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WORKING PAPER

Measuring the International Dimension of Output Volatility

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January 2017
(revised August 2017)

2017/928

D/2017/7012/02

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Abstract

This paper studies output fluctuations in a panel of OECD economies with the aim to decompose the evolution in output volatility into domestic and international factors. To this end we use a factor-augmented dynamic panel model with both domestic and international shocks and spillovers between countries through trade linkages. Changes in the volatility of output growth can be due to time-varying sensitivity to these shocks, changes in the propagation mechanism or shifts in the variances of shocks. We explicitly model cross-sectional dependence in the variance equation by specifying a common factor structure in the volatility of domestic shocks. The results show that while the size of international shocks and spillovers does not decrease in most countries, the volatilities of domestic shocks share a clear common decreasing trend. Hence, the ‘Great Moderation’ appears to be mainly driven by a decline in the volatility of domestic shocks rather than smaller international shocks.

JEL classification: C32, E32, F44

Keywords: Volatility, business cycle, Bayesian model selection

*The authors would like to thank the participants of the Society for Nonlinear Dynamics and Econometrics 25th Annual Symposium, the 5th Ghent University Empirical Macroeconomics Workshop, the 20th Applied Economics Meeting, the International Association for Applied Econometrics 4th Annual Conference, the 23rd International Panel Data Conference as well as seminar participants at the University of Münster and Ghent University and particularly Freddy Heylen for helpful comments and suggestions. The computational resources (Stevin Supercomputer Infrastructure) and services used in this work were provided by the Flemish Supercomputer Center, funded by Ghent University; the Hercules Foundation; and the Economy, Science, and Innovation Department of the Flemish Government. Martin Iseringhausen gratefully acknowledges financial support from Ghent University’s Special Research Fund (BOF). Correspondence to: Martin Iseringhausen, Ghent University, Faculty of Economics and Business Administration, Campus Tweekerken, Sint Pietersplein 6, 9000 Ghent, Belgium. E-mail: Martin.Iseringhausen@UGent.be.

1 Introduction

The sharp decline in output volatility in most advanced economies since the mid 1980s is one of the most striking stylized facts in modern macroeconomics. First documented for the U.S. by [Kim and Nelson \(1999\)](#) and [McConnell and Perez-Quiros \(2000\)](#), the phenomenon has been so widespread and persistent that it was famously coined the ‘Great Moderation’ by [Stock and Watson \(2003\)](#). Although a large literature has already analyzed the potential sources and consequences of output volatility, this continues to be an area of lively debate.

One strand of the literature has focused on the fundamentals underlying the observed decline in aggregate volatility such as better monetary policy ([Clarida et al., 2000](#)), increased government size and fiscal policy ([Fatas and Mihov, 2001](#)), improved inventory management methods ([Kahn et al., 2002](#)), financial innovation and increased global integration ([Dyman et al., 2006](#)), or demographic changes ([Jaimovich and Siu, 2009](#)). Alternatively, the ‘good luck’ hypothesis brought forward by [Stock and Watson \(2003\)](#) entails the idea that the period from 1980 onwards has simply been characterized by the absence of large shocks hitting economies. Related to this is the question whether the recent Great Recession marks the end of the Great Moderation. While some authors confirm that this is indeed the case (see e.g. [Ng and Wright, 2013](#)), others consider it to be merely a temporary offset of the structural decline in volatility (see e.g. [Clark, 2009](#)).

Starting off from [Blanchard and Simon \(2001\)](#) who show that there has been a global decline in output volatility in G7 countries, with magnitude and timing differing across countries, a second strand of the literature tries to explain trends in aggregate volatility in terms of the ‘geographic origin’, i.e. to what extent these trends are driven by global or country-specific factors. [Stock and Watson \(2005\)](#) estimate a factor-augmented structural VAR where GDP growth is decomposed into common and idiosyncratic shocks as well as spillovers, i.e. shocks that originate in a certain country and subsequently spread to other countries. They find that a decrease in the size of global shocks is responsible for much of the observed decline in business cycle volatility in the G7. [Carare and Mody \(2012\)](#) add evidence that spillovers have become more important since the 1990s and acted as a volatility amplifier during the recent Great Recession. Using a dynamic factor approach, [Kose et al. \(2003\)](#) show that a common world factor is an important source of business cycle volatility in advanced economies. Extending their approach by allowing for time-varying factor loadings and stochastic volatility in the latent factors and idiosyncratic components, [Del Negro and Otrok \(2008\)](#) find no evidence of increased business cycle synchronization. In fact, their results document that a common drop in the volatility of country-specific fluctuations is an important feature of the Great Moderation, but they leave this aspect unmodeled.

In this paper we set up and estimate a factor-augmented dynamic panel data model with time-varying coefficients and stochastic volatilities to decompose aggregate output growth volatility in international and country-specific factors. More specifically, our encompassing empirical framework allows the moderation in volatility to be driven by (i) smaller international shocks; (ii) a moderation in foreign countries that spills over to the remaining countries; (iii) lower contemporaneous sensitivity to international and foreign shocks; (iv) a milder propagation of shocks over time; (v) a common and/or idiosyncratic reduction in the volatility of country-specific shocks. Such a general decomposition has not been done before. Disentangling a country’s output volatility into

its constituent components is of particular importance for policy makers as it provides information on whether the observed change in output volatility is due to one of the country-specific components, which may be under their control, or due to international factors, which are not.

We contribute to the literature in the following three ways. First, we merge the factor-augmented VAR approach of [Stock and Watson \(2005\)](#), by decomposing output growth shocks into country-specific shocks, common shocks and spillovers, and the dynamic factor approach of [Del Negro and Otrok \(2008\)](#), by allowing for time variation in the variance of shocks and time-varying sensitivities to shocks. Second, we further extend these approaches by explicitly modeling a common factor in the volatility of domestic shocks. Hence, next to co-movements in countries' GDP through common growth shocks and spillovers, our model is also able to capture co-movement in the size of country-specific shocks. The idea to model a common component in the volatility of otherwise uncorrelated shocks is not entirely new. [Kim et al. \(2009\)](#) extract macroeconomic uncertainty as the common factor in consumption and dividend growth volatility. [Laurini and Mauad \(2015\)](#) include a common jump factor in a multivariate stochastic volatility model to account for crises and contagion in emerging countries' exchange rates markets. [Herskovic et al. \(2016\)](#) show that not only firms' returns but also their volatilities exhibit a strong common factor structure. However, to the best of our knowledge, we are the first to model a common factor in the volatility of domestic output growth shocks as one of the potential sources of the Great Moderation. Third, we explicitly address model uncertainty. We start by specifying all coefficients and variance parameters as random walks, but then go on and test which time-varying components are relevant model attributes and fall back to a more parsimonious model when appropriate. This not only avoids over-parameterization but will also provide us with information on which components actually contribute to changes in output volatility.

Using quarterly data on the growth rates of real output for 16 advanced countries over the period 1961:Q1 - 2015:Q4, we obtain the following results. First, the volatility of common shocks clearly varies over time - shooting up around the oil crises of the 1970s, the worldwide recession of the early 1990s and the recent Great Recession - but there is no marked evidence of a declining trend. As individual countries' sensitivity to the common shocks and spillovers has remained stable over the sample period, changes in the volatility of the international business cycle component is not what is driving the Great Moderation. Second, the volatility of domestic shocks shows a clear common downward trend across the 16 advanced economies we consider. We identify this as one of the main drivers of the widespread reduction in volatility. Finally, the Great Recession shows up as a temporary increase in the volatility of common shocks and hence does not mark the end of the Great Moderation.

The remainder of the paper is structured as follows: Section 2 introduces our empirical specification and estimation approach. The main estimation results are presented in Section 3 and further documented by means of variance decompositions in Section 4. Section 5 concludes. The appendix contains a detailed description of the estimation methodology.

2 Model and estimation approach

2.1 Empirical specification

Our starting point is the factor-augmented dynamic panel model proposed by [Stock and Watson \(2005\)](#) extended to allow for time-varying coefficients and stochastic volatilities as in [Del Negro and Otrok \(2008\)](#). More specifically,

$$\Delta y_{it} = \alpha_{it} + \sum_{j=1}^p \beta_{it}^j \Delta y_{i,t-j} + \sum_{k=1}^q \gamma_{it}^k \Delta y_{i,t-k}^* + \varepsilon_{it}, \quad (1)$$

where Δy_{it} is real GDP growth for country i in quarter t and Δy_{it}^* is trade-weighted real GDP growth of the trading partners of country i .

Our model has a number of distinct features. First, as outlined in [Stock and Watson \(2005\)](#), Equation (1) can be seen as a vector autoregression (VAR) where the cross-country dimension represents the different variables in the system. The inclusion of Δy_{it}^* corresponds to restricting the coefficients on the lags of foreign GDP growth to be proportional to their respective trade shares. Given the medium-size dataset at hand this solves the dimensionality problem which would arise when including the growth rates for each of the foreign countries separately. Moreover, this weighted average offers a convenient spillover measure.

Second, the model in Equation (1) is structural in the sense that we impose an unobserved component factor structure on the innovations ε_{it} ,

$$\varepsilon_{it} = \phi_{it}^\varepsilon \varepsilon_t^f + \varepsilon_{it}^c, \quad (2)$$

where ε_t^f are common international shocks with country-specific loadings ϕ_{it}^ε and ε_{it}^c are country-specific innovations. These are identified through the assumption that spillovers in Equation (1) happen with at least one-period lag and imposing that ε_{it}^c in Equation (2) is a domestic shock uncorrelated across countries such that all of the contemporaneous cross-country correlation in output growth is induced by the common shock ε_t^f . Thus, this model makes it possible to quantify both the direct effect of common international shocks ε_t^f and the indirect effect of spillovers from (domestic and common) shocks ε_{it} in one country to its trading partners.

Third, we specify all coefficients to vary over time according to driftless random walks:

$$\alpha_{it} = \alpha_{i,t-1} + \eta_{it}^\alpha, \quad \eta_{it}^\alpha \sim \mathcal{N}(0, \sigma_\alpha^2), \quad (3)$$

$$\beta_{it}^j = \beta_{i,t-1}^j + \eta_{it}^{\beta^j}, \quad \eta_{it}^{\beta^j} \sim \mathcal{N}(0, \sigma_{\beta^j}^2), \quad \text{for } j = 1, \dots, p, \quad (4)$$

$$\gamma_{it}^k = \gamma_{i,t-1}^k + \eta_{it}^{\gamma^k}, \quad \eta_{it}^{\gamma^k} \sim \mathcal{N}(0, \sigma_{\gamma^k}^2), \quad \text{for } k = 1, \dots, q, \quad (5)$$

$$\phi_{it}^\varepsilon = \phi_{i,t-1}^\varepsilon + \eta_{it}^{\phi^\varepsilon}, \quad \eta_{it}^{\phi^\varepsilon} \sim \mathcal{N}(0, \sigma_{\phi^\varepsilon}^2). \quad (6)$$

Hence, we allow for changes in the persistence of shocks as measured by β_{it} as well as changes in a country's sensitivity to both spillovers and common shocks as measured by γ_{it} and ϕ_{it}^ε respectively. We also model the intercept α_{it} as a random walk to capture permanent changes in trend output growth. This avoids that low-frequency drifts, such as the productivity slowdown of the early

1970s and slower growth or secular stagnation in the aftermath of the 2008 global financial crisis, bias our decomposition of growth and volatility at the business cycle frequency (Fernald, 2007).

Finally, the variance of both common and country-specific innovations is allowed to vary stochastically over time:

$$\varepsilon_t^f \sim \mathcal{N}(0, e^{g_t}), \quad \varepsilon_{it}^c \sim \mathcal{N}(0, e^{h_{it}}). \quad (7)$$

As one of the stylized facts of the Great Moderation is a global reduction in volatility but without a clear increase in international synchronization of business cycles and with its magnitude and timing varying considerably across countries, country-specific volatilities are likely co-moving even after controlling for common shocks and spillovers. We explicitly model this correlation in the volatilities of country-specific shocks by specifying the log-variance h_{it} as a common factor structure,

$$h_{it} = \phi_{it}^h h_t^f + h_{it}^c, \quad (8)$$

where h_t^f is a common factor with country-specific time-varying loading ϕ_{it}^h and h_{it}^c the remaining idiosyncratic part. Again, we assume that the time-varying volatility components follow independent driftless random walk processes:

$$g_t = g_{t-1} + \eta_t^g, \quad \eta_t^g \sim \mathcal{N}(0, \sigma_g^2), \quad (9)$$

$$h_t^f = h_{t-1}^f + \eta_t^{h^f}, \quad \eta_t^{h^f} \sim \mathcal{N}(0, \sigma_{h^f}^2), \quad (10)$$

$$\phi_{it}^h = \phi_{i,t-1}^h + \eta_{it}^{\phi^h}, \quad \eta_{it}^{\phi^h} \sim \mathcal{N}(0, \sigma_{\phi^h}^2), \quad (11)$$

$$h_{it}^c = h_{i,t-1}^c + \eta_{it}^{h^c}, \quad \eta_{it}^{h^c} \sim \mathcal{N}(0, \sigma_{h^c}^2). \quad (12)$$

Note that while all regression and variance parameters are heterogeneous across countries, for the sake of parsimony the variances of their innovations are assumed homogeneous.

For future use, define $\beta_{it} = (\beta_{it}^1, \dots, \beta_{it}^p)$ and $\gamma_{it} = (\gamma_{it}^1, \dots, \gamma_{it}^q)$. After stacking the unobserved components over cross-sections, i.e. $\beta_t = (\beta_{1t}, \dots, \beta_{Nt})$ and similarly for the other components, further define the vector of time-varying parameters $\lambda_t = (\alpha_t, \beta_t, \gamma_t)$, of time-varying factor loadings $\phi_t = (\phi_t^\varepsilon, \phi_t^h)$ and of stochastic volatilities $\zeta_t = (g_t, h_t^f, h_t^c)$. The vectors λ , ϕ and ζ then refer to λ_t , ϕ_t and ζ_t stacked over time. The innovation variances are combined in the vector $\sigma^2 = (\sigma_\alpha^2, \sigma_\beta^2, \sigma_\gamma^2, \sigma_{\phi^\varepsilon}^2, \sigma_{\phi^h}^2, \sigma_g^2, \sigma_{h^f}^2, \sigma_{h^c}^2)$ with $\sigma_\beta^2 = (\sigma_{\beta^1}^2, \dots, \sigma_{\beta^p}^2)$ and $\sigma_\gamma^2 = (\sigma_{\gamma^1}^2, \dots, \sigma_{\gamma^q}^2)$. In addition $x_t = (\Delta y_t, \Delta y_{t-j}, \Delta y_{t-k}^*)$ represents the data matrix stacked over cross-sections which is further stacked over time to obtain x .

2.2 Identification and normalization

As it stands, the model in Section 2.1 is not identified and thus requires properly chosen normalizations. A first issue that arises is that the products $\phi_{it}^\varepsilon \varepsilon_t^f$ and $\phi_{it}^h h_t^f$ in Equations (2) and (8) are identified but not the relative scale and sign of their constituent components. Multiplying the loadings by a rescaling constant c while dividing the common factor by the same c would leave the product unchanged. As long as the standard deviations of the innovations to both com-

ponents are appropriately adjusted, the two models are equivalent. A standard normalization is therefore to constrain the scale of the factor (see e.g. [Del Negro and Otrok, 2008](#)). However, while being effective in a model with fixed loadings, time variation brings about a new identification issue as the rescaling term can now be a time-varying sequence c_t rather than a constant c . [Del Negro and Otrok \(2008\)](#) argue that in principle this does not pose a formal identification problem as multiplying the time-varying loadings with c_t implies that the rescaled loadings no longer satisfy the model's assumptions. For instance, when transforming ϕ_{it}^ε to $c_t\phi_{it}^\varepsilon$ the innovations $c_t\eta_{it}^{\phi^\varepsilon}$ would no longer satisfy the homoskedasticity assumption of Equation (6). However, while the model is theoretically identified, this can fail in practice for the following reasons. First, the homoskedasticity restriction imposes only a weak constraint on the evolution of the factor loadings when estimating them using the Kalman filter (outlined below), still leaving scope for some time-varying rescaling. Second, although the country-specific loadings ϕ_{it}^ε are assumed to be uncorrelated across cross-sections, the Kalman filter does not impose this when estimating them. Hence, the loadings ϕ_{it}^ε possibly pick up a common business cycle component that should in fact be captured by the common factor ε_t^f . Moreover, a common issue in factor models is that the sign of the factor and the loadings is indeterminate (see e.g. [Del Negro and Otrok, 2008](#); [Kose et al., 2008](#)). A homoskedasticity restriction does not prevent sign switches as multiplying ϕ_{it}^ε and ε_t^f by $c_t = -1$ leaves the variance of the rescaled innovation $c_t\eta_{it}^{\phi^\varepsilon}$ unchanged.

We avoid the above-mentioned identification issues by restricting the cross-sectional averages of the loadings to be 1 in each period:

$$\bar{\phi}_t^\varepsilon = \frac{1}{N} \sum_{i=1}^N \phi_{it}^\varepsilon = 1, \quad \bar{\phi}_t^h = \frac{1}{N} \sum_{i=1}^N \phi_{it}^h = 1, \quad \forall t = 1, \dots, T. \quad (13)$$

This boils down to dividing the original, unnormalized loadings by their cross-sectional average in every period t and assuming that Equations (6) and (11) hold for the rescaled loadings. Note that this normalization scheme implies that a country's loading should be interpreted as being relative to the average loading. Hence, increasing business cycle integration for all countries in the sample should come about through bigger global shocks rather than through an overall increase in the sensitivity to these shocks.

A similar weak identification issue may arise when separating the constituent components of the log-variance h_{it} of domestic shocks in Equation (8). Although, the country-specific random walk processes h_{it}^c are assumed to be uncorrelated across cross-sections, this is not imposed when filtering these sequences using the Kalman filtering approach. Hence, there is some scope for h_{it}^c to pick up common volatility trends that should be captured by $\phi_{it}^h h_t^f$. For this reason, we restrict the cross-sectional average of h_{it}^c to 0 in each period:

$$\bar{h}_t^c = \frac{1}{N} \sum_{i=1}^N h_{it}^c = 0, \quad \forall t = 1, \dots, T. \quad (14)$$

This restriction is consistent with our assumption that all co-movement in countries' log-variances should stem from the common component $\phi_{it}^h h_t^f$ while the remaining idiosyncratic volatilities h_{it}^c should no longer include a common factor.

Note that our normalizations imply that h_t^f will correspond to the panel average log-volatility in each period and hence resembles the Common Correlated Effects (CCE) approach of [Pesaran \(2006\)](#) to estimate panel data models with a common factor structure in the errors. To see this, take cross-sectional averages of h_{it} in Equation (8) and solve for the common factor h_t^f to obtain

$$h_t^f = \frac{1}{\bar{\phi}_t^h} \left(\bar{h}_t - \bar{h}_t^c \right), \quad (15)$$

with \bar{h}_t being the cross-sectional average of h_{it} . Plugging this expression for h_t^f back in Equation (8), we obtain

$$h_{it} = \frac{\phi_{it}^h}{\bar{\phi}_t^h} \left(\bar{h}_t - \bar{h}_t^c \right) + h_{it}^c = \phi_{it}^{h*} \bar{h}_t + h_{it}^{c*}, \quad (16)$$

where $\phi_{it}^{h*} = \phi_{it}^h / \bar{\phi}_t^h$ and $h_{it}^{c*} = h_{it}^c - \phi_{it}^h / \bar{\phi}_t^h \bar{h}_t^c$. It is easily verified that the cross-sectional averages of ϕ_{it}^{h*} and h_{it}^{c*} are 1 and 0, respectively, for each t . This is exactly the normalization we impose in Equations (13) and (14).

2.3 Bayesian estimation and stochastic model specification search

The time-varying dynamic panel model outlined above corresponds to a state space model with the observation equation given by merging Equations (1)-(2) and (7)-(8) and the state Equations (3)-(6) and (9)-(12) describing the laws of motion for the unobserved random walk components. We estimate this model using Bayesian Markov Chain Monte Carlo (MCMC) methods. A detailed description can be found in the appendix.

The main aim of this paper is to determine the time-varying sources of output volatility. While previous research has already dealt with this question, most work relies on structural break tests (e.g. [McConnell and Perez-Quiros, 2000](#)), ad hoc split-sample regressions (e.g. [Stock and Watson, 2005](#)) or imposing time variation on various model components ([Del Negro and Otrok, 2008](#)). Instead of merely assuming time variation from the outset, we will test for which model components the time variation is actually relevant and fall back to a more parsimonious specification when appropriate. More specifically, we will use the Bayesian stochastic model specification search proposed by [Fruehwirth-Schnatter and Wagner \(2010\)](#), as outlined in the next paragraphs.

Non-centered parametrization

The first step is to rewrite the model's time-varying components into a non-centered parametrization. The random walk specification of, for instance, the autoregressive parameters β_{it} in (4) can be reparameterized as

$$\beta_{it}^j = \beta_{i0}^j + \sigma_{\beta^j} \tilde{\beta}_{it}^j, \quad (17)$$

$$\text{with } \tilde{\beta}_{it}^j = \tilde{\beta}_{i,t-1}^j + \eta_{it}^j, \quad \tilde{\beta}_{i0}^j = 0, \quad \eta_{it}^j \sim \mathcal{N}(0, 1), \quad (18)$$

for $j = 1, \dots, p$ and where $\sigma_{\beta j} \tilde{\beta}_{it}^j$ is the time-varying part of β_{it}^j and β_{i0}^j the initial value if β_{it}^j varies over time ($\sigma_{\beta j} > 0$) while being its constant value if there is no time variation ($\sigma_{\beta j} = 0$). The other random walk components $\alpha_{it}, \gamma_{it}, \phi_{it}^\varepsilon, \phi_{it}^h, g_t, h_t^f$ and h_{it}^c in the model can be rewritten in a similar way, with $\alpha_{i0}, \beta_{i0}, \gamma_{i0}, \phi_{i0}^\varepsilon, \phi_{i0}^h, g_0, h_0^f, h_{i0}^c$ referring to the initial values and $\sigma = (\sigma_\alpha, \sigma_\beta, \sigma_\gamma, \sigma_{\phi^\varepsilon}, \sigma_{\phi^h}, \sigma_g, \sigma_{h^f}, \sigma_{h^c})$ to the vector of standard deviations. For future use let $\lambda_0 = (\alpha_0, \beta_0, \gamma_0)$, $\phi_0 = (\phi_0^\varepsilon, \phi_0^h)$ and $\zeta_0 = (g_0, h_0^f, h_0^c)$ be the initial values stacked over cross-sections. Similarly, define the time-varying parts as $\tilde{\lambda} = (\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$, $\tilde{\phi} = (\tilde{\phi}^\varepsilon, \tilde{\phi}^h)$ and $\tilde{\zeta} = (\tilde{g}, \tilde{h}^f, \tilde{h}^c)$.

The non-centered parameterization offers several features that will prove useful for model selection. First, it is not identified as the signs of $\sigma_{\beta j}$ and $\tilde{\beta}_{it}^j$ can be changed while leaving their product unchanged. As a result, the likelihood function is symmetric around zero along the $\sigma_{\beta j}$ dimension. When β_{it}^j varies over time (i.e. $\sigma_{\beta j}^2 > 0$) the likelihood function is bimodal with modes $-\sqrt{\sigma_{\beta j}^2}$ and $\sqrt{\sigma_{\beta j}^2}$. When β_{it}^j is constant (i.e. $\sigma_{\beta j}^2 = 0$) the likelihood function is unimodal around zero. Hence, non-identification of the sign of $\sigma_{\beta j}$ offers an intuitive view on whether β_{it}^j varies over time.

Parsimonious specification

The non-centered parameterization is very useful for formal model selection as, in contrast to the original component β_{it}^j , the transformed process $\tilde{\beta}_{it}^j$ does not degenerate to a time-invariant parameter when $\sigma_{\beta j} = 0$ as the constant part is now represented by β_{i0}^j . This allows us to reformulate the question whether β_{it}^j is time-varying as a more standard variable selection problem. To this end, [Fruehwirth-Schnatter and Wagner \(2010\)](#) define the parsimonious specification as

$$\beta_{it}^j = \beta_{i0}^j + \delta_{\beta j} \sigma_{\beta j} \tilde{\beta}_{it}^j, \quad (19)$$

where $\delta_{\beta j}$ is a binary indicator that is either 0 or 1. If $\delta_{\beta j} = 0$, $\tilde{\beta}_{it}^j$ is excluded from the model and $\sigma_{\beta j}$ is set to zero. If $\delta_{\beta j} = 1$, $\tilde{\beta}_{it}^j$ is included and $\sigma_{\beta j}$ is estimated.

Defining similar binary indicators for the other time-varying components and collecting all of them in the vector $\mathcal{M} = (\delta_\alpha, \delta_\beta, \delta_\gamma, \delta_{\phi^\varepsilon}, \delta_{\phi^h}, \delta_g, \delta_{h^f}, \delta_{h^c})$, with $\delta_\beta = (\delta_{\beta^1}, \dots, \delta_{\beta^p})$ and $\delta_\gamma = (\delta_{\gamma^1}, \dots, \delta_{\gamma^q})$, the specification of the model is described by a combination of the elements in \mathcal{M} .

Gaussian prior centered at zero

Our Bayesian estimation procedure requires choosing prior distributions for the time-invariant parts of the parameters in λ_0 , ϕ_0 , and ζ_0 , for the innovation variances in σ^2 and for the probabilities of the binary indicators in \mathcal{M} being 1. It is well known that when using the standard inverse Gamma prior distribution for the variances in σ^2 , the choice of the shape and scale that define this distribution has a strong influence on the posterior, especially when the true value of the variance is close to zero. More specifically, as the inverse Gamma distribution does not have probability mass at zero, using it as a prior distribution tends to push the posterior density away from zero. This is particularly problematic as we want to decide whether the model's parameters are time-varying, i.e. whether their innovation variances are zero or not. Due to the fact that in the non-centered parameterization, as outlined above, the standard deviations σ of the innovations

to the random walk processes enter as regression parameters, we can replace the commonly used inverse Gamma prior for σ^2 by a Gaussian prior centered at zero for σ .

We therefore use a Gaussian prior distribution for all parameters. First, we choose an uninformative prior $\mathcal{N} \sim (0, 1)$ for the time-invariant part of the parameters λ_0 and of the stochastic volatilities ζ_0 while using $\mathcal{N} \sim (1, 1)$ for the time-invariant part of the factor loadings ϕ_0 . The latter is consistent with our chosen normalization scheme, introduced in Section 2.2, that imposes the cross-sectional average of the factor loadings to be 1. Second, the prior distributions for the standard deviations σ to the various random walk components are centered around zero with the variance chosen such that the prior distribution has support over the range of relevant parameter values, given the scale of the data and the fact that most of the time-varying parameters capture slow long-run developments. The exact prior choices for σ are provided in the left part of Table 1. Finally, for each of the binary indicators in \mathcal{M} we choose a Bernoulli prior distribution where each indicator has a prior probability $p_0 = 0.5$ of being 1. Robustness of the results will be checked with respect to the prior choices.

3 Estimation results

In this section we present our main estimation results. Following Stock and Watson (2005), all reported results are obtained setting $p = 4$ and $q = 1$ in Equation (1). Experimenting with alternative lag structures shows that the results are robust with respect to this choice. After discussing the data used, we start off with the stochastic model specification search to test for time variation in the various model components. Next, we present the results for the chosen parsimonious specification. We end with a number of robustness tests.

3.1 Data

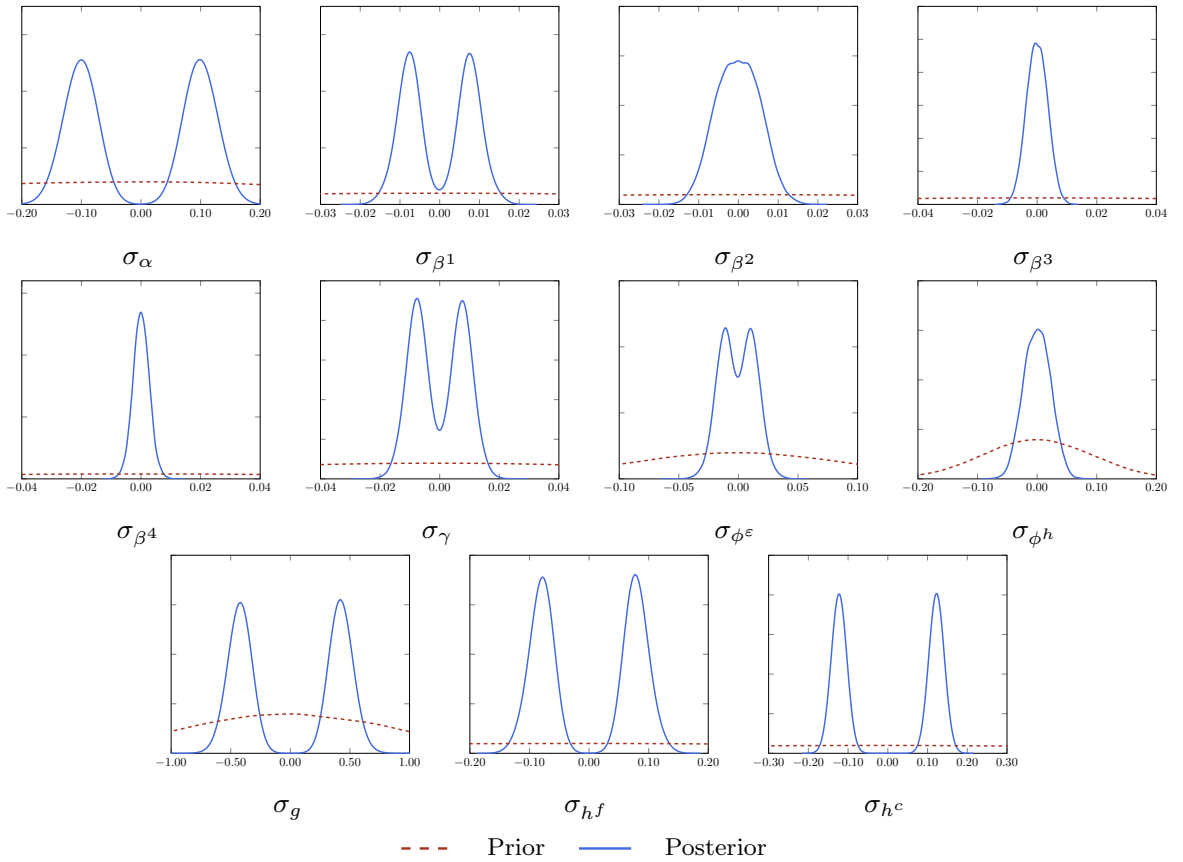
We estimate the model outlined in Section 2 using quarterly data for 16 advanced economies over the period 1961:Q1 - 2015:Q4. The included countries are: Australia, Austria, Belgium, Canada, Finland, France, Germany, Italy, Japan, Netherlands, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States.¹ As our focus is on economic fluctuations over the business cycle horizon, we follow Stock and Watson (2005) and filter out high frequency (quarter-to-quarter) fluctuations by measuring economic growth Δy_{it} as the year-on-year growth rate of real GDP. Real GDP is taken from the OECD Quarterly National Accounts database. Trade-weighted growth rates, which serve as the spillover measure in Equation (1), are calculated as $\Delta y_{it}^* = \sum_{j=1, j \neq i}^N w_{it}^j \Delta y_{it} \forall i = 1, \dots, N, t = 1, \dots, T$, where $w_{it}^j = (\text{EX}_{it}^j + \text{IX}_{it}^j) / (\text{EX}_{it} + \text{IM}_{it})$ is the share of country j in total gross trade of country i at time t . Gross trade is taken from the OECD Quarterly International Trade database. Section 3.5 reports robustness checks with respect to the country sample, using quarter-on-quarter growth rates, as well as estimating the model with annual instead of quarterly data.

¹Quarterly growth data are also available for Ireland and Norway over the sample period. However, we excluded these countries as the construction of quarterly from annual data by the OECD induced significant interpolation issues for these two countries. We further discuss the sensitivity of our results with respect to the construction of quarterly data in Section 3.5.

3.2 Results stochastic model specification search

We start by estimating an unrestricted model with all binary indicators in \mathcal{M} set to one to generate posterior distributions for the standard deviations σ of the innovations to the 11 non-centered components of interest. As the sign of these standard deviations is not identified, a bimodal posterior distribution is a first indication of time variation. Posterior densities are plotted in Figure 1, while Table 1 reports the median and percentiles of the absolute value of the standard deviations.

Figure 1: Posterior densities of the standard deviations σ (all binary indicators set to 1)



A number of interesting features stands out. First, the standard deviations of the innovations to each of the three stochastic volatility components g_t , h_t^f , and h_{it}^c all have a clear-cut bimodal posterior density with no probability mass at zero. This suggests that changes in the size of both common and domestic shocks have played an important role in the evolution of aggregate output growth volatility over the last five decades. The variance of global shocks g_{it} is subject to the largest innovations, with a posterior median absolute standard deviation of around 0.5. But also the variance of domestic shocks h_{it} varies over time, induced by time variation in both the common volatility factor h_t^f and the purely country-specific component h_{it}^c .

Table 1: Summary information for the prior and posterior distributions of the standard deviations σ

Parameter		Prior variance	Posterior		
		V_0	median	5%	95%
Std. of long-run growth	σ_α	0.5^2	0.133	0.089	0.183
Std. of AR(1) coefficient	σ_{β^1}	0.1^2	0.009	0.004	0.016
Std. of AR(2) coefficient	σ_{β^2}	0.1^2	0.004	0.000	0.012
Std. of AR(3) coefficient	σ_{β^3}	0.1^2	0.003	0.000	0.008
Std. of AR(4) coefficient	σ_{β^4}	0.1^2	0.002	0.000	0.006
Std. of spillover sensitivity	σ_γ	0.1^2	0.009	0.002	0.016
Std. of loadings common growth shocks	$\sigma_{\phi^\varepsilon}$	0.1^2	0.013	0.002	0.029
Std. of loadings common volatility factor	σ_{ϕ^h}	0.1^2	0.017	0.002	0.047
Std. of SV common growth shocks	σ_g	1.0^2	0.534	0.392	0.705
Std. of common volatility factor	σ_{h^f}	1.0^2	0.103	0.077	0.144
Std. of idiosyncratic volatility	σ_{h^c}	1.0^2	0.155	0.128	0.182

Notes: The prior distribution is $\mathcal{N}(0, V_0)$. The posterior distribution is for the absolute value of the standard deviations.

Second, there appears to be some time variation in the persistence of output growth. The posterior distribution of the standard deviation of innovations to β_{it}^1 shows clear bimodality, although there is still probability mass left at zero and the absolute posterior median of 0.01 is rather small. For β_{it}^2 , β_{it}^3 and β_{it}^4 there is no sign of time variation. Third, there is also some evidence that the sensitivity to spillovers γ_{it} varies over time. The posterior density of σ_γ is bimodal but also has considerable probability mass at zero. Fourth, accounting for a time-varying mean growth rate α_{it} proves to be necessary as there is clear bimodality in the posterior distribution of σ_α . Finally, we do not find convincing evidence for changes in countries' sensitivity to common growth shocks and to the common volatility factor. The posterior density of $\sigma_{\phi^\varepsilon}$ is bimodal but also has significant probability mass at zero while that of σ_{ϕ^h} is clearly unimodal.

Inspecting the posterior density of the innovation standard deviations only provides a first idea on the presence of time variation. As a more formal test, we next sample the stochastic binary indicators in \mathcal{M} together with the other parameters in the model. Table 2 reports the posterior probabilities for the binary indicators being one. These probabilities are calculated as the fraction of draws in which the stochastic model specification search prefers a model which allows for time variation in the corresponding parameter. To check the robustness over alternative prior variances for the innovation standard deviations σ , we multiply V_0 as reported in Table 1 by a scaling factor v_0 . Hence, the middle row (where $v_0 = 1$) corresponds to our baseline scenario, while the first two rows (where $v_0 < 1$) and the last two rows (where $v_0 > 1$) imply more and less informative priors, respectively. Rows 3 and 5 further check the robustness with respect to alternative values for the prior inclusion probability p_0 .

Overall, the results in Table 2 confirm our earlier conclusions. Under the baseline prior scenario ($p_0 = 0.5$; $v_0 = 1$) we find strong evidence of time variation in the stochastic volatility components, the intercept and to a slightly lesser extent in the AR(1) coefficient.

Table 2: Posterior inclusion probabilities for the binary indicators \mathcal{M} over different prior variances for σ and different prior inclusion probabilities p_0

Priors		Posterior inclusion probabilities										
		Intercept	AR coefficients				Spillovers	Loadings		Stochastic volatilities		
p_0	v_0	δ_α	δ_{β^1}	δ_{β^3}	δ_{β^3}	δ_{β^4}	δ_γ	$\delta_{\phi^\varepsilon}$	δ_{ϕ^h}	δ_g	δ_{hf}	δ_{hc}
0.5	0.1	1.00	0.84	0.15	0.11	0.08	0.51	0.61	0.41	1.00	1.00	1.00
0.5	0.5	1.00	0.80	0.10	0.05	0.04	0.32	0.42	0.28	1.00	1.00	1.00
0.1	1	1.00	0.21	0.01	0.00	0.00	0.08	0.07	0.03	1.00	1.00	1.00
0.5	1	1.00	0.81	0.11	0.04	0.03	0.28	0.33	0.23	1.00	1.00	1.00
0.9	1	1.00	0.93	0.43	0.28	0.22	0.75	0.78	0.72	1.00	1.00	1.00
0.5	2	1.00	0.69	0.07	0.04	0.02	0.25	0.29	0.21	1.00	1.00	1.00
0.5	10	1.00	0.46	0.03	0.01	0.01	0.16	0.16	0.09	1.00	1.00	1.00

Notes: The prior distribution for each of the elements in σ is $\mathcal{N}(0, v_0 V_0)$ with V_0 the variance in the baseline scenario as reported in Table 1. p_0 is the prior inclusion probability of the binary indicators. The posterior inclusion probabilities are calculated as the average selection frequencies over all iterations of the MCMC.

Moreover, the inclusion probabilities for the stochastic volatility components and the intercept are completely unaffected by the prior choice. For the AR(1) parameter, the inclusion probability only falls below 50% when using a very loose prior for the innovation standard deviation ($v_0 = 10$) or a low prior inclusion probability ($p_0 = 0.1$). For the other components there is much less evidence in favor of time variation. Only for the spillover parameter and the factor loadings, the posterior inclusion probabilities exceed 50% when using a very informative prior for the innovation standard deviation ($v_0 = 0.1$)² or a high prior inclusion probability ($p_0 = 0.9$).

The main conclusions of our tests for time variation are roughly in line with previous findings and discussions in the literature. First, [Antolin-Diaz et al. \(2017\)](#) have recently shown that long-run growth in the U.S. is characterized by a slowly but persistently decreasing pattern. A similar result can be found in [Berger et al. \(2016a\)](#). Second, the discussion whether persistence as measured by the AR coefficients has changed over time is more controversial. Using rolling regression approaches and different break tests for U.S. data, [Blanchard and Simon \(2001\)](#) and [Stock and Watson \(2003\)](#) do not find evidence for changes in the dynamics of output growth. In contrast, [Galí and Gambetti \(2009\)](#) document changes in the conditional and unconditional moments of several components of output in the U.S. Third, also the evidence concerning convergence to a common business cycle is mixed. [Kose et al. \(2008\)](#) find evidence for convergence among industrialized countries while [Doyle and Faust \(2002\)](#) conclude that this has not been the case. Finally, we more formally confirm the observation by [Del Negro and Otrok \(2008\)](#) that there has been co-movement in the volatility of country-specific shocks.

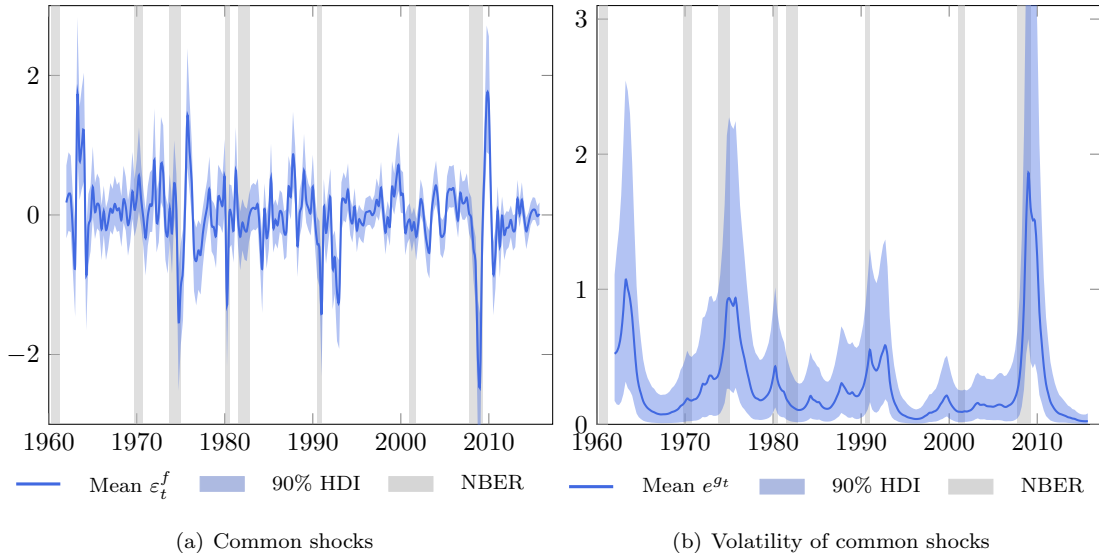
²The increase in the posterior probabilities when using a smaller (and vice versa higher) prior variance for σ may appear counter intuitive, but results from the fact that in this case more weight is given to less extreme (and hence more likely) values of σ when calculating the marginal likelihood of the model with time variation. Moreover, by allowing for less time variation the competing models become more similar in their marginal likelihoods causing a tendency for the posterior inclusion probability to shrink towards the prior $p_0 = 0.5$.

3.3 Parameter estimates and unobserved components

In this section we present the posterior distributions of constant parameters and time-varying components in the parsimonious specification. To get further insights into the role played by the various model components, a variance decomposition of the evolution of total volatility will be presented in Section 4.

Based on the results of the model selection presented in the previous section, our parsimonious specification allows for time variation in the different variance components, the intercept and the AR(1) coefficient. The other parameters are fixed to be invariant over time. We have experimented with models allowing for time variation in the spillover parameters γ_{it} and the factor loadings ϕ_{it}^ε , for which some evidence of time variation existed, but we found no clear trends in these parameters. Moreover, the behaviour of the other components was largely unaffected by these changes.

Figure 2: Common growth shocks and their volatility



Note: HDI is the 90% highest density interval. The gray bars indicate National Bureau of Economic Research (NBER) recessions.

Common shocks and spillovers

Figure 2 plots the posterior means and 90% highest density intervals (HDI) of the common shocks ε_t^f along with their time-varying volatility. Several periods characterized by large common shocks correspond to well-known events. We clearly identify the oil crises of 1973/1974 and 1979/1980, the worldwide recession of the early 1990s and the recent Great Recession of 2007 - 2009. The timing of most U.S. recessions, which are indicated by the gray bars in Figure 2, coincides with the occurrence of large negative global economic shocks. The only exceptions are the relatively mild recession in 1969/1970, the early 1980s recession due to the Federal Reserve's contractionary monetary policy, as well as the recession in the early 2000s associated with the burst of the dot-com bubble and the September 11th attacks, which are all U.S. recessions that do not show up as global shocks.

In line with the stochastic model specification search in Section 3.2, Figure 2(b) shows considerable variation in the volatility of common growth shocks over time. Periods with larger common shocks are followed by more tranquil times and vice versa. There is however no sign of a decreasing trend. This shows that a reduction in the size of common shocks is not the major driver of the observed decrease in volatility across advanced economies. This is in line with the different timing in the volatility decline across countries. Also note that the Great Recession shows up as a temporary increase in the volatility of common shocks and hence does not mark the end of the Great Moderation according to our results. The stochastic model specification search also showed that country-specific sensitivities to the common shocks did not change over the sample period. The left hand side of Table 3 therefore presents the time-invariant factor loadings ϕ_{i0}^ε .

Table 3: Posterior distributions of factor loadings and spillover sensitivities

	Growth: Loadings and Spillovers						Volatility: Loadings		
	ϕ_{i0}^ε	Percentiles		γ_{i0}	Percentiles		ϕ_{i0}^h	Percentiles	
	median	5%	95%	median	5%	95%	median	5%	95%
Australia	0.37	0.11	0.61	0.11	0.03	0.20	1.10	0.39	1.83
Austria	1.16	0.95	1.38	0.44	0.32	0.57	0.70	-0.03	1.49
Belgium	0.97	0.79	1.15	0.37	0.28	0.47	0.72	-0.03	1.40
Canada	0.75	0.56	0.93	0.24	0.14	0.33	0.57	-0.16	1.30
Finland	1.38	1.03	1.72	0.34	0.17	0.51	1.03	0.25	1.77
France	1.00	0.86	1.13	0.29	0.19	0.39	1.05	0.29	1.76
Germany	1.50	1.27	1.75	0.27	0.12	0.43	1.38	0.70	2.07
Italy	1.06	0.85	1.26	0.40	0.27	0.54	1.38	0.70	2.11
Japan	1.02	0.68	1.35	0.12	0.02	0.24	1.26	0.53	1.97
Netherlands	1.19	0.98	1.43	0.42	0.28	0.56	0.70	0.01	1.45
Portugal	0.93	0.67	1.18	0.18	0.06	0.30	1.14	0.43	1.92
Spain	0.72	0.56	0.87	0.15	0.06	0.24	1.67	0.95	2.38
Sweden	1.39	1.11	1.68	0.36	0.20	0.52	1.09	0.45	1.81
Switzerland	1.01	0.81	1.21	0.23	0.13	0.34	0.85	0.11	1.63
U.K.	0.90	0.68	1.11	0.07	-0.04	0.18	1.02	0.21	1.81
U.S.	0.67	0.48	0.86	0.04	-0.06	0.14	0.32	-0.35	1.00

Most countries exhibit very similar sensitivity to common shocks, i.e. the posterior median is close to one for the majority of countries in our sample. Australia constitutes a clear exception. Its median factor loading of 0.37 signals a partial decoupling of the international business cycle. Also Canada, Spain and the U.S. seem to be somewhat less sensitive to global shocks. Other countries like Finland, Germany, and Sweden seem to be particularly sensitive as indicated by a median factor loading clearly exceeding one.

Next, we turn to evaluating the role of spillovers. The middle part of Table 3 presents summary results for the posterior distributions of γ_{i0} , which is the country-specific sensitivity to lagged trade-weighted average growth rates $\Delta y_{i,t-1}^*$. The stochastic model specification search showed

these sensitivities to be constant over time. Nevertheless, there are significant differences across countries. On the one hand, some European countries appear to be particularly sensitive to growth spillovers transmitted via the trade channel. Those include small open economies such as Austria, Belgium, Finland, Netherlands and Sweden but also Italy. On the other hand, we find the more closed economies Australia, Japan, the U.K. and the U.S. to be much less affected by spillovers.

The left hand side of Table 4 reports average pairwise correlation coefficients of the original output growth rates Δy_{it} and the model’s residuals $\hat{\varepsilon}_{it}^c$, respectively. As expected, output growth is positively correlated with an average pairwise correlation coefficient of around 0.5. For the residuals this crumbles to -0.02. This shows that common shocks and spillovers are sufficient to capture most of the cross-country correlation in output growth rates.

Table 4: Cross-sectional correlation in output growth and its volatility

	Growth		Volatility		
	Δy_{it}	$\hat{\varepsilon}_{it}^c$	$\Delta \text{Var}(\Delta y_{it})$	$\Delta e^{(\phi_{it}^h h_t^f + h_{it}^c)}$	$\Delta e^{h_{it}^c}$
Avg. corr.	0.53	-0.02	0.74	0.43	-0.04
5th perc.	0.34	-0.19	0.44	0.09	-0.28
95th perc.	0.74	0.14	0.94	0.75	0.24

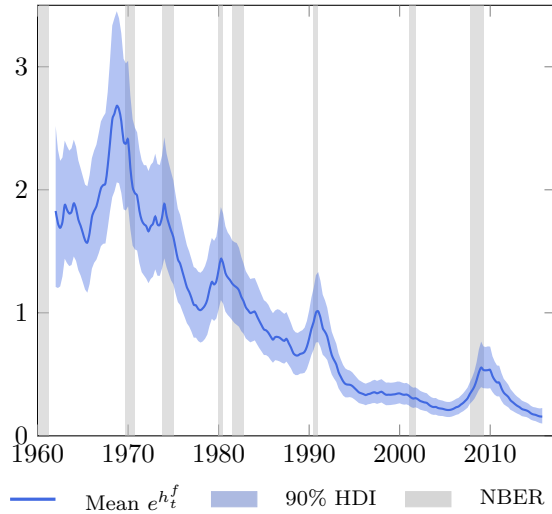
Notes: Reported are the average along with the 5th and 95th percentiles of the country-by-country cross-correlations. The total variance series $\text{Var}(\Delta y_{it})$ is calculated from our model estimates using a simulation-based approach. Details can be found in Section 4. As the estimated stochastic volatilities are non-stationary by construction, we report correlations for first-differenced series to avoid spurious correlations.

Domestic shocks: common and idiosyncratic volatility

The right hand side of Table 4 provides a first view on the correlation structure of output volatility across our 16 advanced economies. First, with a correlation of 0.74, co-movement in output volatility is clearly present. Second, the correlation coefficient of 0.43 for changes in the volatilities of domestic shocks suggests that there is an important role for commonality in the size of country-specific shocks for the overall correlation structure in output volatility. Third, the last column in Table 4 confirms that there is no significant correlation left in the idiosyncratic volatilities h_{it}^c .

We will now take a closer look at the evolution and composition of the volatility of domestic shocks. The model specification search indicated that the country-specific loadings ϕ_{it}^h on the common volatility factor h_t^f do not exhibit time variation, while both the common volatility factor itself and the remaining idiosyncratic volatility h_{it}^c were found to vary over time. Figure 3 plots the posterior mean and HDI of the common volatility factor. The variances of domestic growth shocks clearly share a common downward trend. This implies that the observed widespread decline in output volatility is induced by a common drop in the size of domestic rather than global shocks or spillovers. Although already suggested by the results in Del Negro and Otrok (2008), this paper is the first to actually model and quantify such a common volatility factor.

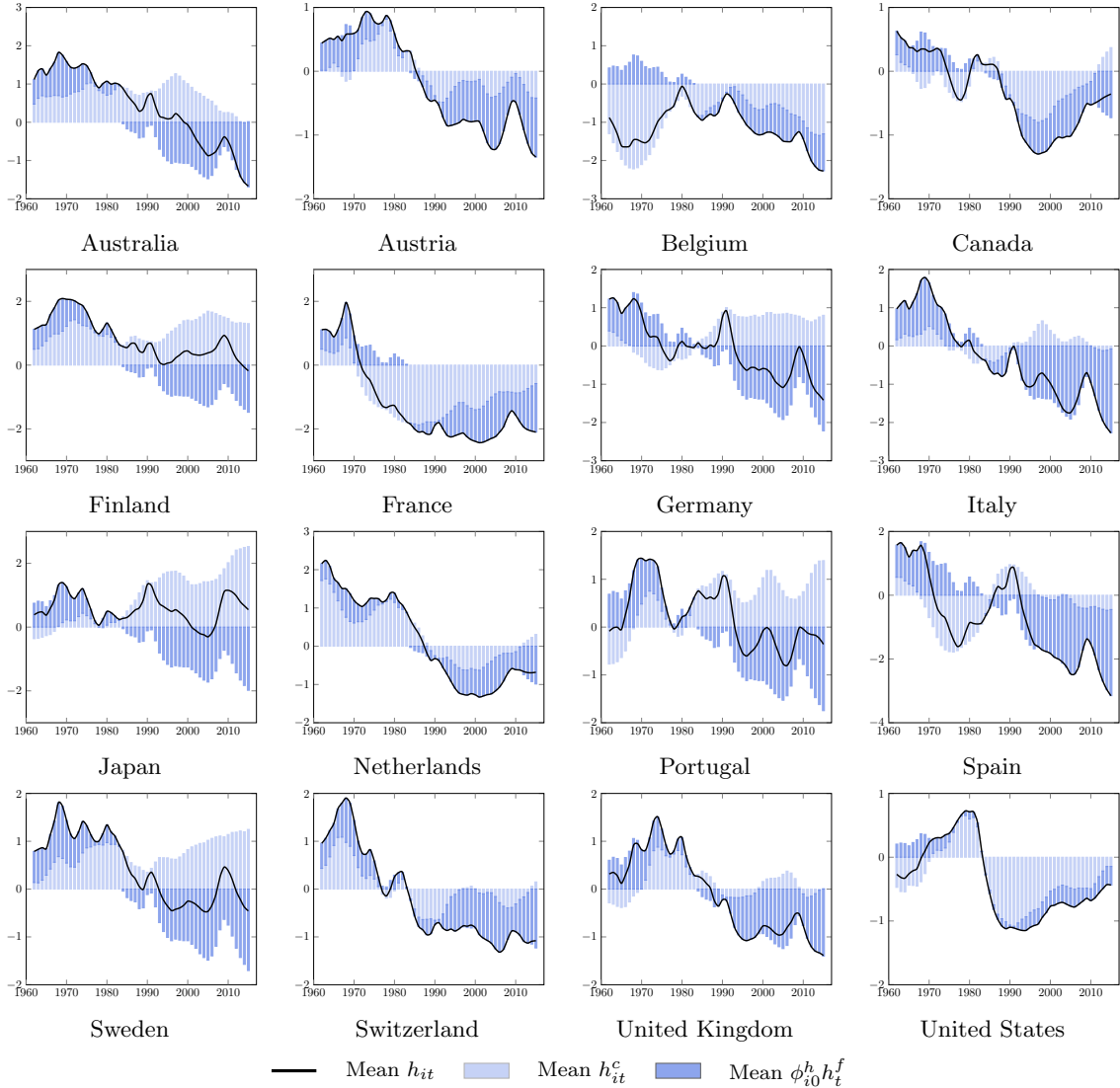
Figure 3: Common factor in the variance of domestic shocks



Next to a clear downward trend, common volatility also shoots up around the time that major global shocks occur. This suggests that the turmoil caused by a large global shock is further amplified through a global increase in country-specific macroeconomic uncertainty. Results for the loadings ϕ_{i0}^h on the common volatility factor h_t^f are presented in Table 3. Next to exhibiting no significant time variation, as indicated by the results from Section 2.3, they are also rather similar across countries, i.e. the posterior distributions include 1 in nearly all countries. Note however that the loadings are not very precisely estimated, as indicated by the relatively wide 5% and 95% posterior percentiles reported in Table 3. The U.S. is the only country for which the factor loadings ϕ_{i0}^h are significantly smaller than 1 with a posterior median of around 0.3.

Figure 4 presents a decomposition of the evolution in the total volatility of domestic shocks into the contribution of the common factor and the idiosyncratic component for each of the 16 considered countries. The relative importance of these two components clearly varies over countries. First, and most striking is the fact that the sharp drop in volatility in the U.S. seems to be purely country-specific in the sense that its particular pronounced pattern is fully captured by the idiosyncratic volatility component. Second, idiosyncratic volatility has remained much more stable in countries like Australia, Finland, Italy or Sweden. Hence, in these countries the observed drop in the volatility of domestic shocks is almost exclusively accounted for by the common volatility factor.

Figure 4: Decomposition of (log-)volatility of domestic shocks into common and idiosyncratic part

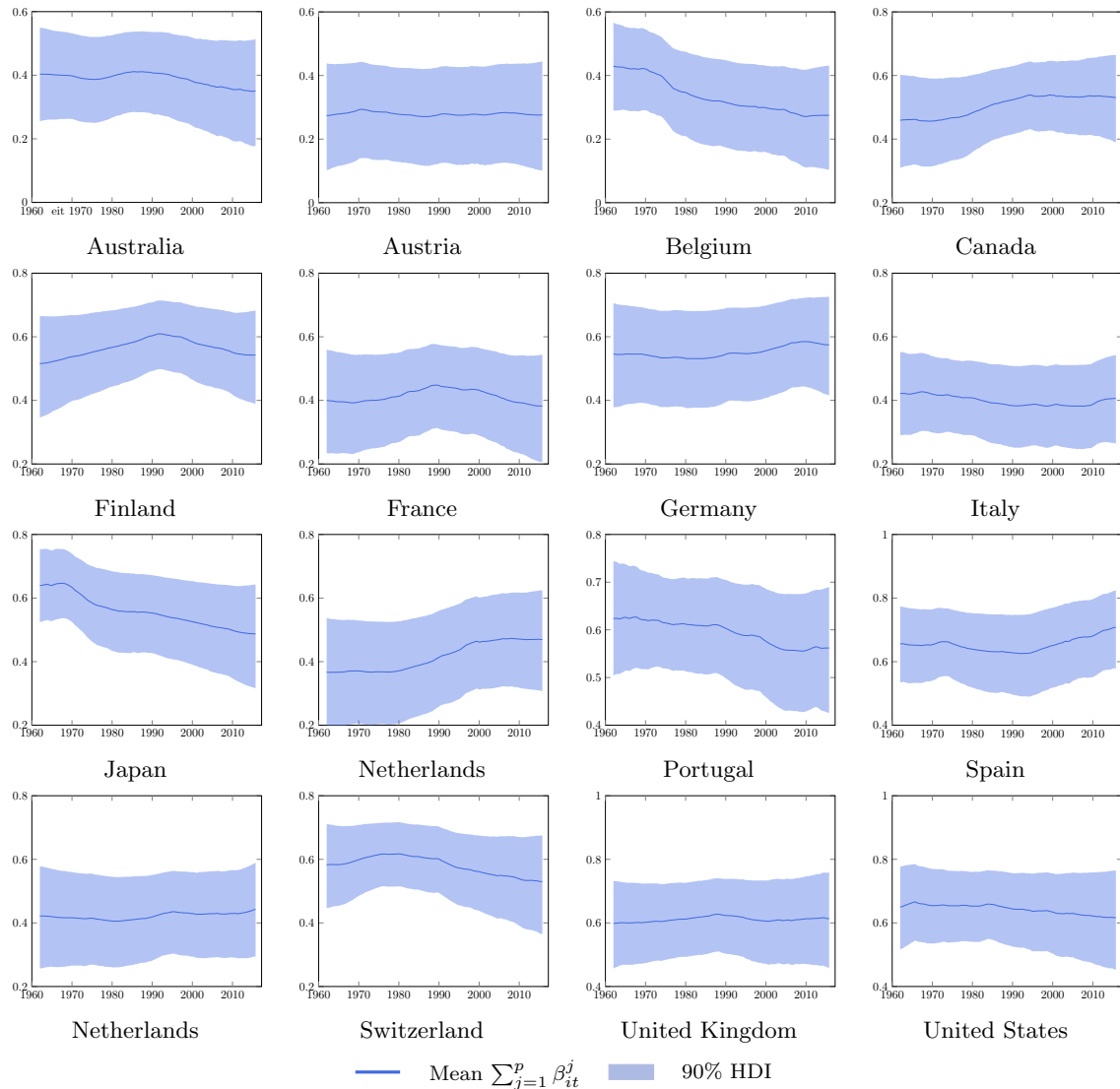


Third, in other countries like Germany, Japan and Portugal an increase in the idiosyncratic component has counteracted the decrease in the common component, rendering the total drop in volatility less pronounced. In Germany for instance, idiosyncratic volatility increased towards the reunion in 1990. In Japan, the idiosyncratic component increased steadily since the beginning of the 1990s, a period of low growth and stagnation commonly called ‘the lost decades’.

Propagation of shocks

To get an idea about the time-varying persistence in output growth, Figure 5 plots the sum of the AR coefficients. Based on the results of the stochastic model specification search, only β_{it}^1 varies over time while β_{it}^2 , β_{it}^3 and β_{it}^4 are fixed to be constant. Despite some moderate changes, the persistence is relatively stable in most countries, showing that changes in the propagation mechanism in Equation (1) are not the main source of the Great Moderation.

Figure 5: Sum of AR coefficients



3.4 Discussion

Although we do not explicitly link the evolution in the various time-varying components, parameters and sensitivities to underlying macroeconomic and other fundamentals, our results nevertheless provide some insights into the relevance of competing explanations for the common drop in volatility.

Absence of large common shocks: ‘good luck’

At least the ‘good luck’ hypothesis does not seem to be fully reconcilable with our findings. While Figure 2 reveals that the 1980s and the late 1990s/early 2000s are characterized by relatively small common shocks, suggesting that a less hostile international environment and hence ‘good luck’ has temporarily contributed to lower volatility in all countries, we identify the permanent

common drop in the volatility of domestic shocks plotted in Figure 3 as the main driver of the Great Moderation. It is hard to argue that ‘good luck’ is driving the volatility of domestic shocks to a permanently lower level in all countries at the same time.

Advances in fiscal and monetary regimes: ‘good policy’

A common and gradual adoption of ‘best practices’ in fiscal and monetary policy methods provide much more likely explanations. With respect to fiscal policy, output volatility may be reduced as a result of governments committing to fiscal rules rather than using discretionary interventions (Fatas and Mihov, 2003). However, while all countries in our sample have implemented various types of fiscal rules at certain points in time during our sample period (see Bova et al., 2015, for an overview), in most cases these became only active during the 1990s or even later, which is long after the global drop in output volatility has set in. Pertaining to monetary policy, a common moderating effect may stem from a general movement towards a more systematic response to shocks ultimately increasing credibility of monetary policy (Clarida et al., 2000). Note that the U.S., with the aggressive disinflation policy under Federal Reserve Chairman Paul Volcker in the early 1980s, adopting this ‘new’ monetary policy earlier, or more abrupt, compared to most of the other countries in our sample may explain why it has a much lower loading on the common volatility factor. This global component may then simply reflect the delayed, or more gradual, adoption of U.S. monetary policy by the other countries in our sample. The fact that the factor loading for Canada is also relatively low may be explained by the fact that the Bank of Canada followed the U.S. lead on raising interest rates more swiftly in an attempt to resist downward pressure on the exchange rate. Note that as the majority of countries in our sample are members of the EMU or euro area, the common factor may to some extent also be due to the gradual movement towards a common European monetary policy, eventually resulting in the installment of the ECB in 1998 and the introduction of the euro in 1999. However, this does not explain the high factor loadings of countries like Australia and Japan.

Improved inventory management

Structural improvements at the firm level related to better management of inventories have been discussed as a likely explanation for the Great Moderation from the beginning (Blanchard and Simon, 2001; Kahn et al., 2002). These allow firms in all countries to better smooth the impact of shocks, and may thus also be driving our common volatility factor. However, this cannot explain the low factor loadings in the U.S. and Canada. One could further argue that improved inventory management, as well as the policy changes discussed above, are developments that should be reflected in the propagation of shocks rather than their size. Our results indicate that smaller shocks were much more important than changes in the way the economy responds to these shocks. However, as pointed out by Bean (2009) “shocks are not measured directly, only their consequences are”. Sims (2012) draws the analogy with the use of fire extinguishers in response to a kitchen fire. If these are used quickly and effectively, the fire can easily be suppressed. The observed damage would, however, be very different if the extinguishers had not been used. Although the shock that caused the fire is the same in both cases, the observed consequences are very different. In this

view, structural changes and good policy that reduce the economy’s vulnerability to shocks may very well show up as smaller shocks in the data.

Shifts in the sectoral composition

Most advanced economies have displayed significant changes in their sectoral composition over our sample period, moving away from agriculture and manufacturing towards services. As the latter sector is known to be less volatile than the former two, an increase in the share of services is expected to dampen aggregate volatility. The empirical results in [Moro \(2012\)](#) and [Burren and Neusser \(2013\)](#) indeed suggest that the shift to services accounts for up to 30% of the decline in U.S. volatility, while [Carvalho and Gabaix \(2013\)](#) argue that the low-frequency decline in U.S. volatility observed from 1960 to 1990 can be accounted for almost entirely by the downfall of a handful of heavy-manufacturing sectors. This explanation is supported by our empirical findings to the extent that the start of our estimated common drop in the volatility of country-specific shocks around 1970 coincides with the start of the de-industrialization in most OECD economies. However, the magnitude of some of the factor loadings is not fully in line with what one would expect from the literature. The more rapid de-industrialization in countries such as the U.K. and the U.S. compared to Germany and Japan (as documented by e.g. [Nickell et al., 2008](#)) is not reflected in higher factor loadings on the common volatility factor for the former two countries. This suggests that also other explanations are driving our results. Evaluating the contribution of sectoral shifts to the Great Moderation is ideally done by extending and applying our model to sectoral data. We leave this for future research.

3.5 Robustness checks

In this section, we briefly discuss the outcome of a number of robustness tests. Full results are available from the authors on request.

Dropping countries with questionable quality of quarterly data

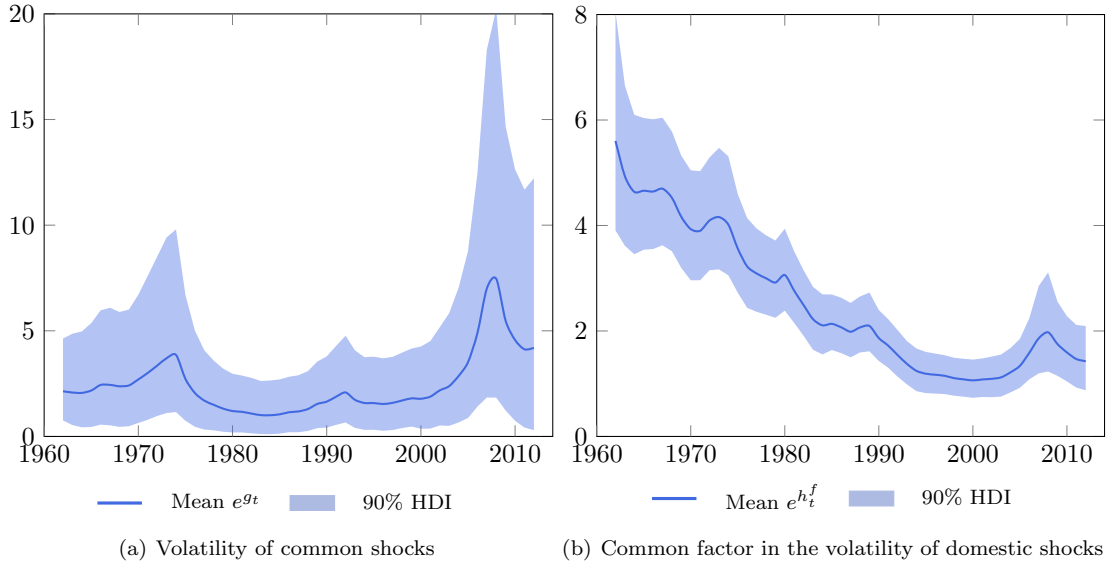
We start with discussing the robustness of our results with respect to data quality. It should be noted that some caution is needed when using longer series of quarterly real GDP. As early GDP data is only available on an annual basis for the majority of countries, the OECD uses interpolation methods to construct quarterly data. Depending on the length of the interpolated period and the complexity of the applied method, this results in artificial volatility patterns for some countries. Although the country-specific error terms ε_{it}^c in Equation (2) and volatility terms h_{it}^c in Equation (8) should be able to capture most of these artificial fluctuations, they may still distort the results of our empirical model. This is especially the case for the relative importance of common and domestic movements in volatility. While interpolation methods are used over periods of different length at the beginning of the sample for all countries except Australia, the U.K. and the U.S., the data looks most suspicious for Belgium and Portugal. Hence, we re-estimate our model dropping these countries from the sample. While the main results are qualitatively not affected, the correlation in the volatility across countries as well as the persistence of growth

rates as measured by the AR coefficients appear to be somewhat sensitive to the cross-sectional dimension of the sample. This is not surprising as excluding countries that, due to interpolation, have less volatile growth rates in the beginning of the sample leads to a strengthening of the cross-country correlation structure. In addition, breaks in the series when the interpolation period ends may affect the AR coefficients implying less evidence of changes in the propagation mechanism when the affected countries are excluded.

Annual data

As an additional robustness check we also use annual data. This greatly improves the quality of the data but also reduces the sample size significantly. Despite the resulting higher estimation uncertainty, Figure 6 demonstrates that the general patterns in the volatility of global shocks and in the common volatility factor still show up. Again no clear trend is visible in the volatility of common shocks, while the volatility of domestic shocks shows a clear common downward trend. For the sake of brevity we do not present results of the tests for time variation. However, for the two factors plotted in Figure 6 time variation was found to be relevant whereas this was not the case for the other model components.

Figure 6: Robustness of volatility estimates to annual data



Quarter-on-quarter growth rates

We also test the robustness of our results with respect to the way growth rates are calculated. While we use year-on-year rates in our baseline estimation, other papers rely on annualized quarter-on-quarter rates (e.g. Del Negro and Otrok, 2008; Berger et al., 2016b). Overall, results and interpretation remain qualitatively unchanged but we find that the high frequency noise present in quarter-on-quarter data tends to blur the correlation structure in the growth rates, i.e. the role

of common shocks and spillovers is found to be smaller. As a result, the correlation structure in the volatility of domestic shocks is even stronger in this case.

Alternative lag structure

Regarding the lag structure of the model we test a number of different specifications. Our baseline version with four lags of own GDP and one lag of foreign countries GDP follows [Stock and Watson \(2005\)](#). Generally, including less own lags or more lags of foreign GDP does not change the results significantly.

Heterogeneous standard deviations of innovations

Although the random walk components α_{it} , β_{it} , γ_{it} , ϕ_{it}^ε , ϕ_{it}^h and h_{it}^c are fully heterogeneous, for efficiency reasons our baseline model assumes that the corresponding innovation variances σ_α^2 , σ_β^2 , σ_γ^2 , $\sigma_{\phi^\varepsilon}^2$, $\sigma_{\phi^h}^2$ and $\sigma_{h^c}^2$ are homogeneous across countries.

Table 5: Posterior inclusion probabilities for the heterogeneous binary indicators in \mathcal{M}

	Intercept	AR coefficients				Spillovers	Loadings		SV
	δ_α^i	$\delta_{\beta^1}^i$	$\delta_{\beta^2}^i$	$\delta_{\beta^3}^i$	$\delta_{\beta^4}^i$	δ_γ^i	$\delta_{\phi^\varepsilon}^i$	$\delta_{\phi^h}^i$	$\delta_{h^c}^i$
Australia	0.67	0.43	0.18	0.32	0.28	0.78	0.37	0.50	0.90
Austria	0.84	0.14	0.45	0.36	0.44	0.29	0.33	0.59	0.89
Belgium	0.56	0.90	0.38	0.22	0.10	0.15	0.58	0.52	0.97
Canada	0.45	0.08	0.08	0.10	0.10	0.11	0.27	0.50	0.91
Finland	0.73	0.31	0.30	0.15	0.10	0.25	0.61	0.51	0.77
France	0.32	0.33	0.15	0.17	0.15	0.50	0.45	0.64	0.80
Germany	0.30	0.12	0.14	0.11	0.15	0.20	0.39	0.51	0.36
Italy	0.99	0.21	0.20	0.19	0.12	0.30	0.47	0.48	0.88
Japan	1.00	0.14	1.00	0.25	0.12	0.73	0.59	0.47	0.18
Netherlands	0.97	0.15	0.11	0.13	0.21	0.13	0.83	0.47	0.97
Portugal	0.99	0.56	0.26	0.46	0.18	0.37	0.43	0.48	0.96
Spain	1.00	0.16	0.11	0.12	0.11	0.22	0.36	0.45	0.98
Sweden	0.59	0.18	0.14	0.12	0.26	0.47	0.36	0.47	0.74
Switzerland	0.82	0.45	0.23	0.16	0.17	0.23	0.35	0.45	0.95
U.K.	0.12	0.06	0.07	0.08	0.08	0.06	0.48	0.48	0.94
U.S.	0.34	0.12	0.07	0.07	0.08	0.09	0.23	0.43	0.96

Notes: The prior distribution for each of the elements in σ^i is $\mathcal{N}(0, V_0)$ with V_0 the variance in the baseline scenario as reported in [Table 1](#). The prior inclusion probability p_0 of the binary indicators equals the baseline value of 0.5. The posterior inclusion probabilities are calculated as the average selection frequencies over all iterations of the MCMC.

This further implies that the stochastic model specification search outlined in [Section 2.3](#) tests for panel-wide time variation in the parameters rather than in each country separately, i.e. the binary indicators δ_α , δ_β , δ_γ , $\delta_{\phi^\varepsilon}$, δ_{ϕ^h} and δ_{h^c} are fixed to be the same for all countries in the

panel. As this homogeneity assumption may be violated in practice, we have re-estimated the model allowing the innovation variances and binary indicators to differ across countries. The results are found to be robust to this heterogeneous specification. First, the evidence for time variation in the common components g_t and h_t^f is not significantly affected. While the posterior inclusion probability of the stochastic volatility component g_t in the common shocks remains at 1, the probability for the common volatility factor h_t^f drops slightly to 0.84.

Second, the posterior inclusion probabilities for the heterogeneous binary indicators reported in Table 5 still show strong support for time variation in the idiosyncratic volatility component h_{it}^c and in the long-run mean growth rate α_{it} . Evidence for a changing propagation mechanism as measured by the AR coefficients β_{it} remains very weak. Moreover, inclusions probabilities for time-varying sensitivities to spillovers γ_{it} and common growth shocks ϕ_{it}^s remain below 0.5 in most countries. While the posterior indicator mean for the loadings on the common volatility factor ϕ_{it}^h has been around 0.2 in the homogeneous case, most countries now show values of around 0.5. This signals a relatively limited amount of information in the data with respect to possible time variation in these loadings such that the posterior inclusion probabilities tend towards the prior probability of 0.5.

The role of changing trade patterns

As we use time-varying trade weights, our spillover measure Δy_{it}^* in Equation (1) may be partly driven by changing trade patterns. However, when inspecting the evolution of trade weights, for most countries there is no clear trend visible. Only some country pairs show increasing/decreasing trade shares, e.g. the share of the U.K. in total gross trade of Australia decreased from around 40% to 6% over the sample period while over the same period Japan’s share in total trade of Australia increased from 15% to 42%. Moreover, the trade weights do not exhibit significant structural breaks at the time of the ‘Great Trade Collapse’ in late 2008, indicating that this was a synchronized drop in trade across countries. Nevertheless, we check the robustness of our results by re-estimating the model using fixed trade weights (at both their mean values as well as the first values in the sample). None of the results were significantly affected, though.

4 Variance decomposition

In this section we use a variance decomposition to illustrate the relative importance of the various model components for explaining the overall evolution in output growth volatility. The variance decomposition approach most commonly used in the dynamic factor model literature (see e.g. Kose et al., 2003; Del Negro and Otrok, 2008) is based on what is generally known as model-implied variance. This means that for each point in time t , each country i , and each iteration of the MCMC the model estimates are used to calculate implied (long-run equilibrium) variances. Since common and idiosyncratic components are independent by assumption, total variance is additive and the variance shares of interest can be straightforwardly calculated. However, through the inclusion of lagged foreign GDP growth in Equation (1), the spillover channel in our empirical specification, implies that the different model components are not mutually independent anymore.

This requires a slight adjustment of the procedure, as outlined below.

4.1 A simulation-based approach

To calculate the contribution of common shocks, spillovers and domestic shocks to the overall variance of output growth, first rewrite the model in Equations (1)-(2) as follows:

$$\Delta y_{it} - \Delta y_{it}^0 = \Delta y_{it}^1 + \Delta y_{it}^2 + \Delta y_{it}^3, \quad (20)$$

$$\text{with } \Delta y_{it}^0 = \sum_{j=1}^p \beta_{it}^j \Delta y_{i,t-j}^0 + \alpha_{it}, \quad (21)$$

$$\Delta y_{it}^1 = \sum_{j=1}^p \beta_{it}^j \Delta y_{i,t-j}^1 + \sum_{k=1}^q \gamma_{it}^k \Delta y_{i,t-k}^*, \quad (22)$$

$$\Delta y_{it}^2 = \sum_{j=1}^p \beta_{it}^j \Delta y_{i,t-j}^2 + \phi_{it}^{\varepsilon} \varepsilon_t^f, \quad (23)$$

$$\Delta y_{it}^3 = \sum_{j=1}^p \beta_{it}^j \Delta y_{i,t-j}^3 + \varepsilon_{it}^c. \quad (24)$$

Conditional on the model estimates we can use Equations (21)-(24) to calculate Δy_{it}^0 and generate samples for Δy_{it}^1 , Δy_{it}^2 and Δy_{it}^3 by (i) drawing ε_t^f and ε_{it}^c from their distributions in Equation (7) and (ii) calculating the spillover terms $\Delta y_{i,t-k}^*$ using lagged simulated growth rates.³ Doing this in each draw of the MCMC, we obtain $J - B$ simulated samples of Δy_{it} and its constituent components Δy_{it}^0 , Δy_{it}^1 , Δy_{it}^2 and Δy_{it}^3 .

Next, for each component, each country and each point in time we compute the sample variance over the $J - B$ draws. Note that by construction, our spillover component Δy_{it}^1 is not independent of the common and domestic shock components Δy_{it}^2 and Δy_{it}^3 . This is because both a global shock f_{t-1} and a domestic shock $\varepsilon_{i,t-1}^c$ will still be present in the components Δy_{it}^2 and Δy_{it}^3 but at the same time feed into the spillover component Δy_{it}^1 through their impact on trade-weighted growth $\Delta y_{i,t-1}^*$. Although the resulting covariance terms $Cov(\Delta y_{it}^1, \Delta y_{it}^2)$ and $Cov(\Delta y_{it}^1, \Delta y_{it}^3)$ are small, we assign them to the spillover component to make sure that the components' variances sum up to the total variance. Note that the covariance between the common and domestic shock components is zero by assumption. Hence, our variance decomposition is given by

$$\underbrace{Var(\Delta y_{it} - \Delta y_{it}^0)}_{\text{Total}} = \underbrace{Var(\Delta y_{it}^1) + 2Cov(\Delta y_{it}^1, \Delta y_{it}^2) + 2Cov(\Delta y_{it}^1, \Delta y_{it}^3)}_{\text{Spillovers}} + \underbrace{Var(\Delta y_{it}^2)}_{\text{Common shocks}} + \underbrace{Var(\Delta y_{it}^3)}_{\text{Country-specific shocks}}. \quad (25)$$

Our simulation-based approach offers several advantages. First, it allows to separate the contribution of spillovers from that of global and domestic shocks. Second, the dynamics of the model are fully taken into account. This is not the case when calculating model-implied variances as these

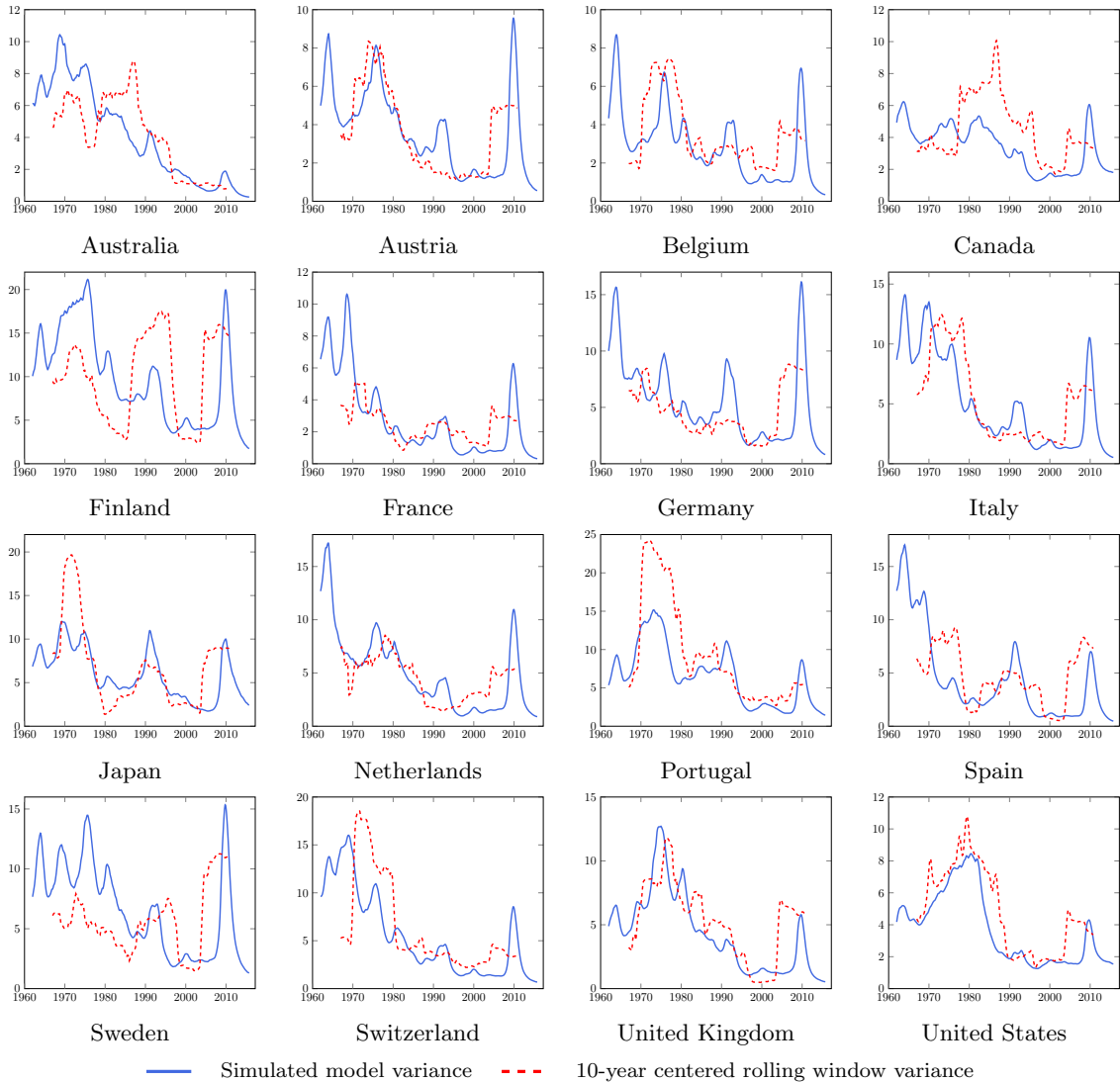
³Each of the components is initialized at zero with a burn-in period of 10 quarters. The parameter values used over the burn-in period are set equal to their mean values over the first 5 years of the sample.

are typically long-run equilibrium measures ignoring short-run dynamics. Third, by simulating the model in every draw of the MCMC, parameter uncertainty is explicitly taken into account.

4.2 Simulated model-based versus rolling window volatility

To assess the adequacy of our model and simulation approach, Figure 7 plots the simulated total variance of GDP growth along with the commonly used 10-year centered rolling window variance. With a decreasing trend in all countries, starting either at the beginning of the sample or somewhere in the 1970s or 1980s, the Great Moderation clearly shows up in the two alternative volatility measures.

Figure 7: Simulated model-based versus rolling window variance



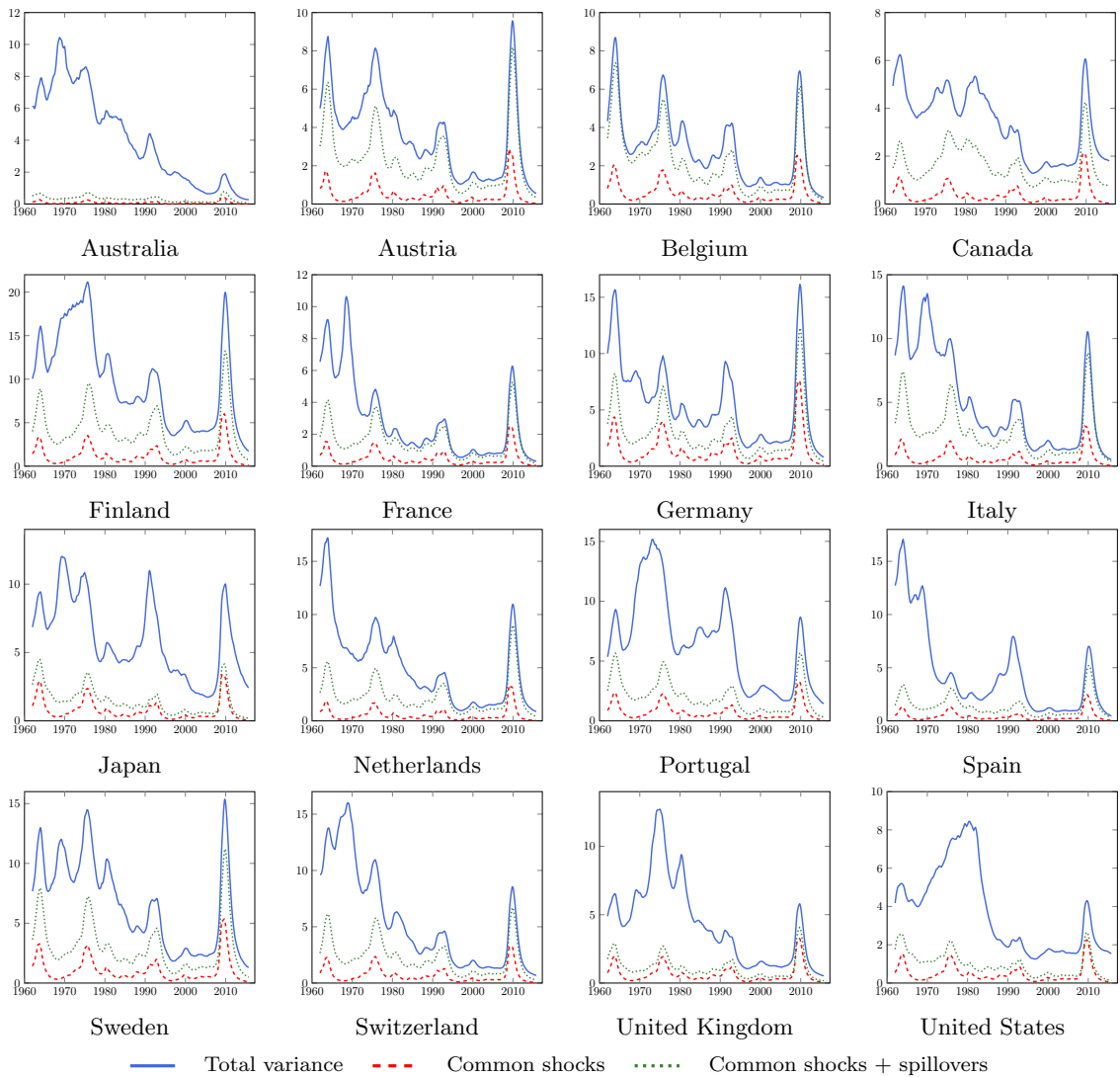
However, our model-based approach seems to be much more accurate in timing the changes. Although the centered rolling window measure is able to pick up the timing of the long-run decline in volatility in most countries, by partly relying on future realized volatility more sudden events

like the Great Recession are predated by a number of years. Using an entirely backward-looking window as an alternative will tend to postdate most events. Also note that the rolling window variance remains high(er) at the end of the sample for most countries and hence is not yet able to show that the Great Recession induced only a temporary volatility increase.

4.3 Results variance decomposition

Figure 8 decomposes total volatility into the contributions of global shocks, spillovers and domestic shocks.

Figure 8: Simulation-based variance decomposition



The plots reveal large differences across countries and time with respect to the importance of these three components. First, driven by considerable cross-sectional variation in the sensitivities reported in Table 3, the contribution of common shocks and spillovers differs widely across countries. Output volatility in small open economies like Austria and Belgium is almost entirely

driven by these international components, leaving only a minor role for domestic shocks. Figure 8 reveals that in these countries the decline in total volatility mainly spills over from the less volatile output growth of their trading partners. To a lesser extent, a similar pattern emerges in Canada and even in larger economies like France, Germany and Italy. At the other end of the spectrum, output volatility in Australia is almost entirely driven by domestic shocks.

Table 6: Average variance shares of the different components over two subsamples (in %)

	Global shocks		Spillovers		Domestic shocks	
	1962-1983	1984-2015	1962-1983	1984-2015	1962-1983	1984-2015
Australia	1.39	4.70	4.87	11.48	93.75	83.82
Austria	9.93	15.41	44.37	57.10	45.70	27.49
Belgium	15.82	20.47	60.13	45.20	24.06	34.33
Canada	8.40	12.35	35.98	43.28	55.62	44.37
Finland	8.94	13.50	28.54	39.23	62.52	47.27
France	14.55	24.95	40.53	58.60	44.91	16.45
Germany	20.13	25.77	32.37	35.77	47.50	38.46
Italy	9.56	17.85	35.68	53.60	54.76	28.55
Japan	12.65	11.81	16.87	10.21	70.47	77.98
Netherlands	7.23	15.94	27.59	56.51	65.17	27.56
Portugal	10.42	13.53	25.06	22.75	64.52	63.71
Spain	10.77	15.85	29.77	41.67	59.47	42.48
Sweden	11.74	19.10	26.43	36.53	61.83	44.37
Switzerland	9.67	19.31	27.51	44.09	62.82	36.60
U.K.	9.99	22.71	10.89	17.75	79.11	59.53
U.S.	9.55	17.40	15.12	14.05	75.32	68.55

Also in Japan, Spain, Switzerland, the U.K. and the U.S. international shocks are relatively less important compared to the other countries in the sample. Second, for most countries, a decline in the volatility of domestic shocks is an important source of the overall volatility decline. This is most prominently the case for Australia, Finland, France, Germany, Italy, Netherlands, Spain, Sweden, Switzerland, the U.K. and the U.S. Third, because of the decline in the volatility of domestic shocks in most countries, international shocks and spillovers contribute more to overall output volatility towards the end of the sample. This can also be observed from Table 6 where we report the average variance shares for the 1962 - 1983 and 1984 - 2015 subsamples.

5 Concluding remarks

This paper has investigated the sources of output volatility within a time-varying factor-augmented dynamic panel model with stochastic volatility for a panel of 16 OECD countries over the period 1961:Q1 - 2015:Q4. Our empirical specification allows output growth in a particular country to be driven by global shocks, spillovers and domestic shocks. Changes in the volatility of output

growth can stem from a time-varying sensitivity to each of these shocks, changes in the propagation mechanism or shifts in the variances of shocks. As a novel model component we allow for a common factor in the volatility of domestic shocks. We start with a Bayesian stochastic model specification search to determine for which of the model's components the time variation is actually relevant. The results clearly indicate that both the volatility of global and domestic shocks vary over time. There is some evidence of time variation in the propagation mechanism, while the sensitivities to spillovers and global shocks are found to be constant. Next, we estimate the parsimonious model specification, restricting parameters for which no relevant time variation was found to be constant over the sample period. The results show that although the volatility of global shocks varies over time it does not exhibit a clear downward trend. It mainly reflects periods of worldwide turmoil, temporarily shooting up around the oil crises of the 1970s, the worldwide recession of the early 1990s and the recent Great Recession. Hence, the latter does not mark the end of the Great Moderation. As individual countries' sensitivities to the common shocks and spillovers have also remained stable over the sample period, changes in the volatility of the international business cycle component is not what is driving the Great Moderation. In contrast, the volatilities of domestic shocks show a clear common downward trend. We identify this as the main driver of the widespread reduction in volatility.

The focus of this paper has been a decomposition of output and in particular its volatility into domestic versus international components. Obviously, a better understanding of the underlying drivers of output volatility further requires linking these components to macroeconomic and other fundamentals. While our paper does not elaborate on that, it is an essential first step towards a better understanding of output volatility as it signals that there is an important common factor in the volatility of domestic shocks. When this unobserved common volatility factor is correlated with the alleged country-specific determinants, ignoring it will lead to inconsistent estimates. Future research on output volatility and its determinants will therefore have to account for cross-sectional dependence in the variance equation.

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Appendix A General outline of the Gibbs sampler

Our MCMC scheme to jointly sample the binary indicators $\mathcal{M} = (\delta_\alpha, \delta_\beta, \delta_\gamma, \delta_{\phi^\varepsilon}, \delta_{\phi^h}, \delta_g, \delta_{hf}, \delta_{hc})$, the time-invariant parameters $\mathcal{P} = (\lambda_0, \phi_0, \zeta_0, \sigma)$ and the time-varying unobserved state variables $\mathcal{S} = (\tilde{\lambda}, \tilde{\phi}, \tilde{\zeta}, \varepsilon^f)$ is as follows:

1. Sample the binary indicators in \mathcal{M} together with the constant parameters \mathcal{P} conditional on the time-varying states \mathcal{S} . Restricted elements in σ , i.e. for which the corresponding binary indicator in \mathcal{M} is zero, are set to zero.
2. Sample the time-varying states \mathcal{S} conditional on the binary indicators \mathcal{M} and the time-invariant parameters \mathcal{P} . States in \mathcal{S} which are not selected by the stochastic specification search, i.e. for which the corresponding binary indicator in \mathcal{M} is zero, are sampled from their prior random walk distribution.
3. Perform a random sign switch for \mathcal{S} and the corresponding standard deviations in σ . This sign switch is suggested by [Fruehwirth-Schnatter and Wagner \(2010\)](#) to amplify the sign indeterminacy of σ and, hence, its potential bimodality.

Starting from an arbitrary set of initial values, sampling from these blocks is iterated J times and after a sufficiently long burn-in period B , the sequence of draws $(B + 1, \dots, J)$ can be taken as a sample from the joint posterior distribution of interest $f(\mathcal{M}, \mathcal{P}, \mathcal{S}|x)$. The results reported in the paper are based on 50,000 iterations with 10,000 draws being discarded as burn-in.

Appendix B Detailed Gibbs sampling algorithm

In this section we provide details on the MCMC building blocks.

Block 1: Sampling the binary indicators in \mathcal{M} and the parameters in \mathcal{P}

For notational convenience, let us define a general regression model

$$w = z^{\mathcal{M}} b^{\mathcal{M}} + e, \quad e \sim \mathcal{N}(0, \Sigma), \quad (\text{A-1})$$

where w is a $NT \times 1$ vector including observations on a dependent variable w_{it} stacked over time and cross-sections and z an unrestricted predictor matrix. The corresponding unrestricted parameter vector is denoted b . $z^{\mathcal{M}}$ and $b^{\mathcal{M}}$ are then the restricted predictor matrix and the restricted parameter vector that exclude those elements in z and b for which the corresponding indicator in \mathcal{M} is 0. Furthermore, Σ is a diagonal matrix with elements σ_{it}^2 that may vary over both cross-sections and time to allow for heteroskedasticity of a known form.

As in [Fruehwirth-Schnatter and Wagner \(2010\)](#) we first marginalize over the parameters in b when sampling \mathcal{M} and next draw b conditional on the sampled indicators \mathcal{M} . The posterior distribution of \mathcal{M} can be obtained using Bayes' rule as

$$f(\mathcal{M}|\mathcal{S}, w) \propto f(w|\mathcal{M}, \mathcal{S})p(\mathcal{M}), \quad (\text{A-2})$$

with $p(\mathcal{M})$ being the prior probability of \mathcal{M} and $f(w|\mathcal{M}, \mathcal{S})$ being the marginal likelihood of the regression model (A-1) where the effect of the parameters $b^{\mathcal{M}}$ has been integrated out. Under the normal conjugate prior $b^{\mathcal{M}} \sim N(a_0^{\mathcal{M}}, A_0^{\mathcal{M}})$, the closed form solution for the marginal likelihood $f(w|\mathcal{M}, \mathcal{S})$ is

$$f(w|\mathcal{M}, \mathcal{S}) \propto \frac{|\Sigma|^{-0.5} |A_T^{\mathcal{M}}|^{0.5}}{|A_0^{\mathcal{M}}|^{0.5}} \exp \left(-\frac{1}{2} (w' \Sigma^{-1} w + (a_0^{\mathcal{M}})' (A_0^{\mathcal{M}})^{-1} a_0^{\mathcal{M}} - (a_T^{\mathcal{M}})' (A_T^{\mathcal{M}})^{-1} a_T^{\mathcal{M}}) \right), \quad (\text{A-3})$$

with

$$a_T^{\mathcal{M}} = A_T^{\mathcal{M}} \left((z^{\mathcal{M}})' \Sigma^{-1} w + (A_0^{\mathcal{M}})^{-1} a_0^{\mathcal{M}} \right), \quad (\text{A-4})$$

$$A_T^{\mathcal{M}} = \left((z^{\mathcal{M}})' \Sigma^{-1} z^{\mathcal{M}} + (A_0^{\mathcal{M}})^{-1} \right)^{-1}. \quad (\text{A-5})$$

Following [George and McCulloch \(1993\)](#), instead of using a multi-move sampler in which all elements in \mathcal{M} are sampled simultaneously, we use a single-move sampler in which each of the binary indicators δ_r (for $r = \alpha, \beta, \gamma, \phi^\varepsilon, \phi^h, g, h^f, h^e$) in \mathcal{M} is sampled from $f(\delta_r|\mathcal{M}_{/\delta_r}, \mathcal{S}, w)$. Given these general definitions, Block 1 of the MCMC algorithm splits up as follows:

Block 1(a): Sampling the binary indicators $\delta_\alpha, \delta_\beta, \delta_\gamma$ and parameters $\lambda_0, \sigma_\alpha, \sigma_\beta, \sigma_\gamma$

In this block we first sample the binary indicators $\delta_\alpha, \delta_\beta$ and δ_γ , marginalizing over the parameters for which variable selection is performed while conditioning on the time-varying states in \mathcal{S} . Using the parsimonious non-centered specification introduced in Equation (19), the model in (1) can be written in the linear regression format (A-1), whereas an observation at point t , w_t , is defined as

$$\underbrace{\Delta y_t - \phi_t^\varepsilon \varepsilon_t^f}_{w_t} = \underbrace{\left[I_N \quad \text{diag}(\Delta y_{t-1}) \quad \text{diag}(\Delta y_{t-1}^*) \quad \delta_\alpha \tilde{\alpha}_t \quad \delta_\beta \tilde{\beta}_t \Delta y_{t-1} \quad \delta_\gamma \tilde{\gamma}_t \Delta y_{t-1}^* \right]}_{z_t^{\mathcal{M}}} \underbrace{\begin{bmatrix} \alpha_0 \\ \beta_0 \\ \gamma_0 \\ \sigma_\alpha \\ \sigma_\beta \\ \sigma_\gamma \end{bmatrix}}_{b^{\mathcal{M}}} + \varepsilon_t^c,$$

where I_N denotes the identity matrix with dimension N and the restricted vectors $z_t^{\mathcal{M}}$ and $b^{\mathcal{M}}$ exclude those elements for which the corresponding binary indicator is zero. For the sake of notational convenience we include only one lag of Δy_{it} and Δy_{it}^* , i.e. we set $p = q = 1$, but the algorithm can be straightforwardly extended to allow for higher order dynamics. Using Equations (7) and (8), the covariance matrix Σ is constructed as a diagonal matrix with elements e^{hit} . The marginal likelihood $f(w|\delta_\alpha, \delta_\beta, \delta_\gamma, \mathcal{S})$ can then be calculated as in Equation (A-3) such that using the posterior distribution of \mathcal{M} defined in Equation (A-2) the binary indicators $\delta_\alpha, \delta_\beta$ and δ_γ can

be sampled one at a time from the Bernoulli distribution with probability

$$p(\delta_r = 1|\delta_{/r}, \mathcal{S}, w) = \frac{f(\delta_r = 1|\delta_{/r}, \mathcal{S}, w)}{f(\delta_r = 0|\delta_{/r}, \mathcal{S}, w) + f(\delta_r = 1|\delta_{/r}, \mathcal{S}, w)},$$

for $r = \alpha, \beta, \gamma$.

The time-invariant parameters $\lambda_0 = (\alpha_0, \beta_0, \gamma_0)$ and the unrestricted (i.e. for which the corresponding binary indicator is one) standard deviations $\sigma_\alpha, \sigma_\beta, \sigma_\gamma$ can next be sampled from their posterior distribution $N \sim (a_T^M, A_T^M)$ with a_T^M and A_T^M given by (A-4) and (A-5). The restricted (i.e. for which the corresponding binary indicator is zero) standard deviations are set to zero.

Block 1(b): Sampling the binary indicator $\delta_{\phi^\varepsilon}$ and parameters ϕ_0^ε and $\sigma_{\phi^\varepsilon}$

Using the parsimonious non-centered specification for ϕ_{it}^ε , Equations (1)-(2) can be written in the general linear regression format (A-1) as

$$\underbrace{\Delta y_t - \alpha_t - \beta_t \odot \Delta y_{t-1} - \gamma_t \odot \Delta y_{t-1}^*}_{w_t} = \underbrace{\begin{bmatrix} \varepsilon_t^f I_N & \delta_{\phi^\varepsilon} \varepsilon_t^f \tilde{\phi}_t^\varepsilon \end{bmatrix}}_{z_t^M} \underbrace{\begin{bmatrix} \phi_0^\varepsilon \\ \sigma_{\phi^\varepsilon} \end{bmatrix}}_{b^M} + \varepsilon_t^c,$$

where \odot is the element-wise (Hadamard) product of two vectors. Using Equations (7) and (8), the covariance matrix Σ is again a diagonal matrix with elements e^{hit} . As in Block 1(a), the binary indicator $\delta_{\phi^\varepsilon}$ is first sampled from the Bernoulli distribution and next the time-invariant parameters ϕ_0^ε and the unrestricted standard deviation $\sigma_{\phi^\varepsilon}$ are sampled from their posterior distribution. When $\delta_{\phi^\varepsilon} = 0$, we set $\sigma_{\phi^\varepsilon} = 0$.

Block 1(c): Sampling the binary indicator δ_g and the parameter σ_g

Conditional on ε_t^f , the stochastic volatility component g_t enters the model in a non-linear way:

$$\varepsilon_t^f = e^{g_t/2} \tilde{\varepsilon}_t^f, \quad \tilde{\varepsilon}_t^f \sim \mathcal{N}(0, 1).$$

Following Kim et al. (1998) this expression can be linearized by taking the natural-log of the squares

$$\ln((\varepsilon_t^f)^2 + c) = g_t + \tilde{\varepsilon}_t^f, \quad (\text{A-6})$$

where $c = 0.001$ is an offset constant and $\tilde{\varepsilon}_t^f = \ln(\tilde{\varepsilon}_t^f)^2$. The latter follows a log-chi-square distribution that can be approximated by a mixture of M normal distributions as follows

$$f(\tilde{\varepsilon}_t^f) = \sum_{j=1}^M q_j f_{\mathcal{N}}(\tilde{\varepsilon}_t^f | m_j - 1.2704, v_j^2),$$

where q_j is the component probability of a specific normal distribution with mean $m_j - 1.2704$ and variance v_j^2 . This mixture can equivalently be expressed as

$$\tilde{\epsilon}_t^f | (l_t^g = j) \sim \mathcal{N}(m_j - 1.2704, v_j^2), \quad \text{with} \quad \Pr(l_t^g = j) = q_j.$$

with l_t^g a mixture indicator that can be sampled from

$$p(l_t^g = j | g_t, \tilde{\epsilon}_t^f) \propto q_j f_{\mathcal{N}}(\tilde{\epsilon}_t^f | g_t + m_j - 1.2704, v_j^2),$$

with the values for q_j , m_j , and v_j^2 for $M = 10$ taken from Table 1 in [Omori et al. \(2007\)](#).

Using a non-centered parsimonious specification for g_t , Equation (A-6) can be written in the general regression format (A-1) as

$$\underbrace{\left(\ln \left((\epsilon_t^f)^2 + 0.001 \right) - (m_{l_t^g} - 1.2704) \right)}_{w_t} = \underbrace{\begin{bmatrix} 1 & \delta_g \tilde{g}_t \end{bmatrix}}_{z^{\mathcal{M}}} \underbrace{\begin{bmatrix} g_0 \\ \sigma_g \end{bmatrix}}_{b^{\mathcal{M}}} + \tilde{\epsilon}_t^f,$$

with the covariance matrix Σ of $\tilde{\epsilon}^f$ a diagonal matrix with elements $v_{l_t^g}^2$. Similar to the approach in Block 1(a), this representation can now be used to first draw the binary indicator δ_g from a Bernoulli distribution and next sample the time-invariant parameter g_0 and the (unrestricted) shock standard deviation σ_g from their posterior distribution. When $\delta_g = 0$, we set $\sigma_g = 0$.

Block 1(d): Sampling the binary indicators δ_{ϕ^h} , δ_{h^c} and parameters ϕ_0^h , h_0^c , σ_{ϕ^h} , σ_{h^c}

Similar to the approach in Block 1(c) we start by linearizing the error term ϵ_{it}^c with respect to the stochastic volatility component h_{it}

$$\ln((\epsilon_{it}^c)^2 + c) = h_{it} + \tilde{\epsilon}_{it}^c = \phi_{it}^h h_t^f + h_{it}^c + \tilde{\epsilon}_{it}^c, \quad (\text{A-7})$$

where $\epsilon_{it}^c = \Delta y_{it} - \alpha_{it} - \beta_{it} y_{i,t-1} - \gamma_{it} y_{i,t-1}^* - \phi_{it}^{\epsilon} \epsilon_{it}^f$ and $\tilde{\epsilon}_{it}^c$ again follows a log-chi-square distribution that can be approximated using a mixture of normals with mixture indicators l_{it}^h sampled as outlined in Block 1(c). Using a non-centered parsimonious specification for both ϕ_{it}^h and h_{it}^c , Equation (A-7) can be written in the general regression format (A-1) as

$$\underbrace{\left(\ln \left((\epsilon_{it}^c)^2 + 0.001 \right) - (m_{l_{it}^h} - 1.2704) \right)}_{w_t} = \underbrace{\begin{bmatrix} h_t^f I_N & I_N & \delta_{\phi^h} \tilde{\phi}_t^h h_t^f & \delta_{h^c} \tilde{h}_t^c \end{bmatrix}}_{z_t^{\mathcal{M}}} \underbrace{\begin{bmatrix} \phi_0^h \\ h_0^c \\ \sigma_{\phi^h} \\ \sigma_{h^c} \end{bmatrix}}_{b^{\mathcal{M}}} + \tilde{\epsilon}_t^c,$$

with the covariance matrix Σ of $\tilde{\epsilon}^c$ being a diagonal matrix with elements $v_{l_{it}^h}^2$. Using this equation, the binary indicators δ_{ϕ^h} and δ_{h^c} are first drawn from a Bernoulli distribution as described above and the time-invariant parameters ϕ_0^h and h_0^c as well as the (unrestricted) standard deviations σ_{ϕ^h} and σ_{h^c} are sampled from their posterior distribution. Restricted standard deviations are again

set to zero.

Block 1(e): Sampling the binary indicator δ_{h^f} and parameters h_0^f, σ_{h^f}

Now using a non-centered parsimonious specification for h_t^f , Equation (A-7) can be written in the general regression format (A-1) as

$$\underbrace{\left(\ln((\varepsilon_t^c)^2 + 0.001) - (m_{l_t^h} - 1.2704) - h_t^c \right)}_{w_t} = \underbrace{\begin{bmatrix} \phi_t^h & \delta_{h^f} \phi_t^h \tilde{h}_t^f \end{bmatrix}}_{z_t^{\mathcal{M}}} \underbrace{\begin{bmatrix} h_0^f \\ \sigma_{h^f} \end{bmatrix}}_{b^{\mathcal{M}}} + \tilde{\varepsilon}_t^c.$$

The binary indicator δ_{h^f} is drawn from a Bernoulli distribution as described above whereas the time-invariant parameter h_0^f and the (unrestricted) standard deviation σ_{h^f} are sampled from their posterior distribution. When $\delta_{h^f} = 0$, we set $\sigma_{h^f} = 0$.

Block 2: Sampling the time-varying states \mathcal{S}

In this block we use the forward-filtering and backward-sampling approach of Carter and Kohn (1994) to sample the time-varying states in \mathcal{S} . To this end, we first specify a general state space model of the following form as given in Durbin and Koopman (2012)

$$w_t = Z_t \kappa_t + e_t, \quad e_t \sim \mathcal{N}(0, H_t), \quad (\text{A-8})$$

$$\kappa_{t+1} = T_t \kappa_t + R_t \eta_t, \quad \eta_t \sim \mathcal{N}(0, Q_t), \quad (\text{A-9})$$

where w_t is an $N \times 1$ vector of observations (stacked over cross-sections) and κ_t an unobserved state vector. The matrices Z_t, T_t, H_t, Q_t are assumed to be known (conditioned upon). The error terms e_t and η_t are assumed to be serially uncorrelated and independent of each other at all points in time. As Equations (A-8) and (A-9) constitute a linear Gaussian state space model, the unknown state variables κ_t can be filtered using the standard Kalman filter. Sampling $\kappa = [\kappa_1, \dots, \kappa_T]$ can then be done using the algorithm outlined in Carter and Kohn (1994). Given this general state space model, Block 2 splits up as follows:

Block 2(a): Sampling the time-varying states $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ and the common shocks ε^f

The time varying parameters $\tilde{\alpha}_{it}$, $\tilde{\beta}_{it}$, $\tilde{\gamma}_{it}$ and the standardized common shocks $\tilde{\varepsilon}_t^f$ can be sampled from the general state space model in Equations (A-8)-(A-9) upon defining

$$\underbrace{(\Delta y_t - \alpha_0 - \beta_0 \odot \Delta y_{t-1} - \gamma_0 \odot \Delta y_{t-1}^*)}_{w_t} = \underbrace{\begin{bmatrix} \sigma_\alpha I_N & \sigma_\beta \text{diag}(y_{t-1}) & \sigma_\gamma \text{diag}(y_{t-1}^*) & \phi_t^\varepsilon e^{g_t/2} \end{bmatrix}}_{Z_t} \underbrace{\begin{bmatrix} \tilde{\alpha}_t \\ \tilde{\beta}_t \\ \tilde{\gamma}_t \\ \tilde{\varepsilon}_t^f \end{bmatrix}}_{\kappa_t} + \underbrace{\varepsilon_t^c}_{e_t},$$

$$\underbrace{\begin{bmatrix} \tilde{\alpha}_{t+1} \\ \tilde{\beta}_{t+1} \\ \tilde{\gamma}_{t+1} \\ \tilde{\varepsilon}_{t+1}^f \end{bmatrix}}_{\kappa_{t+1}} = \underbrace{\begin{bmatrix} I_N & 0 & 0 & 0 \\ 0 & I_N & 0 & 0 \\ 0 & 0 & I_N & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{T_t} \underbrace{\begin{bmatrix} \tilde{\alpha}_t \\ \tilde{\beta}_t \\ \tilde{\gamma}_t \\ \tilde{\varepsilon}_t^f \end{bmatrix}}_{\kappa_t} + \underbrace{\begin{bmatrix} I_N & 0 & 0 & 0 \\ 0 & I_N & 0 & 0 \\ 0 & 0 & I_N & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{R_t} \underbrace{\begin{bmatrix} \eta_t^{\tilde{\alpha}} \\ \eta_t^{\tilde{\beta}} \\ \eta_t^{\tilde{\gamma}} \\ \eta_t^{\tilde{\varepsilon}^f} \end{bmatrix}}_{\eta_t},$$

with $H_t = I_N e^{(\phi_t^h h_t^f + h_t^c)}$ and $Q_t = I_{3N+1}$.

Conditional on the time-invariant parameters in \mathcal{P} , the centered time-varying parameters α , β and γ can then be reconstructed using Equations (17)-(18). Conditional on g_t the common shocks ε_t^f can be calculated as $\varepsilon_t^f = e^{g_t/2} \tilde{\varepsilon}_t^f$.

Block 2(b): Sampling the state \tilde{g}

Using the non-centered version of Equation (A-6), the time-varying state \tilde{g}_t can be sampled from the state space model

$$\underbrace{\left(\ln \left((\varepsilon_t^f)^2 + 0.001 \right) - (m_{v_t^g} - 1.2704) \right) - g_0}_{w_t} = \underbrace{\begin{bmatrix} \sigma_g \end{bmatrix}}_{Z_t} \underbrace{\begin{bmatrix} \tilde{g}_t \end{bmatrix}}_{\kappa_t} + \underbrace{\tilde{\varepsilon}_t^c}_{e_t},$$

$$\underbrace{\tilde{g}_{t+1}}_{\kappa_{t+1}} = \underbrace{\begin{bmatrix} 1 \end{bmatrix}}_{T_t} \underbrace{\begin{bmatrix} \tilde{g}_t \end{bmatrix}}_{\kappa_t} + \underbrace{\begin{bmatrix} 1 \end{bmatrix}}_{R_t} \underbrace{\begin{bmatrix} \eta_t^{\tilde{g}} \end{bmatrix}}_{\eta_t},$$

with $H_t = v_{v_t^g}^2$ and $Q_t = 1$.

Block 2(c): Sampling the states $\tilde{\phi}^h$ and \tilde{h}^c

Using a non-centered version of Equation (A-7), the time-varying states $\tilde{\phi}_{it}^h$ and \tilde{h}_{it}^c can be sampled from the state space model

$$\underbrace{\left(\ln \left((\varepsilon_t^c)^2 + 0.001 \right) - (m_{v_t^h} - 1.2704) - \phi_0^h h_t^f - h_0^c \right) - g_0}_{w_t} = \underbrace{\begin{bmatrix} \sigma_{\phi^h} h_t^f I_N & \sigma_{h^c} I_N \end{bmatrix}}_{Z_t} \underbrace{\begin{bmatrix} \tilde{\phi}_t^h \\ \tilde{h}_t^c \end{bmatrix}}_{\kappa_t} + \underbrace{\tilde{\varepsilon}_t^c}_{e_t},$$

$$\underbrace{\begin{bmatrix} \tilde{\phi}_{t+1}^h \\ \tilde{h}_{t+1}^c \end{bmatrix}}_{\kappa_{t+1}} = \underbrace{\begin{bmatrix} I_N & 0 \\ 0 & I_N \end{bmatrix}}_{T_t} \underbrace{\begin{bmatrix} \tilde{\phi}_t^h \\ \tilde{h}_t^c \end{bmatrix}}_{\kappa_t} + \underbrace{\begin{bmatrix} I_N & 0 \\ 0 & I_N \end{bmatrix}}_{R_t} \underbrace{\begin{bmatrix} \eta_t^{\tilde{\phi}^h} \\ \eta_t^{\tilde{h}^c} \end{bmatrix}}_{\eta_t},$$

with $H_t = v_{i_t^h}^2$ and $Q_t = I_{2N}$.

In order to implement the normalizations on ϕ_{it}^h and h_{it}^c as described in Subsection 2.2, we follow the approach outlined in Doran (1992), i.e. augment the Kalman filter such that the estimates satisfy chosen restrictions. For sake of brevity, the description above only outlines the general estimation procedure but does not elaborate on the normalizations.

Block 2(d): Sampling the state \tilde{h}^f

Again using a non-centered version of Equation (A-7), the time-varying state \tilde{h}^f can be sampled from the following state space model

$$\underbrace{\left(\ln((\varepsilon_t^c)^2 + 0.001) - (m_{i_t^h} - 1.2704) - h_t^c - \phi_t^h h_0^f \right)}_{w_t} = \underbrace{\left[\phi_t^h \sigma_{h^f} \right]}_{Z_t} \underbrace{\left[\tilde{h}_t^f \right]}_{\kappa_t} + \underbrace{\varepsilon_t^c}_{e_t},$$

$$\underbrace{\left[\tilde{h}_{t+1}^f \right]}_{\kappa_{t+1}} = \underbrace{\left[1 \right]}_{T_t} \underbrace{\left[\tilde{h}_t^f \right]}_{\kappa_t} + \underbrace{\left[1 \right]}_{R_t} \underbrace{\left[\eta_t^{\tilde{h}^f} \right]}_{\eta_t},$$

with $H_t = v_{i_t^h}^2$ and $Q_t = 1$.

Block 2(e): Sampling the state $\tilde{\phi}^\varepsilon$

The linear state space model used in this block to sample the time-varying states $\tilde{\phi}^\varepsilon$ takes the following form

$$\underbrace{\left(\Delta y_t - \alpha_t - \beta_t \odot \Delta y_{t-1} - \gamma \odot \Delta y_{t-1}^* - \phi_0^\varepsilon \varepsilon_t^f \right)}_{w_t} = \underbrace{\left[\sigma_{\phi^\varepsilon} \varepsilon_t^f I_N \right]}_{Z_t} \underbrace{\left[\tilde{\phi}_t^\varepsilon \right]}_{\kappa_t} + \underbrace{\varepsilon_t^c}_{e_t},$$

$$\underbrace{\left[\tilde{\phi}_{t+1}^\varepsilon \right]}_{\kappa_{t+1}} = \underbrace{\left[I_N \right]}_{T_t} \underbrace{\left[\tilde{\phi}_t^\varepsilon \right]}_{\kappa_t} + \underbrace{\left[I_N \right]}_{R_t} \underbrace{\left[\eta_t^{\tilde{\phi}^\varepsilon} \right]}_{\eta_t},$$

with $H_t = I_N e^{(\phi_t^h h_t^f + h_t^c)}$ and $Q_t = I_N$. In order to implement the normalization on ϕ_{it}^ε as described in Subsection 2.2, we again use the approach of Doran (1992).

Block 3: Random sign switch for the standard deviations in σ

Perform a random sign switch for each of the standard deviations in σ and the corresponding states in \mathcal{S} , e.g. with probability 0.5 the signs of both the respective standard deviation and the state are changed while remaining unchanged with the same probability.