Research Article

Applied Stochastic Models in Business and Industry

Received XXXX

(www.interscience.wiley.com) DOI: 10.1002/sim.0000
MOS subject classification: XXX; XXX

Nonparametric Conditional Autoregressive Expectile model via Neural Network with applications to estimating financial risk

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The parametric conditional autoregressive expectiles (CARE) models have been developed by [1] to estimate expectiles that can be used to assess value at risk (VaR) and expected shortfall (ES). The challenge lies in parametric CARE modeling is specification of a parametric form. To avoid any model misspecification, we propose a nonparametric CARE model via neural network. The nonparametric CARE model can be estimated by a classical gradient based nonlinear optimization algorithm. We then apply the nonparametric CARE model to estimating VaR and ES of six stock indices. Empirical results for the new model is competitive with those classical models and parametric CARE models. Copyright © 2015 John Wiley & Sons, Ltd.

Keywords: Expectiles; Quantile; Neural network; Nonparametric conditional autoregressive expectiles; Value at risk; Expected shortfall

1. Introduction

An accurate risk measure is crucial for portfolio and asset pricing in financial risk management. Value at risk (VaR) has become the standard measure of financial market risk for its simplicity and accuracy. According to [2], VaR is defined as the maximum potential loss on the portfolio over a prescribed holding period with a confidence level $100 \times (1-\theta)\%$. Therefore, assessing VaR amounts to estimating negative tail quantiles of the underlying return distribution, i.e. $VaR(1-\theta) = -Q(\theta)$. Based on this intuition, Engle and Manganelli [3] advanced the conditional autoregressive value at risk (CAViaR) class of models to give an accurate VaR estimation. VaR, however, is not a coherent risk measure proposed by [4] for the lack of subadditivity and convexity. As an alternative risk measure, Expected shortfall (ES) is known to have better properties than VaR and is coherent, see [5]. ES is defined to be the conditional expectation of the loss given that it exceeds the VaR, which can be estimated by expectiles. A new class of univariate expectile models: conditional autoregressive expectiles (CARE), an analogue of CAViaR, introduced by [1] to estimate time varying or conditional ES. No matter what CAViaR or CARE modeling is, the specification of the functional form is arbitrary and certain models are challenging to find an appropriate one for giving an accurate financial risk measure. For example, there are four CAViaR models in [3]: adaptive CAViaR, symmetric absolute value CAViaR, asymmetric slope CAViaR, and indirect GARCH(1,1) CAViaR and its counterparts in CARE models of [1], two asymmetric slope CARE models with high order lag terms in [6]. [6] proposed a downside risk measure, the expectile-based Value at Risk (EVaR), a more sensitive measure of the magnitude of extreme losses than the conventional quantile-based VaR (QVaR). moreover extend the EVaR to conditional EVaR and propose various CARE specifications as well as establishing the asymptotic results of [7] to allow for stationary and weakly dependent data.

Most applications of quantile regression or expectile regression to predict financial risk have relied on linear or simple parametric nonlinear models, see [8, 9]. While both complexity of dependency and misspecification of functional form need to address, artificial neural network (ANN) and support vector machine (SVM) are often used to discover a complex nonlinear dependence between the inputs and outputs, see [10] for more details. White [11] derived the consistency of nonparametric conditional quantile estimators based on ANN. Taylor [12] extended ANN-based linear quantile regression to a quantile regression neural

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network (QRNN). Cannon [13] improved the QRNN model by using the Huber norm to approximate the tilted absolute value error function which is not differentiable at 0 and developed the 'qrnn' package in the R programming language. However, these existing work focuses solely on quantile estimation.

While conditional expectile is an alternative and powerful tool for risk measurement, in this paper we propose a new model, named nonparametric conditional autoregressive expectiles (NCARE) via neural network. Without specifying the functional form, the NCARE model can be used to estimate the potential nonlinear dynamics in expectiles, which provides the basis for conditional ES estimation. It is worth to note that our NCARE method is different from [14]. Although they have proposed a nonparametric approach to model EVaR, the expectiles are only modeled using covariates rather than the lagged predicted expectiles. In contrast, we consider the first order lag of prediction expectile and propose a flexible nonparametric CARE model, which not only overcome the withdraw that exogenous economic and investment related factors are being ignored, but also account for the complexity and potential nonlinearity hidden in real-world data. As an illustration, we apply the proposed NCARE model to assess the ES of six stock indices. The empirical results show that the NCARE model is quite flexible in describing dynamics of various financial time series and generally outperforms some classical models and the parametric CARE models in terms of the accuracy in VaR and ES measure.

The paper is organized as follows. We briefly review the estimation methods of VaR and ES in Section 2. We present NCARE model specification, model estimation and model selection in Section 3. The empirical applications are reported in Section 4. Section 5 makes a summary and concluding comments.

2. Methods for estimating financial risk

In this section we review some classical methods of estimating financial risk mainly on VaR and ES.

2.1. RiskMetrics model

RiskMetrics methodology to VaR calculation developed by [15] has been widely used in financial risk management. Let y_t denote a portfolio return with the distribution function $F_Y(y) = P(Y \le y)$. According to the definition of VaR, the VaR of y_t with the confidence level $100 \times (1-\theta)\%$ is the negative of the θ-th quantile of F_Y: $VaR_Y(1-\theta) = -Q_Y(\theta)$. In application, RiskMetrics assumes that y_t follows the conditional normal distribution $y_t|\mathcal{F}_{t-1} \sim N(\mu_t, \sigma_t^2)$ and can be described as follows

$$\begin{cases} y_t = \mu_t + \epsilon_t, \quad \epsilon_t = \sigma_t z_t \\ \sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda)\epsilon_{t-1}^2, \end{cases} \tag{1}$$

where $\mathcal{F}_{t-1}$ denotes the information set available at time t, μ_t is the conditional mean, σ_t is the conditional variance and evolves over time according to the exponentially weighted moving average (EWMA) model with a weighting parameter λ, ϵ_i is a random disturbance term, and the residual sequence z_t is usually set to follow the standard normal distribution.

Under the assumptions in RiskMetrics, financial risk with the confidence level $100 \times (1-\theta)\%$ can be estimated by

$$\begin{cases} VaR_t(1-\theta) = -\mu_t - \sigma_t z(\theta) \\ ES_t(1-\theta) = -\mu_t - \sigma_t E[z|z < z(\theta)], \end{cases} \tag{2}$$

where $z_\theta = F_z^{-1}(\theta)$ is the θth quantile of the standard normal distribution.

2.2. GARCH-EVT model

The GARCH-EVT model proposed by [16] and [17] combines GARCH models to estimate the current volatility and EVT for estimating the tail of the innovation distribution of the GARCH model. The model has been widely used to estimate extreme financial risk, see [18].

As we know, the EWMA model is a special case of a generalized autoregressive conditional heteroscedasticity model proposed by [19] with the GARCH(1,1) form

$$\begin{cases} y_t = \mu_t + \epsilon_t, \quad \epsilon_t = \sigma_t z_t, \\ \sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{cases} \tag{3}$$

where $z_t \sim iid.N(0,1)$ and $\omega, \alpha_1, \beta_1$ are parameters underestimate. We set the conditions on parameters: $\omega > 0, \alpha_1 > 0, \beta_1 > 0$ and $\alpha_1 + \beta_1 < 1$, to ensure strong positivity and stationarity of the conditional variance.

If F represents the distribution function of the residual series z_t, the conditional excess distribution function in [20] can be obtained as follows

$$F_u(y) = \Pr(z - u \le y | z > u) = \frac{F(z) - F(u)}{1 - F(u)}, \tag{4}$$

where u is a given threshold, $0 \le y < z_F - u$, $z_F < \infty$ is the right endpoint of F and $y = z - u$. For a large class of underlying distribution functions F, the conditional excess distribution function $F_u(y)$, for u is large, is well approximated by the generalized

Pareto distribution (GPD)

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\sigma}\right)^{-1/\xi}, & \text{if } \xi \neq 0 \\ 1 - \exp^{-y/\sigma}, & \text{if } \xi = 0, \end{cases} \tag{5}$$

where ξ and σ are called the shape and scale parameters, respectively. From equation (4), we have

$$F(z) = (1 - F(u))\, F_u(y) + F(u). \tag{6}$$

If we use the random proportion of the data $(n - n_u)/n$ to estimate $F(u)$ and use $G_{\xi,\sigma}(y)$ to approximate $F_u(y)$, we get the tail estimator

$$F(z) = 1 - \frac{n_u}{n}\left(1 + \frac{\xi}{\sigma}(z - u)\right)^{-1/\xi}, \tag{7}$$

for $z > u$. Here, n_u is the number of observations above u in all n observations.

The negative inverse of (7) with a probability θ gives the VaR

$$VaR_z(1-\theta) = -u - \frac{\sigma}{\xi}\left[\left(\frac{n}{n_u}(1-\theta)\right)^{-\xi} - 1\right] \tag{8}$$

For $\xi < 1$, using the relationship between the ES and VaR, we obtain

$$ES_z(1-\theta) = \frac{VaR_z(1-\theta)}{1-\xi} + \frac{\sigma - \xi u}{1-\xi} \tag{9}$$

Borrowing the idea of equation (2), the financial risk of y_t with the confidence level $100 \times (1-\theta)\%$ can be further estimated by

$$\begin{cases} VaR_t(1-\theta) = -\mu_t - \sigma_t[-VaR_z(1-\theta)] \\ ES_t(1-\theta) = -\mu_t - \sigma_t[-ES_z(1-\theta)]. \end{cases} \tag{10}$$

2.3. GJR-GARCH model

The GJR-GARCH model of [21], which follows equation (3), is modified by the negative impact of leverage and analyzes positive and negative shocks on the conditional variance asymmetrically with

$$\sigma_t^2 = \omega + \sum_{i=1}^{p}\left(\alpha_i \epsilon_{t-1}^2 + \gamma_i I_{t-i}\epsilon_{t-1}^2\right) + \sum_{j=1}^{q}\beta_j \sigma_{t-j}^2, \tag{11}$$

Consequently, VaR and ES at time t can be calculated through equation (3), where volatility σ_t is estimated by using GJR-GARCH model (11).

2.4. Quantile and CAViaR models

It is well known that the θ-th quantile can be obtained by a simple optimization problem

$$Q_Y(\theta) = \arg\min_q E[\rho_\theta^{(Q)}(Y - q)], \tag{12}$$

where, $\rho_\theta^{(Q)}(u)$ is an asymmetric loss function defined as

$$\rho_\theta^{(Q)}(u) = (\theta - I(u < 0)) \cdot u, \tag{13}$$

and $I(\cdot)$ is the indicator function. Based on this formula, the θ-th quantile can be estimated through quantile regression method introduced by [22].

A recent proposed VaR method using quantile regression is the CAViaR models of [3]. The CAViaR models are mainly used to estimate conditional VaR based on the relationship: $VaR(1-\theta) = -Q(\theta)$. A generic CAViaR specification might be expressed by

$$Q_t(\theta) = \omega + \sum_{i=1}^{m}\alpha_i Q_{t-i}(\theta) + \sum_{j=1}^{n}\beta_j f(y_{t-j}), \tag{14}$$

where $Q_t(\theta)$ is the conditional θ-th quantile, $\omega, \alpha_i, \beta_j$ are parameters to be estimated, and f is a function of a finite number of lagged values of observations. The commonly used CAViaR models, include adaptive CAViaR, symmetric absolute value CAViaR, asymmetric slope CAViaR, and indirect GARCH(1,1) CAViaR, are respectively stated as follows

$$\text{CAViaR-1}: Q_t(\theta) = Q_{t-1}(\theta) + \alpha[\theta - I(y_{t-1} < Q_{t-1}(\theta))], \tag{15}$$

$$\text{CAViaR-2}: Q_t(\theta) = \omega + \alpha Q_{t-1}(\theta) + \beta|y_{t-1}|, \tag{16}$$

$$\text{CAViaR-3}: Q_t(\theta) = \omega + \alpha Q_{t-1}(\theta) + \beta_1(y_{t-1})^+ + \beta_2(y_{t-1})^-, \tag{17}$$

$$\text{CAViaR-4}: Q_t(\theta) = (1 - 2I(\theta < 0.5))(\omega + \alpha Q_{t-1}(\theta)^2 + \beta y_{t-1}^2)^{1/2}, \tag{18}$$

where $(y_{t-1})^+ = \max(y_{t-1}, 0)$ and $(y_{t-1})^- = -\min(y_{t-1}, 0)$.

2.5. Expectile and CARE models

The loss function $\rho_\theta^{(Q)}(u)$ defined by (13) is a piecewise linear function. [7] considered an asymmetric quadratic loss function

$$\rho_\tau^{(E)}(u) = |\tau - I(u < 0)| \cdot u^2, \tag{19}$$

which yields expectile by optimizing

$$Expectile_Y(\tau) = \arg\min_e E\left[\rho_\tau^{(E)}(Y - e)\right], \tag{20}$$

where $\tau \in [0, 1]$ determines the degree of asymmetry of the loss function. The τ-th expectile of Y can be estimated by using asymmetric least squares (ALS) regression, which is the asymmetric version of ordinary least squares method or the least squares analogue of quantile regression. It is interesting that there is a corresponding relationship between quantile and expectile. For each τ-th expectile, there is a corresponding θ-th quantile, though τ is typically not equal to θ. For any $\theta \in [0, 1]$, let $\tau(\theta)$ satisfy

$$Expectile_Y(\tau(\theta)) = Q_Y(\theta). \tag{21}$$

The one-to-one mapping from expectiles to quantiles via the relation between $\tau(\theta)$ and $Q_Y(\theta)$ in [23] is

$$\tau(\theta) = \frac{\theta \cdot Q(\theta) - \int_{-\infty}^{Q(\theta)} y dF(y)}{2E(Y) - 2\int_{-\infty}^{Q(\theta)} y dF(y) - (1 - 2\theta)Q(\theta)} \tag{22}$$

While it is possible for two assets under different returns distributions to have the same QVAR, their EVAR will likely be different due to the dependence of expectiles on $F(y)$ and the extreme values of Y ([6]). For our specific case, we let Y under Gaussian distribution, for all assets. In the procedure of optimizing, we use the mean optimal values of parameter τ, 1.493% and 0.182% obtained by [1] for $\theta = 5\%$ and 1% respectively, as our initial values of τ.

Expectiles can be directly used to estimate VaR by using the one-to-one mapping in (22) and $Q_Y(\theta) = Expectile_Y(\tau(\theta))$. It can also be used to estimate ES based on the link between expectiles and ES derived by [1]. The estimation method can be expressed as

$$ES(1 - \theta) = \left(1 + \frac{\tau}{(1 - 2\tau)\theta}\right) Expectile(\tau) - \frac{\tau}{(1 - 2\tau)\theta} E(Y). \tag{23}$$

With Y defined to be a residual term with mean 0, equation (23) reduces to the following

$$ES(1 - \theta) = \left(1 + \frac{\tau}{(1 - 2\tau)\theta}\right) Expectile(\tau). \tag{24}$$

Then, it is nature to estimate conditional ES by using the following expression

$$ES_t(1 - \theta) = \left(1 + \frac{\tau}{(1 - 2\tau)\theta}\right) Expectile_t(\tau). \tag{25}$$

Therefore, conditional expectiles do make sense for accurate estimation of conditional ES. Similar to the structure of the CAViaR models, four parametric conditional autoregressive expectiles models with their corresponding formula as follows are adaptive CARE, symmetric absolute value CARE, asymmetric slope CARE, and indirect CARE:

$$\text{CARE-1}: Expectile_t(\tau) = Expectile_{t-1}(\tau) + \alpha[\tau - I(y_{t-1} < Expectile_{t-1}(\tau))], \tag{26}$$

$$\text{CARE-2}: Expectile_t(\tau) = \omega + \alpha Expectile_{t-1}(\tau) + \beta|y_{t-1}|, \tag{27}$$

$$\text{CARE-3}: Expectile_t(\tau) = \omega + \alpha Expectile_{t-1}(\tau) + \beta_1(y_{t-1})^+ + \beta_2(y_{t-1})^-, \tag{28}$$

$$\text{CARE-4}: Expectile_t(\tau) = (1 - 2I(\tau < 0.5))(\omega + \alpha Expectile_{t-1}(\tau)^2 + \beta y_{t-1}^2)^{1/2}. \tag{29}$$

The differences between parametric CARE models and CAViaR models lie in that response variable is expectile at τ in parametric CARE models but is quantile at θ in CAViaR models. The performance and accuracy of theses parametric models in practical application lie in specification of the functional form. Except additional information and experience are provided, parametric model specification is usually an challenging issue in practice.

3. Nonparametric conditional autoregressive expectile model

While nonparametric approaches are useful for dealing with model misspecification, in this section we introduce nonparametric conditional autoregressive expectile (NCARE) model via neural network, which can be viewed as a nonparametric version of CARE model proposed by [1].

3.1. Model specification

From expression (20), the conditional expectiles can be estimated by minimizing the expectation of a quadratic loss fucntion. Thus, we may consider the sample counterpart and define its empirical loss as

$$EL(\tau) \equiv \frac{1}{T}\sum_{t=1}^{T}\rho_\tau^{(E)}(y_t - Expectile_t(\tau)), \tag{30}$$

where T is the sample size, and the loss function, $\rho_\tau^{(E)}$, is defined as in (19).

We then take nonparametric specification for the conditional expectile, $Expectile_t(\tau)$, in the NCARE model through the standard multilayer perceptron ANN. Inspired by the parametric CARE models, we use the predictors $|y_{t-1}|, Expectile_{t-1}$ as inputs and $Expectile_t$ as output, which is depicted in Figure 1. It is worth to note that our proposed NCARE model is an open framework and is flexible to use more lagged predictors, $|y_{t-1}|, \cdots, |y_{t-p}|; Expectile_{t-1}, \cdots, Expectile_{t-q}$, as inputs. It is certain that these additional explanatory variables can be added into the NCARE model to improve its predictive power. For contrast and simplicity, we just discuss the use of first order lag here.

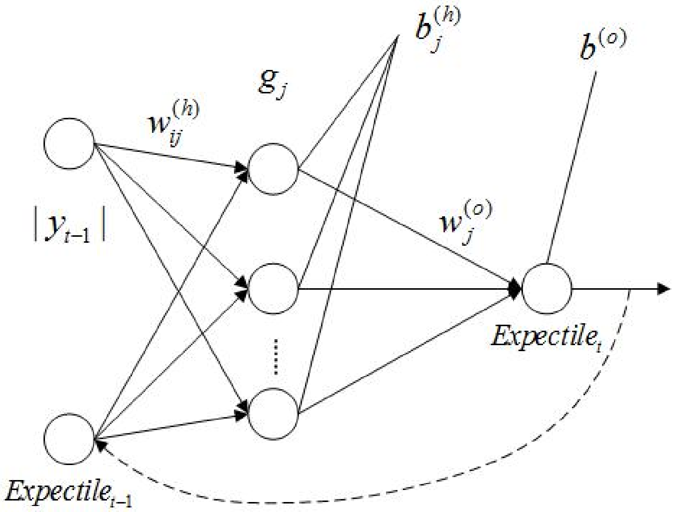


Figure 1. Schematic diagram showing a NCARE model with two predictors.

We first calculate j-th hidden layer node: $g_{j,t}$ as

$$g_{j,t} = f^{(h)}\left(w_{1j}^{(h)}|y_{t-1}| + w_{2j}^{(h)}Expectile_{t-1} + b_j^{(h)}\right), \tag{31}$$

where $f^{(h)}$ denotes the hidden layer transfer function, which is given by applying the hyperbolic tangent, a sigmoidal transfer function, $\boldsymbol{w}^{(h)} \equiv (w_{1j}^{(h)}, w_{2j}^{(h)})'$ denotes the hidden layer weight vector, and $\boldsymbol{b}^{(h)} \equiv (b_1^{(h)}, b_2^{(h)}, \cdots, b_J^{(h)})'$ denotes the hidden layer bias vector. We then calculate output layer: $Expectile_t$ by

$$Expectile_t(\boldsymbol{\gamma}) = f^{(o)}\left(\sum_{j=1}^{J} w_j^{(o)} g_{j,t} + b^{(o)}\right), \tag{32}$$

where $f^{(o)}$ is a transfer function in the output layer, which is often chosen as the identity function, $\boldsymbol{\gamma} \equiv [(\boldsymbol{w}^{(h)})', (\boldsymbol{w}^{(o)})', (\boldsymbol{b}^{(h)})', b^{(o)}]'$ is a vector of all parameters, $\boldsymbol{w}^{(o)} \equiv (w_1^{(o)}, w_2^{(o)}, \cdots, w_J^{(o)})'$ is a weight vector, and $b^{(o)}$ is a bias.

If the empirical loss in (30) is used to train the ANN in Figure 1, then outputs are estimates of the conditional expectiles. It is clear that the NCARE model is flexible to represent nonlinear predictor-response relationships without prior specification of the functional form.

3.2. Model estimation

Unlike the tilted absolute value loss function $\rho_\tau^{(Q)}$ in [22] and [24], the asymmetric quadratic loss function $\rho_\tau^{(E)}$ is differentiable everywhere for the reason is that the curve of $\rho_\tau^{(E)}$ is smoothed at each τ, see Figure 2. Therefore, weights and biases in the NCARE model can be estimated by a standard gradient based nonlinear optimization algorithm. Regarding the optimization routine, a quasi Newton BFGS algorithms is used by MATLAB functions 'fminsearch' and 'fminunc', while the loops to compute the recursive expectile functions are coded in C. The procedure for optimizing the NCARE model at each τ is as follows:

Step 1. Generate 10^4 vectors of parameters from uniform random number generator: $U[-0.5, 0.5]$.

Step 2. Compute the regression expectile (RE) function defined by (32) for each of these vectors and select 10 vectors that produce the lowest RE criterion as initial values for the following optimization routine.

Step 3. Feed each of these initial values to the quasi Newton algorithm with tolerance level set to $1e-10$ and choose the vector producing the lowest RE criterion as the final parameter vector.

Step 4. Obtain conditional expectiles through equations (31) and (32) with the final parameter vector.

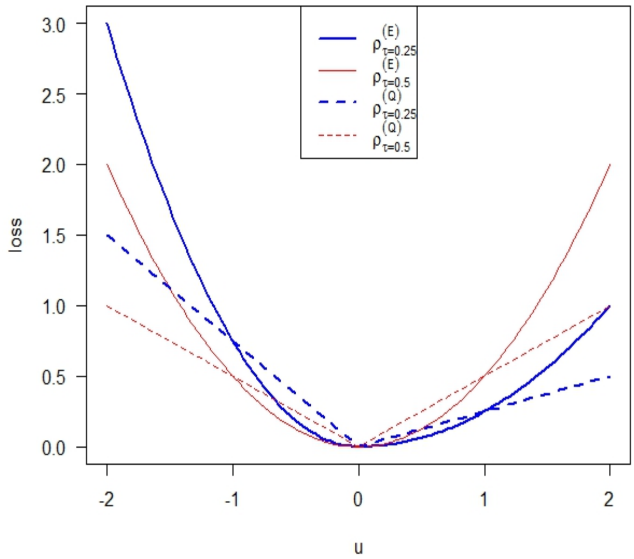


Figure 2. Asymmetric linear loss functions and asymmetric quadratics loss function at two different τ's.

3.3. Model selection

In our NCARE model, the number of hidden layer nodes J determines the overall complexity of the model. A model with large J is too complicated might overfit training data, and vice versa. An important issue in NCARE modeling is to find an appropriate J. To this end, we define the generalized approximate cross validation (GACV) criterion as

$$GACV(J) = \frac{\sum_{t=1}^{T} \rho_\tau^{(E)}(y_t - Expectile_t(\tau))}{T - df}, \tag{33}$$

where df is a measure of the effective dimensionality of the fitted model, and can be estimated by the SURE divergence formula $\sum_{i=1}^{n} \frac{\partial Expectile_t(\tau)}{\partial y_t}$ proposed in [25]. The optimal J^* at each τ is the J that minimizes the value of $GACV(J)$. The GACV criterion is found by [26] and [27] to be superior to the commonly used SIC (Schwarz information criterion) and AIC (Akaike information criterion) under regular conditions.

4. Empirical applications

To illustrate the performance of the proposed NCARE model, we conduct empirical application to estimating VaR and ES of six stock indices.

4.1. Data and backtesting methods

Our analysis uses daily observations of six stock indices: the US S&P500 Index (S&P500), the Financial Times and Stock Exchange 100 Index (FTSE100), the Japanese Nikkei225 Index (Nki225), the Taipei Weighted Index (TWI), the Hongkong Hang Seng Index (HSI) and the Shanghai Composite Index (SHCI) from 7 January 2002 to 25 January 2016. This interval delivered overall 3,076 log returns which are computed as 100 times the first difference of the log transformation of the index, i.e. $r_t = 100 \times (\ln p_t - \ln p_{t-1})$. In order to demonstrate the robustness of our proposed NCARE method, we apply the method under different selected sample intervals, such as including the financial crisis period or not. In our particular case, we cut the whole sample with two cut points, 4 January 2007 and 4 January 2011, and obtain three different selected sample intervals instead. We choose the sample interval in which these indices are relatively more volatile so as to help to illustrate the usefulness of our method. Table 1 collects the summary statistics of these daily log returns in each sample interval. Most of these returns are negative skewed except for the return of SHCI and GSPC in 2002-2006 and the return of HSI in 2007-2010. The distributions of each return in different sample intervals are significant distinguished, which shows the necessity of robustness check, for example, SHCI got a right-skewed and fat-tailed distribution for its positve skewnwss and higher kurtosi than other returns in the first sample interval 2002-2006, however, it yielded a negative skewness and the lowest kurtosi in 2007-2010.

Table 1. Summary statistics of stock indices log returns in each sample interval.

Sample interval	Stock indices	Number	Mean	Median	Min	Max	Std. Dev.	Skew.	Kurt.
2002-2006	SHCI	1109	3.843	-2.842	0.014	0.329	0.574	0.702	4.947
	TWI	1109	2.382	-3.002	0.006	0.319	0.565	-0.137	2.604
	S&P500	1109	2.421	-1.842	-0.001	0.198	0.445	0.198	3.098
	HSI	1109	1.759	-1.817	0.004	0.189	0.435	-0.064	1.034
	FTSE100	1109	2.564	-2.427	-0.010	0.223	0.472	-0.206	5.201
	Nki225	1109	2.491	-2.270	0.008	0.318	0.564	-0.192	0.903
2007-2010	SHCI	882	3.924	-4.020	-0.016	0.895	0.946	-0.395	1.960
	TWI	882	2.834	-2.925	0.002	0.470	0.686	-0.411	2.203
	S&P500	882	4.450	-4.113	-0.020	0.551	0.742	-0.351	5.495
	HSI	882	5.823	-5.899	0.006	0.856	0.925	0.084	6.391
	FTSE100	882	3.678	-3.552	-0.011	0.441	0.664	-0.117	4.479
	Nki225	882	5.748	-5.260	-0.026	0.705	0.840	-0.220	6.873
2011-2015	SHCI	1085	2.434	-3.853	0.006	0.427	0.654	-0.870	5.725
	TWI	1085	1.937	-2.494	-0.007	0.185	0.430	-0.331	2.992
	S&P500	1085	2.012	-2.995	0.011	0.188	0.433	-0.569	4.978
	HSI	1085	2.379	-2.614	-0.008	0.266	0.516	-0.307	2.822
	FTSE100	1085	1.713	-2.076	-0.003	0.181	0.426	-0.366	2.481
	Nki225	1085	3.225	-4.844	0.020	0.379	0.616	-0.663	5.882

In our empirical analysis, the first 739 observations are used as training data for model estimation and the remaining 370 observations are left as test data for the out-of-sample evaluation in the first interval 2002-2006, while 588 /294 and 723/362 are used for the rest two intervals, separately. Following a common practice, we use the mean, $\bar{r}$, of the in-sample returns instead of estimating the conditional mean of each series in the analysis. All methods (RiskMetrics, GJR-GARCH, GARCH-EVT, CAViaR, parametric CARE, and NCARE) are all applied to the resultant residuals, $y_t = r_t - \bar{r}$.

To evaluate the accuracy of VaR estimation, we use the likelihood ratio (LR) test of [28] and Interval Forecast Test of [29] to backtesting, which uses the same log-likelihood testing framework as [28]. To distinguish these two tests, we name them as uc.LR and cc.LR, respectively. Define the observed proportion of failures as

$$p = \frac{N}{T} = \frac{1}{T}\sum_{t=1}^{T} I(-y_t > VaR_t(1-\theta)). \tag{34}$$

The ideas of both uc.LR test and cc.LR test are to check whether $H_0 : p = p^*$, where $p^* = \theta$ denotes the expected probability of failures. Under the null hypothesis, the corresponding uc.LR statistic

$$uc.LR = 2\ln\left[(1-p)^{T-N}p^N\right] - 2\ln\left[(1-\theta)^{T-N}\theta^N\right], \tag{35}$$

is asymptotically $\chi^2(1)$ distributed.

Based on the uc.LR test, [29] proposed the cc.LR test which is extended to include a separate statistic for independence of exceptions. The test defines an indicator variable

$$I_t = \begin{cases} 0 & \text{if no violation occurs} \\ 1 & \text{if violation occurs} \end{cases}$$

Following the definition, the test statistic for independence of exceptions is

$$ind.LR = -2\ln\left[(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}\right] - 2\ln\left[(1-\pi_0)^{n_{00}}\pi_0^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{n_{11}}\right] \tag{36}$$

where $n_{ij}, i, j = 0, 1$ denote the number of times that $I_{t-1} = i, I_t = j$ occurs; $\pi_i, i = 0, 1$ represent the probability that a violation occurs conditional on the previous day, that is $\pi_0 = \frac{n_{01}}{n_{00}+n_{01}}$, $\pi_1 = \frac{n_{10}}{n_{00}+n_{11}}$ and $\pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}}$.

Under the null hypothesis, the corresponding conditional coverage cc.LR statistic:

$$cc.LR = uc.LR + ind.LR \tag{37}$$

is asymptotically $\chi^2(2)$ distributed.

For ES evaluation, we follow the bootstrap method in [16] and define residuals

$$res_t = \frac{-y_t^* - ES_t}{VaR_t}, \tag{38}$$

where $y_t^* \equiv \{y_t| - y_t > VaR_t\}$ for $t = 1, 2, \cdots, T^*$. According to [16], the residual series is an i.i.d with mean 0. To check whether $H_0 : \mu = 0$, where μ is the mean of the residuals, we bootstrap from the residual series B times and obtain bootstraped sample $\left(res_t^{(1)}, res_t^{(2)}, \cdots, res_t^{(B)}\right)$. Then, we construct statistics for bootstraped sample and observed sample respectively with forms

$$t_i = \frac{\overline{res^{(i)}}}{\sigma^{(i)}/\sqrt{T^*}}, i = 1, 2, \cdots, B, \tag{39}$$

and

$$t_o = \frac{\overline{res_o}}{\sigma_o/\sqrt{T}}, \tag{40}$$

where $\overline{res^{(i)}}$, $\sigma^{(i)}$ are mean and standard deviation of bootstraped sample, while $\overline{res_o}$, σ_o are values of observed sample. Therefore, the p-value of the bootstrap test is

$$p_{value} = 2\min\left\{\frac{1}{B}\sum_{i=1}^{B} I(t_i \geq t_o), \frac{1}{B}\sum_{i=1}^{B} I(t_i \leq t_o)\right\}. \tag{41}$$

4.2. NCARE model

In NCARE modeling, we chose the commonly used sigmoidal function and linear function in hidden layer and output layer respectively. For the initial expectilas calculation, we use the inverse function of (25):

$$Expectile_1(\tau) = ES_1(1-\theta) \times \frac{(1-2\tau)\theta}{(1-2\tau)\theta + \tau}, \tag{42}$$

where $ES_1(1-\theta)$ is the average of the values that exceed the $VaR_1(1-\theta)$, and $VaR_1(1-\theta)$ takes the negative value of sample θ-th quantile. As set in (21), the τ-th expectile implies that the proportion of in-sample observations lying below the expectile is θ. We set θ as 5% and 1% because they are widely used in practice. For example, Morgan reposts its daily VaR at 5% level and the Bank of International Settlements considers VaR for assessing the adequacy of bank capital at the tail level of 1%. Therefore, the optimal value of τ is derived by optimizing

$$\tau^* = \arg\min_{\tau} |P[y_t \leq Expectile_t(\tau)] - \theta|, \tag{43}$$

and the results are reported in Table 2.

Table 2. Optimal values of τ for different stock indices derived from a given θ-th quantile.

Sample interval	θ(%)	τ(%)					
		SHCI	TWI	S&P500	HSI	FTSE	Nik225
2002-2006	5	1.800	2.394	2.558	1.757	3.020	2.034
	1	0.368	0.338	0.380	0.585	0.387	0.492
2007-2010	5	1.840	0.904	2.328	2.393	2.661	3.284
	1	0.136	0.216	0.350	0.737	0.180	0.505
2011-2016	5	4.101	2.159	2.476	2.650	2.700	2.687
	1	0.312	0.334	0.557	0.575	0.315	0.455

Note: The expectile level is the mean value of the in-sample estimation.

The results reported in Table 2 shows these expectile levels do not change largely to affect the final results. We then estimate NCARE model parameters using the procedure as described in Section 3.2. The optimal number of hidden layer nodes in Table 3 for each stock index return is selected through the GACV criterion. The number of hidden nodes across two θ's and six series is 2,3 or 4, which indicates that we do not need much complicated neural network structure in real applications. Figure 3, 4 and 5 present the out-of-sample 95% and 99% VaR and ES estimates for six stock indices produced by our NCARE model for each sample interval. The VaR and ES estimates can be seen to change with the volatility of the returns. The curve of ES always lies below that of VaR indicates that the former evaluates the financial risk in a conservative way.

Table 3. Optimal numbers of hidden nodes in NCARE model determined by GACV at $\theta = 5\%$ and $\theta = 1\%$.

Sample interval	Stock indices	$\theta = 5\%$		$\theta = 1\%$	
		Number of nodes	GACV	Number of nodes	GACV
2002-2006	SHCI	2	0.077	3	0.032
	TWI	4	0.051	2	0.037
	S&P500	2	0.017	2	0.005
	HSI	2	0.043	2	0.016
	FTSE100	3	0.026	4	0.010
	Nki225	3	0.064	3	0.027
2007-2010	SHCI	2	0.220	3	0.089
	TWI	2	0.127	2	0.051
	S&P500	2	0.064	3	0.022
	HSI	3	0.078	3	0.032
	FTSE100	3	0.062	3	0.023
	Nki225	2	0.089	4	0.033
2011-2016	SHCI	4	1.144	3	0.086
	TWI	2	0.043	3	0.018
	S&P500	3	0.044	3	0.020
	HSI	3	0.068	2	0.028
	FTSE100	2	0.049	2	0.017
	Nki225	2	0.094	3	0.043

4.3. VaR and ES results

For comparison, we use six types of models: RiskMetrics, GJR-GARCH, GARCH-EVT, CAViaR, parameter CARE and NCARE, to estimate VaR. For implementation of these models, we estimate RiskMetrics model, GJR-GARCH, GARCH-EVT model and CAViaR models presented in Section 2.4 following the procedures in [15], [16] and [3], respectively. The procedure to implement parametric CARE models is similar to but different from those of [1]. We estimate parametric CARE models not for the moving window sample but for the in-sample observations and recursively predict VaR and ES for out-of-sample.

Table 4, 5 and 6 present the values of the proportion of failures measure of each method for 95% and 99% VaR estimation for each sample interval. The final two columns (NS1 and NS2) report a count for the number of both uc.LR test and cc.LR test for which the null is rejected or significant at the 5% level. The closer the proportion of failures to 5% under 95% confidence level or 1% under 99% confidence level, the better the model is.

In terms of the first sample interval 2002-2006, based on the results of uc.LR test, only GJRGARCH, CAViaR-Asymmetric Slope, CAViaR-Indirect GARCH(1,1) and our proposed NCARE model perform well for the 95% VaR estimation. Moreover, although CAViaR-Indirect GARCH(1,1) yields relatively better estimations than the other models, the results are still not comparable to our NCARE model's. Regardless of those four well-performed models, CAViaR-Adaptive and the four CARE models perform relative poor among the rest, which all have not less than 4 values of NS. Under both confidence levels, we also find that it is often challenging to obtain an accurate evaluation for VaR of S&P500, which is similar to [1] and the evaluation for VaR of TWI in our particular case. Our NCARE model, however, has been successfully applied to these two stock indexes and gives interesting results.

In the second interval 2007-2010, which includes the financial crisis period, similarly, although CAViaR-Symmetric Absolute Value yields relatively better estimations than the other models, the results are still not comparable to our NCARE model's. Moreover, it is not surprising that RiskMetrics and GARCH-EVT model perform inferior than their performances in the first interval, with 5 values of NS for the 95% VaR estimation and 6 values of NS for the 99% VaR estimation under both uc.LR test and cc.LR test. The estimation results indicate that those two methods are insufficient while coming down to the crisis evaluation.

In terms of 2011-2016, it may be noticed that CAViaR-Indirect GARCH(1,1) is performing slightly better than our NCARE model yet only with stock TWI and HSI for 95% VaR estimation. However, under a stricter cc.LR test, our NCARE model is the only one that performs well, whilst others yield more or less values of NS. Interestingly, the overall poorer performance are yielded in the third sample interval compared to the first sample interval and even the second interval including financial crisis interval. This phenomenon may occur because of the Europe's sovereign debt crisis since the end of 2009. The financial crisis has been lasting over a long time span, which is a challenging task, especially when the extreme quantiles change.

Note that, in all the three sample intervals, both 95% VaR and 99% VaR derived from the family of parametric CARE models are generally poor compared to the other models, estimated via both uc.LR test and cc.LR test. The both zero values of NS imply that the NCARE model gives accurate VaR evaluations of all six stock indices for its flexible ability via exploring the nonlinear dynamics in various financial time series under different selected sample intervals, no matter including the financial

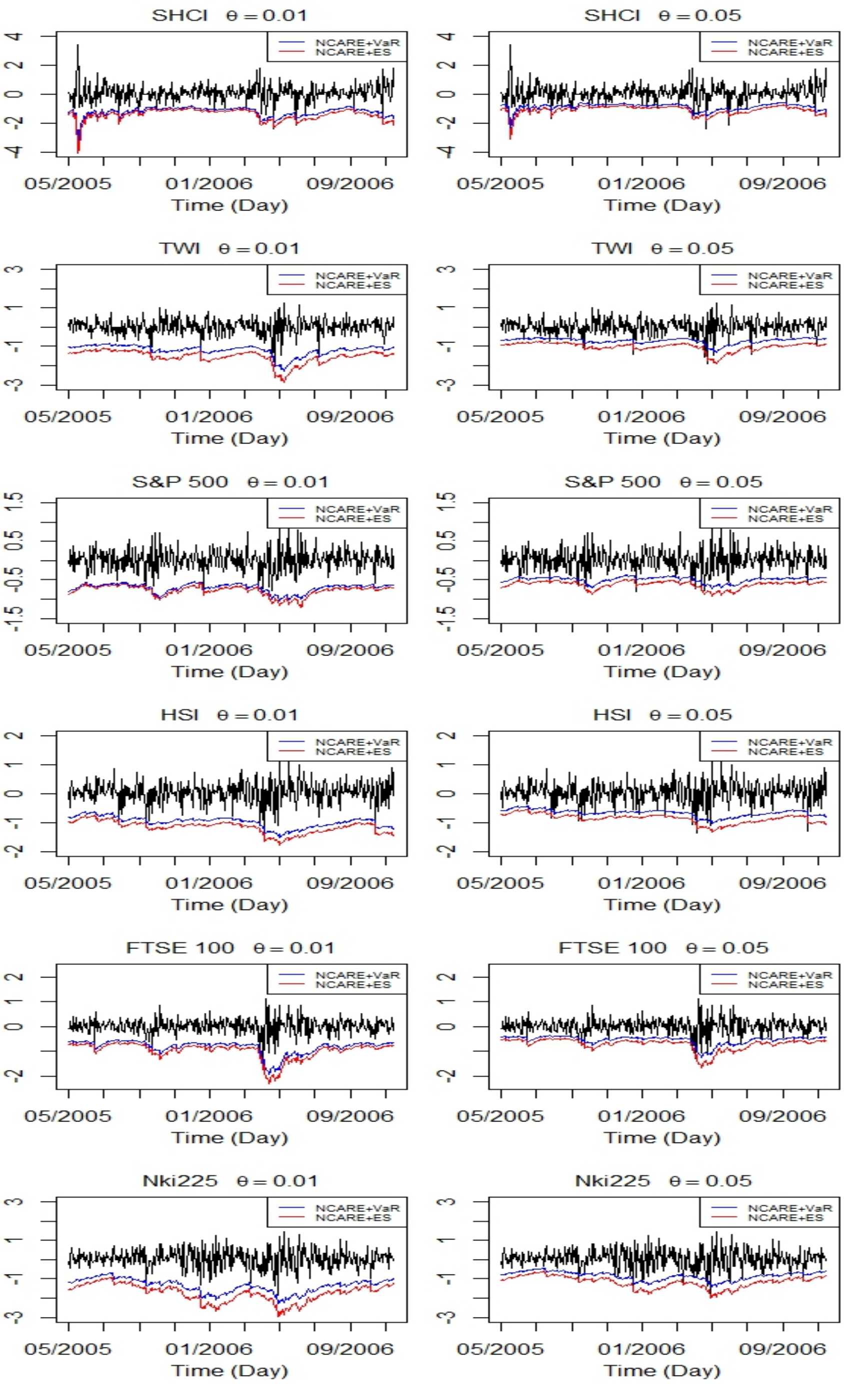


Figure 3. Stock index returns for the 370 out-of-sample days with VaR and ES estimates from the NCARE model for sample interval 2002-2006.

crisis period or not.

In spite of the strong appeal of CAViaR approach to VaR estimation, how to use this approach to assess ES is still unknown yet. So, we compare our NCARE method with those parametric CARE models for estimating ES, and, CARCH-EVT models, GJR-GARCH and RiskMetrics model are implemented for comparison as well. In Table 7, 8 and 9, we report p-values for the bootstrap test for the out-of-sample estimates of 95% and 99% ES for each sample interval. As with VaR evaluation, such as Table 4, the number of NS is placed in the final column of Table 7, 8 and 9. For the p-values of bootstrap test, the higher the value is, the better the model is.

For each of methods in sample interval 2002-2006, we find that the total number of rejections for six series at two different confidence levels are all zero, except for GJR-GARCH and CARE-Adaptive model for 99% VaR evaluation. Interestingly, ES evaluation in the second sample interval, which includes financial crisis period, is relatively poor performed compared to the other

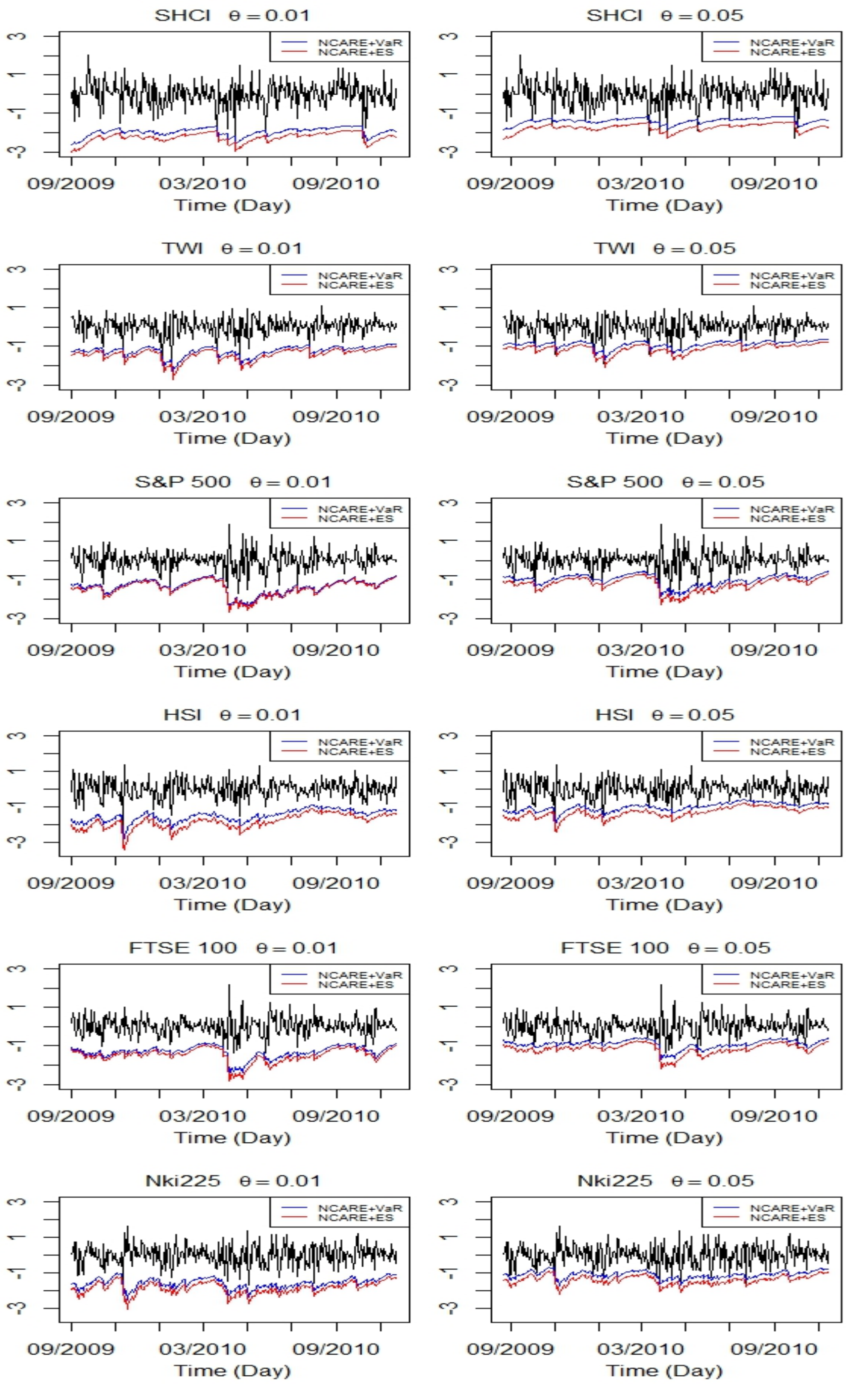


Figure 4. Stock index returns for the 370 out-of-sample days with VaR and ES estimates from the NCARE model for sample interval 2007-2010.

two sample intervals. The reason why ES are affected more by the financial crisis probably is that ES probes the risk associated with extreme events, whereas VaR is blind to any tail risk with a probability of occurrence smaller than the chosen confidence level.

It is not surprising that parametric CARE model performances better in ES evaluation than in VaR evaluation because the optimized object in CARE model is expectiles, which is corresponding to ES directly. In general, the NCARE model performs better than the parametric CARE models and the other three methods in ES evaluation. This implies that there may be some complex nonlinear structure in dynamics of these six stock indices which can not be appropriately described by the existing four parameter CARE models. To give a reasonable explanation of stylized facts in finance, we should go further to find the real functional form of CARE model.

The best model for VaR evaluation for each sample interval is recommended by the value of proportion of failures in Table

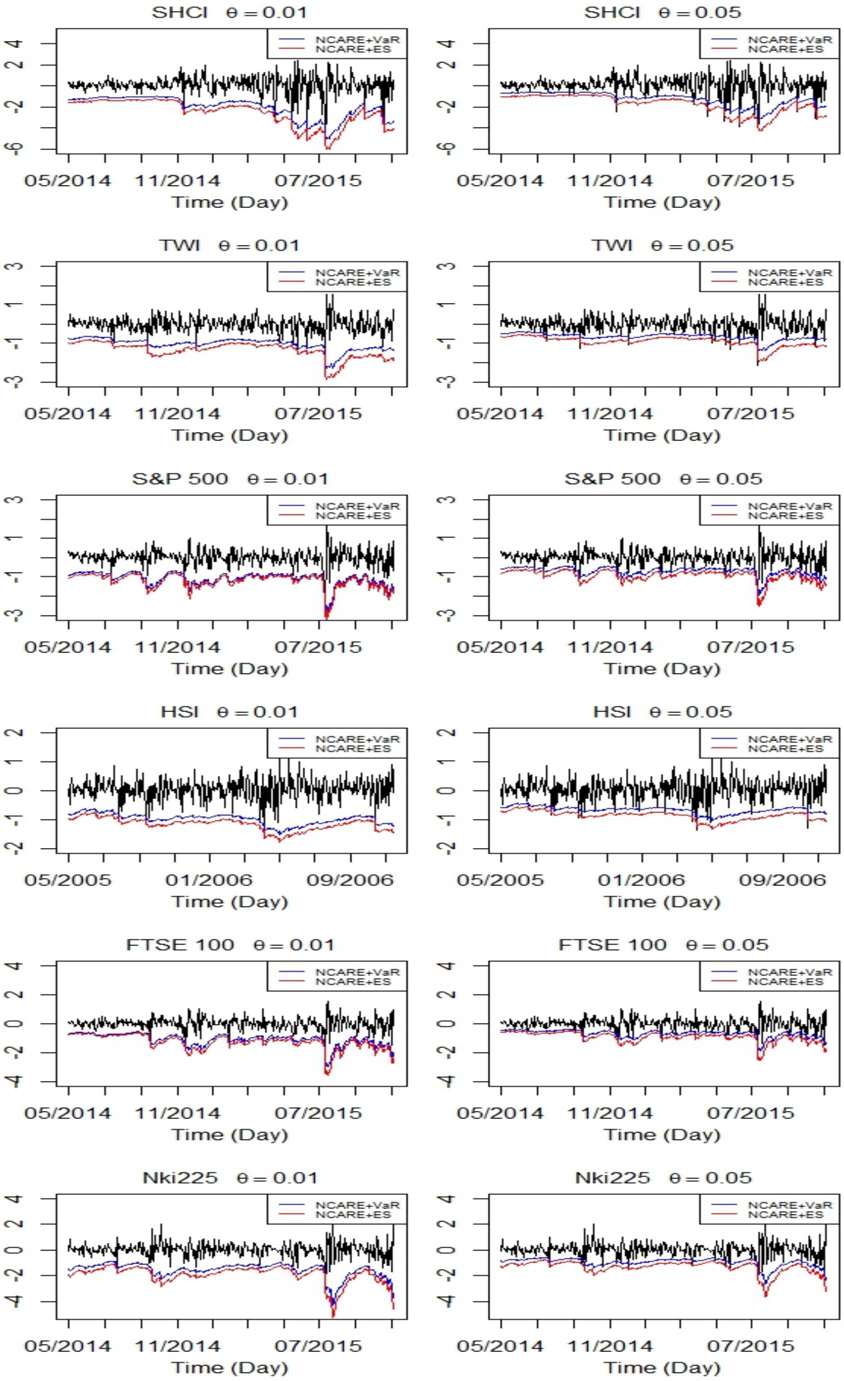
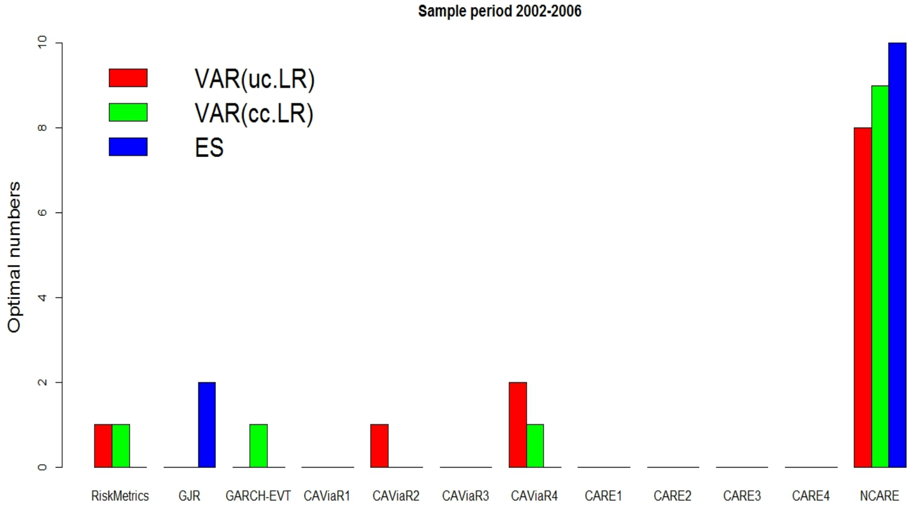
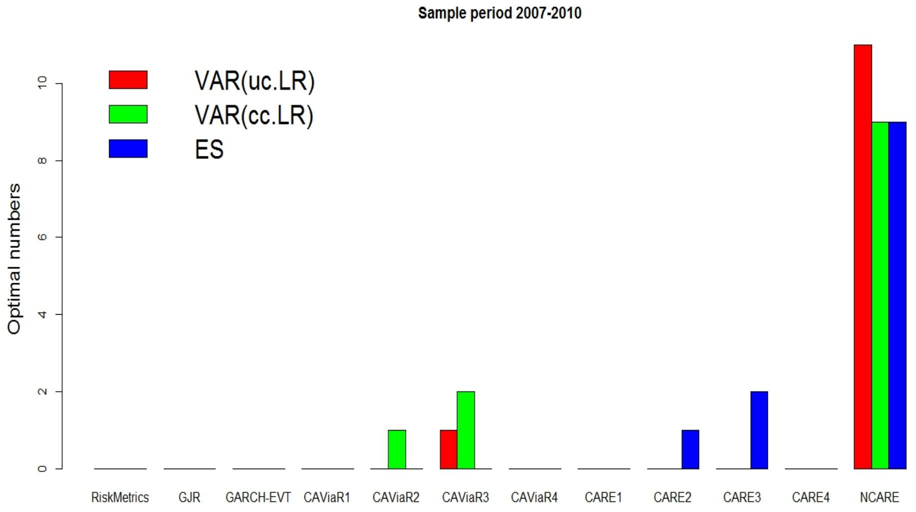


Figure 5. Stock index returns for the 370 out-of-sample days with VaR and ES estimates from the NCARE model for sample interval 2011-2016.

4, 5 and 6, while the best model for ES evaluation for each sample interval by the bootstrap p-values are presented in Table 7, 8 and 9. The results show that the NCARE model are at most recommended best model for both in VaR evaluation and ES evaluation.

Figure 6 counts the numbers of best for each model for each sample interval. It is clear that the NCARE model outperform the other models (RiskMetrics, GARCH-EVT, GJR-GARCH, CAViaR and parametric CARE). Only CAViaR-Symmetric Absolute Value model in CAViaR family has been recommended for all three sample intervals, while the other methods have been recommended only once during the three intervals. Interestingly, CARE models are never recommended for the best model in VaR evaluation. Note that, our proposed NCARE model has been highly recommended, especially under uc.LR test for the second sample interval 2007-2010, which includes the financial crisis period. The finding implies that the proposed NCARE model is a competitive and alternative way for both VaR and ES evaluation, especially for the challenging financial crisis period.





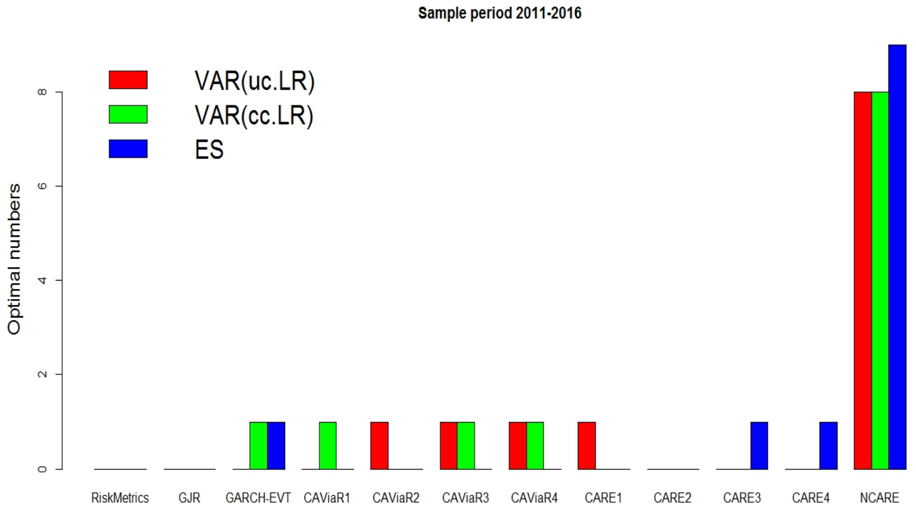


Figure 6. The optimal numbers of each model for VaR and ES evaluation of six stock indices at two confidence levels for each sample interval.

Table 4. Evaluation of estimators of 95% and 99%VaR. Proportion of failures for 370 out-of-sample estimates of 5% and 1% conditional quantile in sample interval 2002-2006.

Models	SSEI			TWI			S&P500			HSI			FTSE100			Nki225			NS	
	F	P1	P2	F	P1	P2	F	P1	P2	F	P1	P2	F	P1	P2	F	P1	P2	NS1	NS2
	$\theta = 1\%$																			
RiskMetrics	1.44	0.16	0.00	1.80	0.02	0.00	1.35	0.26	0.14	1.53	0.10	0.04	2.07	0.00	0.01	1.71	0.03	0.06	3	4
GJRGARCH	1.08	0.87	0.23	1.63	0.27	0.13	1.80	0.02	0.01	1.36	0.52	0.22	1.08	0.87	0.43	1.36	0.52	0.22	1	1
GARCH-EVT	0.54	0.33	0.07	0.27	0.09	0.04	0.27	0.09	0.04	0.27	0.09	0.04	0.00	0.00	0.00	0.00	0.00	0.00	2	5
CAViaR models																				
CAViaR-1	0.81	0.52	0.47	0.54	0.09	0.01	0.36	0.01	0.01	0.81	0.52	0.33	0.45	0.04	0.01	0.63	0.19	0.05	1	3
CAViaR-2	1.17	0.57	0.23	0.90	0.74	0.20	1.08	0.78	0.53	0.72	0.33	0.12	0.81	0.52	0.22	0.90	0.74	0.20	0	0
CAViaR-3	1.62	0.06	0.02	0.99	0.98	0.56	1.26	0.40	0.27	0.81	0.52	0.22	1.08	0.78	0.44	0.90	0.74	0.20	0	1
CAViaR-4	0.72	0.33	0.04	0.99	0.98	0.56	1.08	0.78	0.44	0.90	0.74	0.20	0.99	0.98	0.77	0.81	0.52	0.22	0	1
Parametric CARE																				
CARE-1	0.45	0.04	0.02	0.27	0.00	0.00	0.00	0.00	0.00	0.27	0.00	0.00	0.09	0.00	0.00	0.27	0.00	0.00	6	6
CARE-2	0.36	0.01	0.01	0.45	0.04	0.02	0.45	0.04	0.02	0.18	0.00	0.00	0.18	0.00	0.00	0.09	0.00	0.00	6	6
CARE-3	0.54	0.09	0.02	0.36	0.01	0.00	0.63	0.19	0.12	0.00	0.00	0.00	0.18	0.00	0.00	0.09	0.00	0.00	4	5
CARE-4	1.53	0.10	0.04	1.18	0.57	0.27	0.45	0.04	0.02	1.35	0.26	0.14	0.36	0.01	0.01	0.09	0.00	0.00	3	4
NCARE	0.99	0.98	0.87	1.08	0.78	0.74	1.08	0.87	0.67	0.99	0.98	0.87	0.81	0.52	0.47	1.08	0.78	0.52	0	0
	$\theta = 5\%$																			
RiskMetrics	5.23	0.72	0.51	4.69	0.64	0.56	5.05	0.93	0.22	6.16	0.09	0.11	6.59	0.02	0.02	5.23	0.72	0.94	1	1
GJRGARCH	4.07	0.40	0.23	2.98	0.06	0.03	4.07	0.40	0.23	4.07	0.40	0.62	4.87	0.85	0.38	3.52	0.17	0.30	0	1
GARCH-EVT	5.42	0.71	0.25	2.98	0.06	0.03	2.98	0.06	0.03	4.07	0.40	0.62	1.90	0.00	0.00	2.71	0.03	0.05	2	3
CAViaR models																				
CAViaR-1	6.58	0.02	0.07	3.88	0.08	0.13	3.25	0.00	0.02	5.60	0.37	0.18	2.89	0.00	0.00	3.43	0.01	0.03	4	3
CAViaR-2	5.14	0.83	0.49	3.43	0.01	0.04	3.88	0.08	0.02	4.60	0.54	0.48	4.24	0.24	0.21	4.33	0.30	0.58	1	2
CAViaR-3	5.23	0.72	0.75	4.42	0.37	0.28	4.42	0.37	0.07	4.69	0.64	0.49	4.69	0.64	0.49	4.51	0.45	0.74	0	1
CAViaR-4	5.32	0.62	0.79	4.60	0.54	0.48	4.33	0.30	0.04	4.69	0.64	0.49	4.69	0.64	0.56	4.33	0.30	0.58	0	0
Parametric CARE																				
CARE-1	1.26	0.00	0.00	1.44	0.00	0.00	0.72	0.00	0.00	1.35	0.00	0.00	0.72	0.00	0.00	1.35	0.00	0.00	6	6
CARE-2	2.98	0.06	0.00	5.60	0.37	0.18	1.81	0.00	0.00	1.26	0.00	0.00	3.43	0.01	0.03	1.26	0.00	0.00	4	5
CARE-3	1.99	0.00	0.00	1.53	0.00	0.00	1.71	0.00	0.00	4.33	0.30	0.00	4.69	0.64	0.56	1.26	0.00	0.00	4	5
CARE-4	4.33	0.30	0.00	3.25	0.00	0.02	1.81	0.00	0.00	1.26	0.00	0.00	2.89	0.00	0.00	2.71	0.03	0.05	5	5
NCARE	5.23	0.72	0.80	4.69	0.64	0.66	4.96	0.96	0.75	4.87	0.85	0.38	5.15	0.90	0.79	4.33	0.30	0.58	0	0

NOTE: (1) CAViaR-1, CAViaR-2, CAViaR-3, and CAViaR-4 denote CAViaR-Adaptive, CAViaR-Symmetric Absolute Value, CAViaR-Asymmetric Slope, and CAViaR-Indirect GARCH(1,1) model; (2) CARE-1, CARE-2, CARE-3, and CARE-4 denote CARE-Adaptive, CARE-Symmetric Absolute Value, CARE-Asymmetric Slope, and CARE-Indirect GARCH(1,1) model; (3) F denotes proportion of failures while P1 and P2 stand for p-values obtained via uc.LR test and cc.LR test separately; (4)NS1 and NS2 represent a count for the number of uc.LR test and cc.LR test separately for which the null is rejected at 5% significance level.

5. Conclusions

In this paper, we have introduced NCARE as a means of using ANN to estimate the conditional expectiles, which serves as a flexible model for VaR and ES calculation. Our approach can be viewed as nonparametric version of CARE method proposed by [1] and [6]. The proposed NCARE model is a basic model to represent the nonlinear relationships among variables without prior specification of the functional form. An appealing feature of the NCARE model is that the objective function is differentiable everywhere and the model can be estimated by standard gradient based nonlinear optimization algorithm. The numerical results of the new method are promising in terms of the performance of both VaR and ES evaluation.

Acknowledgements

This work was supported by the National Natural Science Foundation of PR China (71490725, 71071087, 71101134), the National Social Science Foundation of PR China (15BJY008) and the Humanity and Social Science Foundation of Ministry of Education of PR China (No. 14YJA790015).

The second author would like to thank Prof. Engle and Prof. Manganelli for sharing their Matlab and C codes of the CAViaR approach.

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Table 5. Evaluation of estimators of 95% and 99%VaR. Proportion of failures for 294 out-of-sample estimates of 5% and 1% conditional quantile in sample interval 2007-2010.

Models	SSEI			TWI			S&P500			HSI			FTSE100			Nki225			NS	
	F	P1	P2	F	P1	P2	F	P1	P2	F	P1	P2	F	P1	P2	F	P1	P2	NS1	NS2
	$\theta=1\%$																			
RiskMetrics	2.61	0.00	0.00	3.06	0.00	0.00	3.29	0.00	0.00	1.48	0.19	0.11	2.50	0.00	0.00	2.27	0.00	0.00	5	5
GJRGARCH	0.79	0.52	0.04	1.69	0.27	0.11	2.72	0.00	0.00	0.34	0.19	0.07	1.35	0.56	0.23	1.35	0.56	0.23	1	2
GARCH-EVT	0.34	0.19	0.11	0.00	0.00	0.00	0.00	0.00	0.00	1.59	0.11	0.05	0.34	0.19	0.07	0.00	0.00	0.00	3	3
CAViaR models																				
CAViaR-1	0.34	0.02	0.01	0.91	0.78	0.16	0.79	0.52	0.04	0.79	0.52	0.04	0.79	0.52	0.14	1.59	0.11	0.03	1	4
CAViaR-2	0.68	0.31	0.08	0.91	0.78	0.16	1.48	0.19	0.07	0.68	0.31	0.11	0.68	0.31	0.12	0.57	0.16	0.07	0	0
CAViaR-3	0.68	0.31	0.08	0.79	0.52	0.04	0.91	0.78	0.66	0.45	0.07	0.02	0.68	0.31	0.08	0.91	0.78	0.16	0	2
CAViaR-4	0.68	0.31	0.12	0.79	0.52	0.14	0.79	0.52	0.23	0.34	0.02	0.00	0.68	0.31	0.20	1.14	0.69	0.50	1	1
Parametric CARE																				
CARE-1	0.11	0.00	0.00	0.11	0.00	0.00	0.23	0.01	0.00	0.34	0.02	0.00	0.23	0.01	0.00	0.23	0.01	0.00	6	6
CARE-2	0.11	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.11	0.00	0.00	0.11	0.00	0.00	0.45	0.07	0.00	5	6
CARE-3	0.23	0.01	0.00	0.34	0.02	0.00	0.23	0.01	0.00	0.23	0.01	0.00	0.23	0.01	0.00	0.11	0.00	0.00	6	6
CARE-4	0.11	0.00	0.00	0.57	0.16	0.06	0.23	0.01	0.00	0.23	0.01	0.00	0.23	0.01	0.00	0.11	0.00	0.00	5	5
NCARE	1.36	0.56	0.34	1.02	0.95	0.77	1.14	0.69	0.50	1.02	0.95	0.77	0.91	0.78	0.67	0.91	0.78	0.66	0	0
	$\theta=5\%$																			
RiskMetrics	6.58	0.04	0.07	6.47	0.05	0.16	7.38	0.00	0.00	5.22	0.76	0.89	6.36	0.08	0.20	7.60	0.00	0.00	3	2
GJRGARCH	3.61	0.10	0.04	4.02	0.18	0.20	6.43	0.73	0.93	4.02	0.18	0.11	4.77	0.75	0.23	6.83	0.55	0.83	0	1
GARCH-EVT	2.38	0.02	0.00	2.04	0.01	0.00	2.01	0.00	0.00	1.61	0.00	0.00	2.01	0.00	0.00	0.34	0.00	0.00	6	6
CAViaR models																				
CAViaR-1	2.84	0.00	0.00	3.97	0.15	0.33	4.65	0.63	0.47	5.33	0.65	0.18	4.20	0.26	0.03	3.97	0.15	0.01	1	3
CAViaR-2	3.97	0.15	0.33	3.97	0.15	0.33	4.54	0.53	0.23	3.75	0.07	0.16	4.20	0.26	0.50	4.43	0.43	0.60	0	0
CAViaR-3	3.06	0.00	0.00	4.43	0.43	0.71	4.54	0.53	0.23	3.97	0.15	0.31	4.54	0.53	0.64	5.45	0.55	0.83	0	1
CAViaR-4	2.95	0.00	0.00	4.20	0.26	0.47	4.54	0.53	0.23	3.86	0.11	0.23	4.31	0.34	0.12	4.65	0.63	0.47	0	1
Parametric CARE																				
CARE-1	1.59	0.00	0.00	1.25	0.00	0.00	2.04	0.01	0.00	1.14	0.00	0.00	1.25	0.00	0.00	1.36	0.00	0.00	6	6
CARE-2	6.47	0.05	0.16	2.38	0.02	0.00	1.14	0.00	0.00	1.25	0.00	0.00	1.14	0.00	0.00	1.25	0.00	0.00	5	5
CARE-3	1.14	0.00	0.00	4.02	0.18	0.11	6.36	0.08	0.20	1.02	0.00	0.00	1.14	0.00	0.00	1.25	0.00	0.00	4	4
CARE-4	3.97	0.15	0.33	1.14	0.00	0.00	1.02	0.00	0.00	3.86	0.11	0.23	3.97	0.15	0.33	3.97	0.15	0.33	2	2
NCARE	5.45	0.55	0.12	4.54	0.53	0.81	5.22	0.76	0.54	4.88	0.87	0.86	5.62	0.85	0.44	4.77	0.75	0.95	0	0

NOTE: same as Table 4.

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Table 6. Evaluation of estimators of 95% and 99%VaR. Proportion of failures for 294 out-of-sample estimates of 5% and 1% conditional quantile in sample interval 2011-2016.

Models	SSEI			TWI			S&P500			HSI			FTSE100			Nki225			NS	
	F	P1	P2	F	P1	P2	F	P1	P2	F	P1	P2	F	P1	P2	F	P1	P2	NS1	NS2
	$\theta = 1\%$																			
RiskMetrics	2.56	0.00	0.00	2.68	0.00	0.00	2.86	0.00	0.00	2.68	0.00	0.00	2.77	0.00	0.00	1.85	0.01	0.00	6	6
GJRGARCH	3.87	0.00	0.00	2.76	0.00	0.00	1.93	0.11	0.08	3.54	0.00	0.00	2.53	0.01	0.01	1.77	0.11	0.05	4	4
GARCH-EVT	2.03	0.00	0.01	0.83	0.74	0.20	0.00	0.00	0.00	0.28	0.10	0.07	0.00	0.00	0.00	0.28	0.10	0.07	3	3
CAViaR models																				
CAViaR-1	3.78	0.00	0.00	0.83	0.56	0.32	0.74	0.36	0.26	0.09	0.74	0.36	0.74	0.36	0.09	6.18	0.00	0.00	2	2
CAViaR-2	11.35	0.00	0.00	1.48	0.14	0.17	1.29	0.36	0.32	0.83	0.56	0.32	1.57	0.08	0.04	0.83	0.56	0.14	1	2
CAViaR-3	1.57	0.08	0.05	1.20	0.52	0.28	0.74	0.36	0.26	1.21	0.52	0.28	1.75	0.02	0.05	0.74	0.36	0.26	1	0
CAViaR-4	2.58	0.00	0.00	1.57	0.08	0.12	0.83	0.56	0.32	1.01	0.96	0.21	1.38	0.23	0.03	1.29	0.36	0.26	1	2
Parametric CARE																				
CARE-1	0.92	0.80	0.20	0.28	0.00	0.00	0.18	0.00	0.00	0.18	0.00	0.00	0.28	0.00	0.00	0.18	0.00	0.00	5	5
CARE-2	0.37	0.02	0.00	0.18	0.00	0.00	0.28	0.00	0.00	0.28	0.00	0.00	0.55	0.11	0.02	0.74	0.36	0.00	4	5
CARE-3	0.74	0.36	0.18	0.09	0.00	0.00	0.09	0.00	0.00	0.09	0.00	0.00	0.37	0.02	0.00	0.18	0.00	0.00	5	5
CARE-4	0.65	0.21	0.11	0.09	0.00	0.00	0.09	0.00	0.00	0.28	0.00	0.00	0.55	0.11	0.02	0.18	0.00	0.00	4	5
NCARE	0.83	0.74	0.53	0.92	0.80	0.28	0.83	0.56	0.32	1.20	0.52	0.64	1.20	0.52	0.28	0.83	0.56	0.32	0	0
	$\theta = 5\%$																			
RiskMetrics	5.81	0.23	0.45	6.27	0.06	0.07	6.09	0.11	0.24	6.37	0.05	0.13	7.29	0.00	0.00	20.20	0.00	0.00	2	2
GJRGARCH	7.18	0.07	0.02	6.35	0.26	0.48	6.08	0.36	0.56	5.52	0.65	0.90	7.73	0.03	0.08	4.14	0.44	0.67	1	1
GARCH-EVT	4.70	0.79	0.94	3.87	0.30	0.50	1.38	0.00	0.00	4.42	0.61	0.00	3.59	0.20	0.34	3.31	0.12	0.21	1	2
CAViaR models																				
CAViaR-1	7.38	0.00	0.00	3.23	0.00	0.01	3.63	0.11	0.26	4.43	0.38	0.45	3.97	0.11	0.26	4.34	0.31	0.48	2	2
CAViaR-2	6.92	0.01	0.00	4.52	0.46	0.40	5.44	0.51	0.27	4.84	0.76	0.72	5.17	0.80	0.78	4.89	0.87	0.36	1	1
CAViaR-3	6.73	0.01	0.00	4.61	0.55	0.76	4.80	0.76	0.90	5.07	0.91	0.55	4.31	0.65	0.84	4.24	0.24	0.50	1	1
CAViaR-4	1.75	0.00	0.00	4.43	0.38	0.57	4.61	0.55	0.82	4.52	0.46	0.22	4.89	0.87	0.95	5.54	0.43	0.48	1	1
Parametric CARE																				
CARE-1	2.86	0.00	0.00	1.20	0.00	0.00	1.38	0.00	0.00	1.48	0.00	0.00	1.11	0.00	0.00	1.38	0.00	0.00	6	6
CARE-2	7.20	0.00	0.00	1.85	0.00	0.00	7.18	0.07	0.02	4.34	0.31	0.48	1.85	0.00	0.00	1.85	0.00	0.00	4	5
CARE-3	6.35	0.26	0.48	1.75	0.00	0.00	1.11	0.00	0.00	1.66	0.00	0.00	3.63	0.11	0.26	1.29	0.00	0.00	4	4
CARE-4	4.43	0.38	0.57	1.94	0.00	0.00	1.29	0.00	0.00	1.57	0.00	0.00	1.94	0.00	0.00	0.55	0.11	0.02	4	5
NCARE	5.07	0.91	0.77	4.89	0.87	0.68	5.07	0.91	0.99	4.84	0.76	0.92	5.07	0.91	0.77	4.80	0.76	0.90	0	0

NOTE: same as Table 4.

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Table 7. Evaluation of estimators of 95% and 99% ES corresponding to $\theta = 5\%$ and $\theta = 1\%$. Bootstrap test p-values based on 370 out-of-sample estimates of conditional ES in sample interval 2002-2006.

Models	SSEI	TWI	S&P 500	HSI	FTSE	Nki225	NS
	$\theta = 1\%$						
RiskMetrics	0.28	0.31	0.23	0.29	0.23	0.21	0
GJRGARCH	0.03	0.03	0.26	0.28	0.12	0.18	2
GARCH-EVT	0.11	0.11	0.15	0.14	0.26	0.22	0
Parametric CARE							
CARE-1	0.00	0.03	0.11	0.24	0.00	0.00	4
CARE-2	0.29	0.75	0.12	0.44	0.32	0.82	0
CARE-3	0.18	0.77	0.09	0.19	0.22	0.71	0
CARE-4	0.16	0.71	0.15	0.13	0.57	0.82	0
NCARE	**0.62**	**0.82**	**0.74**	**0.57**	**0.82**	**0.97**	0
	$\theta = 5\%$						
RiskMetrics	0.34	0.18	0.17	0.32	0.21	0.12	0
GJRGARCH	0.24	0.18	0.25	0.28	**0.74**	**0.39**	0
GARCH-EVT	0.12	0.14	0.14	0.14	0.18	0.17	0
Parametric CARE							
CARE-1	0.12	0.12	0.00	0.11	0.00	0.26	2
CARE-2	0.17	0.17	0.09	0.11	0.14	0.11	0
CARE-3	0.17	0.10	0.10	0.19	0.12	0.13	0
CARE-4	0.16	0.11	0.10	0.10	0.11	0.17	0
NCARE	**0.73**	**0.43**	**0.50**	**0.63**	0.35	0.29	0

NOTE: (1) CARE-1, CARE-2, CARE-3, and CARE-4 denote CARE-Adaptive, CARE-Symmetric Absolute Value, CARE-Asymmetric Slope, and CARE-Indirect GARCH(1,1) model; (2)NS represents a count for the number of Bootstrap test for which the null is rejected at 5% significance level.

Table 8. Evaluation of estimators of 95% and 99% ES corresponding to $\theta = 5\%$ and $\theta = 1\%$. Bootstrap test p-values based on 294 out-of-sample estimates of conditional ES in sample interval 2007-2010.

Models	SSEI	TWI	S&P 500	HSI	FTSE	Nki225	NS
	$\theta = 1\%$						
RiskMetrics	0.34	0.30	0.18	0.22	0.20	0.17	0
GJRGARCH	0.03	0.10	0.06	0.12	0.16	0.11	1
GARCH-EVT	0.14	0.16	0.08	0.19	0.07	0.07	0
Parametric CARE							
CARE-1	0.00	0.10	0.00	0.11	0.03	0.21	3
CARE-2	0.02	0.22	0.06	**0.45**	0.11	0.19	1
CARE-3	0.02	0.26	0.06	0.13	**0.29**	0.48	1
CARE-4	0.04	0.30	0.09	0.11	0.18	0.24	1
NCARE	**0.34**	**0.33**	**0.30**	0.22	0.20	**0.66**	0
	$\theta = 5\%$						
RiskMetrics	0.18	0.26	0.15	0.30	0.17	0.60	0
GJRGARCH	0.19	0.27	0.27	0.18	0.30	0.59	0
GARCH-EVT	0.42	0.09	0.07	0.20	0.17	0.16	0
Parametric CARE							
CARE-1	0.08	0.07	0.15	0.11	0.23	0.18	0
CARE-2	0.07	0.56	0.12	0.66	0.25	0.62	0
CARE-3	**0.94**	0.47	0.51	0.80	0.35	0.28	0
CARE-4	0.52	0.34	0.14	0.93	0.28	0.33	0
NCARE	0.56	**0.80**	**0.60**	**0.95**	**0.35**	**0.74**	0

NOTE: same as Table 7

Table 9. Evaluation of estimators of 95% and 99% ES corresponding to $\theta = 5\%$ and $\theta = 1\%$. Bootstrap test p-values based on 362 out-of-sample estimates of conditional ES in sample interval 2011-2016.

Models	SSEI	TWI	S&P 500	HSI	FTSE	Nki225	NS
	$\theta = 1\%$						
RiskMetrics	0.07	0.06	0.06	0.09	0.12	0.07	0
GJRGARCH	0.20	0.26	0.31	0.33	0.17	0.09	0
GARCH-EVT	0.12	0.05	0.23	0.16	0.23	0.18	0
Parametric CARE							
CARE-1	0.06	0.02	0.27	0.47	0.22	0.21	0
CARE-2	0.05	0.18	0.70	0.24	0.12	0.19	0
CARE-3	0.10	0.16	0.23	0.22	0.08	0.19	0
CARE-4	0.08	0.19	**0.73**	0.21	0.12	0.22	0
NCARE	**0.25**	**0.29**	0.34	**0.50**	**0.30**	**0.31**	0
	$\theta = 5\%$						
RiskMetrics	0.19	0.22	0.23	0.21	0.28	0.18	0
GJRGARCH	0.16	0.23	0.39	0.22	0.25	0.40	0
GARCH-EVT	0.27	**0.48**	0.07	0.41	0.22	0.42	0
Parametric CARE							
CARE-1	0.12	0.12	0.09	0.19	0.10	0.12	0
CARE-2	0.91	0.10	0.10	0.09	0.13	0.15	0
CARE-3	**0.39**	0.11	0.23	0.11	0.11	0.18	0
CARE-4	0.21	0.10	0.07	0.16	0.15	0.07	0
NCARE	0.19	0.24	**0.64**	**0.47**	**0.61**	**0.97**	0

NOTE: same as Table 7