

# Integration of Torque Blending and Slip Control using Nonlinear Model Predictive Control

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Antilock Braking System (ABS) is an important active safety feature in preventing accidents during emergency braking. Electrified vehicles which include both hydraulic and regenerative braking systems provide the opportunity to implement brake torque blending during slip control operation. This study evaluates the design and implementation of a new torque allocation algorithm using a Nonlinear Model Predictive Control (NMPC) strategy that can run in real-time, with results showing that wheel-locking can be prevented while also permitting for energy recuperation.

Topics / Brake, active safety, full-electric vehicles and hybrid electric vehicles

## 1. INTRODUCTION

Electrified vehicles are equipped with redundant braking actuators, namely hydraulic brakes and a regenerative braking system. This creates the opportunity for research into brake torque blending for Antilock Braking System (ABS) in a hybrid braking system.

Several articles report torque blending algorithms using optimization methods. In [1] an adaptive slip controller is proposed and the brake torque allocation is designed using Control Allocation (CA). In [2] a linear slip controller is designed and a linear Model Predictive Control (MPC) strategy is employed to allocate the brake torque between the two actuators. The cascaded strategy in [2] is replaced with a combined strategy for slip control and torque blending using linear MPC in [3]. Static brake torque allocation by Daisy Chain (DC) which requires less computational effort as compared to optimization methods is proposed and tested on a hardware-in-the-loop simulator in [4].

The problem of integrating slip control and brake torque allocation includes important nonlinearities in both the system dynamics and constraints. The emergence of real time nonlinear solvers make it possible to treat the optimization as a constrained nonlinear problem instead of using linear approximations. To the authors' best knowledge, there is currently no work on integrated slip control and brake torque allocation using Nonlinear Model Predictive Control (NMPC) with real time implementation capability. This work therefore presents a slip control and torque blending strategy incorporated in single NMPC formulation.

## 2. PREDICTIVE CONTROL STRATEGY

For the NMPC formulation a single-wheel model is employed. Assume that the continuous-time model is

$$\dot{V}_x = \frac{F_x}{m}, \quad (1a)$$

$$\dot{s}_x = -\frac{F_x}{V_x} \left( \frac{R_w^2}{J_w} + \frac{s_x}{m} + \frac{1}{m} \right) + \frac{R_w}{J_w V_x} T_{tot}, \quad (1b)$$

with  $V_x$  the wheel's forward velocity,  $s_x$  the longitudinal slip,  $F_x$  the tyre's longitudinal force,  $T_{tot}$  the total torque applied on the wheel and  $R_w$ ,  $J_w$  and  $m$  the wheel's radius, moment of inertia and mass respectively. In the above model,  $F_x$  is set as a function of  $s_x$  through a simplified version of Pacejka's Magic Formula (MF) [5]:

$$F_x = F_z D \sin(C \tan^{-1}(B s_x)), \quad (2)$$

where  $B$ ,  $C$  and  $D$  are the MF's factors and  $F_z$  the vertical force on the tyre. For our blending strategy, the  $T_{tot}$  is the summation of the torque from the electric motor  $T_e$  and the torque from the hydraulic brake  $T_h$ . Then if the single wheel model (1) is augmented with the trivial equalities  $dT_e = T_e^{rate}$  and  $dT_h = T_h^{rate}$ , the following nonlinear system is obtained

$$\dot{x} = f(x, u), \quad (3)$$

with  $x = [V_x \ s_x \ T_e \ T_h]^T$  and  $u = [T_e^{rate} \ T_h^{rate}]^T$ . Using system (3) the NMPC problem with sampling time  $T_s$  and prediction horizon  $N$  is

$$\min_u \sum_{k=0}^{N-1} \left\| \begin{bmatrix} V_{xk} - V_x^{ref} \\ s_{xk} - s_x^{ref} \end{bmatrix} \right\|_Q^2 + \|u_k\|_R^2$$

$$\text{s. t. } \begin{aligned} x_0 &= x_{current}, \\ x_{k+1} &= g(x_k, u_k), \quad k = 0, \dots, N-1, \\ \underline{x} &\leq x_k \leq \bar{x}, \\ \underline{u} &\leq u_k \leq \bar{u}, \end{aligned}$$

where we choose to penalize the  $V_x$  and  $s_x$  errors only from a given reference, along with the control effort in the form of a penalty on the torque rates  $T_e^{rate}$  and  $T_h^{rate}$ . In this way we do not explicitly set references for the

electric motor torque  $T_e$  and the hydraulic brake torque  $T_h$ , but rather leave the NMPC find the appropriate values according to the given  $V_x$  and  $s_x$  references, the torque and torque rate constraints, and the chosen weight matrices  $Q > 0$  and  $R > 0$ .

To solve the NMPC problem online, the Real Time Iteration (RTI) scheme available as part of the ACADO Toolkit [6] is employed, which allows for small computational times as demonstrated in the simulation study of the next section.

Finally, in order to generate the reference wheel speed  $V_x^{ref}$  we can use steady-state analysis. Given a longitudinal slip reference  $s_x^{ref}$ , a longitudinal force reference can be computed using (2) and from that a constant acceleration target  $a_x^{ref}$  from (1a). Then the reference wheel speed is simply given by

$$V_x^{ref} = V_{current} + T_s N a_x^{ref}.$$

### 3. SIMULATION STUDY

In this section we present preliminary results using the NMPC strategy from section 2 on a single-wheel model in Simulink, with the motor and hydraulic brake modelled as simple 1<sup>st</sup> order delays (time constants of  $\tau_e = 0.03$  and  $\tau_h = 0.09$  respectively). The wheel and tyre parameters used can be found in Table 1.

Table 1 Wheel and Tyre Parameters

wheel total mass	$m$	362.5 kg
wheel moment of inertia	$J_w$	1.04 kgm <sup>2</sup>
wheel radius	$R_w$	0.3 m
MF's stiffness factor	$B$	7
MF's shape factor	$C$	1.6
MF's peak value	$D$	1

For the NMPC we set  $T_s = 10ms$  and  $N = 50$ . The rate limits for the motor and the brake are set to  $\frac{1000}{0.1}$  Nm/s and  $\frac{1000}{0.3}$  Nm/s respectively and the weights in the cost function are chosen such that we penalize large longitudinal slip errors, while also we give priority to the use of the electric motor for the torque delivery:  $Q = diag([10, 100000])$ ,  $R = diag([0.08, 0.072])$ .

In the presented scenario the wheel is initially free-rolling with initial speed of 30m/s on a dry road ( $\mu = 1$ ) and after 1s a slip target of  $s_x = -0.1$  is demanded. As we can see from Fig 1, the NMPC strategy successfully regulates the longitudinal slip (Fig 1(b)) by distributing the  $T_e$  and  $T_h$  torques (Fig 1(c)), with an obvious preference for the electric motor. It is also worth noting at this point that the commanded torques from the controller are very close to the actual torques as delivered by the actuators, a result of including the torque rate constraints in the NMPC formulation. Finally, as we can see from Fig 1(d), the time to compute the solution using a rather standard laptop (i5-2520M at 2.50Ghz with 8GB of memory) takes around 1ms, which is much lower than the sampling time of 10ms.

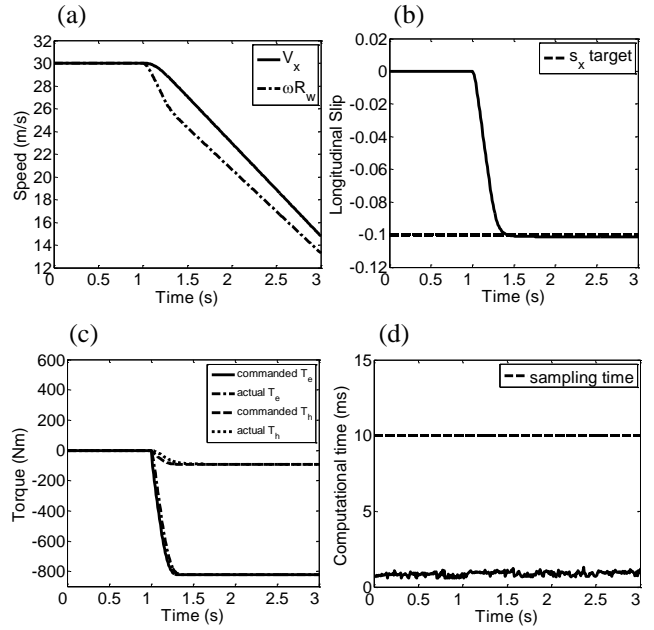


Fig. 1 Simulation results

### 4. SUMMARY & ONGOING WORK

A new unified slip controller and brake torque integration strategy has been proposed in this work using NMPC formulation. Preliminary result shows that real time implementation of NMPC for slip control by different braking actuators can be deployed. Actuator dynamics and constraints have been taken into consideration in the optimization formulation. Ongoing work will include brake torque range limit and validate using high fidelity model.

### REFERENCES

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