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# Distributed Fault Estimation with Randomly Occurring Uncertainties over Sensor Networks

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This paper is concerned with the distributed fault estimation problem for a class of uncertain stochastic systems over sensor networks. The norm-bounded uncertainty enters into the system in a random way governed by a set of Bernoulli distributed white sequence. The purpose of the addressed problem is to design distributed fault estimators, via available output measurements from not only the individual sensor but also its neighboring sensors, such that the fault estimation error converges to zero exponentially in the mean square while the disturbance rejection attenuation is constrained to a give level by means of the $\mathcal{H}_\infty$ performance index. Intensive stochastic analysis is carried out to obtain sufficient conditions for ensuring the exponential stability as well as prescribed $\mathcal{H}_\infty$ performance for the overall estimation error dynamics. Simulation results are provided to demonstrate the effectiveness of the proposed fault estimation technique in this paper.

**Keywords:** distributed fault estimation; randomly occurring uncertainties; sensor networks

## 1. Introduction

Over the past decades, fault diagnosis and isolation (FDI) and fault-tolerant control (FTC) problems have attracted much attention owe to the increasing security and reliability demand of modern control systems. However, in practice, it is generally difficult to obtain the accurate information of the size and shape of the fault from an FDI strategy only. It is fortunate that fault estimation technique is capable of providing the exact information of the size of the fault, thereby helping reconstruct the fault signals. As such, fault estimation is further needed for the purpose of active fault tolerant control. So far, considerable research attention has been devoted to the theoretical research on the fault estimation problem, and a variety of fault estimation approaches have been developed in existing literature, see e.g., (Shen et al., 2013; Ding, 2008; Ertiame et al., 2015; Fekih and Seelem, 2015; Wei et al., 2013; Bouibed et al., 2014; Jiang et al., 2006; Yan and Edwards, 2007) and the references therein. For example, in (Jiang et al., 2006; Xu et al., 2012; Zhang et al., 2008), the problems of fault estimation have been investigated by applying adaptive fault diagnosis observers that can improve the rapidity of fault estimation. In (Yan and Edwards, 2007), the sliding mode observer-based fault estimation method has been presented

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to realize fault reconstruction. In (Shen et al., 2013; Dong et al., 2014), the finite-horizon fault estimators have been designed for time-varying systems. It is worth mentioning that, in the existing literature concerning system fault estimation problems, nearly all fault estimators have been designed in a centralized way, which may cause lower reliability than distributed framework.

Recently, because of the rapid development of wireless sensor networks, the distributed filtering or estimation problem for sensor networks has received considerable research interests from system analysts, computer scientists and communication engineers, and a great number of estimator design algorithms have been proposed in the literature. Recent advancement on the distributed filtering in sensor networks is referred to (Ding et al., 2012, 2014; Dong et al., 2013; Shen et al., 2011). Different from the centralized filtering framework, distributed estimation schemes over sensor networks have more redundancies and higher reliability since, whenever some sensors occur non-recoverable fault, other sensors can still provide the estimation signal. For distributed filtering/estimation problems, the information available on an individual node of the sensor network is not only from its own measurement but also from its neighboring sensors measurements according to the given topology. As such, the key issues in designing distributed filters depend upon how to cope with the complicated coupling issues between one sensor and its neighboring sensors and how to reflect such couplings in the filter structure specification. It should be pointed out that, compared to the fruitful results on the distributed state estimation problems over sensor networks, the corresponding results on distributed fault estimation problems are still a challenging work due probably to the mathematical/computational complexities.

On the other hand, with rapid development of network technologies, the randomly occurring phenomena induced by networks have been thoroughly investigated for filtering, control and fault detection problems of networked systems. However, in comparison with those frequently investigated network-induced phenomena including packet dropouts (Hu et. al., 2013; Wang et al., 2012), communication delays (Shen et al., 2013; Luo et al., 2015; Hu et. al., 2013; Wei et al., 2014), signal quantization (Wang et al., 2013), randomly occurring sensor saturations (Wang et al., 2012) and randomly occurring nonlinearities (Ding et al., 2012, 2015; Dong et al., 2015; Hu et. al., 2013), the randomly occurring uncertainties in the control/estimation communities have not yet received much research attention despite its practical significance in wireless mobile communications. In reality, the parameter uncertainties may occur in a probabilistic way and are randomly changeable in terms of their types and/or levels due to the random occurrence of networked-induced phenomena such as random network-induced structural changes, repairs of components, changing subsystem interconnections or sudden environment changes. Note that in (Dong et al., 2015; Hu et. al., 2014), some initial research results have been presented to study the reliable control and state estimation for systems with randomly occurring uncertainties. Unfortunately, the distributed fault estimation problem has not been properly investigated so far for the sensor networks, not to mention the case where randomly occurring uncertainties are also involved. Thus, the main purpose of this paper is to fill in this gap.

To summarize, in this paper, we focus on investigating the distributed fault estimation problem with randomly occurring uncertainties over sensor networks. A set of distributed fault estimators are designed such that the estimation error dynamic is exponentially mean-square stable and achieves a prescribed $\mathcal{H}_\infty$ performance. The rest of this paper is arranged as follows. In Section 2, a class of discrete-time dynamic plant with a network of $n$ sensors is introduced and the problem under consideration is formulated. In Section 3, based on the semi-definite programme method and the Lyapunov stability theory, a new sufficient condition is obtained for the solvability of the considered distributed fault estimation problem. In Section 4, a simulation example is presented to illustrate the main results acquired. Finally, conclusions are drawn in Section 5.

**Notation**. The notation used in this paper is fairly standard except where otherwise stated. $\mathbb{R}^n$ denotes the $n$ dimensional Euclidean space and $\mathbb{R}^{n\times m}$ represents the set of all $n \times m$ real matrices. $l_2[0,\infty)$ is the space of square summable sequences. The notation $X \geq Y$ (respectively, $X > Y$) where $X$ and $Y$ are real symmetric matrices, denotes that $X-Y$ is positive semi-definite (respectively, positive definite). $M^T$ represents the transpose of $M$. $I$ and 0 denote the identity matrix and zero matrix, respectively with compatible dimension. $\|x\|$ describes the Euclidean norm of a vector $x$. In symmetric block matrices, "$*$" is used as an ellipsis for terms induced by symmetry. $\mathbf{1}_n := [1, 1, \ldots, 1]^T \in \mathbb{R}^n$. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

## 2. Problem Formulation

In this paper, we suppose that the $n$ sensor nodes are distributed in space on the basis of a fixed network topology represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ of order $n$ with the set of nodes $\mathcal{V} = 1, 2, \ldots, n$, the set of edges $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$, and the weighted adjacency matrix $\mathcal{A} = [a_{ij}]$ with nonnegative adjacency element $a_{ij}$. An edge of $\mathcal{G}$ is denoted by ordered pair $(i, j)$. The adjacency elements associated with the edges of the graph are positive, i.e., $a_{ij} > 0 \Longleftrightarrow (i, j) \in \mathcal{E}$ which means that sensor $i$ can obtain information from sensor $j$. Also, we assume that $a_{ii} = 1$ for all $i \in \mathcal{V}$, and therefore $(i, i)$ can be regarded as an additional edge. The set of neighbors of node $i \in \mathcal{V}$ plus the node itself are denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$.

Consider the following class of discrete-time stochastic uncertain systems:

$$x(k+1) = (A + \alpha(k)\Delta A)x(k) + D\omega(k) + Gf(k), \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state vector which cannot be observed directly; $\omega(k) \in \mathbb{R}^{n_w}$ is the disturbance input belonging to $l_2[0,\infty)$; $f(k) \in \mathbb{R}^l$ is the fault to be detected.

The real-valued matrix $\Delta A$ represents the norm-bounded parameter uncertainty of the following structure

$$\Delta A = H_a F(k) N \tag{2}$$

where $H_a$ and $N$ are known real constant matrices and $F(k)$ is an unknown matrix function satisfying the following condition

$$F^T(k)F(k) \leq I \tag{3}$$

The stochastic variable $\alpha(k) \in \mathbb{R}$ in (1), which characterizes the phenomenon of randomly occurring uncertainties, is a Bernoulli distributed white sequence taking values on either 0 or 1 with

$$\text{Prob}\{\alpha(k) = 1\} = \bar{\alpha}, \qquad \text{Prob}\{\alpha(k) = 0\} = 1 - \bar{\alpha} \tag{4}$$

where $\bar{\alpha} \in [0, 1]$ is a known constant.

In this paper, the measurement outputs from the $i$th sensor are described by

$$y_i(k) = C_i x(k) + E_i v(k) + H_i f(k) \tag{5}$$

where $y_i(k) \in \mathbb{R}^{n_y}$ is the output measured by sensor $i$ from the plant, $v(k) \in \mathbb{R}^{n_v}$ is the measurement noise signal which is assumed to be arbitrary belonging to $l_2[0,\infty]$. Throughout the paper, we assume that all the matrices mentioned above, i.e., $A, D, G, C_i, E_i$ and $H_i$, are known matrices with appropriate dimensions.

In this paper, the following structure is adopted on sensor node $i$:

$$\begin{cases} \hat{x}_i(k+1) = A\hat{x}_i(k) + \sum_{j\in\mathcal{N}_i} a_{ij}K_{ij}(y_j(k) - C_j\hat{x}_j(k)) \\ r_i(k) = \sum_{j\in\mathcal{N}_i} a_{ij}H_{ij}(y_j(k) - C_j\hat{x}_j(k)) \end{cases} \tag{6}$$

where $\hat{x}_i(k) \in \mathbb{R}^{n_x}$ is the state estimate on sensor node $i$ and $r_i(k) \in \mathbb{R}^l$ is the so-called residual that is compatible with the fault vector $f(k)$. Here, matrices $K_{ij}$ and $H_{ij}$ are the fault estimator parameters on node $i$ to be determined. The initial values of estimator $\hat{x}_i(0)$ $(i = 1, 2, \cdots, n)$ are assumed to be known vectors.

**Remark 1.** *The fault estimator structure in (6) establishes the communications between sensor node i and its neighboring nodes, in which the sensor nodes are distributed over a spatial region. It is worth mentioning that (6) represents a quite general estimator model structure. To see this, we assume $n = 1$, the fault estimator (6) can be reduced to*

$$\begin{aligned} \hat{x}_1(k+1) =& A\hat{x}_1(k) + K_{11}(y_1(k) - C_1\hat{x}_1(k)), \\ r_1(k) =& H_{11}(y_1(k) - C_1\hat{x}_1(k)) \end{aligned} \tag{7}$$

*which has been widely adopted for fault estimator design in the literature.*

For convenience of later analysis, we denote

$$\begin{aligned} &\bar{A} = I_n \otimes A, \quad \Delta\bar{A} = \mathbf{1}_n \otimes \Delta A, \quad \bar{C} = \mathrm{diag}\{C_1, C_2, \ldots, C_n\}, \\ &\bar{D} = \mathbf{1}_n \otimes D, \quad \bar{E} = \begin{bmatrix} E_1^T \ E_2^T \ \cdots \ E_n^T \end{bmatrix}^T, \quad \bar{G} = \mathbf{1}_n \otimes G, \\ &\tilde{H} = \begin{bmatrix} H_1^T \ H_2^T \ \cdots \ H_n^T \end{bmatrix}^T, \quad \tilde{\alpha}(k) = \alpha(k) - \bar{\alpha}, \quad g = \bar{\alpha}(1-\bar{\alpha}) \end{aligned} \tag{8}$$

and

$$\begin{aligned} &\bar{K} = [\bar{K}_{ij}]_{n\times n} \quad \text{with} \quad \bar{K}_{ij} = \begin{cases} a_{ij}K_{ij}, & i = 1, 2, \ldots, n; \quad j \in \mathcal{N}_i \\ 0, & i = 1, 2, \ldots, n; \quad j \notin \mathcal{N}_i \end{cases} \\ &\bar{H} = [\bar{H}_{ij}]_{n\times n} \quad \text{with} \quad \bar{H}_{ij} = \begin{cases} a_{ij}H_{ij}, & i = 1, 2, \ldots, n; \quad j \in \mathcal{N}_i \\ 0, & i = 1, 2, \ldots, n; \quad j \notin \mathcal{N}_i \end{cases} \end{aligned} \tag{9}$$

Obviously, since $a_{ij} = 0$ when $j \notin \mathcal{N}_i$, $\bar{K}$ and $\bar{H}$ are two matrices that can be expressed as

$$\bar{K} \in \mathscr{T}_{n_x\times n_y}, \quad \bar{H} \in \mathscr{T}_{l\times n_y} \tag{10}$$

where $\mathscr{T}_{p\times q} = \{\bar{U} = [U_{ij}] \in \mathbb{R}^{np\times nq} \mid U_{ij} \in \mathbb{R}^{p\times q},\ U_{ij} = 0 \ \text{if} \ j \notin \mathcal{N}_i\}$.

Letting $\eta(k) = \begin{bmatrix} x^T(k) \ e^T(k) \end{bmatrix}^T$, $\xi(k) = \begin{bmatrix} \omega^T(k) \ v^T(k) \ f^T(k) \end{bmatrix}^T$, $\tilde{r}_i(k) = r_i(k) - f(k)$, $\tilde{r}(k) = \begin{bmatrix} \tilde{r}_1^T(k) \ \tilde{r}_2^T(k) \ \cdots \ \tilde{r}_n^T(k) \end{bmatrix}^T$, $e(k) = \begin{bmatrix} e_1^T(k) \ e_2^T(k) \ \cdots \ e_n^T(k) \end{bmatrix}^T$ and $e_i(k) = x(k) - \hat{x}_i(k)$, we have the following augmented system to be investigated:

$$\begin{cases} \eta(k+1) = \tilde{A}\eta(k) + \tilde{\alpha}(k)\Delta\mathcal{A}\eta(k) + \mathcal{D}\xi(k) \\ \tilde{r}(k) = \mathcal{M}\eta(k) + \mathcal{N}\xi(k) \end{cases} \tag{11}$$

where

$$\begin{aligned}
&\tilde{A} = \mathcal{A} + \bar{\alpha}\Delta\mathcal{A} - C, \quad \mathcal{A} = \begin{bmatrix} A & 0 \\ 0 & \bar{A} \end{bmatrix}, \quad \Delta\mathcal{A} = \begin{bmatrix} \Delta A & 0 \\ \Delta\bar{A} & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 \\ 0 & \bar{K}\bar{C} \end{bmatrix}, \\
&\mathcal{D} = \begin{bmatrix} D & 0 & G \\ \bar{D} & -\bar{K}\bar{E} & \bar{G} - \bar{K}\tilde{H} \end{bmatrix}, \quad \mathcal{M} = \begin{bmatrix} 0 & \bar{H}\bar{C} \end{bmatrix}, \quad \mathcal{N} = \begin{bmatrix} 0 & \bar{H}\bar{E} & Q \end{bmatrix}, \quad Q = \bar{H}\tilde{H} - I.
\end{aligned} \tag{12}$$

Our aim in this paper is to design a set of fault estimator of the form in (6) on each node $i$ of the sensor network for system (1). In other words, we are going to find the parameters $K_{ij}$ and $H_{ij}$ such that the following two requirements are satisfied simultaneously:

a) (exponentially mean-square stability) The zero-solution of the augmented system (11) is exponentially mean-square stable;

b) ($\mathcal{H}_\infty$ performance) Under zero initial conditions, for a given disturbance attenuation level $\gamma > 0$ and all nonzero $\xi(k)$, the fault estimation error $\tilde{r}(k)$ from (11) satisfies the following condition:

$$\frac{1}{n}\sum_{k=0}^{\infty} E\{\|\tilde{r}(k)\|^2\} < \gamma^2 \sum_{k=0}^{\infty} \|\xi(k)\|^2 \tag{13}$$

**Remark 2.** *The average $\mathcal{H}_\infty$ performance (13) over the n estimators for n sensors is a constraint adopted from the classical $\mathcal{H}_\infty$ control theory. It means that the average energy gains from the average energy of all disturbances on the target plant and sensor network to the average energy of all fault estimation errors should be less than a given disturbance attenuation level $\gamma$. Such an average $\mathcal{H}_\infty$ performance index is more appropriate to quantify the overall performance of the distributed estimators than the conventional central $\mathcal{H}_\infty$ performance constraint.*

## 3. Main Results

In this section, let us investigate the distributed fault estimation for system (1) with $n$ sensors whose topology is determined by the given graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. The following lemmas will be needed in establishing our main results.

**Lemma 1.** *(Schur Complement) Given constant matrices $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$ where $\mathcal{S}_1 = \mathcal{S}_1^{\mathrm{T}}$ and $0 < \mathcal{S}_2 = \mathcal{S}_2^{\mathrm{T}}$, then $\mathcal{S}_1 + \mathcal{S}_3^{\mathrm{T}}\mathcal{S}_2^{-1}\mathcal{S}_3 < 0$ if and only if*

$$\begin{bmatrix} \mathcal{S}_1 & \mathcal{S}_3^{\mathrm{T}} \\ \mathcal{S}_3 & -\mathcal{S}_2 \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -\mathcal{S}_2 & \mathcal{S}_3 \\ \mathcal{S}_3^{\mathrm{T}} & \mathcal{S}_1 \end{bmatrix} < 0. \tag{14}$$

**Lemma 2.** *(S-procedure) Let $S = S^{\mathrm{T}}$ and $H$ and $E$ be real matrices of appropriate dimensions with $F$ satisfying $FF^T \le I$, then $S + HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} < 0$, if and only if there exists a positive scalar $\varepsilon > 0$ such that $S + \varepsilon^{-1}HH^{\mathrm{T}} + \varepsilon E^{\mathrm{T}}E < 0$ or equivalently,*

$$\begin{bmatrix} S & H & \varepsilon E^{\mathrm{T}} \\ H^{\mathrm{T}} & -\varepsilon I & 0 \\ \varepsilon E & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{15}$$

**Lemma 3.** *Let $P = diag\{P_1, P_2, \ldots, P_n\}$ with $P_i \in \mathbb{R}^{p\times p}$ $(1 \le i \le n)$ being invertible matrices. If $X = PW$ for $W \in \mathbb{R}^{np\times nq}$, then we have $W \in \mathscr{T}_{p\times q} \Longleftrightarrow X \in \mathscr{T}_{p\times q}$.*

The following theorem guarantees that the estimation error system (11) is exponentially mean-square stability with an $\mathcal{H}_\infty$ performance $\gamma$.

**Theorem 1.** *For given fault estimator parameters $K_{ij}$, $H_{ij}$ and a prescribed $\mathcal{H}_\infty$ index $\gamma > 0$, the estimating dynamics in (11) is exponentially mean-square stable and also satisfies the $\mathcal{H}_\infty$ performance constraint (13) if there exists a positive definite matrix $P > 0$ satisfying*

$$\Pi = \begin{bmatrix} \tilde{A}^T P\tilde{A} + g\Delta\mathcal{A}^T P\Delta\mathcal{A} + \frac{\mathcal{M}^T\mathcal{M}}{n} - P & * \\ \mathcal{D}^T P\tilde{A} + \frac{\mathcal{N}^T\mathcal{M}}{n} & \mathcal{D}^T P\mathcal{D} + \frac{\mathcal{N}^T\mathcal{N}}{n} - \gamma^2 I \end{bmatrix} < 0. \tag{16}$$

*Proof.* Choose the following Lyapunov function for system (11):

$$V(\eta(k)) = \eta^T(k)P\eta(k) \tag{17}$$

The difference of the Lyapunov function is given as follows:

$$\Delta V(\eta(k)) = \mathbb{E}\{V(\eta(k+1))|\eta(k)\} - V(\eta(k)) \tag{18}$$

Calculating the difference of $V(\eta(k))$ along the trajectory of system (11) and taking the mathematical expectation with $\xi(k) = 0$, we have

$$\begin{aligned} \mathbb{E}\{\Delta V(\eta(k))\} &:= \mathbb{E}\Big\{\eta^T(k+1)P\eta(k+1) - \eta^T(k)P\eta(k)\Big\} \\ &= \mathbb{E}\Bigg\{\eta^T(k)(\tilde{A}^T P\tilde{A} + g\Delta\mathcal{A}^T P\Delta\mathcal{A} - P)\eta(k)\Bigg\} \end{aligned} \tag{19}$$

It follows from (16) that $\tilde{A}^T P\tilde{A} + g\Delta\mathcal{A}^T P\Delta\mathcal{A} - P < 0$ and, subsequently,

$$\mathbb{E}\{\Delta V(\eta(k))\} \leq -\lambda_{\min}(-(\tilde{A}^T P\tilde{A} + g\Delta\mathcal{A}^T P\Delta\mathcal{A} - P))\mathbb{E}\{\| \eta(k) \|^2\}. \tag{20}$$

Finally, we can confirm from Lemma 1 of (Wang et al., 2012) that the dynamic system (11) is exponentially mean-square stable.

To establish the $\mathcal{H}_\infty$ performance, we assume zero initial conditions and introduce the following index:

$$\begin{aligned} &\mathbb{E}\Big\{\Delta V(\eta(k))\Big\} + \frac{1}{n}\mathbb{E}\Big\{\|\tilde{r}(k)\|^2\Big\} - \gamma^2\|\xi(k)\|^2 \\ &= \mathbb{E}\Bigg\{\eta^T(k)(\tilde{A}^T P\tilde{A} + g\Delta\mathcal{A}^T P\Delta\mathcal{A} - P)\eta(k) + 2\eta^T(k)\tilde{A}^T P\mathcal{D}\xi(k) + \xi^T(k)\mathcal{D}^T P\mathcal{D}\xi(k) \\ &\quad + \frac{1}{n}(\eta^T(k)\mathcal{M}^T\mathcal{M}\eta(k) + 2\eta^T(k)\mathcal{M}^T\mathcal{N}\xi(k) + \xi^T(k)\mathcal{N}^T\mathcal{N}\xi(k)) - \gamma^2\xi^T(k)\xi(k)\Bigg\} \\ &= \eta^{*T}(k)\Pi\eta^*(k) \end{aligned} \tag{21}$$

where

$$\eta^*(k) = \Big[\eta^T(k)\ \xi^T(k)\Big]^T \tag{22}$$

Furthermore, it follows from (16) that

$$\mathbb{E}\Big\{\Delta V(\eta(k))\Big\} + \frac{1}{n}\mathbb{E}\Big\{\|\tilde{r}(k)\|^2\Big\} - \gamma^2\|\xi(k)\|^2 < 0 \tag{23}$$

for all nonzero $\xi(k)$ .

By considering zero initial conditions, the above inequality implies that

$$\frac{1}{n}\sum_{k=0}^{\infty} E\{\|\tilde{r}(k)^2\|\} < \gamma^2 \sum_{k=0}^{\infty} \|\xi(k)\|^2 \tag{24}$$

which is equivalent to (13), and the proof is now complete. □

We are now in a position to provide the specific design method for distributed fault estimator in the following theorem.

**Theorem 2.** *Let a positive scalar $\gamma > 0$ be given. For the discrete-time stochastic uncertain system (1) and sensors (5), the fault estimator dynamics in (11) is exponentially mean-square stable and satisfies the $\mathcal{H}_\infty$ performance constraint (13) if there exist a positive definite matrix $P > 0$, the matrix $\bar{X} \in \mathbb{R}^{(n+1)n_x \times nn_y}$, $\bar{H} \in \mathbb{R}^{nl \times nn_y}$ and a positive constant scalar $\epsilon$ satisfying*

$$\begin{bmatrix} -P & * & * & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * & * & * \\ P\mathcal{A} - \bar{X}\hat{R}_0 & P\hat{D}_0 + \bar{X}\hat{R}_1 & -P & * & * & * & * \\ \sqrt{\frac{1}{n}}\bar{H}\hat{R}_0 & -\sqrt{\frac{1}{n}}(\bar{H}\hat{R}_1 + \hat{N}_0) & 0 & -I & * & * & * \\ 0 & 0 & 0 & 0 & -P & * & * \\ 0 & 0 & \bar{\alpha}\bar{\mathcal{H}}_a^T P^T & 0 & \sqrt{g}\bar{\mathcal{H}}_a^T P^T & -\epsilon I & * \\ \epsilon N_a & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix} < 0 \tag{25}$$

*where*

$$\begin{aligned} &\hat{R}_0 = \begin{bmatrix} 0 & \bar{C} \end{bmatrix},\ \hat{\varepsilon}_0 = \begin{bmatrix} 0 & I \end{bmatrix}^T,\ \hat{R}_1 = \begin{bmatrix} 0 & -\bar{E} & -\tilde{H} \end{bmatrix},\ \bar{\mathcal{H}}_a = \mathbf{1}_{n+1} \otimes H_a, \\ &\hat{D}_0 = \begin{bmatrix} D & 0 & G \\ \bar{D} & 0 & \bar{G} \end{bmatrix},\ \hat{N}_0 = \begin{bmatrix} 0 & 0 & I \end{bmatrix},\ N_a = \begin{bmatrix} N & 0 \end{bmatrix} \end{aligned} \tag{26}$$

*and the other parameters are defined in (8) and (12). Moreover, if the above inequality is feasible, the matrix $\bar{K}$ is given as follows:*

$$\bar{K} = (\hat{\mathcal{E}}_0^T P \hat{\mathcal{E}}_0)^{-1}\hat{\mathcal{E}}_0^T \bar{X} \tag{27}$$

*Accordingly, the desired filter parameters $K_{ij}$ and $H_{ij}$ ($i = 1, 2, \ldots, n, j \in \mathcal{N}_i$) can be obtained from (9).*

*Proof.* In order to reduce unnecessary conservatism, we rewrite the parameters in Theorem 1 in the following form

$$C = \hat{\varepsilon}_0\bar{K}\hat{R}_0,\ \mathcal{D} = \hat{D}_0 + \hat{\varepsilon}_0\bar{K}\hat{R}_1,\ \mathcal{M} = \bar{H}\hat{R}_0,\ \mathcal{N} = -\bar{H}\hat{R}_1 - \hat{N}_0. \tag{28}$$

Applying the Schur Complement Lemma, the inequality (16) can be expressed as

$$\Gamma_1 = \begin{bmatrix} -P & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * \\ P\mathcal{A} + \bar{\alpha} P \Delta\mathcal{A} - P\hat{\varepsilon}_0 \bar{K}\hat{R}_0 & P\hat{D}_0 + P\hat{\varepsilon}_0 \bar{K}\hat{R}_1 & -P & * & * \\ \sqrt{\frac{1}{n}}\bar{H}\hat{R}_0 & \sqrt{\frac{1}{n}}(-\bar{H}\hat{R}_1 - \hat{N}_0) & 0 & -I & * \\ \sqrt{g} P\Delta\mathcal{A} & 0 & 0 & 0 & -P \end{bmatrix} < 0 \tag{29}$$

Noticing $\Delta\mathcal{A} = \begin{bmatrix} \Delta A & 0 \\ \Delta\bar{A} & 0 \end{bmatrix} = \mathbf{1}_{n+1} \otimes (H_a F(k) N_a)$, we rewrite (29) in terms of Lemma 2 as follows:

$$\Gamma_1 = \Gamma + H_b F(k) N_b + N_b^T F^T(k) H_b^T < 0 \tag{30}$$

where

$$\Gamma = \begin{bmatrix} -P & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * \\ P\mathcal{A} - P\hat{\varepsilon}_0 \bar{K}\hat{R}_0 & P\hat{D}_0 + P\hat{\varepsilon}_0 \bar{K}\hat{R}_1 & -P & * & * \\ \sqrt{\frac{1}{n}}\bar{H}\hat{R}_0 & \sqrt{\frac{1}{n}}(-\bar{H}\hat{R}_1 - \hat{N}_0) & 0 & -I & * \\ 0 & 0 & 0 & 0 & -P \end{bmatrix},$$

$$H_b = \begin{bmatrix} 0 \; 0 \; \bar{\alpha}(\mathbf{1}_{n+1} \otimes H_a)^T P^T \; 0 \; \sqrt{g}(\mathbf{1}_{n+1} \otimes H_a)^T P^T \end{bmatrix}^T,$$

$$N_b = \begin{bmatrix} N_a \; 0 \; 0 \; 0 \; 0 \end{bmatrix}, \tag{31}$$

Letting $P = \mathrm{diag}\{P_1, P_2, \ldots, P_n\}$ and noting $P\hat{\mathcal{E}}_0 \bar{K} = \bar{X}$, from Lemma 3, it is easy to verify that the conditions $\bar{K} \in \mathscr{T}_{n_x \times n_y}$ is satisfied. Applying S-procedure, (25) can be obtained by (30) after some straightforward algebraic manipulations. The proof of this theorem is now complete. □

## 4. Numerical Example

In this section, a numerical example is given to verify the effectiveness of the proposed distributed fault estimation with randomly occurring uncertainties over sensor networks.

The sensor network is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, with the set of nodes $\mathcal{V}$=$\{1, 2, 3, 4\}$, set of edges $\mathcal{E}$=$\{(1, 1), (2, 1), (2, 2), (3, 1), (3, 3), (4, 2), (4, 3), (4, 4)\}$, and the following adjacency matrix

$$\mathcal{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

The system data are given as follows:

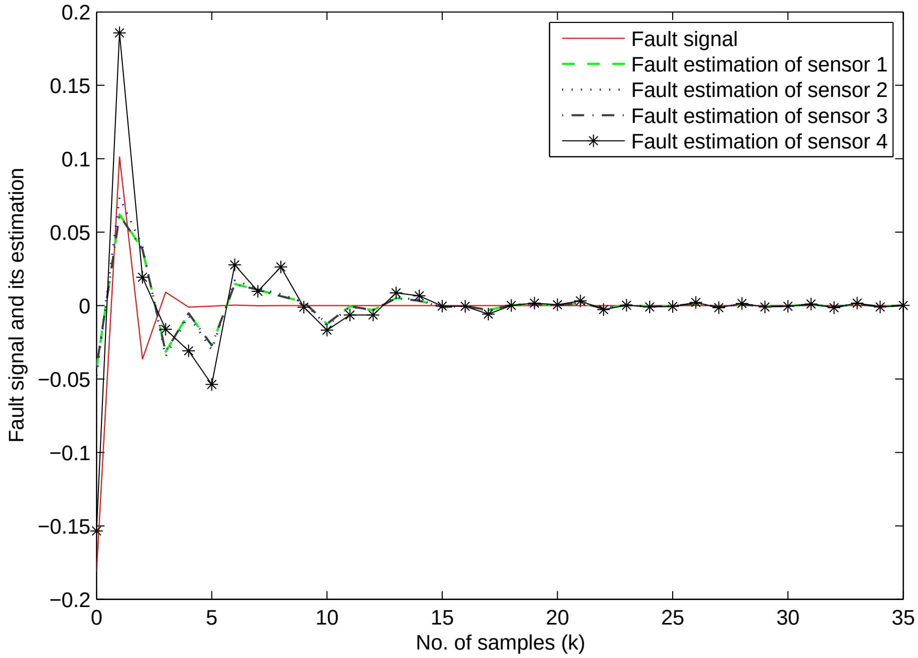


Figure 1. Fault signal and its estimate.

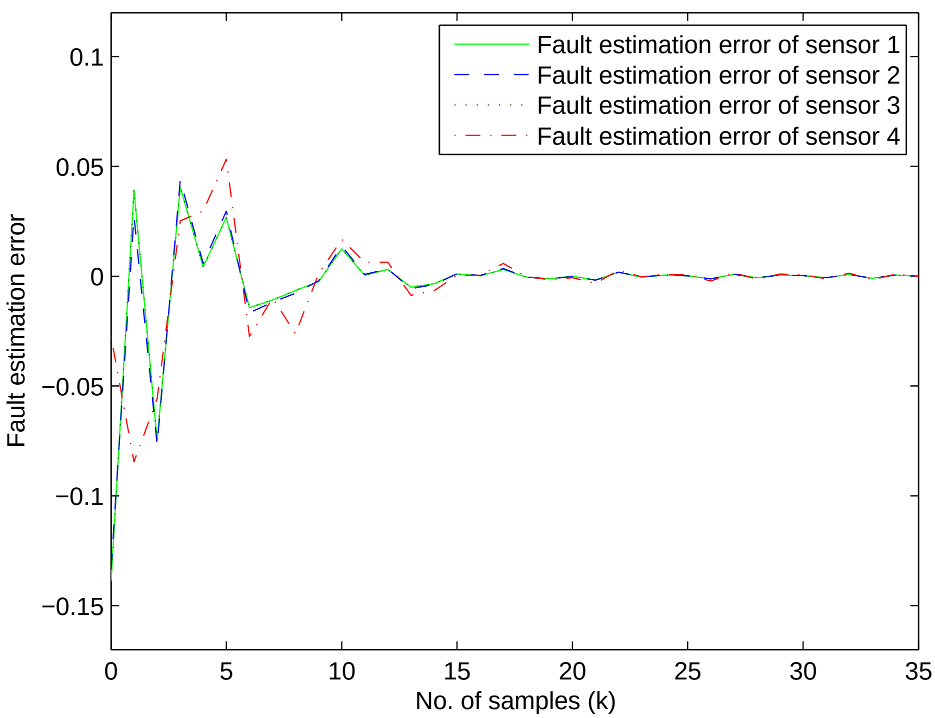


Figure 2. Fault estimation error.

$$A = \begin{bmatrix} 0.6 & 0.2 & 0 \\ 0 & -0.7 & -0.4 \\ 0.2 & 0.4 & -0.6 \end{bmatrix},\ D = \begin{bmatrix} 0.6\ 0.8\ 0.1 \end{bmatrix}^T,\ G = \begin{bmatrix} 0.2\ 0.2\ 0.3 \end{bmatrix}^T,$$

$$F(k) = \sin(0.6k),\ H_a = \begin{bmatrix} 0.2\ 0.2\ 0.1 \end{bmatrix}^T,\ N = \begin{bmatrix} 0.1\ 0.2\ 0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.2\ 0.1\ 0.1 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.2\ 0\ 0.2 \end{bmatrix},\ C_3 = \begin{bmatrix} 0.5\ 0.5\ 0.2 \end{bmatrix},$$

$$C_4 = \begin{bmatrix} 0.6\ 0.2\ 0.1 \end{bmatrix},\ E_1 = 0.2,\ E_2 = 0.2,\ E_3 = 0.1,\ E_4 = 0.3,$$

$$H_1 = 0.1,\ H_2 = 0.2,\ H_3 = 0.4,\ H_4 = 0.1.$$

In this example, the probability of the randomly occurring uncertainty is taken as $\bar{\alpha} = 0.8$, the following parameters of the desired distributed fault estimator can be obtained by solving (25) in Theorem 2:

$$K_{11} = \begin{bmatrix} 1.1735 \\ -6.8438 \\ -6.2781 \end{bmatrix},\ K_{21} = \begin{bmatrix} -0.2993 \\ -5.3521 \\ -2.0396 \end{bmatrix},\ K_{22} = \begin{bmatrix} 0.7697 \\ -4.8283 \\ -0.6783 \end{bmatrix},\ K_{31} = \begin{bmatrix} -1.2197 \\ -1.9445 \\ -1.8405 \end{bmatrix},$$

$$K_{33} = \begin{bmatrix} 0.1427 \\ -1.2091 \\ 0.1277 \end{bmatrix},\ K_{42} = \begin{bmatrix} -0.4696 \\ -1.6319 \\ -0.7980 \end{bmatrix},\ K_{43} = \begin{bmatrix} -0.0289 \\ -0.0998 \\ -0.0499 \end{bmatrix},\ K_{44} = \begin{bmatrix} 0.0073 \\ -5.9634 \\ -0.3638 \end{bmatrix},$$

$$H_{11} = 0.5802,\ H_{21} = 0.5398,\ H_{22} = 0.0405,\ H_{31} = 0.5806,$$

$$H_{33} = 0.0408,\ H_{42} = 0.3692,\ H_{43} = 0.0167,\ H_{44} = 0.1907.$$

The optimal performance index is $\gamma^* = 1.021$. In the numerical example, the exogenous disturbance inputs $\omega(k) = \exp(-0.2k)\sin(k)$, $v(k) = \frac{\sin(10k+1)}{3k+1}$, the initial value of the state $x(0)$ is selected as $\begin{bmatrix} 0.6\ 0.3\ 0.6 \end{bmatrix}^T$. The fault to be estimated is $f(k) = \exp(-k)\sin(10k)$. Fig. 1 plots the simulation result on the fault signal and its estimate. Fig. 2 shows the fault estimation error. The simulation result has confirmed the effectiveness of the distributed fault estimation technique presented in this paper.

## 5. Conclusions

In this paper, we have dealt with the distributed fault estimation problem for a class of uncertain stochastic systems over sensor networks. The randomly occurring uncertainties are modeled by the Bernoulli distributed white sequences with known conditional probabilities. The distributed fault estimators have been designed for the fault dynamics to be exponentially mean-square stable and the fault estimation errors to satisfy the $\mathcal{H}_\infty$ performance constraint. Finally, a simulation example has been presented to illustrate the main results obtained.

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## References


Shen, B., S. X. Ding, and Z. Wang. 2013. "Finite-horizon fault estimation for linear discrete time-varying systems with delayed measurements." *Automatica* 49(1): 293–296.

Ding, S. X. 2008. *Model-based Fault Diagnosis Techniques: Design Schemes, Algorithms, and Tools*. Berlin: Springer.

Ertiame, A. M., D. Yu, F. Yu, and J. B. Gomm. 2015. "Robust fault diagnosis for an exothermic semi-batch polymerization reactor under open-loop." *Systems Science and Control Engineering: An Open Access Journal* 3(1): 14–23.

Fekih, A., and S. Seelem. 2015. "Effective fault-tolerant control paradigm for path tracking in autonomous vehicles." *Systems Science and Control Engineering: An Open Access Journal* 3(1): 177–188.

Wei, G., C. Wang, and F. Han. 2013. "A gain-scheduled approach to fault-tolerant control for discrete-time stochastic delayed systems with randomly occurring actuator faults." *Systems Science and Control Engineering: An Open Access Journal* 1(1): 82–90.

Bouibed, K., L. Seddiki, K. Guelton, and H. Akdag. 2014. "Actuator and sensor fault detection and isolation of an actuated seat via nonlinear multi-observers." *Systems Science and Control Engineering: An Open Access Journal* 2(1): 150–160.

Jiang, B., M. Staroswiecki, and V. Cocquempot. 2006. "Fault accommodation for nonlinear dynamic systems." *IEEE Transactions on Automatic Control* 51(9): 1578–1583.

Yan, X., and C. Edwards. 2007. "Nonlinear robust fault reconstruction and estimation using a sliding mode observer." *Automatica* 43(9): 1605–1614.

Xu, D., B. Jiang, and P. Shi. 2012. "Nonlinear actuator fault estimation observer: An inverse system approach via a T-S fuzzy model." *Applied Mathematics and Computer Science* 22(1): 183–196.

Dong, H., Z. Wang, S. X. Ding, and H. Gao. 2014. "Finite-horizon estimation of randomly occurring faults for a class of nonlinear time-varying systems." *Automatica* 50(12): 3182–3189.

Zhang, K., B. Jiang, and V. Cocquempot. 2008. "Adaptive observer-based fast fault estimation." *Int. J. Control, Automation, and Systems* 6(3): 320–326.

Ding, D., Z. Wang, H. Dong, and H. Shu. 2012. "Distributed $\mathcal{H}_\infty$ state estimation with stochastic parameters and nonlinearities through sensor networks: the finite-horizon case." *Automatica* 48(8): 1575–1585.

Ding, D., Z. Wang, and B. Shen. 2014. "Recent advances on distributed filtering for stochastic systems over sensor networks." *International Journal of General Systems* 43(3-4): 372–386.

Ding, D., Z. Wang, B. Shen, and H. Dong. 2015. "Envelope-constrained $\mathcal{H}_\infty$ filtering with fading measurements and randomly occurring nonlinearities: the finite horizon case." *Automatica* 55: 37–45.

Luo, Y., G. Wei, Y. Liu, and X. Ding. 2015. "Reliable $\mathcal{H}_\infty$ state estimation for 2-D discrete systems with infinite distributed delays and incomplete observations." *International Journal of General Systems* 44(2): 155–168.

Dong, H., Z. Wang, S. X. Ding, and H. Gao. 2015. "Fnite-horizon reliable control with randomly occurring uncertainties and nonlinearities subject to output quantization." *Automatica* 52: 355–362.

Hu, J., Z. Wang, B. Shen, and H. Gao. 2013. "Quantized recursive filtering for a class of nonlinear systems with multiplicative noises and missing measurements." *International Journal of Control* 86(4): 650–663.

Hu, J., Z. Wang, B. Shen, and H. Gao. 2013. "Gain-constrained recursive filtering with stochastic nonlinearities and probabilistic sensor delays." *IEEE Transactions on Signal Processing* 61(5): 1230–1238.

Hu, J., D. Chen, and J. Du. 2014. "State estimation for a class of discrete nonlinear systems with randomly occurring uncertainties and distributed sensor delays." *International Journal of General Systems* 43(3-4): 387–401.

Wei, G., F. Han, L. Wang, and Y. Song. 2014. "Reliable $\mathcal{H}_\infty$ filtering for discrete piecewise linear systems with infinite distributed delays." *International Journal of General Systems* 43(3-4): 346–358.

Dong, H., Z. Wang, and H. Gao. 2013. "Distributed $\mathcal{H}_\infty$ filtering for a class of Markovian jump nonlinear time-delay systems over lossy sensor networks." *IEEE Trans. Industrial Electronics* 60(10): 4665–4672.

Shen, B., Z. Wang, Y. S. Hung, and G. Chesi. 2011. "Distributed $\mathcal{H}_\infty$ filtering for polynomial nonlinear stochastic systems in sensor networks." *IEEE Trans. Industrial Electronics* 58(5): 1971–1979.

Wang, Z., H. Dong, B. Shen, and H. Gao. 2013. "Finite-horizon $\mathcal{H}_\infty$ filtering with missing measurements and quantization effects." *IEEE Transactions on Automatic Control* 58(7): 1707–1718.

Wang, Z., B. Shen, and X. Liu. 2012. "$\mathcal{H}_\infty$ filtering with randomly occurring sensor saturations and missing measurements." *Automatica* 48(3): 556–562.

Wang, Z., B. Shen, H. Shu, and G. Wei. 2012. "Quantized $\mathcal{H}_\infty$ control for nonlinear stochastic time-delay systems with missing measurements." *IEEE Transactions on Automatic Control* 57(6): 1431–1444.