ABSTRACT

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ABSTRACT

This dissertation addressed the relationship between levels of Piagetian cognitive development and algebraic reasoning. A correlational analysis was conducted to show the relationship between levels of Piagetian cognitive development and algebraic reasoning and also to show the relationship between levels of Piagetian cognitive development and algebra course grades. High-school students were chosen because they are at the age approximation Piaget predicted children would transition from concrete operations to formal operations. An intervention followed with a small group of students to accelerate their transition to formal operations. The types of strategies used and the errors made during the intervention were observed and calculated.

The objectives of this study were as follows: (1) to determine whether there is a relationship between Piagetian levels of cognitive development and the level of algebraic reasoning in high-school freshmen, (2) to determine whether there is a relationship between Piagetian levels of cognitive development and grades in algebra class in high-school freshmen, (3) to determine whether the intervention group had a statistically significantly greater change in level of Piagetian cognitive development from the transitional stage between concrete operations and formal operations to formal operations than the comparison group, (4) to determine what types of strategies the intervention students used who successfully shifted to formal operations, and (5) to

determine the patterns of errors of the intervention students who did not successfully shift to formal operations.

The results indicate that there is a significant positive relationship between the Piagetian level of cognitive development and levels of algebraic reasoning in high-school freshmen but not between Piagetian levels of cognitive development and algebra course grades. The results did not show that the students who participated in the intervention had a greater change in the level of Piagetian cognitive development than the students who did not receive the intervention. The results showed that the intervention students who successfully shifted to formal operations used algebraic strategies more than 50% of the time. The students who participated in the intervention and did not successfully shift to formal operations primarily made pattern errors or made errors when writing arithmetic equations to solve problems.

NORTHERN ILLINOIS UNIVERSITY

AN INTERVENTION TO ADVANCE PIAGETIAN LEVELS OF COGNITIVE DEVELOPMENT AND ALGEBRAIC REASONING IN HIGH-SCHOOL STUDENTS

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DEDICATION

To Grandma Pulver, for starting me on this journey

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CHAPTER I

INTRODUCTION

Jean Piaget is probably best known to psychologists and educators for his four stages of cognitive development. Attempts have been made, since his research was discovered in the United States in the 1950s, to include Piaget's theories in curricula, for example, the "new math" programs funded by National Science Foundation grants that were developed in the 1960s as a part of the curriculum reform movement that coincided with the demise of progressive education. These programs were in part based on Piaget's theories as was evidenced by his collaborator, Dr. Barbel Inhelder's participation in the Woods Hole Conference at Cape Cod in 1959 (Bruner, 1960). Today, many ideas have remained from that reform movement, such as having children learn to think and problem solve for themselves; however, Piaget's basic theory of cognitive development has been de-emphasized as other constructivist theorists, such as Lev Vygotsky, have gained prominence. In addition, many textbooks are being written with expectations that students are at a higher stage of cognitive development, based on Piaget's guidelines, than they have necessarily achieved (O'Hara, 1975; Wolfe, 2000).

Based on his experimentation with children, Piaget segmented intellectual development into four basic periods: sensorimotor, preoperational, concrete operations, and formal operations (Piaget, 1936/1952). The sensorimotor stage occurs from approximately birth to age one and one half. In this stage a child learns circular reactions, imitation, and finally object permanence. The preoperational stage occurs at approximately age one and one half and continues until about age five. It is at this stage that children will begin the symbolized thought process by using their imagination for games. Next is the stage of concrete operations, which lasts from about age 5 through age 12. At the concrete level a child will learn the conservation of mass, weight, and volume. Then at approximately age 12 it is theorized that a child will move on to the formal operational stage, which includes abstract thinking, like an adult. Piaget claims that the child will move through these stages sequentially. Although Piaget has attached some basic age approximations at which each stage might occur, it is stressed that "the appearance of any particular operation is *stage-dependent*. It is not, however, *age-dependent*" (Hilgard & Bower, 1975, p. 325).

Piaget is often misinterpreted on this point and is often taken for a maturationist who believes in strict age guidelines. Progression through the stages is based on the physical and social environment of the child. It does not occur at a specific age, nor does it occur without external stimulation such as teaching, discussion, or social interaction (Elkind, 1976; McGrath, 1980).

Although Piaget's theories have often been contested, his results of experimentation for the stages of development have been replicated with different cultures, although not necessarily at the same ages that Piaget found (Elkind, 1961a-e). It has been found that even if they occur at different ages, the stages still proceed in the same order and build upon one another (Athey & Rubadeau, 1970).

Because the transition from concrete to formal operations in Piaget's levels of cognitive development is primarily based on a change in logical reasoning, in a mathematics classroom students would have more success and achieve more if they were at a higher stage of cognitive development. This seems rather obvious, and many mathematics textbook publishers assume this to be the case.

At this point in time, the majority of the literature supports the theory that there is a relationship between Piagetian levels of cognitive development and mathematics achievement. Vaidya and Chansky (1980) found this to be the case with second, third, and fourth graders. Al-Dokheal (1983) supports this theory for sixth-grade Saudi Arabian males. Bloland and Michael (1984) and Bitner (1991) surveyed high-school students to show the same positive relationship. Ablard and Tissot (1998) examined reasoning levels of gifted second through sixth graders to show their readiness for higher levels of mathematics. Finally, Wolfe (2000) found a significant relationship between levels of cognitive development and math achievement in nontraditional college students.

The methods of these studies proceeded in a similar manner. The researchers tested the specified group of students for Piagetian levels of cognitive development, assessed math achievement levels, and subsequently statistically analyzed the relationship between the two sets of scores. A variety of measures were used for both types of tests that were age or grade appropriate.

Although all of these studies showed a significant positive relationship between Piagetian levels of cognitive development and math achievement, there are some limitations. A number of these conclusions were based on studies with small sample sizes. For example, Vaidya and Chansky (1980) tested a group of 102 suburban students across three grade levels: second, third, and fourth grades. Bitner (1991) administered tests to a group of 102 rural students ranging from grades 9 through 12. Also, Ablard and Tissot (1998) sampled a group of 150 students in grades 2 through 6, approximately 30 students per grade level.

Along with the limitation in the results due to sample size, there is also the difficulty in generalizing the results to a larger group such as "all elementary students" or "all high-school students." This is especially the case in the studies in which the sample population was narrow based on the location the population was drawn from (Bitner, 1991; Vaidya & Chansky, 1980). Al-Dokheal (1983), however, both examined and generalized only to sixth-grade male Saudi Arabian students, based on his random sample of 230 students in a large school district.

Yet, despite the methodological limitations, the varying demographic characteristics of the participants in the different studies, and the different tests used to measure cognitive development or mathematics achievement, the results were remarkably robust across several studies from different cultures (Ablard & Tissot, 1998; Al-Dokheal, 1983; Bitner, 1991; Vaidya & Chansky, 1980). All of the studies reviewed showed a positive relationship between mathematics achievement and Piagetian levels of cognitive development.

Algebraic Reasoning

More specifically, however, the branch of mathematics that receives a great deal of attention in the field of education at the high-school level is algebra, in particular because it has been reported that approximately 95% of 17-year-old high-school students have taken an algebra course (Perie, Moran, & Lutkus, 2005). Moreover, in the high-school curricula of today, it is expected that most students will take an algebra course of some sort, typically as freshmen, and many students who plan to attend college will take a second algebra course as juniors. It is also expected that these students apply algebraic methods in a geometry class and even in a pre-calculus class or beyond at the high-school level. However, based on recent National Assessment of Educational Progress (NAEP) testing results, only 7% of 17-year-olds who were tested showed proficiency in solving multistep problems and using algebra (Perie, Moran, & Lutkus, 2005).

Yet, at a time when students do not appear to be achieving in algebra, there are heightened expectations on schools and especially math teachers due to the federal government's latest 2001 revision to the Elementary and Secondary Education Act of 1965, more commonly known as No Child Left Behind (NCLB) (Moyer-Packenham, 2004). Consequently, it has become even more important for high-school students to show algebraic reasoning skills and to pass state standardized tests. However, research has shown that students experience a cognitive gap as they transition from arithmetic to algebra (Filloy & Rojano, 1989; Goodson-Espy, 1998; Herscovics, 1989; Herscovics & Linchevski, 1994; Kieran, 1988, 1989; Linchevski & Herscovics, 1996; MacGregor & Stacey, 1997; Pillay, Wilss, & Boulton-Lewis, 1998; Sfard & Linchevski, 1994). Specifically, problem areas include the inability to use algebraic equations in problem solving (Goodson-Espy, 1998; Herscovics, 1989; Kieran, 1989), an inability to work with variables (Herscovics & Linchevski, 1994; MacGregor &

Stacey, 1997), difficulty with using the equals sign as equivalency rather than a command to find an answer (Herscovics & Linchevski, 1994; Pillay, Wilss, & Boulton-Lewis, 1998), using an algebraic approach as opposed to an arithmetic approach to solving equations (Kieran, 1988), difficulty using proper operations to solve algebraic equations and group like terms appropriately (Linchevski & Herscovics, 1996), and seeing beyond the computational process of solving algebraic equations to the abstract (Sfard & Linchevski, 1994).

Statement of the Problem

To date, it is not clear what creates this cognitive gap between arithmetic and algebraic reasoning. Possible explanations for this gap could include the curricula, the teaching methods, or the maturity level of the students. It appears that even though there are students who are successful in using algebraic reasoning well past the Algebra I course, there are some students who pass Algebra I classes without being able to apply it later (Perie, Moran, & Lutkus, 2005). According to Flavell (1963), research shows that children can be trained to master a task but may not be able to apply the learning of that particular task at a later date. One possibility for this lack of transfer of knowledge is that the cognitive structures to support abstract reasoning are not fully developed. Adolescents learning algebra at ages 12 to 14 may not be ready for full accommodation of this knowledge due to the abstract nature of algebra because these students may not have reached the level of formal operations (Piaget, 1975/1985).

Research has shown that through intervention, the progression from concrete to formal operations can be accelerated (Adey & Shayer, 1990; Iqbal & Shayer, 2000; Shayer & Adey, 1992a, 1992b) and can result in increased math achievement over time (Adey & Shayer, 1993; Shayer & Adey, 1993). It would benefit teachers not only to be able to teach to the level of the students but also aid in the transition from concrete into formal operations through classroom instruction.

Purpose

The purpose of this study was twofold. First, the study replicated findings of past studies with a sample of American suburban high-school students to determine if those who achieve higher levels of algebraic reasoning are functioning at a higher operational mode according to Piaget's levels of cognitive development. A second purpose was to develop and test an intervention designed to shift students from the transitional stage between the level of concrete operations and formal operations to the formal operations stage.

Research Questions

The following questions were addressed:

1. Is there a relationship between Piagetian levels of cognitive development and the level of algebraic reasoning in high-school freshmen? It was predicted that there is a statistically significant positive relationship between the Piagetian

level of cognitive development and level of algebraic reasoning. Prior research has shown a positive relationship between math achievement and Piagetian levels of cognitive development (Al-Dokheal, 1983; Bitner, 1991; Bloland & Michael, 1984; Vaidya & Chansky, 1980; Wolfe, 2000), so it would follow that a similar relationship would exist when specifically examining algebraic reasoning.

2. Is there a relationship between Piagetian levels of cognitive development and grades in algebra class in high-school freshmen? It was predicted that there is a statistically significant positive relationship between the Piagetian level of cognitive development and algebra course grades. Because it has been shown there is a relationship between math achievement and levels of cognitive development, it would follow that a similar relationship would exist between the grades students earn in class and their level of cognitive development.
3. Did the intervention group have a statistically significantly greater change in level of Piagetian cognitive development from the transitional stage between concrete operations and formal operations to formal operations than the comparison group with the typical instruction over a 12-week period of time? It was predicted that an intervention can shift the level of Piagetian cognitive development from the transitional stage between concrete operations and formal operations to formal operations. In their interventions with middle-school science students, Shayer and Adey (1992a, 1992b, 1993) were successful in raising the Piagetian level of cognitive development by embedding logical reasoning lessons into the everyday science curriculum.

4. Of those who received the intervention, how were the students who successfully shifted from the transitional stage between concrete operations and formal operations to formal operations through academic intervention working through the information? Were they using arithmetic strategies or algebraic strategies to solve problems? What processes did they undergo to overcome making errors?
5. Of those who received the intervention, what were the patterns of errors of the students who did not successfully shift from the transitional stage between concrete operations and formal operations to formal operations through academic intervention?

Definitions

Cognitive development was defined by using Piaget's levels of cognitive development. This was determined using the Group Assessment of Logical Thinking (GALT), which measures the level of Piagetian cognitive development. In particular, this study concentrated on the concrete operational level, the formal operational level, and the transition from concrete to formal operations.

The level of algebraic reasoning was determined with a pretest that is given to all of the students upon entering high school as freshmen and assesses mastery of fraction operations, decimal operations, proportions, solving one- and two-step algebraic equations, order of operations, measures of central tendency, area, perimeter, and application of concepts in word problems.

Arithmetic methods were defined as those methods with no use of variables and by use of strategies such as unwinding or reversing the arithmetic or by using arithmetic operations rather than using patterns. Algebraic methods were defined as those methods using variables or patterns to solve the problems.

Delimitations

Delimitations to this study include the sample of participants since it was a convenience sample based on accessibility. The population was chosen from one large suburban high school with a fairly homogeneous ethnic background and was limited to the students available to the researcher from her own freshman classes.

Significance of the Study

Currently, many high-school algebra students are being asked to learn material involving abstract reasoning, which is considered to be at the formal operational level. If the proposed hypotheses are correct, a student functioning at the concrete level, or even in the transitional period between concrete and formal reasoning, is going to encounter difficulty fully accommodating the information taught in a typical algebra course.

The results of this study will be informative to teachers interested in discerning their students' cognitive functioning. If it is the case that the majority of students in a class are at one particular stage of development, then curricula can be modified accordingly so that the students can learn what they are capable of learning rather than trying to push them ahead to a concept they are not ready to learn.

If the intervention is effective, it could be used to provide specific examples of the type of instruction that teachers should focus on with pre-algebra students to prepare the students for the level of abstract reasoning required to fully assimilate the course material in an algebra course. This research will provide teachers with an intervention that advances the students' cognitive development and allows them to maximize their performance in algebra.

CHAPTER II

REVIEW OF LITERATURE

Introduction

Jean Piaget's theories of cognitive development have been shown to have a positive relationship with math achievement (Ablard & Tissot, 1998; Al-Dokheal, 1983; Vaidya & Chansky, 1980). In particular, Piaget's theories have been found to relate to algebraic reasoning as students taking an algebra course require abstract reasoning in order to be successful (Bitner, 1991; Bloland & Michael, 1984; Wolfe, 2000). Yet research has shown that there is a cognitive gap as students transition from arithmetic to algebra (Filloy & Rojano, 1989; Goodson-Espy, 1998; Herscovics, 1989; Herscovics & Linchevski, 1994; Kieran, 1988, 1989; Linchevski & Herscovics, 1996; MacGregor & Stacey, 1997; Pillay, Wilss, & Boulton-Lewis, 1998; Sfard & Linchevski, 1994). However, through wide-scale interventions in science classes, students have shifted from the Piagetian level of cognitive development of concrete operations to formal operations (Adey & Shayer, 1990; Iqbal & Shayer, 2000; Shayer & Adey, 1992a, 1992b, 1993).

Piaget's Theories of Cognitive Development

A seminal theory of cognitive development is Jean Piaget's four-stage theory of cognitive development. Based on his experimentation with children, Piaget segmented intellectual development into four basic periods: sensorimotor, preoperational, concrete operations, and formal operations (Piaget, 1936/1952). His theories of constructivism and active learning have also been applied to education in the United States, first with the "new math" programs in the early 1960s (Bruner, 1960), then again in the late 1960s and early 1970s in the open school movement (Elkind, 1976). These theories have recently reappeared in the current trend of "discovery learning" and "problem-based learning" in schools today (Goldsmith, 1999).

Constructivism

Piaget's first opportunity to work with children came about in Paris when he worked at a laboratory continuing Alfred Binet's work on intelligence testing. While working in this laboratory, Piaget quickly became bored with the testing of children; however, he became greatly interested in the thought processes children used when he observed many of them coming up with similar "wrong" answers to questions (Elkind, 1976). These observations on children's thinking became the basis of his research for the rest of his life (Sawada, 1972). He had intended to investigate his questions about the thought processes of children and then move on to other problems. But the more research he did on children's conceptions of the world, the more he realized that he would have to continue to move back to the earliest moments of human existence, hence his study of infants (Sawada, 1972). Just like other researchers who also studied infants had done, Piaget used his own children (Piaget, 1945/1951, 1936/1952, 1937/1954). However, Piaget's basic presumption was quite different from those who had done prior research. Instead of assuming that there was a basic reality the infant copies from and becomes familiar with, Piaget assumed that the infant was constructing a unique reality. This is the basis of his theory of constructivism that is often referred to today. What Piaget meant by "constructivism" is that "the child constructs reality out of his experiences with the environment" (Elkind, 1976, p. 59).

Specifically in constructivism, a new concept is transformed by the child's own way of thinking; it is not necessarily an exact copy of the concept. This is possibly why teachers often have difficulty teaching new concepts to children if the children have nothing in their prior knowledge with which to connect. The point at which children are constructing new theories is also when they misinterpret many ideas. The children will put these new theories into terms or relate them to something that they do already understand, but the ideas may not connect quite right in an adult logic. It may not be until much later in the child's development that these ideas finally get sorted out properly in a more logical manner.

Cognitive Development

In Piaget's theory of cognitive development, he has broken down intellectual development into four basic periods: sensorimotor, preoperational, concrete operational, and formal operations. He claims that the child will move through these stages sequentially. Although Piaget has attached some basic age approximations at which each stage might occur, it is stressed that "the appearance of any particular operation is *stage-dependent*. It is not, however, *age-dependent*" (Hilgard & Bower, 1975, p. 325).

Piaget is often misinterpreted on this point and is often mistaken for a maturationist who believes in strict age guidelines. However, he has argued against biological maturation theories of development as they are commonly stated because these biological maturation theories are based on the idea that children will progress instinctively. Progression through Piaget's stages is based on the physical and social environment of the child. This progression through the stages does not occur at a specific age, nor does it occur without external stimulation (Elkind, 1976; McGrath, 1980).

The first period of development, the period of sensorimotor intelligence, typically begins at birth and lasts through approximately age one and one half. There are six stages within this first period of development. Stage one is exercising ready-made sensorimotor equipment such as sucking, crying, elimination, and gross body activity. Assimilation is the act of fitting a new idea in to what is already in existence even if it contradicts another idea already possessed in the schema. Accommodation is the process of adapting to the new idea. Neither assimilation nor accommodation is differentiated at this point. The second stage includes primary circular reactions. An example if this is when an infant stumbles upon a new activity and repeats it over and over. Assimilation and accommodation become differentiated at this point. In stage three, secondary circular reactions occur. This is when the infant begins to act and wait for a result to occur. The infant will also begin to do things intentionally. The fourth stage includes coordination of the secondary schemas when the infant will move objects to reach another behind it. The infant will also do a great deal of imitating new responses such as copying movements that other people make or the movements of a character on television. The fifth stage is that of tertiary circular reactions, which is when the infant will explore a new object using experimentation to see what is new about the object. The infant also begins to learn the use of means to an end. The sixth and final stage includes the invention of new means through mental combinations. This is demonstrated when an infant will go after an object by going around a barrier even if the distance is then farther away from the goal. The infant begins to infer causes from observing effects and infer effects from observing causes. This is also when the infant will invent new applications of things by learning them in different contexts (Hilgard & Bower, 1975). It is commonly found during the sensorimotor period that the game of peek-a-boo is so amusing to the infant because the concept of object permanence has not yet been developed. So when an object is hidden, the infant believes the object has completely disappeared. This concept comes just prior to the next stage of preoperational intelligence.

The preoperational stage, which Piaget posits begins at approximately age one and one half and lasts until about age five, starts with the internalization of imitation when the child starts visualizing images. This leads to the symbolic thought process. At this time pretend games such as playing dress up and pretending to become a princess become important because the child is beginning to be able to use a symbol to represent a real object. Egocentrism is a large part of this stage because the child still feels that he/she is the center of everything. The child does not have the capability yet to see beyond what happens to oneself. This is the point when parents get very frustrated when trying to complete a task such as balancing a checkbook without interruption because the child is in a sense blind to whatever else is going on and will continue to disrupt the activity.

At approximately age five the child moves into the concrete operational stage of development. It is at this point in time that learning by doing becomes very important in order to successfully continue development. In addition, it is during this stage that children learn the conservation principles of mass, weight, and volume. The experiments to test for successful transition to the concrete operational stage are commonly found in most psychology textbooks. For the conservation of matter experiment a child is shown a ball of clay that is rolled out into a cylindrical shape. The concept of conservation of matter is understood when a child realizes that both shapes have the same amount of material and weigh the same. The conservation of volume is noted by the child's ability to recognize when an orange-colored drink is poured into a tall, thin cup and then when it is poured into a short, wide cup there is the same amount of liquid in both containers. These are concepts which need to come sequentially, and they cannot be forced onto the child to understand before the child is ready for it. Also at this stage the child learns the concepts of seriation, or arranging objects in order; classification, or sorting according to some quality; and correspondence, or grouping.

Beyond this, at age 12 or later, the child moves on to the formal operation stage. At this point the child begins to think more like adults on an abstract level. It is at this time that the child learns grouping concepts such as identity, negation, reciprocal, and correlation. Deductive reasoning and systematic planning are also results of this stage allowing the child to use hypothetical thinking and consider the consequences of actions. In this stage, the child can also solve problems using proportional reasoning, probabilistic reasoning, correlational reasoning, and combinatorial logic. This allows the child to solve mathematical problems using proportions, probability, permutations, and combinations. The child can also conduct science experiments and prove hypotheses using the scientific method of changing only one variable at a time. This level of formal operations only comes when the rest of the sequence prior to it is complete, and it is sometimes possible for someone to never reach the stage of formal operations. Subsequent studies (Kuhn, Langer, Kohlberg, & Haan, 1977; Renner et al., 1976) have shown that this is the case, as only 30 to 35% of adults reach the level of formal operations.

Piaget's theories have often been contested, yet the results of his experimentation for the stages of development have been replicated with different cultures, although not necessarily at the same ages that Piaget found (Elkind, 1961a-e). Elkind was able to reproduce Piaget's theories on the conservation of mass, weight, and volume (Elkind, 1961b), classification (Elkind, 1961d), the development of abstract right-left conceptions (Elkind, 1961a), comparing quantities (Elkind, 1961c), and the development of abstract conceptions of mass, weight, and volume in junior and senior high-school students (Elkind, 1961e). It has also been found that even if they occur at different ages, the stages still proceed in the same order and build upon one another (Athey & Rubadeau, 1970; Corman & Escalona, 1969; Green, 1978; Raven & Guerin, 1975).

Application to Education

Although Piaget himself did not provide many explanations as to how his theories of cognitive development can be applied to the field of education, in the United States in the 1960s these theories were applied in "new math" programs funded by National Science Foundation (NSF) grants as part of the curriculum reform movement that came about as a result of the demise of progressive education. Piaget's collaborator, Dr. Barbel Inhelder, was a key participant in the Woods Hole Conference on Cape Cod in 1959 that produced some of these programs (Bruner, 1960). However, the "new math" programs were not necessarily altogether successful on a large scale because they were too different from the previous methods of teaching math, such as using direct instruction and lecture-based learning.

Piaget's name is also often cited when referring to the open school movement of the late 1960s and early 1970s (Elkind, 1976). The open, or informal, classroom was based on student-centered learning activities in which students had the opportunity to learn by themselves or with a small group at their own rate by doing structured activities in or even outside of the classroom. The school building itself did not necessarily need to be "open" in the sense of having walls or not, as long as the students were provided the freedom to move about and actively work on their activity.

However, Piaget stressed that children need direction in their play in order for it to be active learning and not just a manipulation of materials. But in reality, the schools without walls were not successful primarily due to many teachers letting their students run free to do whatever they wanted to with the hands-on manipulatives provided in the classroom. Most teachers were not given enough training prior to being put into an open classroom, and the government pulled out its support before the teachers could learn how to be effective leaders in this environment.

More recently, Piaget's ideas regarding active learning have resurfaced under the guise of "discovery learning" and "problem-based learning" especially in the areas of math and science. In particular, Piaget's theories have been applied in the development of the Everyday Mathematics series by the University of Chicago School Mathematics Project in 1998 and the Mathematics: Modeling in Our World by the Consortium for Mathematics and Its Applications in 1998 which incorporate discovery-based learning and are both supported by the NSF and the National Council of Teachers of Mathematics (Goldsmith, 1999).

Today many ideas have remained from the reform movement of the 1960s, such as having children learn to think and problem solve for themselves. However, in the field of education, Piaget's basic theory of cognitive development has been de-emphasized while other constructivist theorists, such as Lev Vygotsky, have gained prominence.

Vygotsky's Theories of Cognitive Development

Vygotsky's social interactionist theory is based on the premise that education occurs through social contact, guides an individual toward a higher level of learning, and is thus based on biology and culture (Vygotsky, 1936/1962, 1978). His theory states that learning is a continuous process and that the key is mastering the signs and symbols of the culture. His primary focus is to explain changes in different levels of psychological functioning. The child can move on to higher levels of learning based on the child's zone of proximal development. Vygotsky (1978) defines the zone of proximal development as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). First, the child is introduced to a new concept at a level slightly higher than that which the child can accomplish on his/her own. Then with practice the child moves up to this higher level of thinking. This process of moving to a higher level of thinking is called scaffolding, which is when children learn concepts beyond their understanding with the aid of another. Concepts build upon one another, thus creating the image of a painter's scaffolding.

Vygotsky's theories of cognitive development have been applied in the educational setting. In particular, his theories are evident in the practice of cooperative learning, in which a student functioning at a lower level is paired with a higher functioning student so that the higher level student might bring the lower level student to a higher point in that student's zone of proximal development. Although

Vygotsky's theories are currently used in the classroom, his theories are difficult to apply because they contrast the traditional teaching method of recitation teaching and require more planning on the part of the teacher (Hausfather, 1996).

Piaget and Vygotsky

Piaget's theories are often compared and contrasted with those of Vygotsky. At first glance, Vygotsky appears to contradict a good portion of Piaget's work with his zone of proximal development and social learning theories (Bell-Gredler, 1986). However, researchers have shown that the theories of Piaget and Vygotsky can complement one another and work together (Shayer, 2003). Although Piaget's theories focus on biological maturation making the development of some skills possible, it has been argued that one cannot move from one stage of cognitive development to the next without external stimulation such as teaching, discussion, or social interaction (Elkind, 1976; McGrath, 1980).

One major difference between the theories of Piaget and Vygotsky lies primarily in the order in which development and learning occur. Piaget's theories are based on development preceding learning, whereas Vygotsky's theories are sequenced such that learning precedes development. In addition, Howe (1996) stated that "Piagetian thought is characterized by the view that the driving force in development is internal while Vygotskian thought is characterized by the view that the driving force is external" (p.42). Piaget is often criticized because his four-stage theory of development ends at adolescence and does not include adult learning. Yet it is often explained that not everyone reaches the last stage of abstract and logical reasoning in all areas. One may also function at different stages depending on what topic is at hand and what a student's particular level of understanding might be. If a student has a high level of expertise in a particular topic, then the student may function at that higher level; if not, the student may function at a lower level. Vygotsky's theory shows a contrasting continuous development using the zone of proximal development instead of stages defined roughly by age.

Another major difference in the two cognitive theories is that Piaget's theory is based on biology, and Vygotsky's is based on a combination of biology and culture. According to Piaget, children progress through the stages at their own biological rate, and when they are developmentally ready they can learn higher level concepts. The primary goal then is to eventually achieve a level of abstract reasoning. However, Piaget states that development ends in adolescence, whereas Vygotsky claims that learning could continue on into adulthood so long as one is being pushed into that zone of proximal development. Vygotsky compares different cultures and feels that the culture one lives in can set limits to the cognitive level one might attain. He also puts a strong emphasis on the learning of symbols. Speech becomes the most important activity in cognitive development in order to learn. Vygotsky does outline basic stages of development for sign use and the development of speech, but unlike Piaget he does not attach ages to it. These stages are perhaps more along the lines of general tendencies rather than actual time frames.

The two theories also diverge on the learning process. Piaget's theories are based on a child's independent experimentation in which the child learns primarily alone with some guidance from others. At the other extreme, Vygotsky states that a child learns from others through scaffolding and interactions with the world.

Despite the differences, Shayer (2003) would argue that the theories and works of Piaget and Vygotsky complement each other, such that "Vygotsky would have needed Piaget's descriptions of development had he gone on in the work of improving schooling and had Piaget wanted to convert his (correct) intuitions about the importance of collaborative learning among peers into school practice…he would have needed to draw on the work of Vygotsky" (p. 478).

Both Vygotsky's and Piaget's theories have a place in education today. Specifically, Piaget's theories are being applied specifically to math and science curricula through "problem-based learning" and "discovery learning" (Goldsmith, 1999), while Vygotsky's are seen in general teaching strategies such as cooperative learning and reciprocal teaching.

Cognitive Development and Math Achievement

A number of studies support the notion that there is a relationship between Piagetian levels of cognitive development and math achievement. Vaidya and Chansky (1980) found this to be the case with second, third, and fourth graders. Al-Dokheal (1983) supports this theory for sixth-grade Saudi Arabian males. Bloland and Michael (1984) and Bitner (1991) surveyed high-school students to show the same positive relationship. Ablard and Tissot (1998) examined reasoning levels of gifted second through sixth graders to show their readiness for higher levels of mathematics.

Finally, Wolfe (2000) found a significant relationship between levels of cognitive development and math achievement in nontraditional college students.

Vaidya and Chansky (1980) found a positive relationship between levels of cognitive development and math achievement with students between second and fourth grades. Even though the focus of this study was with students who have not yet reached the concrete level of cognitive development, this study supports the notion of a budding relationship between the level of cognitive development and math achievement as early as second grade.

Piagetian levels of cognitive development were assessed using the Conservation Test Battery, which classifies children as high or low concrete operational (Vaidya & Chansky, 1980). Additionally, they assessed level of field independence using the Children's Embedded Figures Test and math achievement using the Stanford Achievement Test. They found that of the 102 students in second through fourth grades, field independence was positively correlated to math achievement and operativity had a positive correlation to math achievement for those students in grade 2. From this, Vaidya and Chansky concluded that due to these relationships, it is important for teachers to be aware of results such as these so they can individualize instruction to the specific learner with a higher success rate.

Ablard and Tissot (1998) worked with sixth-grade students who were learning at a concrete level of cognitive development. These students were tested using the quantitative subtest of the School and College Ability Test (SCAT), which predicts scores on the Scholastic Aptitude Test (SAT), and the Arlin Test of Formal Reasoning (ATFR), which indicates a level of Piagetian cognitive development. It was found that these gifted students scored similar to students four grade levels above them and showed a statistically significant relationship between the SCAT and ATFR scores, thus showing a correlation between the level of Piagetian cognitive development and math ability for academically talented students from second to sixth grades. Proficiency scores on the one particular section of the SCAT were also found to be a predictor of success in algebra since those students were also classified as being at the level of formal operations.

Al-Dokheal (1983) focused on a specific group of students, sixth-grade males in Saudi Arabia. This study surveyed 230 boys using the Arnold Math Problem Solving Test for a level of ability in math problem solving and the Piaget Reasoning Test, which classified students by their level of Piagetian cognitive development. It was found that all of these students scored at the early or late concrete operational level and there was a positive correlation between the scores of the two tests, thus showing a correlation between Piagetian level of cognitive development and math ability in this particular subgroup of the population.

Both Bloland and Michael (1984) and Bitner (1991) surveyed high-school students and showed that there is a relationship between Piagetian levels of cognitive development and math achievement. Bloland and Michael (1984) tested a sample of 290 ninth- and tenth-grade students in a first-year algebra course. They concluded that the test of Piagetian cognitive development that was used "could be expected to show considerable validity in forecasting success in the first-year algebra course" (p. 941). It was found that there was a significant correlation between Piagetian developmental level using the ATFR and the final exam and final course grades in the algebra course.

There was also a significant difference in levels of performance between the students found to be functioning at the concrete versus formal operational level.

Bitner (1991) surveyed 101 students from grades 9 through 12 and found levels of formal reasoning, as assessed by the Group Assessment of Logical Thinking (GALT), to be predictive of success in math and science. Specifically, knowledge about the function of controlling variables, or identifying variables in a scientific experiment, explained the most variance in math achievement. Also of interest, only 18% of the students surveyed were found to be functioning at a level of formal operations. However, Bloland and Michael (1984) demonstrated that there was a negative correlation between age and Piagetian level of cognitive development, with 61% of the youngest one fourth of the students performing at the level of formal operations and only 27% of the oldest one fourth of the students were functioning at that level. Moreover, there was a negative correlation between age and final exam scores as well as between age and the algebra course grades. It was found in this case that age was a predictor of success in the algebra course based on that negative correlation as the younger students were more successful than the older students.

In a study by Eaves, Vance, Mann, and Parker-Bohannon (1990), 38 students from kindergarten through grade 12 were tested on levels of mathematics achievement using the Keymath Revised Measurement and reasoning level on the Cognitive Levels Test (CLT). It was found that the abstract reasoning score of the CLT was a significant predictor of math achievement. Similar results were found in a study of students in grades kindergarten through grade 2 (Eaves, Darch, Mann, & Vance, 1990).

It was found that at the college level, the level of Piagetian cognitive development using the GALT had a high to moderate level of correlation with the mathematics portion of the SAT for students in a remedial math course (Berenson, Best, Stiff, & Wasik, 1990). However, the GALT was not found to be a predictor of the final grade in class. One conclusion drawn from this particular study was that although these students passed high-school math courses, they may have memorized algebraic procedures without a thorough understanding of the underlying concepts and without using a level of formal operations. This made it difficult for these students to use those skills when taking the college math placement test, and thus they were placed in remedial math courses. This is consistent with Flavell's (1963) theory that children can be trained to master a particular task but then may not be able to reproduce and apply that learning at a later point in time. This conclusion has been demonstrated again in recent results from the National Assessment of Educational Progress testing, which determined that only seven percent of 17-year-olds who were tested were proficient in solving multistep problems and using algebra (Perie, Moran, & Lutkus, 2005). Sfard and Linchevski (1994) agree with this conclusion in their qualitative study of students working through algebraic equations and inequalities. They found that high-school students can work with problems by applying standard algorithms but are unable to see the abstract ideas in the symbols. Their suggested solution was to change how algebra is being taught, so students can discover their own algorithms and then be able to apply them at a more abstract level.

In addition, Wolfe (2000) found a significant relationship between levels of Piagetian cognitive development and math achievement also using the GALT and tests of arithmetic and algebraic reasoning in nontraditional college students. Wolfe surveyed 264 adult college students age 22 and older. Approximately one third of the students were found to be at the level of formal operations, with another third at the concrete level and the rest were found to be transitioning between the two. This supports the prior studies that have shown that only 30 to 35% of adults reach the level of formal operations (Kuhn, Langer, Kohlberg, & Haan, 1977; Renner et al., 1976). Wolfe also found no correlation between age and level of Piagetian cognitive development in these adults.

Morris and Sloutsky (1998) showed the importance of instruction in developing abstract reasoning and that this type of reasoning does not always develop naturally. In two studies, Morris and Sloutsky gave Russian and English students algebraic tasks to complete, and then the student work was analyzed for use of algebraic reasoning. The students were also interviewed and asked to explain their solutions. Morris and Sloutsky's analysis showed that many students attending regular high schools were not developing formal operations without instruction; although, it was also found that prolonged instruction with an emphasis on algebraic deductive reasoning may contribute to making the transition into formal operations.

To date, the majority of the literature agrees that there exists a relationship between Piagetian levels of cognitive development and math achievement (Ablard & Tissot, 1998; Al-Dokheal, 1983; Vaidya & Chansky, 1980). In particular, a number of studies found a significant relationship between levels of cognitive development and algebraic reasoning in both high-school and college students (Bitner, 1991; Bloland & Michael, 1984; Wolfe, 2000).

Algebraic Reasoning

With the increased national focus on academic achievement, in particular in mathematics, due to the 2001 revision of the Elementary and Secondary Education Act of 1965, or No Child Left Behind (NCLB), schools and educators are being put in a position to increase achievement levels on standardized tests (Moyer-Packenham, 2004). At the high-school level, this often means success in algebra because 95% of 17-year-old students take at least one algebra course in high school (Perie, Moran, & Lutkus, 2005). Algebraic topics appear frequently on high-school-level standardized tests and are also expected to be applied in other high-school math courses such as geometry, trigonometry, a second higher level algebra course, pre-calculus, or calculus. Most college-bound students will take a minimum of two years of algebra in high school.

Yet, research in math education has shown the existence of a cognitive gap as students transition from arithmetic to algebra (Filloy & Rojano, 1989; Goodson-Espy, 1998; Herscovics, 1989; Herscovics & Linchevski, 1994; Kieran, 1988, 1989; Linchevski & Herscovics, 1996; MacGregor & Stacey, 1997; Pillay, Wilss, & Boulton-Lewis, 1998; Sfard & Linchevski, 1994). This cognitive gap can have a negative impact on the success of students who are enrolled in an algebra course and are not ready to be learning at that level.

Filloy and Rojano (1989) coined the term "didactic cut" to describe what others have referred to as a "cognitive gap" between arithmetic and algebraic thinking. According to Filloy and Rojano, this cut is located at the transition between solving algebraic equations with one unknown on one side of the equation, which can be completed arithmetically, and solving algebraic equations with unknowns on both sides of the equation, which requires an algebraic thought process to solve. Herscovics and Linchevski (1994) described the cognitive gap as "the students' inability to operate spontaneously with or on the unknown" (p. 59).

One specific problem area is the inability of students to use algebraic equations in problem solving (Goodson-Espy, 1998; Herscovics, 1989; Kieran, 1989). Goodson-Espy found that college students who had transitioned to a level of algebraic thinking operated at higher levels of reflective abstraction than those who had not successfully completed that transition. In one study, the students were given seven word problems and were asked to solve the problems during unstructured interviews. Those students who were found to be at a higher level of reflective abstraction showed the ability to write and solve algebraic equations for the word problems, unlike those who were still functioning at a lower level and were still using arithmetic reasoning (Goodson-Espy, 1998).

In a similar study of word problems, MacGregor and Stacey (1993) collected data from 281 ninth graders from a free-response algebra test and from 1,048 eighth through tenth graders on a multiple-choice test item. They found the students were unable to translate the words into an equation even after the problems had been written in such a way as to eliminate common reversal errors such as writing $8y = z$ instead of $y = 8z$ for the question "The number y is eight times the number z" (MacGregor & Stacey, 1993, p. 222).

Another area of difficulty for students is working with variables. Herscovics and Linchevski (1994) interviewed a class of 22 seventh graders on solving algebraic equations with variables. They found that most students used arithmetic rather than algebra to solve the equations at this level and were unable to work with the variable as an unknown, as opposed to something that had to be replaced by a number. It was also the case that the students at this level were unable to view the equals sign as a statement of equivalency as opposed to a command to find an answer. This has been found to be the case in other studies as well (Pillay, Wilss, & Boulton-Lewis, 1998).

Pillay et al. (1998) also concluded that a pre-algebra course is a necessity in grade 8 or 9 to make the transition from arithmetic to algebra. In a three-year longitudinal qualitative study, they followed 51 students from grade 7 through grade 9. The students in grades 7 and 8 did not have a complete understanding of the commutative and distributive laws necessary to solve algebraic equations. However, by grade 9, they were able to function at a pre-algebraic level as they had a satisfactory understanding of both laws.

MacGregor and Stacey (1997) sought the origin of the misinterpretation of variables. After testing a large number of students and following the progress of 156 11- to 12-year-olds over two years, they drew some conclusions. They attributed these errors of misinterpreting variables not only to the level of cognitive development but also to teaching methods and to the student interpretations. In particular the students made errors with the use of variables in applied geometric formulas in which a variable represents a specific measurement as opposed to creating algebraic equations for word problems in which variables do not function in the same manner.

Researchers have attempted to investigate and document the development of algebraic reasoning. Starting with elementary students between the ages of 8 and 10, Schliemann and Carraher (2002) found that when algebraic concepts were put into a context the students were familiar with, they were able to solve and graph linear equations of the type found in a high-school algebra text. They also found students often had more success when they were answering oral questions put into a familiar context such as money than when asked to do the same problem with pencil and paper because they then reverted to relying on school-taught algorithms. For example, a young Brazilian street vendor could correctly calculate the prices of goods when on the street, but when given the same problems to complete on paper, the young vendor had a lower success rate.

English and Sharry (1996) attempted to define the development of algebraic abstraction through classifying types of algebraic equations. Ten students in grades 10 and 12 of varying ability levels were given the task of classifying 21 algebraic equations. They found that 12^{th}-grade, above-average students using analogical reasoning to classify algebraic equations were unable to do so at an abstract level. Even after five years of applying algebraic skills in math classes, when classifying the equations these students focused more on the computational process used to solve the equations and not how the equations were related algebraically.

Linchevski and Herscovics (1996) made an attempt to close the cognitive gap through an individualized teaching experiment. In this experiment, six seventh-grade students of different ability levels participated in five lessons intended to teach students to group terms involving literal symbols and solve algebraic equations with unknowns on both sides of the equation. In the lessons, instead of using examples that would immediately require use of formal operations, the researchers began with a transition at a more concrete level. However, rather than closing the gap, they found that obstacles continued to exist, although the students did come up with their own procedures to solve the equations with one-on-one guidance.

Nathan and Koedinger (2000) discovered that how students reason algebraically is not necessarily consistent with the beliefs of teachers and researchers or the textbooks that are used in algebra classrooms. They noted that students did better on word problems and solving algebraic equations when given the opportunity to use strategies such as unwinding or reversing the arithmetic, rather than the formal step-by-step process of solving an algebraic equation with unknowns. This was in contrast with how teachers and researchers ranked problems in order of difficulty. The teachers and researchers ranked the problems according to how difficult they would be to write the algebraic equation and solve, which would require a higher level of abstract thought, while the students were able to solve some of these more difficult problems using primarily arithmetic skills and without even writing an algebraic equation.

Interventions to Shift to Formal Operations

Shayer and Adey (1992a, 1992b, 1993) demonstrated that interventions to shift from a Piagetian level of cognitive development of concrete operations to formal operations work with long-term effects with middle-school students in the science classroom. The primary study they conducted was a part of the Cognitive

Acceleration Through Science Education (CASE) project in the United Kingdom (Adey & Shayer, 1990).

The CASE project was a large-scale program that occurred in the 1980s in eight middle schools in the United Kingdom. The interventions were embedded into the regular science curriculum with lessons related to ten formal operations tests. These lessons took up no more than 25% of the normal science class time. Control classes were also used at each school. The lessons themselves were taught at a rate of one 60- to 80-minute lesson every two weeks for two years. The teachers were given all classroom materials such as notes, worksheets, and problems and bridged the CASE lessons to the regular science curriculum.

Students were tested for their level of Piagetian cognitive development prior to the intervention and again at the end. They were also given posttests on science achievement. Results showed the experimental group had significantly greater gains in levels of cognitive development than the control group. Students were tested again one and two years later on levels of cognitive development, and the experimental group maintained the gains in cognitive development made during the intervention. At two and three years after the initial intervention, half of the students in the experimental group also showed gains in math and English achievement as well as in science.

More recently in a similar study, Iqbal and Shayer (2000) showed that cognitive growth could be accelerated with a group of secondary students from age 11 to 13 in Pakistan. Using the science lessons and training methods from the CASE program in four schools, the students showed positive gains in their cognitive development. This study was originally conducted in response to research that showed that the level of cognitive development required for the Pakistan science curriculum for this age group was far above the level of cognitive development of the students.

Summary

Through Jean Piaget's work, the terms "constructivism," "concrete operation," and "formal operations" have meaning and application to education. Piaget's stage theory of cognitive development has been a recurrent topic in college psychology classes, and now Lev Vygotsky's zone of proximal development is becoming more well known and has also been applied in educational settings. Although Piaget's and Vygotsky's theories seem contrasting at first, their theories can work together to scaffold the learning of abstract concepts to advance students from the Piagetian level of cognitive development of concrete operations to formal operations as has been done through intervention in science classes (Adey & Shayer, 1990; Shayer & Adey, 1992a, 1992b, 1993).

As math, and in particular algebra, requires a logical thinking process, it would indicate that a student will have a higher level of math achievement at a higher Piagetian stage of cognitive development. In particular, because there is a great deal of abstract thought required in algebra, it would suggest that a student who has transitioned to the level of formal operations would have more success than the student who has not yet made that transition. According to Piaget's age approximations in his levels of cognitive development, freshman-level students from ages 13 to 15 have transitioned or are transitioning from concrete operations into formal operations. Many freshmen take an algebra course as their first high-school math course, which typically requires abstract reasoning and thus formal operations, making this grade level important to study. Yet as research has shown, only 30 to 35% of adults reach the level of formal operations (Kuhn, Langer, Kohlberg, & Haan, 1977; Renner et al., 1976). Therefore, adolescents learning algebra at ages 12 to 14 may not be ready for full accommodation of this knowledge due to the abstract nature of algebra because these students may not have reached the level of formal operations (Piaget, 1975/1985). However, with interventions such as those completed by Adey and Shayer (1990) and Iqbal and Shayer (2000), students may be able to make the transition from concrete to formal operations and become better prepared to learn the abstract topic of algebra.

One of the primary purposes of the current study is to create an intervention to accelerate the level of Piagetian cognitive development in high-school algebra students so that they can learn algebraic concepts at an abstract level rather than relying on the memorization of algorithms. If students have a more in-depth understanding of algebraic topics then they will be able to apply these concepts in their future math courses such as geometry, trigonometry, a second higher level algebra course, pre-calculus, or calculus, which are courses that students are taking with higher frequency each year (Perie, Moran, & Lutkus, 2005).

CHAPTER III

METHODS

Hypothesis

The purpose of this study was twofold. First, the study replicated findings of past studies with a sample of American suburban high-school students to determine if those who achieved higher levels of algebraic reasoning were functioning at a higher operational mode according to Piaget's levels of cognitive development. The second purpose was to develop and test an intervention designed to shift students from the transitional stage between concrete operations and formal operations to the formal operations stage and determine what types of strategies the students used to problem solve. According to Piaget's age approximations in his levels of cognitive development, most freshman-level students from ages 13 to 15 have made or are transitioning from concrete operations into formal operations. However, subsequent studies (Kuhn, Langer, Kohlberg, & Haan, 1977; Renner et al., 1976) have shown that these age guidelines do not always apply as only 30 to 35% of adults reach the level of formal operations. Many freshmen take an algebra course as their first high-school math course, which typically requires abstract reasoning and thus formal operations, making this grade level important to study. It is also important for educators to understand a student's cognitive development in order to provide a compatible curriculum to the child. In this study, the level of cognitive development was analyzed with freshman students in algebra.

Research Questions

The following questions were addressed:

1. Is there a relationship between Piagetian levels of cognitive development and the level of algebraic reasoning in high-school freshmen? It was predicted that there is a statistically significant positive relationship between the Piagetian level of cognitive development and level of algebraic reasoning.
2. Is there a relationship between Piagetian levels of cognitive development and grades in algebra class in high-school freshmen? It was predicted that there is a statistically significant positive relationship between the Piagetian level of cognitive development and algebra course grades.
3. Did the intervention group have a statistically significantly greater change in level of Piagetian cognitive development from the transitional stage between concrete operations and formal operations to formal operations than the comparison group with the typical instruction over a 12-week period of time? It was predicted that an intervention can shift the level of Piagetian cognitive development from the transitional stage between concrete operations and formal operations to formal operations.
4. Of those who received the intervention, how were the students who successfully shifted from the transitional stage between concrete operations

and formal operations to formal operations through academic intervention working through the information? Were they using arithmetic strategies or algebraic strategies to solve problems? What processes did they undergo to overcome making errors?

5. Of those who received the intervention, what were the patterns of errors of the students who did not successfully shift from the transitional stage between concrete operations and formal operations to formal operations through academic intervention?

Participants

A sample of 86 high-school freshmen Algebra I students from a Chicago suburban school, both male and female, were solicited to participate in this study. In this particular school, students are initially tracked as freshmen into their math courses based on scores from a district placement test and teacher recommendations and continue on to their sophomore courses based on grades and teacher recommendations.

The school population is made up of approximately 77.2% Caucasian, 11.4% Asian/Pacific Islander, 6% Black, and 4.3% Hispanic, .2% Native American, and .8% Multiracial/Ethnic students. Most of the students come from upper middle-income homes, and the school has a 1.3% low-income rate. There is a total enrollment of 3791 students and a 99.4% graduation rate.

Instrumentation

All participating students were asked to complete a General Information Questionnaire, the Group Assessment of Logical Thinking, and a pretest of algebraic reasoning during the second week of school. Students' first-quarter grades for their algebra course were collected as well.

General Information Questionnaire

The General Information Questionnaire (see Appendix D) was used to assess demographic characteristics of the students such as age (years and months), gender, and ethnicity.

Group Assessment of Logical Thinking

The Group Assessment of Logical Thinking (GALT) (see Appendix E) is a tool for determining Piagetian levels of cognitive development of children and young adults in grade 6 through college (Roadrangka, Yeany, & Padilla, 1982). It has been used successfully in studies to determine Piagetian levels of cognitive development for high-school (Bitner, 1991) and college students (Berenson, Best, Stiff, & Wasik, 1990; Wolfe, 2000) and was standardized with a group of students from grade 6 through college (Roadrangka, 1986). The GALT was designed to report Piagetian levels of cognitive development, specifically along the continuum from the concrete to formal operational stages. It was developed to be used as a group test and eliminate the need for the one-on-one interviews and demonstrations that are typically used to measure levels of cognitive development.

The GALT includes 12 logico-mathematical items covering six Piagetian concepts. These items were chosen by the original authors of the GALT from a pool of 21 questions that were narrowed down to 12 for a test that could be completed in one class period. Two of the 12 items measure skills at the concrete level while the other 10 items measure skills at the formal operational level of cognitive development. The six Piagetian concepts that are tested include the following: conservation, proportional reasoning, controlling variables, probabilistic reasoning, correlational reasoning, and combinatorial logic. The GALT has 10 multiple-choice items and two open-ended items. The multiple-choice items include an answer and a reason. In order for those questions to be answered correctly, both the answer and reason must be correct. The two open-ended questions are the combinatorial logic questions, which are items 11 and 12. In order for item 11 to be answered correctly there can be only one error or omission, and for item 12 to be answered correctly there must be two or fewer errors or omissions.

The GALT in final form was administered to a group of 628 students from grade 6 through college (Roadrangka, 1986). With this population, a construct validity coefficient equal to .80 was obtained using convergent validation with the Test of Piagetian Interview Tasks. Also from this sample, the criterion-related validity between the GALT and the Test of Integrated Process Skills was found to be .71. Reliability using Cronbach's alpha for internal consistency of the scores from this original population was .85 (Roadrangka, 1986).

Test of Algebraic Reasoning

The test to determine algebraic reasoning was a diagnostic test (see Appendix C) that all of the algebra students were given at the start of the school year to confirm correct placement in courses. There were 35 questions on the test, and the students had one class period to complete the test. They could not use a calculator on the test. All of the questions were a review of material taught at the middle-school level such as fraction operations, real number operations, evaluating algebraic expressions and inequalities, writing algebraic expressions, opposites, absolute value, the distributive property, and application of concepts in word problems. This test was created using information from the algebra textbook used in the algebra course (Larson, Kanold, & Stiff, 1997) and had been used with all of the students taking algebra for the past three years. The test is aligned with the National Council of Teachers of Mathematics (NCTM) principles and standards for algebra (NCTM, 2000) through the use of "symbolic algebra to represent situations and to solve problems" (p. 395) and by having the students "recognize and generate equivalent forms for simple algebraic expressions and solve linear equations" (p. 395).

Intervention

The purpose of the intervention was to move students who scored at the transitional level of cognitive development between concrete and formal operations to the level of formal operations. The intervention included tasks pertaining to five of the six tasks associated with formal operations: proportional reasoning, controlling variables, probabilistic reasoning, correlational reasoning, and combinatorial logic (see Appendix H).

For proportional reasoning, students were given examples of proportions and were asked to work through the problems while providing their reasons to the researcher. Examples include the following: 3/5 = y/20; 25/16 = x/40; You have to read a 220-page book. It takes you 15 minutes to read 10 pages. How long will it take you to read the whole book?; Give two triangles with corresponding sides of lengths 3, 4, 5 and 4.5, 6, x, find the missing length.

For controlling variables, students were given two examples of experimentation. The first experiment was to balance a cardboard clown on a pencil using a variety of given materials. The second experiment was to make a paper helicopter spin as slowly and then as quickly as possible. The students were asked to work through the experiment while providing reasons to the researcher.

For probabilistic reasoning, students were given examples of probability and were asked to work through the problems while providing their reasons to the researcher. Examples include the following: You have a bag containing 8 red marbles, 10 blue marbles, and 4 white marbles. What is the probability of choosing a blue marble? What is the probability of choosing a red or white marble?

For correlational reasoning, students were shown a drawing of two different shapes of three different sizes colored in two corresponding colors and asked to define the variables and move to the definition of "relationship." They were then given further examples with shapes of different sizes and colors and were asked to classify them and provide their reasoning.

For combinatorial logic, students were given examples of combinations and permutations and were asked to work through the problems while providing their reasons to the researcher. Examples include the following: What is the number of possible orders for a track relay with four members? What is the number of possible orders for 12 people to sit around a table? How many ways can you choose two of six of your friends to go out to the movies with you? How many ways can you choose three people from a group of five?

Students were given these problems in written form with an increasing difficulty level throughout the sessions. They were given oral feedback as they worked on each individual question, and if they provided an incorrect answer, they were provided with another example, possibly in a different format for better understanding. They were also asked to write their reasons down when asked for them.

Procedure

During the 2006-2007 school year, each potential participant was given a voluntary informed consent form to be signed by both the student and the student's parent or guardian to indicate explicit consent (see Appendix A). After all of the potential participants returned the consent form to their math teacher, the study was conducted. Each participant was given a questionnaire to compile specific demographic statistics. Each child also completed the GALT to determine his or her Piagetian level of cognitive development. This took place in the students' math class, one 45-minute class period, during the regular school day. There was no penalty for nonparticipation. The nonparticipants were given an alternate assignment on the day the study was conducted. A small-group intervention was then conducted with eight students who, based on the GALT score, were at the transitional stage between concrete and formal operations with a goal of moving them to the level of formal operations. Students were chosen to participate in the intervention using a stratified random sample based on gender and score on the GALT. These students were offered some extra credit toward their final second-quarter and third-quarter grades in class for participating in the intervention and were also given a gift card to a local book store. Eight intervention meetings occurred outside of regular class time before school, so students would not lose regular instructional time, and lasted approximately 30 minutes one time per week over a course of 12 weeks. After approximately 6 weeks and 12 weeks of intervention strategies, all of the students who had previously tested at the transitional level were asked to be retested using the GALT, including the students who were a part of the intervention and those who were either not chosen to be a part of the intervention or were asked but chose not to participate in the intervention.

Students were given the test of algebraic reasoning the day before taking the initial administration of the GALT. This was also taken during class time because it was used by the teachers to verify correct placement in the course.

Grades for the algebra course were also collected at the end of the first quarter for all of the students who initially took the GALT.

Data Analysis

Four types of quantitative data were collected in this study. There was demographic data from the General Information Questionnaire, level of algebraic reasoning, first-quarter grades, and the information collected from the administration of the Group Assessment of Logical Thinking (GALT).

Demographic data included age, gender, and ethnic group to describe the sample population tested. Percentages were calculated according to the data collected.

The level of algebraic reasoning was recorded as a score out of 100 points possible based on the number of answers correct on the algebraic reasoning test. A score above 70 shows competency of the material on the test.

First-quarter grades were recorded as a percentage out of 100 based on the weighting of grades in the algebra course. Grades for the course were weighted with 70% based on tests and quizzes and 30% based on homework and participation.

One can receive a score ranging from 0 to 12 on the GALT based on the number of answers correct. These scores determine the level of cognitive development at which one is functioning. A score from 0 to 4 indicates a concrete level of cognitive development, a score of 5 through 7 indicates that one is in the transitional stage from concrete to formal operations, and a score of 8 to 12 indicates functioning at the formal operations level of cognitive development. These scores aided in testing the original predictions of the study.

Correlational analysis was used to determine if there is a significant positive relationship between the level of algebraic reasoning and the overall GALT score. This tested the statistical significance of the first prediction, which was that there was a significant positive relationship between Piagetian levels of cognitive development and the level of mathematics achievement. Correlational analysis was also used to test the second prediction, which was that there was a significant positive relationship between the GALT score and the grade in the algebra course. The third prediction was tested using a repeated-measures analysis of the change in the initial, week 6, and week 12 GALT scores between the students receiving the intervention and the other students who initially were at the transitional level between concrete operations and formal operations to determine if the intervention was successful. The statistical program SPSS was used for the statistical analyses.

Qualitative analyses were used to answer the fourth question, which was to determine how the students who successfully shifted from the transitional stage between concrete operations and formal operations to formal operations processed the information and what processes they went through to overcome their errors. During the intervention, observations of the students were written down immediately after the meetings. The observations and written data from the students were coded using content analysis according to whether the students were using primarily arithmetic methods or algebraic methods to work through the problems (Herscovics & Linchevski, 1994). Arithmetic methods were classified as those using no variables and by using strategies such as unwinding or reversing the arithmetic or by going through each step using an arithmetic operation rather than using patterns to work through a problem. Algebraic methods were classified as those using variables or patterns to solve the problems. Coding was done by hand and statistical data, including percentages and averages, were calculated. A negative case analysis was conducted to verify the original hypothesis. The algebraic and arithmetic cases were compared to those that do not fit in either category to determine whether the original hypothesis was correct or needed to be modified. A member check was also done in order to confirm the meaning of the responses of the participants. The participants were asked to verify their responses when it was not clear whether the response was an algebraic or arithmetic response.

A qualitative analysis was also used to answer the fifth question, which was to determine the patterns of errors that occurred in the students who did not successfully transition from the transitional stage between concrete operations and formal operations to formal operations. The observations and written data from the students were used to find the common errors that occurred and coded using content analysis according to the types of errors made. The analytic categories for the types of errors were based on prior research and included the incorrect use of algebraic equations in problem solving (Goodson-Espy, 1998; Herscovics, 1989; Kieran, 1989), an incorrect use of variables in a problem (Herscovics & Linchevski, 1994; MacGregor & Stacey, 1997), difficulty with using the equals sign as equivalency rather than a command to find an answer (Herscovics & Linchevski, 1994; Pillay, Wilss, & Boulton-Lewis, 1998), and difficulty using proper operations to solve algebraic equations and group like terms appropriately (Linchevski & Herscovics, 1996). Coding was done by hand

and statistical data such as percentages and averages were then calculated. A negative case analysis was conducted to verify the original hypothesis. The types of errors were compared to those that did not fit in any category to determine whether the original hypothesis was correct or needed to be modified. A member check was also done in order to confirm the meaning of the responses of the participants. The participants were asked to verify their responses when their work was not clearly shown.

CHAPTER IV

RESULTS

Of the 86 high-school freshman Algebra I students solicited to participate in the study, 82 (95.3%) turned in the permission slip, and 76 (88.4%) of those chose to participate. Of those who chose to participate, 39 (51.3%) were female; 37 (48.7%) were male. The students classified their ethnicities as follows: 68.4% Caucasian, 14.5% Asian, 3.9% Hispanic, 2.6% African American, and 10.5% other.

All 76 students completed the General Information Questionnaire, the Group Assessment of Logical Thinking (GALT), and the test of algebraic reasoning (the day prior to taking the GALT). Of the 76 students tested with the GALT for their level of Piagetian cognitive development, 38 (50%) scored at the concrete operational level, 27 (35.5%) were found to be at the transitional level between concrete and formal operations, and 11 (14.5%) scored at the formal operational level. Twelve of the students scoring at the transitional level on the GALT had the lowest transitional score, 5 out of 12; seven students scored 6 out of 12, and eight students scored the highest transitional score, 7 out of 12. Twelve of the 27 students scoring at the transitional level were asked to participate in the small-group intervention, four from each transitional score level. Eight of those 12 students responded that they would participate, and all eight students completed all eight of the interventions as well as both retests of the GALT. In addition, 17 of the other 19 students scoring at the transitional level, who did not participate in the intervention, also completed both retests of the GALT. Internal consistency reliability for the GALT using Cronbach's alpha from the scores of this study was .537.

Analyses of Research Questions

Cognitive Development and Algebraic Reasoning

1. *Is there a relationship between Piagetian levels of cognitive development and the level of algebraic reasoning in high-school freshmen? It was predicted that there is a statistically significant positive relationship between the Piagetian level of cognitive development and level of algebraic reasoning.*

There was a statistically significant correlation between Piagetian level of cognitive development and the level of algebraic reasoning, $r = .404, p < .001, R^2 = .163$. The GALT scores ranged from 1 to 10 while the algebraic reasoning scores ranged from 61 to 100. There was a difference in the scores of the female students and the male students. The GALT scores for the females ranged from 1 to 8 while for males they ranged from 1 to 10. The algebraic reasoning scores for females ranged from 61 to 100 while for males they ranged from 71 to 100. Means and standard deviations for each assessment are shown in Table 1. When comparing the Piagetian level of cognitive development for females, there was not a statistically significant correlation with algebraic reasoning, $r = .296, p = .067, R^2 = .088$. However, for males there was a statistically significant correlation between the Piagetian level of cognitive development and algebraic reasoning, $r = .389$, $p = .017$, and $R^2 = .151$. A graph of this correlation can be found in Figure 1. Although not in the original hypothesis, gender differences were tested based on research that has shown that males have consistently scored better than females in math, and the differences between them become more apparent in high school (American Association of University Women, 1992; Hyde, Fennema, & Lamon, 1990).

Table 1

Means and Standard Deviations of GALT Scores and Algebraic Reasoning Scores for Males (n = 37) and Females (n = 39)

	Male		Female		Total	
Variable	M	SD	M	SD	M	SD
GALT Score	5.76	2.229	4.10	1.744	4.91	2.161
Algebraic Reasoning	87.84	8.318	82.62	9.952	85.16	9.502

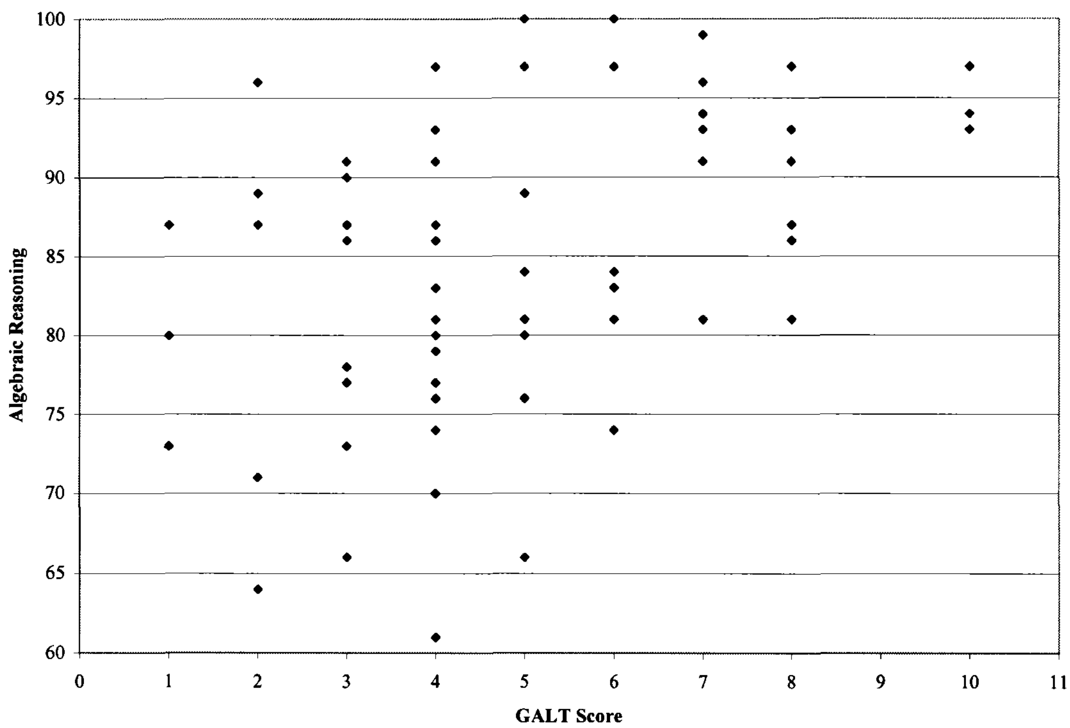


Figure 1. Relationship between Piagetian level of cognitive development and algebraic reasoning.

Cognitive Development and Algebra Course Grades

2. *Is there a relationship between Piagetian levels of cognitive development and grades in algebra class in high-school freshmen? It was predicted that there is a statistically significant positive relationship between the Piagetian level of cognitive development and algebra course grades.*

There was not a statistically significant correlation between the level of Piagetian cognitive development and quarter-one grades in algebra class, $r = .117$, $p = .318$, $R^2 = .014$. Means and standard deviations are given in Table 2. The course grades for quarter one ranged from 65 to 99. The quarter-one course grades for females ranged from 65 to 99. The quarter-one course grades for males ranged from 66 to 97.

Table 2

Means and Standard Deviations of Quarter One Algebra Course Grades for Males (n = 37) and Females (n = 38)

Variable	Male M	Male SD	Female M	Female SD	Total M	Total SD
Course Grades	84.68	8.148	85.76	6.816	85.23	7.472

There was not a statistically significant correlation between the level of Piagetian cognitive development and quarter one course grades for females or males, $r = .245$, $p = .139$, $R^2 = .060$, and $r = .096$, $p = .570$, $R^2 = .009$, respectively. The graph of this correlation can be found in Figure 2.

Course grades were recorded such that an A was from 91 to 100 percent, a B was from 81 to 90 percent, a C was from 71 to 80 percent, a D was from 65 to 70 percent, and an F was any score below a 65. Frequencies of course grades are reported in Figure 3.

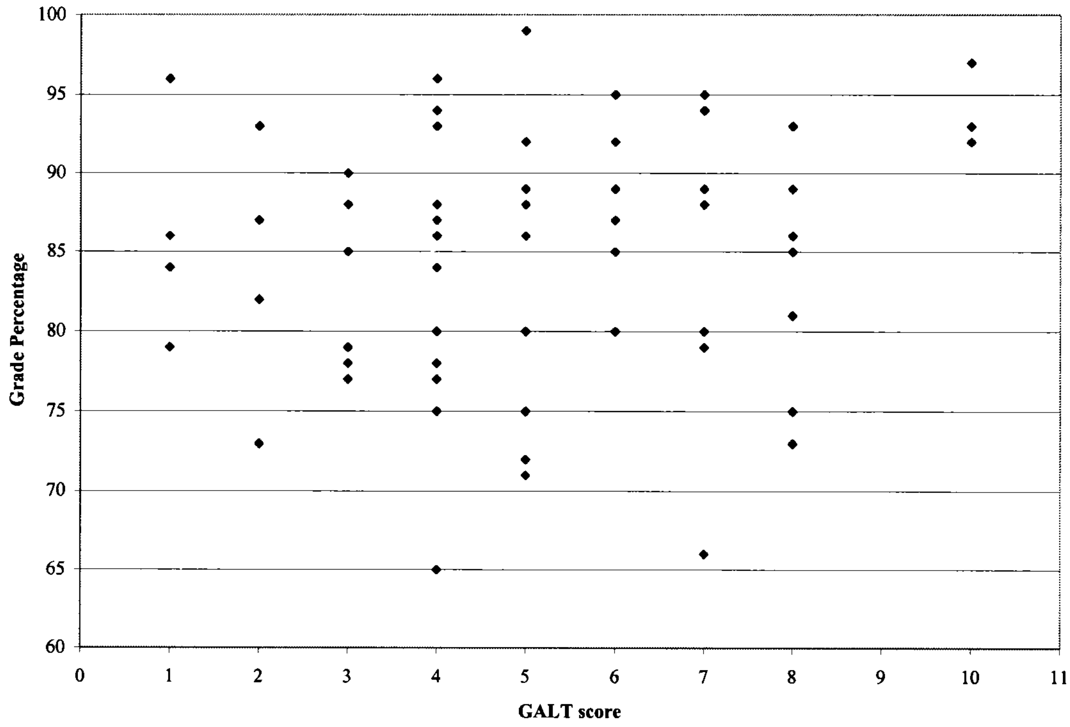


Figure 2. Relationship between Piagetian level of cognitive development and quarter one algebra course grade percentages.

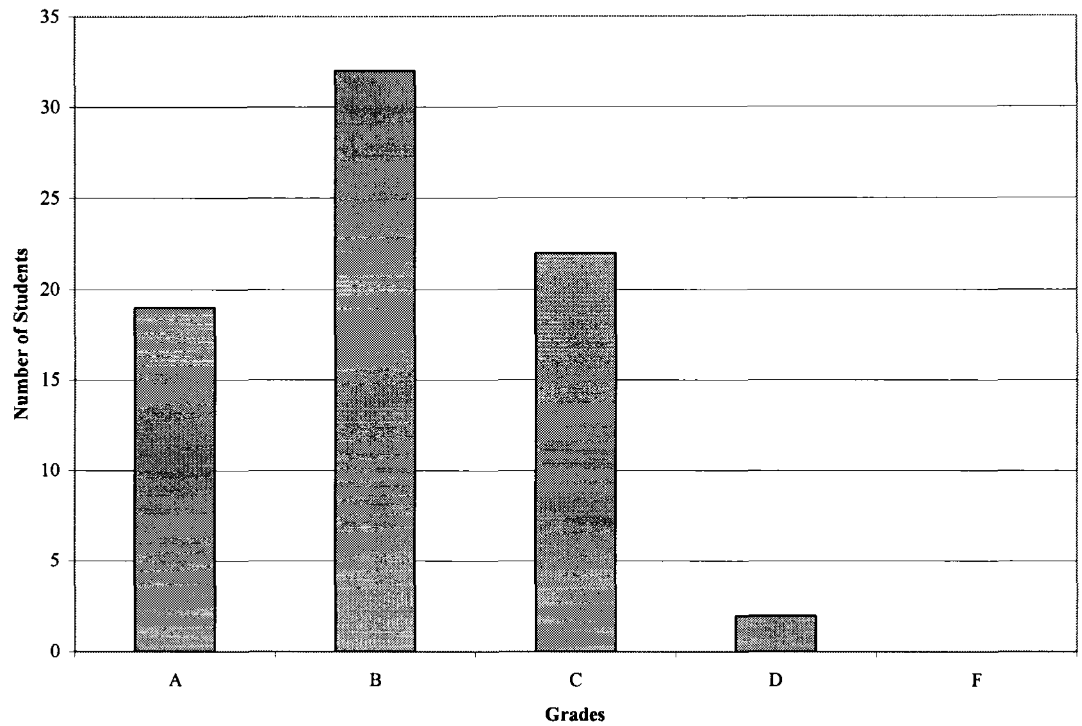


Figure 3. Frequency of algebra course grades for quarter one.

Cognitive Development of Intervention and Non-Intervention Students

3. *Does the intervention group have a statistically significantly greater change in level of Piagetian cognitive development from the transitional stage between concrete operations and formal operations to formal operations than the comparison group with the typical instruction over a 12-week period of time? It was predicted that an intervention can shift the level of Piagetian cognitive development from the transitional stage between concrete operations and formal operations to formal operations.*

Tests also showed that there was not a statistically significant difference in the change in the scores of the initial, 6-week, and 12-week administrations of the GALT between the intervention and non-intervention students, $F(2, 23) = .409$ and $p = .667$, $\eta^2 = .017$. There was a significant linear trend in the GALT scores of both the intervention and non-intervention students averaged across both groups, $F(1, 23) = 23.372$ and $p < .001$, $\eta^2 = .504$. Means and standard deviations are reported in Table 3. A graph showing the linear trend can be found in Figure 4. In comparing the intervention and the non-intervention groups, the effect size of the initial GALT administration was $d = .411$ and the second administration was $d = .389$, both of which show between a small and medium indication of the strength of the difference of the means between the two groups. The third administration of the GALT had an effect size of $d = .056$, which showed virtually no indication of the strength of the difference in the means between the intervention and non-intervention groups. Means and standard deviations of algebraic reasoning scores and quarter-one course grades for the intervention and non-intervention groups are reported in Table 4, and demographic data for both groups can be found in Table 5.

Although there was not a statistically significant difference in scores between the students who participated in the intervention and those who did not, six of the eight intervention students did increase their scores on the GALT from the initial to the third administration while the other two maintained their scores on the GALT. Of the six who increased their scores, three of them increased it to a level of formal operations with a score of eight or higher. On the second administration of the GALT, six of the eight students not only had higher scores on the GALT, they also had scores that were at the level of formal operations, but only two of them continued to improve

Table 3

Means and Standard Deviations of GALT Scores for Intervention Students (n = 8) and Non-Intervention Students (n = 17)

	Initial		Week 6		Week 12	
Variable	M	SD	M	SD	M	SD
Intervention	6.00	0.926	7.25	1.909	7.50	1.414
Non-Intervention	5.65	0.786	6.47	2.095	7.41	1.734
Total	5.76	0.831	6.72	2.031	7.44	1.609

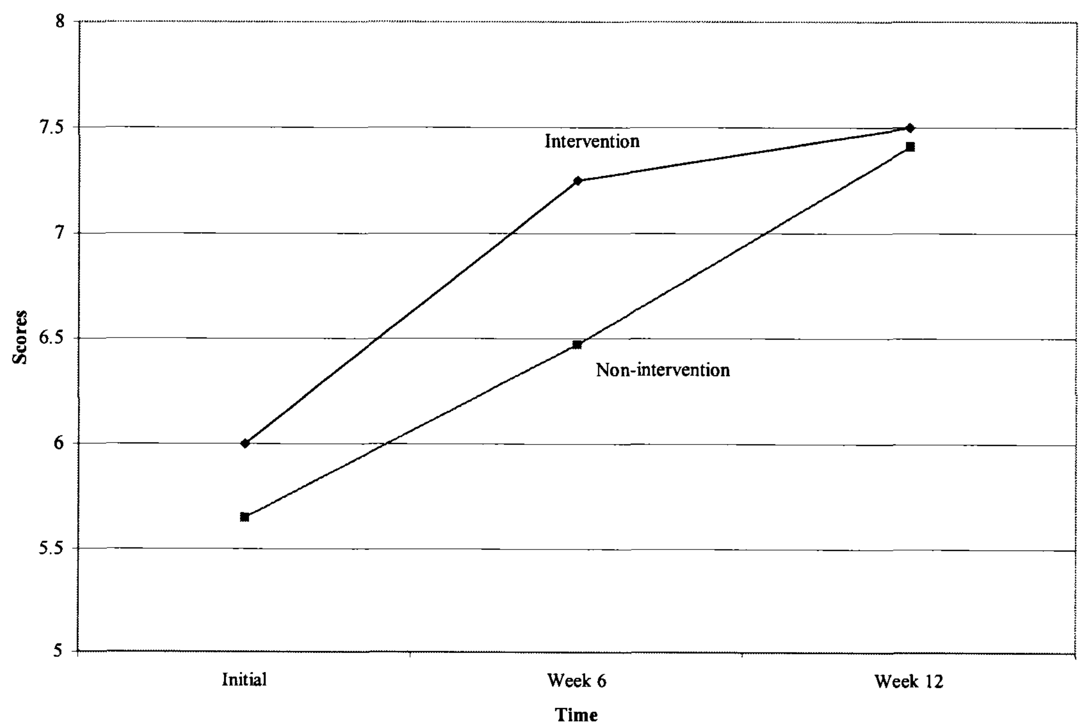


Figure 4. Means of GALT scores for intervention and non-intervention students.

Table 4

Means and Standard Deviations of Algebraic Reasoning Scores and Quarter One Algebra Course Grades for Intervention Students (n = 8) and Non-Intervention Students (n = 17)

	Intervention		Non-Intervention	
Variable	M	SD	M	SD
Algebraic Reasoning	90.25	10.620	86.71	9.790
Course Grades	89.50	4.986	83.65	8.760

Table 5

Demographic Data for Intervention Students (n = 8) and Non-Intervention Students (n = 17)

	Intervention		Non-Intervention	
	n	%	n	%
Gender				
Male	3	37.5	10	58.8
Female	5	62.5	7	41.2
Ethnicity				
Caucasian	6	75.0	12	70.6
Asian	1	12.5	2	11.8
Hispanic	0	0.0	2	11.8
African American	0	0.0	0	0.0
Other	1	12.5	1	5.9

on the final administration while the other four scored back at the transitional level between concrete and formal operations (see Table 6). All student names are reported as pseudonyms in the analyses.

Strategies Used and Patterns of Errors During the Intervention

The eight intervention meetings took place over a 12-week time period that also included both retests of the GALT as well as the school's two-week winter break. The meetings were held before school in a classroom that was not in use until the fifth period of the school day. The students were divided into two groups based on when they could make it before school, with four students in each group. However, there were occasions when students would need to attend the other meeting that week due to conflicts in their schedules.

At each meeting, the students were asked to complete the intervention activity for that week and fill in all necessary information on paper. The researcher walked around while they were working and answered questions as they were completing the intervention activity. The researcher also took detailed notes about the students and the intervention after the students completed the activity and left the room.

The first intervention activity was a worksheet on solving proportions. There were six problems of increasing difficulty that the students needed to solve and provide an explanation of how they arrived at their answers. Three of the students had difficulty with solving the equations until they were informed that answers did not need to be whole numbers. Only one student, Sarah, got all six answers correct.

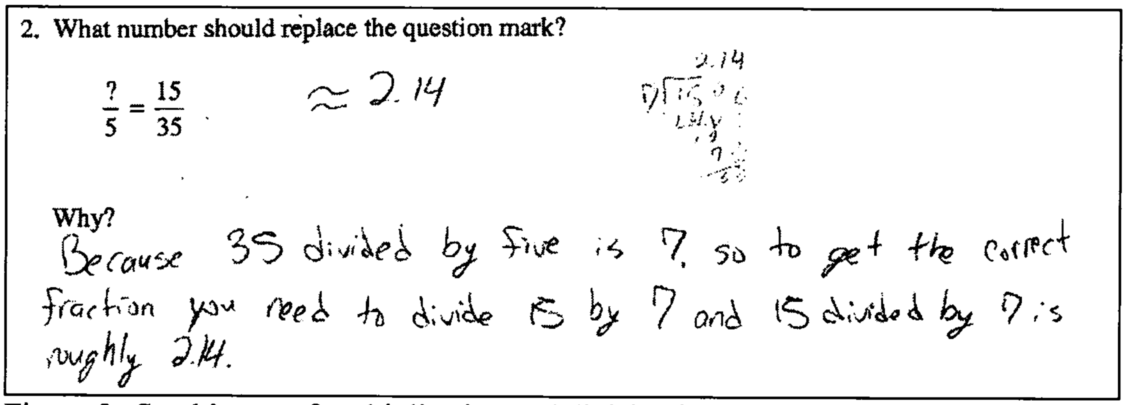
Table 6

GALT Scores for Intervention Students

	Initial	Week 6	Week 12
Beth	7	8*	10*
Cathy	5	3	7
John	5	8*	7
Melanie	6	9*	6
Ricky	6	6	8*
Sam	7	8*	9*
Sarah	7	8*	7
Valerie	5	8*	6
Mean	6.00	7.25	7.50
Standard Deviation	0.926	1.909	1.414

Note. Scores of 0 to 4 are concrete operational, 5 to 7 are transitional, and 8 to 12 are formal operational.
*Scored at level of formal operations.

However, she also used arithmetic processes of multiplication and division to solve all of the problems (see Figure 5). Ricky and Melanie also solved all six problems using only arithmetic but only got three out of six correct. The rest of the students incorporated algebraic equations in at least two out of the six problems by setting up algebraic equations and using cross-multiplication to solve them (see Figure 6). All of the other students also received scores of four or five correct out of six. The most common errors on this intervention were setting up an incorrect arithmetic equation or calculating incorrectly.



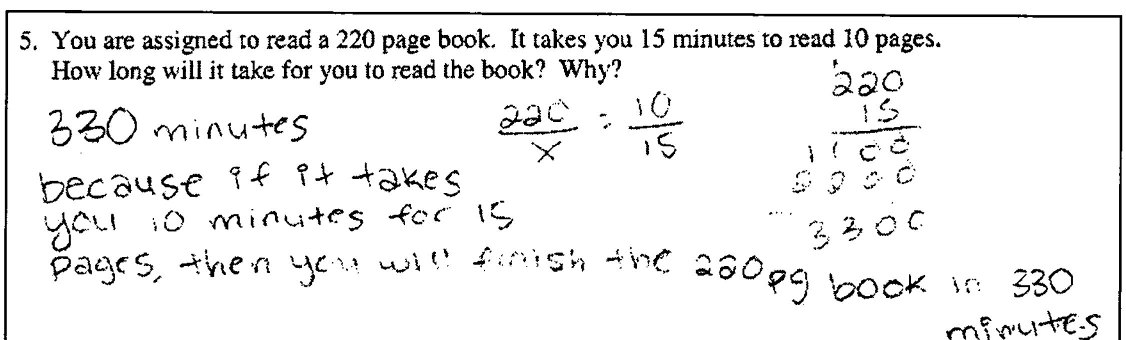
2. What number should replace the question mark?

$\frac{?}{5} = \frac{15}{35}$ ≈ 2.14

Why?

Because 35 divided by five is 7, so to get the correct fraction you need to divide 15 by 7 and 15 divided by 7 is roughly 2.14.

Figure 5. Sarah's use of multiplication and division in the first intervention activity.



5. You are assigned to read a 220 page book. It takes you 15 minutes to read 10 pages. How long will it take for you to read the book? Why?

330 minutes $\frac{220}{x} = \frac{10}{15}$

because if it takes you 10 minutes for 15 pages, then you will finish the 220pg book in 330 minutes

Figure 6. Beth's use of cross-multiplication in the first intervention activity.

The second intervention activity involved the use of scientific experimentation to balance a cardboard clown on a pencil using different materials such as cotton balls, clothespins, pipe cleaners, paper clips, and coins. All of the students were able to get the clown to balance with varying levels of difficulty. Ricky and Sam figured out how to make it balance on the first try without using trial and error; however both came up with different methods. Ricky used two paper clips and coins attached to the clown hands while Sam hung clothespins on the hands. All of the students wrote up their process as having used trial and error, except for John, who wrote, " The first thing I did was a control," referring to using the scientific method to solve the problem.

The third intervention took place over a three-week period of time and involved finding theoretical and experimental probability. The first activity was a coin toss activity. Based on what they wrote when comparing the theoretical probability of one half to their experimental results after flipping a coin 50 times, all of the students showed a clear understanding of what the experimental probability should have been even if their numbers did not come out to exactly one half. The second activity was a replacement activity that involved having the letters "MISSISSIPPI" in a bag, and they were asked to draw the letters out and calculate the theoretical and experimental probabilities. Four out of the eight students made errors and stated that their probabilities were not close to the theoretical when in fact they were. These students stated that the fractional probabilities were very different, and they did not change the fractions into decimals to make an accurate comparison. The third activity was finding the theoretical and experimental probabilities of rolling two dice. In this activity, two of the eight students made errors on their comparisons due to an incorrect comparison of the fractions. All of these activities were classified as using arithmetic methods due to the fact that there were no algebraic equations to write or solve to find or compare the theoretical and experimental probabilities.

The fourth intervention activity was a worksheet designed to have the students determine variables for a collection of shapes and figure out what another shape would look like if one were added to the collection. There were three problems in increasing difficulty. The first set of shapes only had one variable to be identified, the second set of shapes had two variables to be identified, and the final set of shapes also had two variables, but the description of the shape to be added would depend on one of the variables. Variable was defined to the students verbally as "what makes the shapes vary, or what makes them the same/different." Four of the eight students made at least one error in determining the variables, classified as a "pattern error," and two of those students made errors in two of the three questions. One student, Ricky, made errors due to thinking there could only be one of each type of shape with the same variables in the collection of shapes without repeating the same pattern.

The fifth intervention activity was a worksheet that had the students solving permutation and combination problems. The permutation problems involved putting a series of objects in order while the combination problems involved choosing a certain number of objects from a larger group of objects. Two of the students needed clarification as to whether order would be important when completing the combination problems. This intervention session was the only one that involved any direct instruction from the researcher to the whole group. One of the permutation problems would have had the students listing 120 different ways to order five textbooks. The students started listing them but then complained that there would be too many to list, so the researcher gave them the formula for finding the number of permutations as well as the formula for combinations so the students could first calculate the number of combinations before listing the actual combinations. Getting these problems correct was classified as using algebraic methods due to the process involved to get the correct answer. Three out of the eight students got all eight problems correct. Of the students who had incorrect answers, three of the students made pattern errors while the other two either made calculation errors or did not read the directions properly.

The final intervention activity was another scientific experimentation activity designed to have the students control variables. In this activity the students needed to create a paper helicopter and then modify it to make it spin as fast as possible and then as slow as possible by changing the weight of the paper, changing the size of the helicopter, or by adding weights to the bottom of the helicopter. The students were given an explanation of what the three variables were in this experiment before starting the experiment. Based on their written explanation, four of the eight students used a scientific method to complete the experiment, while the others did not. The students who used the scientific method methodically tried all of the variables to determine how they affected the speed of the helicopter.

A member check was conducted throughout the intervention process by the researcher checking each student's work as it was turned in and asking the students to either show more work if there was not enough work shown or asking them to write a more thorough statement about their process in completing the problem.

4. *Of those who received the intervention, how were the students who successfully shifted from the transitional stage between concrete operations and formal operations to formal operations through academic intervention working through the information? Are they using arithmetic strategies or algebraic strategies to solve problems? What processes did they undergo to overcome making errors?*

The strategies that the students used in the intervention were categorized as using arithmetic/non-scientific methods, using algebraic/scientific methods, or neither. All of the students except for one used algebraic/scientific methods more than half of the time. There did not appear to be any noticeable difference in percentages between the students who did successfully transition to formal operations and those who did not. The students who transitioned to formal operations used arithmetic strategies an average of 40.67% of the time, algebraic strategies an average 57.67% of the time, and neither strategy an average of 1.67% of the time; whereas the students who did not transition used arithmetic strategies an average of 38.00% of the time, algebraic strategies an average of 54.80% of the time, and neither strategy an average of 7.20% of the time. However, most of the algebraic or scientific methods were methods that had been taught to the students at some point during a middle-school math or science class. Individual results are reported in Table 7.

Based on previous research, the hypothesized original categories for errors included an incorrect use of algebraic equations in problem solving, an incorrect use of variables in a problem, difficulty with using the equals sign as equivalency rather than a command to find an answer, and difficulty using proper operations to solve algebraic equations and group like terms appropriately. However, the students did not make all of these types of errors and did make some others, so the categories were modified to include the errors they did make. The patterns of errors for the intervention students were categorized as one of the following: an incorrect algebraic equation, a pattern error, a calculation error, or an incorrect arithmetic equation.

Of these four types of errors, the use of an incorrect arithmetic equation had the highest frequency with 12 errors. This was followed by 11 pattern errors, nine calculation errors, and one incorrect algebraic equation. The first activity on solving proportional reasoning produced the highest number of calculation errors; both the first activity and the third activity on probabilistic reasoning tied for the highest number of errors in the use of an incorrect arithmetic equation. The fourth and fifth activities on correlational reasoning and combinatorial logic respectively had the highest numbers of pattern errors in the intervention.

Of the three students who tested at the level of formal operations on the final administration of the GALT, Beth and Sam had the fewest number of errors in the entire intervention, one and three respectively, and their errors were only in the calculation category. Both of these students had also scored at the formal operations level at both the week-6 and week-12 administrations of the GALT. The third student to reach formal operations at the end of the intervention was Ricky, and he had scored the same on the first and second administrations of the GALT while moving up to formal operations only on the final administration of the GALT. Ricky also had the highest number of errors throughout the course of the intervention. Beth and Sam did not need to overcome making errors since the only errors they made were calculation

Table 7

Percentages of Types of Strategies Used During the Intervention

	Arithmetic %	Algebraic %	Neither %
Transitional			
Cathy	27	55	18
John	27	64	9
Melanie	50	50	0
Sarah	50	41	9
Valerie	36	64	0
Formal Operational			
Beth	36	64	0
Ricky	50	50	0
Sam	36	59	5
Mean	39.00	55.88	5.13
Standard Deviation	9.842	8.442	6.556

errors, which could have been corrected if they had use of a calculator. Ricky did not overcome making errors, as he did have the highest number of errors through the intervention. He did become more adept at asking questions through the course of the intervention to help understand the directions or the process he needed to follow to complete the activity.

5. *Of those who received the intervention, what were the patterns of errors of the students who did not successfully shift from the transitional stage between concrete operations and formal operations to formal operations through academic intervention?*

The students who did not successfully transition to formal operations had a variety of different errors. Only Cathy made the error of using an incorrect algebraic equation. The other errors that were made by the students who did not transition were as follows: using an incorrect pattern to solve the problem, using an incorrect arithmetic equation, and basic calculation errors such as multiplication and division errors. The most common error was using an incorrect arithmetic equation to solve the problem. A breakdown of these errors by student can be found in Table 8.

Summary of the Results

The results confirmed the first hypothesis by showing there is a positive relationship between Piagetian levels of cognitive development and the level of algebraic reasoning in high-school freshmen. When separated by gender, these results

Table 8

Numbers of Errors for Intervention Students

	Algebra	Pattern	Calculation	Arithmetic	Total
Transitional					
Cathy	1	2	0	0	3
John	0	0	2	3	5
Melanie	0	0	2	2	4
Sarah	0	4	0	1	5
Valerie	0	1	1	3	5
Formal Operational					
Beth	0	0	1	0	1
Ricky	0	4	0	3	7
Sam	0	0	3	0	3
Mean	0.13	1.38	1.13	1.50	4.13
Standard Deviation	0.354	1.768	1.126	1.414	1.808

were also confirmed specifically for males but not for females. The results failed to confirm the second hypothesis because there was not a statistically significant positive relationship between the Piagetian level of cognitive development and algebra course grades in high-school freshmen. The third hypothesis was also not confirmed by the data since there was not a greater change in the level of Piagetian cognitive development for the students who did receive the intervention than for the students who did not receive the intervention. However, there was evidence of a linear trend in the level of cognitive development averaged across both groups.

The results of the fourth research question showed that the three intervention students who successfully shifted to formal operations were not consistent in their methods of solving problems; however, they all used algebraic strategies at least half of the time. Of those three students, the two who tested at formal operations at both the second and third administrations of the GALT made only calculation errors.

The analysis of the fifth research question demonstrated that the intervention students who did not successfully shift to formal operations on the final administration of the GALT made primarily pattern errors or errors in writing the arithmetic equation to solve the problem. In addition they had a few calculation errors as well.

CHAPTER V

DISCUSSION

This chapter begins with a summary of the results of the study and is followed by an interpretation of the results. The implications for research are then discussed followed by the implications for practice, the limitations of the study, suggestions for future research, and a final conclusion.

Summary of the Results

Quantitative analyses showed a positive relationship between the Piagetian level of cognitive development and levels of algebraic reasoning in high-school freshmen. However, statistical analyses did not confirm a significant relationship between Piagetian levels of cognitive development and algebra course grades. Nor did the results confirm that the students who participated in the intervention had a greater change in the level of Piagetian cognitive development than the students who did not receive the intervention.

Qualitative analyses showed that the students who participated in the intervention and successfully shifted from the transitional stage between concrete and formal operations to formal operations used algebraic strategies more than 50% of the time. In addition, two of those students, who had also tested at the level of formal operations at the half-way point of the study, only made calculation errors during the intervention activities.

Analyses of the error patterns showed that the students who participated in the intervention and did not successfully shift from the transitional stage between concrete and formal operations to formal operations primarily made pattern errors or made errors when writing an arithmetic equation to solve the problems during the intervention activities.

Interpretation of the Results

At the start of this study, half of the students tested at the concrete level of cognitive development. This does not match with Piaget's age approximations for concrete and formal operations as all of the students tested were between 13 and 15 years old, yet Piaget theorized that children would begin to move to formal operations at approximately age 12. However, this result is supported by studies that have shown that only 30 to 35% of adults have reached the level of formal operations (Kuhn, Langer, Kohlberg, & Haan, 1977; Renner et al., 1976). Therefore it is possible that some of the students who tested at the low end of concrete operations at the beginning of the study may never transition to the level of formal operations.

The results of this study showed a statistically significant correlation between algebraic reasoning and Piagetian level of cognitive development in high-school algebra students. These results support previous research which has shown a relationship between math achievement and Piagetian level of cognitive development with elementary students (Ablard & Tissot, 1998; Eaves, Darch, Mann, & Vance, 1990; Eaves, Vance, Mann, & Parker-Bohannon, 1990; Vaidya & Chansky, 1980). Previous research has shown a relationship between cognitive development and math achievement with different aged children. Al-Dokheal found this relationship with sixth-grade Saudi Arabian males. Bloland and Michael (1984) and Bitner (1991) surveyed high-school students and found the same positive relationship between levels of cognitive development and mathematics achievement. Wolfe (2000) also confirmed a positive relationship between level of Piagetian cognitive development and math achievement with nontraditional college students.

In addition, the fact that there was not a statistically significant relationship between the level of Piagetian cognitive development and the grades in algebra class is supported by the work of Berenson, Best, Stiff, and Wasik (1990), even though this contradicts the original hypothesis of the current study. Berenson et al. (1990) had found that although the students were successful in their math courses, the level of Piagetian cognitive development was not a significant predictor of their final grade in class. It is possible the students in the current study were functioning in a similar fashion to the students in the Berenson et al. study. It is also possible that they were memorizing algorithms to succeed in class but did not thoroughly understand all of the concepts as was shown in the research by Sfard and Linchevski (1994). The lack of relationship between Piagetian cognitive development and grades in algebra class in this study is also consistent with the work of Flavell (1963), who theorized students can be trained to master tasks without a complete understanding but then cannot apply them at a later point in time. In addition, in the current study, the student grades were weighted with 70% based on test and quiz scores and the other 30% was based on homework scores. So a student who was not necessarily successful on tests or quizzes could still perform well in the class by working hard on homework, which was not graded based on the percentage correct the way tests and quizzes were.

The results of the intervention in this study contrast the work done by Shayer and Adey (1992a, 1992b, 1993). They showed that interventions in science classes to shift the level of cognitive development do have long-term effects, whereas the current study was not successful in advancing the intervention students to the level of formal operations more frequently than those students who did not participate in the intervention. However, the Shayer and Adey experiment was a large-scale longitudinal project that took place over the course of two school years with the intervention embedded into the science curriculum. The intervention lessons were 60- to 80-minute lessons and took place once every two weeks for a total of 30 lessons. It also took place across eight schools. The current study took place outside of class over the course of a 12-week time period with one 20- to 30-minute intervention every week. Yet in the current study, even though the intervention students did not all transition into formal operations, they all maintained or increased their level of cognitive development over the 12-week time period. With more intervention time and activities embedded into the math or science curricula, it is possible that more of the intervention students would have maintained a level of formal operations between the six-week and twelve-week retests of the Group Assessment of Logical Thinking (GALT) as seven of the eight intervention students tested at the level of formal operations on at least one of the two retests. Also, there was a larger increase in the level of cognitive development for the intervention students than the non-intervention students between the initial and six-week administration of the GALT. The effect size of the initial and six-week GALT results showed there was between a small to medium power in the difference of the means in both the first and second GALT administrations and almost no power in the difference of the means of the final GALT administration. In addition, the non-intervention students also increased their level of cognitive development, although at a slower rate than the intervention students. All of the students participating in the study, including both the intervention and the non-intervention students, were concurrently taking both an algebra course and a biology course, which could have impacted the results of the intervention because all of the students were receiving other instruction requiring abstract reasoning in both of these courses. Another consideration is because all of the students participating in the study were between the ages of 13 and 15, and thus within the range of Piaget's age approximation for transitioning to formal operations, they could have been impacted by their own biological maturity and not any external influences with their increased level of cognitive development. Also, because the intervention took place over 12 weeks it is possible that the students were able to remember the questions on the GALT from one retest to the next and learn from their mistakes, although they were not given the correct answers at any time during the study. There was also a less than standard reliability found for the GALT using the scores of all of the students in the study, which is a possible explanation for the non-significant test results.

The strategies used by the students during the intervention were also inconsistent with the research of Goodson-Espy (1998), who found that students who were functioning at a lower level of reflective abstraction were using primarily arithmetic reasoning. The results of the current study showed no noticeable difference between the students functioning at a level of formal operations and those who were still at the transitional stage between concrete and formal operations.

The patterns of errors made by the students who had not yet transitioned into formal operations were consistent with prior research, such that the students made errors in the incorrect use of algebraic equations (Goodson-Espy, 1998; Kieran, 1989). It was also found that the students made errors in setting up the correct arithmetic equation and in using the correct patterns, neither of which were errors referred to in prior research. Both the errors of using an incorrect algebraic equation and using an incorrect pattern show that those students were still working at a concrete level when doing the activities. For incorrect algebraic equations, the students were not using the abstract level of thinking necessary to understand the relationship of the given information and what they were being asked to find in order to write an algebraic equation with a variable in it. Also, when writing proportional equations, those students were relying on the previously taught and possibly memorized algorithm of using cross-multiplication of the fractions to solve the equation. In finding the correct patterns for the permutation and combination activities the students often were not realizing which permutations or combinations were the same due to reversal of letters or being able to find all of the different possible orders because they were unable to see the patterns using abstract reasoning. Also, when asked to define variables, the students often did so in a concrete manner and were unable to see the patterns that required abstract reasoning. In addition, in the probability activity, the students were unable to compare equivalent fractions, which also requires an abstract level of thought.

Overall, with this specific group of eight students it is quite possible that the intervention was not effective in advancing their level of Piagetian cognitive development. Even though seven of the intervention students did attain a level of formal operations on either retest of the GALT, only three tested at that level on the final retest. But at the same time, the non-intervention students also increased their scores on the GALT. That the intervention was not effective could be due to the fact that the intervention was completed outside of class and was not as intense a program as the one found in the Shayer and Adey research (1992a, 1992b, 1993). Also, the students were taking two other courses requiring abstract thinking, algebra and biology, at the same time as the intervention that may have had a confounding impact on their level of cognitive development.

Implications for Research

The current study used intervention activities of a mathematical nature to advance the level of Piagetian cognitive development that could be applied to larger group and classroom use. This is different than previous research that has attempted to transition students to the level of formal operations using primarily Piagetian tasks (Lawson, Blake, & Nordland, 1976; Lawson & Wollman, 1976; Siegler, Liebert, & Liebert, 1973) or science-based activities (Lawson & Snitgen, 1982; Shayer and Adey, 1992a, 1992b, 1993). The intervention activities in this study can be expanded and can be incorporated into math curricula for middle-school or high-school math courses and then can be used with a larger number of students. The activities are also a base for a Vygotskian approach in the classroom because they begin at a level that the students can accomplish on their own and move to a level that requires a higher level of thinking. In this study, the students worked alone on the activities, but in the classroom the lower level students could work with partners that are already at a higher level of cognitive development. In addition, during the intervention, the students were asked to analyze their work and summarize what they did, which was an important process in helping the students reflect on their thought processes in completing the activities.

This study was also able to show an in-depth analysis of the qualitative data. By working with a small number of students, the researcher was able to work with each student individually during the intervention and teach them in a way that would not have been possible with a larger group of students. Also, the analysis of the types of strategies the students used and the types of errors the students made was able to be completed at a much higher level of detail than with a larger group. By using her own students, the researcher also had a stronger rapport with the students and was able to talk to them more informally. By doing this she was able to have them follow through with a complete written response of each intervention, which could have been more difficult with students the researcher did not know as well. The students also had more incentive to do what the researcher asked because she was also their teacher, and, even as freshmen, many students still have a desire to impress their teacher.

Implications for Practice

Even without a full intervention, the GALT could also be used by classroom teachers so they have a better understanding of what level their students are functioning at so they can cater their teaching to the specific learning level of their students. By using the GALT or some type of cognitive development assessment tool at the start of the school year to determine the level the students are functioning at, the daily lessons can then be differentiated to meet the needs of the different levels of learners in the classroom.

In addition, the use of more hands-on activities such as the intervention activities in this study could be incorporated into math and/or science lessons to enhance the learning of the students. Also, by incorporating more written response activities and more class discussions about the problem-solving process used in the activities, the students are then forced to apply their knowledge and reflect upon what they have learned. However, in order for students to be more successful in a freshman-level algebra course, it would be better for these types of activities to be incorporated into the elementary and/or middle-school levels. By doing more activities like this the students will be more prepared when entering high school to use abstract thinking. Many schools are already taking this initiative by implementing programs such as the Piaget-based Everyday Mathematics series by the University of Chicago School Mathematics Project and the Mathematics: Modeling in Our World by the Consortium for Mathematics and Its Applications, which are designed with these types of activities in mind.

However, once students do reach the high-school algebra course, the teaching methods for the typical algebra course may need to change to continue to stimulate the learning of students who have been brought up with more enriched activities, as most algebra courses are still taught using more lecture-based methods. By incorporating more problem-based-learning activities into the algebra curriculum, the students will be able to explore real-life situations and use algebraic methods and abstract thinking to solve the problems.

Limitations

The timing of the final administration of the GALT may have been problematic, especially for the intervention students, due to the fact that it was given two weeks after a two-week winter break and there was only one intervention meeting between the winter break and the final administration of the GALT. The intervention students may have benefited from a review of the previous intervention material after a break away from it.

Yet, the non-intervention students also increased their level of cognitive development, so it is possible that because the students took the GALT three times in a relatively short period of time some of them may have remembered some of the questions from one time to the next.

Also, the small number of students who were a part of the intervention group limited the power for detecting statistically significant results when comparing the initial GALT and final GALT scores. In addition, the demographics of the intervention group and the non-intervention group were not evenly matched. To overcome these limitations, a greater number of participants would have to have been initially screened because the number of students was limited by how many students initially tested at the transitional level between concrete and formal operations.

The timing of the study could also be considered a limitation. Had the study taken place during a different time of the year, such as during the summer, the students would not have been impacted by effects of the other classes they were taking, such as algebra and biology, that may have also contributed to an increase in the level of abstract reasoning. If this study had been completed when the students were not in school, it would have ensured that the results were not confounded with other instructional effects. However, during the summer the researcher would have then been limited by the accessibility to the students and would not have been able to meet with the students on a consistent basis or been able to give them any incentive, such as the extra credit in class, to continue to meet.

The intensity of the intervention might also be considered a limitation because it was only implemented one time per week, and it occurred in a before-school setting that had no real relationship with any of their other classes, other than the extra credit that was being offered toward their math grade. The skills that the students were learning were not being consistently reinforced in the classroom as they were not topics that were a part of the algebra curriculum. It may be that an intervention such as this one needs to be incorporated into the math curriculum as Shayer and Adey (1992a, 1992b, 1993) successfully did in science classrooms on a large scale. In addition, the meeting times were such that the bulk of the meetings, five of them, occurred prior to the first retake of the GALT and then was followed by two more meetings, a two-week winter break, and one more meeting prior to the final administration of the GALT. Thus the intervention students may have been at a disadvantage due to not having consistent intervention meetings before the final retake of the GALT. This result was also shown with the effect size difference between the three GALT administrations. There was between a small to medium power in the difference of the means in the first and second GALT administrations, whereas there was almost no power in the difference of the means of the final GALT administration. The intervention students showed a faster increase in their level of cognitive development than the non-intervention students between the initial and six-week GALT administrations, but there was not as much of an increase between the six-week and twelve-week GALT administrations.

Also, the offering of extra credit to the students as an incentive to participate in the study may have been a limitation because the students may not have put their best effort into the activities. This could have been an impact on the first administration of the GALT, the intervention activities, and both the middle and final administrations of the GALT because the students were given extra credit to participate in all of these activities. The students may have been more interested in boosting their grades with the extra credit than in producing the best results for the research data.

Suggested Future Research

Although the results were not statistically significant, this study should be replicated because the results of this study did show that seven of the eight intervention students tested at a level of formal operations at either the midpoint of the intervention or at the conclusion. Yet the transition was not yet stable enough to result in a statistically significantly greater change than the control group. However, some changes to consider would be to use a larger group of students both for the initial administration of the GALT and for the intervention. Also, it may be beneficial to embed the intervention into the math curriculum so the concepts can be reinforced on a day-to-day basis rather than having independent intervention sessions on only a weekly basis that do not relate to each other or to the current math curriculum. For the students in this study, the concepts used in the intervention were topics covered in the middle-school math and science curriculum in this particular district. However, it was apparent that overall the students did not have a full understanding of many of the concepts other than the algorithm for solving proportions, finding probability, and basic scientific method. An intervention such as this in the middle-school pre-algebra course would prepare the students better for the level of abstract reasoning necessary for taking an algebra course.

Conclusion

For long-term success and understanding of algebraic concepts, students need to be able to reason abstractly and thus need to be functioning at a level of formal operations. However, based on the results of this study it is evident that not all students enrolled in freshman-level algebra courses are functioning at this level. Many students are still functioning at a concrete level and thus are not likely to fully comprehend the material and will be unable to apply it later when they are enrolled in a higher level high-school math course such as geometry or pre-calculus. These students may also have difficulty when they are asked to apply algebraic concepts on state standardized tests or college placement exams. It is also possible that some of the students may never reach the level of formal operations (Kuhn, Langer, Kohlberg, & Haan, 1977; Renner et al., 1976).

Although the current study did not show definitively that small-group intervention was effective for advancing the Piagetian level of cognitive development in high-school algebra students, there was some measurable improvement in the intervention group that indicated the intervention was having a positive effect. It is possible that by embedding the intervention into the curriculum, so the concepts are reinforced on a daily basis, the students would have better success. Prior research has shown instruction is an important factor in developing abstract reasoning (Morris & Sloutsky, 1998). It would also seem that a change in curriculum may be needed so students can do more discovery-based learning in algebra as suggested by Sfard and Linchevski (1994).

In order for students to succeed in high-school mathematics, they need a strong foundation in algebra as algebraic concepts and their applications are found in all higher level high-school math curricula such as geometry, trigonometry, and pre-calculus. It is also the case that many of these courses are requirements for high-school graduation and then for acceptance into college. Thus it is necessary for these students to make the transition from concrete to formal operations in order to be successful in high-school mathematics. It is important for teachers and educators to take note of this and modify some of the current methodology in math classes so students are required to use abstract reasoning instead of skill-based knowledge, especially in algebra courses. The long-term effects of this would hopefully be evidenced by a higher number of students graduating, being accepted into college, and placing out of remedial-level math courses in college, as well as higher state standardized test scores. In this day and age with a college degree being a minimum requirement for many jobs, having these students succeed in their high-school math courses would be the first step to success and having them become productive members of society.

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APPENDIX A

INITIAL PARENT AND STUDENT CONSENT FORM

August 30, 2006

Dear Parent or Guardian:

I am conducting a study for my Doctoral dissertation on cognitive development and algebraic reasoning in high school students. The purpose of this study is to determine whether there is a relationship between cognitive development and algebraic reasoning, and if such a relationship is found to exist then attempt to accelerate cognitive development to improve algebraic reasoning. It is hoped that this study will help educators to improve algebraic understanding in the classroom.

With your permission and your child's permission, your child or ward will be asked to respond to questionnaires on demographic information and logical thinking during a portion of his/her math class. It should take about 35-40 minutes to complete both questionnaires. Based on the results of the questionnaire, a small group of students may be selected for further study, and permission to participate in the small group will be asked at that time.

Your child may choose not to participate or complete the questionnaires. There are no penalties for not participating or withdrawing early from the study.

Your child's responses will remain confidential. Results of the questionnaires will be reported in group form only, and no names will be stored with the responses given (their names will be deleted from the questionnaire).

If you have any questions about this study, please contact me at ( ) . If you have any questions regarding your son's/daughter's rights as research participants, please call the Northern Illinois University Office of Research Compliance at (815) 753-8588. Please indicate on the form attached whether or not your child/ward may participate in this study and have him/her return it to me in class.

Thank you,
Michelle R. Wesolowski

Parent Consent Form: An Intervention to Advance Piagetian Levels of Cognitive Development and Algebraic Reasoning in High-School Students

Responsible Faculty Member: Dr. Janet Holt
Department of Educational Technology, Research and Assessment
Northern Illinois University
(815)753-8523

I have read and understand the letter attached and agree for my child to participate in this study on cognitive development and algebraic reasoning. I can request a copy of this form.

Name of child (please print) ______________________________

Parent or Guardian signature ______________________________ Date __________

I have read and understand the letter attached and agree to participate in this study on cognitive development and algebraic reasoning.

Student signature ____________________________________ Date __________

APPENDIX B

INITIAL STUDENT ASSENT FORM

September 7, 2006

Dear Student:

I am conducting a study for my Doctoral dissertation on cognitive development and algebraic reasoning in high school students. The purpose of this study is to determine whether there is a relationship between cognitive development and algebraic reasoning, and if such a relationship is found to exist then attempt to accelerate cognitive development to improve algebraic reasoning. It is hoped that this study will help educators to improve algebraic understanding in the classroom.

With your permission, you will be asked to respond to questionnaires on demographic information and logical thinking during a portion of your math class. It should take about 35-40 minutes to complete both questionnaires. Based on the results of the questionnaire, a small group of students may be selected for further study, and permission to participate in the small group will be asked at that time.

You may choose not to participate or complete the questionnaires. There are no penalties for not participating or withdrawing early from the study.

Your responses will remain confidential. Results of the questionnaires will be reported in group form only, and no names will be stored with the responses given (names will be deleted from the questionnaire).

If you have any questions about this study, please contact me at ( ) . If you have any questions regarding your rights as a research participant, please call the Northern Illinois University Office of Research Compliance at (815) 753-8588. Please indicate on the form attached whether or not you will participate in this study.

Thank you,
Michelle R. Wesolowski

Student Assent Form: An Intervention to Advance Piagetian Levels of Cognitive Development and Algebraic Reasoning in High-School Students

Responsible Faculty Member:
Dr. Janet Holt
Department of Educational Technology, Research and Assessment
Northern Illinois University
(815)753-8523

I have read and understand the letter attached and agree to participate in this study on cognitive development and algebraic reasoning.

Name of student (please print)___________________________

Student signature ___________________________ Date __________

APPENDIX C

GENERAL INFORMATION QUESTIONNAIRE

ID: _________

GENERAL INFORMATION QUESTIONNAIRE

1. What is your birth date? Month _____________ Date _______ Year _______

2. What is your gender?
   - Male
   - Female

3. How would you classify your ethnicity?
   - Caucasian
   - Hispanic
   - African American
   - Asian
   - Other

4. In which math course are you currently enrolled?
   - Practical Math I
   - Algebra I Part I
   - Algebra I
   - Geometry
   - Honors Geometry
   - Algebra II/Trigonometry
   - Honors Algebra II/Trigonometry
   - Other

APPENDIX D

GROUP ASSESSMENT OF LOGICAL THINKING

GALT

GROUP TEST OF LOGICAL THINKING

Developed by:

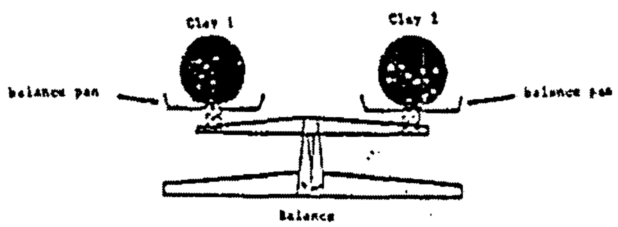
Vantipa Roadrangka
Russell H. Yeany
Michael J. Padilla
University of Georgia
Athens, Georgia 30602

December 1982

Item 1

<u>Piece of Clay</u>

Tom has two balls of clay. They are the same size and shape. When he places them on the balance, they weigh the same.



The balls of clay are removed from the balance pans. Clay 2 is flattened like a pancake.

WHICH OF THESE STATEMENTS IS TRUE?

a. The pancake-shaped clay weighs more.

b. The two pieces weigh the same.

c. The ball weighs more.

REASON

1. You did not add or take away any clay.

2. When clay 2 was flattened like a pancake, it had a greater area.

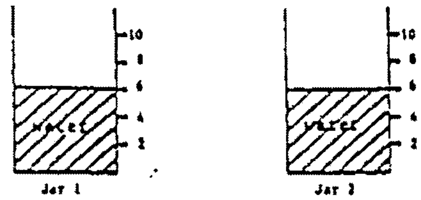
3. When something is flattened, it loses weight.

4. Because of its density, the round ball had more clay in it.

Item 2

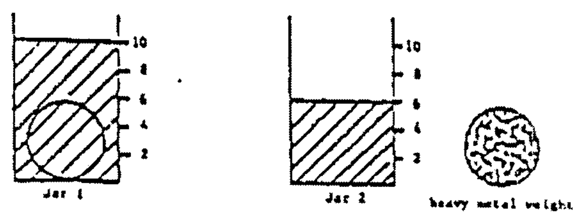
<u>Metal Weights</u>

Linn has two jars. They are the same size and shape. Each is filled with the same amount of water.



She also has two metal weights of the same volume. One weight is light. The other is heavy.

She lowers the light weight into jar 1. The water level in the jar rises and looks like this:



IF THE HEAVY WEIGHT IS LOWERED INTO JAR 2, WHAT WILL HAPPEN?

a. The water will rise to a higher level than in jar 1.
b. The water will rise to a lower level than in jar 1.
c. The water will rise to the same level as in jar 1.

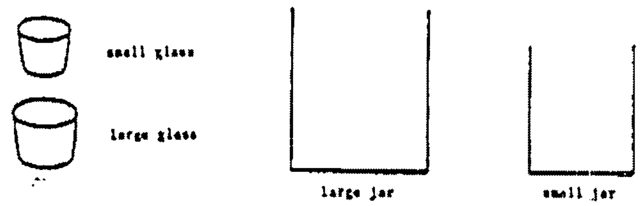
REASON

1. The weights are the same size so they will take up equal amounts of space.
2. The heavier the metal weight, the higher the water will rise.
3. The heavy metal weight has more pressure, therefore the water will rise lower.
4. The heavier the metal weight, the lower the water will rise.

Item 3

Glass Size # 2

The drawing shows two glasses, a small one and a large one. It also shows two jars, a small one and a large one.



It takes 15 small glasses of water or 9 large glasses of water to fill the large jar. It takes 10 small glasses of water to fill the small jar.

HOW MANY LARGE GLASSES OF WATER DOES IT TAKE TO FILL THE SAME SMALL JAR?

a. 4

b. 5

c. 6

d. other

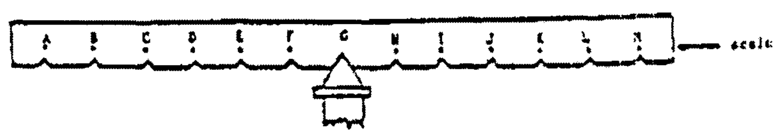
REASON

1. It takes five less small glasses of water to fill the small jar. So it will take five less large glasses of water to fill the same jar.
2. The ratio of small to large glasses will always be 5 to 3.
3. The small glass is half size of the large glass. So it will take about half the number of small glasses of water to fill up the same small jar.
4. There is no way of predicting.

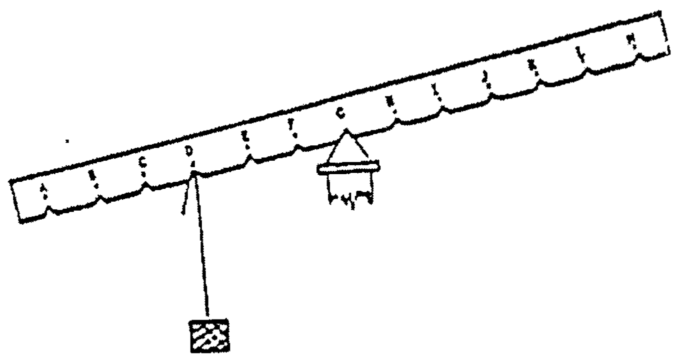
Item 4

<u>Scale # 1</u>

Joe has a scale like the one below.



When he hangs a 10-unit weight at point D, the scale looks like this:



WHERE WOULD HE HANG A 5-UNIT WEIGHT TO MAKE THE SCALE BALANCE AGAIN?

a. at point J

b. between K and L

c. at point L

d. between L and M

e. at point M

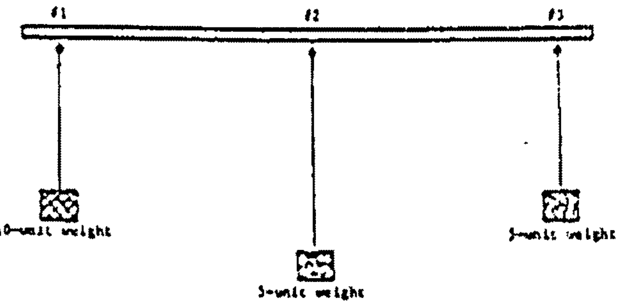
REASON

1. It is half the weight so it should be put at twice the distance.
2. The same distance as 10-unit weight, but in the opposite direction.
3. Hang the 5-unit weight further out, to make up its being smaller.
4. All the way at the end gives more power to make the scale balance.
5. The lighter the weight, the further out it should be hung.

Item 5

Pendulum Length

Three strings are hung from a bar. String #1 and #3 are of equal length. String #2 is longer. Charlie attaches a 5-unit weight at the end of string #2 and at the end of #3. A 10-unit weight is attached at the end of string #1. Each string with a weight can be swung.



Charlie wants to find out if the length of the string has an effect on the amount of time it takes the string to swing back and forth.

WHICH STRING AND WEIGHT WOULD HE USE FOR HIS EXPERIMENT?

a. string #1 and #2

b. string #1 and #3

c. string #2 and #3

d. string #1, #2, and #3

e. string #2 only

REASON

1. The length of the strings should be the same. The weights should be different.

2. Different lengths with different weights should be tested.

3. All strings and their weights should be tested against all others.

4. Only the longest string should be tested. The experiment is concerned with length not weight.

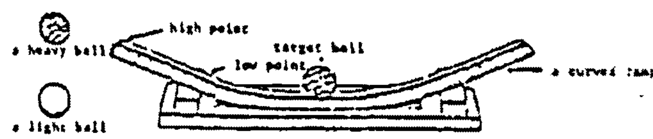
5. Everything needs to be the same except the length so you can tell if length makes a difference.

Item 6

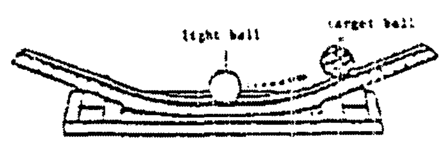
Ball #1

Eddie has a curved ramp. At the bottom of the ramp there is one ball called the target ball.

There are two other balls, a heavy and a light one. He can roll one ball down the ramp and hit the target ball. This causes the target ball to move up the other side of the ramp. He can roll the balls from two different points, a low point and a high point.



Eddie released the light ball from the low point. It rolled down the ramp. It hit and pushed the target ball up the other side of the ramp.



He wants to find out if the point a ball is released from makes a difference in how far the target goes.

TO TEST THIS WHICH BALL WOULD HE NOW RELEASE FROM THE HIGH POINT?

a. the heavy ball
b. the light ball

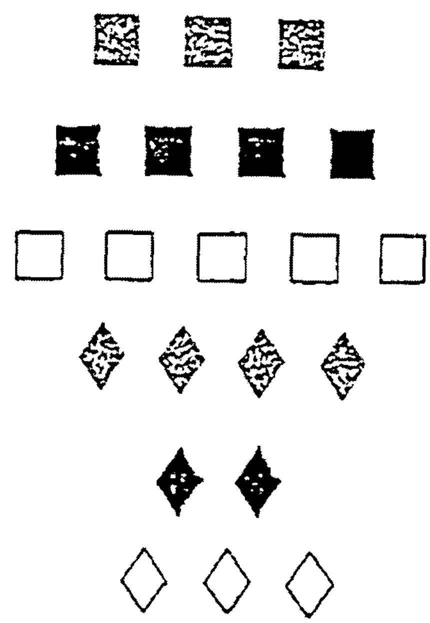
REASON

1. He started with the light ball he should finish with it.
2. He used the light ball the first time. The next time he should use the heavy ball.
3. The heavy ball would have more force to hit the target ball farther.
4. The light ball would have to be released from the high point in order to make a fair comparison.
5. The same ball must be used as the weight of the ball does count.

Item 7

<u>Squares and Diamonds #1</u>

In a cloth sack, there are



3 spotted wooden squares

4 black wooden squares

5 white wooden squares

4 spotted wooden diamonds

2 black wooden diamonds

3 white wooden diamonds

All of the square pieces are the same size and shape. The diamond pieces are also the same size and shape. One piece is pulled out of the sack. WHAT ARE THE CHANCES THAT IT IS A SPOTTED PIECE?

a. 1 out of 3
b. 1 out of 4
c. 1 out of 7
d. 1 out of 21
e. other

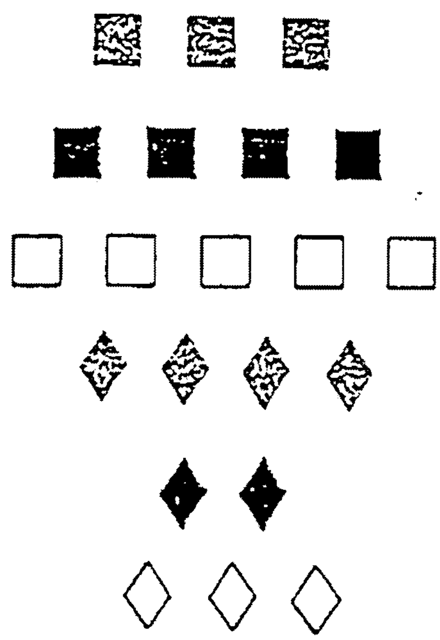
REASON

1. There are twenty-one pieces in the cloth sack. One spotted piece must be chosen from these.
2. One spotted piece needs to be selected from a total of seven spotted pieces.
3. Seven of the twenty-one pieces are spotted pieces.
4. There are three sets in the cloth sack. One of them is spotted.
5. 1/4 of the square pieces and 4/9 of the diamond pieces are spotted.

Item 8

<u>Squares and Diamonds #2</u>

In a cloth sack, there are



3 spotted wooden squares

4 black wooden squares

5 white wooden squares

4 spotted wooden diamonds

2 black wooden diamonds

3 white wooden diamonds

All of the square pieces are the same size and shape. The diamond pieces are also the same size and shape. Reach in and take the first piece you touch.

WHAT ARE THE CHANCES OF PULLING OUT A SPOTTED DIAMOND OR A WHITE DIAMOND?

a. 1 out of 3
b. 1 out of 9
c. 1 out of 21
d. 9 out of 21
e. other

REASON

1. Seven of the twenty-one pieces are spotted or white diamonds.
2. 4/7 of the spotted and 3/8 of the white are diamonds.
3. Nine of the twenty-one pieces are diamonds.
4. One diamond piece needs to be selected from a total of twenty-one pieces in the cloth sack.
5. There are 9 diamond pieces in the cloth sack. One piece must be chosen from these.

Item 9

<u>The Mice</u>

A farmer observed the mice that live in his field. He found that the mice were either fat or thin. Also, the mice had either black tails or white tails.

This made him wonder if there might be a relation between the size of a mouse and the color of its tail. So he decided to capture all of the mice in one part of his field and observe them. The mice that he captured are shown below.

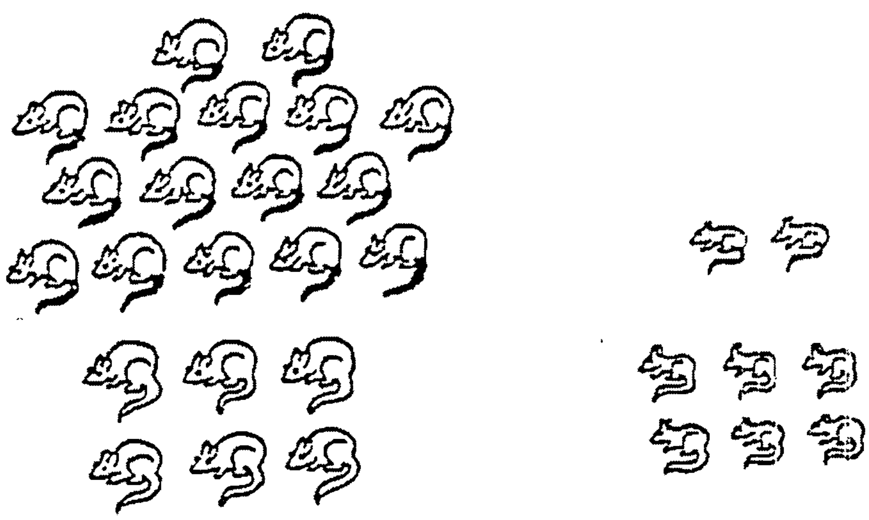
DO YOU THINK THERE IS A RELATION BETWEEN THE SIZE OF THE MICE AND THE COLOR OF THEIR TAILS (THAT IS, IS ONE SIZE OF MOUSE MORE LIKELY TO HAVE A CERTAIN COLOR TAIL AND VICE VERSA)?

a. Yes

b. No

REASON

1. 8/11 of the fat mice have black tails and 3/4 of the thin mice have white tails.
2. Fat and thin mice can have either a black or a white tail.
3. Not all fat mice have black tails. Not all thin mice have white tails.
4. 18 mice have black tails and 12 have white tails.
5. 22 mice are fat and 8 mice are thin.



Item 10

The Fish

Some of the fish below are big and some are small. Also some of the fish have wide stripes on their sides. Others have narrow stripes.

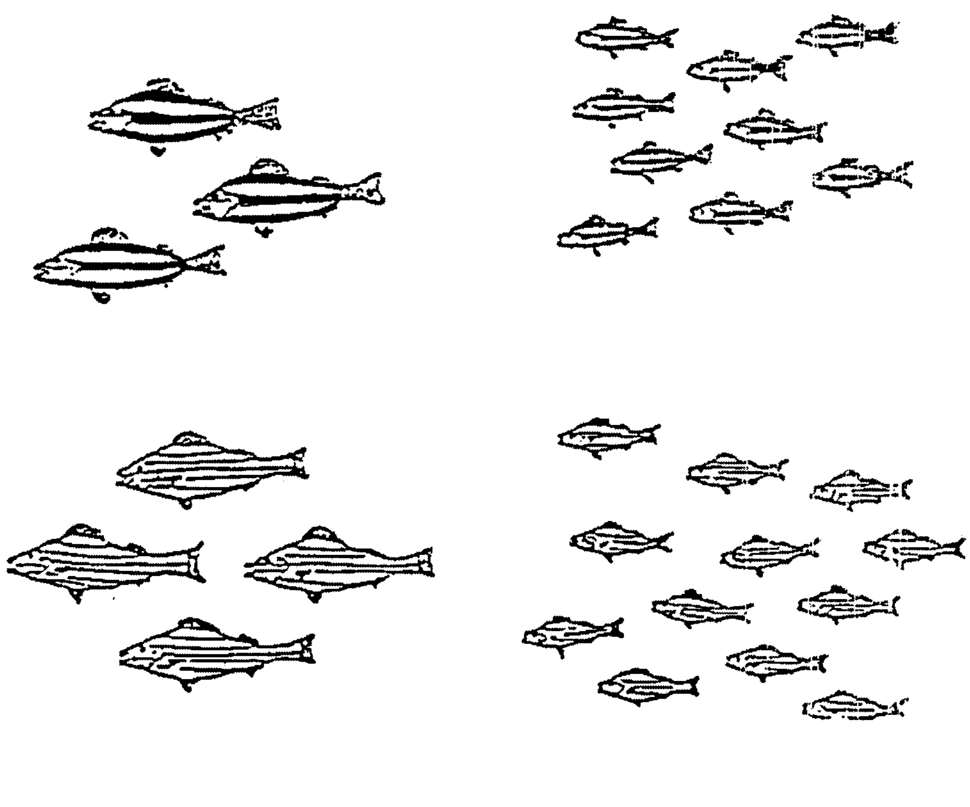
IS THERE A RELATIONSHIP BETWEEN THE SIZE OF THE FISH AND THE KIND OF STRIPES IT HAS (THAT IS, IS ONE SIZE OF FISH MORE LIKELY TO HAVE A CERTAIN TYPE OF STRIPES AND VICE VERSA)?

a. Yes

b. No

REASON

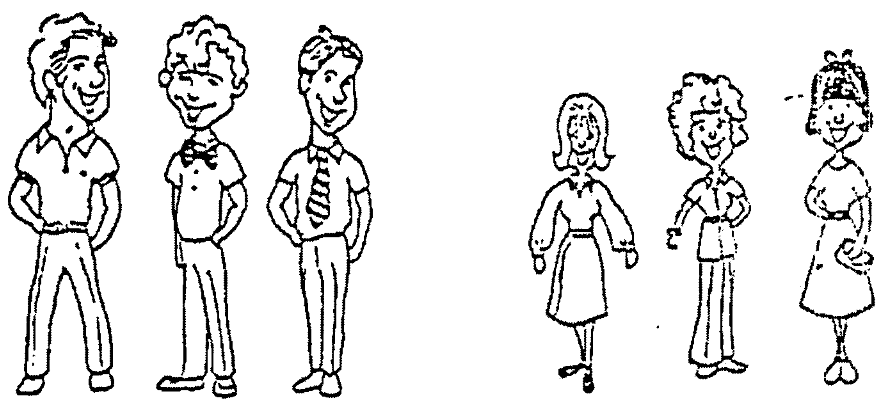
1. Big and small fish can have either wide or narrow stripes.
2. 3/7 of the big fish and 9/21 of the small fish have wide stripes.
3. 7 fish are big and 21 are small.
4. Not all big fish have wide stripes and not all small fish have narrow stripes.
5. 12/28 of fish have wide stripes and 16/28 of fish have narrow stripes.



Item 11

The Dance

After supper, some students decide to go dancing. There are three boys: ALBERT (A), BOB (B), and CHARLES (C), and three girls: LOUISE (L), MARY (M), and NANCY (N).



ALBERT (A) BOB (B) CHARLES (C) LOUISE (L) MARY (M) NANCY (N)

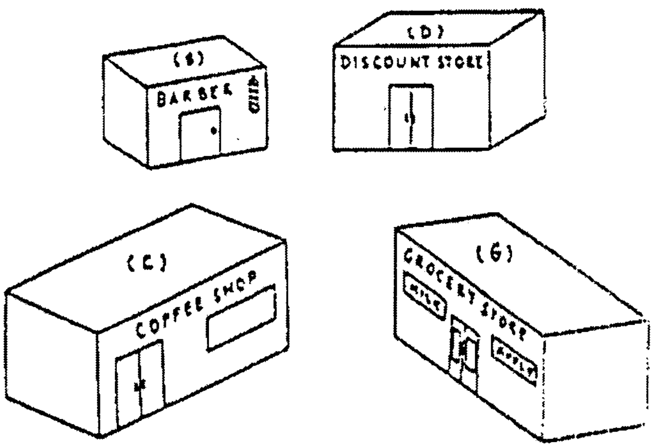
One possible pair of dance partners is A-L, which means ALBERT and LOUISE.

LIST ALL OTHER POSSIBLE COUPLES OF DANCERS. BOYS DO NOT DANCE WITH BOYS, AND GIRLS DO NOT DANCE WITH GIRLS.

Item 12

The Shopping Center

In a new shopping center, 4 stores are going to be placed on the ground floor. A BARBER SHOP (B), a DISCOUNT STORE (D), a GROCERY STORE (G), and a COFFEE SHOP (C) want to locate there.



One possible way that the stores could be arranged in the 4 locations is BDGC. Which means the BARBER SHOP first, the DISCOUNT STORE next, then the GROCERY STORE and the COFFEE SHOP last.

LIST ALL THE OTHER POSSIBLE WAYS THAT THE STORES CAN BE LINED UP IN THE FOUR LOCATIONS.

GROUP ASSESSMENT OF LOGICAL THINKING
ANSWER SHEET

Instructions: For items 1-10 you are to choose the best answer and reason for selecting that answer. Indicate your answer by darkening the letter and number corresponding to the test item.

ITEM	BEST ANSWER	REASON
1. Piece of Clay	(A) (B) (C)	(1) (2) (3) (4)
2. Metal Weights	(A) (B) (C)	(1) (2) (3) (4)
3. Glass Size #2	(A) (B) (C) (D)	(1) (2) (3) (4)
4. Scale #1	(A) (B) (C) (D) (E)	(1) (2) (3) (4) (5)
5. Pendulum Length	(A) (B) (C) (D) (E)	(1) (2) (3) (4) (5)
6. Ball #1	(A) (B)	(1) (2) (3) (4) (5)
7. Squares and Diamonds #1	(A) (B) (C) (D) (E)	(1) (2) (3) (4) (5)
8. Squares and Diamonds #2	(A) (B) (C) (D) (E)	(1) (2) (3) (4) (5)
9. The Mice	(A) (B)	(1) (2) (3) (4) (5)
10. The Fish	(A) (B)	(1) (2) (3) (4) (5)

11. The Dance

Place your answers below:

__A-L__ ______ ______ ______ ______ ______

______ ______ ______ ______ ______ ______

______ ______ ______ ______ ______ ______

______ ______ ______ ______ ______ ______

______ ______ ______ ______ ______ ______

12. The Shopping Center

Place your answers below:

__BDGC__ ________ ________ ________ ________

________ ________ ________ ________ ________

________ ________ ________ ________ ________

________ ________ ________ ________ ________

________ ________ ________ ________ ________

________ ________ ________ ________ ________

GROUP ASSESSMENT OF LOGICAL THINKING
ANSWER KEY

Below are the correct responses for the best answer and reason. For items 1-10, the item is considered correct only if the best answer and reason are both correct.

ITEM	BEST ANSWER	REASON
1. Piece of Clay	(B)	(1)
2. Metal Weights	(C)	(1)
3. Glass Size #2	(C)	(2)
4. Scale #1	(E)	(1)
5. Pendulum Length	(C)	(5)
6. Ball #1	(B)	(4)
7. Squares and Diamonds #1	(A)	(3)
8. Squares and Diamonds #2	(A)	(1)
9. The Mice	(A)	(1)
10. The Fish	(B)	(2)

For "The Dance" and "The Shopping Center" students must (1) show a pattern and (2) have no more than one error or omission for "The Dance" and no more than two errors or omissions for "The Shopping Center." Below are samples of possible patterns students may exhibit.

11. The Dance

Place your answers below:

A-L	B-L	C-L	____	____	____
A-M	B-M	C-M	____	____	____
A-N	B-N	C-N	____	____	____

12. The Shopping Center

Place your answers below:

BDGC	DBGC	GDCB	CGBD	____
BDCG	DBCG	GDBC	CGDB	____
BGDC	DCBG	GCBD	CDBG	____
BGCD	DCGB	GCDB	CDGB	____
BCGD	DGBC	GBDC	CBDG	____
BCDG	DGCB	GBCD	CBGD	____

APPENDIX E

TEST OF ALGEBRAIC REASONING

Name_______________

Algebra I - Basic Skills Test

*****Be sure to show all work. Remember, no work, no credit!!!*****
Please write your answer on the answer blank.

Evaluate the expression.

1. $[22 \div (9 + 2)]$ ___________
2. $3 \bullet 2^2 - 4$ ___________
3. $6 \bullet 4 \div 3 - 8 \div 2$ ___________

4. Evaluate : $4x (y - 7)$; when $x = 5$ & $y = 9$. ___________
5. Evaluate : $15 - 2x^2 \div 8$; when $x = 6$. ___________

Write an algebraic expression or equation for the given verbal statements.

6. "the product of 7 and x, subtracted from twelve" ___________________
7. "the sum of two times a number and 8 is 14" ___________________
8. "four less than eight times a number" ___________________

Decide whether the given number is a solution to the equation or inequality. (Be sure to show your work and write yes or no!!!)

9. $3x + 2 = 10 + x$; 4

____

10. $x^2 - 2x + 8 > 45$; 7

____

11. You just turned 16 and want to purchase a car. You have $375 for a down payment and decide to buy a $1300 car. You have 20 months to pay it off. How much are each of your monthly payments?

____

12. Write the following numbers in increasing order. (Smallest to largest!)

$$\frac{1}{2}, \frac{-2}{3}, 0, -2, \frac{5}{3}$$

____

State the opposite of each number.

13. -5

_____

14. 3.5

_____

Find the absolute value of each.

15. $|3.2|$ ______ 16. $\left|-\frac{3}{5}\right|$ ______

Evaluate each of the following.

17. 8 + (-4) ______
18. -5 + (-3) + 4 ______
19. -[8 + 9-7] ______

20. -13 - (-12) - 11 ______
21. 3 - 7 - 9 ______
22. (-6)(8) ______

23. (-2)(-1)(10) ______
24. 18 ÷ - 2 ______
25. $\frac{-72}{9}$ ______

26. -36 ÷ -6 ______
27. $\frac{-6}{5} + \frac{2}{5} + \frac{-3}{5}$ ______
28. $\frac{-2}{3} \div \frac{-1}{2}$ ______

Use the distributive property to simplify.

29. 4(10 - 3x) ______
30. -13(x - 1) ______
31. x (x + 2) ______

Simplify the quotients.

32. $\dfrac{24x+18}{-2}$ ____________

33. $\dfrac{9x-27}{3}$ ____________

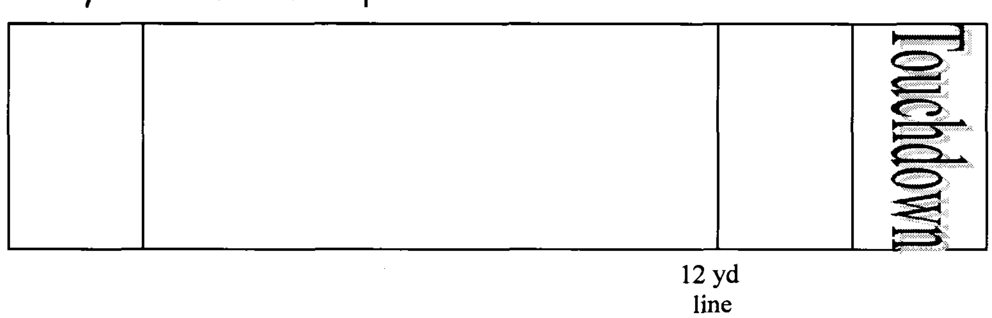
34. A first grade class is going on a trip to an amusement park. The park requires groups to have one adult for every eight children. There are 3 teachers, 2 parents and 36 children planning to go on the field trip. Are there enough adults to meet the park's requirements? Justify your answer.

____

____

____

35. Suppose it is the game that decides the high school football championship. Our team is behind by five points and needs a touchdown to win. Starting on the opponents 12 yard line, Our teams final four plays result in a gain of 8 yards from a completed pass, a loss of 4 yards on a quarterback sack, 2 yards gained by the fullback and a 7 yard gain on a quarterback sneak as time runs out. Does your team win? Explain!!!



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____

____

APPENDIX F

INTERVENTION PARENT AND STUDENT CONSENT FORM

October 26, 2006

Dear Parent or Guardian:

I am conducting a study for my Doctoral dissertation on cognitive development and algebraic reasoning in high school students. The purpose of this study is to determine whether there is a relationship between cognitive development and algebraic reasoning, and if such a relationship is found to exist then attempt to accelerate cognitive development to improve algebraic reasoning. It is hoped that this study will help educators to improve algebraic understanding in the classroom.

With your permission and your child's permission, your child or ward will be asked to participate in twelve one-on-one or small-group meetings. These will take place before or after school typically for 20-30 minute sessions each time.

Your child may choose not to participate. There are no penalties for not participating or withdrawing early from the study.

Your child's responses will remain confidential. Elaborations and comments resulting from the meetings will be reported using pseudonyms.

If you have any questions about this study, please contact me at ( ) . If you have any questions regarding your son's/daughter's rights as research participants, please call the Northern Illinois University Office of Research Compliance at (815) 753-8588. Please indicate on the form attached whether or not your child/ward may participate in this part of the study and have him/her return it to me in class.

Thank you,
Michelle R. Wesolowski

Parent Consent Form: An Intervention to Advance Piagetian Levels of Cognitive Development and Algebraic Reasoning in High-School Students (Part II)

Responsible Faculty Member: Dr. Janet Holt
Department of Educational Technology, Research and Assessment
Northern Illinois University
(815)753-8523

I have read and understand the letter attached and agree for my child to participate in this study on cognitive development and algebraic reasoning. I can request a copy of this form.

Name of child (please print) ____________________________

Parent or Guardian signature ____________________________ Date __________

I have read and understand the letter attached and agree to participate in this study on cognitive development and algebraic reasoning.

Student signature ____________________________ Date __________

APPENDIX G

INTERVENTION STUDENT ASSENT FORM

November 1, 2006

Dear Student:

I am conducting a study for my Doctoral dissertation on cognitive development and algebraic reasoning in high school students. The purpose of this study is to determine whether there is a relationship between cognitive development and algebraic reasoning, and if such a relationship is found to exist then attempt to accelerate cognitive development to improve algebraic reasoning. It is hoped that this study will help educators to improve algebraic understanding in the classroom.

With your permission, you will be asked to participate in twelve one-on-one or small-group meetings. These will take place before or after school typically for 20-30 minute sessions each time.

You may choose not to participate. There are no penalties for not participating or withdrawing early from the study.

Your responses will remain confidential. Elaborations and comments resulting from the meetings will be reported using pseudonyms.

If you have any questions about this study, please contact me at ( ) . If you have any questions regarding your rights as research participants, please call the Northern Illinois University Office of Research Compliance at (815) 753-8588. Please indicate on the form attached by signing the first line whether or not you will participate in this part of the study and return it to me in class.

Thank you,
Michelle R. Wesolowski

Student Assent Form: An Intervention to Advance Piagetian Levels of Cognitive Development and Algebraic Reasoning in High-School Students (Part II)

Responsible Faculty Member: Dr. Janet Holt
Department of Educational Technology, Research and Assessment
Northern Illinois University
(815)753-8523

I have read and understand the letter attached and agree to participate in this intervention on cognitive development and algebraic reasoning.

Name of student (please print)____________________________

Student signature ______________________________ Date __________

Student signature ______________________________ Date __________

Student signature ______________________________ Date __________

Student signature ______________________________ Date __________

Student signature ______________________________ Date __________

Student signature ______________________________ Date __________

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Student signature ______________________________ Date __________

Student signature ______________________________ Date __________

Student signature ______________________________ Date __________

Student signature ______________________________ Date __________

Student signature ______________________________ Date __________

APPENDIX H

INTERVENTION

Intervention #1
Proportional Reasoning

Materials: Worksheet

Activity: Students will complete a worksheet on proportions.

Intervention #1

ID: ____________________
Date: ________________

1. How many hearts are needed in place of the question mark?

$$\frac{♥♥♥♥♥}{☺☺} = \frac{?}{☺☺☺☺}$$

Why?

2. What number should replace the question mark?

$$\frac{?}{5} = \frac{15}{35}$$

Why?

3. How many diamonds are needed in place of the question mark?

$$\frac{☺☺☺}{?} = \frac{☺☺☺☺☺☺☺☺}{♦♦♦♦}$$

Why?

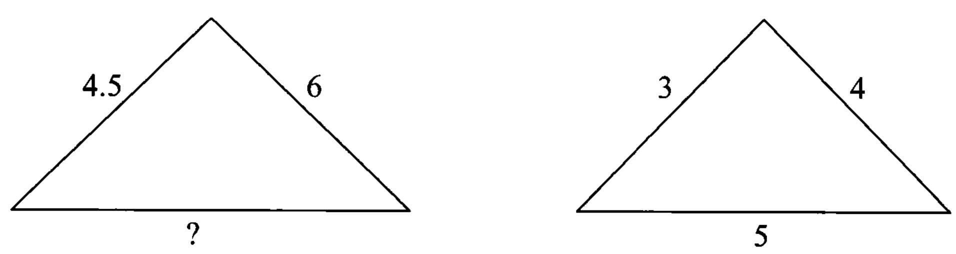
4. What number should replace the question mark?

$$\frac{15}{8} = \frac{6}{?}$$

Why?

5. You are assigned to read a 220 page book. It takes you 15 minutes to read 10 pages. How long will it take for you to read the book? Why?

6. Find the missing side length of the similar triangles and explain why.



Intervention #2
Controlling Variables

Materials: Balancing clown glued to cardboard
Clothespins
Cotton balls
Pipe cleaners
Pencil/pen
Paper clips
Coins
Worksheet

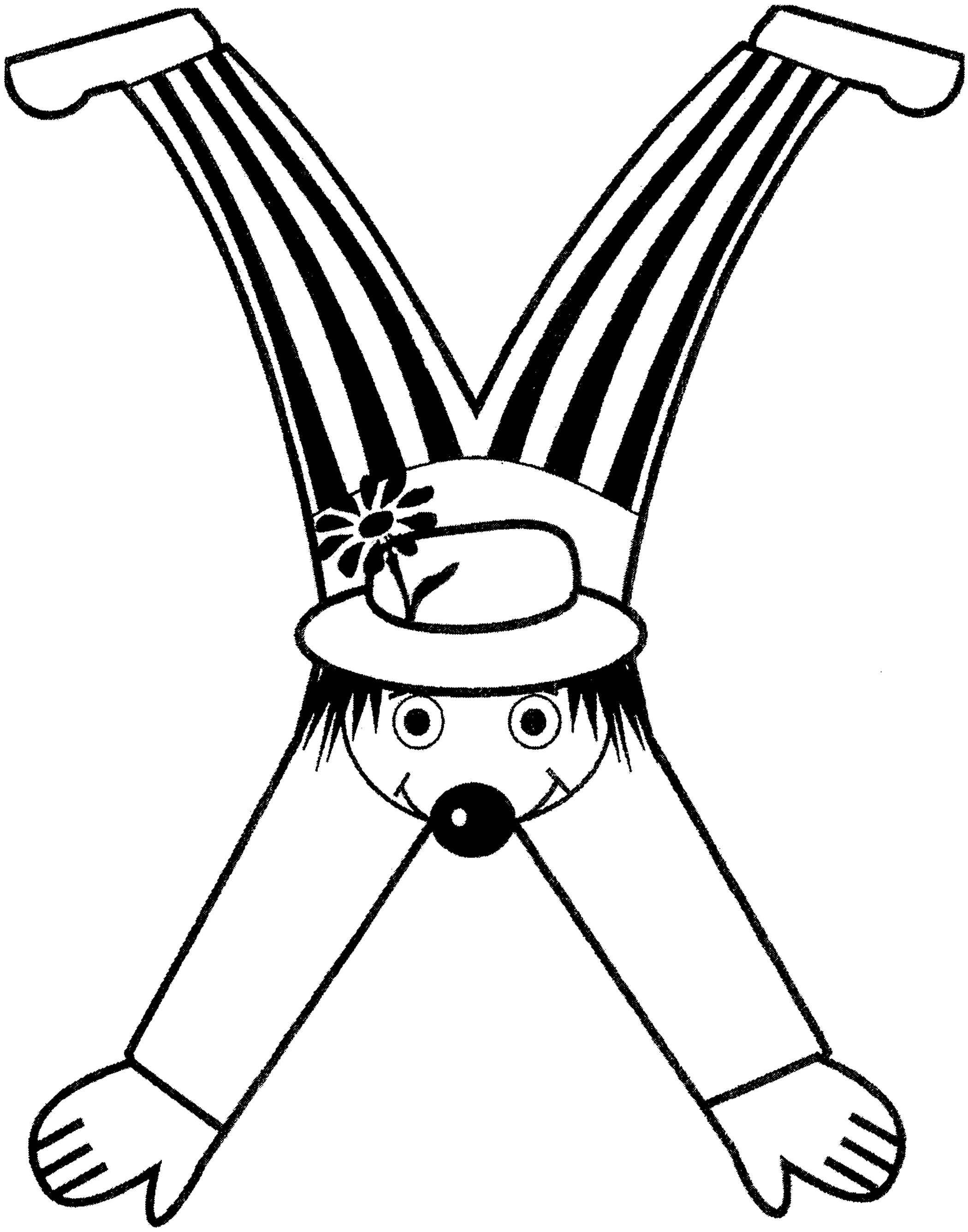
Activity: The researcher will give each student a balancing clown, glue and a bag containing 4 clothespins, 4 cotton balls, 4 pipe cleaners, 8 paper clips, and 6 pennies. The students will be asked to attempt to balance the clown on their pencil or pen. After finding that they cannot get the clown to balance, they will be told to use the materials in the bag to help balance the clown. They will also be asked to write down and explain everything they do in their experiment to get the clown to balance. If the students are not changing just one variable at a time, the researcher will intervene and ask why and guide them towards changing one variable at a time.

Intervention #2

ID: ____________________
Date: ______________

Activity:

What are you doing to get the clown to balance on your pencil?



Intervention #3
Probabilistic Reasoning

Materials: 1 coin
1 bag of letters spelling "Mississippi"
2 6-sided dice
Worksheet

Activity: Students will be given a worksheet with directions to follow to find theoretical and experimental probability in three activities: flipping a coin, drawing letters out of a bag, and rolling dice.

Intervention #3

ID: ________________
Date: ____________

FLIPPING A COIN

You have a coin with 2 sides (heads and tails).

1. What is the probability of getting tails when flipping a coin? Write the probability as a fraction.

   PROBABILITY (flipping tails) = ________________

2. Flip a coin 50 times and record the outcomes in the frequency table below.

Side of Coin	Tally	Frequency
Heads		
Tails		

3. Find the experimental probability of getting heads or tails.

   EXPERIMEMTAL PROBABILITY (flipping heads) = ______________

   EXPERIMEMTAL PROBABILITY (flipping tails) = ______________

4. How does the experimental probability compare to the probability that you found in step 1?

CHOOSING A LETTER

Each letter in MISSISSIPPI is written on a separate piece of paper and put into a bag. You randomly choose a piece of paper from the bag.

M I S S I S S I P P I

1. Find the probability of each event. Write the probability as a fraction.

Probability (choosing an M) = ______ Probability (choosing an I) = ______

Probability (choosing an S) = ______ Probability (choosing a P) = ______

2. Pick a letter out of the bag 50 times. Record a tally for each letter picked and the corresponding frequencies in the frequency table below.

Letter Picked	Tally	Frequency
Letter M		
Letter I		
Letter S		
Letter P		

3. Find the experiment probability of choosing each letter.

EXPERIMENTAL PROBABILITY (choosing an M) =________________

EXPERIMENTAL PROBABILITY (choosing an I) =________________

EXPERIMENTAL PROBABILITY (choosing an S) =________________

EXPERIMENTAL PROBABILITY (choosing an P) =________________

4. How does the experimental probability compare to the probability that you found in step 1?

ROLLING DICE

Two dice are rolled. Find the probability that the sum of the resulting numbers is 7.

1. To find the sums of the numbers that can result when two dice are rolled, complete the table below.

	1	2	3	4	5	6
1	**2**	**3**				
2	**3**	**4**				
3						
4						
5						
6						

2. Find the probability of a sum of 7:

   PROBABILITY(of a sum of 7) = $\frac{\text{Number of rolls that have a sum of 7}}{\text{Total number of rolls}}$ =

3. Roll two dice 50 times. Record the sums of the resulting numbers and the corresponding frequencies in the frequency table below.

Sum	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Tally											
Frequency											

4. Find the experimental probability of the sum of 7.

   EXPERIMENTAL PROBABILITY=_______________

5. How does the experimental probability compare to the probability that you found in step 2?

Intervention #4
Correlational Reasoning

Materials: Worksheet

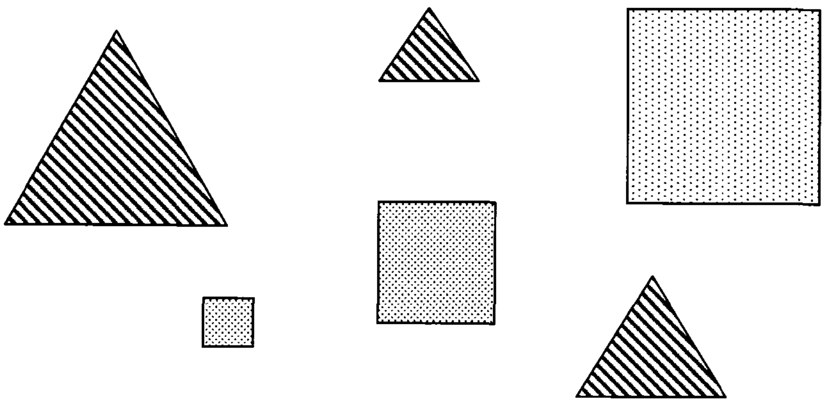
Activity: The students will be given a worksheet with a collection of shapes and will be asked what the variables are for the shapes. The students will be led to the definition of relationship and will be asked to show the relationship between the variables color and shape (Adey & Shayer, 1993).

Intervention #4

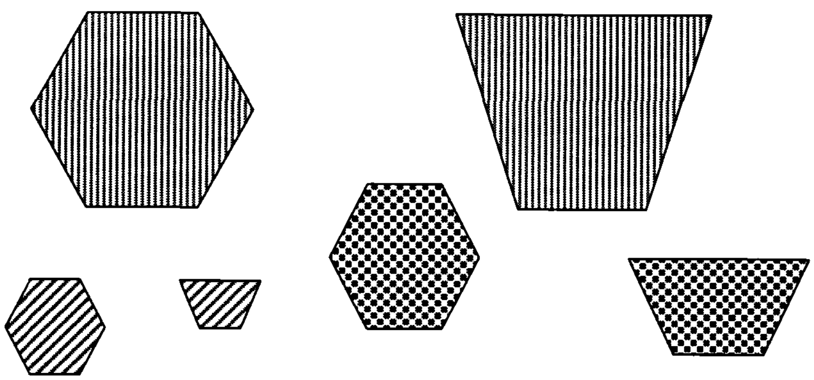
ID: ______________

Date: __________

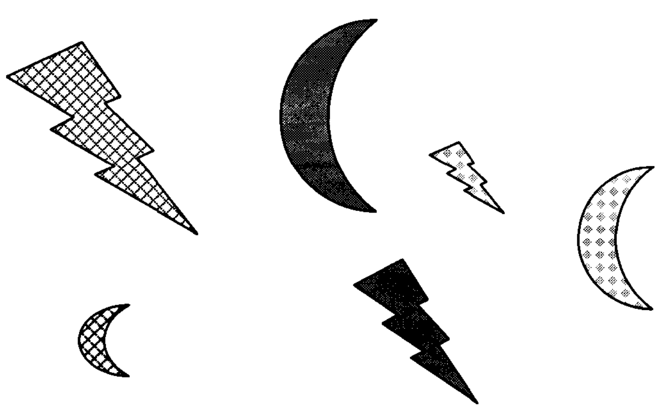
1. What are the variables for these shapes? If you drew another triangle, how would it look? Why?



2. What are the variables for these shapes? If you were to add a medium octagon, how would it look? Why?



3. What are the variables for these shapes? If you were to add another moon, how would it look? Why?



Intervention #5
Combinatorial Logic

Materials: Worksheet

Activity 1: Students will be asked to complete a worksheet on permutations.

Activity 2: Students will be asked to complete a worksheet on combinations.

Intervention #5

ID: ____________________
Date: ____________

Activity 1:

1. List the ways that you can order the letters in CAT.

2. List the ways that you can order the following objects: ■ ● ♦

3. List the possible orders that 4 runners, Allie, Barbara, Caroline, and Deliah, can run on a relay.

4. List the possible orders that five textbooks can be sitting on the floor of your locker (Algebra, English, Spanish, Biology, and History).

Activity 2:

1. List the combinations if you choose two symbols from the set {♥, ●, ♦}.

2. List the combinations if you choose three colors from the set {red, blue, white, green}.

3. List the combinations if you choose two numbers from the set {2, 4, 6, 8}.

4. List the combinations if you have 5 people going rafting (Alex, Bob, Christine, Doug, and Ellie), but only 3 can fit in a raft at one time.

Intervention #6
Controlling Variables

Materials: Helicopter pattern
Different weights of paper (construction, copy, and loose leaf)
Different sizes of paperclips

Activity: The students will be given two patterns to create a helicopter and will be asked to find a way to make it spin as fast as possible and as slow as possible. They will be asked to write down and explain what they did to make the helicopter spin faster and slower. They will also be asked to keep track of what materials they used to create the helicopter and what size weights they used if they used any. If the students are not changing just one variable at a time, the researcher will intervene and ask why and guide them towards changing one variable at a time.

Intervention #6

ID: ________________
Date: __________

Activity:

Part 1 – make the helicopter spin as fast as you can.

What type of paper did you use for the helicopter?

What size helicopter did you make?

Did you use any weights on the helicopter? If so, what size and where did you place them?

What changes did you make to the helicopter to make it go faster?

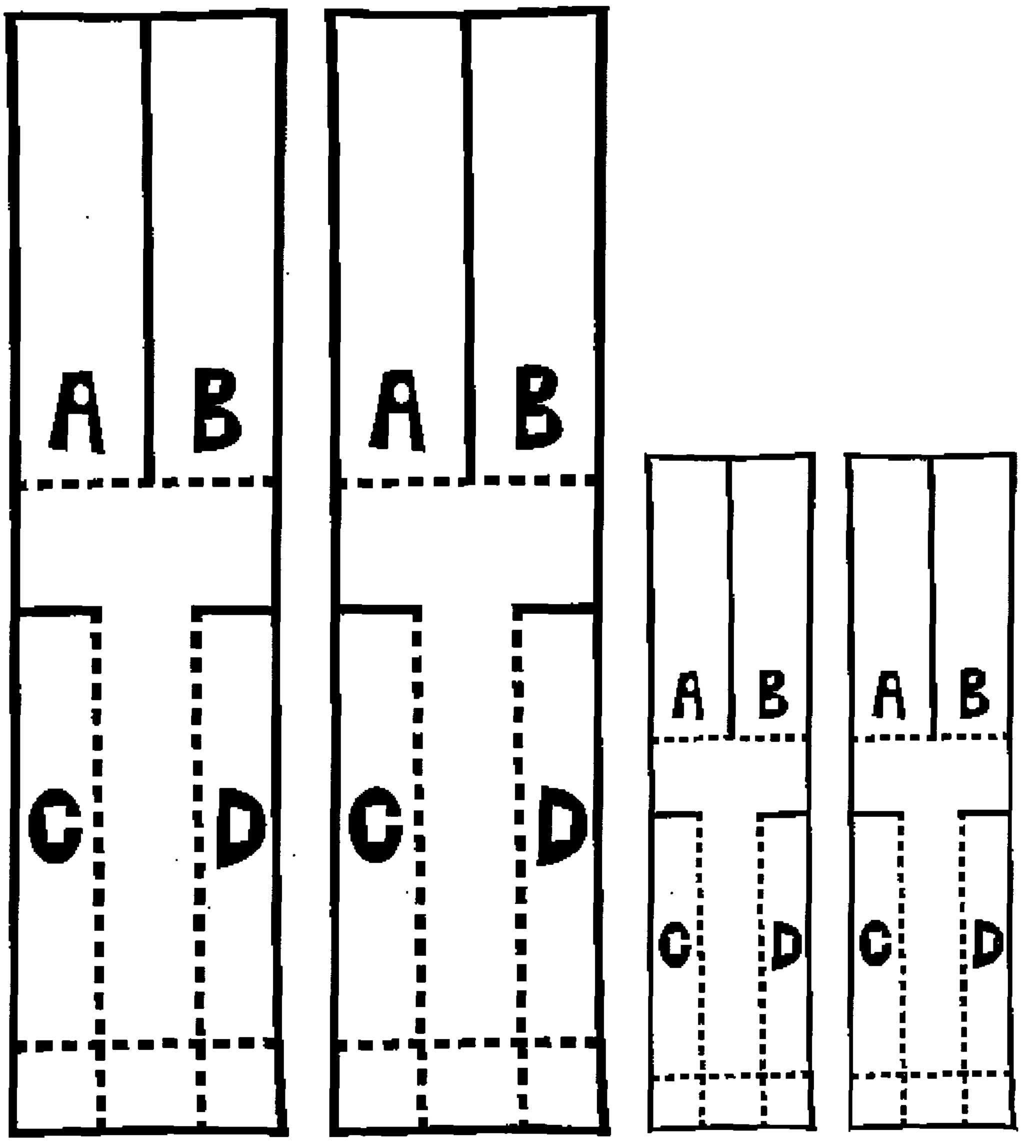
Part 2 – make the helicopter spin as slow as you can.

What type of paper did you use for the helicopter?

What size helicopter did you make?

Did you use any weights on the helicopter? If so, what size and where did you place them?

What changes did you make to the helicopter to make it go slower?


A
B
C
D
A
B
C
D
A
B
C
D
A
B
C
D

APPENDIX I

GALT RETAKE PARENT AND STUDENT CONSENT FORM

November 27, 2006

Dear Parent or Guardian:

I am conducting a study for my Doctoral dissertation on cognitive development and algebraic reasoning in high school students. The purpose of this study is to determine whether there is a relationship between cognitive development and algebraic reasoning, and if such a relationship is found to exist then attempt to accelerate cognitive development to improve algebraic reasoning. It is hoped that this study will help educators to improve algebraic understanding in the classroom.

Earlier in the school year your child or ward responded to a questionnaire for this study. With your permission and your child's permission, your child or ward will be asked to retake the questionnaire about logical thinking two additional times during the course of the school year. This will take place before or after school. It should take about 30-35 minutes each time to complete the questionnaire.

Your child may choose not to participate or complete the questionnaire. There are no penalties for not participating or withdrawing early from the study.

Your child's responses will remain confidential. Results of the questionnaire will be reported in group form only, and no names will be stored with the responses given (their names will be deleted from the questionnaire).

If you have any questions about this study, please contact me at ( ) . If you have any questions regarding your son's/daughter's rights as research participants, please call the Northern Illinois University Office of Research Compliance at (815) 753-8588. Please indicate on the form attached whether or not your child/ward may participate in this study and have him/her return it to me in class.

Thank you,
Michelle R. Wesolowski

Parent Consent Form: An Intervention to Advance Piagetian Levels of Cognitive Development and Algebraic Reasoning in High-School Students (Retake)

Responsible Faculty Member: Dr. Janet Holt
Department of Educational Technology, Research and Assessment
Northern Illinois University
(815)753-8523

I have read and understand the letter attached and agree for my child to participate in this study on cognitive development and algebraic reasoning. I can request a copy of this form.

Name of child (please print) ______________________________

Parent or Guardian signature ______________________________ Date __________

I have read and understand the letter attached and agree to participate in this study on cognitive development and algebraic reasoning.

Student signature ____________________________________ Date __________

APPENDIX J

GALT RETAKE STUDENT ASSENT FORM

December 5, 2006

Dear Student:

I am conducting a study for my Doctoral dissertation on cognitive development and algebraic reasoning in high school students. The purpose of this study is to determine whether there is a relationship between cognitive development and algebraic reasoning, and if such a relationship is found to exist then attempt to accelerate cognitive development to improve algebraic reasoning. It is hoped that this study will help educators to improve algebraic understanding in the classroom.

Earlier in the school year you responded to a questionnaire for this study. With your permission, you will be asked to retake the questionnaire about logical thinking two additional times during the course of the school year. This will take place before or after school. It should take about 30-35 minutes each time to complete the questionnaire.

You may choose not to participate or complete the questionnaire. There are no penalties for not participating or withdrawing early from the study.

Your responses will remain confidential. Results of the questionnaire will be reported in group form only, and no names will be stored with the responses given (their names will be deleted from the questionnaire).

If you have any questions about this study, please contact me at ( ) . If you have any questions regarding your son's/daughter's rights as research participants, please call the Northern Illinois University Office of Research Compliance at (815) 753-8588. Please indicate on the form attached whether or not you will participate in this study.

Thank you,
Michelle R. Wesolowski

Student Assent Form: An Intervention to Advance Piagetian Levels of Cognitive Development and Algebraic Reasoning in High-School Students

Responsible Faculty Member: Dr. Janet Holt
Department of Educational Technology, Research and Assessment
Northern Illinois University
(815)753-8523

I have read and understand the letter attached and agree to participate in this study on cognitive development and algebraic reasoning.

Name of student (please print)____________________________

Student signature _____________________________ Date __________

Student signature _____________________________ Date __________