David Hayes, Mohammad M. Lone, James Whidborne, and Etienne Coetzee. "Evaluating the Rationale for Folding Wing Tips Comparing the Exergy and Breguet Approaches", 55th AIAA Aerospace Sciences Meeting, AIAA SciTech Forum, (AIAA 2017-0464) http://dx.doi.org/10.2514/6.2017-0464

55th AIAA Aerospace Sciences Meeting, 9-13 January 2017, Grapevine, Texas, USA

Evaluating the Rationale for Folding Wing Tips Comparing the Exergy and Breguet Approaches

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The design and development processes for future aircraft aims to address the environmental and efficiency challenges needed to facilitate the engineering of concepts that are far more integrated and require a multidisciplinary approach. This study investigates the benefit of incorporating span extension wing tips onto future aircraft configurations as a method of providing improved aerodynamic efficiency, whilst allowing the extension to fold on the ground to meet airport gate size constraints. Although the actuated wing tips are not studied in detail, the focus of this study is to compare two different methods of analysis that can be used to identify the benefit and limitations of adding such devices. The two methods considered are a quasi-steady implicit energy analysis based on the Breguet Range Equation and an explicit energy analysis based on the first and second laws of thermodynamics known as Exergy Analysis. It has been found that both methods provide agreeable results and have individual merits. The Breguet Range Equation can provide quick results in early design, whilst the Exergy Analysis has been found to be far more extensive and allows the complete dynamic behaviour of the aircraft to be assessed through a single metric. Hence, allowing comparison of losses from multiple subsystems.

Nomenclature

α	Angle of attack (rad)
Δh	Fuel heating value (Jkg^{-1})
η	Efficiency value $(-)$
Λ	Sweep angle (rad)
$\mathcal{V}$	Volume (m^3)
μ	Chemical potential of substance $(Jmol^{-1})$
$\tilde{m}$	Penalised system mass (kg)
A	Wing aspect ratio $(-)$
b	Wing span (m)
C_f	Skin-friction drag coefficient $(-)$
C_L	Lift coefficient $(-)$
C_{D_0}	Parasitic zero-lift drag $(-)$
C_{D_i}	Lift induced drag $(-)$
C_{L_α}	Lift coefficient at given angle of attack (rad^{-1})
D	Drag (N)
e	Oswald efficiency factor $(-)$
F	General Force (N)

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f	Thrust Specific Fuel Consumption ($kgN^{-1}s^{-1}$)
F_T	Thrust (N)
h_f	Enthalpy of formation ($Jmol^{-1}$)
I_{sp}	Engine Specific Impulse (s)
L	Lift (N)
M	Mach number ($-$)
P	Pressure (Pa)
P_f	Penalty factor ($-$)
Q_F	Component form factor[1] ($-$)
Q_i	Component interference factor[1] ($-$)
R	Aircraft Range (m)
S	Wing reference area (m^2)
s	Entropy (JK^{-1})
s^o	Standard molar entropy ($Jmol^{-1}K^{-1}$)
S_w	Wing wetted area (m^2)
T	Temperature (K)
t	Time (s)
U	Internal energy (J)
V	Velocity, with components u, v, w in x, y, z respectively (ms^{-1})
w	Work done on a system (J)
W_0	Aircraft Operational Weight Empty (OWE) (kg)
W_1	Mass constant payload and fuel reserves (kg)
W_2	Aircraft mass at beginning of cruise (kg)
W_f	Mass of fuel burnt during cruise (kg)
X	Exergy (J)
x_k	Environmental composition ratio ($-$)
y_k	Mass ratio ($-$)

I. Introduction

A trend is apparent in Figure 1, that with the evolution in commercial aircraft there has been a general increase in wing aspect ratio with each design iteration, with some exceptions such as that of the Airbus A380. Increased aspect ratio improves aerodynamic efficiency (lift-to-drag ratio), providing a more energy efficient aircraft. However, the Advisory Council for Aviation Research and Innovation in Europe (ACARE)[2] targets for *protecting the environment and the energy supply* by 2050 are driving efficiency improvements that will not be met with the evolutionary improvements seen independently in aerodynamic, propulsion and structure technologies. Thus, the next generation of aircraft need to be revolutionary in technology and configuration, seen in the development of conceptual Blended Wing Body (BWB), Prandtl box wing and very High Aspect Ratio Wing (HARW) designs.

The NASA Subsonic Ultra Green Aircraft Research (SUGAR) Volt[3] (Figure 2a), proposes a HARW hybrid propulsion vehicle with a high wing span to provide an enhanced lift-to-drag ratio over current generation short haul aircraft. This results in a configuration that builds towards meeting the ACARE targets. Yet, these next generation aircraft are unlikely to be seen in commercial application before 2030, so aircraft manufacturers are resorting to short term efficiency improvements to current conventional aircraft. Similar improvements to aerodynamic efficiency as seen on the SUGAR concept can be found with the incorporation of span extension technologies. Where the wing span of aircraft is typically limited by the gate constraints of airports, the addition of winglets has provided the semblance of a larger wing span (hence aspect ratio) without changing the aircraft maximum span. This reduces induced drag but does not provide the additional lifting surface a true wing extension would.

In addition to the gate constraints, HARW aircraft also face challenges when it comes to in-flight loading. As the wings are larger in span and more slender, the wing internal loads are increased during gust incidents and aerodynamic loading in cruise. This results in the requirement of a stiffer wing box or as seen on the SUGAR concept, the addition of a truss which adds weight and drag to the aircraft. This means that careful accounting of the benefits and drawbacks of the improvement in lift-to-drag ratio is needed to ensure the benefits of HARW configurations are achieved.

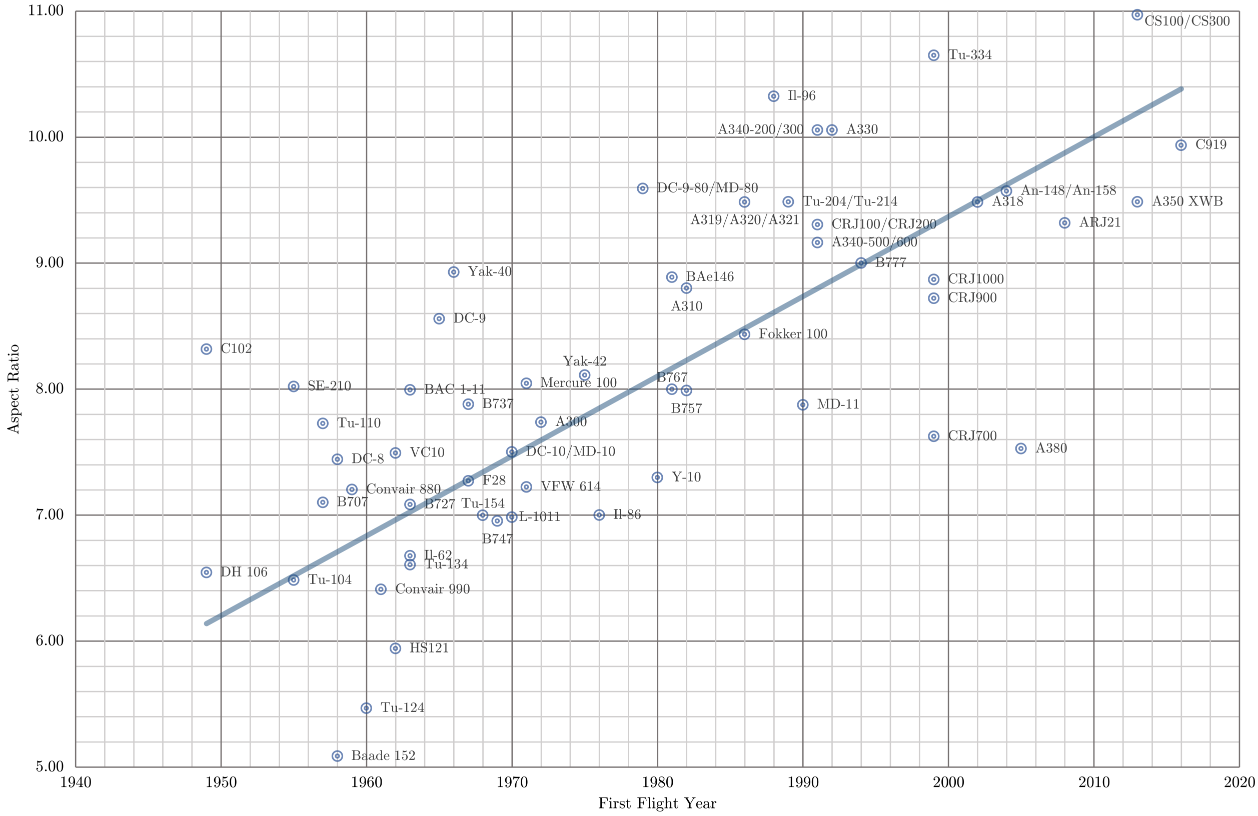
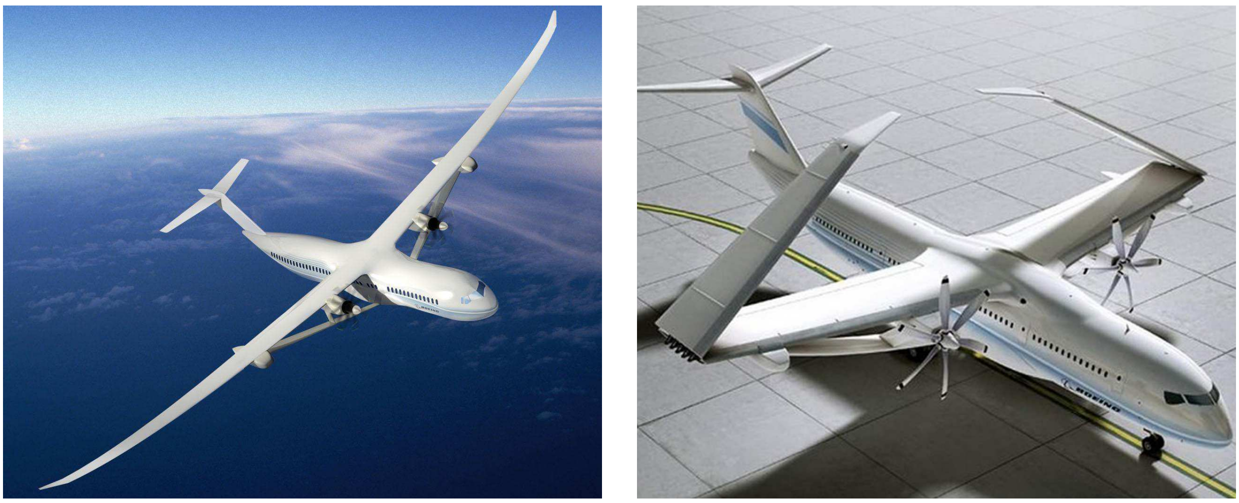


Figure 1. Trend of Commercial Aircraft Aspect Ratio vs First Flight Year

Extending the wing span of a conventional aircraft provides the prospect of the aircraft not satisfying gate constraints, as such the extended wingspan can be designed to fold to meet previous gate constraints, as considered by the SUGAR aircraft (Figure 2b). To date ground folding wing span technology has not seen wide proliferation on commercial aircraft. One reason for this is that changing the aerodynamic profile of a wing requires some form of actuation system, which must be powered and adds weight to the aircraft. Thus,



(a) Extended wing configuration

(b) Folded wing aircraft configuration

Figure 2. NASA Subsonic Ultra Green Aircraft Research[3]

any performance benefit morphing provides must outweigh the penalty due to additional weight and power requirements, in addition to the previously discussed additional weight and drag from structural changes.

This body of work will look to justify the integration of ground folding span extension technology by comparing two different analysis methods, the Breguet Range Equation and a novel exergy analysis method. The long-established Breguet Range Equation is an implicit energy solver used to maximise the range of an aircraft for a given fuel load based on lumped mass parameters. The second law explicit energy solver, exergy analysis, is a natural view of an aircraft as a system that converts chemical energy (fuel) into useful work to accomplish its mission. With the aim to optimise a system by minimising the loss in useful energy accompanied with irreversible energy flows into and out of a system along paths of mass flow, heat transfer, and work.

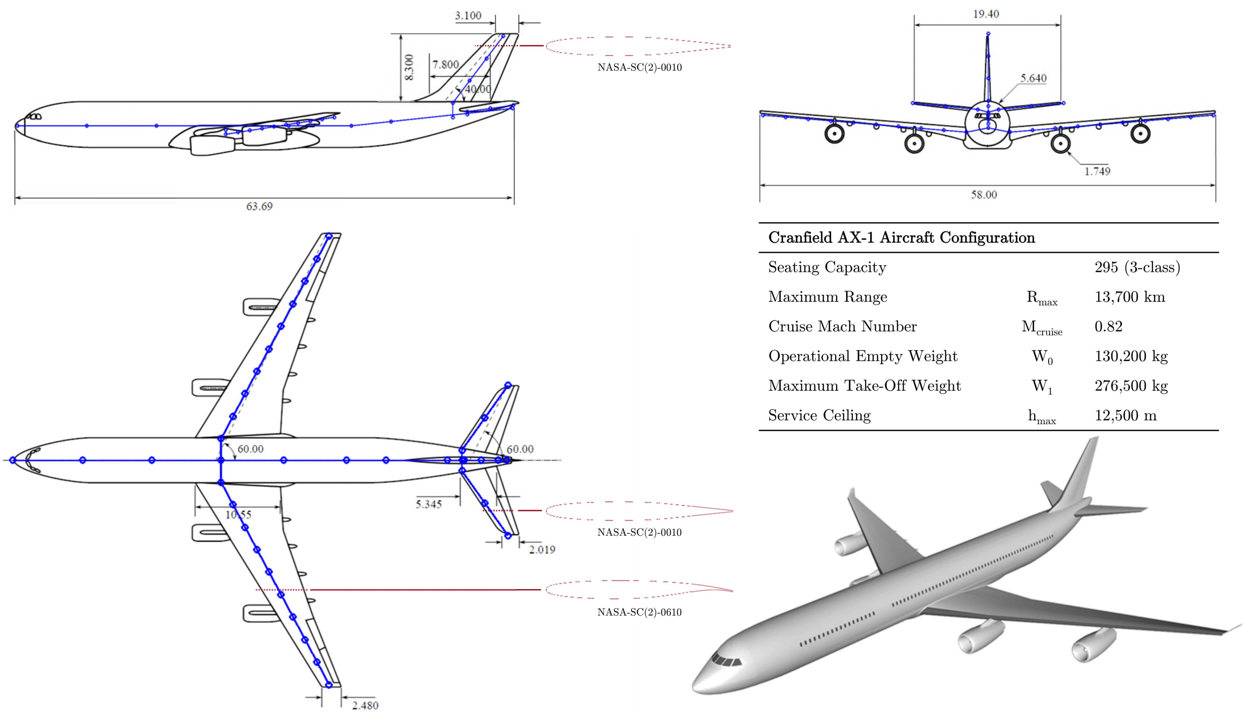
This study is inspired by two separate bodies of work: that of Von Spakovsky[4] who used exergy analysis as a comparative measure to justify morphing wing technology for future military aircraft, and that of Cooper,[5] who as part of the Claret programme used the Breguet approach to analyse the effectiveness of various morphing wing tip devices. Both methods showed the addition of morphing technology was beneficial to overall aircraft performance under certain conditions. However, it is not clear which method proves to be the most useful analysis tool during the design process. The analysis and discussion presented in this paper aims to provide further clarity in this area.

II. Analysis Method

A. Geometry

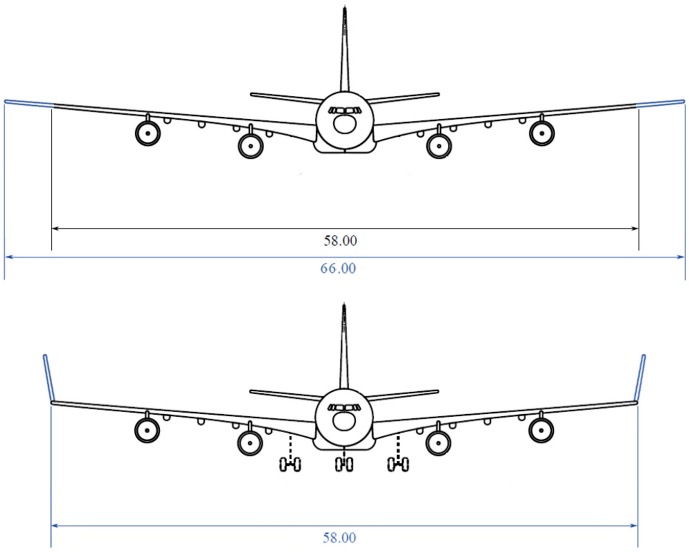
The test case used in this study looks to improve the energy efficiency of a long haul aircraft by providing span extension to a baseline configuration. The geometry used in both analysis methods is the Cranfield AX-1 aircraft[6] (Figure 3), a generic long-haul commercial aircraft.

The baseline aircraft will be compared against a modified AX-1 with an increased wing span of up to 12 metres (6 metres on each semi-span), using both the Breguet and Exergy approaches. The extended wing



Cranfield AX-1 Aircraft Configuration		
Seating Capacity		295 (3-class)
Maximum Range	R_{max}	13,700 km
Cruise Mach Number	M_{cruise}	0.82
Operational Empty Weight	W_0	130,200 kg
Maximum Take-Off Weight	W_1	276,500 kg
Service Ceiling	h_{max}	12,500 m

Figure 3. Cranfield AX-1 Aircraft Configuration



Parameter	Symbol	Units	Wing
Reference Area	S	m^2	357.10
Span	b	m	58.0
Root Chord	C_r	m	9.83
Tip Chord	C_t	m	2.48
Sweep Angle (LE)	Λ_{LE}	rad	0.5577
Fuselage radius	s_0	m	2.793
Exposed Area	S_{exp}	m^2	302.2
Wetted Area	S_{wet}	m^2	621.3
Aspect Ratio	A	-	9.42
MAC	$\bar{C}$	m	6.889
Span location of $\bar{C}$	$\bar{Y}$	m	11.61
Taper Ratio	λ	-	0.252
Sweep Angle ($\frac{c}{4}$)	$\Lambda_{\frac{c}{4}}$	rad	0.5107
Thickness Ratio	$\frac{t}{c}$	-	0.1525

Figure 4. Span extension of Cranfield AX-1 Aircraft Configuration

span, b, will increase the aspect ratio, A, for a constant wing reference area, S as:

$$A = \frac{b^2}{S} \tag{1}$$

The reader is referred to Figure 4 for an example of the proposed wing extension from a 58 metres span to 66 metres. Given the International Civil Aviation Organization (ICAO) aerodrome reference codes[7] the 66 metre wingspan exceeds the *category E* gate constraint, thus to keep the aircraft category E compliant the wing tips would need to fold when the aircraft is grounded. A weight penalty will be added to the extended aircraft to account for the additional structure required and the actuator weight and power required to fold the wing when on the ground.

The AX-1 has a wing that is cranked in four locations, as such the analysis is done using the *equivalent wing* method as outlined in ESDU 76003. This creates a geometry for an equivalent trapezoidal wing which is required because the calculations performed in the Prandtl-Glauert analysis are only valid for swept trapezoid wings. The analysis is done in MATLAB/Simulink.

B. Flight Envelope

Commercial aircraft flight envelopes are dominated by the cruise phase of flight and as such it is typical to perform comparative studies at just these conditions. Assuming a constant cruise flight condition allows clear comparison between the methods without the additional divergence due to a variable reference state.

C. Prandtl-Glauert Aircraft Modelling

In order to generate the same aerodynamic coefficients for both analysis methods a low fidelity strip theory model was built based on Glauert theory[8] for the two dimensional aerofoil and extended to a three dimensional wing to determine lift and drag using Prandtl's[9] empirical relationships. For example the lift coefficient, C_L is given based on the aerofoil zero lift angle of attack, α_0 and the cruise angle of attack, α, via the following relationship:

$$C_L = C_{L_\alpha}(\alpha - \alpha_0) \tag{2}$$

where

$$C_{L_\alpha} = \frac{2\pi A}{2 + \sqrt{A^2\left(1 + \tan^2\Lambda - M^2\right) + 4}}$$

The drag coefficient, C_D, is the sum of the parasitic (zero lift) drag, C_{D_0}, and the lift-induced drag, C_{D_i}, which is proportional to the lift coefficient, defined as:

$$C_D = C_{D_0} + C_{D_i} \tag{3}$$

where

$$C_{D_0} = \frac{\Sigma\left(C_f Q_F Q_i S_w\right)_c}{S}$$

and

$$C_{D_i} = \frac{C_L^2}{\pi e A}$$

Note that drag due to shock waves and additional parasitic drag from nacelles and engines is not considered in this analysis.

The initial cruise aircraft weight, W_2, is the sum of the Operational Weight Empty (OWE), W_0, the total of the flight constant payload and fuel reserves, W_1, and the fuel mass burnt off during the cruise flight, W_f, given as $W_2 = W_0 + W_1 + W_f$. During concept design, details are not available for the wing extension, such as the additional structural requirements and the actuation system. To account for these associated weights a *weight penalty* is added to the aircraft OWE for extended wing configurations. The penalised OWE, $\tilde{W}_0$, is increased by a proportion of the baseline OWE, as a function of the increase in wing root bending moment, $L_0 \frac{\frac{b_i}{2}}{\frac{b_0}{2}}$, which occurs from the new load distribution. The magnitude of this weight penalty is varied using a weight factor, P_f, where $0 < P_f < 2.0$, giving a penalised initial aircraft weight, $\tilde{W}_2$, as:

$$\tilde{W}_2 = \tilde{W}_0 + W_1 + W_f \tag{4}$$

where

$$\tilde{W}_0 = W_0 + W_0 P_f \left(L_0 \frac{\frac{b_i}{2}}{\frac{b_0}{2}} - 1 \right)$$

D. Breguet Range Equation

The traditional method used to assess the effectiveness of span extension wing tips is to maximise the range (R) of an aircraft, for a given set of aircraft parameters, utilising the Breguet Range Equation for steady cruise. This is given as a function of the propulsive specific impulse, I_{sp}, as follows:

$$R = Vt_1 = (VI_{sp}) \left(\frac{L}{D}\right) \left(\ln \frac{\tilde{W}_2}{\tilde{W}_0 + W_1}\right) \tag{5}$$

This provides a comparative range of an aircraft configuration with the extended wingspan against the range of a conventional configuration. Reduced energy intensity provides improved range for a given fuel quantity, which can be achieved by maximising the lift (C_L) to drag (C_D) ratio, as well as the the ratio of initial penalised take off weight ($\tilde{W}_2$) to penalised empty weight ($\tilde{W}_0$) and fixed payload (W_1). If the range is improved for the extended wingspan configuration the design is assumed to be beneficial in terms of energy intensity. For the cruise stage of flight the Breguet variables are assumed constant and these variables are defined using a lumped mass model.

Using the heating values (Δh) of fuel the Breguet equation can be derived in terms of propulsive efficiency. Given the relationships of $I_{sp} = (gf)^{-1}$, thrust $F_T = \dot{m} f^{-1}$ and the rate of energy release $\dot{E} = \dot{m}\Delta h$ the propulsion function is defined as follows:

$$VI_{sp} = V\frac{F_T}{\dot{m}g}\frac{\Delta h}{\Delta h} = \frac{F_T V}{\dot{m}\Delta h}\frac{\Delta h}{g} \tag{6}$$

given

$$\eta_p = \frac{F_T V}{\dot{m}\Delta h} \tag{7}$$

thus

$$\eta_p = \frac{g}{\Delta h} V I_{sp} \tag{8}$$

Now, substituting Equation 8 into Equation 5, the aircraft range can be calculated as follows:

$$R = \frac{\eta \Delta h}{g}\frac{L}{D}\ln\frac{\tilde{W}_2}{\tilde{W}_0} \tag{9}$$

Finally, substituting the relationship for thrust specific fuel consumption, f, in Equation 9, range can be defined in terms of propulsive efficiency (η_p), aerodynamic efficiency (η_a) and the structural efficiency (η_s), as follows:

$$R = \underbrace{\left(\frac{V}{gf}\right)}_{\eta_p} \underbrace{\left(\frac{L}{D}\right)}_{\eta_a} \underbrace{\left(\ln \frac{\tilde{W}_2}{\tilde{W}_0 + W_1}\right)}_{\eta_s} \tag{10}$$

E. Exergy Analysis

Many analysis methods in practice implicitly use energy and the first law of thermodynamics as an optimisation metric. This is typically done by charging aircraft systems for their use of resources, such as vehicle weight as an associated cost in terms of fuel weight. This is the approach implicit in the Breguet Range Equation where the propulsive, aerodynamic and structural efficiency are all looked to be improved and judged against the common range metric. Explicit energy tracking is essentially the same as fuel tracking as fuel is the source of energy. Aircraft systems are unique in that energy is required not just for operation but also for lift generation.

The second law of thermodynamics is sparsely utilised in the commercial aerospace industry outside of propulsion systems. There is however an analysis gap for second law methods such as *exergy analysis*; not to replace the standard first law derived energy method, but to yield additional insight into the irreversibilities and limitations on system performance. When performing an energy study of a system such as an aircraft, the conversion of energy from one form to another is not the only area of interest. The conditions and limitations on such a conversion are another consideration. This is where the application of the second law can provide beneficial insight: as to whether the achieved final solution is near the optimal case or whether the solution is in fact feasible. The integration of the second law into energy analysis is based on identifying the component of the total energy that can be transferred between subsystems, defined as the *exergy* component used as the cost function throughout the system, which can be destroyed through different conversion processes. A convenient interpretation of exergy can be generated by the definitions of Sciubba[10] and Ayres[11] as:

> "*The maximum theoretical useful work obtained if a system is brought into thermodynamic equilibrium with the environment by means of processes in which the system interacts only with this environment. It is not a conserved quantity like energy but it is possible to construct an exergy balance for any energy or materials transformation process, accounting for inputs, process losses, useful products and wastes.*"

Through the traditional energy based approach only the quantity of energy used by each system is obtained. Exergy differs from energy in that it defines the work potential of energy (the quality of the energy) and the ability of a system to receive work from streams of mass, heat and work, based on the environment in which the system resides. By coupling the concept of entropy generation from the second law of thermodynamics to the conservation of mass, momentum and energy from the traditional approach, an analysis method can be developed that identifies the work potential (quality) in the energy flow, as well as providing information on the feasibility and practical boundaries of a process.

Exergy analysis is a time dependent analysis that can be undertaken over the whole mission profile, where the exergy required to move and operate the actuation system will be compared to the reduced exergy destruction from the lower induced drag. Applying such a method to the extended wingspan example detailed above is just one possible use of such a tool. It can be extended to higher level wing optimisation and to even higher level aircraft systems analysis where the exergy source (typically fuel) is mapped throughout the flight mission to highlight areas of exergy destruction. For a more extensive background on exergy the reader is referred to Camberos[12] and Hayes.[13]

1. *Exergy Post-Processing*

The exergy post-processing of the Prandtl-Glauert model, defines the usable energy content (exergy) in the system at every time iteration during the flight. On a microscopic level all exergy, as energy, can be described as either kinetic or potential. However, in engineering it is simpler and clearer to discuss exergy in macroscopic terms and sub-divide these exergies into other forms with clear mathematical expressions. The total exergy of an aircraft can be divided into four forms as follows:

$$X_{sys} = \sum X_i = X_{ph} + X_{ke} + X_{pe} + X_{ch} \tag{11}$$

These are the physical exergy (X_{ph}), the kinetic exergy (X_{ke}), the potential exergy (X_{pe}) and the chemical exergy (X_{ch}). Physical exergy is the classic thermodynamic *free energy*[a], based on the internal energy, U, pressure, P, volume, $\mathcal{V}$, temperature, T and entropy, s. Kinetic exergy is the exergy associated with movement, be it the motion of waves, electrons, atoms, molecules or substances. Potential exergy is the exergy of state (e.g. position), where the system has a disparity in some form to its environment which enables it to do or receive work. Chemical exergy, a form of potential energy, is typically the primary exergy that is available in fuel. With these definitions Equation 11 can be expanded as follows:

$$X_{sys} = (U - U_0) + P_0(\mathcal{V} - \mathcal{V}_0) - T_0(s - s_0) + X_{ke} + X_{pe} + X_{ch} \tag{12}$$

As energy flows between systems via work or heat, except in the hypothetical reversible process, the quality of energy (exergy) decreases as entropy is produced (associated only with heat transfer). The Guoy-Stodola identity represents the *principle of decreasing exergy*, that is the generation of entropy always signifies equivalent destruction of exergy, defined as:

$$X_{des} = T_0 s_{gen} \geq 0 \tag{13}$$

Note that exergy destroyed is a positive quantity for any actual process and becomes zero for a reversible process. Exergy destroyed represents the lost work potential and is also called irreversibility or lost work.

2. Exergy use by Aerodynamic Systems

Concepts such as heat and work transfer are easily read across to aircraft systems such as propulsion systems or environmental control systems (ECS). However, it is less clear how the second law analysis is used on purely aerodynamic systems. Exergy is transferred throughout the system including the airframe, but what needs to be identified are how the aerodynamics uses and converts exergy and the causes of entropy generation /exergy destruction. Taking a crude view of an airframe, it has two primary purposes: (a) to house the payload and (b) to convert part of the forward thrust from the engines into lift. In doing this the airframe generates drag (with contributions from vortex, parasitic and wave drag components), that generates entropy which accounts for a loss in useful energy.

Entropy generation or exergy destruction due to aircraft aerodynamics are typically over shadowed by the exergy destruction within the propulsion system. This does not however mean there is no purpose to optimise aircraft aerodynamics. As it may be the case, reducing exergy destruction due to drag is more cost effective than reducing total engine exergy destruction. Exergy analysis also proves to be a useful tool for wing optimisation when the aerodynamics are considered in isolation from the rest of the aircraft.

In steady level cruise flight, assuming lift and thrust compensate the aircraft's weight and drag respectively, aircraft lift coefficient can be derived from the penalised mass, $\tilde{m}$ and expressed as follows:

$$C_L = \frac{2\tilde{m}g}{\rho u_\infty^2 S} \tag{14}$$

where u_∞ is the cruise forward speed. The aircraft drag coefficient (assuming subsonic, thus zero wave drag) is typically expressed as Equation 3 and expanded as follows:

$$C_D = C_{D_0} + C_{D_i} = C_{D_0} + \frac{C_L^2}{\pi e A} \tag{15}$$

where e is the Oswald efficiency factor. The rate of work done (power), $\dot{w}$, on a body to move through a fluid is given as, $\dot{w} = FV$, where F is the driving force of the body at velocity V. So, the rate of exergy use can be similarly defined as $\dot{X} = F\,(V - V_0)$, where the reference state velocity V_0. For the purposes of level unaccelerated cruise flight the rates of exergy use of interest are the exergy use to overcome drag:

$$\dot{X}_D = D\,(u_\infty - \cancel{u_0}) \tag{16}$$

[a] An unconstrained extension of Gibbs and Helmholtz Free Energy

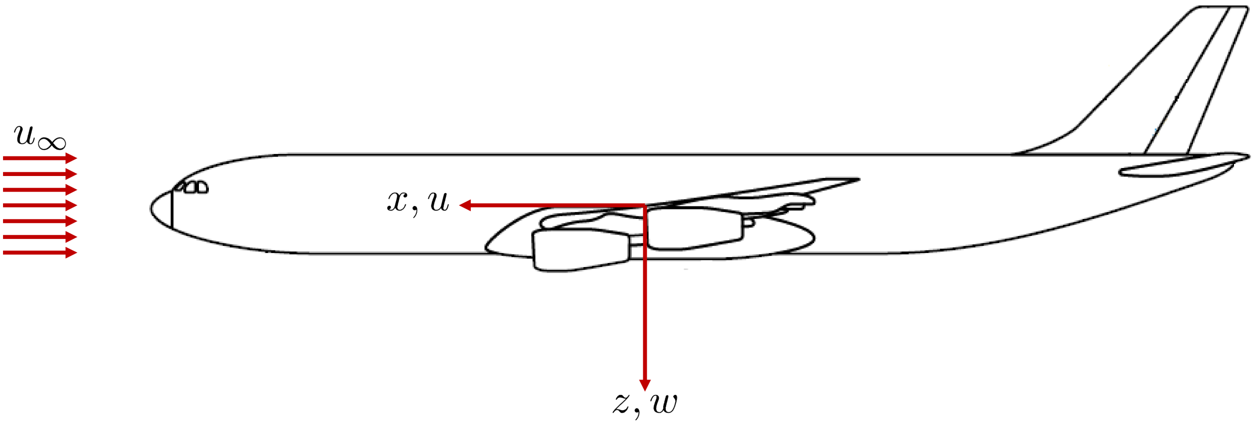


Figure 5. Aircraft axis in cruise condition

where the reference velocity, u_0, is given as zero and the exergy required to keep the system in flight, also known as the exergy of lift, which can be expressed as follows:

$$\dot{X}_L = L\left(\cancel{w} - w_0\right) \tag{17}$$

where as defined by Paulus,[14] the reference velocity w_0 cannot equal zero. Given steady cruise flight, $w = 0$, if the reference velocity was also equal to zero it suggests no exergy input is required to maintain level flight and keep the system mass aloft, which cannot be the case. To illustrate this, take the drag force due to only the lift-induced drag:

$$D = \frac{1}{2}\rho u_\infty^2 S C_{D_L} = \frac{\rho u_\infty^2 S C_L^2}{2\pi e A} \tag{18}$$

The exergy used to overcome drag given in Equation 16 can now be defined as:

$$\dot{X}_D = \left(\frac{\rho u_\infty^2 S C_L^2}{2\pi e A}\right)(u_\infty) \tag{19}$$

For ideal flight it is assumed the thrust from propulsion (equal to drag) is converted completely to lift, without any losses (thus no parasitic drag), such that $\dot{X}_D = \dot{X}_L$, so from Equation 17:

$$L\left(-w_0\right) = -\tilde{m} g w_0 = \left(\frac{\rho u_\infty^3 S C_L^2}{2\pi e A}\right) \tag{20}$$

Giving the reference velocity, w_0, in the lift axis, z, as (simplified using C_L^2 from Equation 14)

$$w_0 = -\left(\frac{\rho u_\infty^3 S C_L^2}{2\pi e A \tilde{m} g}\right) = -\left(\frac{2\tilde{m} g}{\pi e A \rho S u_\infty}\right) \tag{21}$$

This derivation shows that for a mass in flight, the ideal *optimised* system has zero parasitic drag, but the exergy use due to lift is not zero and minimised for a wing with an elliptical distribution, such that $e = 1$, giving the exergy due to lift in level cruise as:

$$\dot{X}_{min} = \dot{X}_{D_{min}} = \dot{X}_{L_{min}} = \tilde{m} g \left(\frac{2\tilde{m} g}{\pi A \rho S u_\infty}\right) \tag{22}$$

This solution is only for an idealised system, in practice additional losses and inefficiencies need to be accounted for, including the parasitic drag and the non-elliptical lift distribution, $e < 1$. The second law exergy aerodynamic efficiency can then be stated as follows:

$$\eta_{a_{II}} = \frac{\dot{X}_{min}}{\dot{X}} \tag{23}$$

For accelerated flight, $T \neq D$, so rate of increase in exergy can be defined as:

$$\dot{X}_{sys} = \dot{X}_T - \dot{X}_L - \dot{X}_{C_{D_0}} \tag{24}$$

For cruise flight, $\dot{X}_{sys} = 0$, however unlike the above derivation, thrust is not fully converted into lift because there are additional losses due to parasitic drag, therefore:

$$0 = \dot{X}_T - \dot{X}_L - \dot{X}_{C_{D_0}} \tag{25}$$

Finally, during climb or descent the assumption that $w = 0$ is no longer valid, so:

$$\dot{X}_{sys} = \dot{X}_T - L\,(w - w_0) - \dot{X}_{C_{D_0}} \tag{26}$$

If the aircraft is an isolated system within the reference environment, such that there is no energy recovery from the aircraft wake, such as that found in formation flying patterns, Oswatitsch's[15] expression linking power to entropy can be used. Firstly define, the steady flow of viscous, compressible gases, including turbulent flows with steady mean values around a body, using the following equation:

$$F = -\iint \left[\rho u w_n + p\cos(\alpha)\right] df \tag{27}$$

Oswatitsch[15] stated that drag was simply generation of entropy, and thus the generation rate of such entropy would be the rate of exergy destruction. Thus, the drag force can be defined as an integral of entropy flow, as follows:

$$\dot{X}_{des} = u_\infty F = T_\infty \iint_F (s - s_\infty)\,\rho w_n df \tag{28}$$

where

> *"The power required to move a body immersed in a fluid with the constant velocity u_∞ is equal to the temperature of the approach flow times the flow of entropy through an area which includes all entropy changes caused by the body"*[15]

The statement is related to the Guoy-Stodola thermodynamic theorem (Equation 13) which states that the decrease of useful work of a thermal machine is equal to the entropy change of the system times the surrounding temperature. In this case no useful work is done and $u_\infty F$ corresponds to the lost energy. The increase of entropy flow represents the increase of entropy per unit time of the whole system.[15]

3. *Chemical Exergy*

The chemical exergy component is more complex than other forms of exergy, as at the environmental state the chemicals themselves may be stable (thus no work potential), but when combined together may release exergy. Thus Camberos,[12] states that:

> "*chemical exergy of a pure chemical compound is equal to the maximum amount of work obtainable when a compound is brought from the environmental state to the dead state, characterised by the same environmental conditions of temperature and pressure, but also by the concentration of reference substances in a standard environment*"

Chemical exergy arises when there is a disequilibrium between the resource and environment leading to a chemical potential. This could be a potential created by a concentration gradient of species freely available in the environment, such as oxygen, carbon dioxide and methane. Or exergy arises from a non-environmental species; fuels typically fall into this category. In both scenarios work can be extracted when the resource and environment are bought into chemical equilibrium. Conversion from chemical to other forms of exergy is not without losses. In addition to the exergy losses through heat generation (entropy production) via reactions such as combustion, irreversibility is generated as the environmental species are released to the environment at their environmental dead state.

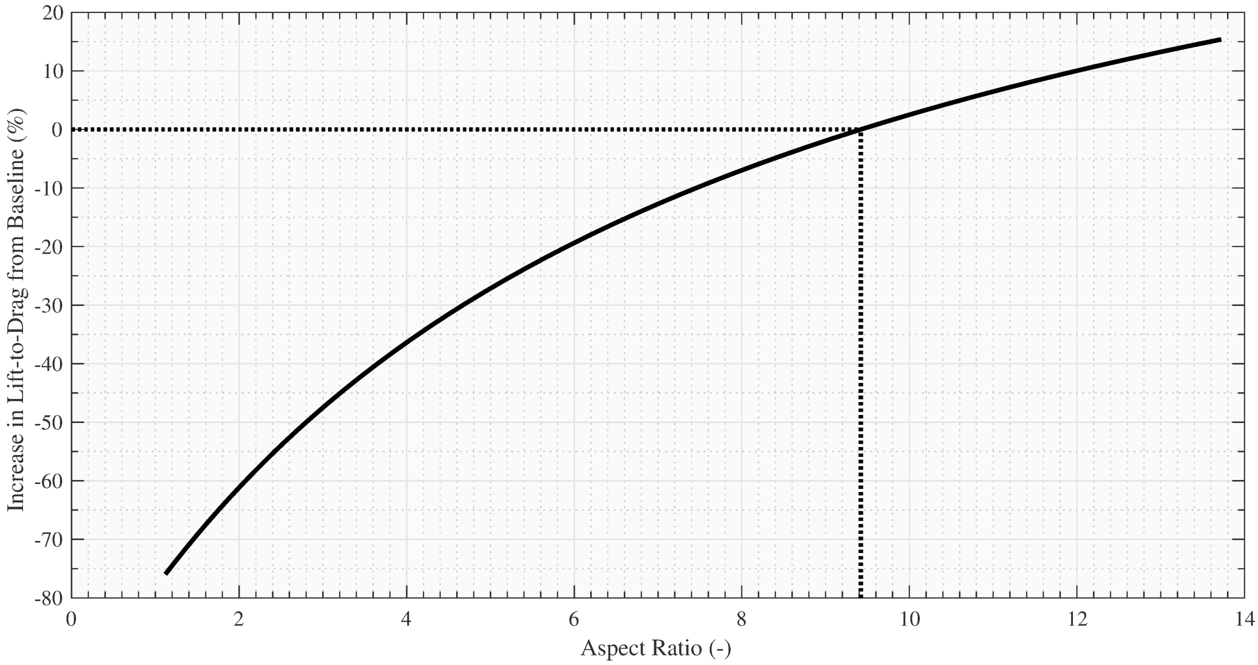


Figure 6. Cranfield AX-1 lift-to-drag ratio against increasing aspect ratio (constant S)

Camberos[12] formulates a mass derived chemical exergy (X_{CH}) (equal to the mole derived function of Simpson[16]) as:

$$X_{CH} = \sum_{i}^{n} y_i \left(\mu_k - \mu_{k_0}\right) \tag{29}$$

where the exergy is a function of the chemical potential (μ_k) and mass ratio (y_i). The chemical potential as presented from Szargut,[17] is a function of the enthalpy of formation (h_f^o) with losses from entropy production ($T_0 s_k^o$) in this combustion process, given by the following relationship:

$$\mu_k = \left(h_f^o - T_0 s_k^o\right) + RT_0 \ln x_k \tag{30}$$

Note that (μ_{k_0}) is the chemical potential at the environmental dead state where $T = T_0$ and $P = P_0$.

III. Results and discussion

A. Verification of Prandtl-Glauert Model

Given the simplification of the aircraft geometry to an equivalent wing and the use of Prandtl's[9] empirical relationships with defined assumptions, it is important to verify the output from the model before post-processing with either the Breguet or Exergy solvers. Given the main output as the aerodynamic lift and drag forces, along with the equivalent geometry, Figure 6 shows that with an increasing aspect ratio the lift-to-drag ratio also increases. The trend is verified against published results in Raymer [1, Figure 3.6]

B. Breguet Analysis

The Breguet analysis uses Equation 10 with a constant propulsive efficiency, η_p, and a variable aerodynamic, η_a, and structural, η_s efficiency based on the extended wing span and weight penalty. The percentage increase in the range compared to the baseline AX-1 against the OWE percentage mass increase is given for four different aspect ratio aircraft in Figure 7. The graph can be used in such a way that if the design of say an aspect ratio 10.4 aircraft has an additional mass increase of less than 6.5% of the baseline OWE, the design will have an improved range. However, if the additional structural and wing mass in greater than the 6.5% threshold the additional mass has a larger detrimental affect on aircraft efficiency than the aerodynamic benefits. With an increase in aspect ratio the improvements in the aerodynamic efficiency are greater and as such, the mass penalty threshold is higher before the cross over to a less efficient design to the baseline.

C. Exergy Analysis

The initial exergy reserves are calculated by the exergy of the jet fuel and/or batteries on board the aircraft. The exergy of these sources is then mapped through each conversion process with the exergy destruction highlighted at each stage, to the point of complete exergy destruction.

The combustion of standard commercial aircraft fuel, Jet A ($C_{12}H_{23}$), is given as follows:

$$C_{12}H_{23} + 17.75O_2 \rightarrow 12CO_2 + 11.5H_2O$$

Assuming environment of $T_0 = 298K$ and $P_0 = 100kPa$ standard composition of air for x_k, the exergy released in combustion of Jet A can be obtained as follows:

$$X_c = \mu_{C_{12}H_{23},TM} + 17.75\mu_{O_2,0} - 12\mu_{CO_2,0} - 11.5\mu_{H_2O,0}$$
$$X_c = 7.42 \times 10^6 \quad \text{J/mol} = 44.34 \quad \text{MJ/kg}$$

Note that assuming a constant enthalpy of formation and standard molar entropy with pressure variation, the chemical exergy of the fuel changes with altitude (variable temperature) as shown in Figure 8, where the sea level chemical exergy content (at 298K) and cruise flight exergy content (at 217K) are highlighted. This relationship suggests a decrease in propulsion efficiency with increasing altitude, where in cruise the exergy available is 98% that available at sea level. A trend that is comparable to that seen with in use turbofan engine efficiencies.

During cruise flight an exergy flow diagram (based on the visual method of Paulus[14]) is given in Figure 9, which shows the exergy rate (J/s) of different aircraft systems and how it is transferred between each system.

To compare the exergy results to the Breguet results, Figure 10 plots the same data as in Figure 7 for the Breguet output, but on the second y-axis the rate of exergy use during cruise is plotted against the weight penalty for the same aspect ratios as the Breguet method. The exergy results show the baseline AX-1

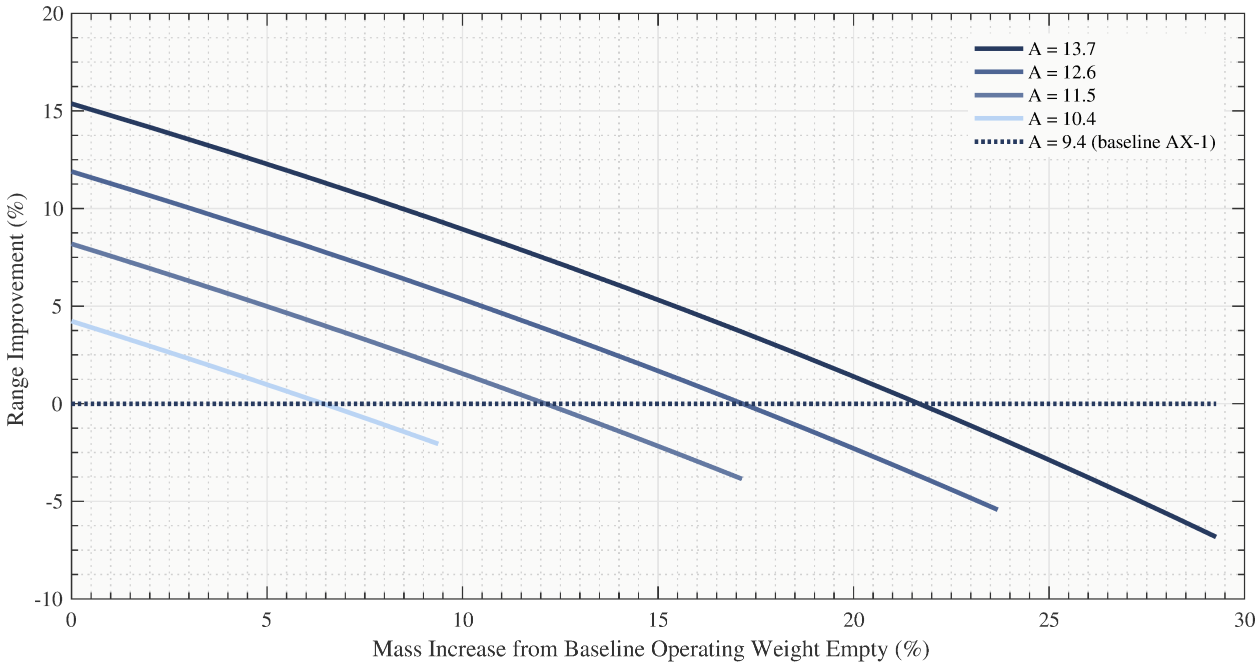


Figure 7. Weight penalty requirements generated from the Breguet approach

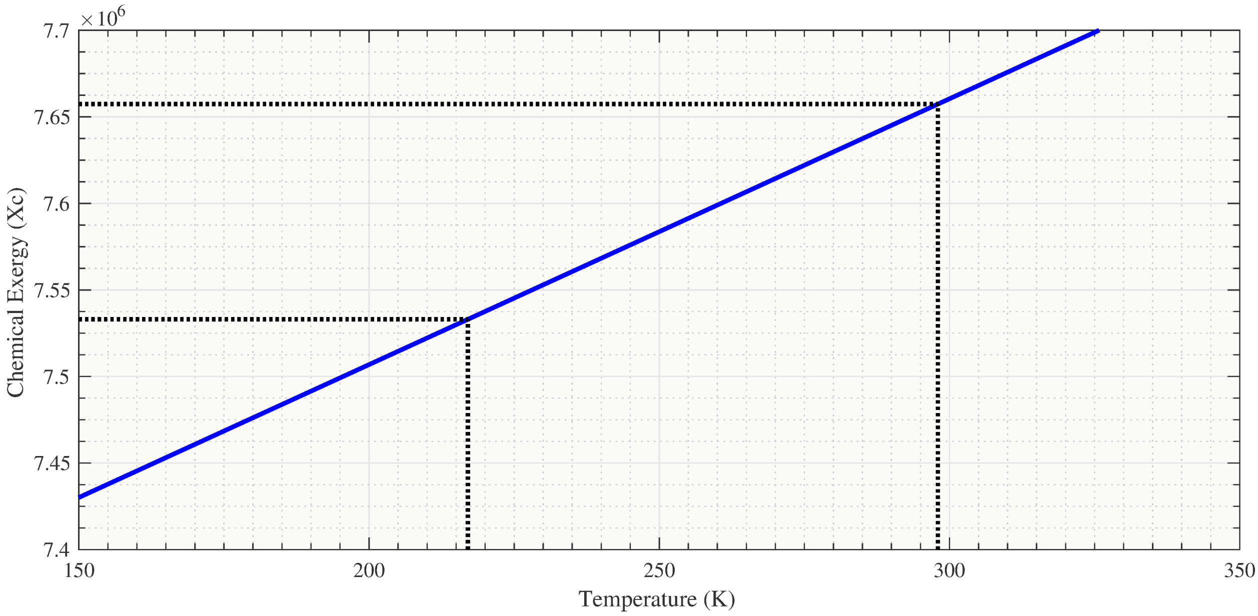


Figure 8. Variance of fuel chemical exergy with environment temperature

exergy rate during cruise, and how with a low weight increase the higher aspect ratios provide a more exergy efficient design. At the same weight increase points as the Breguet method the transition is seen between a more fuel efficient design to a less efficient design against the baseline.

One of the key advantages of the Exergy method over the Breguet approach is shown in Figure 11, where each energy using process (those modelled are combustion, aircraft sub-systems, parasitic drag and

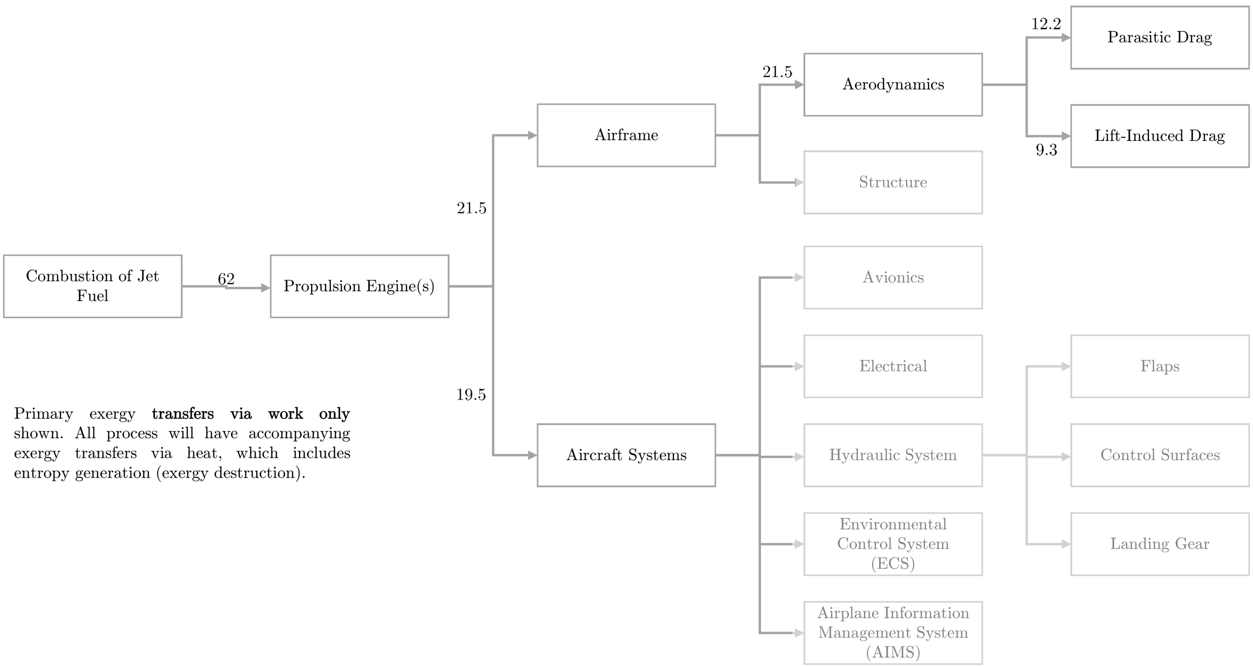


Figure 9. Exergy Flow Diagram for Baseline Cranfield AX-1 Configuration

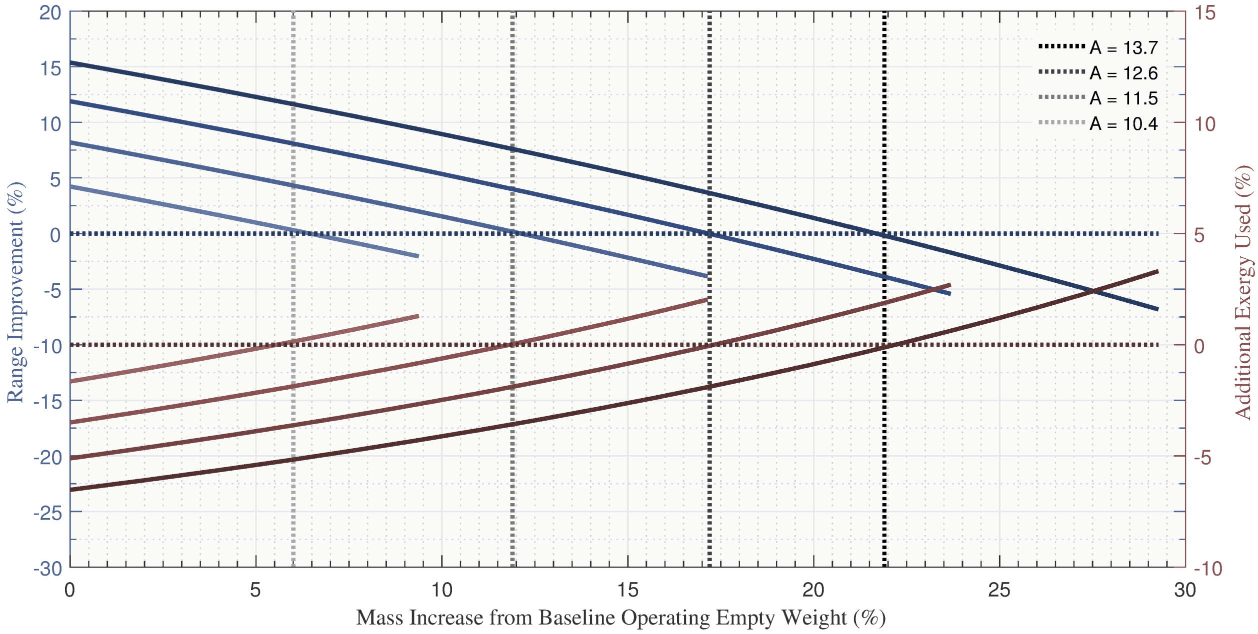


Figure 10. Comparison of the Exergy and Breguet results

lift induced drag) can be compared directly to different configurations under a common design metric of exergy.

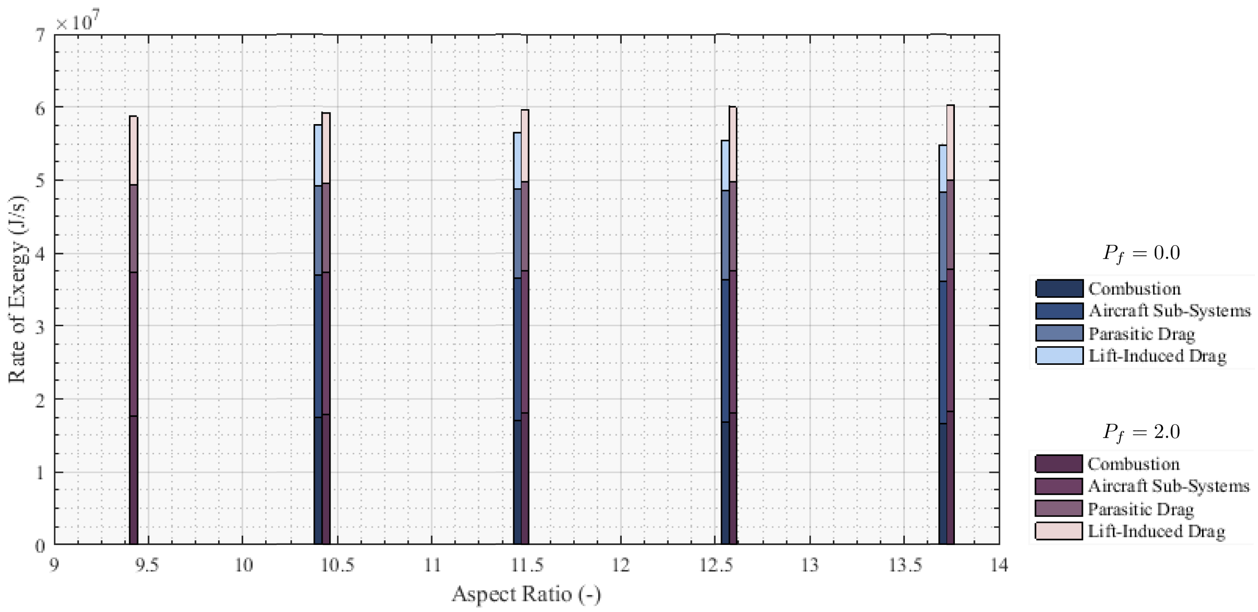


Figure 11. Exergy destroyed through different process in aircraft with variable aspect ratios

IV. Conclusions

The use of implicit and explicit energy analysis methods for the incorporation of span extended technologies into future aircraft configurations have been studied through analysis performed on a test case. Two approaches have been used: the Breguet Range Equation and the second law based Exergy Analysis. The latter leads to a methodology that can support the design of the complete vehicle as a system of systems in a common mathematical framework. A critical part of this is the development of a decomposition strategy where all the subsystem components can be optimized to a system-level common metric. It has been shown that both the Breguet and the Exergy method provide suitable output to compare different in-flight morphing mechanisms under a single metric. However, the exergy method provides a more detailed analysis method which allows energy losses to be compared to any of the aircraft's subsystems.

However, it should be noted that in order to understand the benefit or detriment of the wing tip the full mission profile must be modelled. Thus, a quasi-steady Breguet approach will be adopted for future analysis, where the mission profile is discretized into multiple phases of flight, during which the Breguet variables are assumed constant. An improved range over this quasi-steady mission profile would therefore indicate that the morphing wing tip is providing a more energy efficient aircraft.

Further development focuses on the inclusion of a Breguet analysis of a conventional configuration aircraft compared to that of an increased span configuration of the same aircraft with an incorporated folding mechanism. The two methods will then be discussed and compared to recommend which method should be used at different stages of design and at different stages of the flight envelope. In doing so, expanding past the cruise focused analysis in this paper.

Also, consider a morphing device that significantly changes the aspect ratio of the wing. In current research the wing is treated as a rigid body[18] and the benefits of drag reduction are compared to the losses in morphing the wing to this position. However, the aeroelastic properties of the wing have also been significantly altered, affecting flight and structural dynamic responses. It is for this reason the incorporation of an active and passive gust load alleviation (GLA) device in new generation HARW aircraft is sought for, and why in such an analysis the energy stored in the deformed bodies is accounted for, a capability exergy analysis has.

Acknowledgements

This research is supported by an Engineering and Physical Sciences Research Council (EPSRC) Industrial Cooperative Award in Science & Technology (CASE) grant in collaboration with Airbus Group.

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