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Preliminary report on the analysis of the stresses
in a die-bolster combination.

- by -

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Summary

An analysis is presented of the stresses in a carbide die-steel
bolster combination. Results from a computer treatment of this analysis
are given in tabular and graphical form. Suggestions are made as to the
choice of interface diameters, and a nomogram is drawn enabling the max-
imum allowable interference to be selected.

1. Introduction

This report considers die-bolster combinations, of the type used in cold heading operations, in which a carbide die is shrunk into a steel bolster.

The result of this shrink fit is to induce a compressive hoop stress into the carbide at the bore of the die. The hoop stress is reduced when the die is working and the interference between the die and bolster is selected so that the stress remains compressive under working conditions in order to reduce die wear. In addition the interference induces a tensile hoop stress and a compressive radial stress in the bolster, which have maximum values at the inner face of the bolster and are increased under working conditions.

In designing such a die-bolster combination the criteria to be considered are that the stresses at the interface should not lead to failure of the bolster, the stresses in the carbide at the bore of the die should not be excessive and the fluctuating stresses, due to working, imposed on a high mean stress level should not lead to a fatigue failure.

2. Stresses resulting from interference fit

By applying the Lame equations together with the known boundary conditions it can be shown that the radial pressure Prr between the die and bolster is given by (using the notation tensile stress+ve compressive stress-ve).

$$P_{rr} = \frac{-\Delta r}{\dfrac{r_2}{E_B}\left[\dfrac{r_3^2 + r_2^2}{r_3^2 - r_2^2} + \mu_B\right] + \dfrac{r_2}{E_B}\left[\dfrac{r_1^2 + r_2^2}{r_2^2 - r_1^2} - \mu_D\right]} \qquad \text{--------- 1}$$

where Δr is radial interference

r_1 is inner die radius

r_2 is interface radius

r_3 is outer bolster radius

E_B is modulus of elasticity of the bolster material

E_D "   "   "   "   "   "   die   "

μ_B is poissons ratio of the bolster material

μ_D "   "   "   "   "   die   "

Similarly the maximum (tensile) hoop stress in the bolster occurring at the interface, is given by,

$$p_{\theta\theta} = \frac{r_3^2 + r_2^2}{r_2^2 - r_3^2} \, p_{rr} \qquad\qquad -------- 2$$

and the maximum (compressive) hoop stress in the die, which occurs at the inner surface of the die is given by,

$$\sigma_{\theta\theta} = \frac{-2r_2^2}{r_1^2 - r_2^2} \, p_{rr} \qquad\qquad -------- 3$$

3. Stresses due to working

If under working conditions the pressure exerted on the die by the material being deformed is "k" then again by applying the Lame equations it can be shown that the radial stress (compressive) at the interface of the die and bolster, induced by this pressure, is given by,

$$p'_{rr} = \frac{-k(r_3^2\ r_1^2 - r_2^2\ r_1^2)}{r_2^2\ (r_3^2 - r_1^2)} \quad\text{---------- 4}$$

and that the corresponding hoop stress (tensile) is given by,

$$p'_{\theta\theta} = \frac{k(r_2^2\ r_1^2 + r_3^2\ r_1^2)}{r_2^2(r_3^2 - r_1^2)} \quad\text{---------- 5}$$

Similarly the hoop stress (tensile) at the bore of the die is given by,

$$\sigma'_{\theta\theta} = \frac{k(r_3^2 + r_1^2)}{(r_3^2 - r_1^2)} \quad\text{---------- 6}$$

4. Design criteria

The design criteria are:-

a) Under working conditions the hoop stress at the die bore should be compressive so as to reduce wear.

$$(\sigma_{\theta\theta} + \sigma'_{\theta\theta}) \not> 0 \qquad\qquad \text{---------- 7}$$

where $\sigma_{\theta\theta}$ is the hoop stress due to interference

$\sigma'_{\theta\theta}$ is the hoop stress (additional) due to working.

b) The combined stresses at the inner face of the bolster should not lead to failure of the bolster. Using the maximum shear stress criterion this can be expressed as,

$$(p_{\theta\theta} + p'_{\theta\theta}) - (p_{rr} + p'_{rr}) \not> Y_B \qquad\qquad \text{---------- 8}$$

where $p_{\theta\theta}$ is the hoop stress in the bolster due to interference p_{rr} is the radial stress in the bolster due to interference and $p'_{\theta\theta}$ and p'_{rr} are the additional hoop and radial stresses respectively induced by working. Y_B is the yield strength of the bolster material.

c) The stress level in the carbide which will be a maximum under non-working conditions should not lead to failure of the die.

$$- (\sigma_{\theta\theta}) \not> Y_c \qquad\qquad \text{---------- 9}$$

where Y_c is the yield strength of the carbide.

d) The stress levels in both the die and bolster should be such that a fatigue failure is unlikely during the life of the die.

Inspection of a number of failed dies has shown that catastrophic failure has predominantly been caused by excessive stresses at the inner surface of the bolster and this analysis will be primarily concerned with these stresses.

5. Computer Programme

As they stand it is difficult to draw any useful conclusions from equations 1 - 9 and for this reason a computer study was made using actual values for the various parameters to determine their effect.

In order to reduce the computer time and to obtain as much information as possible from the computer output the equations given above were stated in a more general form.

From equations 1 - 6 it can be seen for geometrically similar dies the stresses are independent of the actual dimensions of the die combination and that if the following substitutions are made

$$a = r_2/r_1$$
$$b = r_3/r_1$$
$$.001\, n = \Delta r/r_1$$

the equations become

$$p_{rr} = \frac{- .001\, n}{\dfrac{a}{E_B}\left[\dfrac{b^2 + a^2}{b^2 - a^2} + \mu_B\right] + \dfrac{a}{E_D}\left[\dfrac{a^2 + 1}{a^2 - 1} - \mu_D\right]} \qquad\text{---------- 10}$$

$$p_{\theta\theta} = \frac{- p_{rr} [a^2 + b^2]}{[b^2 - a^2]} \qquad \text{----------} \; 11$$

$$\sigma_{\theta\theta} = \frac{p_{rr} \, 2a^2}{a^2 - 1} \qquad \text{----------} \; 12$$

$$p'_{rr} = \frac{k(a^2 - b^2)}{a^2(b^2 - 1)} \qquad \text{----------} \; 13$$

$$p'_{\theta\theta} = \frac{k(a^2 + b^2)}{a^2(b^2 - 1)} \qquad \text{----------} \; 14$$

$$\sigma'_{\theta\theta} = \frac{k(b^2 + 1)}{(b^2 - 1)} \qquad \text{----------} \; 15$$

where E_B, E_D, μ_B, μ_D represents the physical properties of the die and bolster materials, "a, b & n" the dimensions of the die combination in ratio terms ("n" represents .001 in interference/inch of die bore) and "k" the working pressure in the die which will be dependent on the work material and to some extent the geometry of the die.

As the most common cause of die failure is rupture of the bolster due to overstressing at the interface the programme (Fig. 1) was arranged to work through equations 10-15 and using the condition

$$(p_{\theta\theta} + p'_{\theta\theta}) + (p_{rr} + p'_{rr}) = Y'_B \qquad \text{------------} \ 16$$

(where $Y'_B < Y_B$ and represents the maximum allowable stress level in the bolster material) to calculate the maximum permissable value of "n" for one set of material properties and a given value of "a" and "b" .

The computer output gave printed values of "a, b, n" and $(\sigma_{\theta\theta} + \sigma'_{\theta\theta})$, the hoop stress at the die bore, for three different values of Y'_B representing three grades of bolster material, for a range of values of "a" and "b" .

In practice it seems that the ratio of the die bore to the o.d. of the bolster varies between 1 : 4 and 1 : 12 and so nine values of "b" were used in the computation to cover this range. Similarly nine values of "a" were taken for each value of "b" so that a range of interface diameters between that of the die bore and that of the bolster o.d. were covered.

The material constants used in the programme were taken as:-

$$
\begin{array}{llll}
E_B & = & 13400 \text{ tpi} & \\
E_D & = & 35700 \text{ tpi} & \\
Y'_B & a & 70 \text{ tpi} & \text{(Hov)} \\
& b & 80 \text{ tpi} & \text{(Crovan)} \\
& c & 110 \text{ tpi} & \text{(Maraging steel)} \\
\mu_B & = & 0.3 & \\
\mu_D & = & 0.3 & \\
k & = & 30 \text{ tpi} &
\end{array}
$$

(the value of"k" which is a function of the shear flow stress
of the work material will vary from one product to the next, but
a value of 30tpi was chosen to be sufficiently high to cover the
materials in conventional use).

As these material constants (except for poissons ratio which
will always have a similar value) appear at the end of the programme
as part of the data tape the programme can conveniently be adapted
for other die or work materials should the occasion arise.

6. Results

The computer output is given in Fig. 2. The first column
gives blocks of nine values of "a" under the appropriate value of
"b" , columns 2, 4 and 6 give the values of "n" and columns 3, 5
and 7 the values of the hoop stress under working conditions at the
bore of the die for the three grades of bolster steels. It can be
seen from this figure that as the interface diameter is decreased
so the allowable interference is also decreased but that the comp-
ressive hoop stress at the die bore, which tends to improve wear
resistance, increases.

The first criterion of design of a die-bolster combination
is that the interfence should not be so great that the bolster fails
at the interface. The maximum allowable interference for any con-
figuration can be obtained from figure 2 but it can more easily be
chosen graphically. Figures 3, 4 and 5 show values of "n"$(\Delta r/.001r_1)$,
plotted against "a", (r_2/r_1), for all value of "b" for the three
bolster materials respectively. It can be seen that the points,
which have a certain amount of scatter, are more or less independent
of"b", (r_3/r_1) and that if a curve is drawn along the lower boundary
of the points this will give the same maximum value of "n" for all
values of "b", $(4 \leqslant b \leqslant 12)$. Using these figures of "n" against "a"
it is possible to construct a nomogram figure 6 from which the maximum
allowable interference can be read directly for a known die bore and
interface diameter.

(In using this nomogram the scales can be considered as in
units of inches or tenths of inches so that they can be used for a
wide range of die sizes. A guide to remember, so that the final
answer is not out by a factor of ten, is that the interference should
be in the region of 7/1000, 5/1000 and 4/1000 inches per inch diameter
for Maraging, Crovan and Hov bolster steels respectively).

The second design criterion is that the hoop stress at the bore of the die in the non-wroking condition should not be excessive. Figure 7, 8 and 9 have been prepared from the data in figure 2 and show the hoop stress at the bore plotted against the interface radius, which is in terms of percentage wall thickness. (i.e. $\frac{r_2 - r_1}{r_3 - r_1} \times 100$)

The area to the right and above the points represents the condition under which the bolster will fail, the area below the dotted lines represents the condition under which the hoop stress will become tensile when the die is working. From these figures it is possible to read off the percentage carbide to be used in a die-bolster combination to give a certain stress level in the bore. In general it is considered that the wearing properties of carbide are increased by a compressive hoop stress but obviously if this stress were too high the carbide would crimble. Unfortunately there is no quantitative information as to the effect of a compressive stress on wear resistance.

In view of this lack of information the best approach would seem to be to make an intelligent guess at the optimum stress level and then to vary this in light of experience.

The figures show the high compressive stresses which it is possible to obtain by using Maraging steel as a bolster material.

The equation 1 - 9 enable the mean stresses and the fluctuating stresses to be estimated so that an analysis of the fatigue behaviour can be investigated when information as to the fatigue properties of die and bolster materials become available.

Conclusions

The report gives an analysis of the stresses in a die bolster combination together with a computer programme enabling these stresses to be calculated for given die and bolster materials.

Although in many cases the interface diameter selected for a particular die will depend on such parameter as the size of the bolt or rivet head to be made, figures 7, 8 and 9 can be used for selecting a bolster material and interface diameter to give a predetermined stress level at the bore of the die.

Once the interface diameter and bolster material have been chosen the nomogram in figure 6 can be used to determine the maximum allowable interference which can be tolerated without overstressing the bolster.

RADIAL INTERFERENCE

```
D
N
RADIAL INTERFERENCE

J1.0
STOP
v30=TAPE6
n1=4
2)Xv1=n1
PRINTv1,3040

n2=1
1)v2=n2
v3=v1-1
v3=v3/10
v3=v3Xv2
v3=v3+1
PRINTv3,3042

v4=v1Xv1
v5=v3Xv3
v6=v4+v5
v7=v2-v5
v8=v6/v7
v9=v8+0.3
v9=v9Xv3
v9=v9/v30

v10=v5+1
v11=v5-1
v12=v10/v11
v12=v12-0.3
v12=v12Xv3
v12=v12/v3I
v12=v12+v9
v12=0.001/v12

v13=-v12
v14=v12Xv8
v15=2Xv13
v15=v15Xv5
v15=v15/v11

v15=v32Xv7
v16=-v16
v16=v16/v5
v17=v4+1
v18=v4-1
v16=v16/v18
v19=v32Xv6
v19=v19/v5
v19=v19/v18

v20=v32Xv17
v20=v20/v18
n3=0
3)v21=v(33+n3)+v16
v21=v21-v19
v23=v14-v13
v23=v21/v22
PRINTv23,4042

v24=v15Xv23
v24=v24+v30
PRINTv24,4061
n3=n3+1
→3,n3≠3
n2=n2+1
→1,n2≠10
n1=n1+1
→2,n1≠13
TAPE
(→0)
```

****
****

+13400 +35700 +30 +70 +80 +110

(STOP)

****
****

FIGURE 1.

FIGURE 2.

FIG.3. 'n' AGAINST 'a' FOR HOV BOLSTER.
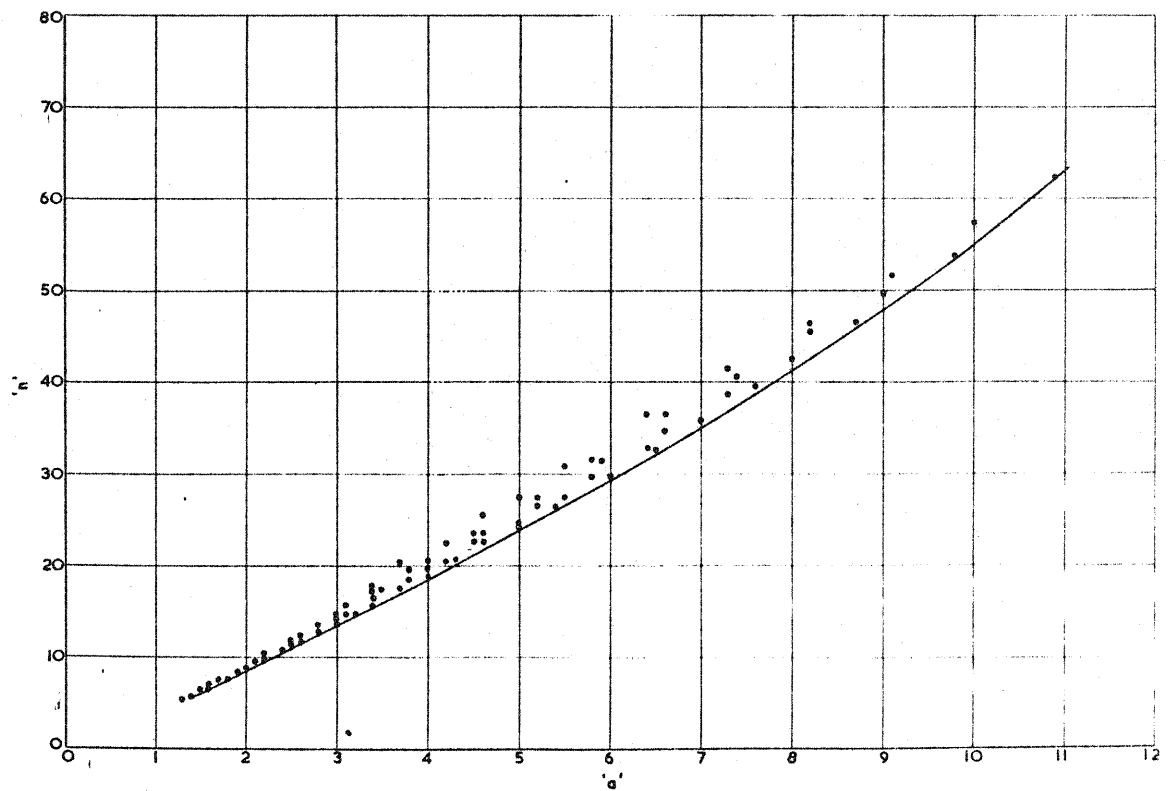
FIG.4. 'n' AGAINST 'a' FOR CROVAN BOLSTER.
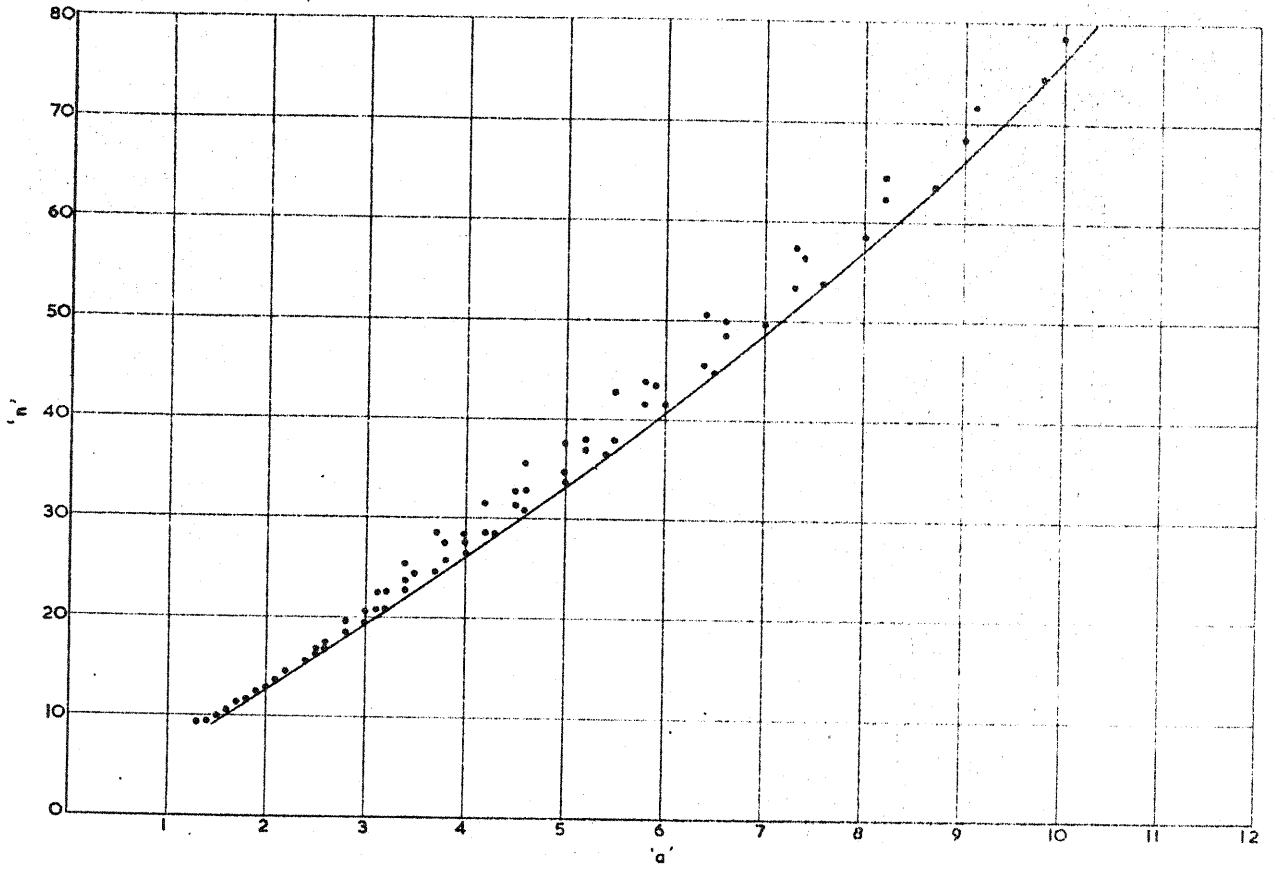
FIG.5. 'n' AGAINST 'a' FOR MARAGING BOLSTER.

INTERFERENCE ON DIAMETER
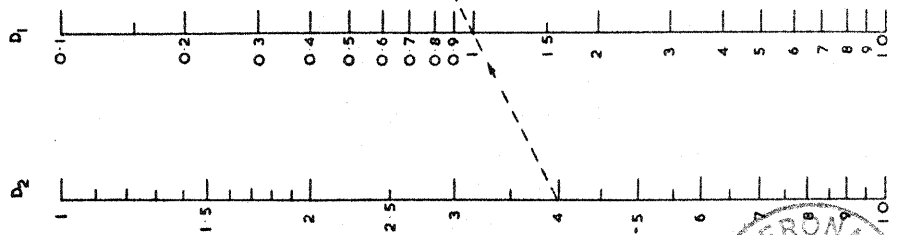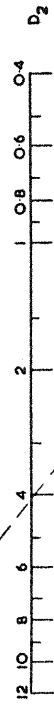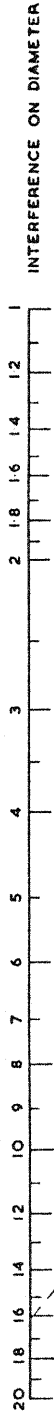
NOTE: WHEN D_1 AND D_2 ARE IN INCHES INTERFERENCE IS IN $1/1000$ INCHES

CARBIDE

STEEL

D_2

D_1

D_2
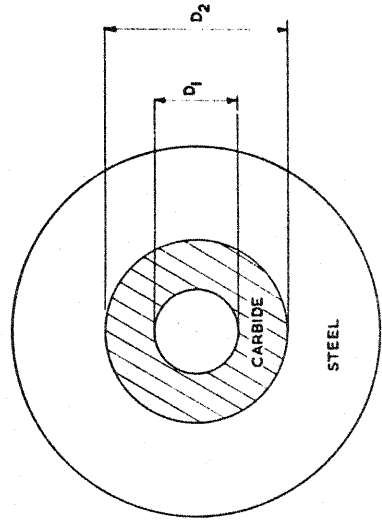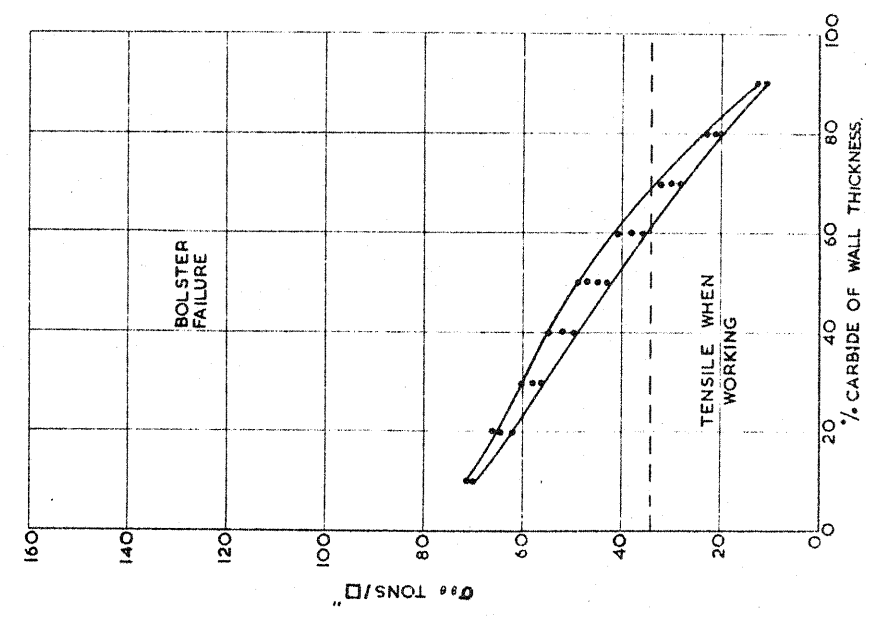
MARAGING

CROVAN

HOV

D_1

D_2

FIG. 6.
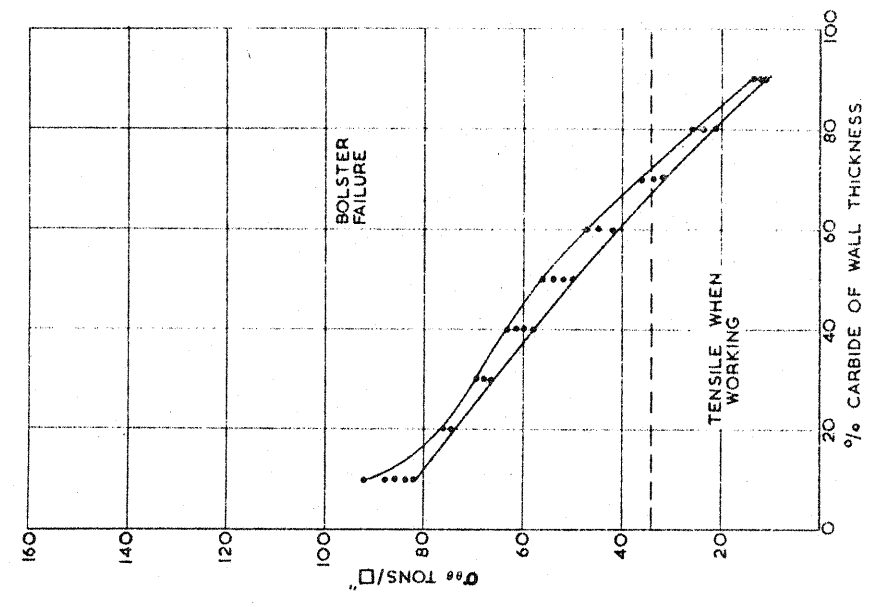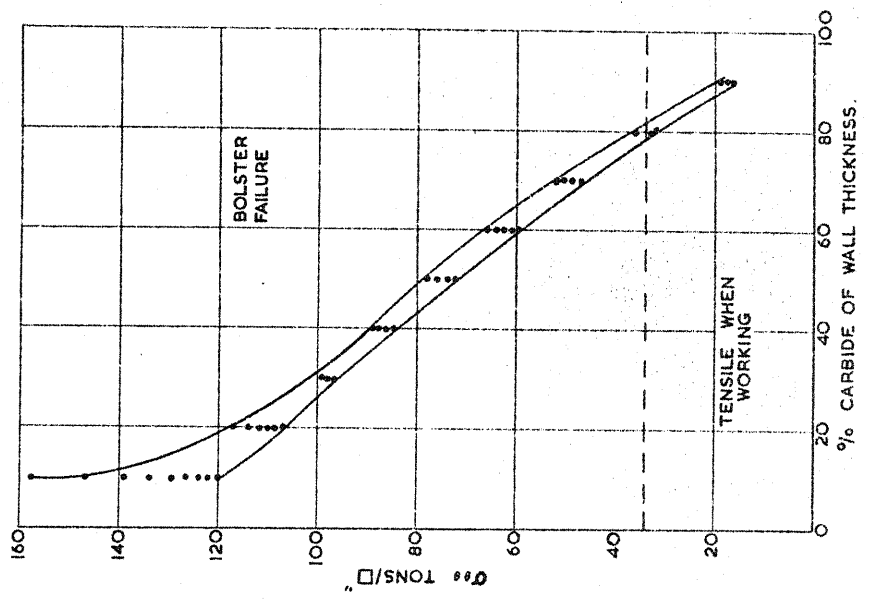
FIG.9. STRESS AT BORE - MARAGING BOLSTER.

FIG.8. STRESS AT BORE - CROVAN BOLSTER.

FIG.7. STRESS AT BORE - HOV BOLSTER.