Вісник Тернопільського національного технічного університету
Scientific Journal of the Ternopil National Technical University
2016, № 3 (83)
ISSN 1727-7108. Web: ***visnyk.tntu.edu.ua***

UDC 629.02:620.19(043.2)

NUMERICAL REALIZATION OF PROBABILISTIC MODEL OF COMPOSITE MATERIAL TAKING INTO ACCOUNT THE DAMAGE EVOLUTION AT HIGH IMPACT VELOCITIES

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Summary. *A numerical simulation of strength of plates of a fiber-reinforced multicomponent composite material under impact with collision velocities ranging from 20 to 1500 m/s was made using the probabilistic model within the framework of the finite element method. Peculiarities of the process of energy absorption and plate damage dimensions depending on the impact velocity are analyzed from the point of view of key physical damage processes. A comparison of numerical results and experimental data on research of damage of composite plates after impact was carried out.*

Key words: *fiber-reinforced composites, damage evolution, probabilistic modeling, high-velocity impact, strain rate*

Received 19.05.2016

Introduction and problem setting. The modeling of impact and damage of flat hybrid-fiber composite plates is mainly carried out by means of analysis of dependency of the strength on stress conditions as well as the peculiarities of damage evolution for each certain type of studied composites [1-3].

Construction composites, which are reinforced with infinite fibers, are formed as multi-layer plates or shells of similar [4-6] or entirely heterogeneous [3, 7, 8] laminated plastic layers. Analysis of the obtained layered composites requires application of classic theory of laminated plastic plates, which is based on the theory of plates and shells under condition of Kirchhoff-Love hypothesis [3, 9, 10], or improved theories of multi-layer plates and shells [11-13]. Hence, the dependence of forces *N* and moments *M* in the laminate plane on strains ε^0 in the medial plane and second derivatives κ of the middle plate deflection on coordinate directions being indicated as $i = 1, j = 2$ and lying simultaneously in the plane of orthotropic laminated plastic and in relevant planes of symmetry plates is presented as follows [14]:

$$\left[\frac{N}{M}\right] = \left[\frac{A \;\; : B}{B \;\; : D}\right]\left[\frac{\varepsilon^0}{\kappa}\right], \tag{1}$$

where, if Q_{ij}^l is stiffness matrix which corresponds to an *l-th* layer whose external surface is at distance z_l from the medial laminate plane and *m* is the total number of such layers:

$$A_{ij} = \sum_{l=1}^{m} Q_{ij}^l \cdot \left(z_l - z_{l-1}\right); \quad B_{ij} = \frac{1}{2} \cdot \sum_{l=1}^{m} Q_{ij}^l \cdot \left(z_l^2 - z_{l-1}^2\right);$$
$$D_{ij} = \frac{1}{3} \cdot \sum_{l=1}^{m} Q_{ij}^l \cdot \left(z_l^3 - z_{l-1}^3\right). \tag{2}$$

However, the main difficulty during modeling of fiber- and textile-reinforced laminated

Corresponding author: Vyacheslav Astanin; e-mail: astanin@nau.edu.ua composites is a proper selection of equation for stress-deformed state of the layers' material that is represented in case of elastic interaction with a dependence of stresses σ on strains ε

$$\sigma_i = Q_{ij} \cdot \varepsilon_j , \; i, j = 1,2,6 . \tag{3}$$

Here, in order to avoid an ambiguity in recording of equations for 2D and 3D cases, the Voigt notation is used (e.g. [15, 16] etc.), so the elements of the stress tensor

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}, \tag{4}$$

which describes a spatial stressed state are represented as follows:

$$\sigma_{11} = \sigma_1; \quad \sigma_{22} = \sigma_2; \quad \sigma_{12} = \sigma_{21} = \sigma_6 . \tag{5}$$

Then in case of flat stressed state assuming that

$$\sigma_{33} = \sigma_{23} = \sigma_{32} = \sigma_{13} = \sigma_{31} = 0 , \tag{6}$$

the other elements of the tensor (4), in particular elements (5), can be written in a vector form:

$$\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{bmatrix}. \tag{7}$$

Similar notations are used for strains ε. Then, at one hand, the correctness of record (3) is retained, which assumes summation over repeated indices i, j , and at other hand, the stiffness matrix Q_{ij} expression for laminate layer material is simplified:

$$Q_{ij} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}. \tag{8}$$

More complicated behavior of an orthotropic material, which is damaged internally due to the action of applied loading, but whose undamaged regions continue exerting elastic resistance to this loading, can be described with a more general comparing to (3) model [17-19]:

$$\sigma_i = f[\tilde{Q}_{ij}(d_{ij}), \varepsilon_j], \; i, j = 1,2,6 , \tag{9}$$

where the arguments of functional dependence are strains ε_j and the damaged stiffness tensor $\tilde{Q}_{ij}$, which, in turn, depends on certain damage parameters d_{ij} (e.g. [16, 20-23]).

Unlike the tensor of initial undamaged stiffness Q_{ij}, which is represented by means of mechanical characteristics of the material, such as tensile E and shear G elastic moduli and also Poisson coefficients ν_{12} and ν_{21} in the laminate plane as follows [15]:

$$Q_{ij} = \begin{bmatrix} \dfrac{E_{11}}{1-\nu_{12}\cdot\nu_{21}} & \dfrac{\nu_{21}\cdot E_{11}}{1-\nu_{12}\cdot\nu_{21}} & 0 \\ \dfrac{\nu_{12}\cdot E_{22}}{1-\nu_{12}\cdot\nu_{21}} & \dfrac{E_{22}}{1-\nu_{12}\cdot\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}, \tag{10}$$

the tensor of damaged stiffness $\tilde{Q}_{ij}$ is [24, 25]:

$$\tilde{Q}_{ij} = \frac{1}{D}\begin{bmatrix} (1-d_{11})\cdot E_{11} & (1-d_{11})\cdot(1-d_{22})\cdot\nu_{21}\cdot E_{11} & 0 \\ (1-d_{11})\cdot(1-d_{22})\cdot\nu_{12}\cdot E_{22} & (1-d_{22})\cdot E_{22} & 0 \\ 0 & 0 & (1-d_{12})\cdot G_{12}\cdot D \end{bmatrix}, \tag{11}$$

where $D = 1-(1-d_{11})\cdot(1-d_{22})\cdot\nu_{12}\cdot\nu_{21}$; d_{ij} are damage parameters in corresponding directions; indices 11 and 22 indicate the directions along and across the fibers (or along the principal and along perpendicular fibers, i.e. warp and weft directions) correspondingly, and index 12 shows the values of mechanical characteristics and damage parameters in the laminate’s plane.

A general concept of modeling of damage and fracture processes for composite materials during impact, which is based on [27, 28], is described in [26]. It is grounded on investigation of probabilistic processes of fracturing and restoration of bonds between structural elements of a material on nano and micro level and describes the probabilistic dependence of damage parameters d_{ij} on current parameters of stress-deformed material state and environment conditions. It is carried out on the basis of analysis of probabilistic by their nature physical phenomena of redistribution of impact energy in space and time that reflect gravitational and energy properties of studied material elements.

Such analysis can principally be realized on the basis of known data about general physical properties of the material, like sound speed, critical temperature or melting temperature, critical strain of fracturing at given strain rate.

An appropriate probabilistic model was also suggested in [26] for the calculation of resulting probability of material damage. In accordance to this model the resulting function, which determines the probability of material damage P_d, can be described with a general formula:

$$P_d(\varepsilon,\dot{\varepsilon},T) = (1-\frac{1}{2\pi\cdot L}\cdot e^{-\left(\frac{(\varepsilon-d\mu_{\varepsilon})^2}{2\cdot(dS_{\varepsilon})^2}+\frac{(\dot{\varepsilon}-d\mu_{\dot{\varepsilon}})^2}{2\cdot(dS_{\dot{\varepsilon}})^2}+\frac{(T-d\mu_T)^2}{2\cdot(dS_T)^2}\right)})^2, \tag{12}$$

where $L = dS_{\varepsilon}\cdot dS_{\dot{\varepsilon}}\cdot dS_T$; $d\mu_i$ are shifts of maximum of function of damage probability along the strain axis ε, strain rate $\dot{\varepsilon}$, temperature T at index values $i = \varepsilon,\dot{\varepsilon},T$ correspondingly; dS_i is a root-mean-square deviation for mentioned arguments, which is found by characteristics of critical states of the material, which correspond to its complete fracture as described in [26], or to the onset of its intensive fructuring that will be described later.

Besides a method for calculation of damage probability is presented in [26] by means of separately calculated (and therefore calculated with different shift and root-mean-square deviation) probabilities of fracture P_f and restoration P_r of the bonds in material. The detailed investigation of both approaches will be published in the next issues. The damage parameters d_{ij} are calculated as a certain realization of the random variable according to the function (12).

Application of the proposed model at low strain rates agrees with experimental results obtained by other scholars, as it is shown in [26]. The objective in the presented in this article research was stated to realize numerical implementation of the proposed probabilistic material model as well as analysis of damage and energy consumption of hybrid-fiber plates at high velocities of impact.

Peculiarities of realization and results of numerical calculations. The task of the given research project was solved on example of damage and fracture modeling of flat hybrid-fiber composite plates with diameter 200 mm at direct central impact with a steel sphere with diameter 8.75 mm. The modeling was carried out using "Abaqus" finite-element software, features of used calculation methods are described in [29].

The calculation algorithm for material's behavior at loading according to the suggested probabilistic model [26] was realized as a user's subroutine that was automatically linked to the general calculation algorithm within the mentioned software as in [30]. The mechanic characteristics of a hybrid-fiber material on the basis of glass fiber and polypropylene matrix, that are given in Table 1, were used as initial data for calculations within the proposed probabilistic model [26] and within a traditional model for fiber-reinforced composites described in [24, 25].

Main mechanical properties of material in statics as well as their general physical properties were determined experimentally within this research project and supplemented with some data from [16, 20, 31, 32]. The calculation values for the moduli of elasticity of separately fibers and matrix were obtained using known composite elasticity characteristics by solving of a problem which is reciprocal to the one investigated in [33]. Besides, more accurate data about properties of propylene matrix were used from [34, 36]. Features due to peculiarities of plaiting, namely of the textile reinforcement, were also taken into consideration according to [37-41].

Temporary stressed state of the material is calculated on each iteration taking into account the deformations obtained during previous calculation step [42]. Simulated contour plots of specific absorbed energy of material damage at impact velocities 140 m/s (*a, b*), 500 m/s (*c, d*) and 1000 m/s (*e, f*) according to deterministic (*a, c, e*) and suggested probabilistic (*b, d, f*) models are represented in Figure 1. Simulated contour plots of total material damage in the directions of warp and weft fibers according to deterministic (*a, b*) and suggested probabilistic (*c, d*) models at impact velocities 280 m/s (*a, c*) and 140 m/s (*b, d*) are represented in Figure 2. Figure 3 shows damaged areas measured using ultrasound scanning of specimens which have been tested at the same impact velocities experimentally, that is described in detail below.

The impact of interacting bodies was modeled on the basis of a 3D model [43, 44]. The geometry of interacting bodies was defined in the first step, loadings were applied and degrees of freedom were defined [45]. The impactor was modeled as an absolutely rigid body as in specially carried out supplementary calculations it was shown that the influence of vibrations of an elastic steel sphere of the predetermined dimensions upon the impact process is negligible due to the fact that sound speed in metals overcomes greatly the studied velocity range, meanwhile taking into account of its elasticity results in excessive computational costs. So only its absolutely rigid surface was simulated as a non-deformable shell. The impactor's mass was therefore determined experimentally by means of weighting and introduced as given data for the numerical procedure as a mass associated with a sole reference point of the modeled with the absolutely rigid shell impactor in its center of gravity.

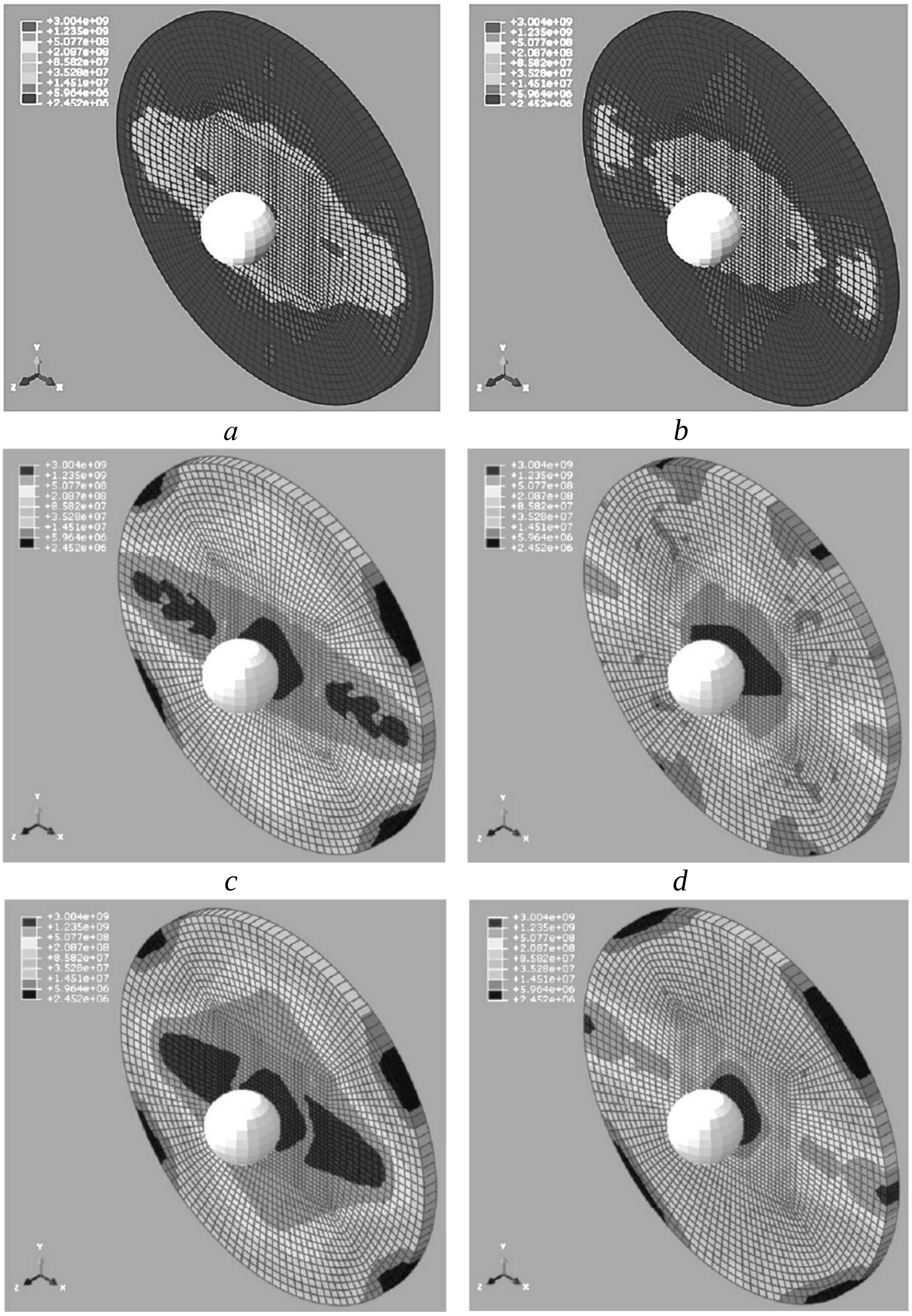
Shell elements are used if one object's dimension is much less than any of the two other ones and if stresses along through-thickness direction are neglected, which is possible under Kirchhoff-Love hypothesis for the theory of plates and shells as it was considered in [46-48]. The shell elements of general purpose, which are aimed at modeling of both thin and thick plates, were selected for finite elements of the composite plate. The used FEM software suggests the possibility of such elements' usage for calculations by means of the method of explicit analysis.

Table № 1

Mechanical characteristics of material

№ з/п	Index	Value	Unit	Material Characteristics
1.	E_1	$13{,}8\cdot10^9$	Pa	Tensile modulus of elasticity along warp direction
2.	E_2	$11{,}5\cdot10^9$	Pa	Tensile modulus of elasticity along weft direction
3.	ν_{12}	0,09	-	Poisson coefficient in plane of laminated plastic
4.	G_{12}	1,05	Pa	Shear modulus of elasticity in plane of laminated plastic
5.	G_{13}	0,597	Pa	Shear modulus of elasticity perpendicular to the plane of laminated plastic
6.	G_{23}	0,559	Pa	
7.	ρ	1490	kg/m^3	Density
8.	R_1^t	$283\cdot10^6$	Pa	Strength along warp direction under tension
9.	R_1^c	$125\cdot10^6$	Pa	Strength along warp direction under compression
10.	R_2^t	$279\cdot10^6$	Pa	Strength along weft direction under tension
11.	R_2^c	$103\cdot10^6$	Pa	Strength along weft direction under compression
12.	R_{12}	$44\cdot10^6$	Pa	Strength in plane of laminated plastic under shear
13.	E_1^t	70145	J	Fracture energy along warp direction under tension
14.	E_1^c	13380	J	Fracture energy along warp direction under compression
15.	E_2^t	9913	J	Fracture energy along weft direction under tension
16.	E_2^c	7355	J	Fracture energy along weft direction under compression
17.	v_0	190	m/sec	Ballistic limit of material
18.	$c_{зв}^{\perp}$	1300	m/sec	Velocity of transverse sound wave in material
19.	ε_1^t	0,00319	-	Critical strain along warp direction under tension
20.	ε_1^c	-0,00319	-	Critical strain along warp direction under compression
21.	ε_2^t	0,00131	-	Critical strain along weft direction under tension
22.	ε_2^c	-0,00097	-	Critical strain along weft direction under compression
23.	γ_{12}	0,04190	-	Critical shear strain in plane of laminated plastic
24.	γ_{23}	0,03400	-	Critical shear strain perpendicular to the plane of laminated plastic
25.	γ_{31}	0,02700	-	

Composite plate itself was modeled with shell elements [49, 50]. In order to mesh it into finite elements the geometry of the circular plate was split into a central part, which is usually modeled in works dedicated to impact against orthotropic plates, for example [51], with a square shape, and also rays connecting the part's angles with plate edges were introduced. Selection among finite elements, which are provided within the used FEM software, for plates' modeling also suggested two available types of shell elements, namely shell and continuous shell elements. As plate geometry unlike, for example, a cylindrical shell, requires fixed dimensions of each element along the plate's thickness (i.e. there are no initial large distortions of the surface), so the simple shell finite elements were chosen.



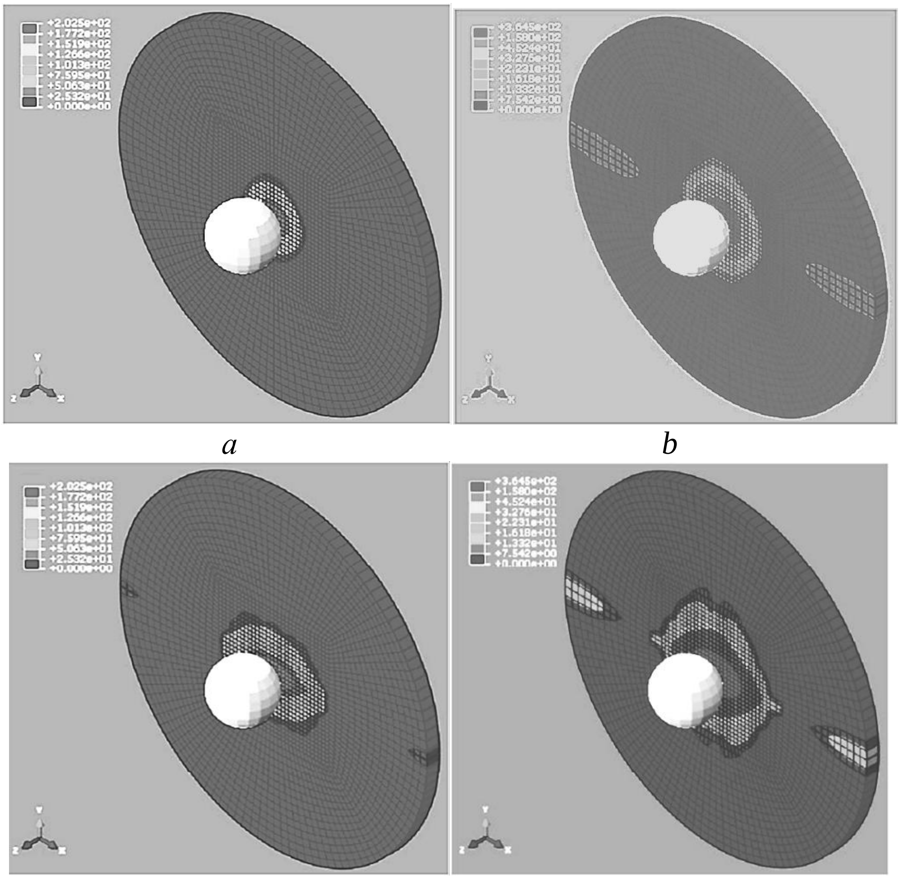
a *b*

c *d*

e *f*

Figure 1. Simulated contour plots of specific absorbed energy of material damage at impact velocities 140 m/s (*a*, *b*), 500 m/s (*c*, *d*) and 1000 m/s (*e*, *f*) in accordance with deterministic (*a*,*c*, *e*) and suggested probabilistic (*b*, *d*, *f*) model

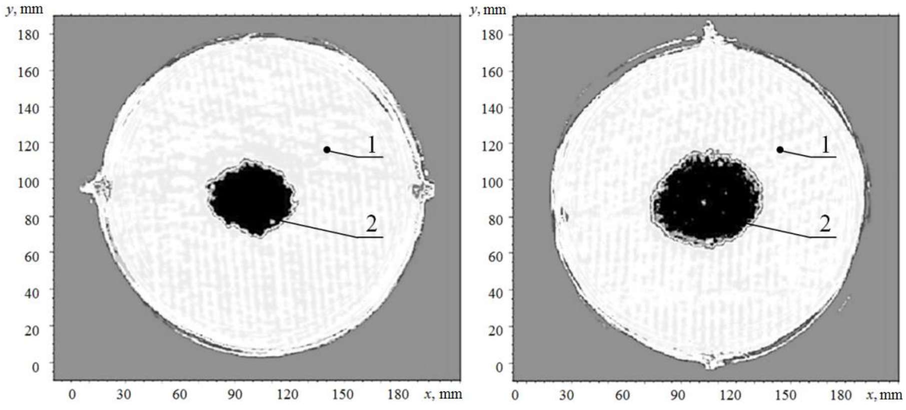
For solution of static problems modeling of the system's state at some moment in time requires selection of a not too small value of the increment for time integration and the numerical modeling provides acceptable errors only by means of implicit integration method. At the same time dynamic problems considered in this research can be solved with sufficient accuracy by means of explicit integration method assuming direct solving of equations that link functionally the system state during the preceding and the current time moment [29, 52, 53].



a *b*

c *d*

Figure 2. Simulated contour plots of cumulative material damage in the warp and weft fiber directions in accordance with deterministic (*a*, b) and suggested probabilistic (c, d) model at impact velocities 140 m/c (*a*, c) and 280 m/s (b, d)



a *b*

Figure 3. Damage of round plates of material according to data of ultrasound scanning at impact velocities 140 m/s (*a*) and 280 m/s (*b*): 1 – plate, 2 – area damage during the impact

As a result of impact, the plate is considerably linearly deformed in tension; therefore S4R shell elements are used that take into consideration the finite membrane deformations. Unlike the other similar types of finite elements within the used software, this type provides both high accuracy and preserving of the numerical resources. Kinematic boundary conditions for the system of interacting bodies include definition of initial velocity of the impactor towards the plate. Besides, the plate clamping along its edge is modeled. Time integration step is selected by means of test calculations in order to avoid numerical instability of calculation, particularly to avoid the typical for problems of sideward collision errors due to excessive rotations of the finite elements under loading comparing to the initially provided by the software limit value of allowed rotations, which was obtained during one step of solution [54].

The model of composite material includes a series of laminated plastic layers located at angle to each other; this angle is relevant to the structure of experimentally tested specimens i.e. $[0/90]_s$. For each layer parameters of material were provided as given data for the directions along warp and weft fibers. The contact between the impactor and material was modeled to be absolutely hard and the friction was neglected, as due to software specifications, the accounting of friction resulted into problems of time interval selection for single calculation step [55].

The characteristics of the cross section of the composite plate include the data about its cumulative thickness, definition of the reference shell surface as being linked to the medial shell plane, thickness and orientation of constituent laminate layers as well as indicators to each layer's material [29, 30]. The model of fiber-reinforced composite according to [24, 30, 56] was taken as basis for programming the user's subroutine; in terms of strain, damage and stress calculations it was modified according to the listed above approaches and algorithm of probabilistic calculation.

Analysis of results of the numerical simulation and their comparison to experimental data. According to the simulated contour plots of energy of material damage, which are presented in Figure 1, in the whole investigated range of impact velocities from 20 to 1500 m/s the energy absorbtion is realized by all the finite elements within the included into deforming and damaging area of the specimen. It became possible due to the fact that though the damage value of every finite element is probabilistic, but the probability of certain realization of its values depends at the same time on interaction conditions deterministically.

An important feature of obtained simulated contour plots of specific absorbed damage energy comparing to those obtained using the traditional model is that the value of this energy (Figure 1) for identically remote from the impact epicenter areas of the plate are considerably less in the first case, and damage values (Figure 2) are larger. It means that probabilistic model facilitates forecasting of damage localization within the impact area, especially at high collision velocities from 400 to 1500 m/s. It can be explained as follows.

For the case of a quasi-static interaction it is enough to utilize the traditional criterion of material fracture alongside with strength parameters. But high-speed deformation demands accounting for the strain rate as an important task. Manual correction of the strength limit depending on strain rate does not solve the problem at high deformation velocities due to exaggerated non-linearity of this dependence at high values of the latter one [57], though the experimental definition of material properties by means of standard methods and using the standard certified equipment is problematic at high strain rates [58, 59].

Thus, general dependencies of probabilistic approach according to the formula (4) and Figure 8, represented in [26], facilitate solving of this problem by coming closer to modeling of real physical processes of material damage.

As the level of absorbed damage energy is directly interconnected to creation of an impact crater, a hole or cracks and fiber pull-out, it is important to compare the experimental and numerical results concerning the plate damage dimensions.

Special experimental investigations were carried out to determine the internal structure of the damaged specimen after impacts causing both surface damage or through-out piercing.

The tests were carried out by means of ultrasound scanning. Figure 3 represents the scaled tested plates with dimensions indicated, where darker areas correspond to regions where ultrasound was scattered or dissipated while going through the plate because of its dispersion due to significant cracking and delamination in these regions. Solid undamaged material is represented by lighter areas. Particularly, the diversification of probabilistic damage parameters along deformation directions (i.e. along and across the principle reinforcing fibers and in the plane of the laminated plastic) allows modeling the difference in longitudinal and transversal directions of the damaged area.

The comparison of Figure 2 and Figure 3 shows that the model allows obtaining of predictions about the damaged area dimensions which are close to real experimental data of ultrasound scanning at the same impact velocities. Exactly these values, absorbed energy and material damage, determine the material’s protective properties.

Conclusions. FEM-implementation of the previously proposed in [26] and based on [27, 28] probabilistic model of a fiber-reinforced material is realized. The algorithm for simulating the behavior of a material under loading, implemented in the framework of software for analysis of structures using the finite element method, was verified for an example of simulation of a normal high-velocity impact of flat plates of a textile-reinforced hybrid-garn material. In this work a range of high impact velocities (20-1500 m/s) is considered.

Due to the use of the probabilistic model, which was developed in order to adequately describe real physical processes of damage of bonds between structural elements of a composite, which determine its strength and behavior under loading, the problem of taking into account of the non-linearity of the dependency of the strength on the strain rate at high velocities was solved. The research results show that usage of the probabilistic model made it possible to model the damage localization at high impact velocities, as well as the peculiarities of the damaged area form due to the unequal distribution of the mass fraction of fibers in two directions of orthotropy of the textile-reinforced composite.

Analysis of experimental data on characteristics of damage formation associated with cracks and delamination of composites and theoretical analysis of known structural material models suggest the possibility of principle generalization of the proposed concept of probabilistic modeling of the behavior of a multi-component material at different velocities of interaction on a wide range of advanced composites, as well as traditional anisotropic metal materials, which are used e.g. in aviation, on-ground transportation vehicles, construction.

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УДК 629.02:620.19(043.2)

ЧИСЕЛЬНА РЕАЛІЗАЦІЯ ЙМОВІРНІСНОЇ МОДЕЛІ КОМПОЗИЦІЙНОГО МАТЕРІАЛУ З УРАХУВАННЯМ ЕВОЛЮЦІЇ ПОШКОДЖЕННЯ ПРИ ВИСОКИХ ШВИДКОСТЯХ УДАРУ

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Резюме. *Методом скінченних елементів із застосуванням ймовірнісної моделі проведено чисельний розрахунок на міцність пластин багатокомпонентного волоконнозміцненого композиційного матеріалу при ударі в діапазоні швидкостей зіткнення від 20 до 1500 м/с. Проаналізовано особливості динаміки енергопоглинання і розмірів пошкодження пластини залежно від швидкості удару з урахуванням фізичних процесів пошкодження, які їх визначають. Проведено порівняння результатів чисельного розрахунку з експериментальними даними з дослідження пошкодження композиційних пластин після удару.*

Ключові слова: *волоконнозміцнені композити, еволюція пошкодження, ймовірнісне моделювання, високошвидкісний удар, швидкість деформації*

Отримано 19.05.2016