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Determination of effective stress-effective strain relationship for use as a machinability index

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SUMMARY

In recent work it has been shown that the effective stress effective strain relationship of a work material is important in determining the shear angle in orthogonal cutting. This note describes the method of obtaining this relationship.

Introduction

In recent papers[1,2] it was suggested from theoretical considerations that the machinability of metals, in terms of the shear angle in orthogonal cutting, was dependent on the ratio of the flow stress (k) and the slope of the shear stress-shear strain curve (m) at the mean strain rate of cutting. A material with a low value for the ratio m/k was expected to machine better than one having a high value. Although high rates of strain are encountered in metal cutting experimental evidence was given to show that the relative machinability of materials could be assessed from the values of m/k taken from the results of conventional compression tests. This note describes the method of obtaining the ratio m/k from compression tests used at Cranfield.

Test Procedure

The test pieces used are cylindrical having a diameter of .02 in. and a height of .225 in. and are compressed in a testing machine with a capacity of 15000 lb. Before placing a test piece between the compression platterns the ends are lubricated by rubbing them with a graphite pencil and then lightly smearing with oil. The test piece is then loaded in increments of 500 lb. until 60% reduction is reached. Between each loading the test piece is removed from the machine, its height measured and its ends re-lubricated. The advantages of incremental over continuous loading techniques are that the effect of friction between the ends of the test piece and the compression platterns, which causes barrelling, is minimised by lubricating the ends between each loading, the effect of creep can be overcome by maintaining the load at the desired value for a given time and as the measurements of height are made zero load no allowance has to be made for the elastic strains of the test piece or machine.

Theory

The effective stress for a three dimensional state of stress can be defined as

$$\sigma_{eff} = \frac{\sqrt{2}}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \qquad (1)$$

where σ_1, σ_2 and σ_3 are the principal stresses.

The corresponding invariant of natural strain rate, the effective strain rate, can be defined as

$$\epsilon'_{eff} = \frac{\sqrt{2}}{3}\sqrt{(\epsilon_1' - \epsilon_2')^2 + (\epsilon_2' - \epsilon_3')^2 + (\epsilon_3' - \epsilon_1')^2} \qquad (2)$$

where ϵ_1', ϵ_2', and ϵ_3' are the principal strain rates. By integration the effective strain is

$$\epsilon_{eff} = \int \epsilon'_{eff} dt$$

$$= \frac{\sqrt{2}}{3} \int \sqrt{(\epsilon'_1 - \epsilon'_2)^2 + (\epsilon'_2 - \epsilon'_3)^2 + (\epsilon'_3 - \epsilon'_1)^2} dt \qquad (3)$$

In uniaxial compression

$$\sigma_2 = \sigma_3 = 0 \qquad (4)$$

which by substitution in equation (1) gives

$$\sigma_{eff} = \frac{\sqrt{2}}{2} \sqrt{2\sigma_1{}^2} = \sigma_1 \qquad (5)$$

By symmetry and incompressibility

$$\epsilon'_2 = \epsilon'_3 = -\frac{\epsilon'_1}{2} \qquad (6)$$

so that equation (3) becomes

$$\epsilon_{eff} = \int \epsilon'_1 dt = \epsilon_1 \qquad (7)$$

Thus in uniaxial compression the true stress natural strain curve gives the relation between effective stress and effective natural strain.

If a cylinder of original height h_o and original diameter d_o is compressed to a height h by a load W, the effective stress, assuming constant volume, is given by

$$\sigma_{eff} = \frac{4Wh}{\pi d_o{}^2 h_o} \qquad (8)$$

and the effective natural strain by

$$\epsilon_{eff} = \log_e \left[\frac{h}{h_o}\right] \qquad (9)$$

Although equations (8) and (9) ignore the effects of barrelling (an extrapolation technique which allows for these effects has been described by Watts and Ford[3]) they have been found to give sufficiently accurate results for steels.

The process of orthogonal cutting is one of plane strain for which equation (1) reduces to

$$\sigma_{eff} = \sqrt{3}\,\tau_{max} \tag{10}$$

where τ_{max} is the maximum shear stress which during plastic deformation is equal to the flow stress k, and equation (3) becomes

$$\epsilon_{eff} = \frac{1}{\sqrt{3}} \int \gamma_{max} dt = \frac{\gamma max}{\sqrt{3}} \tag{11}$$

where γ_{max} is the maximum shear strain.

Most effective stress-effective strain curves are of the form shown in figure 1, which is for steel CX 45, and have a high initial slope which falls to a steady value at high strains. To standardise the m/k values obtained from such curves it has been our practice to take the effective stress at an effective strain of 0.5 (this corresponds to a shear strain of approximately unity and is of the order found in cutting) as the basis for determining k. As it has been shown[4] that it is the slope of the curve at high strains rather than the initial slope that is important in cutting the mean slope of the curve above an effective strain of 0.2 has been used for calculating m.

Appendix 1

As an example of the method of calculation let us take the results given in Table 1 for CX 45.

For a load, W, of 4000 lb.

h_o = .223 in.

h = .203 in.

d_o = .20 in.

Hence the effective stress is

$$\sigma_{eff} = \frac{4.4000 \quad .203}{\pi.(0.2)^2.223} = 116 \times 10^3 \text{ psi}$$

and the effective strain

$$\epsilon_{eff} = \log_e \left[\frac{.203}{.223}\right] = 0.1 \text{ (compressive)}$$

By calculating in this manner for each load the stress-strain curve shown in figure 1 can be obtained. From this curve

σ_{eff} (at ϵ_{eff} 0.5) = 148×10^3 psi

and the slope (above ϵ_{eff} 0.2) = 53×10^3 psi

By combining equations (10) and (11) it can be seen that

$$m/k = \frac{\text{slope of effective stress-effective strain curve}}{\sqrt{3}\ \text{effective stress (at a given strain)}}$$

Hence the m/k value for CX 45 is

$$m/k = \frac{53 \times 10^3}{\sqrt{3}\ 148 \times 10^3} = .21$$

References

1. P.L.B. Oxley and M.J.M. Welsh — Calculating the shear angle in orthogonal metal cutting from fundamental stress-strain-strain rate properties of the work material. 4th MTDR Conference 1963.

2. P.L.B. Oxley — Introducing strain-rate dependent work material properties into the analysis of orthogonal cutting. CIRP Report 1964.

3. A.B. Watts and H. Ford — Proc. I. Mech. E. 169,1141 (1955)

4. P.L.B. Oxley, A.G. Humphreys and A. Larizadeth — Proc. I. Mech. E. 175, 18,881 (1961)

Table 1

Compression test for steel CX 45

Original height 0.223 in. Original diameter 0.20 in.

Load lb.	Height in.	Effective stress 10^3psi	Effective strain
0	.223	0	0
500	.223	16	0
1000	.223	32	0
1500	.223	48	0
2000	.221	63	.01
2500	.2185	78	.02
3000	.215	92	.04
3500	.2105	105	.06
4000	.203	116	.1
4500	.193	124	.15
5000	.182	130	.2
5500	.171	134	.26
6000	.161	138	.33
6500	.152	141	.39
7000	.144	144	.44
7500	.137	147	.49
8000	.130	148	.54
8500	.124	150	.59
9000	.1185	152	.63
9500	.114	155	.68
10000	.109	157	.71
10500	.1055	158	.75
11000	.102	160	.78

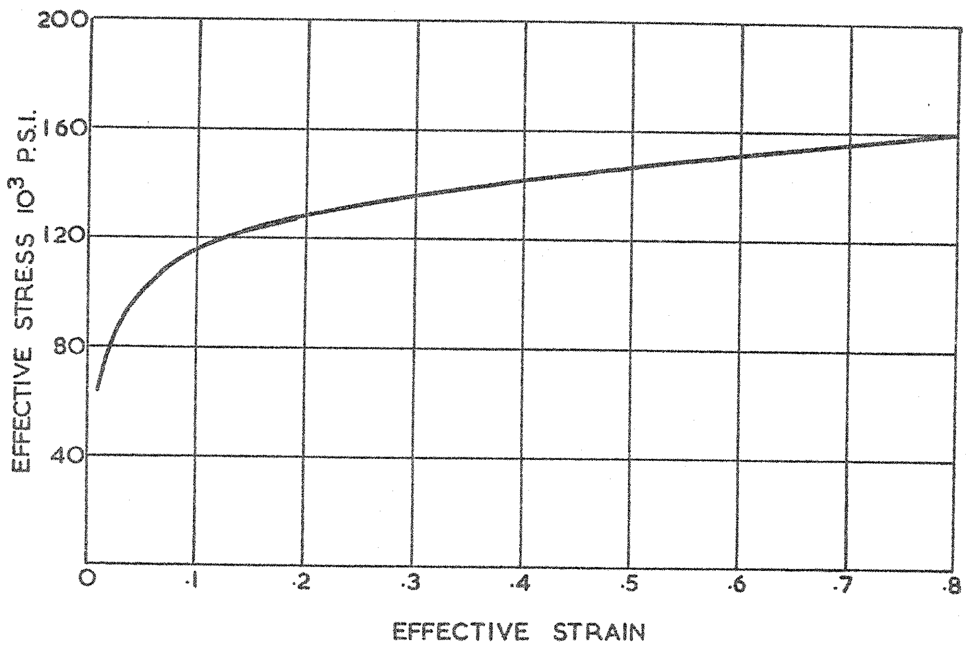

FIG I STRESS STRAIN CURVE CX 45