring below critical limits can be adequately described using the normal distribution. There are weather elements that are bounded, such as precipitation which cannot be negative. The Gamma distribution has been found to provide useful estimates of the probability of bounded elements. Also, there often is interest in the value of the extreme value of some meteorological event during a certain period. Theoretical frequency distributions that fit the occurrence of extreme values have been applied to extreme wind speed data. A list of selected references describing some of the theoretical frequency distributions, with applications to Missouri data, will be found in the supplemental bibliography at the end of this appendix.

Further information on any of the sources of meteorological data described above, or on possible applications of mathematical or statistical analysis of meteorological data can be obtained by writing to: Weather Bureau State Climatologist, Post Office Box 117, Columbia, Mo.

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