NORTHERN ILLINOIS UNIVERSITY

Design and Material Selection of a CO_2-Powered Beverage Cooler

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By

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ABSTRACT:

While training for a triathlon, 6-hour bike rides come with the territory. As a result, protein-based drinks are a must to keep nourished. Currently, the only way to keep a drink from spoiling is to have an insulated bottle. However, this does not actively keep the drink cool; it simply slows the rate of warming. This experiment was designed to create a drink holder that would use CO_2 to actively cool a drink to maintain a desired temperature. To do this, coiled copper tubing was inserted into an insulated housing with a CO_2 source attached to the other end. Between the combination of the insulated housing and CO_2 bursts, the rate at which the drink warms is slowed and even reversed when CO_2 is used. We hypothesized the concept that CO_2 would actively cool the drink inside the bottle, and experimentation proved this to be accurate. Once proof of concept was established, the next step was to optimize when and how much CO_2 was needed to maintain the desired temperature inside the bottle. Ultimately, smaller double bursts were found to be the most effective way of maintaining the temperature of the drink. Depending on the size of the CO_2 bottle used, drinks are able to maintain proper temperatures up to 3-4 times longer than by insulation alone. This is significant because it shows the concept doesn't just work, but it makes a considerable difference from merely insulating the bottle. This experiment provided positive results, allowing future work to be focused on the ergonomics and practicality of the final design.

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Acknowledgements

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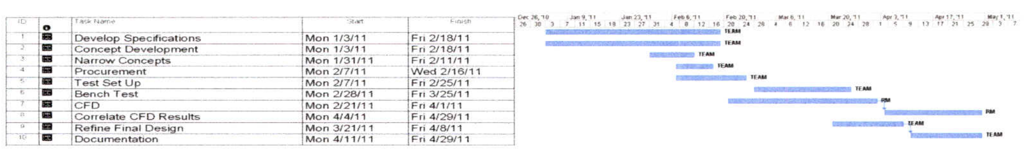
Abstract

While training for a triathlon, 6-hour bike rides come with the territory. As a result, protein-based drinks are a must to keep nourished. Currently, the only way to keep a drink from spoiling is to have an insulated bottle. However, this does not actively keep the drink cool; it simply slows the rate of warming. This experiment was designed to create a drink holder that would use CO_2 to actively cool a drink to maintain a desired temperature. To do this, coiled copper tubing was inserted into an insulated housing with a CO_2 source attached to the other end. Between the combination of the insulated housing and CO_2 bursts, the rate at which the drink warms is slowed and even reversed when CO_2 is used. We hypothesized the concept that CO_2 would actively cool the drink inside the bottle, and experimentation proved this to be accurate. Once proof of concept was established, the next step was to optimize when and how much CO_2 was needed to maintain the desired temperature inside the bottle. Ultimately, smaller double bursts were found to be the most effective way of maintaining the temperature of the drink. Depending on the size of the CO_2 bottle used, drinks are able to maintain proper temperatures up to 3-4 times longer than by insulation alone. This is significant because it shows the concept doesn't just work, but it makes a considerable difference from merely insulating the bottle. This experiment provided positive results, allowing future work to be focused on the ergonomics and practicality of the final design.

Chapter 1: Introduction

For any competitive triathlete, training is of the upmost importance. At times, training requires 6+ hour bicycle rides. This is no easy feat, and the body must be replenished during this time to make sure it does not give out. According to endurance sport scientist Dr. Steve Born, any endurance athlete working out for more than two hours at a time must consume protein [1]. This is due to the body starting to burn lean muscle; triathletes prevent this by consuming protein drinks. However, as with any milk-based product, flavor and quality will degrade rapidly when the drink gets too warm. Currently, there is no product on the market that will hold drinks to a bike and keep them cool beyond just insulating them. Our design uses CO_2 liquid as well as conventional insulation to actively cool drinks to keep them below the 7.2°C spoiling point, preventing them from going bad.

In this problem our group saw a need for a product that would actively keep a drink cool while out on long rides. In order to complete this project on time a new Gantt chart needed to be created. The Gantt chart can be seen below in Figure 1.



ID	Task Name	Start	Finish
1	Develop Specifications	Mon 1/3/11	Fri 2/18/11
2	Concept Development	Mon 1/3/11	Fri 2/18/11
3	Narrow Concepts	Mon 1/31/11	Fri 2/11/11
4	Procurement	Mon 2/7/11	Wed 2/16/11
5	Test Set Up	Mon 2/7/11	Fri 2/25/11
6	Bench Test	Mon 2/28/11	Fri 3/25/11
7	CFD	Mon 2/21/11	Fri 4/1/11
8	Correlate CFD Results	Mon 4/4/11	Fri 4/29/11
9	Refine Final Design	Mon 3/21/11	Fri 4/8/11
10	Documentation	Mon 4/11/11	Fri 4/29/11

Figure 1: Gantt chart

This Gantt chart was crucial to getting the project going. Without having a pre-planned timeline, the project would probably not been finished on time.

When first starting this project one of the most important steps was to develop standards and specifications. According to US law, milk must be stored under 7.2°C in order to be considered Grade A; above this temperature the rate of spoilage starts to double for every 10°C rise [2]. This told us that our design must be able to keep the drink below this point for at least four hours. Long life is important, considering the average triathlete will ride their bicycle ten hours per week all year round. This comes out to roughly 520 hours of use per year. In order to achieve long life the group decided that the design must last for 10,000 hours.

Chapter 2: Product Design

As the original concept slowly evolved into the final design things changed quite a bit. However, the final design exceeded all of our original specification requirements, allowing a drink to be kept cool for longer than 6 hours. You can see the final design below in Figure 2.

Figure 2: Final Design

As seen in Figure 2, the setup mounts behind the bike seat with the CO_2 bottle positioned upside down in order to draw liquid and not gas. It is crucial for the design to draw liquid in order to cool more effectively. The preliminary designs used copper coils to run the CO_2 from the tank to the drink holders; by using cast aluminum and an aluminum sleeve we were able to eliminate the need to for tubing. This can be seen below in Figure 3.

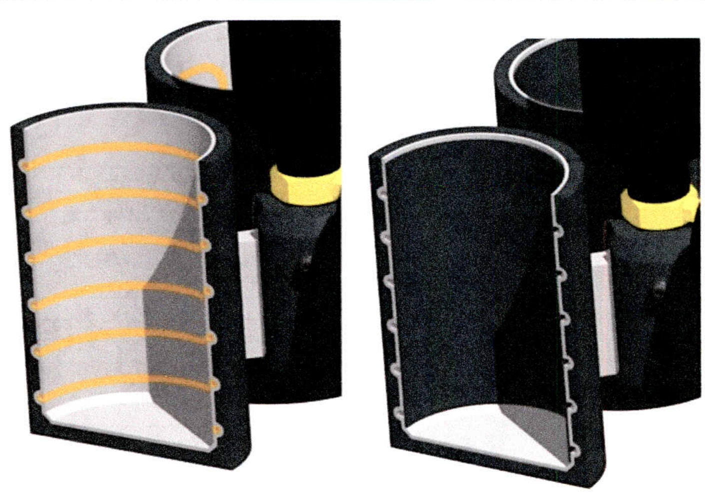


Figure 3: Copper tubes vs. cast aluminum with sleeve

This new design presented us with a few major advantages over using copper tubing to deliver CO_2. First off, it would be very difficult and expensive to bend the copper tubing accurately during manufacturing. By welding an aluminum sleeve over cast aluminum channels, it decreases complexity and is more cost effective.

Keeping the project cost effective was a major concern. An aesthetically pleasing and effective design is great, but exorbitant cost can keep a product from being marketable. After doing some research on other drink holders we decided if we could make our design for under $150, we could be successful. In the end we came in well under our target price. Our bill of materials is shown below in Figure 4.

Parts	Quantity	Cost
Aluminum Sleeve	2	$5.00
Cast Aluminum	0.8432 kg	$14.84
Foam Sleeve	2	$1.98
Solenoid	1	$60.00
Hardware	9	$0.90
Total (No Tank)		**$82.72**

Tanks	Cost
4 oz	Range from $15 to $30
9 oz	
12 oz	
16 oz	
20 oz	
24 oz	

Figure 4: Bill of Materials

As you can see from Figure 4 the price of our product depends on what CO_2 tank you choose. However, even with the biggest tank it still almost $30 less than it needed to be. This is a great success, because it leaves lots of room for profit, which is important for a product that will be purchased by a select market.

Overall the final design exceeded our expectations. We hit all of our target specifications and more. With its minimal weight of 3.8 kg, including a full CO_2 tank, it is not so heavy that it will hinder the rider. While this is heavier than most drink holders, it is not an issue, because during races the CO_2 tank can be removed to cut down the weight. This was very important, because the design needed to add benefit to the rider and not hinder them in anyway.

Chapter 3: Product Generation

The experimental procedure entails the collection of temperature readings every second using a USB data logger (DAQ) and a J-Type thermocouple. A sports bottle is filled with water and the thermocouple is placed half way down in the center of the bottle. This is the area where the temperature is the least affected by the temperature fluctuations. The bottle is placed in a holder made from PVC pipes consisting of a copper coil wrapped around the bottle and insulation. A CO_2 tank from the sport of paintball is attached to a valve on the end of the coil, which is wrapped in more insulation. The experiment is adjusted by turning the knob on the valve by hand. To measure the amount of CO_2 used in the cooling process, the setup is weighed prior as well as post experiment. A picture of the setup can be seen in Figure 5 below.

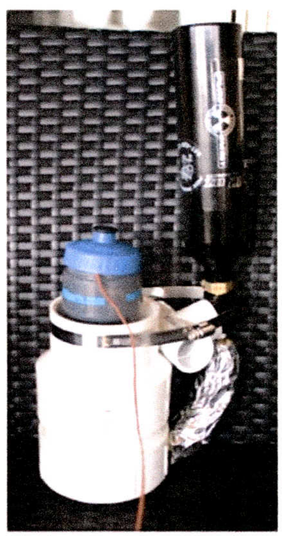


Figure 5: Experimental setup

To establish an estimated steady state temperature within the bottle that is being cooled by 0°C CO_2 half way up the side and the rest at 18°C air, a finite element method calculation was done [3]. The Mathcad programming can be seen in Appendix [1]. Since the bottle is located on the back of a bicycle, it will naturally mix inside, keeping an even temperature throughout. By taking an average of the individual node results from the finite element method, the best prediction was found to be 3.6°C. A similar approach was made using a thermal scan of the bottle with Pro/E. The subsequent picture can be seen in Figure 6.

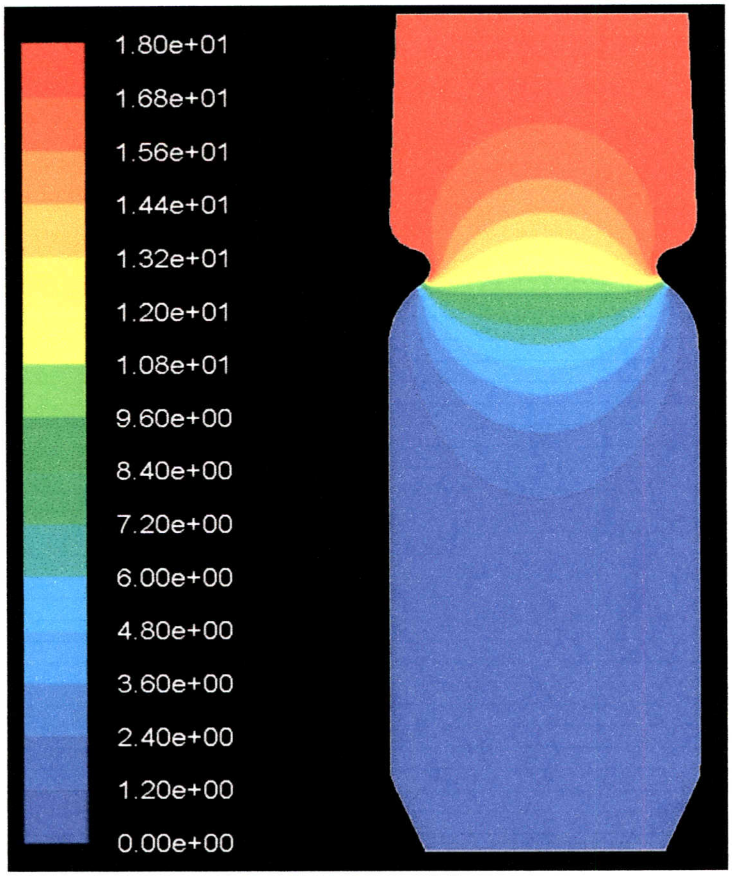
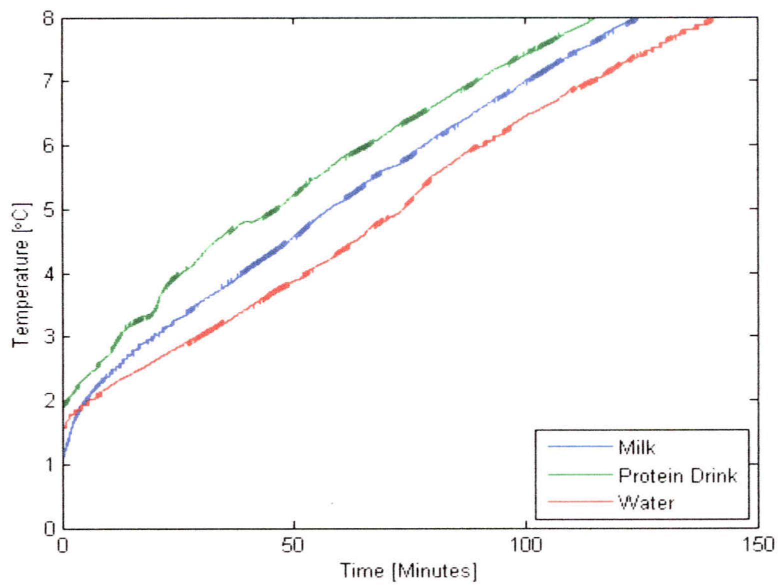


Figure 6: Thermal scan

An average was calculated from this and the value was found to be 6.1°C. These two estimations vary a bit but the concept of being able to cool a bottle with a constant low surrounding temperature seems to be working. In the latter case, convection from the air while riding the bike was omitted, and this accounts for some of the variations. In the subsequent experiments, steady state will never occur, but these results enforce the possibility of the initial guesses of being able to keep a bottle cool.

Since protein drinks are rather costly and messy, water is the preferred medium for experimentation. It is therefore important to make sure that the two liquids behave similarly when cooled. A simple test was carried out where the behavior of these liquids was compared as the temperature inside the bottle changed. As standards for preserving protein drinks are hard to find, milk standards are used since they both contain whey and the correlation between these two liquids was also tested. The behavior of protein drinks, water as well as milk is shown in Graph 1.



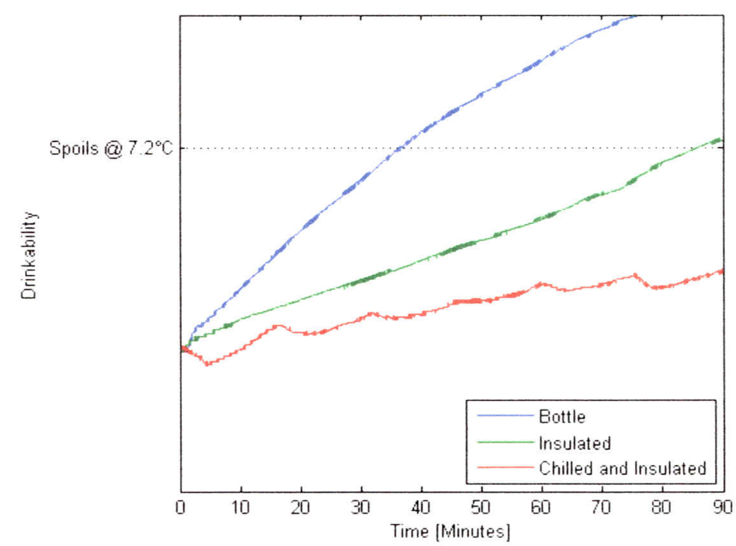
Graph 1: Correlation between Protein Drink, Milk and Water

The slopes seem to behave in the same manner, but to make sure a linear fit was applied through a built in function in MATLAB, which can be seen in Appendix [2]. The values of the slopes all hover around 0.05°C/min, which can be seen in Table 1.

Table 1: Slopes

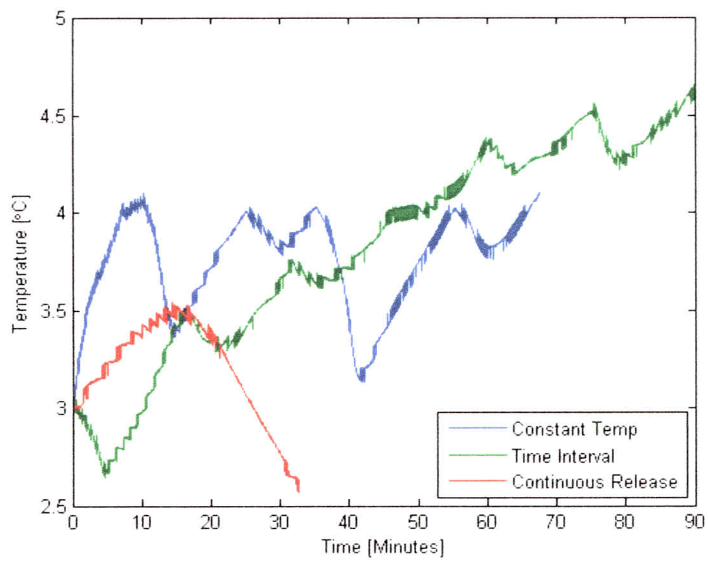
Liquid	°C/Min
Milk	0.050
Protein Drink	0.049
Water	0.046

From these results it is evident that water is a good substitute for protein drinks during experimentation. Also, current health guidelines for milk can be applied to protein drinks to justify temperature ranges for long-term storage. These guidelines state that the temperature needs to stay below 7.2°C for minimal bacteria growth. Currently, there are two options for triathletes to keep their nutrition with them on a long bike ride. The first is just a regular bottle; the second is to buy a coozy and try to insulate the bottle. To determine a timeframe of how long it takes water to reach the spoilage limit, two experiments were conducted. The first allowed a bottle of water starting at 3°C to increase in temperature for about an hour and a half. The second experiment allowed another bottle to do the same, however this time it was placed in an insulated holder. Measurements were taken every second for both experiments and the results were saved. This established a basis for how long a triathlete could be out on a bike and still keep their protein drinks safe. The third experiment and ultimately what needed to be tested was how long a bottle of water could be kept cool while in an insulated holder and chilled with CO_2. The corresponding results can be seen in Graph 2.



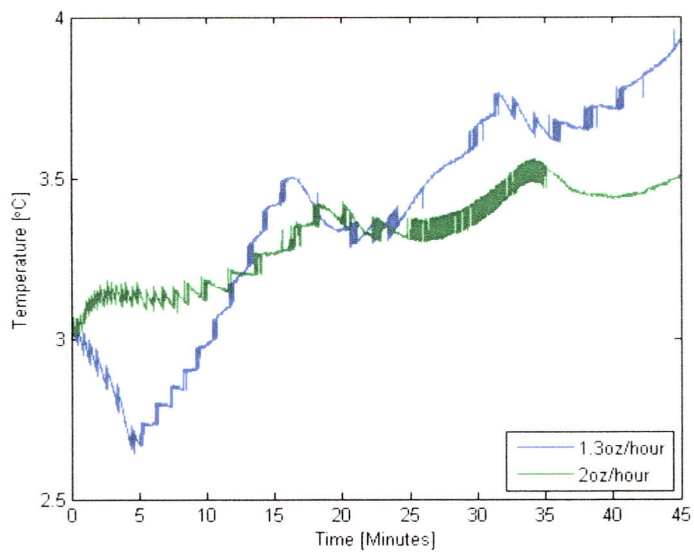
Graph 2: Time before spoilage

Starting with the analysis of just the bottle, it can be seen from the graph that the time it would take the protein drink to reach 7.2°C was approximately 35 min. Insulating the bottle increased this time to about 85 min, while after 90 min the cooled and insulated bottle was not anywhere near the limit. The slope was calculated in MATLAB and had the value of 0.02°C/min. This value is only 40% of the values found in Table 1. After extrapolation beyond the 90 minutes of data the time it would take the protein drink to reach 7.2°C was found to be 210 min or 3.5 hours. This is an increase of two hours over the insulated bottle and 3 hours over the bottle itself. The next step was to try and figure out how much CO_2 and how often it would be released. Several trials were run, but there were three main ideas. The first one was to burst a certain amount of CO_2 whenever the temperature reached a certain value. The second was to burst a set amount of CO_2 in regular time intervals, while the last idea was to release CO_2 continuously. A sample of these trials can be seen in Graph 3.



Graph 3: Samples of release ideas

At first glance, the constant release graph seems to do the best job of actually chilling the protein drink. However, in just 30 minutes, 93g of CO_2 were used up. This corresponds to a little over 3oz, or 75% of our small tank. At this rate, the largest tank would run dry in just over 100 minutes. Though it presents the most desirable of attributes, the valve used in the experiments was not sensitive enough to hone in on the perfect release amount and further experimentation would be needed with different valves to support this alternative. Trying to keep the temperature below a certain value seems like the second best alternative. This way the protein drink would never spoil and the triathletes could fuel when needed. Unfortunately, this alternative used 92g of CO_2 during the experiment, which equates to almost 3oz an hour. With the large tank, this brings the time out on the bike to just about 4 hours. This meets our goal, but a longer time span could mean higher marketability. By releasing the CO_2 at certain time intervals, in this case every 15 minutes, 54g were used. This turns out to be 1.3oz/hour and the large tank would last just over 9 hours. However, this method has a slow steady increase of the temperature of the protein drink which unfortunately crosses the magic 7.2°C in just 3.5 hours as stated earlier. Increasing the release to around 2oz/hour would possibly decrease the slope of the line and subsequently increase the time before spoilage. A comparison with the 1.3oz/hour can be seen in Graph 4.



Graph 4: Different amounts of CO_2 used

A linear fit was applied to this new graph and was found to be 0.01°C/min. That is half of what the 1.3oz/hour had. Extrapolating this, it would now take 420 minutes for the protein drink to reach 7.2°C, or 7 hours. The CO_2 however, would run out in 6 hours. By releasing 2oz of CO_2 per hour, the total time on the bike was increased to just above 6 hours.

After proving that the concept worked, it was time to focus on the structural analysis of the mount to the bike. The Finite Element Analysis (FEA) was conducted on the design of the bike mounted beverage chiller. The analysis was run in Pro/Mechanica running in Wildfire 4. The objective of the analysis was to provide a robust enough structure to the system to allow for 10,000-hour life with the loading being set at 3g.

The material for the main parts of the beverage holder was analyzed as cast aluminum to reduce the overall weight and to meet the life goal. Based on the S-N curve for aluminum, it was determined that in order to meet the life target set forth in the specifications, the material needed to stay below 45 MPa. The S-N curve for aluminum is shown in Figure 7. All of the components in the beverage cooler were set to cast aluminum except for the insulation on the outside of the drink holder.

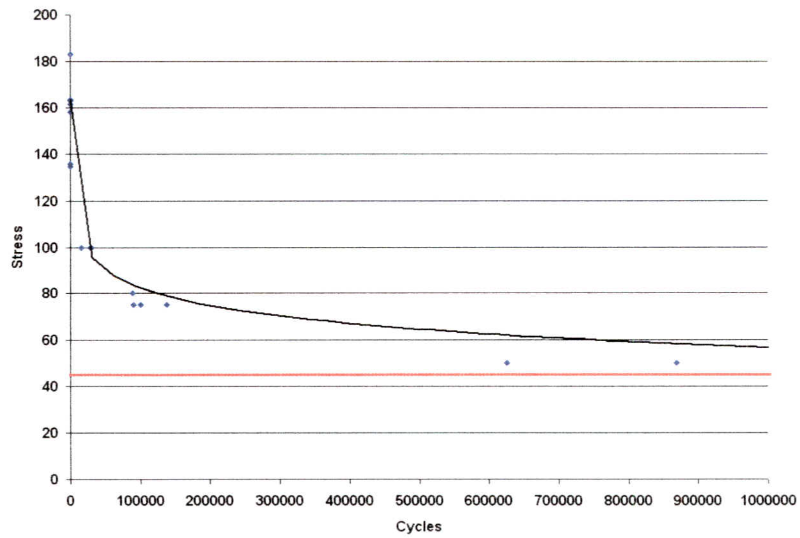


Figure 7: S-N curve for aluminum

The first assumption of the analysis was that the unit would see a max of 3g vertically. With the weight of a full 12-ounce CO_2 tank, two full beverage bottles and the solenoid block, this resulted in a vertical load of about 112 N in the vertical direction. The load was applied at the six mounting holes for the solenoid block. The model was constrained at the saddle tubes used to support the saddle. Both the flat surface and the screw mounting holes were fixed in the x, y and z direction and fixed in rotation for all directions. The FEA setup is shown in Figure 8.

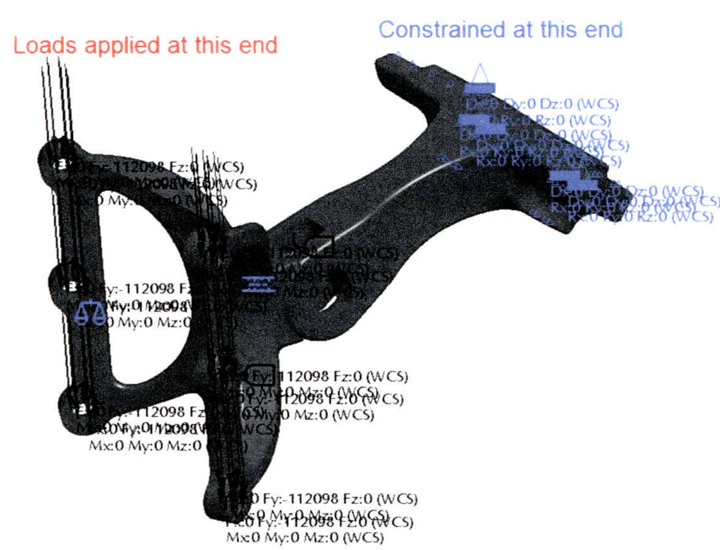


Figure 8: FEA Setup

The area that needed to be analyzed more closely was the link from the bike saddle mounts to the solenoid valve. The analysis was done on these two components only to speed up the iterations. The original setup/design is shown in Figure 9. The max stress in the necked down area of the link was at 60.9 MPa.

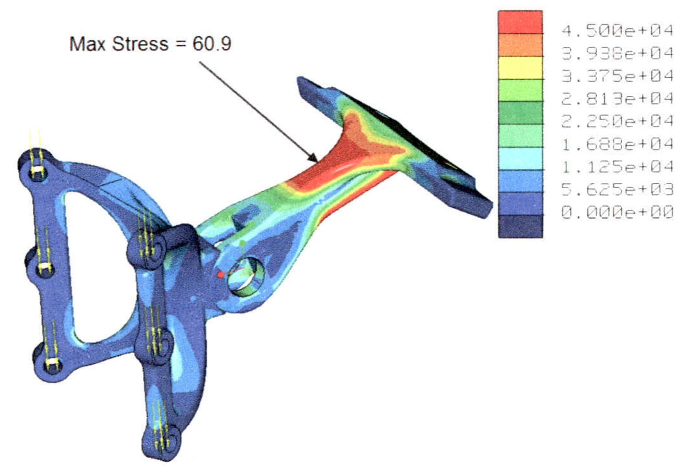


Figure 9: Original mount design

Several iterations were conducted to get the stress in the one area as low as possible. The necked down area of the link was made significantly thicker and the location around the bolt holes were made bigger to allow the stress to flow from the bolt holes to the thinner section of the link. The final iteration is shown in Figure 10. It has a maximum stress in the neck of the link of 22.9 MPa.

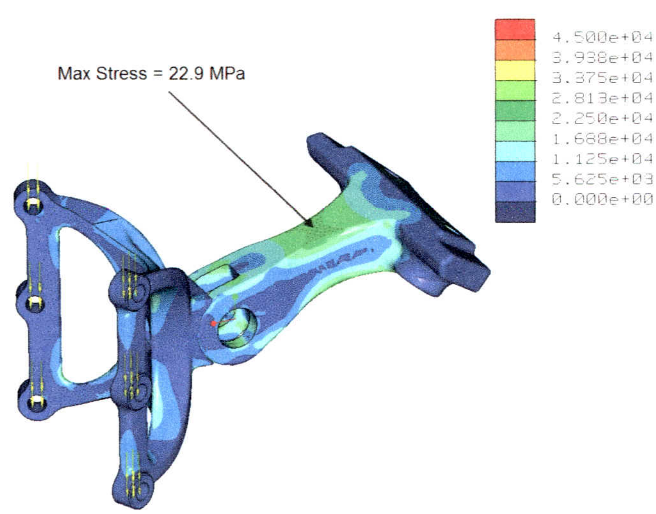


Figure 10: Final mount design

Chapter 4: Product Evaluation

As it was stated in Chapter 2, cost was a major factor in this design. When seeing that drink holders for bikes were going for as much as $170 it was clear we could easily sell this product for a little more due to that is also keeps drinks cool. With that in mind, the assumed sweet spot would be probably to sell it around $200-220. In order to make a decent profit, the cost of the design had to be kept down. It was stated earlier that the goal was to keep the cost under $150 per final unit. Due to refining the design we would be able to make this anywhere from $98-113. This is very important because it allows for substantial profit on each unit sold.

In the original design copper tubes were going to be used to deliver the CO_2 coolant. After refining the design we decided to remove the tubes and cast the channels directly into the aluminum housing with an aluminum sleeve to keep the CO_2 in. This step increased the manufacturability of the design dramatically by taking away the need to bend copper tubing into the required patterns for distribution. It is a lot less complicated to just weld a sleeve into place than work with the tubing. This also increased the durability, in that there are not tubes exposed to bumps and jolts that could deform them over time. With most of the required manufacturing being casting the time is takes to build a unit is very small. Once the molds are made the casting will only take a few minutes. After casting the only other steps required is to weld the sleeve into place, fit the insulation, and screw the mount on.

Given how far this design has progressed in only four months, it could be fairly quickly brought to market. Of course there is a bit more work to do, such as replacing the solenoid with a slow release valve, though this would not take much time. Our product could realistically be brought to market in less than six months if the resources were available for production. This is truly a testament to how fast this project has gone from concept to action.

Discussion and Conclusion

Over the course of this semester we proved that the concept of using liquid CO_2 to actively keep a protein drink cool works. However, this was ultimately a small step in the overall process. The point of senior design is to generate an idea and see it through until the final design. As can be seen in this report, this group did exactly that. It started with the idea to keep a drink cool while on a bike, and then went into the project-planning phase, where it was decided what needed to be done and when it needed to be done by. Next, proof of concept was established by creating a test rig that allowed for basic testing and then more advanced optimization for the CO_2 release. All of this work ultimately leads to the final design, which was focused on decreasing the weight and form of the final product. Despite not being able to make a final prototype this group feels confident that with minimal extra work this beverage cooler could actually be brought to market and be sold successfully. This specific design appeals to a small segment of the population, however the CO_2 cooling system could possibly be applied to many other products out there, such as baby bottles and coolers.

Contemporary Issues

Our time here at NIU has allowed us to grow academically, personally, and professionally. By developing a global perspective regarding the importance of communication, innovation, and service to the technological world, we are set to enter the next stage of our journey to becoming professional engineers. The wide variety of coursework we have taken has prepared us to enter the global engineering world with confidence in our abilities to succeed. Learning how to apply cutting-edge technologies to real-life problems has been instrumental in our understanding of how engineers interact with the environment in a global and social perspective. By continuing to "challenge the process" with regard to lifelong learning, we further poise ourselves to function as mature technical professionals who are apt to tackle more advanced challenges in future employment and/or graduate educational endeavors. We have been fortunate enough, through our access to NIU's great facilities, to learn how to apply sophisticated theoretical and practical knowledge to a wide variety of contemporary applications and processes. The time we have spent at NIU honing our engineering knowledge has allowed us to develop the proper balance of theory, application, and skills needed to integrate existing technologies into new manufacturing processes. This important element has framed our entire baccalaureate experience and will further allow us to thrive in the global engineering marketplace, securing ourselves positions as leaders and mentors to the next generation of young professionals.

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3. Incropera, Frank P., David P. Dewitt, Theodore L. Bergman, and Adrienne S. Lavine. *Fundamentals of Heat and Mass Transfer.* Hoboken, NJ: John Wiley. 2007. Print.

Appendix A: Mathcad Code

$\Delta x := 0.018$

$k := 1$

$h := 5$

$Tc := 0 + 273.15 = 273.15$

$Tinf := 18 + 273.15 = 291.15$

$$C := \begin{pmatrix} -2 h \cdot \Delta x \cdot \frac{Tinf}{k} \\ -2 h \cdot \Delta x \cdot \frac{Tinf}{k} \\ -2 h \cdot \Delta x \cdot \frac{Tinf}{k} \\ 0 \\ 0 \\ -2 h \cdot \Delta x \cdot \frac{Tinf}{k} \\ 0 \\ 0 \\ -2 h \cdot \Delta x \cdot \frac{Tinf}{k} \\ 0 \\ 0 \\ -Tc - 2 h \cdot \Delta x \cdot \frac{Tinf}{k} \\ 0 \\ -Tc \\ 0 \\ -Tc \\ 0 \\ -Tc \\ 0 \\ -Tc \\ 0 \\ -Tc \\ -Tc \\ -2Tc \end{pmatrix}$$

$T := A^{-1} \cdot C - 273.15$

$\mathrm{mean}(T) = 3.576$

T =

	0
0	8.274
1	8.467
2	9.063
3	7.205
4	7.407
5	8.051
6	5.735
7	5.904
8	6.536
9	3.925
10	3.938
11	4.223
12	2.09
13	1.7
14	1.033
15	0.773
16	0.497
17	0.359
18	0.235
19	0.167
20	0.107
21	0.076
22	0.041
23	0.029

$$A := \left[\begin{array}{cccccccccccccccccccccccc}
-2\left(h \cdot \frac{\Delta x}{k} + 2\right) & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -2\left(h \cdot \frac{\Delta x}{k} + 2\right) & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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1 & 0 & 0 & -4 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 2 & -2\left(h \cdot \frac{\Delta x}{k} + 2\right) & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -4 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & -2\left(h \cdot \frac{\Delta x}{k} + 2\right) & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -4 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & -2\left(h \cdot \frac{\Delta x}{k} + 2\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -4 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -4 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -4 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -4 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -4 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -4 & 2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -4 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -4 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -4 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -4 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -4
\end{array}\right]$$

Appendix B: Matlab Code

```
load('WaterTest.txt');
load('MilkInsulated.txt');
load('ProteinInsulated.txt');
load('WaterInsulated.txt');
load('WaterChilled1.txt');
load('WaterChilled2.txt');
load('WaterChilled3.txt');
load('WaterChilled4.txt');
load('Tank1.txt');
load('Tank2.txt');
load('WaterRoomTemp.txt');
load('WaterChilledC1.txt');
load('WaterChilledC2.txt');
load('WaterChilledC3.txt');

figure(1)
plot(MilkInsulated(:,1)/60,MilkInsulated(:,2),ProteinInsulated(:,1)/60,ProteinInsulated(:,2),WaterInsulated(:,1)/
60,WaterInsulated(:,2))
xlabel('Time [Minutes]')
ylabel('Temperature [\circC]')
legend('Milk','Protein Drink','Water','Location','SouthEast')
axis([0 150 0 8])
FitMilk=polyfit(MilkInsulated(:,1)/60,MilkInsulated(:,2),1);
FitProtein=polyfit(ProteinInsulated(:,1)/60,ProteinInsulated(:,2),1);
FitWater=polyfit(WaterInsulated(:,1)/60,WaterInsulated(:,2),1);
Slopes=[FitMilk(1),FitProtein(1),FitWater(1)]';

figure(2)
plot(WaterRoomTemp(:,1)/60,WaterRoomTemp(:,2)-2.9123,WaterInsulated(:,1)/60,WaterInsulated(:,
2)+1.3994,WaterChilled3(:,1)/60,WaterChilled3(:,2)-1.3165)
xlabel('Time [Minutes]')
ylabel('Drinkability')
legend('Bottle','Insulated','Chilled and Insulated','Location','SouthEast')
set(gca,'YGrid','on','YTick',7.2,'YTickLabel',{'Spoils @ 7.2°C'})
axis([0 90 0 10])
FitWaterChilled3=polyfit(WaterChilled3(:,1)/60,WaterChilled3(:,2),1);

figure(3)
plot(WaterChilled1(:,1)/60,WaterChilled1(:,2)-0.8222,WaterChilled3(:,1)/60,WaterChilled3(:,
2)-1.3165,WaterChilledC2(:,1)/60,WaterChilledC2(:,2)-3.397)
xlabel('Time [Minutes]')
ylabel('Temperature [\circC]')
legend('Constant Temp','Time Interval','Continous Release','Location','SouthEast')

figure(4)
plot(WaterChilled3(:,1)/60,WaterChilled3(:,2)-1.3165,WaterChilled4(:,1)/60,WaterChilled4(:,2)-2.5346)
xlabel('Time [Minutes]')
ylabel('Temperature [\circC]')
legend('1.3oz/hour','2oz/hour','Location','SouthEast')
axis([0 45 2.5 4])
FitWaterChilled4=polyfit(WaterChilled4(:,1)/60,WaterChilled4(:,2),1);

figure(5)
plot(WaterChilled2(:,1)/60,WaterChilled2(:,2),WaterChilledC1(:,1)/60,WaterChilledC1(:,2),WaterChilledC3(:,1)/
60,WaterChilledC3(:,2))
xlabel('Time [Minutes]')
ylabel('Temperature [\circC]')
```

Appendix C: List of Figures, Graphs and Tables