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By Wasnaa Witwit, Yitian Zhao, Karl Jenkins & Yifan Zhao

Cranfield University

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An Optimal Factor Analysis Approach to Improve the Wavelet-based Image Resolution Enhancement Techniques

Wasnaa Witwit^α, Yitian Zhao^σ, Karl Jenkins^ρ & Yifan Zhao^ω

Abstract- The existing wavelet-based image resolution enhancement techniques have many assumptions, such as limitation of the way to generate low-resolution images and the selection of wavelet functions, which limits their applications in different fields. This paper initially identifies the factors that effectively affect the performance of these techniques and quantitatively evaluates the impact of the existing assumptions. An approach called Optimal Factor Analysis employing the genetic algorithm is then introduced to increase the applicability and fidelity of the existing methods. Moreover, a new Figure of Merit is proposed to assist the selection of parameters and better measure the overall performance. The experimental results show that the proposed approach improves the performance of the selected image resolution enhancement methods and has potential to be extended to other methods.

Keywords: super-resolution; interpolation; discrete wavelet transform (DWT).

1. INTRODUCTION

Resolution has been always an important property in images and videos. High resolution (HR) image/video has a desired and strong demand in most imaging applications as it contains more details that can be crucial in these applications [1]. Resolution enhancement based on a single low-resolution (LR) image or multiple LR images has been used for different applications in various fields, such as satellite imaging [2]–[5], medical imaging [6], [7], and video enhancement [8]–[10].

Interpolation is one of the most commonly used techniques for increasing the resolution of a digital image [11]–[13]. There are four well-known interpolation methods, namely, nearest neighbor, bilinear, bicubic, and Lanczos. Nearest neighbor interpolation is the simplest method where the intensity of the new location point is assigned as that of the old location point which is the nearest neighbor to the new point. Although it is simple to implement, it produces undesirable artefacts, such as distortion of straight edges. In the bilinear interpolation, the value of a new pixel is interpolated linearly using the four nearest neighbour pixels by taking

a weighted average of these pixels [14]. Bicubic interpolation preserves fine details better and is more complex than bilinear interpolation where sixteen nearest neighbour pixels are used to estimate the value of the new pixel by taking a weighted average of these points. This method is more efficient and accurate and has become the most popular image interpolation method [15]. Lanczos interpolation increases the capability to detect linear features [16]. However, the main drawback of most interpolation-based methods is that the produced images suffer from blurring and staircase artefacts.

Resolution enhancement techniques in the wavelet domain have attracted more and more investigations to address the problems associated with conventional interpolation methods. Wavelet-Zero Padding (WZP) is relatively simple to implement and is capable of outperforming the conventional interpolation methods but it commonly introduces artefacts such as smoothing and ringing in the neighbourhood of edges in the reconstructed HR image. Addressing this problem, a Cycle-Spinning (CS) based WZP method was proposed [17]. Hidden Markov Tree (HMT) based resolution enhancement method is capable of modelling the statistical relationships between coefficients at different scales [18], but the main drawback is that the used Gaussian model does not take into account to keep track of the sign coefficients since the Gaussian is symmetrical around zero and the signs of these coefficients are randomly generated. To reduce this shortcoming, a refined HMT based method was proposed in [19], where the magnitude parameters are estimated using the HMT model, and the sign parameters are estimated based on a higher correlation among the parameters between a high-pass filtered version of the LR image and the high-frequency sub-bands. A Directional Cycle-Spinning (DCS) method was introduced in [20], where approximates of edge orientation information are derived from a wavelet decomposition of the LR image and used to affect the choice of CS parameters. It can refine better edge orientation and prevent staircase artefacts. More recently, a new dual-tree complex wavelet transform (DT-CWT) technique [4] based on non-local-means (NLM) filter and Lanczos interpolation was proposed for resolution enhancement of satellite images. In this

Author ^α ^ρ ^ω: School of Aerospace, Transport and Manufacturing, Cranfield University, Bedfordshire MK43 0AL, UK.

e-mail: yifan.zhao@cranfield.ac.uk

Author ^σ: Beijing Engineering Research Center of Mixed Reality and Advanced Display, School of Optics and Electronics, Beijing Institute of Technology, Beijing, China.

method, the high-frequency coefficients produced by CWT and the input image are interpolated using the Lanczos interpolation. A Demirel-Anbarjafari Super Resolution (DASR) method [21] was proposed based on Discrete Wavelet Transform (DWT), where three high-frequency components produced by DWT as well as the input image are interpolated using the bicubic interpolation. An updated DASR technique was proposed in [2] with its application in satellite images. Although DWT has been used to preserve the high-frequency details of the image, but downsampling in each of the DWT sub-bands and then the interpolation of the high-frequency sub-bands generate information loss in each of these sub-bands. More recently, a technique based on DWT and stationary wavelet transform (SWT) [22] was proposed to correct the estimated high-frequency sub-bands produced using DWT by adding the high-frequency sub-bands obtained by using SWT.

A major limitation of most above methods is that the assumptions they make are not always satisfied for real applications. For example, the detail of a physical object that an optical instrument can reproduce in an image has limits that are mandated by laws of physics, whether formulated by the diffraction equations in the wave theory of light or the Uncertainty Principle for photons in quantum mechanics. There is no such a well-accepted model can fully describe the underlying mechanism. This mechanism can also be various case by case. In other words, the superior of one method than other methods claimed in the literatures is conditional. Although it has been reported that the performance of resolution enhancement methods can be affected by the methods to produce the low-resolution image, and other factors [16], there is very

limited literatures investigating how to utilise these factors to assess and improve the resolution enhancement performance. Addressing this problem, this paper proposes an Optimal Factor Analysis method to increase the applicability and fidelity of the existing methods.

Although the authors are aware that machine-learning-based super-resolution methods have attracted more and more interests recently [23]–[28], this paper focuses on wavelet-based methods and interpolation methods only. Section 2 initially identifies the important factors that affect the performance and analyses corresponding importance, and then proposes the new method as well as a new Figure of Merit to assist the selection of parameters. Section 3 presents the results of quantitative analysis using the proposed method and associated discussions. Conclusions are presented in the final section.

II. METHOD

a) Important Factors

Table 1 summarises the reviewed wavelet-based image resolution enhancement techniques in terms of the way to evaluate their performance. The inconsistency of assumption of the considered factors for each individual technique has been observed. For example, the considered methods make the assumption that the observed LR image is produced by either applying a low-pass filtering and then downsampling, or achieving the low-frequency (LL) sub-band of DWT. For some methods, the description of these factors is either neglected or unclear. The performance of these methods is unknown when such an assumption is not satisfied. A method to compare the resolution enhancement methods in a more comprehensive

Table 1: Summary of different wavelet-based resolution enhancement techniques in terms of performance assessment

Techniques	Input LR Image	Scale Factor	Interpolation Method	Wavelet Function	Test Image
WZP-CS [17]	LL sub-band of DWT	2 & 4	N/A	Db.9/7	Lena, Elaine, Baboon, and Peppers
WZP-DCS [20]	Low-pass filtering and downsampling	2 & 4	N/A	Db.9/7	Lena, Elaine, Baboon, and Peppers
HMT [18]	Downsampling of HR image	N/A	N/A	N/A	Lena
HMT [5]	Downsampling of HR image	2	N/A	N/A	Lena
HMT [19]	low-pass filtering and downsampling	2 & 4	N/A	Db.9/7	Lena, Elaine, Baboon, and Peppers
CWT [3]	LL sub-band of DWT	2 & 4	Bicubic	N/A	5 Satellite Images

DT-CWT [4]	Downsampling of HR image	4	Lanczoc	N/A	1 Satellite Image (Washington DC)
DASR [21]	LL sub-band of DWT	4	Bicubic	Db.9/7	Lena, Elaine, Baboon, and Peppers
DWT-Difference [2]		4	Bicubic	Db.9/7	5 Satellite Images
DWT-SWT [22]		4	Bicubic	Db.9/7	Lena, Elaine, Baboon, and Peppers
DWT-SWT [6]		4	Bicubic	N/A	Lena, Elaine, Head, and Brain

and equitable way is required. Such a method can also be used to further improve the overall performance of existing methods.

Each potential factor that affects the performance has been studied one by one. In order to quantitatively evaluate the performance, the widely used Peak-signal-to-noise-ratio (PSNR) has been employed in this paper, and it can be calculated by

$$PSNR = 10 \log_{10} \left(\frac{L^2}{MSE} \right) \quad (1)$$

where L denotes the maximum fluctuation in the input image. Mean Square Error (MSE) measures the error between the super resolved image ISR and the original HR image $Iorg$. It can be calculated by

$$MSE = \frac{\sum_{i,j} (ISR(i,j) - Iorg(i,j))^2}{M \times N} \quad (2)$$

Table 2: PSNR results for Lena image using different techniques for resolution enhancement from 128×128 to 512×512 for several LR image generation methods

Techniques	PSNR (dB)					
	DWT by DB.9/7	DWT by Haar	Bicubic	Bilinear	Nearest	Low-pass
WZP(haar)	22.36	25.77	25.75	25.19	24.35	25.18
WZP(db.9/7)	24.22	25.75	25.73	25.23	23.21	24.04
Bicubic	22.51	26.31	26.28	25.75	24.80	25.67
Bilinear	22.63	25.53	25.54	24.85	24.87	25.21
Nearest	21.53	24.71	24.61	24.44	22.79	23.97

methods for the input image produced by DWT with the wavelet function db.9/7. For the LR images obtained by DWT (Haar), bicubic, bilinear interpolations and low-pass filtering methods, the bicubic interpolation method has the best performance, but for the LR images produced by the nearest neighbor, the bilinear interpolation technique has the best performance. These observations clearly indicate that the method to produce LR images has significant effect on the performance of different techniques.

ii. Wavelet Function

There are several well-known wavelet families such as Daubechies (db) (db.1 is also referred as Haar),

where M, N denote the width and height of the HR image respectively.

i. The mechanism to produce low-resolution images

It has been identified from the literature review that there are various ways to generate LR image including (a) downsampling of the original HR image through DWT, (b) bicubic interpolation, (c) bilinear interpolation, (d) nearest neighbour, and (e) low-pass filtering. Table 2 shows the resulting PSNR values for the Lena image using different resolution enhancement methods by considering different LR image generation methods. Inspection of Table 2 shows that WZP with the wavelet function db.9/7 has the best performance among the considered

coif2, sym3, and db3 are top three wavelet functions in terms of PSNR values, not the well investigated Haar or bior4.4. This observation is consistent for all three test images. This observation indicates that the selection of wavelet function can play a key role in improving performance. However, in most of existing wavelet-based methods, very few of them has discussed the selection of wavelet function.

iii. *Enlargement Factor*

As shown in Table 1, the performance of most methods are evaluated by a scale factor of 2 or 4. To better evaluate the effectiveness of this factor on performance, this paper considered a

Table 3: PSNR results for three well-known test images (Lena, Baboon, and Elaine) generated using DWT with db.9/7 using different techniques for resolution enhancement from 128×128 to 512×512 of various wavelet families and parameters

Techniques	PSNR (dB)		
	Lena	Baboon	Elaine
Bicubic	22.51	24.21	25.49
Bilinear	22.63	24.23	25.52
Nerest	21.53	23.49	24.40
WZP (haar)	22.36	24.09	25.31
WZP (bior 1.1)	22.36	24.09	25.31
WZP (bior 2.2)	24.19	25.19	27.40
WZP (bior 3.1)	22.64	24.27	25.57
WZP (bior 4.4)	24.22	25.23	27.46
WZP (bior 5.5)	24.13	25.19	27.41
WZP (bior 6.8)	24.22	25.22	27.44
WZP (sym2)	25.32	25.67	28.77
WZP (sym3)	26.45	26.26	30.15
WZP (sym7)	25.59	25.91	28.90
WZP (sym15)	25.56	25.88	29.08
WZP (sym19)	26.15	26.13	29.86
WZP (coif1)	24.16	25.18	27.39
WZP (coif2)	26.56	26.30	30.28
WZP (coif3)	24.08	25.16	27.41
WZP (db2)	25.32	25.76	28.77
WZP (db3)	26.45	26.26	30.15
WZP (db4)	24.21	25.23	27.51

wide range of scale of 2, 4, 8 and 16. The input LR image has been obtained by downsampling using DWT with db.9/7 wavelet function. The produced PSNR values for Lena image are shown in Table 4, inspection of which shows that the WZP method with db.9/7 produces the highest PSNR values for all enlargement factors. Lanczos and bicubic techniques provide higher PSNR values than bilinear technique for the scale of 2 but for the scale of 4 and 8 the bilinear technique produces higher PSNR values. The variation on performance of the





Table 4: PSNR results for Lena image generated using DWT with db.9/7 for enlargement factors of 2, 4, 8 and 16 using different techniques

Techniques	PSNR (dB)			
	Factor 2	Factor 4	Factor	Factor 16
WZP(db.9/7)	32.93	24.22	19.89	17.22
WZP(haar)	26.44	22.36	19.26	16.97
Bicubic	28.05	22.51	19.28	16.98
Lanczos	28.06	22.39	19.16	16.83
Bilinear	27.77	22.63	19.46	17.22
Nearest	26.44	21.53	18.60	16.32

considered methods decreases following the increase of scale factor, which indicates that the scale factor is an important factor to be considered for performance assessment.

iv. Interpolation Function

Because of the obvious weakness, in this paper, the nearest method has been neglected, and bilinear, bicubic and Lanczos have been tested. The input LR image has been produced by downsampling using DWT with db.9/7 wavelet function. The PSNR results for Lena image are shown in Table 5, inspection of which indicates that there is no significant difference in performance for different interpolation methods. Moreover, the interpolation method producing the highest PSNR is not consistent for different methods. These observations indicate that the selection of interpolation function for wavelet-based techniques can affect the performance, but not significantly.

b) Optimal Factor Analysis

The behaviour of resolution enhancement methods has been assessed above by varying one factor and fixing other factors, which aims to identify the important factors but it cannot reveal the best technique with the optimal parameter selection. Addressing this challenge, this paper proposes an Optimal Factors Analysis (OFA) approach in order to increase the performance of the existing methods, and also better assess their overall performance.

OFA considers a resolution enhancement technique, \emptyset , as a Multi-Input and Multi-Output (MIMO) model, which includes 5 inputs variables: the way to produce LR image $LR_a(a=1,2,\dots,A)$, the scale factor $SF_b(b=1,2,\dots,B)$, the testing image $TI_c(c=1,2,\dots,CW)$, the

Table 5: PSNR results for Lena image generated using DWT with db.9/7 for resolution enlargement factor from 128×128 to 512×512 using different techniques

Techniques	PSNR (dB)		
	Bicubic	Lanczos	Bilinear
WZP+CS(db.9/7)	24.23	24.18	24.05
WZP(db.9/7)	24.22	24.18	24.05
WZP(haar)	22.36	22.23	22.52
WZP(coif2)	26.56	26.88	25.60

wavelet function $WF_d(d=1,2,\dots,D)$, and the interpolation method $IM_e(e=1,2,\dots,E)$, where A, B, C, D , and E are the total number of possible states for 5 variables respectively. There are three outputs including the highest PSNR value $PSNR^*$, the optimal wavelet function WF^* and the optimal interpolation method IM^* . The MIMO model can therefore be written as:

$$(PSNR^*, WF^*, IM^*) = F_{\emptyset}(LR_a, SF_b, TI_c, WF_d, IM_e) \quad (3)$$

Depending on the value of A, B, C, D , and E , Eq. (3) can be solved by either an exhausted search or

advanced optimisation techniques. In this paper the Genetic Algorithm was employed.

To better compare the overall performance, this paper introduces a new Figure of Merit (FoM), called Ratio of PSNR (RPSNR) that considers the 'bicubic' interpolation as the baseline. For a testing image TI_c , a way to produce LR images LR_a , and a scale factor SF_b , RPSNR of the technique \emptyset can be written as

$$RPSNR_{\emptyset}(LR_a, SF_b, TI_c) = \frac{\max PSNR_{\emptyset}(LR_a, SF_b, TI_c, WF_d, IM_e)}{PSNR_{\emptyset}(LR_a, SF_b, TI_c, 'bicubic')} \quad (4)$$

A higher *RPSNR* indicates a better performance. To collectively assess the performance of \emptyset over all

considered factors, the averaged *RPSNR* is introduced and expressed as

$$\overline{RPSNR}_{\emptyset} = \frac{1}{A \times B \times C} \sum_{a=1}^A \sum_{b=1}^B \sum_{c=1}^C RPSNR_{\emptyset}(LR_a, SF_b, TI_c) \quad (5)$$

c) *Results and Discussions*

This study considered Six methods to generate input LR images ($A=6$), including DWT with db. 9/7 wavelet function, DWT with Haar wavelet function, bicubic, bilinear, nearest, and low-pass filtering. Three scale factors 2, 4, and 8 ($B=3$) and Three testing ($C=3$) including Lena, Baboon, and Elaine were tested.

Considered wavelet functions include Daubechies (db.1 to db.20), Symlets (sym.2 to sym.20), Coiflets (coif.1 to coif.5) and Biorthogonal (bior1.1 to bior6.8). Considered resolution enhancement techniques can be classified into five groups: interpolation methods and four WZP based methods with different wavelet families (WZP+db,

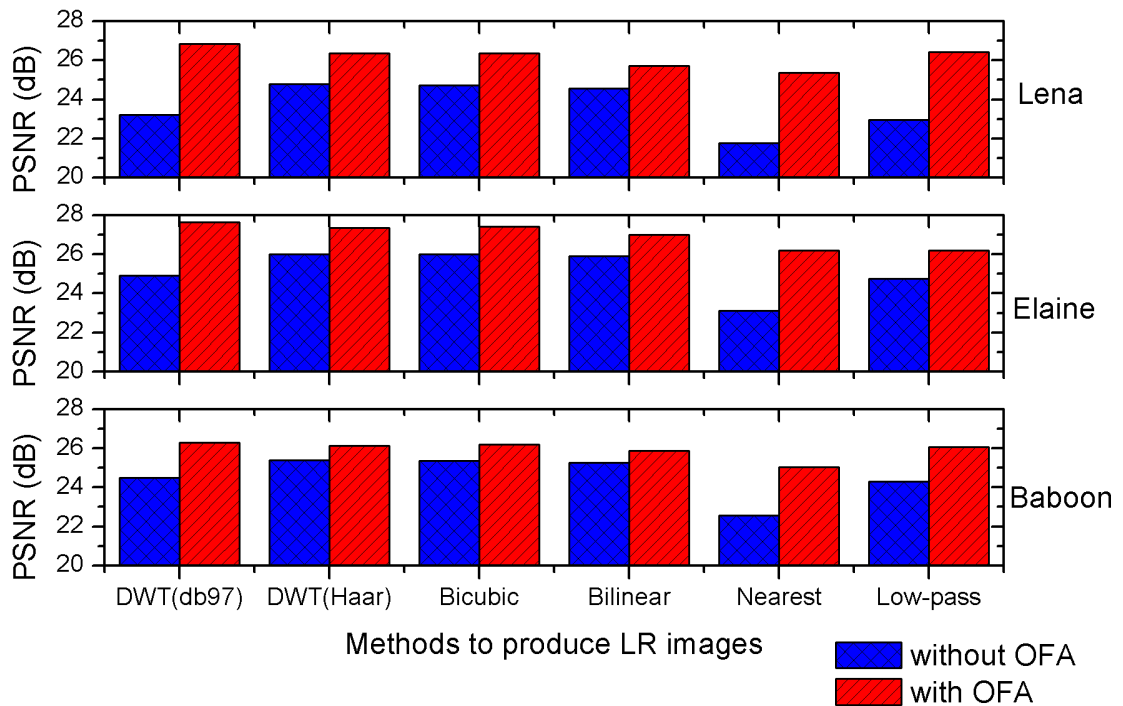


Figure 1: Performance improvement for the WZP technique by applying the proposed OFA method for the scale factor of 4

WZP+sym, WZP+coif, and WZP+bior). Three interpolation methods ($E=3$) were considered, namely bilinear, bicubic and Lanczos.

Fig. 1 illustrates the performance of the WZP method before and after applying the proposed method, where the LR images were super-resolved from 128×128 to 512×512 . The blue and red bars plot the PSNR values before and after applying OFA respectively. It is clearly shown that the proposed method significantly improves the performance for all 7 ways to produce LR image and all three tested images.

Table 6 shows the results including the best-performed method with its parameter selection, as well as the highest PSNR and RPSNR value for different factors. For the LR image obtained from DWT with db. 9/7 wavelet function, the optimal class corresponding

with the optimal interpolation method is WZP using "sym" with bilinear interpolation for the Lena image with scale factor 2. However, for the Baboon and the Elaine images, the best class is WZP using "bior" with bilinear interpolation. For scale factor 4 and 8, the best class with the best interpolation method is WZP using "coif" with Lanczos interpolation for all three images. For

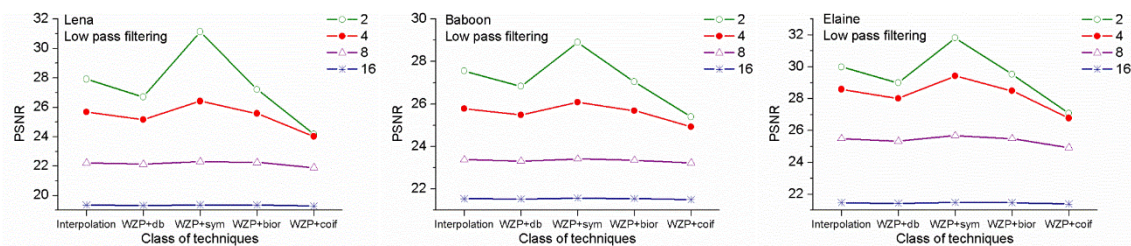
Table 6: Highest PSNR results and RPSNR corresponding with optimal super resolution techniques and interpolation method for Lena, Baboon and Elaine images with three scale factors 2, 4, and 8

Testing image	Scale factor	Methods to produce LR image					
		DWT by DB.9/7	DWT by Haar	Bicubic	Bilinear	Nearest	Low-pass
Lena	2	WZP(sym20) Bilinear 32.98(1.1762)	Interpolation Lanczos 32.63(1.0221)	Interpolation Lanczos 32.48(1.0237)	Interpolation Lanczos 30.85(1.0218)	WZP(sym18) Bilinear 31.22(1.1321)	WZP(sym18) Bilinear 31.18(1.1233)
	4	WZP(coif2) Lanczos 26.88(1.1945)	Interpolation Lanczos 26.56(1.0097)	Interpolation Lanczos 26.58(1.0117)	Interpolation Lanczos 25.89(1.0126)	WZP(sym18) Bicubic 25.40(1.0244)	WZP(sym18) Lanczos 26.45(1.0305)
	8	WZP(coif4) Lanczos 23.14(1.2008)	Interpolation Lanczos 23.05(1.0058)	Interpolation Lanczos 23.09(1.0078)	Interpolation Lanczos 22.64(1.0096)	WZP(sym8) Bilinear 21.76(1.0051)	WZP(sym9) Bicubic 22.35(1.0035)
Baboon	2	WZP(bior4.4) Bilinear 30.09(1.0733)	Interpolation Lanczos 29.65(1.0035)	Interpolation Lanczos 29.68(1.0088)	WZP(sym13) Bilinear 28.98(1.0100)	WZP(sym6) Bilinear 28.09(1.0375)	WZP(sym18) Bilinear 29.21(1.0546)
	4	WZP(coif2) Lanczos 26.44(1.0903)	Interpolation Lanczos 26.34(1.0028)	Interpolation Lanczos 26.40(1.0052)	Interpolation Lanczos 26.04(1.0063)	WZP(sym18) Bilinear 25.25(1.0257)	WZP(sym18) Bicubic 26.22(1.0119)
	8	WZP(coif4) Lanczos 24.22(1.1063)	Interpolation Lanczos 24.06(1.0030)	Interpolation Lanczos 24.10(1.0050)	WZP(bior5.5) Lanczos 23.86(1.0066)	WZP(bior3.1) Bilinear 22.76(1.0219)	WZP(sym6) Bilinear 23.49(1.0038)
Elaine	2	WZP(bior4.4) Bilinear 34.96(1.0824)	Interpolation Lanczos 34.54(1.0043)	Interpolation Lanczos 34.56(1.0073)	Interpolation Lanczos 33.56(1.0084)	WZP(sym6) Bilinear 32.71(1.0402)	WZP(sym18) Lanczos 33.73(1.0652)
	4	WZP(coif2) Lanczos 30.64(1.1785)	Interpolation Lanczos 30.42(1.0090)	Interpolation Lanczos 30.49(1.0096)	Interpolation Lanczos 29.70(1.0111)	WZP(sym18) Bicubic 29.35(1.0259)	WZP(sym18) Lanczos 30.39(1.0323)
	8	WZP(coif4) Lanczos 26.58(1.2371)	Interpolation Lanczos 26.60(1.0121)	Interpolation Lanczos 26.63(1.0124)	Interpolation Lanczos 25.89(1.0139)	WZP(sym17) Bicubic 25.26(1.0069)	WZP(sym17) Bicubic 26.08(1.0073)

the LR image obtained from DWT with Haar, bicubic, and bilinear, the best technique with the highest PSNR value is Lanczos interpolation for most of the cases. For the LR image produced by nearest and low-pass filtering, the best class is WZP using "sym" for almost all cases. These observations conclude that, for the LR image obtained from DWT with db. 9/7 wavelet function, nearest and low-pass filtering, the wavelet-based techniques have the biggest potential to outperform the conventional interpolation methods, due the fact that they have relatively large RPSNR values. For the LR

image obtained from Haar, bicubic, and bilinear, the wavelet-based methods have no significant advantages over the interpolation methods. This justifies that for almost all papers about wavelet-based techniques, the LR image was produced by either DWT with db. 9/7 wavelet function or low-pass filtering.

In order to show the sensitivity for the selection of class of technique with different scale factors, input LR image producing methods and test images, the highest PSNR value for each



(a)

(b)

(c)

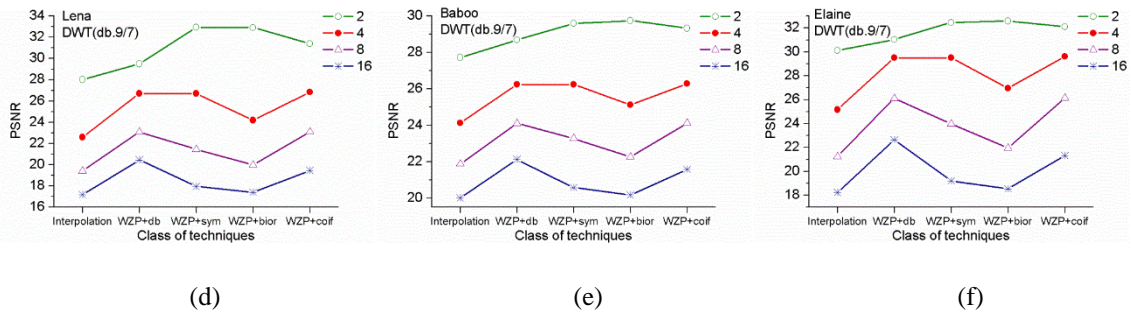


Figure 2: Highest PSNR values for each class of super resolution technique for different test images and low resolution image producing ways. (a) Lena + low pass filtering; (b) Baboon + low pass filtering; (c) Elaine + low pass filtering; (d) Lena + DWT with db. 9/7; (e) Baboon + DWT with db. 9/7; Elaine + DWT with db. 9/7

class of technique has been detected and the results are shown in Fig. 2. The standard deviation (std) for each scale factor has been calculated to describe the performance variation of each class. Table 7 shows the std values for the three test images generated by low-pass filtering and DWT with db.9/7 respectively for scale factors 2, 4, 8 and 16. A high std value indicates that the

selection of class is important because the performance for different classes of techniques is significantly varied. A low std value indicates that the performance for each class of technique is relatively similar. Fig. 2 (a), (b) and (c) illustrate the sensitivity of the class

Table 7: Standard deviation results for three test images (Lena, Baboon, and Elaine) obtained by low-pass filtering and DWT with db. 9/7 for scale factors 2, 4, 8, and 16

	Lena		Baboon		Elaine	
Scale	Low-pass	DWT by db. 9/7	DWT by db. 9/7	Low-pass	DWT by db. 9/7	
Factor	Low-pass					
	pass	9/7	9/7	pass	9/7	
2	2.50	2.16	1.27	0.83	1.71	1.05
4	0.88	1.92	0.43	0.99	0.97	2.01
8	0.16	1.70	0.07	1.03	0.28	2.27
16	0.03	1.40	0.03	0.92	0.04	1.91

selection for Lena, Baboon, and Elaine respectively with the LR image obtained by low-pass filtering. It is observed that if the scale factor is high, the PSNR is low as expected, and importantly the std is low. This observation means that different classes of techniques have similar performance for a larger scale factor and, as a result, the selection of class of techniques is less important. On the contrast, the selection of class of technique is very important if the scale factor is low. To demonstrate the superiority of the technique comparing with others, if the low-resolution image is generated by low-pass filtering, a small scale factor is recommended.

However, for the LR images obtained by DWT with db. 9/7, the result of sensitivity analysis is different, as illustrated in Fig. 2 (d), (e) and (f). The values of std show that the selection of class of technique has significant effect on the results, and it is almost independent on the scale factor. In other words, the selection of scale factor to demonstrate the superior of a new technique is not important. Another observation is that the above conclusions are almost independent on

test images due to the fact that Fig.2 (a), (b) and (c) have similar patterns, as well as Fig.2 (d), (e) and (f).

III. CONCLUSIONS

The wavelet-based image resolution enhancement techniques have been reviewed in this paper, especially the way to assess the performance. The inconsistency of assumptions has been observed, and for some methods, the description of these assumptions is either neglected or unclear. Due to the fact that the laws of physics to generate LR images are unclear and also various case by case, the current ways to assess performance assumptions may result in a biased conclusion. The importance of each factor has then been analysed by varying this factor and fixing other factors. It has been revealed that the way of producing LR image, the variation of wavelet family and its wavelet functions, and the scale factor can substantially affect the performance of techniques. The selection of testing images with different features as well as the selection in of interpolation method can influence

performance moderately. An optimal factor analysis approach has been proposed in this paper in order to improve the performance of existing techniques and better evaluate the overall performance of a technique. The OFA approach selects the optimal technique (including the selection of wavelet family as well as its wavelet functions and interpolation method) by simultaneously varying the way of producing LR image, enlargement factor, and testing images. The quantitative results reveal that the proposed method can significantly improve the performance of the WZP method. It also has potential to be extended to other wavelet-based methods. Results also reveal that the most important factors that have effectiveness on the performance are the method of producing LR image and the selection of wavelet function. For most of existing wavelet-based resolution enhancement techniques, the selection of these factors is very limited or never considered. The experimental results also indicate that the interpolation method has no significant effect in performance and the best interpolation method is not consistent for different techniques. More precisely, the selection of interpolation method for wavelet-based techniques can affect the performance, but this effect is not distinct. For the LR images obtained by downsampling using DWT with db.9/7, nearest neighbour, and low-pass filtering, wavelet-based techniques have the biggest potential to overtake the conventional interpolation methods. However, for the LR images produced by DWT with Haar, Bicubic, and Bilinear interpolation, wavelet-based techniques have no pronounced improvements over conventional interpolation methods. All these observations conclude that in order to assess more comprehensively and equitably for resolution enhancement techniques, variation of LR image generation method, scale factor, and wavelet functions must be considered, otherwise observed performance could be limited and biased.

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REFERENCES RÉFÉRENCES REFERENCIAS

1. Sung Cheol Park, Min Kyu Park, and Moon Gi Kang, "Super-resolution image reconstruction: a technical overview," *IEEE Signal Process. Mag.*, vol. 20, no. 3, pp. 21–36, May 2003.
2. H. Demirel and G. Anbarjafari, "Discrete Wavelet Transform-Based Satellite Image Resolution Enhancement," *IEEE Trans. Geosci. Remote Sens.*, vol. 49, no. 6, pp. 1997–2004, Jun. 2011.
3. H. Demirel and G. Anbarjafari, "Satellite Image Resolution Enhancement Using Complex Wavelet Transform," *IEEE Geosci. Remote Sens. Lett.*, vol. 7, no. 1, pp. 123–126, Jan. 2010.
4. M. Z. Iqbal, A. Ghafoor, and A. M. Siddiqui, "Satellite Image Resolution Enhancement Using Dual-Tree Complex Wavelet Transform and Nonlocal Means," *IEEE Geosci. Remote Sens. Lett.*, vol. 10, no. 3, pp. 451–455, May 2013.
5. Shubin Zhao, Hua Han, and Silong Peng, "Wavelet-domain HMT-based image super-resolution," in *Proceedings 2003 International Conference on Image Processing (Cat. No.03CH37429)*, vol. 3, pp. II–953–6.
6. A. Yavariabdi, C. Samir, and A. Bartoli, "3D medical image enhancement based on wavelet transforms," in *Conference: Medical Image Understanding and Analysis, 2011*, pp. 172–176.
7. X. Lu, Z. Huang, and Y. Yuan, "MR image super-resolution via manifold regularized sparse learning," *Neurocomputing*, vol. 162, pp. 96–104, Aug. 2015.
8. G. Anbarjafari, S. Izadpanahi, and H. Demirel, "Video resolution enhancement by using discrete and stationary wavelet transforms with illumination compensation," *Signal, Image Video Process.*, vol. 9, no. 1, pp. 87–92, Jan. 2015.
9. B. Ning and X. Gao, "Multi-frame image super-resolution reconstruction using sparseco-occurrence prior and sub-pixel registration," *Neurocomputing*, vol. 117, pp. 128–137, Oct. 2013.
10. K. Zhang, G. Mu, Y. Yuan, X. Gao, and D. Tao, "Video super-resolution with 3D adaptive normalized convolution," *Neurocomputing*, vol. 94, pp. 140–151, Oct. 2012.
11. Hsieh Hou and H. Andrews, "Cubic splines for image interpolation and digital filtering," *IEEE Trans. Acoust.*, vol. 26, no. 6, pp. 508–517, Dec. 1978.
12. R. Keys, "Cubic convolution interpolation for digital image processing," *IEEE Trans. Acoust.*, vol. 29, no. 6, pp. 1153–1160, Dec. 1981.
13. J. A. Parker, R. V. Kenyon, and D. E. Troxel, "Comparison of Interpolating Methods for Image Resampling," *IEEE Trans. Med. Imaging*, vol. 2, no. 1, pp. 31–39, Mar. 1983.
14. R. C. Gonzalez and R. E. Woods, *Digital image processing*. NJ: Prentice Hall, 2007.
15. E. Maeland, "On the comparison of interpolation methods," *IEEE Trans. Med. Imaging*, vol. 7, no. 3, pp. 213–217, 1988.
16. S. Azam, F. Tuz Zohra, and M. M. Islam, "A State-of-the-art Review on Wavelet Based Image Resolution Enhancement Techniques: Performance Evaluation Criteria and Issues," *Int. J. Image, Graph. Signal Process.*, vol. 6, no. 9, pp. 35–46, Aug. 2014.
17. A. Temizel and T. Vlachos, "Wavelet domain image resolution enhancement using cycle-spinning," *Electron. Lett.*, vol. 41, no. 3, p. 119, 2005.
18. K. Kinebuchi, D. D. Muresan, and T. W. Parks, "Image interpolation using wavelet based hidden Markov trees," in *2001 IEEE International*

- Conference on Acoustics, Speech, and Signal Processing. Proceedings (Cat. No.01CH37221), vol. 3, pp. 1957–1960.
19. A. Temizel, "Image Resolution Enhancement using Wavelet Domain Hidden Markov Tree and Coefficient Sign Estimation," in 2007 IEEE International Conference on Image Processing, 2007, pp. V – 381–V – 384.
 20. T. Vlachos, "Image resolution upscaling in the wavelet domain using directional cycle spinning," *J. Electron. Imaging*, vol. 14, no. 4, p. 040501, Oct. 2005.
 21. G. Anbarjafari and H. Demirel, "Image Super Resolution Based on Interpolation of Wavelet Domain High Frequency Subbands and the Spatial Domain Input Image," *ETRI J.*, vol. 32, no. 3, pp. 390–394, Jun. 2010.
 22. H. Demirel and G. Anbarjafari, "IMAGE Resolution Enhancement by Using Discrete and Stationary Wavelet Decomposition," *IEEE Trans. Image Process.*, vol. 20, no. 5, pp. 1458–1460, May 2011.
 23. Y. Liang, J. Wang, S. Zhou, Y. Gong, and N. Zheng, "Incorporating image priors with deep convolutional neural networks for image super-resolution," *Neurocomputing*, vol. 194, pp. 340–347, Jun. 2016.
 24. X. Zhao, Y. Wu, J. Tian, and H. Zhang, "Single Image Super-Resolution via Blind Blurring Estimation and Dictionary Learning," *Neurocomputing*, Jun. 2016.
 25. Y. Tang, H. Chen, Z. Liu, B. Song, and Q. Wang, "Example-based super-resolution via social images," *Neurocomputing*, vol. 172, pp. 38–47, Jan. 2016.
 26. J. Li, Y. Qu, C. Li, Y. Xie, Y. Wu, and J. Fan, "Learning local Gaussian process regression for image super-resolution," *Neurocomputing*, vol. 154, pp. 284–295, Apr. 2015.
 27. Jianbing Shen, Yunfan Du, Wenguan Wang, and Xuelong Li, "Lazy Random Walks for Superpixel Segmentation," *IEEE Trans. Image Process.*, vol. 23, no. 4, pp. 1451–1462, Apr. 2014.
 28. J. Shen, Y. Du, and X. Li, "Interactive Segmentation Using Constrained Laplacian Optimization," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 24, no. 7, pp. 1088–1100, Jul. 2014.
 29. R. C. Gonzalez, R. E. Woods, and S. L. Eddins, *Digital Image Processing Using MATLAB*. Pearson Education, 2004.
 30. M. Antonini, M. Barlaud, P. Mathieu, and I. Daubechies, "Image coding using wavelet transform," *IEEE Trans. Image Process.*, vol. 1, no. 2, pp. 205–220, Apr. 1992.