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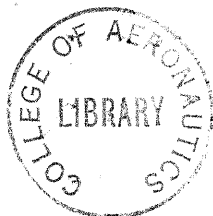
SOLUTION OF A LOAD DIFFUSION PROBLEM
BY RELAXATION METHODS

by

G. VAISEY, M.A., and W. S. HEMP, M.A.

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Solution of a Load Diffusion Problem by Relaxation
Methods

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SUMMARY

The need to generalise the usual assumptions made in the analysis of load diffusion problems has been emphasised by recent experimental work (Ref. 3), which has shown the importance of bending of the edge members. Direct mathematical solution of the plate problems, which arise, is hardly feasible and so in this report a numerical solution using the 'relaxation method' is carried out. Results show the method to be suitable for design purposes, but comparison with experiment still shows the need for further physical generalisations. These will form the subject of future work.

MEP

Introduction

The problem of the diffusion of load from edge members into a panel of stringer reinforced skin has been the subject of numerous theoretical investigations (e.g. Refs 1,2). In the majority of these the assumption is made that the transverse direct strain component can be neglected, with the consequence that the edge members remain straight. In some solutions for example that of Ref. 2, a general two-dimensional system of stress and strain is assumed to exist in the panel, but in this case mathematical difficulties limit the known solutions to the cases where the edge member has infinitely small flexural stiffness. Recent experimental evidence (Ref. 3) has shown however, the importance of edge member bending and the accompanying transverse direct stresses. It is therefore necessary for purposes of design to have available, methods, which will allow for these effects. Since direct mathematical solution does not seem to be feasible, the present report tackles the diffusion problem by the numerical method of 'relaxation'.

Formulation of the Problem

The problem considered in this report is illustrated in Fig. 1. It is the 'classical' diffusion problem of the literature, for the special case of edge members of constant area. The skin is shown as reinforced by stringers of area A_s and pitch a_s . It will be assumed that these members are distributed over the skin to form a uniform 'orthotropic' plate.

The components of displacement in the plate are written (u,v) and the corresponding strain components e_{xx} , e_{yy} and e_{xy} are then given by

$$e_{xx} = \frac{\partial u}{\partial x}, \quad e_{yy} = \frac{\partial v}{\partial y}, \quad e_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \quad \dots(1)$$

The strains of (1) give rise to stresses in the skin and stringers.

Assum / unit length
Assum / unit length

Estimating these as resultants T_1 , T_2 and S per unit length we find,

$$\left. \begin{aligned} T_1 &= \frac{Et}{(1-\sigma^2)} (e_{xx} + \sigma e_{yy}) + \frac{EA_s}{a_s} e_{xx} \\ T_2 &= \frac{Et}{(1-\sigma^2)} (e_{yy} + \sigma e_{xx}) \\ S &= \frac{Et}{2(1+\sigma)} e_{xy} \end{aligned} \right\} \dots\dots(2)$$

where t is the skin thickness, E is Young's modulus and σ Poisson's Ratio. The stringers of course only contribute to T_1 .

The conditions of equilibrium for the plate are

$$\frac{\partial T_1}{\partial x} + \frac{\partial S}{\partial y} = \frac{\partial S}{\partial x} + \frac{\partial T_2}{\partial y} = 0 \dots\dots\dots(3)$$

Substituting from (1) into (2) and from (2) into (3) we find,

$$\left. \begin{aligned} \left\{ 1 + \frac{(1-\sigma^2)A_s}{a_s t} \right\} \frac{\partial^2 u}{\partial x^2} + \left(\frac{1-\sigma}{2} \right) \frac{\partial^2 u}{\partial y^2} + \left(\frac{1+\sigma}{2} \right) \frac{\partial^2 v}{\partial x \partial y} = 0 \\ \left(\frac{1-\sigma}{2} \right) \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \left(\frac{1+\sigma}{2} \right) \frac{\partial^2 u}{\partial x \partial y} = 0 \end{aligned} \right\} (4)$$

Our problem is thus reduced to the solution of (4) subject to the appropriate boundary conditions at the edges of the plate.

The end rib at $x = 0$ will be assumed to be completely flexible in bending. This implies that $(T_1)_{x=0} = 0$. A second condition at this edge follows from the balance of shear input into the rib against the build up of end load within it. This last is given by $EA'(e_{yy})_{x=0}$. We thus find,

$$\left. \begin{aligned} \left\{ 1 + \frac{(1-\sigma^2)A_s}{a_s t} \right\} \frac{\partial u}{\partial x} + \sigma \frac{\partial v}{\partial y} = 0 \\ \frac{\partial^2 v}{\partial y^2} + \frac{t}{2(1+\sigma)A'} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) = 0 \end{aligned} \right\} \text{at } x = 0 \dots\dots\dots(5)$$

The conditions assumed at $x = l$ are those of complete fixity. We shall thus have,

$$u = v = 0 \quad \text{at } x = l \quad \dots\dots\dots(6)$$

On the edge $y = b$ we have first of all the shear input balance and secondly the relation, from the theory of beams, between the transverse loading $(T_2)_{y=b}$ and the displacement $(v)_{y=b}$. These yield,

$$\left. \begin{aligned} \frac{\partial^2 u}{\partial x^2} &= \frac{t}{2(1+\sigma)A} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \\ \frac{\partial^4 v}{\partial x^4} &= - \frac{t}{(1-\sigma^2)I} \left(\frac{\partial v}{\partial y} + \sigma \frac{\partial u}{\partial x} \right) \end{aligned} \right\} \text{at } y = b \quad \dots\dots\dots(7)$$

Since the panel and loading are both symmetrical about Ox, we need only consider the region $y \geq 0$. This requires boundary conditions at $y = 0$. Symmetry demands that both $(v)_{y=0}$ and $(S)_{y=0}$ should be zero and so,

$$v = \frac{\partial u}{\partial y} = 0 \quad \text{at } y = 0 \quad \dots\dots\dots(8)$$

Finally certain special conditions must hold at the corners. At $x = l$ we assume the edge members fully built-in implying.-

$$\frac{\partial v}{\partial x} = 0 \quad \text{at } x = l, \quad y = b \quad \dots\dots\dots(9)$$

At $x = 0$ we have a given end load P in the edge member. We assume too that this member is pinned to the end rib, so that its bending moment will be zero and its shear force equal to the end load in the rib. These conditions yield,

$$\frac{\partial u}{\partial x} = \frac{P}{EA}, \quad \frac{\partial^2 v}{\partial x^2} = 0, \quad \frac{\partial^3 v}{\partial x^3} = - \frac{A'}{I} \frac{\partial v}{\partial y} \quad \dots\dots\dots(10)$$

at $x = 0, \quad y = b$

This completes the mathematical formulation of our problem.

THE RELAXATIONAL SOLUTION

The specific problem solved

Since equations (4) to (10) are solved (approximately) by Relaxation Methods it is necessary to select a skin having specific shape and properties. The quantities chosen were as follows,-

- Length, $l = 51$ ins.
- Width between \bar{c} of booms, $2b = 45$ ins.
- Boom area, $A = 2.23$ ins.²
- Boom moment of inertia, $I = 1.689$ in.⁴
- Skin thickness, $t = 0.0813$ in.
- Stringer area, $A_s = 0.0732$ in.²
- Stringer spacing, $a_s = 2.00$ in.
- End-rib area, $A' = 0.684$ in.²
- Poisson's ratio, $\sigma = 0.3$
- Young's modulus, $E = 10^7$ p.s.i.)
- Loading, $P/A = 4000$ p.s.i. } $\frac{P}{AE} = \frac{4}{10^4}$

These are identical with the quantities appropriate to the test specimen used in the experimental work of Ref. 3.

Non-dimensional transformation of the equations

Since a numerical solution is contemplated it is necessary to put all the equations (4) to (10) into non-dimensional form. With l chosen as representative dimension let

$$\begin{aligned}
 u, v, x, y, t, a_s &= l(\bar{u}, \bar{v}, \bar{x}, \bar{y}, \bar{t}, \bar{a}_s) ; \\
 A, A', A_s &= l^2(\bar{A}, \bar{A}', \bar{A}_s) ; \\
 I &= l^4 \bar{I} .
 \end{aligned}$$

Then all barred quantities are purely numerical. Those substitutions made in equations (4) to (10) leave those equations unaltered apart from the fact that barred quantities now replace their original counterparts; it is therefore unnecessary to re-write the equations. Hereafter we shall work in terms of the transformed equations.

Finite-difference approximations

The next step, in preparation for a relaxational solution, is to convert the governing equations and boundary conditions into finite-difference approximations. The chosen (square) net at nodal points of which values of u and v were found is shown in figure 2. The mesh-size over most of the field is 2 inches, and in fact a solution was first obtained for which the net was uniformly of this size all over. The final solution, incorporating values of u and v on the finer net (of mesh-size 1 inch), gave more detail in the region of stress-concentration and also provided some slight indication of the accuracy to be expected from the results. The net is so arranged that net-lines fall on the boundaries $y = b$ and $x = 0$; 'irregular stars' occur near the other two boundaries. Clearly a rectangular net might have been used (at a cost in more cumbersome arithmetic) which would have presented no irregular stars. We decided against this since the conditions on the boundary $x = l$ and centre-line $y = 0$ are particularly simple making the treatment of 'irregular stars' there straightforward.

Finite-difference approximations to the equations (4) to (10) will be set down next, for use on any square net of mesh-size h inches. Let $h = l\bar{h}$ so that \bar{h} represents the non-dimensional mesh-size.

The numbering scheme of figure 3 will be used to indicate, by suffix notation, the relative positions of values of the functions at a typical nodal point, 0, of the net and at surrounding nodal points.

The governing equations

The governing equations (4) in non-dimensional form and with coefficients evaluated are

$$\left. \begin{aligned} 1.40967 \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + 0.35 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + 0.65 \frac{\partial^2 \bar{v}}{\partial \bar{x} \partial \bar{y}} &= 0 \\ 0.35 \frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + 0.65 \frac{\partial^2 \bar{u}}{\partial \bar{x} \partial \bar{y}} &= 0 \end{aligned} \right\} \dots\dots(4A)$$

Simultaneous diff. eqns.

Typical approximations needed in these equations are those for $\partial^2 \bar{u} / \partial \bar{x}^2$ and $\partial^2 \bar{v} / \partial \bar{x} \partial \bar{y}$ and the following standard expressions were used.-

$$\bar{h}^2 \left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} \right)_0 \doteq \bar{u}_1 + \bar{u}_3 - 2\bar{u}_0$$

$$4\bar{h}^2 \left(\frac{\partial^2 \bar{v}}{\partial \bar{x} \partial \bar{y}} \right)_0 \doteq \bar{v}_5 - \bar{v}_6 + \bar{v}_7 - \bar{v}_8$$

Finite-difference approximations to equations (4A) can now be written down and (after multiplying through by 2 for convenience) two residuals $(F_{\bar{u}}^-)_0$ and $(F_{\bar{v}}^-)_0$, corresponding with the first and second respectively of (4A), are defined by

$$\left. \begin{aligned} (F_{\bar{u}}^-)_0 &\equiv 2.8193 (\bar{u}_1 + \bar{u}_3) + 0.7 (\bar{u}_2 + \bar{u}_4) - 7.0387 \bar{u}_0 \\ &\quad + \{0.325(\bar{v}_5 - \bar{v}_6 + \bar{v}_7 - \bar{v}_8)\} \\ (F_{\bar{v}}^-)_0 &\equiv 0.7 (\bar{v}_1 + \bar{v}_3) + 2(\bar{v}_2 + \bar{v}_4) - 5.4 \bar{v}_0 \\ &\quad + \{0.325(\bar{u}_5 - \bar{u}_6 + \bar{u}_7 - \bar{u}_8)\} \end{aligned} \right\} \dots (11)$$

The reason for the suffixes \bar{u} and \bar{v} in $(F_{\bar{u}}^-)_0$ and $(F_{\bar{v}}^-)_0$ is fairly obvious; the first expression in (11) defining $(F_{\bar{u}}^-)_0$ contains predominantly terms in \bar{u} , the second, defining $(F_{\bar{v}}^-)_0$, terms in \bar{v} . The same notation will be adopted when we define residuals corresponding with the boundary conditions.

The boundary conditions I. Along end-rib and boom

All that has been described so far is standard technique, details of which can be found, for example, in Refs. 4 and 5. We now come to the boundary conditions. Those along the end-rib, equations (5), and along the boom, equations (7), together with the conditions at the corner where the force is applied are somewhat unusual and present the crux of the problem relaxationally.

Leaving aside the corner $x = 0, y = b$ for the present consider the conditions along end-rib and boom. In particular let us examine equations (5). In their non-dimensional form and with coefficients evaluated these become

$$\begin{aligned}
 1.4097 \frac{\partial \bar{u}}{\partial \bar{x}} + 0.3 \frac{\partial \bar{v}}{\partial \bar{y}} &= 0 \\
 \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + 0.0457(51) \left[\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right] &= 0
 \end{aligned}
 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \dots\dots\dots(5A)$$

Equations (5A) hold on the vertical boundary $x = 0$, a piece of which is shown in Figure 4.

First derivatives appear in these equations and the simplest approximation to use, exemplified by $\frac{\partial \bar{u}}{\partial \bar{x}}$, is this.-

$$2\bar{h} \left(\frac{\partial \bar{u}}{\partial \bar{x}} \right)_0 \doteq \bar{u}_1 - \bar{u}_3 \quad \dots\dots\dots(12)$$

An approximation for the second derivate has already been stated so equations (5A) could now be put into finite-difference form. Applied at a point 0 on the boundary (figure 4), values of \bar{u}_3 and \bar{v}_3 appear in the resulting expressions and the node 3 is a 'fictitious' node, i.e. lies outside the field. But we need a pair of equations at 0 which do not involve \bar{u} and \bar{v} at fictitious nodes, and the most obvious way to proceed is to employ the governing equations (4A) together with the boundary conditions (5A) eliminating between them all values at fictitious nodes. (Cf. Ref. 6). The governing equations, applied at 0 in figure 4, introduce values of \bar{u} and \bar{v} at a further number of fictitious points and it becomes necessary to use the boundary conditions not only at 0 itself but at other points such as 2 and 4 in order to get a pair of equations to be satisfied at 0. The same method of elimination could be used along the boom where equations (7) hold. This procedure was investigated at first but was abandoned (perhaps inadvisedly) because the elimination resulted in some very unweildy expressions, particularly

along the boom (where a fourth derivative is present in one of the boundary conditions). It was also difficult to see how to get adequate approximations near the corner where the force acts.

The method finally adopted on these boundaries was the simple one of satisfying the boundary conditions only and avoiding the introduction of values at fictitious nodes by using end-difference formulae where necessary in place of central-differences. [The accuracy of such a procedure ought to be investigated more fully before firm reliance is placed on the results of this report.]

Treating equations (5A) in this way we replace the approximation (12) by

$$2\bar{h} \left(\frac{\partial \bar{u}}{\partial x} \right)_o \doteq 4\bar{u}_1 - 3\bar{u}_o - \bar{u}_9. \quad (\text{see figure 4}). \quad \dots\dots(13)$$

$\left(\frac{\partial \bar{v}}{\partial x} \right)_o$ is approximated similarly and the residual expressions for the end-rib boundary can then be defined by

$$\left. \begin{aligned} (F_{\bar{u}})_o &\equiv 5.6387 \bar{u}_1 - 1.4097 \bar{u}_9 - 4.2290 \bar{u}_o + \{0.3(\bar{v}_2 - \bar{v}_4)\} \text{ indept. of } \bar{h}, \\ (F_{\bar{v}})_o &\equiv 0.0914 \bar{v}_1 - 0.0229 \bar{v}_9 + (\bar{v}_2 + \bar{v}_4) - 2.0686 \bar{v}_o + \{0.0229(\bar{u}_2 - \bar{u}_4)\} \text{ where} \end{aligned} \right\} (14)$$

$\bar{h} = 1/51$, its value for the finer net.

Consider equations (7) next; in non-dimensional form and with coefficients evaluated they become

$$\begin{aligned} \frac{\partial^2 \bar{u}}{\partial x^2} - 0.01402(51) \left[\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y} \right] &= 0 \\ - \frac{\partial^4 \bar{v}}{\partial x^4} - 0.05290(51)^3 \left[\frac{\partial \bar{v}}{\partial y} + 0.3 \frac{\partial \bar{u}}{\partial x} \right] &= 0 \end{aligned} \quad \dots\dots\dots(7A)$$

Now it is $\left(\frac{\partial \bar{u}}{\partial y} \right)_o$ and $\left(\frac{\partial \bar{v}}{\partial y} \right)_o$ which must be approximated by the 3-point end formula illustrated in (13). Central-differences are used for all other derivatives. The (standard) approximation

used for $\frac{\partial^4 \bar{v}}{\partial \bar{x}^4}$ was

$$= \bar{h}^4 \left(\frac{\partial^4 \bar{v}}{\partial \bar{x}^4} \right)_0 \doteq 4(\bar{v}_1 + \bar{v}_3) - (\bar{v}_9 + \bar{v}_{11}) - 6\bar{v}_0.$$

Residuals may now be defined as follows.-

$$\left. \begin{aligned} (F_u)_0 &\equiv 0.0280\bar{u}_4 - 0.0070\bar{u}_{12} + (\bar{u}_1 + \bar{u}_3) - 2.0210\bar{u}_0 - \{0.0070(\bar{v}_1 - \bar{v}_3)\}, \\ (F_v)_0 &\equiv 0.1058\bar{v}_4 - 0.0264\bar{v}_{12} + 4(\bar{v}_1 + \bar{v}_3) - (\bar{v}_9 + \bar{v}_{11}) - 6.0793\bar{v}_0 - \{0.0079(\bar{u}_1 - \bar{u}_3)\}, \end{aligned} \right\} (15)$$

for $\bar{h} = \frac{1}{51}$, its value for the first net.

[Some coefficients in both of (15) need modification when $\bar{h} = \frac{2}{51}$, its value for the coarse net.]

Finally, in detail, the approximations used at the corner $x = 0, y = b$ will be given. Equations (10) hold here; in non-dimensional form and with data inserted these become

$$\left. \begin{aligned} \frac{\partial \bar{u}}{\partial \bar{x}} &= 4/10^4 \\ \frac{\partial^2 \bar{v}}{\partial \bar{x}^2} &= 0 \\ - \frac{\partial^3 \bar{v}}{\partial \bar{x}^3} - 0.40497(51)^2 \frac{\partial \bar{v}}{\partial \bar{y}} &= 0 \end{aligned} \right\} \dots\dots\dots (10A)$$

Corresponding with the first of (10A) an approximation using the 3-point end-formula was adopted, a residual F_u being defined by

$$(F_u)_0 \equiv 4\bar{u}_1 - \bar{u}_9 - 3\bar{u}_0 - \frac{4}{10^4} \cdot (2\bar{h}) \dots\dots\dots (16)$$

The second two equations in (10A) were used together to find a residual expression F_v . The 3-point end-formula was again employed, for $\partial \bar{v} / \partial \bar{y}$, and the approximation taken for $\partial^3 \bar{v} / \partial \bar{x}^3$ (derived with a use of $\partial^2 \bar{v} / \partial \bar{x}^2 = 0$) was

$$\bar{h}^3 \left(\frac{\partial^3 \bar{v}}{\partial \bar{x}^3} \right)_0 \doteq \bar{v}_9 + \bar{v}_0 - 2\bar{v}_1$$

With $\bar{h} = 1/51$ a residual $F_{\bar{v}}$ at this corner can then be defined by

$$\left(F_{\bar{v}}\right)_0 \equiv 0.8099 \bar{v}_4 - 0.2025 \bar{v}_{12} + 2\bar{v}_1 - \bar{v}_9 - 1.6075 \bar{v}_0. \dots\dots\dots(17)$$

The boundary conditions. II. Along built-in end and centre-line

The methods used for dealing with the irregular stars which occur near the built-in end of the skin ($x = l$) and near the centre-line ($y = 0$) were mostly standard and call for no detailed description. The boundary conditions were here applied by using them to eliminate values of \bar{u} and \bar{v} at fictitious nodes which occur in equations (11) when the point 0 lies on either of the net-lines immediately adjacent to these boundaries.

The relaxation process

Knowledge of relaxational technique must be assumed here,^{*} but it is perhaps appropriate to add a few remarks about the technique as applied to the particular problem of this report.

The residual expressions (11), (14), (15), (16) and (17), together with those (not given) for use at the centres of the irregular stars discussed in the last section, form the basis of the relaxation solution.

It is possible, and most convenient, to work the problem in stages, first relaxing \bar{u} only for a time and then switching over to relax \bar{v} alone. The significance of the notation $F_{\bar{u}}$ and $F_{\bar{v}}$ now becomes apparent. While working on \bar{u} , for example, we concentrate on the $F_{\bar{u}}$ residual expressions employing incomplete patterns (derived from these) which involve changes in $F_{\bar{u}}$ residuals only; the terms in \bar{v} , in curly brackets, remain (temporarily) constant at their values obtained from the current distribution of \bar{v} . This is a standard approach which is not invalidated by the particular boundary conditions of the problem under discussion.

* Ref. 4 gives an excellent account of technique.

The problem is of the kind which requires meticulous treatment - especially on the boom and end-rib boundaries. Individual residuals are easily reduced to acceptable magnitudes but large changes in \bar{u} and \bar{v} may still be necessary in order to make the sum of the residuals negligibly small.

RESULTS

The complete solution is not recorded for reasons of space, however figures 5 and 6 give some indication of the nature of the distributions of u and v over the field. Figure 5 shows the variation of u along a few selected lines of constant x ; figure 6 the variation of v along certain lines of constant y . u and v are very smooth functions and the curves run directly through a large number of plotted points.

Figures 7 show the strains by means of contours; since values of e_{xx} , e_{yy} and e_{xy} must be found by differencing these are less accurate than u and v which are computed directly. There is also the difficulty that u varies very rapidly near the corner where the force is applied which makes close estimation of the gradient there impossible. It is perhaps worth recording that the preliminary solution found for u and v - working on a net of mesh-size 2 inches throughout the field - was very little different from the final solution (here presented) which incorporated a finer net in the region of stress-concentration.

Figures 8 and 9 show the variations of strains along boom and end-rib near the corner where the force is applied.

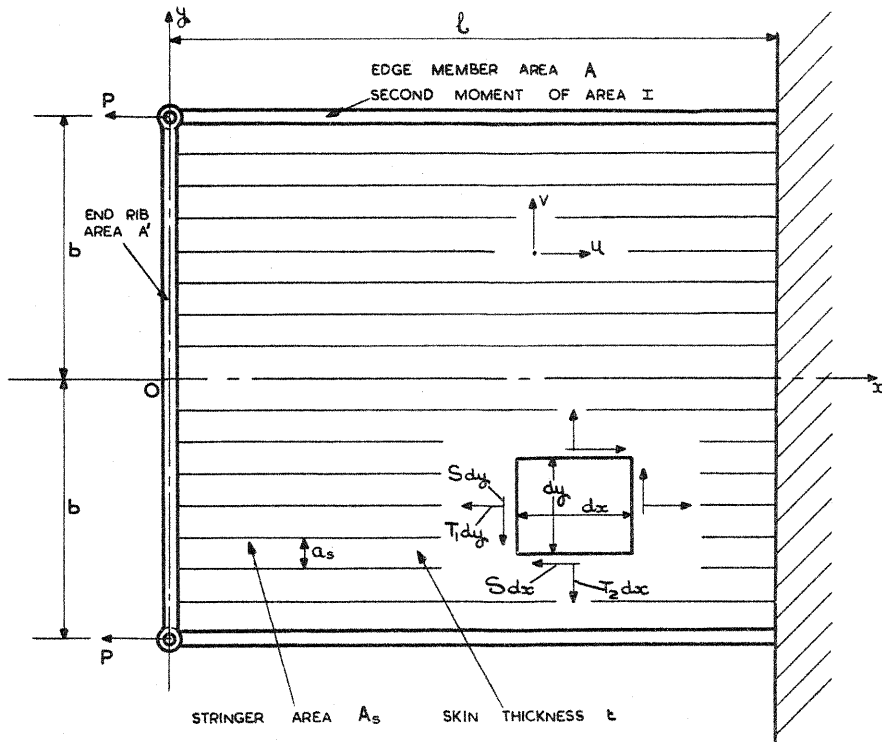
Conclusions

The numerical results obtained in this report show that it is quite feasible to tackle the diffusion problem, even in the complicated case where bending of the edge members is considered, by means of the relaxation method. Comparison with the experimental evidence of Ref. 3 still shows however that further complexity must be introduced into our assumptions, before full agreement can be achieved with experiment. In particular the rigid connection

assumed between plate and edge member must be replaced by an elastic element. This and other developments must await future investigation, but experience to date suggests that the relaxation method will be an adequate tool for the further work.

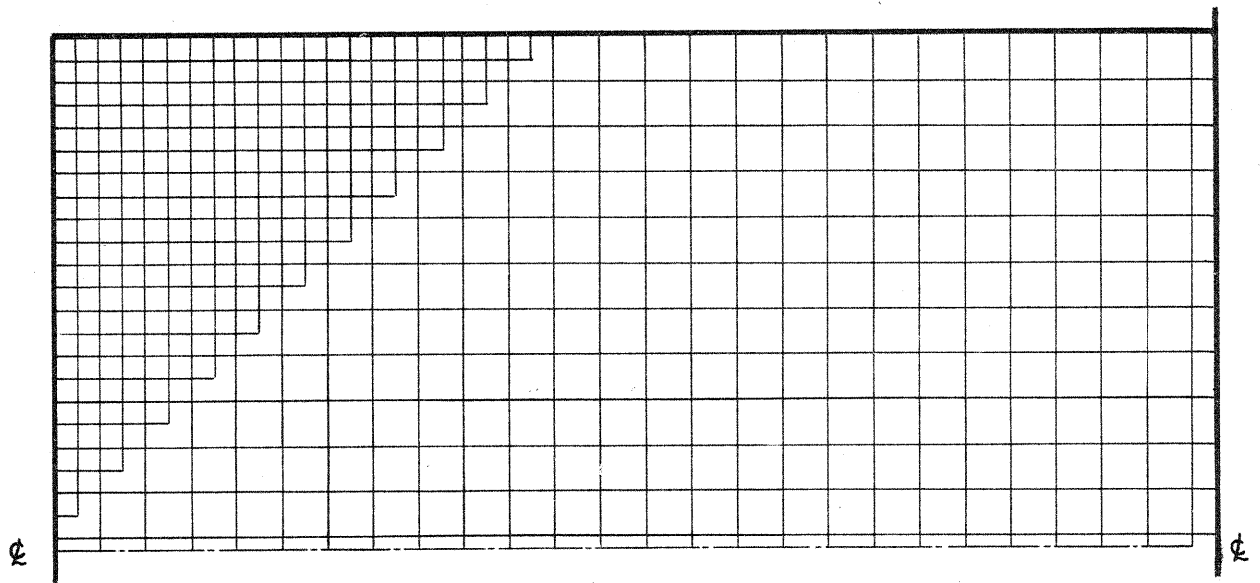
REFERENCES

1. Cox, H.L. Diffusion of loads into monocoque structures.
A.R.C. R. and M. 1860.
2. Mansfield, E.H. Stress distribution in panels bounded by constant stress edge members.
R.A.E. Report Structures 162, 1954.
3. Brown, La Verne W. An experimental investigation into some of the problems associated with stress diffusion in the vicinity of chord-wise cut-outs in the wing, and comparison with existing theories.
College of Aeronautics Report 83, 1954.
4. Allen, D.N.de G. Relaxation methods.
McGraw Hill, 1954.
5. Fox, L., and Southwell, R.V. Relaxation methods applied to engineering problems. VIIa. Biharmonic analysis as applied to the flexure and tension of flat elastic plates.
Phil. Trans. Royal Society, A, Vol. 239, 1945, pp. 419-460.
6. Fox, L. Mixed boundary conditions in relaxational treatment of biharmonic problems.
Proc. Royal Society, A, Vol. 189, 1947, pp. 535-543.



SKETCH OF PROBLEM

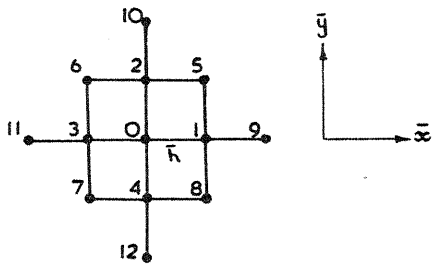
FIG.1.



THE NET USED

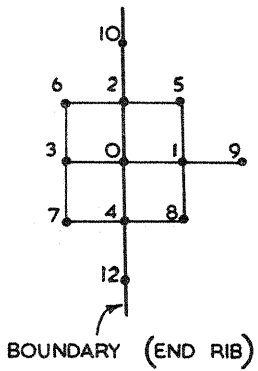
FIG.2.

3. 4. & 5



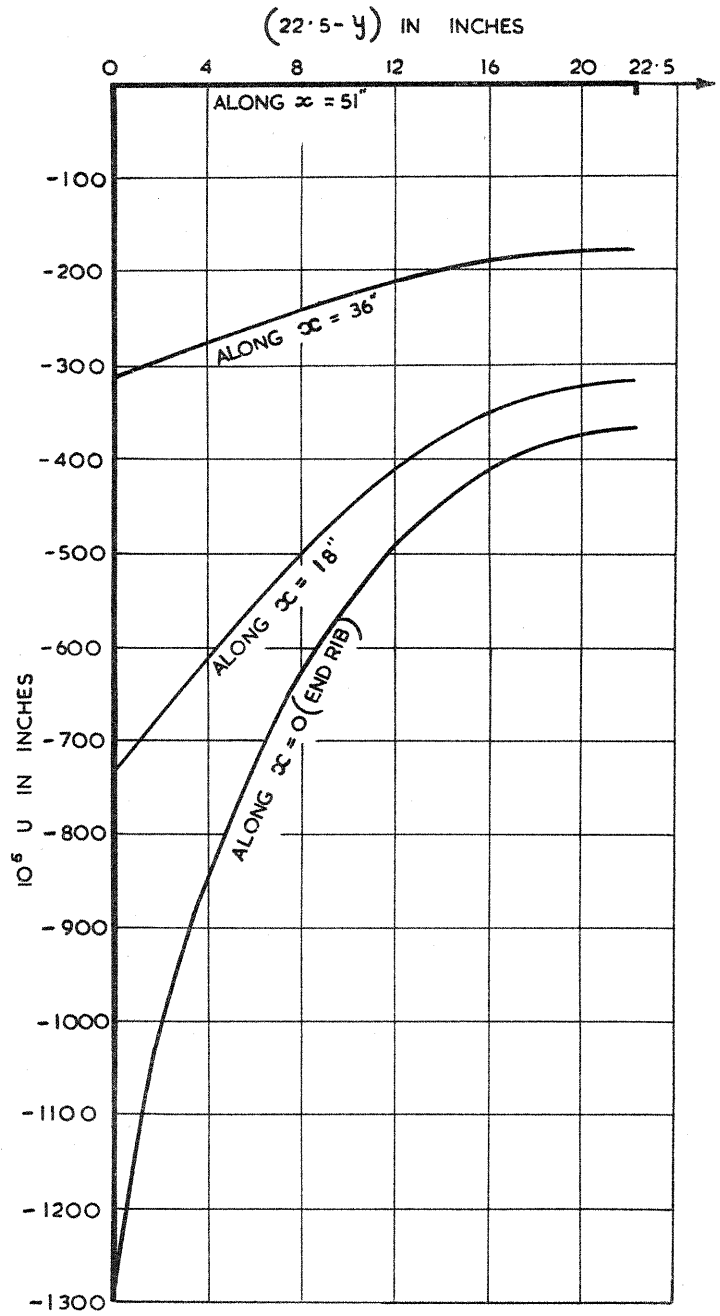
GENERAL NUMBERING SCHEME

FIG.3.



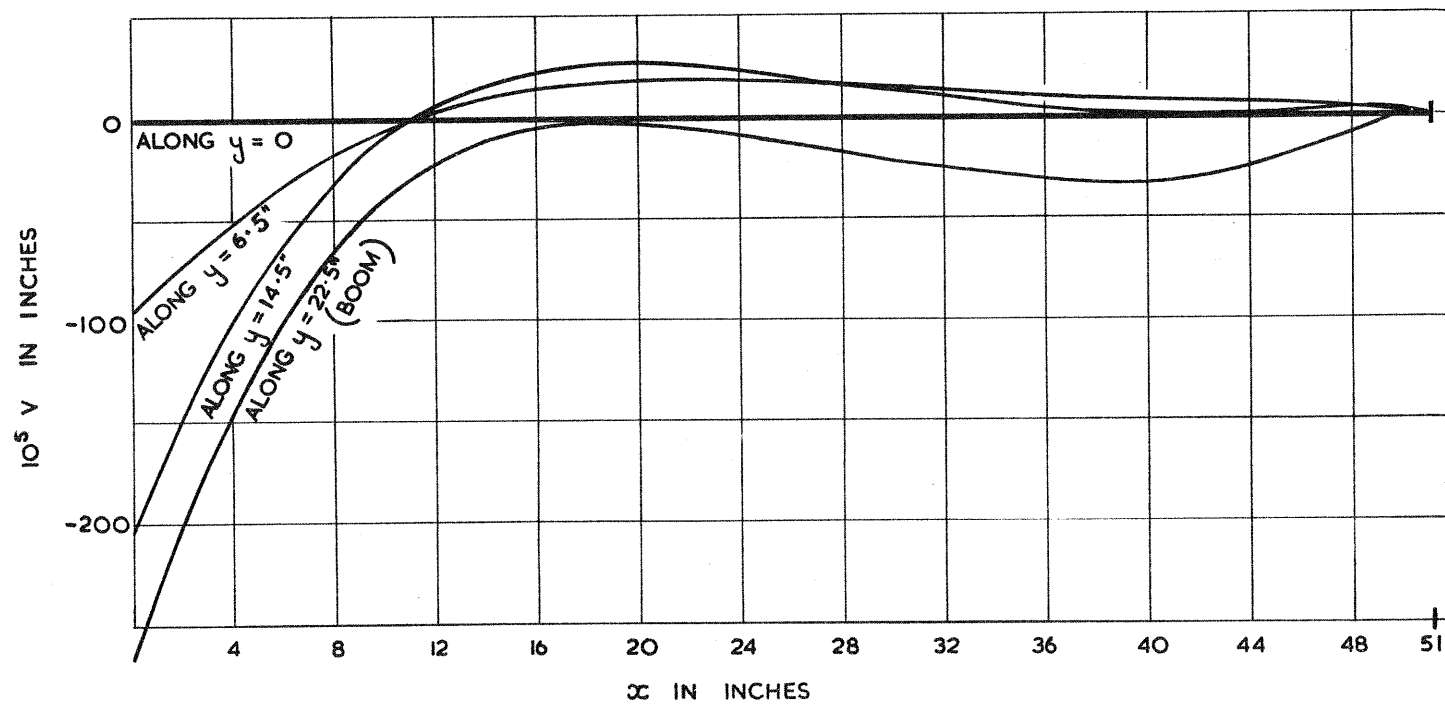
SCHEME IN POSITION ON END-RIB

FIG.4.



u ALONG SELECTED LINES OF CONSTANT x

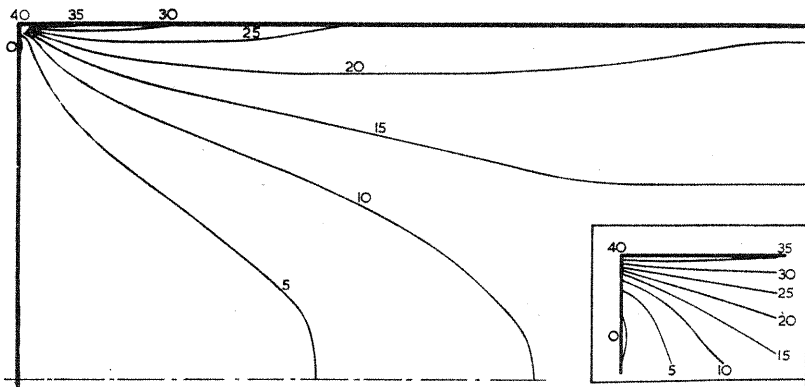
FIG.5.



v ALONG SELECTED LINES OF CONSTANT y

FIG. 6.

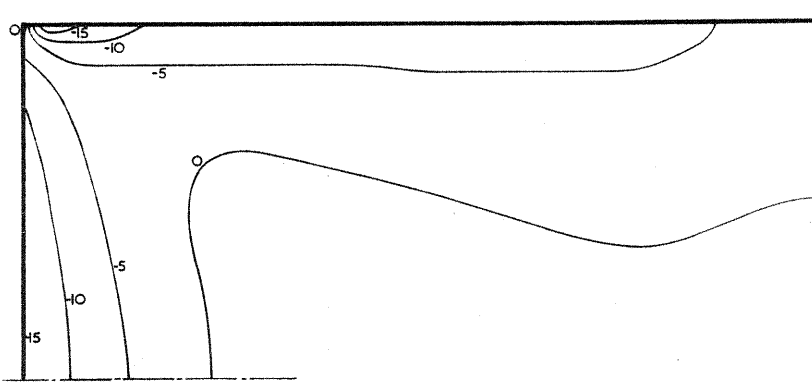
FIGS. 7a 7b & 7c



THE INSET DIAGRAM
SHOWS A REGION
NEAR THE CORNER
MAGNIFIED 4 TIMES

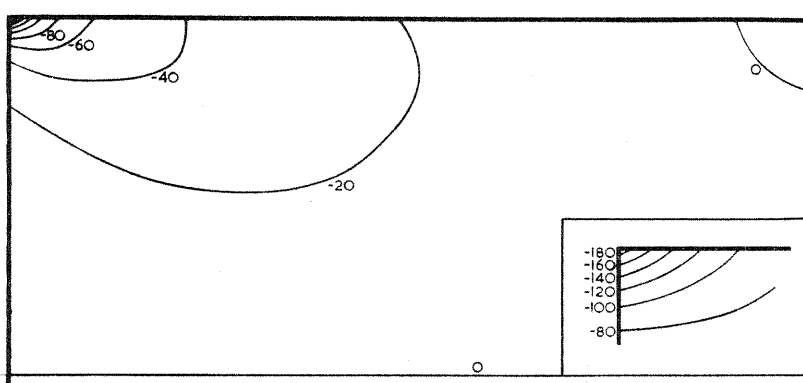
CONTOURS OF CONSTANT e_{xx} ($\times 10^5$)

FIG. 7a



CONTOURS OF CONSTANT e_{yy} ($\times 10^5$)

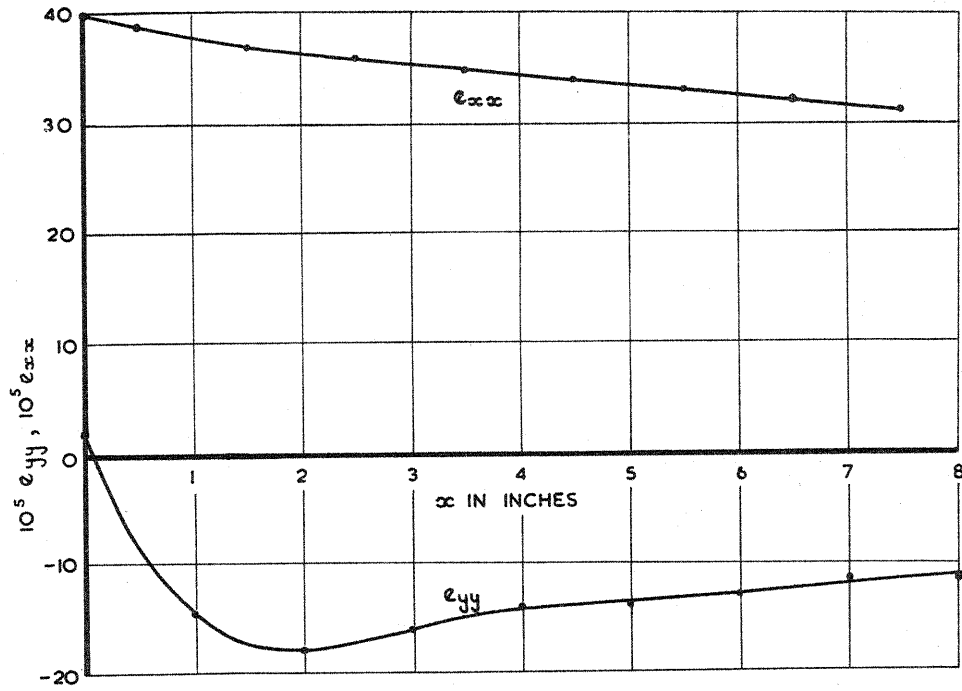
FIG. 7b



THE INSET DIAGRAM
SHOWS A REGION
NEAR THE CORNER
MAGNIFIED 4 TIMES

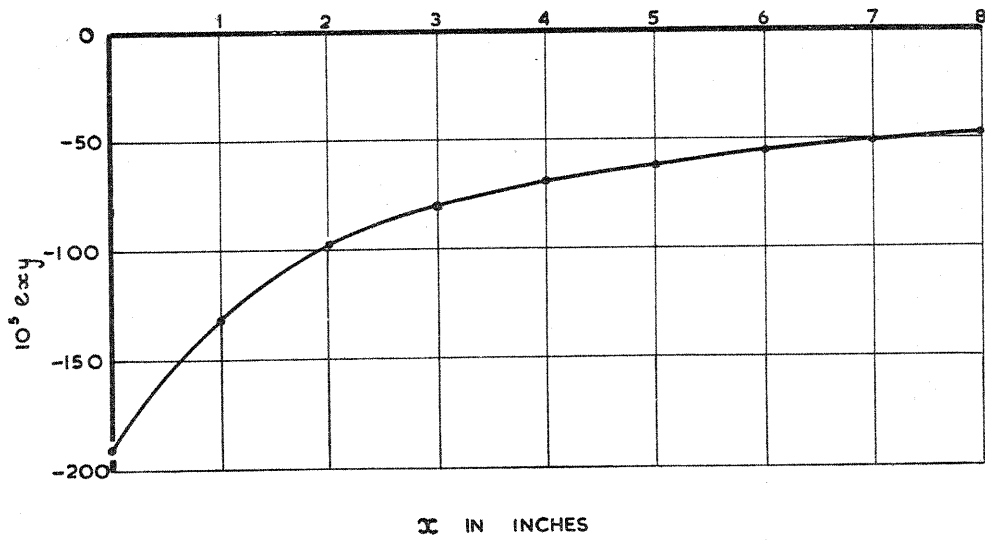
CONTOURS OF CONSTANT e_{xy} ($\times 10^5$)

FIG. 7c



e_{xx} AND e_{yy} ALONG BOOM NEAR
CORNER WHERE FORCE IS APPLIED

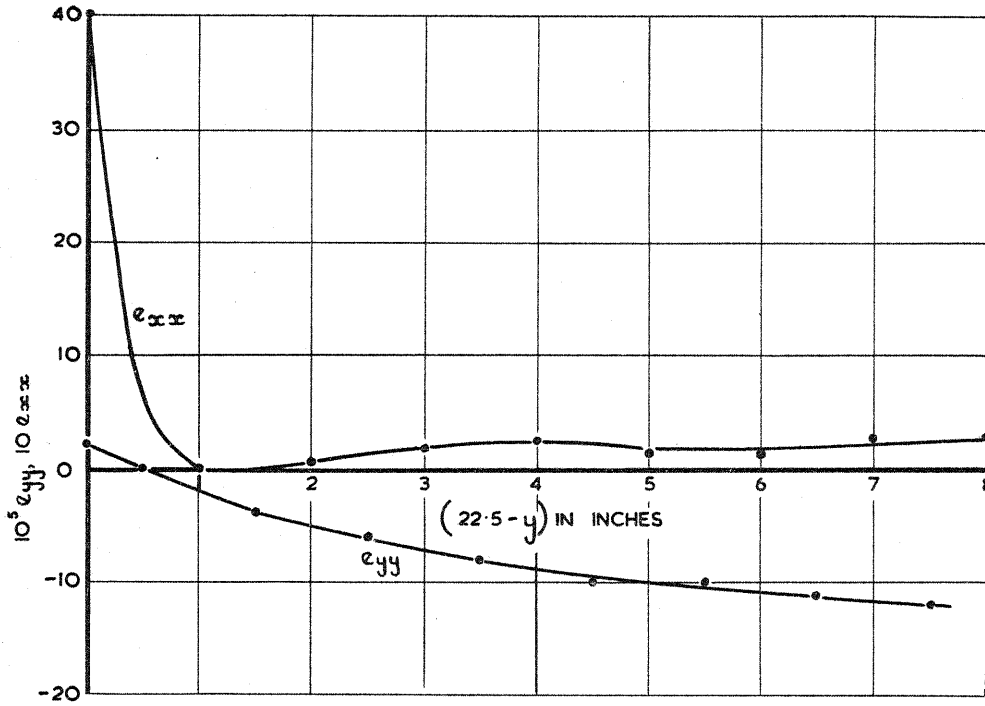
FIG.8a.



e_{xy} ALONG BOOM NEAR
CORNER WHERE FORCE IS APPLIED

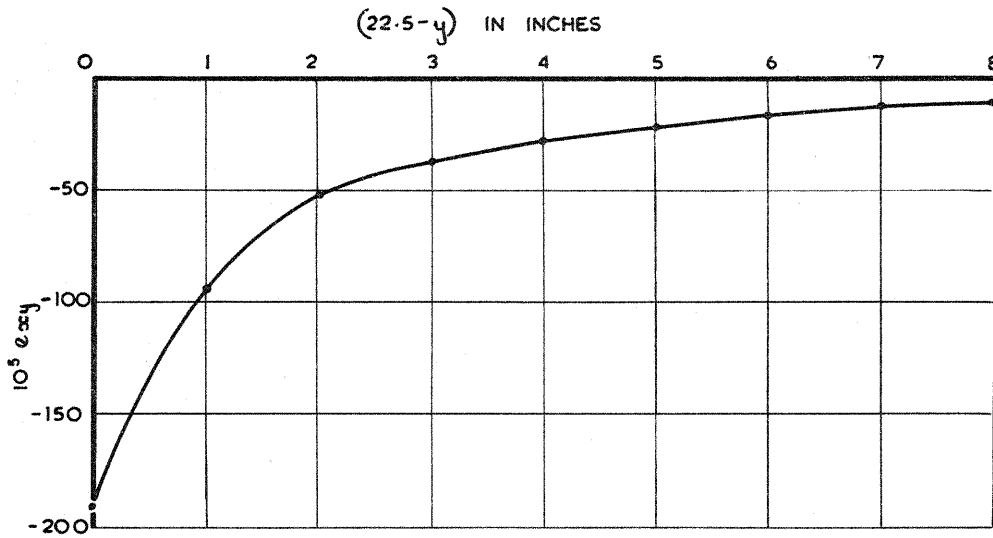
FIG.8b.

FIGS. 9a & 9b



e_{xx} AND e_{yy} DOWN END-RIB NEAR
CORNER WHERE FORCE IS APPLIED

FIG.9a.



e_{xy} DOWN END-RIB NEAR
CORNER WHERE FORCE IS APPLIED

FIG.9b.