

Trends in Mathematics

Swanild Bernstein
Uwe Kähler
Irene Sabadini
Franciscus Sommen
Editors

Modern Trends in Hypercomplex Analysis



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Modern Trends in

Trends in Mathematics

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Preface

The 10th International Conference on Applications of Mathematics, held from August 3 to 7, 2003, in Moscow, Russia, was a very successful event. There is a session on the applications of mathematics in the International ISMA tradition of mixed mathematics, but also the opportunity to have sessions, but also other mathematical topics. In the 1970s, these sessions, in particular, showed a part of the talk in this volume. The direction of the session. The ideas and promising like to point out forms, and scribble alive and running. Ferreira, Kheyfi, an important part. Grigoriev and the

The editor of this volume is grateful to the work of the participants who have seen the light of the local organization for the organization. I thank professor Sommen for his dedication to the journal of Mathematical Analysis.

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Preface

The 10th International ISAAC Congress (International Society for Analysis, its Applications, and Computations), was held at the University of Macau, China from August 3 to August 8, 2015. It has been a tradition that at these conferences there is a session "Clifford and Quaternionic Analysis", starting from the first International ISAAC Congress held at the University of Delaware in 1997. Its tradition of mixing speakers from the area and from related fields as well as using the opportunity of inviting local speakers has not only made it one of the largest sessions, but also contributed to show how active and interesting the field is to other mathematical communities. For a branch of mathematics which started as an active research field in form of a generalization of complex analysis only in the 1970s, these sessions are crucial in promoting and advertising the area, in particular, showing new and interesting directions. It is obvious that only a small part of the talks could find its way into a single volume. Therefore, the papers in this volume present a careful selection of the contributions presented during the session. The editors hope that the present choice of several different aspects and direction of hypercomplex analysis will give the interested reader many new ideas and promising new directions. Among the new directions, the editors would like to point out the overviews on quaternionic spectral theory, Clifford differential forms, and script geometry. That also classic topics in Clifford analysis are well alive and running can be seen in the papers by de Ridder, Eelbode, Eriksson, Ferreira, Kheyfits, Ren and their co-authors. Moreover, also applications are an important part of the field as the papers by Baratchart, Cerejeiras, Guerlebeck, Grigoriev and their co-authors demonstrate.

The editors express their gratitude to the contributors to this volume and to the work of the anonymous referees without which this volume would never have seen the light. They also thank warmly professor Tao Qian as chairman of the local organizing committee of the Conference, and the University of Macau for the organization of such a wonderful event. Furthermore, they would like to thank professor Luigi Rodino, president of the ISAAC society, for his work and dedication to make ISAAC the foremost international organization in the area of Mathematical Analysis.

May 2016, The Editors