Investigating Crack Initiation and Propagation of Concrete in Restrained Shrinkage Circular/Elliptical Ring Test

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Abstract

The restrained ring test, which is recommended by AASHTO and ASTM, has been used for assessing the potential of early-age cracking of concrete and other cement-based materials. Recently, a novel elliptical ring test method has been proposed to replace the circular ring test method for the purpose of shortening ring test duration and observing crack initiation and propagation more conveniently. In order to explore the mechanism of this novel test method, a numerical model is developed to analyze crack initiation and propagation process in restrained concrete rings, in which the effect of concrete shrinkage is simulated by a fictitious temperature drop applied on concrete causing the same strain as that induced by shrinkage. First, an elastic analysis is conducted to obtain the circumferential stress contour of a concrete ring subject to restrained shrinkage. Combined with the fictitious crack model, a fracture mechanics method is introduced to determine crack initiation and propagation, in which crack resistance caused by cohesive force acting on fracture process zone is considered. Finite element analysis is carried out to simulate the evolution of stress intensity factor (SIF) in restrained concrete rings subject to circumferential drying. Cracking age and position of a series of circular/elliptical concrete rings are obtained from numerical analyses which agree reasonably well with experimental results. It is found that the sudden drop of steel strain observed in the restrained ring test represents the onset of unstable crack propagation rather than crack initiation. The results given by the AASHTO/ASTM restrained ring test actually reflects the response of a concrete ring as a structure to external stimulation, in this case restrained concrete shrinkage.

Keywords *concrete; early-age cracking; restrained shrinkage; fracture mechanics; crack propagation process; elliptical ring test.*

Abbreviations

a : crack length

A : exposed surface area of a concrete element

d : steel ring thickness

E : elastic modulus of concrete

f_c : uniaxial compressive strength

f_t : splitting tensile strength of concrete

G_F : fracture energy

K_I^S : stress intensity factor caused by applied load

K_I^σ : stress intensity factor caused by cohesive force

R_0 : inner diameter of circular concrete ring

R_1 : major semi-axes of inner circumference of elliptical concrete ring

R_2 : minor semi-axes of inner circumference of elliptical concrete ring

t : age of concrete

V : volume of a concrete element

w : crack opening displacement

w_0 : stress-free crack opening displacement

w_s : displacement corresponding to the break point in bilinear σ-w relationship

Δa : crack growth length

ΔP : load incensement

σ : cohesive stress

σ_s : stress corresponding to the break point in bilinear σ-w relationship

Introduction

Autogenous, drying or thermal shrinkage often occur in concrete elements or structures, resulting in volume changes of concrete. If volume changes of concrete are restrained, tensile stress may be developed in concrete [1, 2]. Since concrete's low tensile strength provides little resistance to tensile stress, cracking may occur when the developed tensile stress in concrete excesses its tensile strength. These restrained shrinkage cracks are often observed in concrete elements and structures with high surface area-to-volume ratio such as industrial floors, concrete pavements and bridge decks [3]. Cracking in concrete may reduce load carrying capacity, shorten the service life of concrete structures, and accelerate deterioration resulting in increased maintenance costs. Therefore, it is significant to assess cracking potential of concrete mixtures under restrained shrinkage in service.

The restrained circular ring test has been widely used for assessing cracking tendency of concrete and other cement-based materials due to its simplicity and versatility [1, 4-10]. As a standard test method for assessing cracking potential of concrete mixtures under restrained condition, the circular ring test has been recommended by AASHTO (i.e. AASHTO T334-08: *Standard Method of Test for Estimating the Cracking Tendency of Concrete*) and ASTM (i.e. ASTM C1581/C1581M-09a: *Standard Test Method for Determining Age at Cracking and Induced Tensile Stress Characteristics of Mortar and Concrete under Restrained Shrinkage*). Accordingly, in restrained ring test, the effects of specimen size/geometry [11-13]; and moisture gradients and drying condition [12-14] on cracking of concrete or other cement-based materials have been studied. Moreover, novel elliptical ring geometries have been adopted to replace circular ring geometries for investigating shrinkage cracking of mortar and concrete under restrained condition[15-19]. It is generally regarded that, due to stress concentration caused by geometry effect, cracks initiate earlier in elliptical ring specimens than in circular ones though this has yet been validated in those studies. In contrast with the fact that a crack may initiate anywhere in a circular ring specimen, it tends to initiate at certain position in an elliptical ring specimen for a given ring geometry. Therefore, elliptical ring specimens have been employed for assessing cracking tendency of cement-based materials as an improved ring test by some researchers [15-19].

On the other hand, in order to study the mechanism of concrete cracking in restrained circular ring specimens, many theoretical methods have been proposed to predict cracking age of concrete. These methods can be classified as stress analysis approach [12-14, 20-22] and fracture resistance curve approach [9, 11, 23, 24]. In conventional stress analysis approach, cracking age could be obtained by comparing the residual stress in a concrete ring and its tensile strength. However, some researchers [9, 11, 23, 24] pointed out that tensile strength-based failure criterion might not yield accurate results for cracking of concrete at early ages, and a fracture-based failure criterion could appear more appropriate. In this case, the fracture resistance curve approach, which is based on fracture mechanics concepts with consideration of the energy balance, has been widely used by many researches [9, 11, 23-28]. By comparing the energy release rate (*R*-curve) with crack driving energy (*G*-curve), the onset of crack propagating unsteadily in a restrained concrete ring can be determined. It should be noted that the two methods mentioned above have also been used for predicting cracking ages of concrete in restrained elliptical ring tests [29, 30].

Although the fracture resistance curve approach provides a better explanation for what observed in restrained circular/elliptical ring test, it was generally used to predict the critical cracking condition in rings subject to restrained shrinkage. Meanwhile, in restrained circular/elliptical ring test, the age of cracking can be detected from the sudden drop to almost zero in the measured strain of the inner restraining steel ring. Further, the relationship of steel strain and actual residual stress in circular ring test was derived to assess the risk for cracking [31] and to determine the crack growth status in fiber concrete [32]. Generally speaking, concrete is regarded as quasi-brittle material, and there are three distinguished stages in crack propagation process of concrete, i.e. crack initiation, stable propagation and unstable propagation. Restraint disappearing from a steel ring does not suggest the whole crack growth process, i.e. crack initiates and propagates throughout a concrete ring wall.

In line with this, the objective of this paper is to investigate crack initiation and propagation process of concrete under a restrained shrinkage condition, demonstrating the failure mechanism and presenting the quantitative assessment of cracking resistance for concrete in the ring test. By introducing the fictitious crack model [33], the nonlinear properties of concrete, characterized by the cohesive force acting on the fictitious crack, is considered. The existence of the cohesive effect reflects the strain localization and nonlinear properties of quasi-brittle materials like concrete, which is an essential difference between quasi-brittle and brittle materials. Meanwhile, by establishing the equilibrium relationship between the crack driving effect caused by the drying shrinkage and the cracking resistance caused by the quasi-brittle properties of concrete, the crack development and evolution processes are elabrated. Further, the difference between the cracking resistance of concrete, which is represented by the crack initiation, and the structural response under a time-dependent drying stimulation, represented by the unstalbe crack propagation, is clarified. It is expected that the nonlinear fracture mechanics-based numerical model developed in this study will be helpful in exploring the mechanism of restrained circular/elliptical concrete ring test which can be employed for assessing the cracking tendency of concrete and other cement-based materials.

Firstly, a numerical approach was developed in this paper to simulate the shrinkage behavior of concrete in restrained ring specimens subject to drying from their outer circumferential cylindrical surface. A fictitious temperature field, which is derived based on free shrinkage data of concrete prisms, was applied on concrete ring specimens in numerical analyses to simulate the mechanical effect of concrete shrinkage on rings under restrained condition. A crack propagation criterion based on the nonlinear fracture mechanics is introduced to determine the crack propagation, in which crack begins to propagate when the stress intensity factors (SIF) K_{I}^{S} caused by the applied load (i.e., shrinkage effect in case of concrete rings under restrained condition) exceeds K_{I}^{σ} caused by the cohesive stress. Using the proposed method, the whole fracture process including crack initiation, stable and unstable propagation is simulated, and the SIF variations at the different crack propagation stages are analyzed.

2 Experimental Program

The mix proportion for the concrete used for this study was 1:1.5:1.5:0.5 (cement: sand: coarse aggregate: water) by weight, and the maximum of aggregate size was 10 mm.

2.1 Materials Properties

Mechanical properties of concrete, such as elastic modulus *E,* splitting tensile strength f_t and uniaxial compressive strength f_c, at different ages were measured using 100 mm diameter and 200 mm height cylindrical specimens in this study. After curing in sealed cylindrical moulds in normal laboratory environment for 24 h, the cylindrical specimens were de-moulded and moved into an environment chamber with 23℃ and 50% relative humidity (RH) for curing till the desirable ages of testing.

Regression analyses were conducted on the experimental data to obtain continuous functions that can represent the age-dependent mechanical properties, in this case, E and f_t, for the concrete. It was found that elastic modulus, E, of the concrete at early ages can be predicted using Equation 1.

$$E(t)=0.0002t^3-0.0134t^2+0.3693t+12.715 \qquad (t\leq 28) \qquad (1)$$

Splitting tensile strength, f_t, can be predicted using Equation 2.

$$f_t(t)=1.82t^{0.13} \qquad (t\leq 28) \qquad (2)$$

In both equations, t is the age (unit: day) of concrete. The values of E and f_t for concrete at other ages which were not directly measured can be obtained from Eqs. (1) and (2).

2.2 Free Shrinkage Tests

Free shrinkage was measured on concrete prisms with the dimensions of 280 mm in length and 75 mm square in cross section, conforming to ISO 1920-8, subject to drying in the same environment as for curing concrete cylinders. Their longitudinal length change was monitored by a dial gauge, which was then converted into shrinkage. Considering that concrete shrinkage depends on the exposed surface area-to volume (A/V) ratio, four different exposure conditions, i.e. four different values of A/V, were investigated in this study. They are all surfaces sealed, all surface exposed, two side surfaces sealed and three side surfaces sealed, respectively. It should be noted that side surface refers to the surface with the dimensions of 280×75 mm^2. In experiment, double-layer aluminum tape was used to seal the desired surfaces which were not intended for drying. A two side surfaces sealed prism in free shrinkage test is shown in Figure 1. Initial measurement was carried out immediately after the concrete prisms were de-molded at the age of 1 day and the measurements were continuously recorded twice per day until 28 days. Figure 2 shows the measured free shrinkage strain of concrete at various ages under different exposure conditions.

It should be noted that the computation of free shrinkage strain in a concrete ring is an approximate estimation for the purpose of simplification, which match the geometry of the prism to the ring through the ratio of A/V. A more complex and accuracy approach was introduced in Reference [28]. The free shrinkage prisms had an identical volume to surface ratio to that of a short concrete ring with the depth of 75 mm when drying from the top and bottom surfaces. Therefore a direct correlation can be made with the shrinkage properties of the ring. In the case of tall rings drying from outer surface, the shrinkage strain of concrete can be derived by introducing the average humidity profiles and the relationship between shrinkage and relative humidity.

2.3 Restrained ring test

In order to investigate the effects of the inner steel core on cracking age of restrained concrete rings and the mechanism of restrained ring test, two kinds of steel core, which are circular and elliptical, were investigated in current experiment program.

A circular concrete ring with its inner diameter is denoted as R_0 (see Figure 3a), and an elliptical concrete ring, with the major and minor semi-axes of its inner circumference is denoted as R_1 and R_2, respectively (see Figure 3b). According to the study of Zhou et al. [30], compared with traditional circular concrete rings, elliptical concrete rings with R_1/R_2 between 2 and 3 can provide higher degree of restraint leading to shorter cracking period in restrained shrinkage ring test so that to accelerate ring test. Therefore, in this study, for the concrete elliptical ring specimens with a 37.5 mm-thick wall same as that recommended by ASTM C1581/C1581M-09a, the inner major radius, R_1, was chosen as 150 mm and the inner minor radius, R_2, as 75 mm while the radius, R_0, of the inner circumference of the circular rings was designed as 150 mm same as R_1. The wall thickness of restraining steel cores and the height of the specimens were as 12.5 and 75 mm, respectively. Following ASTM C1581/C1581M-09a protocol, the top and bottom surfaces of ring specimens were sealed using two layers of aluminum tape and drying was only allowed through the outer circumferential cylindrical surface of the concrete rings. The test setup and sealed specimens are shown in Figure 4. The strain gauges were then connected to the data acquisition system, and the instrumented ring specimens were finally moved into an environmental chamber after the first day curing for continuous drying under the temperature 23°C and RH 50% till the first crack occurred.

It should be noted that, according to Radlinska et al. [34], specifying the precision of the restrained ring test in terms of standard deviation of measured strain is more promising than of the age of cracking. It is because the variability in the cracking age shows time dependence and it is much higher for the cracking occurred at later age. However, in this study, the cracking age was used to reflect the time-dependent material properties rather than the variability of the ring test. Therefore, it is still employed in this study to compare the cracking tendency of concrete in elliptical and circular ring tests.

3 Numerical Modeling

3.1 Modeling of Restrained Shrinkage

In current research, a derived fictitious temperature field is applied to concrete to represent the mechanical shrinkage effect so that a combined thermal and structural analysis can be adopted to analyze cracking in a concrete ring specimen caused by restrained shrinkage. The restrained circular ring test is recommended by AASHTO and ASTM, for assessing the cracking tendency of concrete under a restrained drying shrinkage condition. However, in the case of concrete deformation caused by large temperature variation, the conventional ring test cannot serve the purpose because the restraining effect varies with the steel ring dimensions caused by varying temperature. Particularly, the restraint will disappear if the concrete expansion occurs under elevated temperature. To overcome these limitations, the dual ring test was proposed by Schlitter et al. [35-37], in which invar with a very low thermal expansion coefficient was used to make the central restraining steel ring. Meanwhile, the autogenous shrinkage can be reduced or eliminated by controlling the temperature reduction in a concrete ring, so that the remaining stress capacity of concrete can be better assessed. The research conducted by Schlitter et al. [35-37] extends the application of the restrained ring test and provide an improved understanding of the cracking potential of a cement mixture. However, it should be noted that the fictitious temperature drop proposed in this study is to numerically simulate the concrete shrinkage effect under drying, as opposed to a real temperature drop experienced in a ring test. For the purpose of the analyses, there is no fictitious temperature drop enforced on the central restraining steel ring therefore it makes no difference whether traditional steel or invar steel is used for making the central restraining steel ring.

With the implementation of the fictitious temperature field, shrinkage of concrete caused by the temperature field is restrained by the inner steel core, resulting in compressive stress developed in the steel core and tensile stress in the concrete ring. The derivation of the fictitious temperature field is elaborated elsewhere [30]. As the result of this exercise, Figure 5 presents the derived relationship between fictitious temperature drop and A/V ratio at 2 days interval for a concrete element irrespective of its geometry/shape. It should be noted that although Fig. 5 only presents the curves at 2 days interval, fictitious temperature drop was actually calculated for each day which was then used to update the input data for FE analyses of concrete rings in this study. For a given concrete ring with certain exposure condition (i.e. certain A/V ratio), the relationship between fictitious temperature drop and concrete age can be derived by linear interpolation from the relationship between A/V ratio and concrete age obtained in Figure 5.

3.2 Crack initiation and propagation

According to experimental observation, in case of a thick ring (i.e., the ring wall thickness is 75 mm), cracking initiates at the outer circumferential surface of a restrained concrete ring, which accords well with the findings of Hossain and Weiss [13] by acoustic emission testing. In case of a thin ring, to the best of the authors' knowledge, there is no explicit experimental evidence that can support whether a crack initiates at its outer or inner circumferential surface. Moon and Weiss [12] studied the residual stress distribution in both thick and thin circular rings by considering the moisture gradient across a concrete ring wall. Based on their results, tensile stress is the highest on the outer circumference of a concrete ring. Due to this, cracking always initiates at the outer circumference surface of a concrete ring when drying from its outer circumferential surface, no matter how thick a concrete wall is if the moisture gradient is taken into account in analyses. It is because that the moisture gradient is very significant with the majority of the drying taking place from the outer circumference of a concrete ring at early ages (i.e. low value of γ, where γ is the product of the moisture diffusion coefficient and drying time). Under this condition, the crack in a restrained concrete ring will be mainly caused by the self-shrinkage due to the moisture gradient rather than from restraining effect from the central steel ring. However, it makes sense that the crack initiation is resulted from the restraining-dominated effect from the central steel ring in case of a thin-wall concrete ring. In which case, the crack will initiate at the inner circumference of a concrete ring. Meanwhile, by considering that a thin ring has a larger A/V ratio than a thick one when drying from its outer circumference surface, the moisture gradient across a thin ring wall can be ignored and the approximate uniform shrinkage across it is assumed. However, it should be noted that it is necessary to further examine the initial crack position of a thin ring under outer circumferential surface drying from experiment.

In this study, cracks are assumed to initiate when the maximum circumferential tensile stress of concrete exceeds its tensile strength f_t. According to the experimental and numerical results conducted by Zhou et al. [30], crack randomly initiates at the inner circumference of a circular ring specimen. While in the case of an elliptical ring specimens with R_1=150 mm, R_2=75 mm, crack initiates close to the vertices on the major axis of the inner elliptical circumference (See Figure 6). Therefore, after crack initiation, a pre-crack, with the length of 2 mm, is set at the position where the maximum circumferential tensile stress occurs. In order to consider the softening behavior in micro-cracks, the fictitious crack model [33] is introduced in the fracture analysis through establishing softening stress (*σ*)-crack opening displacement (*w*) relationship of concrete. In this paper, the bilinear expression for σ-w (see Figure 7) is chosen in the proposed numerical approach which is presented as follows:

According to Peterson [38], σ_s, w_s and w_0 can be determined as follows:

$$\sigma_s = f_t / 3 \tag{3}$$

$$w_s = 0.8 G_F / f_t \tag{4}$$

$$w_0 = 3.6 G_F / f_t \tag{5}$$

Where w_0 is the displacement of the terminal point of *σ*-*w* curve beyond which no stress can be transferred, i.e. the stress-free crack width, w_s and σ_s is the displacement and stress, respectively, corresponding to the break point in the bilinear *σ*-*w* relationship. These parameters and the *σ*-*w* relationship can be derived given the fracture energy G_F and the tensile strength f_t. Here, f_t is obtained from Equation (2) and G_F is from the formula recommended by CEB-FIP model code 2010.

Further, a crack propagation criterion based on nonlinear fracture mechanics is employed for predicting cracking propagation process in circular/elliptical concrete rings under restrained shrinkage. According to this criterion, when the SIF caused by deriving forces exceeds the one by cohesive forces, i.e. K_I^S-$K_I^\sigma \geq 0$, crack will propagate. It represents the competition between the crack driving forces which attempt to open the crack and the cohesive forces which attempt to close the crack [39]. This criterion has been successfully used to simulate the crack propagation in reinforced concrete [40], mode-I and mixed-mode fracture [41, 42] and multiple cohesive crack propagation [43] in concrete. In this study, SIFs caused by deriving forces, i.e. shrinkage effect, and cohesive force are denoted as K_I^S and K_I^σ, respectively. Moreover, in this study, the age-dependent effective elastic modulus of concrete adopted in numerical analyses was taken as 60% of the value obtained from Equation (1) to account for creep effects. Similar measure in taking into account creep effect by reducing elastic modulus was also taken by [14, 30] when analyzing cracking in circular/elliptical concrete rings under restrained shrinkage. In numerical analysis, singular element was used to calculate SIF at the tip of crack. In order to eliminate the effect of friction between concrete and steel, the outer circumferential surface of the steel ring, which contacts the inner circumferential surface of the concrete ring, was coated with a release agent as suggested by ASTM C1581/C1581M-09a when preparing ring tests. Accordingly, in numerical analyses, contact element with zero friction between the contact pair was utilized to simulate this measure in conducting concrete ring tests.

In summary, the following steps were taken in analyzing crack initiation and propagation of a concrete ring subject to restrained shrinkage:

(1) Measure mechanical properties, including f_t and E, of concrete and free shrinkage through a series of concrete prisms with different A/V ratios at various ages. Convert the results of free shrinkage into a relationship between fictitious temperature drop and A/V ratio for a concrete element at various ages.
(2) Derive the fictitious temperature drop for a circular/elliptical ring specimen at various ages by linear interpolation from the relationship between fictitious temperature drop and A/V ratio obtained in step (1).
(3) Conduct an elastic analysis in a concrete ring by applying the corresponding fictitious temperature drop obtained in Step (2) on it, until the maximum circumferential tensile stress of concrete exceeds its tensile strength. Pre-set a 2

mm long fictitious crack on the location where the maximum circumferential tensile stress occurs in a circular/elliptical concrete ring.

(4) Apply cohesive stress on the fracture process zone (FPZ) according to the σ-w relationship. Calculate SIFs at crack tip caused by the cohesive force and shrinkage effect at various ages until K_I^S exceeds K_I^σ.

(5) Add an increment of crack length Δa=2 mm and repeat Step (4) and (5) until crack propagates throughout the section of the concrete ring wall.

4 Results and discussions

According to the numerical analysis results of Zhou et al. [30], the maximum circumferential tensile stress of concrete exceeds its tensile strength at the ages of 12 days for elliptical rings with R_1=150 mm and R_2=75 mm. In the case of circular rings with R_0=150 mm, the cracking age is 15 days. After crack initiation, a pre-crack is located on inner circumference of a circular ring, and the pre-crack is close to the vertices on the major axis of the inner circumference for an elliptical ring. There is cohesive stress acting on the pre-crack, which can be determined by the σ-w relationship specified in Figure 7. Figure 8 illustrates the pre-crack locations in circular and elliptical rings adopted in this study.

In order to elaborate crack propagation after initiation, Figures 9 and 10 present the evolution of SIF caused by shrinkage effect and cohesive stress in circular/elliptical concrete rings. For the circular ring with R_0=150 mm at the cracking age of 15 days, the SIFs at the tip of 2 mm pre-crack satisfy the criterion of K_I^S - K_I^σ=0, which suggests that the crack driving force caused by shrinkage effect can overcome the resistance caused by cohesive force. So the crack can propagate further under that condition.

While, K_I^S - K_I^σ<0 (See Point A in Figure 9a) when the crack propagates to the length of 4 mm, it indicates that the resistance exceeds the crack driving force and the crack cannot propagate further. So in order that the crack is able to move forward, it is necessary to enhance the shrinkage effect of concrete, i.e. the crack driving energy. When concrete gets mature, i.e., from the 15th to the 16th day, the relationship between K_I^S and K_I^σ evolves to K_I^S - K_I^σ>0 corresponding to a crack length of a=4 mm (See Point B in Figure 9b). Moreover, at the age of 16 days, K_I^S becomes always greater than K_I^σ and the difference between K_I^S and K_I^σ (i.e. K_I^S - K_I^σ) keeps increasing with the increase of crack length. This demonstrates that the crack driving force exceeds the resistance which is the cohesive stress tending to close the crack and the crack will propagate throughout the concrete ring wall at the 16th day. Comparing with the scenario of monotonic increasing of K_I^S with the increase of crack length, there are three distinguished stages in the evolution of K_I^σ, i.e., monotonically increasing before crack length reaches 14 mm (i.e. $a \leq 14$ mm), keeping plateaued when crack length is between 14 and 24 mm (i.e. a= 4~24 mm), and monotonically decreasing when crack length exceeds 24 mm (i.e. a>24 mm). The variation of K_I^σ can be explained by the shortening of FPZ when the crack tip is closed to the outer circumferential surface of a concrete ring.

For the elliptical ring with R_1=150 mm and R_2=75 mm, when crack initiates at the age of 12 days, K_I^S - K_I^σ=0 corresponding to a crack length a=2 mm. It indicates that the crack can continue propagating under that condition. In numerical analysis, a new crack length in this case 4 mm is reached by giving a crack length increment Δa=2 mm to the original crack length 2 mm. However, when crack propagates to 4 mm long, it reaches a different scenario. In this case, SIF caused by cohesive force, i.e. K_I^σ, becomes greater than that caused by the driving force, i.e. K_I^S, which in fact is due to shrinkage effect. It indicates that the crack cannot propagate further until the driving force can be enhanced and SIF caused by driving force, i.e. K_{I}^{σ}, becomes greater than SIF caused by cohesive force, i.e. K_{I}^{σ}. As concrete gets mature as drying continues, the accumulative shrinkage effect keeps increasing. At the age of 13 days, K_{I}^{S} eventually becomes greater than K_{I}^{σ} and the deviation between K_{I}^{S} and K_{I}^{σ} becomes greater than 0 thereafter. Therefore the crack will keep propagating till throughout the concrete ring wall.

Besides, before crack starts unstable propagation, the maximum value of K_{I}^{S} - K_{I}^{σ} occurs at the crack length of a=4 mm for a circular ring (See Point A in Figure 9a), and a=8 mm for an elliptical ring (See Point A in Figure 10a). Within one day after crack initiates, it can propagate throughout the concrete ring wall. At the corresponding points B in Figures 9b and 10b, K_{I}^{S} becomes greater than K_{I}^{σ}. Therefore, it can be concluded that the crack lengths at the onset of unstable propagation for circular and elliptical rings are 4 and 8 mm, respectively. The difference in crack length at the onset of unstable propagation in circular and elliptical concrete rings is due to the difference in stress gradient across a concrete ring wall with circular and elliptical geometries. Comparing with a circular ring geometry, the stress gradient in a concrete wall is more significant for an elliptical ring geometry [30]. Meanwhile, the stable propagation length gives a reference to determine an appropriate pre-crack length a_0 for numerical analysis of concrete rings under restrained shrinkage. For example, in the case of a circular ring, the stable propagation of concrete crack cannot be captured by numerical analysis once a_0 exceeds 4 mm under current study condition.

In the standard restrained ring test methods recommended by AASHTO T334-08 and ASTM C1581/C1581M-09a, compressive strain developed in a steel ring caused by shrinkage of mortar or concrete surrounding it is monitored by stain gauges attached on the inner circumferential surface of the steel ring. As concrete is getting mature, it shrinks but is restrained by the inner steel ring. Therefore, tensile stress develops in the concrete ring and compression stress in steel ring. Cracking of the concrete ring is indicated by a sudden drop in the steel ring strain. However, it should be noted that the sudden drop of steel ring strain only represents the release of restraint effect in concrete caused by the inner steel ring, rather than explicitly reflects the statue of crack, i.e. initiation, stable propagation or unstable propagation, in concrete. According to the numerical results conducted in this study, for the purpose of crack propagation, the restraining effect should be strengthened after crack initiation until the driving force caused by restraint shrinkage becomes greater than the resistance force so that crack can continue growing till propagating throughout the concrete ring wall. Therefore, the sudden drop of steel ring strain observed in the test recommended by ASTM C1581/C1581M-09a illustrates the crack unstable propagation, which is regarded as a structural response, rather than the crack initiation, which is regarded as a crack resistance of concrete as a material property. Structural response of a concrete element does depend on its thickness. In this case, the cracking age of a restrained concrete ring depends on its ring wall thickness. However, it should be noted that the period from crack initiation to unstable propagation is very short, which is within one day, in both circular and elliptical rings subject to restrained shrinkage for the cases investigated in this study. Moreover, although K_{I}^{S} is smaller than K_{I}^{σ} after crack initiation, the difference between K_{I}^{S} and K_{I}^{σ} is very marginal with the maximum value of 0.08 MPa·mm$^{1/2}$ in a circular ring (See Point A in Figure 9a) and 0.38 MPa·mm$^{1/2}$ in an elliptical ring (See Point A in Figure 10a). These differences can be ignored compared with the SIFs caused by restrained shrinkage effect, which are about 6.5 MPa·mm$^{1/2}$ in both geometries. Therefore, it is reasonable to accept that restrained shrinkage ring test can approximately determine crack resistance of concrete and other cement-based materials as per AASHTO T334-08 and ASTM C1581/C1581M-09a. According to numerical analysis results, the cracking ages for circular and elliptical rings are 16 and 13 days, respectively, which are the age at the onset of crack unstable propagation. The experimental results of cracking ages with respective to the circular and elliptical rings are about 15 and 10 days, which are determined by a sudden drop in the steel ring strain. The cracking ages predicted through the numerical model agreed reasonably well with experimental results indicating that the proposed numerical model is reliable.

Further, it is well known that crack initiation in concrete can be regarded as material behavior on cracking resistance, which is independent of specimen geometry and boundary condition. However, unstable concrete crack propagation in a restrained concrete ring represents structural response to external stimulation, i.e. it is structure behavior, which depends on specimen geometry and boundary condition. In restrained shrinkage ring test, concrete crack actually experiences the full spectrum of initiation, stable propagation and unstable propagation till propagating throughout the ring wall. The observed sudden strain drop of an inner steel ring reports that a crack propagates throughout a concrete ring wall, i.e. unstable propagation. Then, the results obtained in the ring test indicate structural behavior of a concrete ring subject to restrained shrinkage, rather than material behavior of concrete as per the suggestions of the AASHTO T334-08 and ASTM C1581/C1581M-09a. It is worthy pointing out here that, from the point of view of applying the ring test to assess cracking tendency of concrete as a material property of concrete, the difference in ages at crack initiation (representing material behavior) and unstable propagation (representing structural behavior) is very marginal although the mechanical mechanism is distinguished.

5 Conclusions

In this study, crack initiation and propagation in restrained concrete rings, both circular and elliptical, is analyzed. A fictitious temperature drop is applied on concrete to simulate the shrinkage effect of concrete due to drying. Combined with the fictitious crack model, a numerical mode is established for investigating crack initiation and propagation in circular/elliptical concrete rings under restrained shrinkage for the purpose of assessing cracking tendency of concrete and other cement-based materials as per AASHTO T334-08 and ASTM C1581/C1581M-09a. The evolution of SIF caused by restrained shrinkage and cohesive force for circular and elliptical rings is investigated with respect to concrete age. Based on the results presented in this paper, the following conclusions can be drawn.

(1) The initial cracking ages predicted through the numerical model agree reasonably well with experimental results indicating that the proposed numerical model is reliable. The fracture-based method developed in this study can be employed for simulating crack propagation process in circular/elliptical concrete rings under restrained shrinkage.

(2) There is a stable crack propagation process after crack initiation in restrained circular/elliptical concrete rings. The sudden strain drop measured from the internal surface of a restraining steel ring in a standard ring test indicates that a crack propagates throughout the concrete ring wall. For both circular and elliptical rings in the cases studied of this paper, the crack initiation and propagation throughout the concrete ring wall do not occur at the same time. Concrete cracks can propagate through a ring wall within one day after initiation. However, it should be noted here that the time, from crack initiation to propagation throughout a concrete ring wall,

depends on the type of concrete and the geometry of a ring specimen. In certain cases, this can be much longer than 1 day.

(3) The cracking age of concrete or other cement-based materials, which is characterized as a crack propagating throughout the concrete ring wall in a restrained ring test, depends on the degrees of restraint, resulted from the properties of steel and concrete (i.e. elastic modulus, Poisson's ratio etc.), and the geometry of the ring specimen (i.e. radius of steel ring, thickness of steel and concrete rings, etc.) [14]. So results given by the AASHTO/ASTM restrained ring test actually reflects the response of a concrete ring as a structure to external stimulation, in this case restrained concrete shrinkage. Thus the AASHTO/ASTM restrained ring test actually provides information about the structural behavior of a restrained concrete ring, rather than material behavior of concrete as per the suggestions of the AASHTO T334-08 and ASTM C1581/C1581M-09a.

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Figure captions.

Figure 1. Free shrinkage test setup

Figure 2. Shrinkage strain of concrete obtained from free shrinkage test

Figure 3. Notation of geometrical parameters; **a** circular ring, **b** elliptical ring

Figure 4. Restrained shrinkage ring test; **a** test setup for an elliptical ring, **b** sealed specimens

Figure 5. Derived fictitious temperature drop with respect to A/V ratio for a concrete element

Figure 6. Crack position in an elliptical ring specimen

Figure 7. Bilinear σ-w softening curve for concrete

Figure 8. Pre-crack in concrete rings; **a** circular, **b** elliptical

Figure 9. Evolution of SIFs in a circular ring at the ages of crack initiation and unstable propagation; **a** 15 days, **b** 16 days

Figure 10. Evolution of SIFs in an elliptical ring at the ages of crack initiation and unstable propagation; **a** 12 days, **b** 13 days

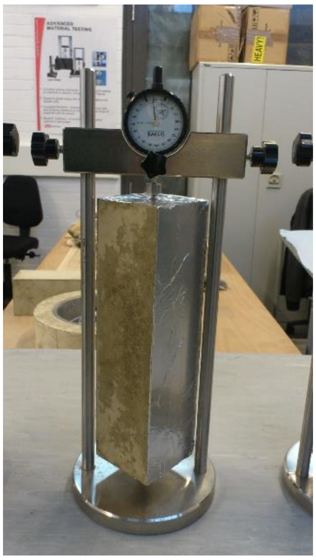


Fig. 1. Free shrinkage test setup

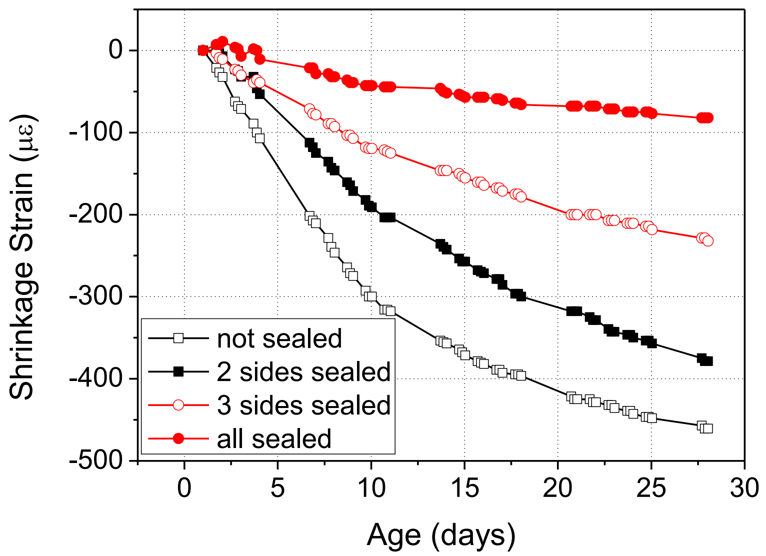
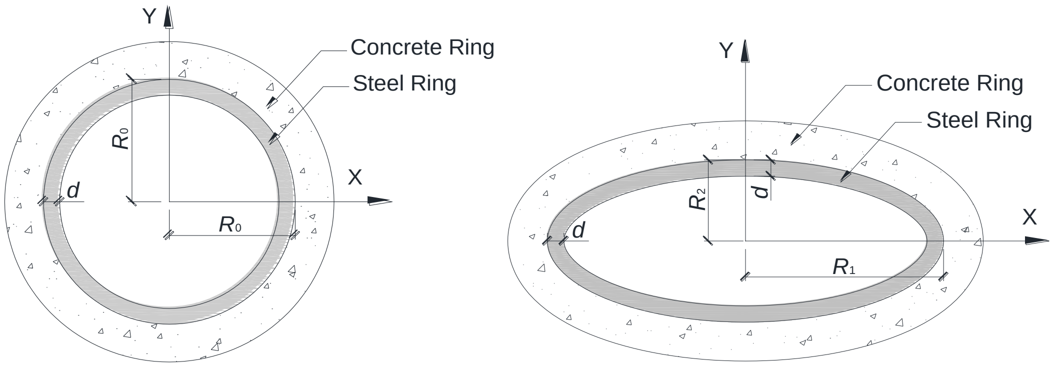


Fig. 2. Shrinkage strain of concrete obtained from free shrinkage test



(a) (b)

Fig. 3. Notation of geometrical parameters; **a** circular ring, **b** elliptical ring

(a) (b)

Fig. 4. Restrained shrinkage ring test; **a** test setup for an elliptical ring, **b** sealed specimens

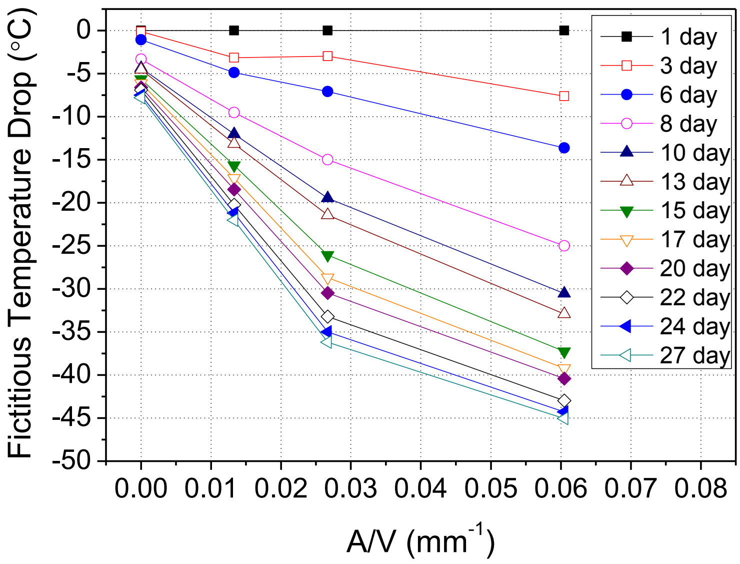


Fig. 5. Derived fictitious temperature drop with respect to A/V ratio for a concrete element

Fig. 6. Crack position in an elliptical ring specimen

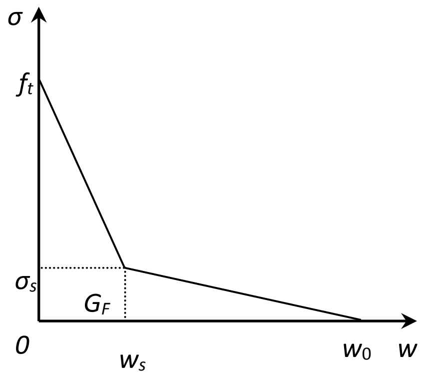
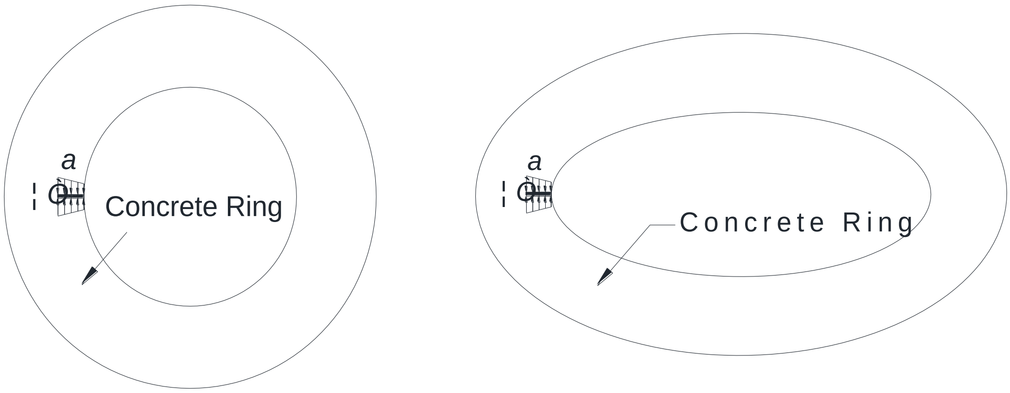
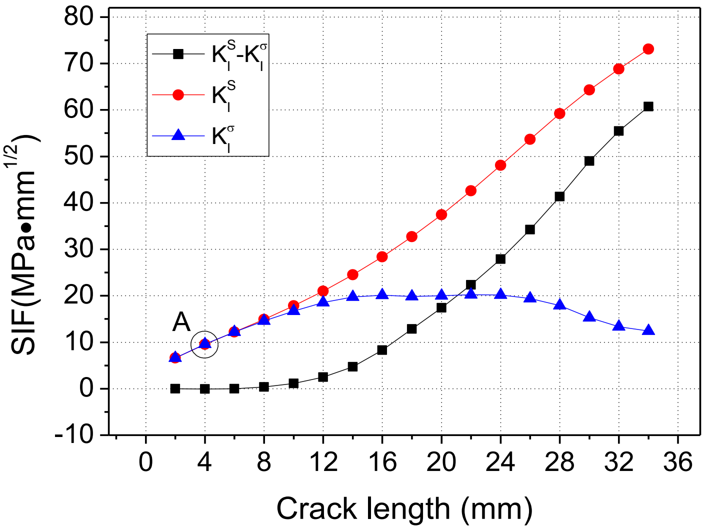


Fig. 7. Bilinear *σ*-*w* softening curve for concrete

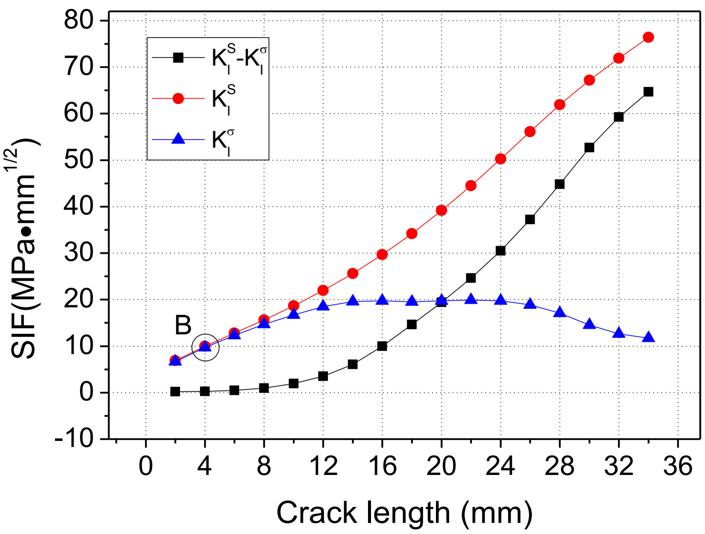


(a) (b)

Fig. 8. Pre-crack in concrete rings; **a** circular, **b** elliptical

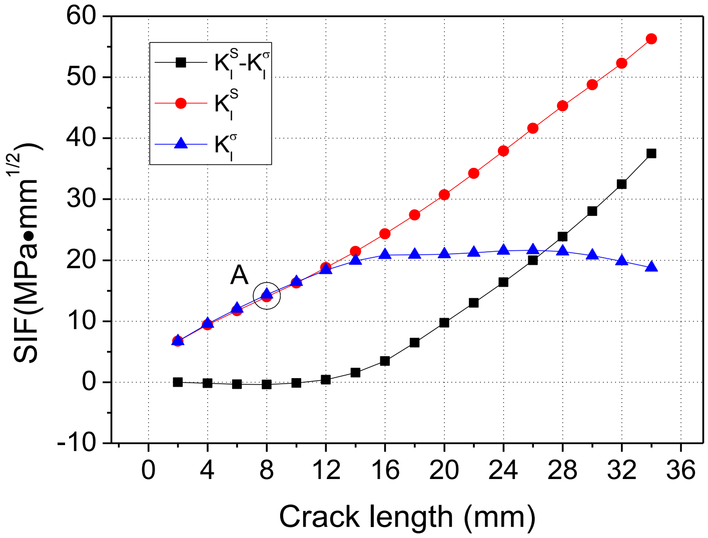


(a)

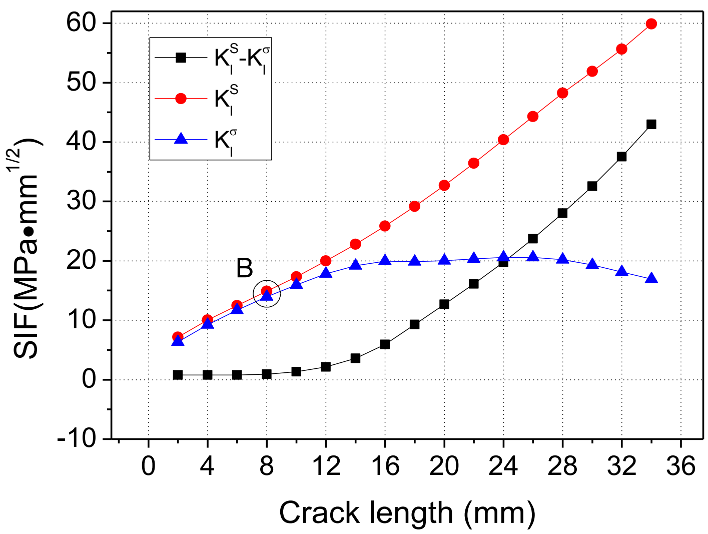


(b)

Fig. 9. Evolution of SIFs in circular ring at the ages of crack initiation and unstable propagation; **a** 15 days, **b** 16 days



(a)



(b)

Fig. 10. Evolution of SIFs in elliptical ring at the ages of crack initiation and unstable propagation; **a** 12 days, **b** 13 days