Continuum damage mechanics in SPH based on particle interaction area.

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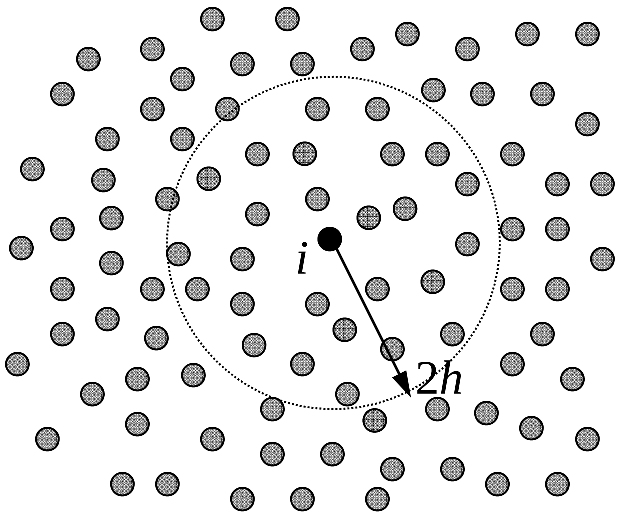
Overview

- Introduction
  - o Smoothed Particle Hydrodynamics (SPH)
  - o Continuum damage mechanics
- Forming a new free surface within SPH
- Particle interaction area concept
- Implementation and test of interaction area concept
  - o Plate impact and damage model used
  - o 1D and 3D plate impact models
- Summary

Smoothed Particle Hydrodynamics (SPH)

- SPH is a numerical technique for the approximate integration of the governing equations of continuum mechanics.
- It is a meshless Lagrangian method (the points move with the material) that uses pseudo-particle interpolation to compute smooth field variables.
- If a function $f(x)$ is known at N discrete points (known as particles), it can be approximated by the summation:

$$\langle f(x)\rangle \approx \sum_{j=1}^{N} \frac{m_j}{\rho_j} f\left(x_j\right) W\left(x - x_j, h\right)$$



- The kernel function W has compact support and so the value of the function at a point depends only on the particles within the support domain of the point.
- The gradient of the function can the be approximated as:

$$\langle \nabla f(x)\rangle \approx \sum_{j=1}^{N} \frac{m_j}{\rho_j} f\left(x_j\right) \nabla W\left(x - x_j, h\right)$$

Smoothed Particle Hydrodynamics (SPH)

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- The governing equations of continuum mechanics can be written in SPH form as:

$$\left\langle \frac{d\rho_i}{dt} \right\rangle = \rho_i \sum_j \frac{m_j}{\rho_j} \left[v_i - v_j \right] \nabla_i W_{ij}$$ Conservation of mass

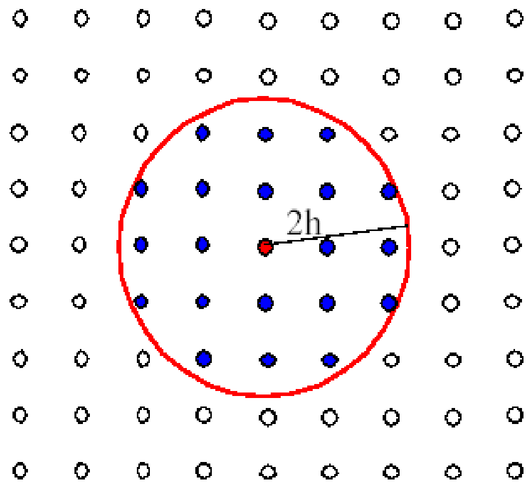
$$\left\langle \frac{dv_i}{dt} \right\rangle = \sum_j m_j \left[\frac{\boldsymbol{\sigma}_i}{\rho_i^2} + \frac{\boldsymbol{\sigma}_j}{\rho_j^2} \right] \nabla_i W_{ij}$$ Conservation of momentum

$$\left\langle \frac{dE_i}{dt} \right\rangle = -\frac{\boldsymbol{\sigma}_i}{\rho_i^2} \sum_j m_j \left[v_i - v_j \right] \nabla_i W_{ij}$$ Conservation of energy

- where the summation is over all neighbour particles and m_j is the mass of particle *j*.

Smoothed Particle Hydrodynamics (SPH)

- In this work we use the Total Lagrangian[1] form of SPH.
  - The conservation equations are written and solved in the material coordinate system, X, instead of the spatial coordinate system, x.
  - The relationship between the two coordinate systems is given by the deformation gradient tensor $\mathbf{F}$.
  - The conservation of momentum equation is therefore written



$$\left\langle \frac{dv_i}{dt} \right\rangle = \sum_j m_j \left[\frac{\mathbf{N}_i}{\rho_i^2} + \frac{\mathbf{N}_j}{\rho_j^2} \right] \nabla_i W \left(X_i - X_j, h_0 \right)$$

  - Where $\mathbf{N} = \left(J\mathbf{F}^{-1}\boldsymbol{\sigma} \right)$ is the nominal stress and $J = \det \mathbf{F}$

[1]R Vignjevic, JR Reveles, JC Campbell, 2006. SPH in a total Lagrangian formalism. *CMES* , 14(3), 181-198.

Continuum Damage Mechanics

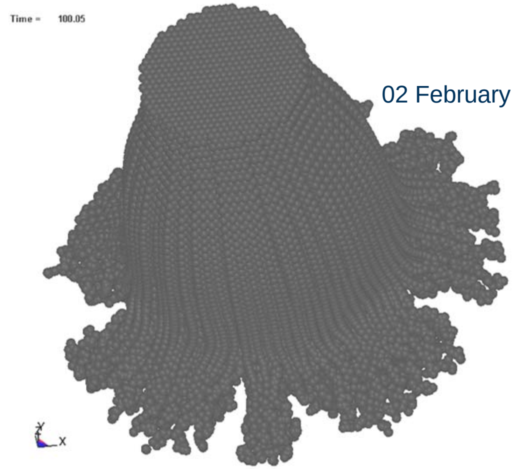
- Continuum damage mechanics (CDM) is an approach for modelling the effect of material damage within the framework of continuum mechanics.
- Underlying concept:
  - mechanical damage in solid materials is due to the creation and growth of micro-cracks or micro-voids.
  - the characteristic length of the cracks is too small to individually represent
  - their influence is homogenised, so the effect of damage is averaged over a volume and represented by a continuous variable related to the density of the defects.
- Considering a 1D problem where a volume element of material is loaded by force F. A scalar damage variable, D, represents the reduction in the cross-section area, S.

$$D = \frac{S - \tilde{S}}{S}$$

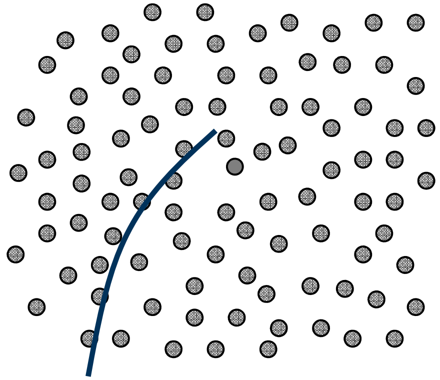
- An effective Cauchy stress, $\tilde{\sigma}$, can then be defined as the stress acting on the effective surface area.

$$\tilde{\sigma} = \frac{\sigma}{(1 - D)}$$

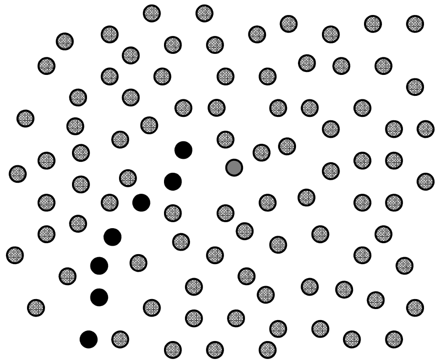
Forming a new free surface



- When the damage variable reaches a critical value, it is assumed that the cracks and voids coalesce to form a new free surface.
- Several approaches are available to describe and treat a new free surface within the original domain.
  - Treat new free surface as geometrical object.
    - Describe geometry of free surface using level set (or similar) modify method to treat resulting discontinuity.

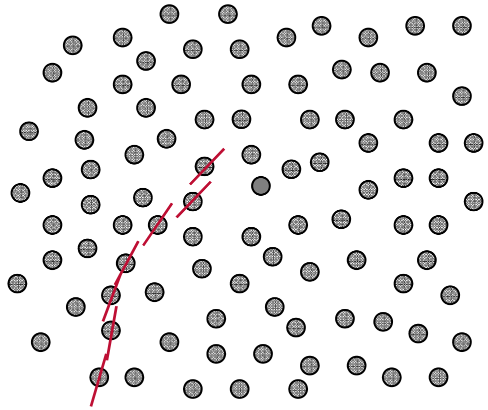


  - Represent free surface as combination of local failures
    - Set failed particle stress to zero

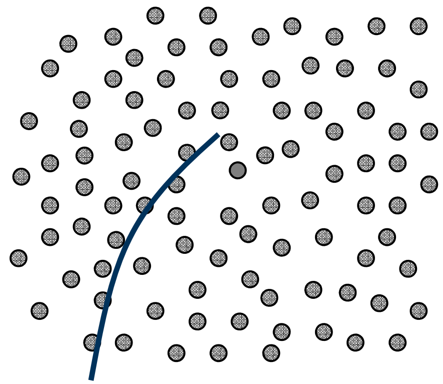


Forming a new free surface

- Represent free surface as combination of local failures
  - Cracked particle approach[1] where failed particle is split in two.



  - Modify particle-particle interaction
    - basis of work presented here



[1]T Rabczuk and T Belytschko. 2004. Cracking particles: A simplified meshfree method for arbitrary evolving cracks. *IJNME*, 61.

Particle interaction area

- The concept of area vectors within the SPH method was outlined by Swegle[1] for the purpose of discussing the tensile instability inherent in any basic SPH description.
- Swegle noted that the fundamental definition of the stress tensor shows that a force exerted on a surface due to stress is given by:

$$\mathbf{F} = \boldsymbol{\sigma} \cdot \mathbf{A}$$

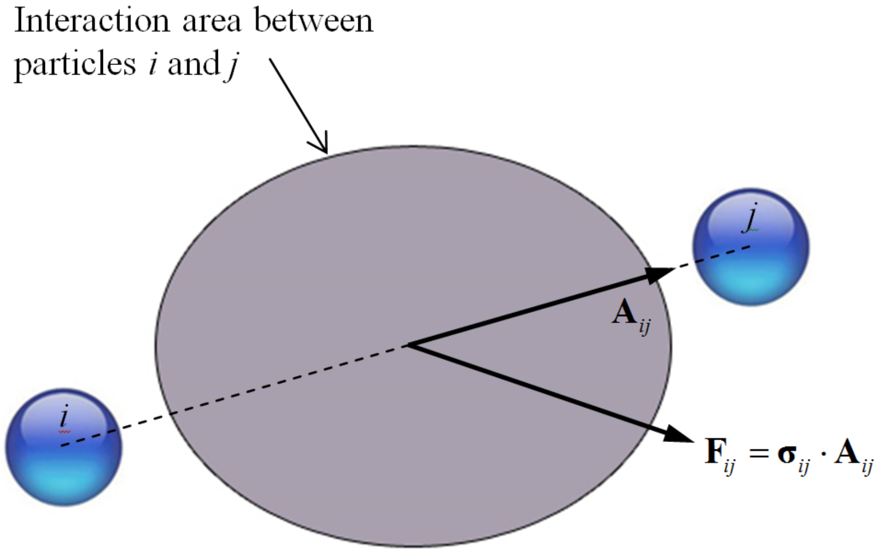
- The conservation of momentum equation can be rewritten as

$$F_i = m_i a_i = -\sum_j m_i m_j \left[\frac{\boldsymbol{\sigma}_i}{\rho_i^2} + \frac{\boldsymbol{\sigma}_j}{\rho_j^2} \right] \nabla_i W_{ij}$$

- Rearranging allows the definition of an interaction area.

$$F_i = -\sum_j \left[\left(\boldsymbol{\sigma}_i\right) \frac{\rho_j}{\rho_i} + \left(\boldsymbol{\sigma}_j\right) \frac{\rho_i}{\rho_j} \right] A_{ij}$$

$$A_{ij} = V_i V_j \nabla_i W_{ij}$$



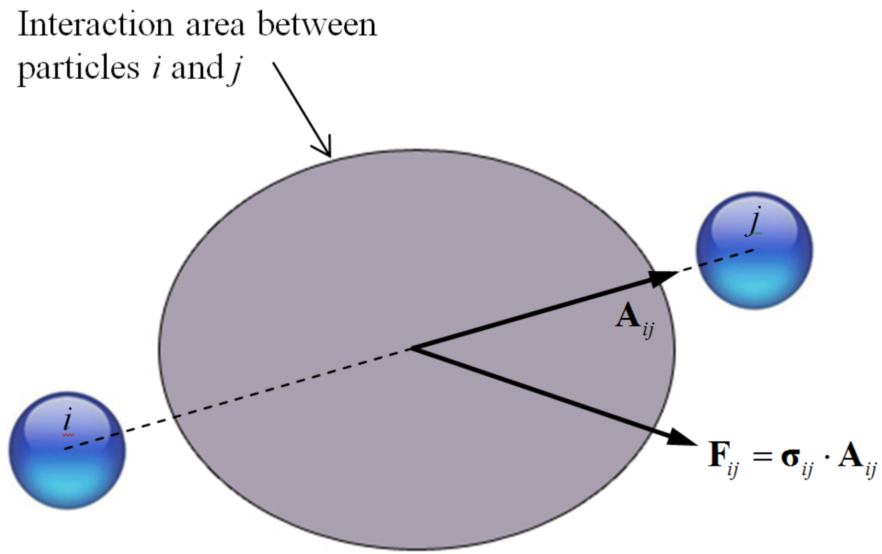
[1] J W Swegle. Conservation of momentum and tensile instability in particle methods. USA: Sandia National Laboratories, 2000. SAND2000-1223

Particle interaction area

- Apply damage to this interaction area,

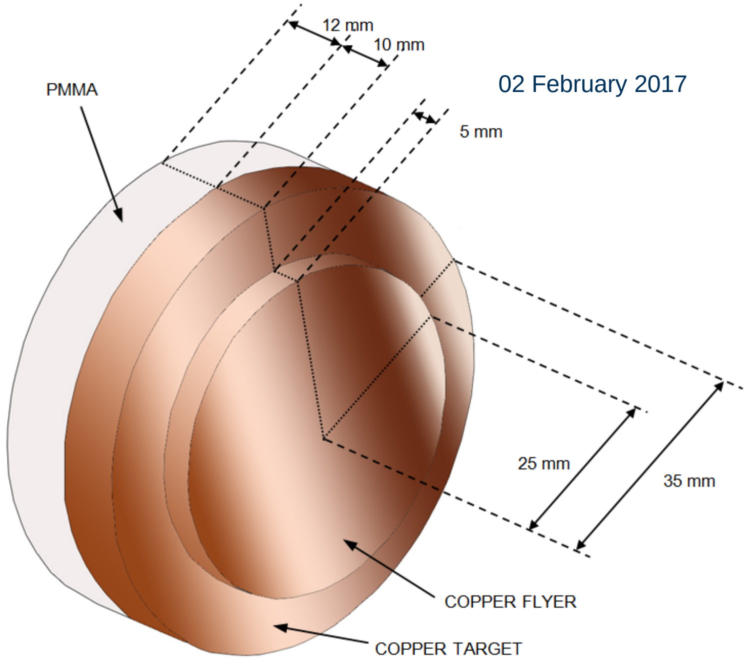
$$F_i = -\sum_j \left[(\boldsymbol{\sigma}_i) \frac{\rho_j}{\rho_i} + (\boldsymbol{\sigma}_j) \frac{\rho_i}{\rho_j} \right] A_{ij} (1 - D_{ij})$$

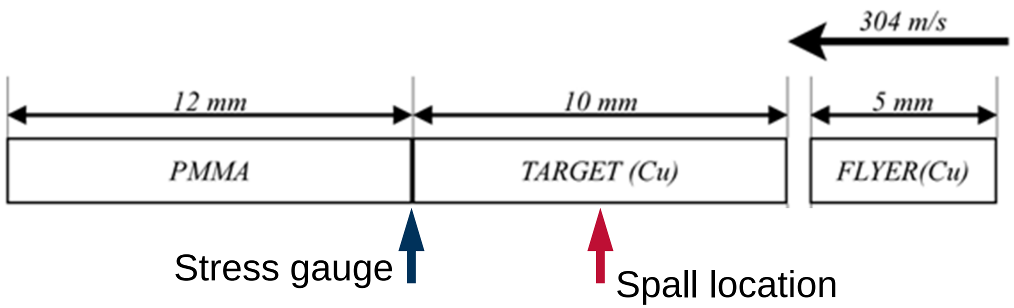
  - when damage reaches its critical value the interaction area is assumed to be zero
- Consistent with CDM, so can use damage laws derived for CDM

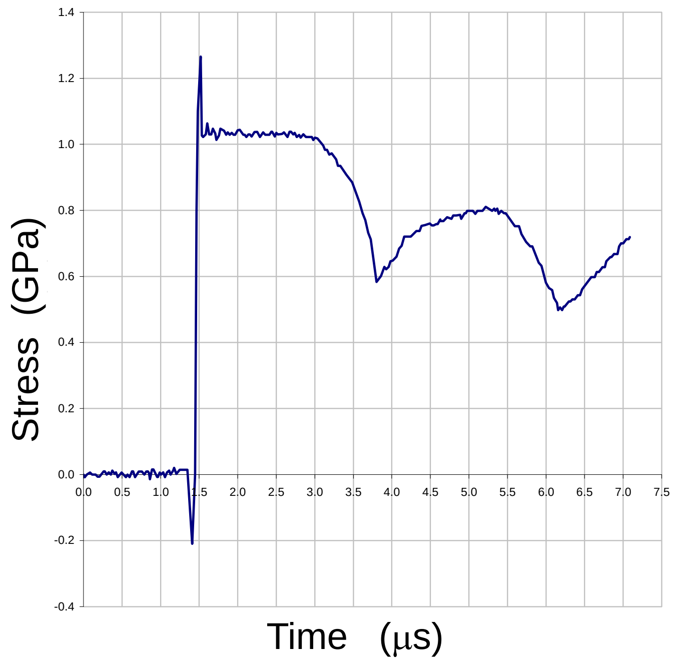


Test problem

- To investigate the behaviour of the interaction area concept we have used 1D, 2D and 3D models of a flyer plate experiment.
- 304 m/s impact of a copper flyer on a copper target.
- Stress gauge mounted in PMMA at interface with copper target.
- Spall plane formed at centre of target plate, pullback signal visible is stress trace.







- Axis of flyer and plate is under 1D strain for duration of initial impact and formation of spall plane

Cochran-Banner spall model

- The Cochran-Banner[1] spall model was used as the damage model for this investigation.
- The model assumes all changes in volume past the material spall strength is due to micro-crack and void growth:

$$D(x,t) = \int_0^t dV/A \qquad dV > 0$$

- For a bond the volume change is assumed to be the average of the two particles

$$\Delta V_{ij} = \frac{dt}{2}\left(V_i \operatorname{tr} \dot{\boldsymbol{\varepsilon}}_i + V_j \operatorname{tr} \dot{\boldsymbol{\varepsilon}}_j\right)$$

- The increment in damage is then

$$\Delta D_{CB} = \frac{\Delta V_{ij}}{\left|A_{ij}\right|}$$

[1]S Cochran, D Banner. Spall Studies in Uranium. *J Appl Phys* 1977;48(7):2729-2737

Plate impact model – 1D solver

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- Without damage model no spall occurs
- With damage model spall occurs at correct location within target
- Shape of pullback signal is not correct

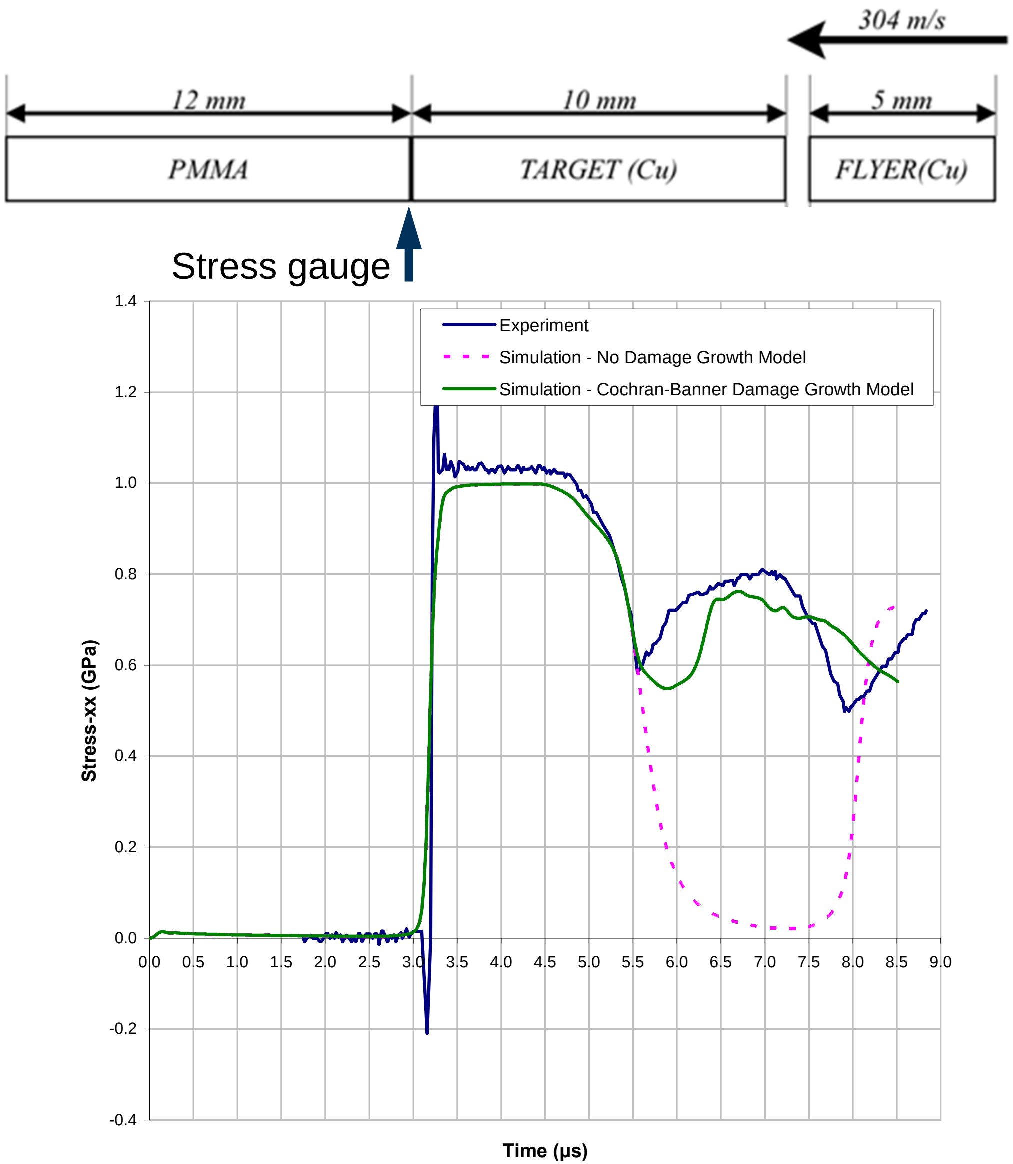
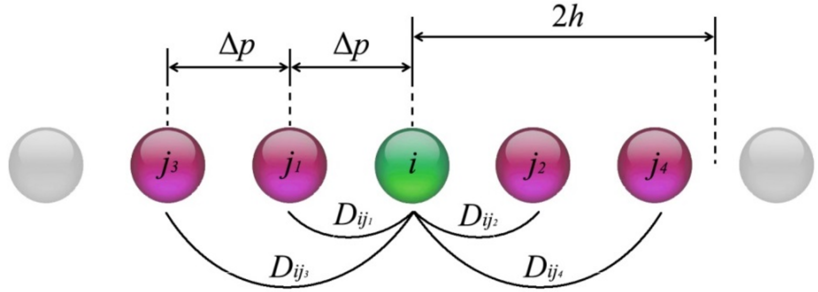


Plate impact model – 1D solver

- A given *i* particle interacts with multiple neighbours in a given direction
- Investigated three criteria for influence of multiple neighbours:
  1. if 1st neighbour failed then 2nd neighbour is also failed
  2. if 2nd neighbour failed then 1st neighbour is also failed
  3. If either fails then other bond is also failed



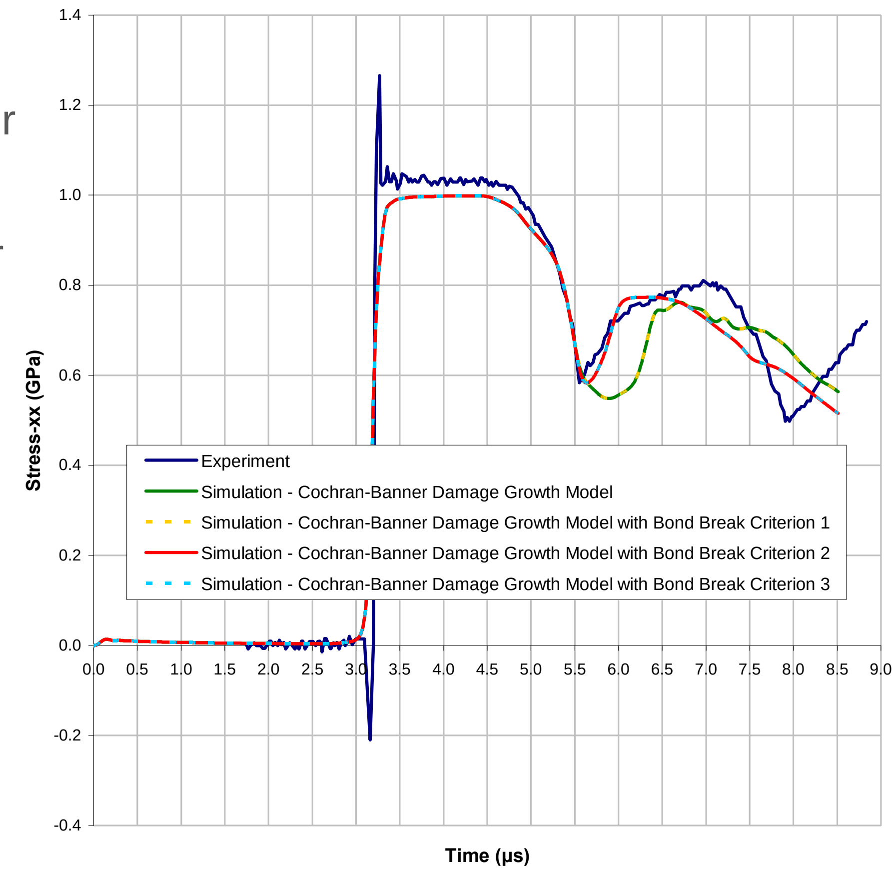
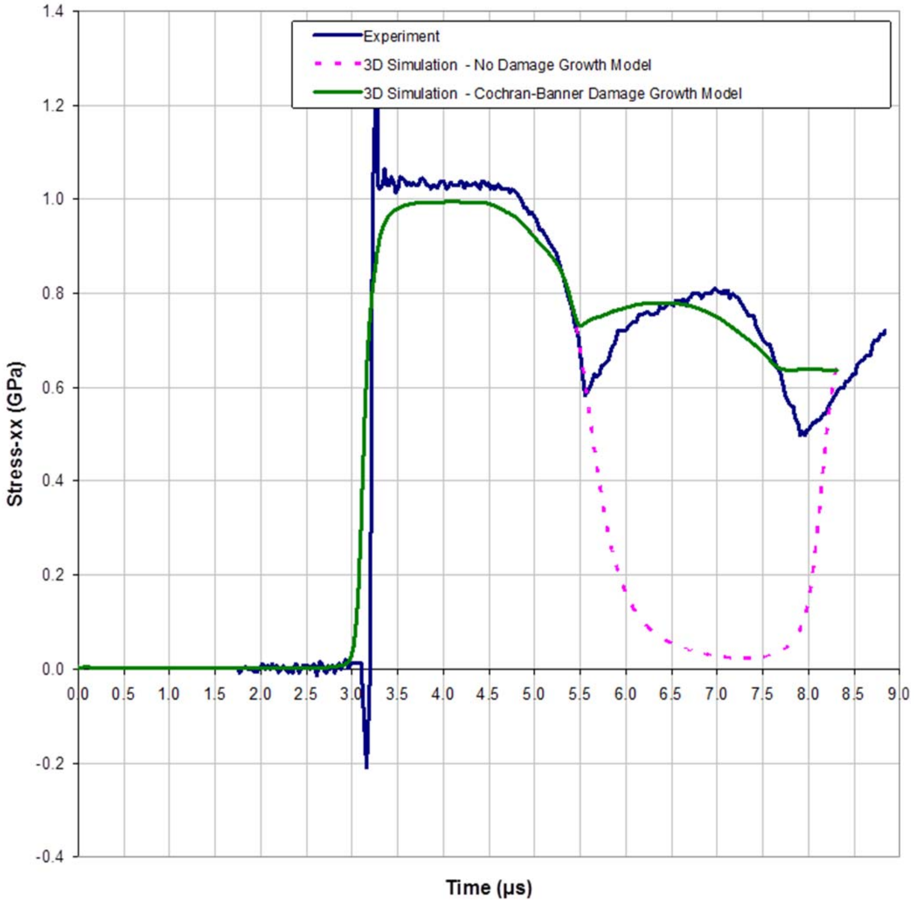
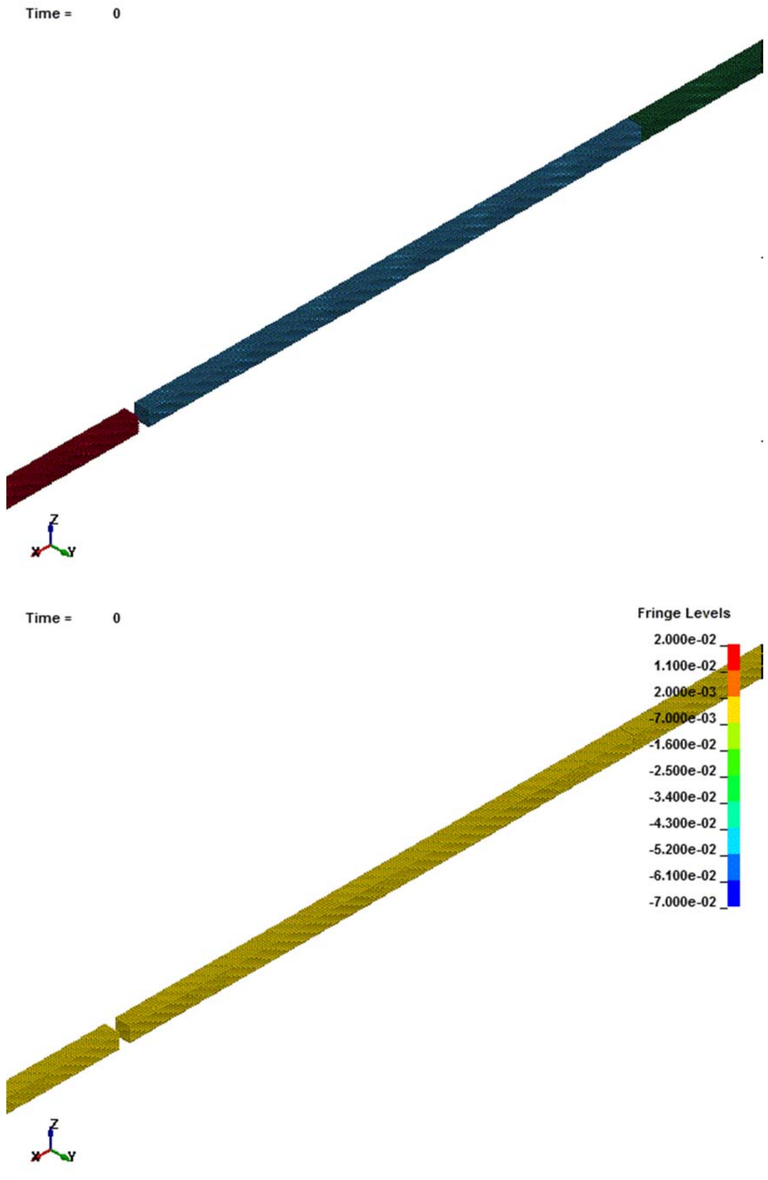


Plate impact model – 3D solver

- 3D solver, symmetry boundary conditions to enforce 1D strain state





Summary

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- Meshless methods offer several approaches to treatment of material damage and failure within the numerical model
- We are investigating the concept of treating damage at the particle-particle interaction level
- Concept of interaction area offers natural approach to treatment of damage based on continuum damage mechanics framework
- Initial tests in 1D and 3D demonstrate feasibility of approach.
- Further work is required to understand influence of damage in neighbour particle-particle bonds in 2D and 3D and investigate ability to represent more geometrically complex failure .