# ASTEROID BELT MULTIPLE FLY-BY OPTIONS FOR M-CLASS MISSIONS 

Joan Pau Sánchez Cuartielles<br>Space Research Group, Centre of Autonomous and Cyber-Physical Systems, Cranfield University, UK. jp.sanchez@cranfield.ac.uk

Alison Gibbings<br>Space System Studies \& Proposals, Science Department, OHB System AG, Germany alison.gibbings@ohb.de<br>Colin Snodgrass<br>Faculty of Science, Technology, Engineering \& Mathematics, Open University, UK colin.snodgrass@open.ac.uk<br>Simon Green<br>Faculty of Science, Technology, Engineering \& Mathematics, Open University, UK<br>simon.green@open.ac.uk<br>Neil Bowles<br>Department of Physics, University of Oxford, UK<br>bowles@atm.ox.ac.uk

Addressing many of the fundamental questions in modern planetary science, as well as in ESA's cosmic vision, requires a comprehensive knowledge of our Solar System's asteroid belt. This paper investigates potential opportunities for medium-class asteroid belt survey missions in the timeframe of 2029-2030. The study has been developed in support to CASTAway Asteroid Spectroscopic Survey mission proposal, which is to be submitted to the latest ESA's medium size mission call. CASTAway envisages the launch of a small telescope with relatively straightforward (i.e. high TRL) remote sensing instrumentation to observe asteroids at a long-range (i.e. point source), but also at a short-range, resolving them at $\sim 10 \mathrm{~m}$ resolution. This paper presents a challenging multiobjective optimization problem and discusses the feasibility of such a mission concept. A baseline trajectory is presented that meets both ESA's medium size mission constraints and the science requirements. The trajectory loops through the asteroid belt during 7 years, visiting 10 objects of a wide range of characteristics, providing sufficient survey time to obtain compositional information for 10,000 s of objects and the serendipitous discovery of also $10,000 \mathrm{~s}$ of $10-\mathrm{m}$ class asteroids. The methodology developed has enabled the exploration of the entire design space for a conservative Soyuz-launch performance, and has found a total of 200 different tour opportunities of the asteroid belt; all compliant with ESA's $5^{\text {th }}$ call for medium size missions.

## I. INTRODUCTION

The exploration of the asteroid main belt (AMB) is of the utmost importance to address many of the fundamental questions in modern planetary science. During the past decades, the major increase on the number of known asteroids ( $>700,000$ AMB objects are known today) have only emphasized the true relevance of the AMB to understand our Solar System [1].

This paper discusses trajectory options for a future science mission dedicated to gain relevant understanding of the AMB to inform solar system formation and evolution theories. Hence, the key science objectives for this potential mission shall be; to understand compositional variation, including volatile reservoirs, throughout the main belt; inform solar system dynamical evolution by exploring the small asteroid population ( $\sim 10$ metre objects); and study the morphology and geological history of asteroids.

The first objective may be achieved with a spacebased survey, which would enable access to spectral
information not available from within the Earth's atmosphere. However, the morphological study of asteroids, and the detection of the smallest objects in the AMB, would most certainly require of an interplanetary probe cruising through the AMB.

CASTAway mission (Comet and Asteroid Space Telescope Away in the asteroid belt) merges these two very different types of space mission concepts; on the one hand, CASTAway is a dedicated space-based telescope, which needs to be placed in a suitable operational orbit, one that maximises the observational opportunities and science return; on the other hand, CASTAway is an interplanetary probe attempting to detect and visit as many asteroids as possible.

The scientific goals of such a mission would thus be to deliver detailed information on the connection between asteroid composition, geomorphology and orbital evolution. More specifically, the mission should test Solar System evolution theories by; 1) providing compositional information for 1,000 s of objects by obtaining spectral data over a wide range of
wavelengths, including key regions not observable through the Earth's atmosphere 2) performing a statistically significant survey of previously unsampled small asteroid belt objects ( $<1 \mathrm{~m}$ to a few tens of metres diameter objects); 3) studying the morphology and geological history from close flybys of a targeted sub-set of objects, at least doubling the number of currently visited main belt asteroids within one single mission (>9 fly-bys).

The study presented in this paper was developed in support of the CASTAway Asteroid Spectroscopic Survey M5 mission proposal. The paper then investigates potential opportunities for medium-class asteroid belt survey missions in the timeframe of 2029+. The launch and other mission constraints, such as cost, are assumed as those corresponding to the ESA's M5 call for medium class mission proposals*.

## II. TRAJECTORY REQUIREMENTS

This section first analyses the requirements and constraints that need to be satisfied by CASTAway's trajectory. The direct requirements on CASTAway trajectory derive from two different sources: on the one hand, from the science objectives and, the other hand, from the boundary conditions imposed by ESA in the M5 call for medium class mission.

## II.I Scientific Requirements

A relatively small and simple telescope ( $\sim 50 \mathrm{~cm}$ ) will be responsible to complete the spectroscopic survey during most of CASTAway's cruise operations. A cut off visual magnitude in survey mode of $\mathrm{V} \geq 13$ should be expected for such an instrument. Therefore, CASTAway would be able to survey large AMB objects ( $>100 \mathrm{~km}$ ) as soon as it is delivered into space. Objects in the order of tens of kilometres diameter however would only be observable from beyond the orbit of Mars. While for smaller objects ( $<10 \mathrm{~km}$ ), CASTAway will need to be within the AMB.

The goal of a survey providing spectral information for $1,000 \mathrm{~s}$ of objects does not seem to pose any serious constraint to the total mission duration. Since, assuming a generous one observation per hour, a survey of 1,000 asteroids would be completed within 2 months of survey operations. Detailed compositional information of 10,000 s of objects could be attained in about 20 months of survey operations, including $10 \%$ of all currently known main belt asteroids with a radius $>10 \mathrm{~km}$. A large fraction of the survey operations shall be completed within the AMB.

On the other hand, CASTAway shall complete the first detailed census of small-scale objects (e.g. $\sim 10 \mathrm{~m}$ diameter) that are not accessible to any current or planned surveys (e.g Gaia, LSST, etc.), opening a

[^0]window on this population for the first time. State-of-the-art star trackers would be responsible for performing this survey. Again, assuming a reasonable limiting magnitude for detection for the star trackers of $\mathrm{V} \geq 14$, a serendipitously discovered object should be within $\sim 100,000 \mathrm{~km}$ (i.e. in solar opposition) from the spacecraft in order to be observable. If CASTAway's trajectory reaches the AMB, a statistically significant number of discoveries should soon be attained. A rough order of magnitude estimate of 50 counts a day should be expected during CASTAway cruise of the AMB.

Finally, CASTAway is required to double the number of AMB objects visited by spacecraft. As of August 2016, only 8 AMB objects have been visited. Note that a total of 12 asteroids have been visited by spacecraft, although, 4 of these are either Mars crossing or near-Earth objects. Ideally, then CASTAway should target 10 or more objects in the AMB. Mars or Earth crossing objects will not be considered in this study. Nevertheless, these can clearly be considered at later stages of the mission design as opportunity targets.

The study presented in this paper focuses on trajectory tours to visit 10 asteroids in the asteroid main belt. It is also of interest to visit a good diversity of types of asteroid sizes, spectra and orbits. Since the number of asteroids with spectral information is currently limited $(\sim 2,500)$, prominent families in the asteroid belt will also be considered as representatives of spectral type.

## II.II M5 Boundary Conditions

Within ESA's Cosmic Vision plan, calls for science mission proposals are regularly issued. The CASTAway proposal is to be submitted to the $5^{\text {th }}$ call for Medium-size missions, i.e. M5 call. It has thus being designed to satisfy all the boundary conditions specified for the M5 call.

Firstly, and most importantly, ESA Cost at Completion (CaC) must be under $550 \mathrm{M} €$. The CaC includes all elements that are nominally funded by ESA, such as the commissioning of the spacecraft, launch and cost of operations. Under typical breakdown, $13 \%$ of the CaC would correspond to the Launch, hence $\sim 72 \mathrm{M} €$. Consequently, a European medium-lift launcher is envisaged for CASTAway's direct insertion into an escape trajectory.

The current European medium-lift launcher is Soyuz. However, Ariane 62 (A62) is called to substitute Soyuz by $\sim 2024$. It is estimated that the launch cost for A62 will range from 70 to $75 \mathrm{M} €$. Unfortunately, there is a lot of uncertainty on the actual launch capabilities of A62, although it is clear by extrapolating the announced A62 GTO capability, that the escape energy (C3) performance will also be much better than that of Soyuz. However, the Soyuz performance is here adopted as a worst case scenario for the baseline trajectory. If sufficiently interesting
feasible mission scenarios can be found for a Soyuzlike performance, then much better options ought to be available for an A62 launch if this was to outperform Soyuz.

Therefore, this paper primarily focuses on trajectory options available for a Soyuz-like performance [2]. However, some insight is also given on the improvements potentially achievable if A62 capability would outperform Soyuz by $50 \%$ in terms of payload mass inserted into an escape trajectory, i.e. C3 capability. This latter scenario is thereafter referred to as A62( $\mathrm{S}^{+}$).

The CASTAway baseline trajectory assumes therefore a direct insertion into an Earth escape trajectory by an A62 with Soyuz-like performance, thereafter referred to simply as Soyuz launch. The values of mass inserted into the hyperbolic escape trajectory as a function of $v_{\infty}$, or C 3 , can be found in the M5 technical annex [2]. Note also that not all declinations of the launch escape hyperbola $\delta_{\infty}$ are allowed, and those that are may strongly constrain the actual performance of the launcher.

While there is no explicit mention of the mission duration in the M5 call, in order for CASTAway to satisfy the CaC , the mission duration is fixed to a maximum of 7 years. This 7 years limit ensures that even considering uncertainties in operational costs, they would roughly be in the order of 60-70 M€ [3], hence satisfying ESA's typical CaC breakdown ( $\sim 15 \%$ for MOC \& SOC).

## III. PRELIMINARY ANALYSIS

This section provides a simplified analysis based on basic astrodynamics principles and coplanar motion. The purpose of the section is to gain some initial insight on what can be realistically considered for CASTAway trajectory, based mainly on launcher capability. Figure 1 shows the potential spacecraft mass that could be delivered into an orbit with periapsis at 1 AU and apoapsis distance at Q. Figure 1 is computed assuming a variable hyperbolic escape velocity $v_{\infty}$ in the Earth's velocity direction. Nominal performance refers to the performance that can be achieved with the reported launcher capability [2, 4], while extended assumes a deep space manoeuvre (DSM) soon after departing Earth using a bipropellant engine of Isp 320 s. Low thrust propulsion is discarded, for the baseline design, on the grounds of complexity and cost. A launch vehicle adapter of 80 kg has been considered [ 5 , 6 ]. Finally, the performance is only extended to a minimum spacecraft dry mass of 800 kg [5].


Figure 1. Launch mass for Soyuz and $\mathrm{A} 62\left(\mathrm{~S}^{+}\right)$as a function of apoapsis. Continuous line indicate apoapsis that can be reached with a direct insertion of the launcher, while dashed line indicate the need for a DSM soon after departing Earth. Curves extend to 800 kg . Four vertical lines indicate apoapsis for which a total of $\mathbf{0 , 2 5 , 5 0}$ and $75 \%$ of the time is spend within the asteroid main belt.

The first noteworthy feature of the results in Figure 1 is the significant difference between the reachable apoapsis for Soyuz and A62( $\mathrm{S}^{+}$) cases. This clearly indicates that any increase of performance, even marginal, from current Soyuz capability would results in an enormous benefit in terms of capability to explore deeper into the AMB. Nevertheless, current Soyuz capability already allows the insertion of nearly 1 tonne of payload into a heliocentric orbit spending about $50 \%$ of the time beyond 1.8 AU .

The option of deploying CASTAway in an orbit fully enclosed by the AMB can also be nearly discarded, although in a later section the use of multiple gravity assist is discussed. At this stage, however, Figure 1 shows that the maximum radius of a circular operational orbit, after a Hohmann transfer with chemical propulsion, would only place CASTAway in a circular 1.45 AU and 1.7 AU orbit for Soyuz and A62( $\mathrm{S}^{+}$) respectively; well below the AMB

Hence, an eccentric heliocentric orbit appears a good option, enabling during a substantial amount of time the exploration of the AMB. The analysis, however, also highlights that the $\Delta v$ budget that can be allocated for manoeuvring and targeting of asteroid flybys will be extremely limited. If CASTAway were, for example, to be inserted into an eccentric heliocentric orbit spending about $50 \%$ of the operational time within interesting regions of the AMB, only about 500 $\mathrm{m} / \mathrm{s}$ would be available to navigate to the 10 or more asteroids. Thus, about $50 \mathrm{~m} / \mathrm{s}$ per asteroids fly-by in average, assuming a spacecraft of 800 kg of dry mass.

Instead, if an A62 $\left(\mathrm{S}^{+}\right)$performance was available by 2029-2030, CASTAway could potentially be launched
into an orbit with apoapsis at 2.75 AU and still have $750 \mathrm{~m} / \mathrm{s}$ to allocate 10 asteroid fly-bys. This would not only represent a $50 \%$ increase in the $\Delta v$ budget to perform an asteroid tour, but also a total of $70 \%$ of the mission time spend within the belt and the capability to reach all prominent families of main belt asteroids.

## IV. FLY-BY TARGETS

As of August 2016, 715,000 AMB objects are known. Exploring all the potential 10 -asteroid tours within the known population would require computing trajectories for more than $10^{58}$ different sequences, i.e. permutations. Even with an efficient Lambert solver, computing this number of trajectories would take many orders of magnitude more than the age of the universe. Hence, it is clear that the search space needs to be massively pruned out.

The very first prune is performed at the workable database for a global search of trajectories. The database of asteroids is pruned to $\sim 100,000$ main-belt asteroids, whose orbital elements and other physical data were downloaded from JPL Small-Body Database ${ }^{\dagger}$. This database is generated by ensuring an adequate diversity of asteroids in size and orbital distribution. The orbital distribution of the pruned database was ensured to remain very similar to that of the unpruned population, in order to ensure an adequate number of representative objects in the Hungaria, inner, middle, outer main belt, Cybele and Hilda regions. Note Mars-crossing or near-Earth objects were not included in this database. The above pruning was performed by ensuring that all objects larger than 10 km were kept in the new database. Only a fraction of the objects $<10 \mathrm{~km}$ were pruned out, by maintaining the relative size proportions between objects $<10 \mathrm{~km}$ and objects $<1 \mathrm{~km}$.

Figure 2 shows a summary schematic of the asteroid database, together with the different regions considered, the approximate regions in orbital space where the different asteroid families are located and the asteroids visited so far by spacecraft. Note that the prominent families are only loosely defined by the upper and lower thresholds of the osculating $a, e$ and $i$ elements in the identified members of each family.
http://ssd.jpl.nasa.gov/sbdb_query.cgi\#x [Last accessed: 24/08/2016]


Figure 2. Asteroid main belt map in semi-major axis $a$ and eccentricity $e$ with prominent families and regions, as well as asteroids previously visited by spacecraft.

Thus, asteroid tours would only be sought for the aforementioned database of $\sim 100,000$. However, the availability of asteroid belt data is expected to increase by an order of magnitude by the launch of the M5 mission. Particularly, GAIA will increase enormously the amount of asteroids with spectral information [7]. The database used here contains a very limited set of 796 objects with spectral information. GAIA is expected to add low resolution spectral data for ~100,000 objects [7], many of these would likely be AMB objects. On the other hand, the Large Synoptic Survey Telescope is expected to increase by one order of magnitude the census of AMB objects [8].

## V. ASTEROID TOUR DESIGN

CASTAway's trajectory design presents a challenging multi-objective optimization problem. On the one hand, the trajectory shall maximise the quality of the surveys (i.e. number of new detections and spectral data). On the other hand, the number of asteroid fly-bys also needs to be maximised. Particularly, the trajectory must fly by 10 or more objects, and the sample of visited asteroids must include a wide range of asteroid types and sizes.

The quality of the survey is driven by the need to spend as much time as possible within the AMB. Particularly, the need to detect a statistically significant number of objects $<10 \mathrm{~m}$ requires CASTAway to be well within the AMB. Similarly, the spectroscopic
survey of the smallest asteroids also requires to be within the belt. Finally, it can also be inferred that the cost associated with performing 10 fly-bys would be minimized by trajectories spending as much time as possible within the AMB. Recall that only asteroids in the AMB are considered here. Aside from the obvious convenience of the targeted asteroid ephemeris, the longer the time spend within the AMB, the larger the likelihood of encountering asteroids sufficiently equispaced to maximize the time of flight (ToF) between each fly-by, and as a consequence minimize the $\Delta v$ cost of the trajectory linking two consecutive asteroid encounters.

The following subsections provide some detail on the methodology followed to solve and design the trajectory for CASTAway. Firstly, following the aforementioned arguments, a simplified problem is tackled: A global optimization of multiple gravity assist trajectories is performed with the objective to maximise the time spent within the AMB. Having identified the promising sequences of planetary swing-bys or gravity assists, the combinatorial problem to choose the right target asteroids is confronted.

## IV.I Time in the AMB

The potential use of multiple gravity assist trajectories (e.g., [9]), to gain access to higher energy trajectories than otherwise would be possible, is assessed here. All the possible permutations of encounters with Venus, Earth and Mars for 1, 2 and 3 swing-by trajectories were analysed [10]. The lower threshold for the periapsis during Mars and Venus hyperbolic passage was set to 300 km altitude, while for Earth was set to 600 km . A global search was then carried out for trajectories maximizing the total TOF within a region extending from 1.8 to 3.5 AU from the Sun. The search was implemented using the standard genetic algorithm solver available within MATLAB global optimization toolbox. Table 1 summarizes the results obtained with 1 single gravity assists (EV, EE, EM), as well as for direct insertion without planetary swing-bys (E).

Table 1. Summary results for trajectory sequences such as: Direct insertion without planetary swing-by (E), Earth-Venus sequence (EV), Earth-Earth sequence (EE) and Earth-Mars sequence (EM).

| $\Delta t_{\text {AMB }}$ | Soyuz |  | A62(S ${ }^{+}$) |  |
| ---: | :---: | :---: | :---: | :---: |
|  | [days] | [\%] | [days] | $[\%]$ |
| E | 1364 | 53 | 1843 | 72 |
| EV | 1014 | 40 | 1369 | 54 |
| EE | 1829 | 72 | 1838 | 72 |
| EM | 1797 | 70 | 1855 | 73 |

Results with 2 or 3 gravity assist (GA) did not provide noticeable improvements and, in many cases, even provided worse values than for 1 single GA. This
is a consequence of two unique features of CASTAway: Firstly, CASTAway does not attempt to rendezvous with any particular object but maximize TOF within the AMB and, secondly, CASTAway mission duration is fixed to a maximum of 7 years.

Note also that, as reported in Table 1, if A62( $\mathrm{S}^{+}$) is considered, not even 1 GA appears to noticeably benefit the total time spend with the AMB ( $\Delta t_{\text {AMB }}$ ). However, for the worse cost scenario of a Soyuzperformance launch, EE and EM sequences do increase the $\Delta t_{\text {AMB }}$. All the optimal solutions summarized in Table 1 begin with a direct insertion launch at relatively high $\mathrm{C} 3, \geq 20 \mathrm{~km}^{2} / \mathrm{s}^{2}$. Finally, recall from Section II.I that 20 months of survey should allow accessing the compositional information for 10,000 s of objects in the AMB. This is exceeded with ample margin in the results in Table 1.

## IV.II Heuristics \& Search

Once a general transfer strategy was identified, i.e. at most 1 Earth or Mars GA, the search for asteroid belt multiple fly-by options was initiated. Computing all possible sequences is an utterly unattainable task, because of the aforementioned computational burden. Therefore, the problem needs to be solved by an adequate heuristic method. This very same computational issue of large combinatorial problems, and examples of heuristics used, can be found in the solutions presented in the Global Trajectory Optimization Competition ${ }^{\frac{\ddagger}{*}}$ (GTOC), particularly for GTOC editions 2, 3, 4, 5 and 7 .

Two different sub-problems can be distinguished for the design of asteroids tours. Firstly, once the set of asteroids that will be visited is fixed, a continuous optimization problem needs to be solved, where the design parameters are the actual dates of each manoeuvre required to complete the asteroid tour with minimum expenditure of propellant. This problem is solved here under the dynamical framework of the patched conics. Asteroid fly-bys are solved as zero distance encounters, where asteroids are assumed to have negligible mass.

On the other hand, choosing the right sequence of asteroids, among the $\sim 100,000$ targets, requires to solve a discrete combinatorial problem. This problem definitively presents the crux of CASTAway trajectory design, and requires to be solved in parallel with the continuous optimization problem, since they are both associated.

Bellman's optimality principle has been successfully applied to solve asteroid tours before by splitting the problem in a sequence of short leg or incremental optimizations [11]. However, here we use instead heuristics to uncouple the relations between the

[^1]two aforementioned problems and solve them separately. This may sound challenging, since one cannot know if a given asteroid sequence is a good feasible tour, unless the continuous problem is also solved for this particular sequence. However, as will be shown here, this can be approximated by using the information of the minimum orbital intersection distance (MOID) judiciously.

More particularly, the following statement drives the search: only asteroids that are close to the spacecraft at any one point during the trajectory can be potentially encountered. However, revising the distance to all asteroids at each instance of time would clearly be inefficient and computationally costly. To assess if an asteroid may be near the spacecraft trajectory at any instance of time during the complete trajectory an algorithm based on S. Milisavljevic [12] is implemented.

The algorithm computes MOIDs between the spacecraft orbit and each asteroid, stores the MOIDs and the eccentric anomalies of the spacecraft and the asteroid at their MOID points. The aforementioned discrete and continuous problem can now be uncoupled by assuming all asteroid fly-bys occur at the asteroid MOID point. Hence, the continuous optimization problem is sub-optimally solved for a fixed Earth departure conditions. The latter can be defined by solving a Lambert arc to transfer CASTAway from the Earth to the first asteroid in the tour; hence, requiring a departure time $t_{\text {dep }}$ and a time of flight for the first leg $\mathrm{TOF}_{1 \text { stL }}$.

Now, the discrete combinatorial problem of choosing a sequence of asteroids can be examined using a simple branch and bound exploration, where the only explored branches are those bounded by some feasibility conditions. These feasibility conditions are then derived as a consequence of the analysis in Section III. This is by understanding the maximum excursion that the spacecraft could potentially perform, based on the $\Delta v$ budget remaining after reaching the first asteroid in the sequence. Hence, the algorithm is initiated by optimizing the Lambert arcs to all first leg possibilities, i.e. $\sim 100,000$ asteroids.

To sum up, we defined two consecutive problems to be solved: firstly, the discrete combinatorial problem (P1), which identifies promising sequence of asteroids to complete a tour to a given fixed number of asteroids, and, secondly, the problem to solve the continuous optimization ( P 2 ), which minimizes the $\Delta v$ of a given sequence by identifying the optimal dates for each manoeuvre. An important feature for this methodology to be efficient is the closeness of the $\Delta v$ estimate of P1 to the actual optimal value computed by P2. Figure 3 shows the confirmation of the efficiency of the methodology. Recall that solving P1 and/or P2 requires a vector of $\geq 9$, each element in this vector being the

ToF for each consecutive asteroid flyby or planetary gravity assist in the sequence. Even in tests where $10^{6}$ suitable randomly generated conditions were entered as a sequence, the lowest $\Delta v$ obtained was $>40 \mathrm{~km} / \mathrm{s}$. Hence, again, this is a strong indication of the merit of the sub-optimal P1 solution.


Figure 3. Summary of P2 optimized solutions and their P1 estimates. Blue error bars indicate 3-sigma distributions at $6,6.25,6.5$ and $6.75 \mathrm{~km} / \mathrm{s}$ P1 results.

P1 was explored using a branch and bound approach. All branches departing from $1^{\text {st }}$ leg solutions with declinations of the escape asymptote $\delta_{\infty}$ between 10 to 5 degrees and whose solution had more than 825 kg of mass at the arrival to the first asteroid were explored. Given the above bounds, the total number of complete first leg branches explored was of 5,675. The total runtime of the explore the design space for 10 FB Chemical Soyuz missions was of 150 days, in a standard desktop computer (Intel Core i7 i7-4790). The actual parallelized exploration took about 1 month.

Each 10 FB sequence identify through P1 as requiring a $\Delta v<7 \mathrm{~km} / \mathrm{s}$ was then passed to the P2 solver, and reoptimized for minimum $\Delta v$. Figure 3 shows the $\sim 11,500$ solutions for which P1 and P2 were solved. The feasibility threshold (i.e. blue dotted line in Figure 3) indicates the total $\Delta v$ estimated as being the maximum that CASTAway could perform. This total $\Delta v$ considers the hyperbolic Earth escape velocity $v_{\infty}$ and all the manoeuvres required to complete the asteroid tour.

Figure 3 clearly shows how the approximations in P1 were able to quickly provide a surprisingly good sub-optimal solution. Similarly, the aforementioned threshold at $7 \mathrm{~km} / \mathrm{s}$, which was adjusted by trial and error, assured that if a P1 solution yielded $\Delta v>7 \mathrm{~km} / \mathrm{s}$, the probability of its P2 optimized counterpart to obtain a $\Delta v<5.5 \mathrm{~km} / \mathrm{s}$ was below $0.3 \%$ (i.e. $3 \sigma$ ).

## IV.I Science Performance Index

While no other performance index but the actual total transfer $\Delta v$ was used to optimize the asteroid tours, it is useful to rank the set of feasible transfers by
means of a single figure of merit giving some understanding of the science return for such a tour. Equation (1) describes the performance index used for this purpose.

$$
\begin{align*}
& J=\frac{\sum_{i=1}^{10} D_{i}[k m]}{20}+n_{D>10 k m}^{o}+n_{\text {spectra }}^{0}  \tag{1}\\
& +n_{\text {regions }}^{0}+n_{\text {regions }}^{0} \cdot\left(n_{\text {regions }}^{0}>2\right)+2 \cdot n_{\text {families }}^{0}
\end{align*}
$$

where $D_{i}$ indicates the diameter of asteroid $i$ in a sequence of 10 asteroid encounters, $n^{0}{ }_{\mathrm{D}>10 \mathrm{~km}}$ refers to the number of objects whose diameter is larger than 10 $\mathrm{km}, n^{0}{ }_{\text {spectra }}$ refers to the number of objects with different spectral type, for those with spectral information available, $n^{0}$ regions the number of asteroids belonging to different AMB regions visited and $n^{0}$ families the number of prominent families visited by the tour. The initial screening of the first solutions indicated that small asteroids had a ubiquitous presence in the asteroid tours, as expected, and so solutions were ranked by the presence of large asteroids. A large variety of asteroid types was attempted to be measured by the remaining terms.

## VI. CATALOGUE OF TRAJECTORIES

A final refinement step is implemented before the solution can be definitively stored as a CASTAway feasible trajectory option. The refinement implements a further optimization of the trajectory with an added DSM after hyperbolic Earth escape. The objective function $G$ of this refinement is:

$$
\begin{equation*}
G=-W\left(C 3, \delta, \Delta v_{\text {total }}\right) \tag{2}
\end{equation*}
$$

where the final mass $W$ takes into consideration the performance of the launcher $(\mathrm{C} 3, \delta)$ and the total $\Delta v$ to compute the final dry mass after the last asteroid fly-by is performed, by means of the rocket equation ( $\mathrm{Isp}=320 \mathrm{~s}$ ). All solutions whose final dry mass is larger than 800 kg are considered as potentially feasible, since these trajectories are likely to accommodate CASTAway dry mass [5]. All potentially feasible solutions are stored into the catalogue. Table 2 summarizes the number of solutions found for Soyuzperformance launch, for both no GA and 1 Mars GA cases.

Table 2. Number of 10 FBs Asteroid Tour Solutions found after the search.

|  | Soyuz Launch |
| ---: | :---: |
| no GA | 142 |
| Mars GA | 58 |

The complete search space for no GA solutions was explored, as well as that for EM sequences. The opportunity to benefit from a Mars gravity assist appears during a short time window between late May and early August in 2029. Because of the Earth-Mars synodic period, the following launch opportunity to
benefit from Mars appears outside the launch window for the M5 call (i.e. 2029-2030). Earth-Earth sequences were not explored in this occasion, since Mars GA appeared to be more beneficial because of both; the possibility to decrease the eccentricity of CASTAway's trajectory and the possibility to avoid resonances that would place CASTAway in superior solar conjunction with the Earth during apoapsis passages.

200 different AMB tours, each with 10 asteroids fly-bys, were found feasible within the M5 boundary constraints, assuming a launch performance of SoyuzFregat. These solutions explore 1,114 different asteroids, thus some repetition occurs in the catalogue. Figure 4 shows how these 1,114 objects spread within the different regions of semimajor axis - eccentricity space. The encounters belonging to trajectories using a gravity assist from Mars are marked with red squares, while black dots identify sequences without gravity assist.


Figure 4. Semimajor axis and eccentricity map of the catalogue of encounters. Small background dots represent the available objects in the search database.

The set of solutions found allows a good spread of asteroids within the Hungaria and Inner main belt objects, but the number of asteroid in the middle and outer main belt is limited. This however is as a result of the limited launch performance, which allows only for apoapis well below 2.5 AU . Any increase of launch performance would change Figure 4 substantially. Also noticible in Figure 4, Mars GA solutions reach asteroids with much lower eccentricity than no-GA solutions.


Figure 5. Histogram of prominent families visited by different asteroid tours.

Figure 5 summarizes the list of prominent families visited with and without the Mars gravity assist. Recall that the number of solutions using Mars gravity assist is one third of those without Mars. Hence, it is evident that the strategy of using Mars gravity assist allows a larger spread of families visited. Nevertheless, none of the solutions found in either case, using Soyuz performance reached prominent families in the outer main belt.

Figure 6 shows the departure date ( $x$-axis) for each feasible solution, together with the mission timespan at the $10^{\text {th }} \mathrm{FB}$. The $x$ axis covers the entire launch window for the M5 call. The 2-month launch opportunity for Mars GA opportunities is clearly depicted in Figure 6.


Figure 6. Summary of launch dates and mission timespan at the $10^{\text {th }} \mathbf{F B}$.

Finally, Figure 7 summarizes the science performance scores, as defined by Eq.(1), and the final dry mass for each feasible solution available in the catalogue. Among all the catalogued solutions, a baseline trajectory was identified for the subsequent concurrent system design workshop at OHB system AG in Bremen, Germany [5]. Details on the baseline trajectory are given in the following section.


Figure 7. Summary of science performance scores and final dry mass for Soyuz launch solutions.

## VII. BASELINE TRAJECTORY

A baseline trajectory was identified among the complete set of solutions based on the number of asteroids of different types and sizes, as well as the time spent within the AMB. Figure 8 summarises the baseline trajectory as in the inertial heliocentric reference frame.

The launch vehicle, i.e. Soyuz, inserts the spacecraft into a hyperbolic Earth escape trajectory with $v_{\infty}$ of $4.60 \mathrm{~km} / \mathrm{s}$ and declination of -6 degrees. According to the M5 Annex [2] (see Figure 1 in document), these represents the insertion of nearly $\sim 1250 \mathrm{~kg}$ into the escape trajectory, from which nearly $\sim 1170 \mathrm{~kg}$ would be the wet mass for the CASTAway system (i.e. 80 kg of adapter). After Earth departure 11 more manoeuvres are required, including 2 deep space manoeuvres after departing Earth and Mars and 9 manoeuvres soon after each asteroid flyby in order to correct the trajectory to encounter the following object within the prescribed maximum distance of $1,000 \mathrm{~km}$. The total $\Delta v$ performed by the on-board propulsion system is of 900 $\mathrm{m} / \mathrm{s}$, delivering a final dry mass of 860 kg .

A single Mars swing-by, with a periapsis of 250 km , is used to increase the total time spent within the AMB. Note that during the refinement process of these solutions, the minimum periapsis at Mars was allowed down to a minimum altitude of 250 km . This minimum altitude was re-defined at this stage following ESA mission operation heritage, such as that of Rosetta successful Mars gravity assists at 250 km altitude on the $25^{\text {th }}$ February 2007 [13]. The effect of the Mars gravity assist is to increase both periapsis and apoapsis, which consequently allows to access a larger set of asteroid families and regions in the AMB.


Figure 8. CASTAway baseline trajectory.

Figure 9 summarizes the timeline of the baseline trajectory. It is interesting to note that FB\#3 still occurs outside the superior solar conjunction configuration for which the accuracy of range and range-rate measurements may be compromised by the solar corona, while in solar elongations below $\sim 3.5$ degrees [14]. FB\#6, FB\#7 and FB\#8 occur 77 days apart from each other, requiring manoeuvres of $27 \mathrm{~m} / \mathrm{s}$ and $11 \mathrm{~m} / \mathrm{s}$ to correct and aim for $\mathrm{FB} \# 7$ and $\mathrm{FB} \# 8$ respectively. The largest manoeuvre performed by the on-board propulsion system is $416 \mathrm{~m} / \mathrm{s}$ as a DSM, two weeks after departing Earth.

## VIII. FURTHER DISCUSSION

The design of multiple asteroid tours is a challenging trajectory optimization problem. Proof of that is its recurrent use in the GTOC, whose $2^{\text {nd }}, 3^{\text {rd }}, 4^{\text {th }}$, $5^{\text {th }}$ and $7^{\text {th }}$ edition used different variants of the problem. All containing a similarly challenging discrete combinatorial problem.

The $4^{\text {th }}$ edition of GTOC, proposed perhaps the problem closets to CASTAway design, on which the number of asteroid fly-bys was also maximized. However, the transfer was constrained by the fact that it had to finish with an asteroid rendezvous (not fly-by) and mission duration was limited to 10 years. The propulsion system was a low thrust and the dry mass of the spacecraft 500 kg . All top 10 participating teams provided solutions with more than 25 asteroid encounters, with a winner trajectory with 44 [11].

While the above result hold promise on the feasibility of CASTAway trajectory design, the realistic
capability of the spacecraft to navigate through the asteroid main belt is severally constrained by a conservative performance envelope outlined for an Mclass mission.

While low thrust propulsion (LT) was discarded on the grounds of complexity and cost, the much higher specific impulses of such a technology would certainly enable much more complex missions than those reported here. Some example low thrust trajectories were also computed during the work reported and demonstrated that the benefits of higher specific impulses for LT by far outweighs the disadvantages of thrust in the order of 100 mN [15].

On the other hand, the baseline scenario assumed a Soyuz-like performance. As has been discussed, this already enables scientifically valuable solutions. However, the potentially higher performances for the next generation of European medium-lift launchers (A62) holds a lot of promise on the much more complex trajectories that could be designed (e.g., [16]). In fact, several 10 s of trajectories with 14 asteroids were found for an $\mathrm{A} 62\left(\mathrm{~S}^{+}\right)$by exploring less than $2 \%$ of the potential design space for this scenario. These solutions could potentially reach to apoapsis beyond 3 AU and thus encounter a much wider menagerie of asteroid types.

## IX. CONCLUSIONS

CASTAway envisages the launch of a small telescope with relatively straightforward (i.e. high TRL) remote sensing instrumentation. One of the main operational modes of CASTAway will perform long-
range (i.e. point source) telescopic survey of over 10,000 objects. However, CASTAway is much more than an asteroid science dedicated space telescope; 10, or more, asteroid fly-bys will be performed during less than 7 years of transfer time. Finally, CASTAway will inform solar system dynamical evolution by providing also $10,000 \mathrm{~s}$ serendipitous discoveries of small 10 metre class asteroids.

CASTAway trajectory loops through the asteroid belt for 3 consecutive orbital revolutions. The mission has been shown to be technically feasible within the M5 boundary constrains, as well as to meet all the science requirements.

It has also been shown that current launch performance allows an entrance level performance for such a mission and that any improvement on the hyperbolic escape capability of the future European medium-lift launchers would potentially result on a much enhanced science return.

Moreover, the study presented has been performed with a reduced population of asteroids. If selected, CASTAway would be launched by 2029+, which
entails an order of magnitude increase on the database of potential candidates. This thus entails a much higher likelihood of visiting targets with high scientific interest.

Finally, the work has focused on ensuring/demonstrating the technical feasibility of CASTAway's trajectory. This has been done by designing a search methodology capable to explore the entire design space for a conservative Soyuz performance launch, and finding good sub-optimal solutions. It is likely that the global optimum has not yet been found in this search. Nevertheless, the work has demonstrated the abundance of feasible trajectories within the envelope of the M5 call.

## ACKNOWLEDGEMENTS

The work reported was supported by the UK Space Agency (NSTP2-GEI1516-020 "CASTPath"). J.P. Sanchez would like to acknowledge, and thank, Marc Tora, Mario Cano, Giacomo Curzi and Rita Neves for their insights and contributions to CASTAway trajectory design.

## REFERENCES

[1] F. DeMeo, C. Alexander, K. Walsh, C. Chapman, R. Binzel, The Compositional Structure of the Asteroid Belt, Asteroids IV, (2015) 13-41.
[2] F.M. Office, M5 Call - Technical Annex (ESA-SCI-F-ESTEC-TN-2016-002), in, ESTEC (ESA), NoordwikjNetherlands, 2016.
[3] J.R. Wertz, D.F. Everett, J.J. Puschell, Space mission engineering: the new SMAD, Microcosm Press, 2011.
[4] Arianespace, Soyuz: User's Manual (Issue 2 - Revision 0), in: E. Perez (Ed.), 2012.
[5] A. Gibbings, B. Bowles, C. Snodgrass, J.P. Sánchez, H. Henning, W. Posselt, A. Braukhane, An Inventory Tour of the Main Asteroid Belt, in: 67th International Astronautical Congress International Astronautical Federation, Guadalajara, Mexico, 26-30 September 2016.
[6] Arianespace, Ariane 5 User's Manual: Issue 5 Revision 1, in: E. Perez (Ed.), 2011.
[7] M. Delbo’, J. Gayon-Markt, G. Busso, A. Brown, L. Galluccio, C. Ordenovic, P. Bendjoya, P. Tanga, Asteroid spectroscopy with Gaia, Planetary and Space Science, 73 (2012) 86.
[8] R. Jones, S. Chesley, A. Connolly, A. Harris, Z. Ivezic, Z. Knezevic, J. Kubica, A. Milani, D.E. Trilling, L.S.S.S. Collaboration, Solar system science with LSST, Earth, Moon, and Planets, 105 (2009) 101-105.
[9] M. Ceriotti, Global optimisation of multiple gravity assist trajectories, in, University of Glasgow, 2010.
[10] M. Cano, Asteroid Belt Survey Mission: Trajectory Design for a Medium Class Telescopic Mission, in: School of Aerospace, Cranfield University, Cranfield, Bedfordshire, UK, 2016.
[11] I.S. Grigoriev, M.P. Zapletin, Choosing promising sequences of asteroids, Automation and Remote Control, 74 (2013) 1284-1296.
[12] Milisavljevic, S., The Proximities of Asteroids and Critical Points of the Distance Function, Serbian Astronomical Journal, (2010).
[13] P. Ferri, A. Accomazzo, E. Montagnon, J. Morales, Rosetta in the year of the swing-bys, Acta Astronautica, 63 (2008) 102-109.
[14] T. Morley, F. Budnik, Effects on Spacecraft Radiometric Data at Superior Solar Conjunction, in: Proceedings 20 th International Symposium on Space Flight Dynamics-20 th ISSFD. Annapolis, MD, USA, 2007.
[15] M. Tora, Low-thrust Trajectory Design for Multiple Fly-by Tours in the Asteroid Belt, in: School of Aerospace, Cranfield University, Cranfield, Bedfordshire, UK, 2016.
[16] G. Curzi, Trajectory Design for Asteroid Belt Tours, in: Department of Industrial Engineering Universita di Bologna, Bologna, Italy, 2016.


[^0]:    http://www.cosmos.esa.int/web/call-for-m5missions [Last accessed: 24/08/2016]

[^1]:    \# http://sophia.estec.esa.int/gtoc_portal/ [Last accessed: 24/08/2016]

