Experimental study of equal biaxial-to-uniaxial compressive strength ratio of concrete at early ages

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ABSTRACT

The ratio of equal biaxial to uniaxial compressive strength of concrete, denoted as β, is an important parameter in the determination of failure criterion for concrete, which has been widely adopted in finite element codes in simulation of fracture and failure of concrete. However, there is no experimental study on β conducted for concretes at early ages. In this study, an experimental study on the uniaxial and equal biaxial compressive strengths of concretes at early ages up to 28 days was carried out using an in-house electro-hydraulic servo-controlled triaxial test machine. Concrete specimens with different coarse aggregate sizes (10mm, 20mm, 30mm) and strength grades (C30, C40, C50) were tested at various ages (6h, 12h, 1d, 3d, 7d, 14d, 28d). The results showed that β decreases with the increase of concrete age. In comparison, there are less significant effects of concrete strength and maximum coarse aggregate size on β. By regression analyses of experimental results, an empirical equation was proposed for β by considering the effects of age on β for concrete at early ages.

Keywords: Equal biaxial-to-uniaxial; Compressive strength ratio; Biaxial compressive strength; Concrete; early age

1. Introduction

Numerical modelling of concrete and other cement-based materials is an efficient tool for the investigation of the static/dynamic behaviour of concrete elements/structures. In this context, the failure envelope plays a significant role in numerical analysis of concrete structures and has been widely studied through experimental and theoretical approaches in the last decades. There are several failure criteria for concrete proposed by researchers. Through calibrating elementary strength data of uniaxial compression, uniaxial tension, and equal biaxial compression from experiment, a three-parameter criterion was proposed by Menetrey and Willam [1]. Based on the fracture theory, a four-parameter criterion was proposed by Hsieh et al. [2] to determine material’s behaviors from initial yielding to fracture failure. Meanwhile, a five-parameter failure criterion [3], which is dependent on three stress-tensor invariants, was proposed through the introduction of a new two-parameter function describing the deviatoric cross section of the failure surface. Recently, aiming at normal strength concrete and high strength concrete in compression-compression-tension, compression-tension-tension, triaxial tension, and biaxial stress states, a unified strength criterion in the principal stress space has been proposed by Ding et al. [4]. Among these, the shape of failure surface in the deviatoric stress space is affected by the out-of-roundness eccentricity parameter, which was recommended as 0.5 for a triangular shape and 1.0 for a circular shape [1]. Meanwhile, the parameter of the out-of-roundness is affected by the curvature of the tensile meridian, so that it is usually calibrated under equal biaxial compression. Therefore, to use the aforementioned failure criterion in numerical analysis, it is necessary to obtain the equal biaxial-to-uniaxial compressive strength ratio, β, of concrete.

For mature normal-strength concrete, many experimental investigations [5-7] have been conducted to derive β with the value of 1.14 [1] widely adopted by the engineering and academic communities. However, with the increase of concrete strength from normal to high strength, it seems that β does not remain constant. According to the study on high-strength concrete by Hussein and Marzouk [8], β decreases with the increase of concrete strength. Further, based on the statistical data obtained from experimental results of concretes with various strengths, Papanikolaou and Kappos proposed a relationship between β and uniaxial concrete strength through a power-law regression curve fitting analysis [9]. According to their research, β decreases from 1.2 to 1.05 when concrete strength grade increases from C20 to C120. In addition to concrete strength, coarse aggregate size is another important factor affecting β. In general, the equal biaxial compressive strength f_{bc} is related to the uniaxial compressive strength f_c of concrete, so that the only variable is f_c in the function of f_{bc}. [10, 11]. It is understandable that f_c is strongly influenced by coarse aggregate size in fresh or hardened concrete [12]. However, it has not been verified by experiment or theoretical analysis that concrete, with the same f_c but different coarse aggregate sizes, exhibits similar f_{bc}. Chen et al. [11] conducted an experimental investigation on biaxial compressive strength for concrete with similar uniaxial compressive strength but different maximum coarse aggregate sizes. Their results indicated that biaxial compressive strength will increase with the increase of coarse aggregate size for concrete with similar uniaxial compressive strength. Meanwhile, aiming at concrete for dam, Wang and Song [13] investigated the normalized biaxial compressive strength of concrete with the maximum coarse aggregate sizes of 20 mm, 40mm and 80mm. Similar conclusions to those drawn by Chen and Leung [11] were reported by them for concrete used for the construction of dams and wet-screened components. However, the quantitative relationship between maximum coarse aggregate and β was not presented in the research of Chen and Song (2009), and Wang and Song (2009), although the variation trend of β was discussed.

It should be noted that the aforementioned research has focused on the behaviour of mature concrete under biaxial compression. Research on early-age concrete under biaxial compression is very limited and only Liu et al. [14] conducted such research but on creep of early-age concrete under biaxial compression. In reality, some massive concrete structures, such as nuclear power plants and docks, is under a multiaxial stress state during construction, i.e. at early ages. Therefore, it is significant to derive the failure criterion in early-age concrete for the purpose of safety evaluation of a concrete structure under construction. β, as a key parameter, affects the out-of-roundness which further determines the shape of failure surface of concrete under biaxial/triaxial loading. Therefore, it is essential to investigate β with respect to concrete age when adopting a failure criterion to assess the safety of a concrete structure during construction. However, for concrete at early ages, to the best of the authors' knowledge, no formula for biaxial compressive strength is reported. Particularly, in the case of early-age concrete with different strength, the study on the effect of maximum coarse aggregate size on β has not been carried out in previous research. Therefore, together with the characteristic of early-age concrete, it is significant to investigate the variation of β for concrete with various strength and coarse aggregate sizes.

In line with this, the objective of this paper is to focus on the variation of equal biaxial-to-uniaxial compressive strength ratio β for early-age concretes. Through measuring the equal biaxial and uniaxial compressive strength of concretes with various strength grades and coarse aggregate sizes, the relationship between equal biaxial-to-uniaxial compressive strength ratio β and concrete age within 28 days was obtained based on the experimental results. Further, the effect of concrete age and maximum aggregate size on β for early-age concretes was analysed, and the specimen failure characteristics under equal biaxial compression was discussed with respect to age for a series of concrete with various strength grades. It is expected that the experimental results presented here can lead to a better understanding of the mechanical properties and failure characteristics of early-age concrete so that the failure criteria can be used to assess the safety and durability of concrete in numerical analyses from the moment of final setting to in service.

2. Experimental Program

2.1 Materials and specimens

Three grades of concretes, i.e. C30, C40 and C50, were prepared to measure their uniaxial and equal biaxial compressive strengths within 28 days. Coarse aggregates with maximum sizes of 10 mm, 20 mm and 30 mm, respectively, were used in preparing each grade of concretes. River sand was used as the fine aggregate. The grade C30 and C40 concretes were made with Grade R42.5 Portland cement (Chinese Standard of Common Portland Cement, GB175-2007 [15]), and the grade C50 concrete was made with Grade R52.5 Portland cement (Chinese standard of Common Portland Cement, GB175-2007 [15]). The mix proportions of the three grades of concretes and their uniaxial compressive strength at 28 days are listed in Table 1. It should be noted that the uniaxial compressive strength listed in Table 1 was obtained on 150 mm cubes conforming to Chinese code of Standard for Test Method of Mechanical Properties on Ordinary Concrete, GB/T 50081-2002 [16], without the friction reducing measure between the loading plate and the specimen surfaces prior to testing. Meanwhile, to obtain the equal biaxial-to-uniaxial compressive strength ratios at different ages, a series of tests on uniaxial and equal biaxial compressive strengths were carried out using 100 mm cubes at the ages of 6h, 12h, 24h, 3d, 7d, 14d and 28d. To eliminate the influence of friction between the loading plate and the specimen surface, friction reducing pads, which were composed of two layers of PVC film and a layer of grease in-between, were inserted between the loading plate and the specimen surface. The 100 mm cubic specimens were cast, demolded and then stored in a curing room at 20℃ and 90% relative humidity. The specimens tested at 6h, 12h and 24h were demolded 1h before testing; the specimens tested at 3d, 7d, 14d and 28d were demolded 24h after casting. A minimum of 3 specimens were tested for each experiment batch, and the average results, denoted as $f_{\mathrm{c,mean}}$ and $f_{b\mathrm{c,mean}}$ were taken as the representative values. The uniaxial strength f_{c} and equal biaxial compressive strength f_{bc} of the grade C30, C40 and C50 concretes at the ages from 6h to 28d are presented in Appendixes A1, A2, B1, B2, C1 and C2, respectively. It should be noted that some specimens were found the existence of some deflects after demolding, e.g. cellular surface and damage of the specimen edges. To ensure the precision of the experimental results, these deflected specimens were gotten rid of the series tests so that there are cases which less than three strength values for some conditions are presented in these Appendixes.

2.2 ***Test apparatus and procedure***

The tests for uniaxial and equal biaxial strength were conducted using a refitted hydraulic servo-controlled true tri-axial test machine, which can apply load in three independent orthogonal directions onto a cubic specimen by two horizontal actuators and one vertical actuator (See Fig. 1). To apply uniform stress to a specimen surfaces, each actuator was equipped with a spherical and self-aligning head. Meanwhile, a compressive platen was attached on each spherical head. The nominal capacity of the loading system is 2000 kN in compression and 500 kN in tension. All specimens were tested in a stress-control mode at a loading rate of 0.1 MPa/s until failure. The loading signals were controlled and recorded by a data acquisition and processing system through a specially allocated amplifier.

3. Experimental Results and Discussion

3.1 Failure Mode

Concrete at different ages shows different failure modes, denoted as mode-I and -II failures, under uniaxial/equal biaxial compression. For concrete at ages 6h and 12h, the specimens failed at mode-I failure. At failure, the mortar on the cube surface spalled, and significant cracking occurred at the interface between aggregates and mortar (See Fig. 2(a)). In the case of biaxial compression at the ages of 6h and 12h, the mortar on the free surfaces (not loaded) spalled but the specimen maintained its integrity. There were some fine cracks on the loading surfaces, which were parallel to the two free surfaces (See Fig. 2(b)). By examining the internal failure shown in Fig. 2(c), it can be seen that the mode-I failure for the concrete at the ages of 6h and 12h was caused by the de-bonding between mortar and coarse aggregates. For the concrete at the ages of 6h and 12h, the incomplete cement hydration resulted in the weak bond between coarse aggregates and cement mortar.

After 12h curing, the mode-II failure occurred in concrete specimens, which is evidently different from mode-I failure. In the case of uniaxial compression, the constraint caused by the friction was reduced since in this test the friction reducing treatment between the loading plate and the specimen surface was adopted, therefore the concrete exhibited typical columnar failure. The cracks, which were perpendicular to the loading surface, propagated across the cube and divided a concrete cube into several independent columns (See Fig. 3(a)). However, the scenario is different in the case of biaxial compression. The load in a certain direction restrained the development of cracks, which were parallel to the loading surfaces and caused by the load in the other direction. Therefore, there were several cracking surfaces parallel to the unloaded surfaces, resulting in damage caused by flaking (See Fig. 3(b)). It should be noted that there are usually different angles between cracking surfaces and non-load surfaces because the internal coarse aggregates prevent crack propagation. Fig. 3 (c) presents the crack details for the concrete at the age of 28 days and shows that some cracks can propagate across the coarse aggregates.

3.2 Effect of Concrete Strength on β

Figs. 4, 5 and 6 illustrate the relationships of f_c, f_{bc} and β with curing age, respectively, for different concrete grades C30, C40 and C50 with various maximum aggregate sizes of 10, 20 and 30 mm. It can be seen from these figures that both the uniaxial and equal biaxial strengths increased with the increase of concrete strength grade. The relationships of the uniaxial and equal biaxial strengths with curing age approximately conform to a logarithmic law. At early ages of hydration, i.e. within 7 days after casting, the uniaxial and equal biaxial strengths increased significantly. Later, both the uniaxial and equal biaxial strengths showed a slow rise to 28 days. Taking the grade C30 concrete with the maximum aggregate size of 20 mm as an example, the uniaxial and equal biaxial strengths were 21.43 MPa and 24.43 MPa, respectively at the age of 7 days. When the age increased to 28 days, their strengths reached 26.17 MPa and 29.90 MPa, representing increases of 22.12% and 22.39%, respectively.

For the variation of β, it can be seen from Figs. 4, 5 and 6 that β decreased with the increase of age. Within 7 days after casting, β decreased dramatically. Later, this value remains almost constant until 28 days. It should be noted that due to the short hydration time, the uniaxial and equal biaxial strengths at the age of 6h showed high discreteness, which results in the high discreteness of β. Except for the points of β at the age of 6h, the remaining data points on the β curves for C30, C40 and C50 concretes were almost overlapping with respect to the same maximum aggregate size. Therefore, in general, the concrete strength has less effect on the variation of β.

3.3 Effect of the Maximum Aggregate Size on β

To study the effect of the maximum coarse aggregate size $d_{\max}$ on f_c, f_{bc} and β, the coarse aggregates with three maximum size of $d_{\max}$ =10 mm, 20 mm and 30 mm, which are widely used in concrete construction, were used for preparing concrete to conduct the analysis. Figs 7, 8, and 9 present f_c, f_{bc} and β with respect to curing age for C30, C40 and C50 concretes with various $d_{\max}$, respectively. It can be seen from these figures that there is no significant effect of $d_{\max}$ on f_c, f_{bc} and β for the three grades of concrete investigated in this study. According to the study on concrete with large coarse aggregate used in dam construction [13], both the uniaxial and biaxial strengths decrease when the maximum aggregate size increases from 40 mm to 80 mm. The decrease can be explained as following: in case that low strength concrete, such as grade C20 concrete, is employed for a dam structure, it is the weak bonding effect at the interface of the aggregates and mortar which determine the overall uniaxial and biaxial strengths of concrete. Meanwhile, more flaws exist at the interface for concrete with larger coarse aggregates. Therefore, the cracks may initiate at the interface and propagate through the interface, that is, the cracks usually bypass the large aggregates during the rupture process of dam concrete [17, 18]. However, for the concrete investigated in this study, i.e. in the case of $d_{\max}\leq 30$ mm, the homogeneity of concrete is better than the one with larger coarse aggregate. On the other hand, these normal strength concretes, i.e. C30, C40 and C50 in practical engineering, provide a better bonding effect than the low strength concrete used in dams. Therefore, the effect of d_{max} on f_c, f_{bc} and β is not significant as discovered in this study.

3.4 Effect of Concrete Age on β

According to previous discussion, the concrete strength and maximum coarse aggregate size have less effect on β when concrete grade ranges from C30 to C50, and d_{max} ranges from 10 to 30 mm. Therefore, based on the experimental results, the relationship of β with age can be obtained through regression analysis, not taking into account the effects of concrete strength and maximum coarse aggregate size. Figure 10 illustrates the values of β at various ages from the experiment. Correspondingly, an expression of β vs. age (t in days) for early age concrete is derived as Eq. (1). According to the fitted results, the value of β obviously decreases up to 7 days after concrete was cast. After that, β almost keeps constant until the age of 28 days, corresponding to a value of 1.15.

$$\beta = 1.38 - 0.07\ln(t-0.25) \quad (0.25 < t \leqslant 28, \text{ in days}) \tag{1}$$

4. Conclusions

In this study, uniaxial and equal biaxial compressive tests were carried out on the early age concrete to investigate the variation of equal biaxial-to-uniaxial compressive strength ratio β with respect to age. By studying normal-strength concrete commonly used in practical engineering, i.e. strength grade ranging from C30 to C50 and a maximum coarse aggregate size ranging from 10 mm to 30 mm, the effect of concrete strength, maximum coarse aggregate size and age on f_c, f_{bc}, and β were discussed. Meanwhile, the different failure modes of concretes with different strength grades under uniaxial and equal biaxial compression were analysed at various early ages from 6 hours up to 28 days. Based on the experimental study, the following conclusions can be drawn:

(1) The failure of the concrete younger than 7 days resulted from the weak bond between mortar and coarse aggregate. For the concrete older than 7 days, columnar damage occurred under uniaxial compression, while flaking damage occurred under equal biaxial compression.

(2) The concrete strength has less effect on the value of β. Meanwhile, the maximum coarse aggregate size $d_{\max}$ ranging from 10 to 30 mm had no effect on f_c, f_{bc} and β.

(3) The effect of concrete age on β is significant, particularly, at early ages. β noticeably decreased within 7 days after concrete was cast, approximately decreasing from 3.5 to 1.2. After that, β remained almost constant up to the age of 28 days, corresponding to a value of 1.15.

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APPENDIX Table1. Concrete mix proportions for different strength grades

Concrete	Maximum aggregate size (mm)	Water	Cement	Sand	Aggregate	Cement grade	f_c (MPa)
		(kg/m^3)					
C30	10	205	331	709	1110	R42.5	34.9
	20	205	331	691	1128	R42.5	37.5
	30	205	336	653	1161	R42.5	38.8
C40	10	220	500	501	1064	R42.5	52.7
	20	215	488	512	1140	R42.5	52.1
	30	210	477	530	1181	R42.5	51.5
C50	10	210	525	496	1054	R52.5	60.8
	20	205	513	507	1130	R52.5	64.5
	30	213	520	467	1200	R52.5	63.8

APPENDIX A1 Uniaxial compressive strength of C30 concrete at different ages

Age	d_{max} (mm)	f_c(MPa)			$f_{c,mean}$ (MPa)	Standard deviation	Coefficient of variation
		Cube1	Cube2	Cube3			
6h	10	0.98	0.83	1.51	1.11	0.36	32.28%
	20	0.81	0.82	0.94	0.86	0.07	8.44%
	30	1.32	1.15	0.98	1.15	0.17	14.78%
12h	10	4.21	4.36	5.19	4.58	0.53	11.51%
	20	2.18	2.57	2.58	2.44	0.23	9.34%
	30	3.05	3.57	3.62	3.41	0.32	9.25%
1d	10	6.64	6.49	6.23	6.45	0.21	3.21%
	20	6.83	6.87	7.78	7.16	0.54	7.50%
	30	6.27	6.41	5.89	6.19	0.27	4.35%
3d	10	17.58	17.15	18.17	17.63	0.51	2.90%
	20	15.76	13.37	15.97	15.03	1.44	9.61%
	30	15.19	15.03	16.58	15.60	0.85	5.46%
7d	10	18.23	16.75	——	17.49	1.05	5.98%
	20	21.10	19.87	23.33	21.43	1.75	8.18%
	30	16.91	15.78	16.70	16.46	0.60	3.65%
14d	10	21.12	18.11	20.35	19.86	1.56	7.87%
	20	22.17	19.34	23.47	21.66	2.11	9.75%
	30	17.97	20.48	20.20	19.55	1.38	7.04%
28d	10	22.72	25.08	24.30	24.03	1.20	5.00%
	20	26.00	27.85	24.65	26.17	1.61	6.14%
	30	24.42	24.38	——	24.40	0.03	0.12%

APPENDIX A2 Equal biaxial compressive strength and equal biaxial-to-uniaxial compressive strength ratio of C30 concrete at different ages

Age	d_{max} (mm)	f_{bc}(MPa)			$f_{bc,mean}$ (MPa)	Standard deviation	Coefficient of variation	$f_{bc,mean}/f_{c,mean}$
		Cube1	Cube2	Cube3				
6h	10	3.48	3.57	3.88	3.64	0.21	5.76%	3.29
	20	3.64	3.73	3.64	3.67	0.05	1.42%	4.28
	30	4.33	4.68	3.10	4.04	0.83	20.56%	3.51
12h	10	6.14	6.54	7.3	6.66	0.59	8.85%	1.45
	20	6.15	5.23	5.25	5.54	0.53	9.48%	2.27
	30	6.96	6.95	6.72	6.88	0.14	1.97%	2.01
1d	10	8.32	9.54	——	8.93	0.86	9.66%	1.38
	20	9.73	9.79	10.11	9.88	0.20	2.07%	1.38
	30	9.25	10.24	10.15	9.88	0.55	5.54%	1.60
3d	10	17.30	16.76	17.44	17.17	0.36	2.09%	0.97
	20	19.20	17.11	18.19	18.17	1.05	5.75%	1.21
	30	17.70	18.32	18.28	18.10	0.35	1.92%	1.16
7d	10	22.10	23.34	25.07	23.50	1.49	6.35%	1.34
	20	23.14	24.52	25.64	24.43	1.25	5.13%	1.14
	30	21.74	22.43	21.75	21.97	0.40	1.80%	1.33
14d	10	20.35	24.25	23.75	22.78	2.12	9.31%	1.15
	20	21.27	26.01	26.74	24.67	2.97	12.04%	1.14
	30	25.26	25.64	24.74	25.21	0.45	1.79%	1.29
28d	10	27.33	27.72	25.65	26.90	1.10	4.09%	1.12
	20	30.14	27.70	31.86	29.90	2.09	6.99%	1.14
	30	26.77	30.41	28.72	28.63	1.82	6.36%	1.17

APPENDIX B1 Uniaxial compressive strength of C40 concrete at different ages

Age	d_{max} (mm)	f_c(MPa)			$f_{c,mean}$ (MPa)	Standard deviation	Coefficient of variation
		Cube1	Cube2	Cube3			
6h	10	2.01	1.92	2.70	2.21	0.43	19.31%
	20	1.75	2.52	1.88	2.05	0.41	20.11%
	30	2.64	2.92	2.59	2.72	0.18	6.55%
12h	10	15.23	11.78	11.84	12.95	1.97	15.25%
	20	13.05	11.44	12.86	12.45	0.88	7.07%
	30	14.12	12.26	14.48	13.62	1.19	8.75%
1d	10	19.22	20.11	15.35	18.23	2.53	13.88%
	20	13.24	15.12	13.82	14.06	0.96	6.85%
	30	17.08	16.05	17.13	16.75	0.61	3.64%
3d	10	23.48	25.10	20.21	22.93	2.49	10.86%
	20	21.48	21.53	19.59	20.87	1.11	5.30%
	30	20.41	21.13	19.46	20.33	0.84	4.12%
7d	10	20.55	20.76	21.04	20.78	0.25	1.18%
	20	32.47	29.69	29.54	30.57	1.65	5.40%
	30	25.41	27.02	28.07	26.83	1.34	4.99%
14d	10	27.30	24.51	25.91	25.91	1.40	5.38%
	20	30.42	29.58	——	30.00	0.59	1.98%
	30	30.78	30.43	31.39	30.87	0.49	1.57%
28d	10	30.91	32.79	——	31.85	1.33	4.17%
	20	32.12	32.22	34.16	32.83	1.15	3.50%
	30	33.50	32.47	35.23	33.73	1.39	4.13%

APPENDIX B2 Equal biaxial compressive strength and equal biaxial-to-uniaxial compressive strength ratio of C40 concrete at different ages

Age	d_{max} (mm)	f_{bc} (MPa)			$f_{bc,mean}$ (MPa)	Standard deviation	Coefficient of variation	$f_{bc,mean}/f_{c,mean}$
		Cube1	Cube2	Cube3				
6h	10	3.48	3.57	3.88	4.60	0.39	8.46%	2.08
	20	3.64	3.73	3.64	4.90	0.64	13.02%	2.39
	30	4.33	4.68	3.10	5.76	0.36	6.19%	2.12
12h	10	6.14	6.54	7.3	15.17	0.54	3.54%	1.17
	20	6.15	5.23	5.25	15.55	1.08	6.93%	1.25
	30	6.96	6.95	6.72	16.38	0.42	2.57%	1.20
1d	10	8.32	9.54	——	20.30	0.49	2.40%	1.11
	20	9.73	9.79	10.11	17.20	2.90	16.84%	1.22
	30	9.25	10.24	10.15	19.11	0.34	1.78%	1.14
3d	10	17.30	16.76	17.44	24.60	1.29	5.25%	1.07
	20	19.20	17.11	18.19	27.07	0.78	2.87%	1.30
	30	17.70	18.32	18.28	23.00	1.34	5.82%	1.13
7d	10	22.10	23.34	25.07	30.55	0.83	2.71%	1.47
	20	23.14	24.52	25.64	36.17	1.09	3.00%	1.18
	30	21.74	22.43	21.75	33.30	1.97	5.91%	1.24
14d	10	20.35	24.25	23.75	28.67	5.88	20.50%	1.11
	20	21.27	26.01	26.74	35.96	1.77	4.91%	1.20
	30	25.26	25.64	24.74	36.30	1.33	3.66%	1.18
28d	10	27.33	27.72	25.65	38.83	1.14	2.95%	1.22
	20	30.14	27.70	31.86	36.83	2.62	7.11%	1.12
	30	26.77	30.41	28.72	37.03	2.25	6.06%	1.10

APPENDIX C1 Uniaxial compressive strength of C50 concrete at different ages

Age	d_{max} (mm)	f_c(MPa)			$f_{c,mean}$ (MPa)	Standard deviation	Coefficient of variation
		Cube1	Cube2	Cube3			
6h	10	0.58	0.64	0.67	0.63	0.05	19.31%
	20	1.92	2.06	2.14	2.04	0.11	20.11%
	30	1.49	1.56	——	1.53	0.05	6.55%
12h	10	3.21	3.33	3.26	3.27	0.06	15.25%
	20	8.43	7.52	8.1	8.02	0.46	7.07%
	30	6.95	7.17	7.19	7.10	0.13	8.75%
1d	10	16.24	17.05	15.21	16.17	0.92	13.88%
	20	14.29	15.48	14.58	14.78	0.62	6.85%
	30	16.18	16.68	17.97	16.94	0.92	3.64%
3d	10	31.67	29.53	30.00	30.40	1.12	10.86%
	20	28.13	28.29	28.28	28.23	0.09	5.30%
	30	30.51	32.45	29.42	30.79	1.53	4.12%
7d	10	27.81	29.27	24.62	27.23	2.38	1.18%
	20	32.80	31.14	30.25	31.40	1.29	5.40%
	30	32.11	33.83	32.05	32.66	1.01	4.99%
14d	10	36.02	37.65	37.32	37.00	0.86	5.38%
	20	34.25	38.17	31.56	34.66	3.32	1.98%
	30	33.81	37.12	35.35	35.43	1.66	1.57%
28d	10	36.80	37.14	39.75	37.90	1.61	4.17%
	20	36.22	38.14	37.55	37.30	0.98	3.50%
	30	37.36	41.61	40.82	39.93	2.26	4.13%

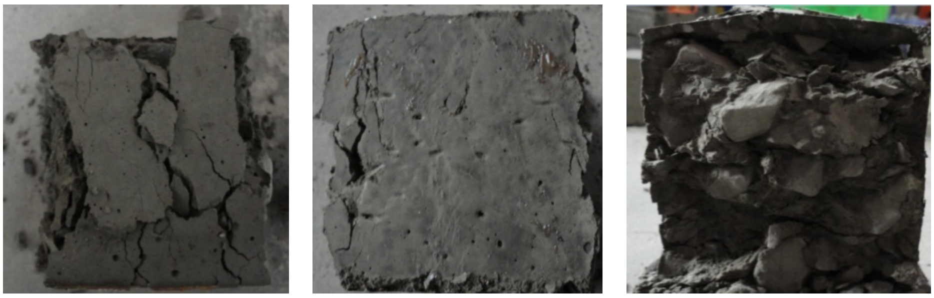
APPENDIX C2 Equal biaxial compressive strength and equal biaxial-to-uniaxial compressive strength ratio of C50 concrete at different ages

Age	d_{max} (mm)	f_{bc} (MPa)			$f_{bc,mean}$ (MPa)	Standard deviation	Coefficient of variation	$f_{bc,mean}/f_{c,mean}$
		Cube1	Cube2	Cube3				
6h	10	2.94	2.80	2.36	2.70	0.30	11.21%	4.29
	20	4.28	5.36	6.04	5.23	0.89	16.98%	2.56
	30	4.46	5.23	5.09	4.93	0.41	8.33%	3.23
12h	10	6.44	6.16	6.41	6.34	0.15	2.43%	1.94
	20	11.92	12.03	12.23	12.06	0.16	1.30%	1.50
	30	12.60	12.27	——	12.44	0.23	1.88%	1.75
1d	10	21.09	21.81	20.60	21.17	0.61	2.88%	1.31
	20	18.26	21.26	19.58	19.70	1.50	7.63%	1.33
	30	21.51	23.53	——	22.52	1.43	6.34%	1.33
3d	10	36.31	37.14	36.05	36.50	0.57	1.56%	1.20
	20	32.50	30.23	33.27	32.00	1.58	4.94%	1.13
	30	36.21	35.63	33.64	35.16	1.35	3.83%	1.14
7d	10	37.32	34.17	36.74	36.08	1.68	4.65%	1.32
	20	39.72	40.76	38.11	39.53	1.34	3.38%	1.26
	30	36.50	39.17	38.71	38.13	1.43	3.74%	1.17
14d	10	42.31	42.76	46.14	43.74	2.09	4.79%	1.18
	20	44.51	40.98	45.32	43.60	2.31	5.29%	1.26
	30	38.97	40.91	41.41	40.43	1.29	3.19%	1.14
28d	10	42.17	47.83	48.62	46.21	3.52	7.61%	1.22
	20	47.93	45.57	42.00	45.17	2.99	6.61%	1.21
	30	45.31	49.49	49.53	48.11	2.42	5.04%	1.20

APPENDIX Figures

(a) (b)

Fig. 1. Testing apparatus: (a) tri-axial test machine; (b) test set up



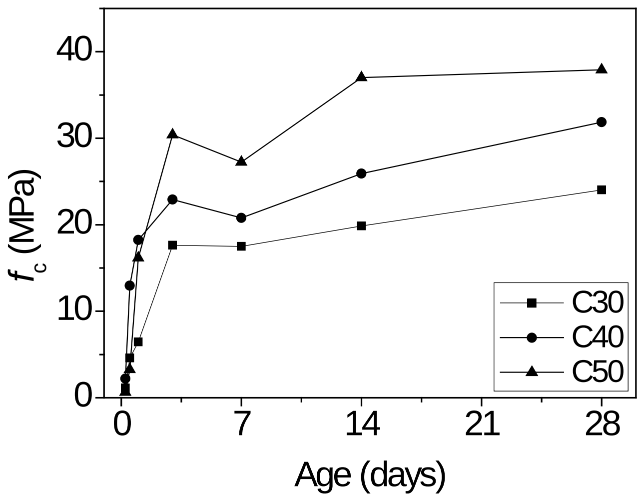
(a) (b) (c)

Fig. 2. Failure mode-I of early age concrete: (a) uniaxial compression; (b) equal biaxial compression; (c) internal feature at the age of 12h

(a) (b) (c)

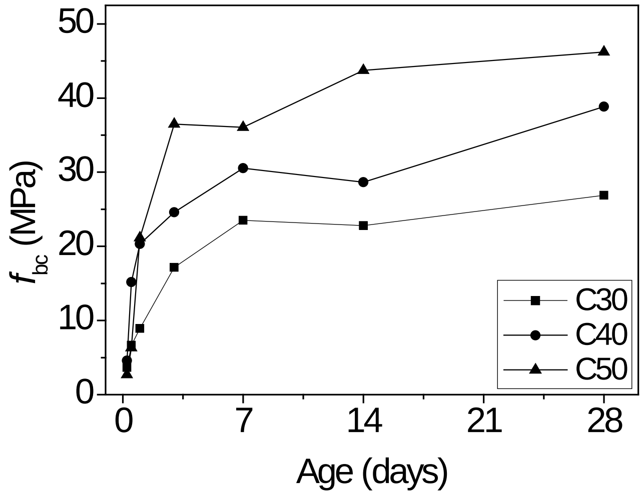
Fig. 3. Mode-II failure of early age concrete: (a) uniaxial compression; (b) equal biaxial compression; (c) crack feature at the age of 28 days

402



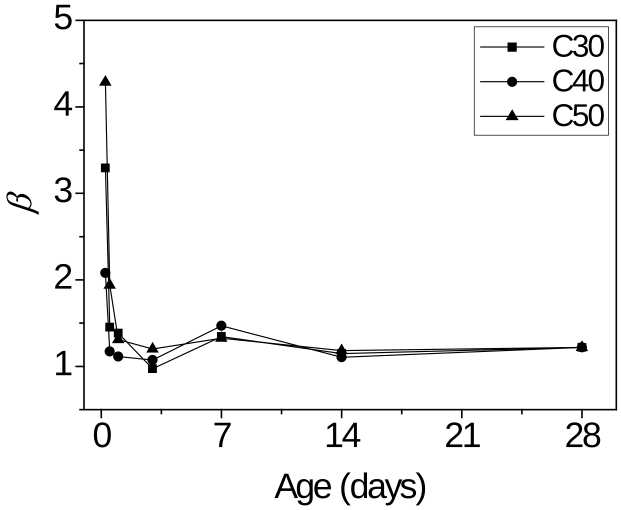
403

404 (a)



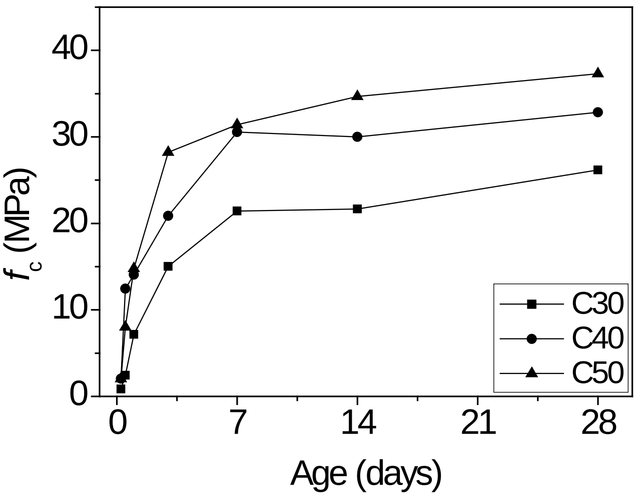
405

406 (b)

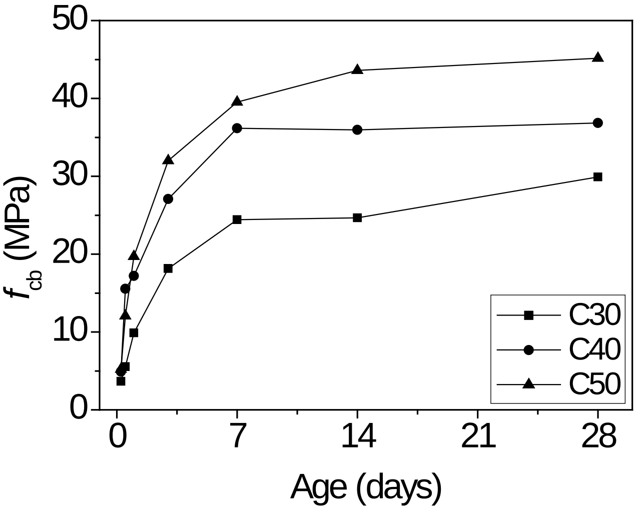


(c)

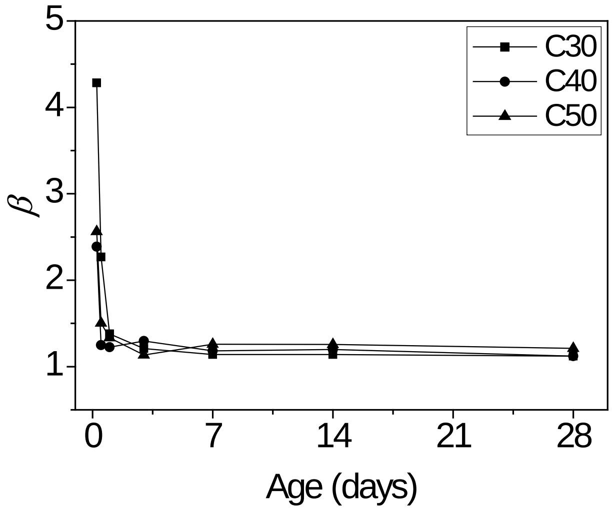
Fig. 4. Effect of concrete strength grade on (a) f_c; (b) f_{bc}; and (c) β with the maximum aggregate size of d_{max} = 10 mm


f_c (MPa)
0
10
20
30
40
0
7
14
21
28
Age (days)
C30
C40
C50

(a)

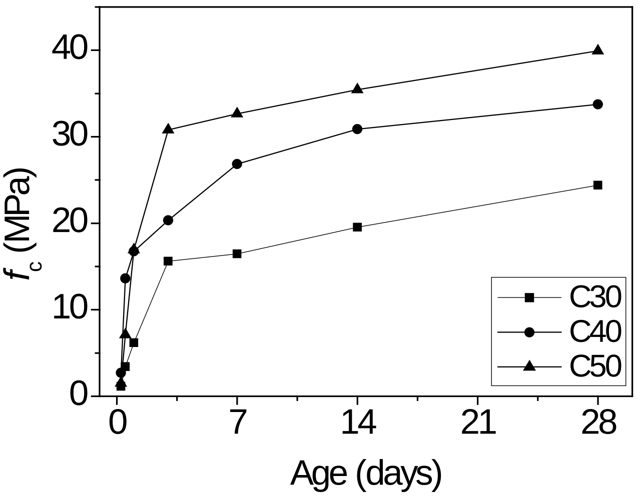

f_{cb} (MPa)
0
10
20
30
40
50
0
7
14
21
28
Age (days)
C30
C40
C50

(b)

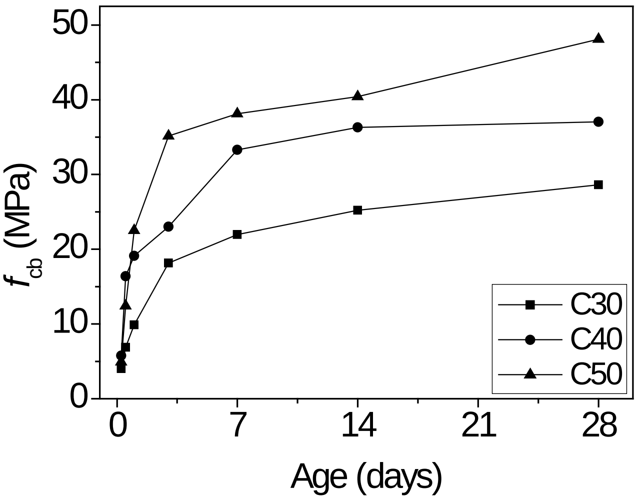


(c)

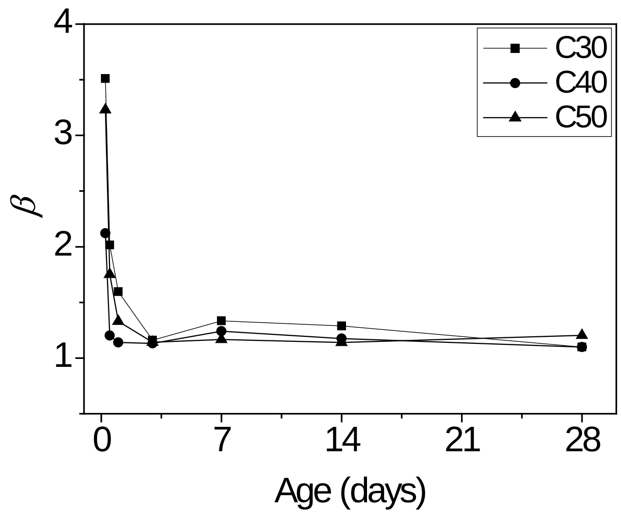
Fig. 5. Effect of concrete strength grade on (a) f_c; (b) f_{bc}; and (c) β with the maximum aggregate size of d_{max} = 20 mm


f_c (MPa)
0
10
20
30
40
0
7
14
21
28
Age (days)
C30
C40
C50

(a)

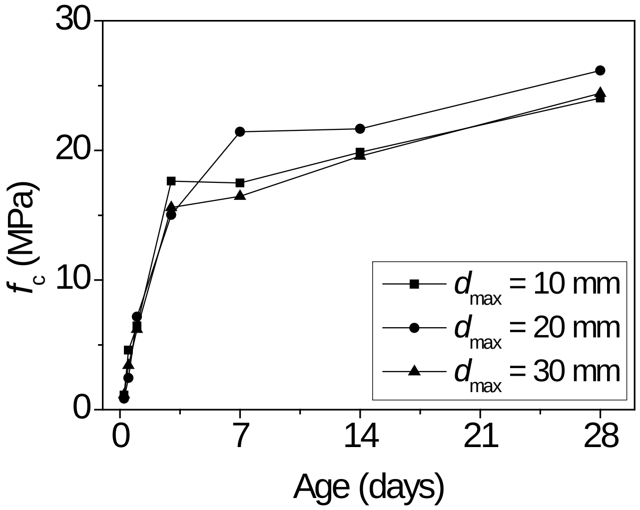

f_{cb} (MPa)
0
10
20
30
40
50
0
7
14
21
28
Age (days)
C30
C40
C50

(b)

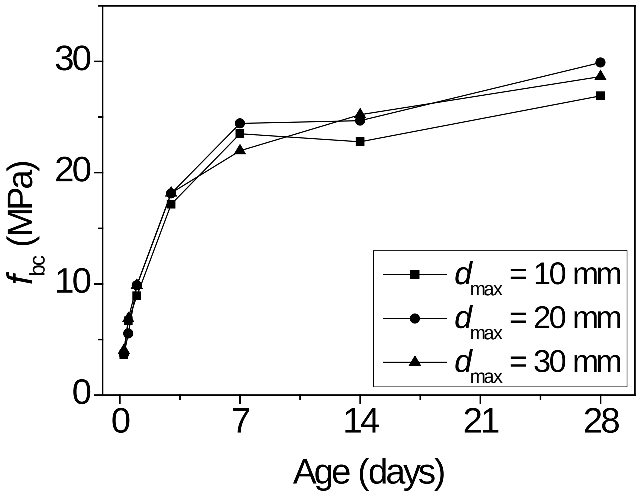


(c)

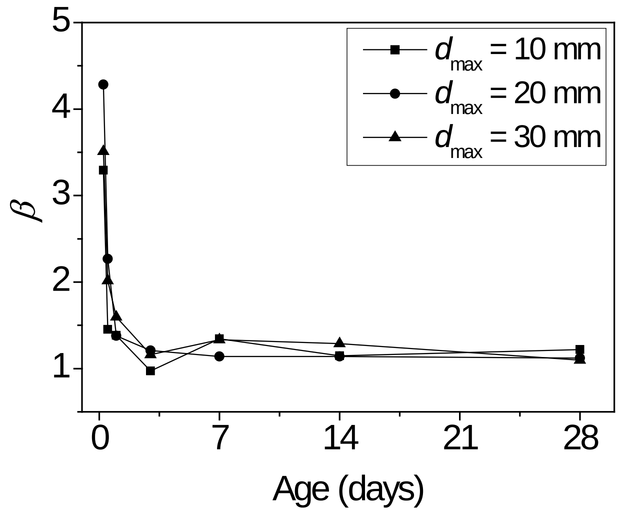
Fig. 6. Effect of concrete strength on (a) f_c; (b) f_{bc}; and (c) β with the maximum aggregate size of d_{max} = 30 mm


f_c (MPa)
0
10
20
30
0
7
14
21
28
Age (days)
d_{max} = 10 mm
d_{max} = 20 mm
d_{max} = 30 mm

(a)

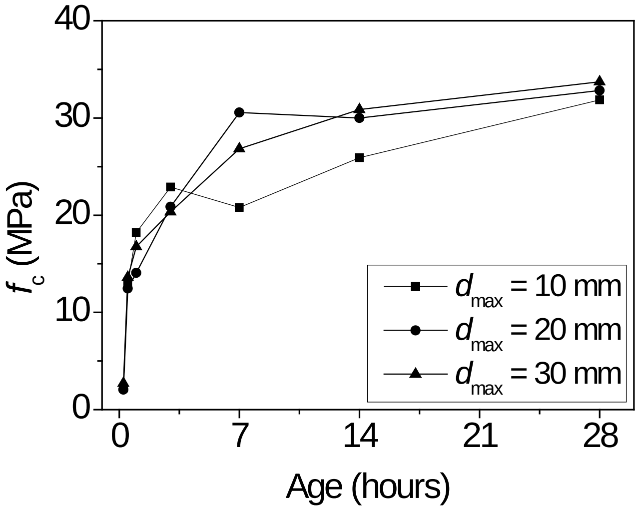

f_{bc} (MPa)
0
10
20
30
0
7
14
21
28
Age (days)
d_{max} = 10 mm
d_{max} = 20 mm
d_{max} = 30 mm

(b)

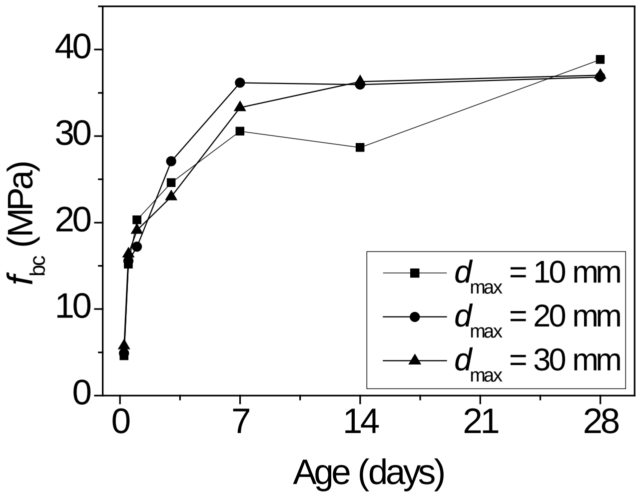


(c)

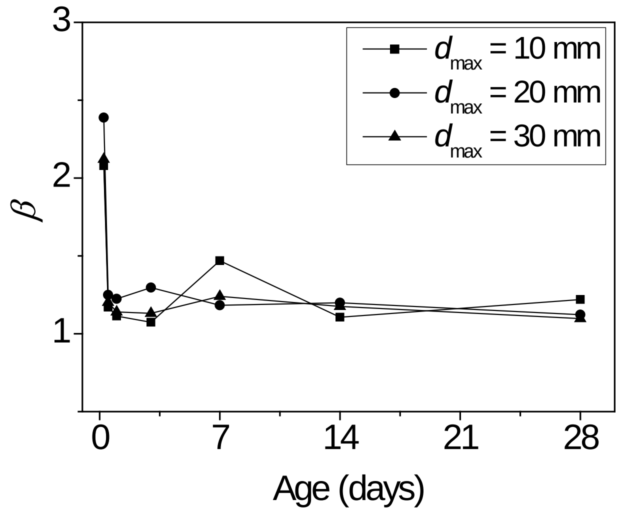
Fig. 7. Effect of $d_{\max}$ on (a) f_c; (b) f_{bc}; and (c) β for C30 concrete


40
30
20
10
0
f_c (MPa)
0
7
14
21
28
Age (hours)
d_{max} = 10 mm
d_{max} = 20 mm
d_{max} = 30 mm

(a)

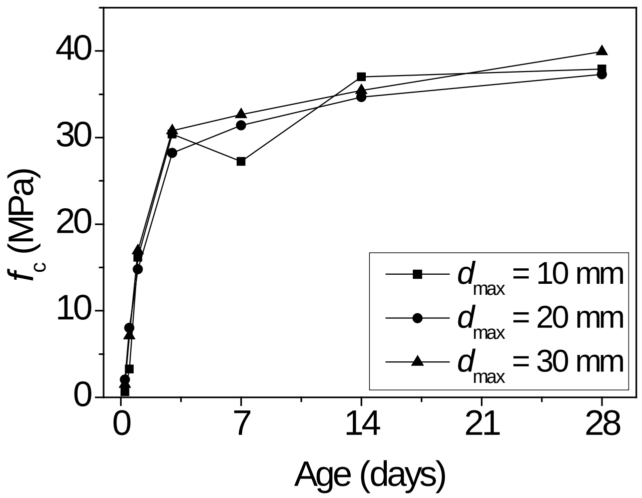

40
30
20
10
0
f_{bc} (MPa)
0
7
14
21
28
Age (days)
d_{max} = 10 mm
d_{max} = 20 mm
d_{max} = 30 mm

(b)

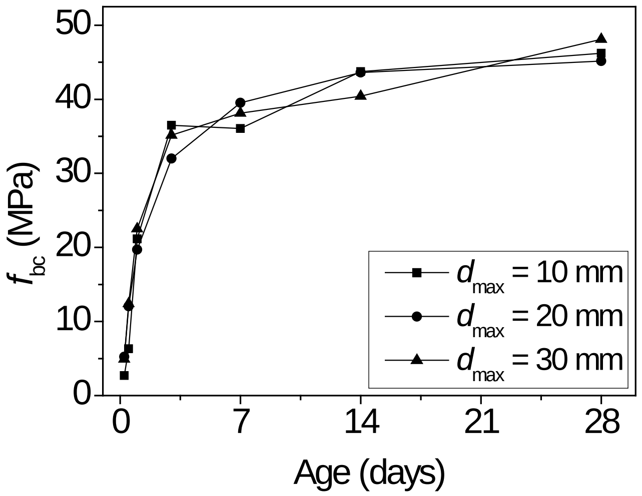


(c)

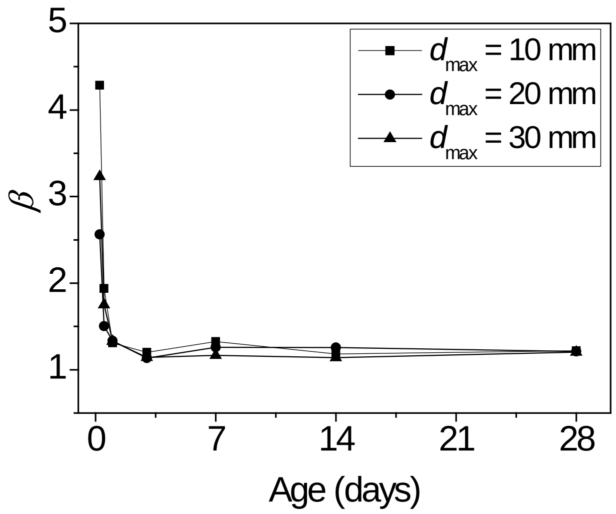
Fig. 8. Effect of d_{max} on (a) f_c; (b) f_{bc}; and (c) β for C40 concrete


f_c (MPa)
0
10
20
30
40
0
7
14
21
28
Age (days)
d_{max} = 10 mm
d_{max} = 20 mm
d_{max} = 30 mm

(a)


f_{bc} (MPa)
0
10
20
30
40
50
0
7
14
21
28
Age (days)
d_{max} = 10 mm
d_{max} = 20 mm
d_{max} = 30 mm

(b)



(c)

Fig. 9. Effect of d_{max} on (a) f_c; (b) f_{bc}; and (c) β for C50 concrete

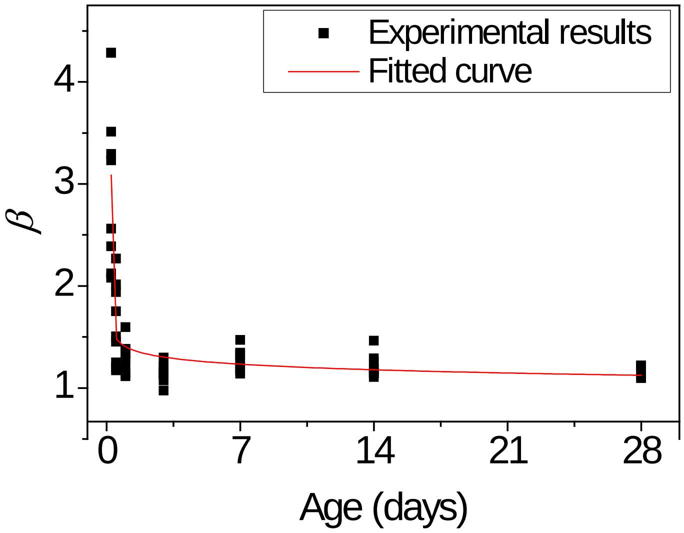


Fig. 10. Fitted curve of *β* based on experimental results