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Declines in Numeracy Skill among University Students: Why Does It Matter?

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Our research on university students' arithmetic skills has shown an interesting trend: The efficiency (i.e., speed and accuracy) with which they solve problems such as 34 + 17 or 456 x 3 has declined by 25 – 40% over the last 20 years. This phenomenon has been observed in students studying in various disciplines in Canada (LeFevre et al., 2014; Standing, 2006; Standing, Sproule, & Leung, 2006) and Northern Ireland (Mulhern & Wylie, 2004). In this article, we summarize these findings and address the question of why reductions in efficient numeracy skills have implications for young adults.

Who needs to have fluent numeracy skills?

Performance on complex mathematical tasks, including problem solving and mathematical reasoning, is an obvious requirement for students in STEM disciplines (i.e., science, technology, engineering, and mathematics). Similarly, training in social sciences such as psychology, sociology, or political science also requires mathematical understanding -- evaluating and conducting research in these fields frequently involves quantitative analyses. Furthermore, regardless of the subject that students studied in university or college, they need to understand quantitative and numerical information in the workplace and in daily life; basic numerical skills are relevant in situations such as health decisions, financial choices, and understanding science policy (e.g., Reyna, Nelson, Han, & Dieckmann, 2009; Rolison, Morsanyi, & O'Connor, 2016). More generally, numeracy skills predict employment and economic success (Bynner & Parsons, 1997; Statistics Canada, 2005; Steen, 1997). Ideally, the mathematical skills required for basic quantitative literacy will be acquired in elementary school, and practiced throughout their schooling as students develop more complex understandings of relevant topics.

Fluency implies that students can quickly and accurately retrieve the answers to basic arithmetic combinations (e.g., $3 \times 8 = 24$; $6 + 9 = 15$), that they understand concepts such as commutativity (e.g., $3 + 4 = 4 + 3$), the distributive property (i.e., $6 \times [2 + 6] = 6 \times 2 + 6 \times 6$), and inversion (e.g., $2 + 3 – 3 = 2$), and that they have knowledge of the procedural rules underlying manipulations of whole numbers, decimals, and fractions. Fluency also implies a sufficient amount of practice with the component skills so that individuals develop speed, accuracy, and flexibility when manipulating numbers. In a recent study we conducted that involved interviews with elementary school teachers, several were concerned with their students' low fluency, a situation that they attributed to limited practice time. More generally, limited time on learning and practicing mathematics is a factor that influences children's early experiences with mathematics and subsequent attitudes and skills (Beilock, Gunderson, Ramirez, & Levine, 2010).

Why have numeracy skills declined?

Curricular changes in many Canadian provinces in the late 1990s represented a step away from numeracy towards a broader scope for mathematics education (O'Shea, 2003). One consequence was that the current curriculum is organized into multiple distinct strands (e.g., number sense and numeration; geometry and spatial sense; measurement, patterning and algebra; and data management and probability) rather than primarily focused on numeration (Craven, 2003; Ross, McDougall & Hogaboam-Gray, 2002). In the current Ontario curriculum there are now 50-75 specific expectations for each grade (Ministry of Education, 2005). A specific expectation is a clear description of a skill the child is expected to acquire by the end of the school year. For example, in Grade 2 the expectations include, "locate whole numbers to 100 on a number line and on a partial number line (e.g., locate 37 on a partial number line that goes from 34 to 41)", (p. 43); "construct tools for measuring time intervals in non-standard units (e.g., a particular bottle of water takes about five seconds to empty)" (p. 45); and "compose and decompose two-dimensional shapes (e.g., I made a picture of a flower from one hexagon and six equilateral triangles)" (p. 47). The large number of topics that must be covered may make it difficult for teachers to provide sufficient practice of fundamental numerical skills. As well, current curriculum documents emphasize multiple strategies to determine basic math facts (e.g., Ontario Ministry of Education, 2005; Western and Northern Canadian Protocol, 2006) that may downplay the importance of fluent calculation. In essence, schooling in mathematics has become broader with less focus (Schmidt, 2012) and the importance of fluency has been de-emphasized. Nevertheless, mathematically skilled students inevitably show greater fluency with numbers than less-skilled students, even when skill distinctions are measured in tasks that do not require speeded performance. Thus, despite the de-emphasis on fluency in the curriculum, it is still an important component of mathematical skill (Douglas & LeFevre, 2016; Price, Mazzocco, & Ansari, 2013).

Why is numerical fluency so consistently associated with mathematical skill?

One reason that fluency and mathematical competence are linked is because efficient access to fundamental knowledge about numbers frees cognitive resources that can be used to process the more complex aspects of mathematical problems (Imbo & LeFevre, 2009; Walcyzk & Griffith-Ross, 2006), regardless of whether fluency involves access to addition facts (Smith-Chant & LeFevre, 2003), recognition of familiar sequences (LeFevre & Bisanz, 1986; Lyons & Beilock, 2011), or confident use of arithmetic principles (Robinson & LeFevre, 2011). Consistent with this perspective, availability of working memory is strongly linked to mathematical performance (DeStefano & LeFevre, 2004; LeFevre, DeStefano, Coleman & Shanahan, 2005; Raghubar, Barnes, & Hecht, 2010). Thus, if fluency reduces working memory demands, students can use these available cognitive resources to choose and execute solutions that involve conceptual or procedural knowledge beyond basic operations.

Individuals who have competent basic numeracy skills are more confident and less anxious about mathematics than individuals with weaker skills (Dowker, Bennett, & Smith, 2012; Jansen et al., 2013; Maloney & Beilock, 2012). This connection between numeracy and perceived competence is important because anxiety about math is also related to reduced attention (Ashcraft & Kirk, 2001; Dowker, Sarkar & Looi, 2016). Lower confidence thus contributes to a vicious cycle: Individuals who have reduced or limited attention during problem solving are at a disadvantage for solving problems or learning new knowledge that has a quantitative focus (Beilock & Carr, 2005). Accordingly, incomplete mastery of simple numerical skills or knowledge in situations that require basic knowledge, for example, calculating that 7 x 8 = 56 as part of an algebra problem, understanding the consequences of multiplying by 0, or comparing two probabilities, can become a roadblock to understanding and acquiring conceptual knowledge and advanced procedural skills.

How much have numeracy skills changed in the past 25 years?

Several studies indicate that fluency skills have decreased substantially. In a long-term comparison of Canadian university students' performance on arithmetic problems like 34 + 17, 52 – 19, and 127 x 9, we found that fluency (i.e., speed and accuracy) on these types of problems in 2010 was about half that of students who participated in the early 1990s (see LeFevre, Penner-Wilger, Pyke, Shanahan, & Deslauriers, 2014 for the detailed data from 1992 to 2005). These students were all between the ages of 18 and 24, in a variety of undergraduate programs, and the analyses controlled for other factors including gender. Performance deficits reflected both a lack of automaticity with the basic facts, as well as uncertainty about the procedures for multiplying or subtracting multi-digit numbers, consistent with the view that arithmetic fluency is essential for higher-level math skills (De Smedt, Holloway, & Ansari, 2011; National Research Council, 2001; Price et al., 2013). Declines in fluency were also correlated with participants' reports of when they started using a calculator, suggesting that technological advances may have contributed to a de-emphasis on basic numeracy skills.

Current students appear to have the conceptual knowledge to calculate simple facts but use a variety of procedures to do the actual calculation. For example, on simple division problems such as 48 ÷ 6, students reported that they retrieved answers from memory only about 40% of the time. They also laboriously worked out solutions by repeated subtraction, by using "known" facts linked to the divisor (e.g., 6 x 6 = 36, plus two more sixes), or simply guessed (Huebner & LeFevre, 2016). Thus, they understood the principles of division, but were inefficient in reaching a solution. These findings are consistent with information about the experiences students have in elementary and middle school mathematics. For example, the current Ontario curriculum was revised in 2005 to state that, by the end of grade 4, students should "multiply to 9 x 9 and divide to 81 ÷ 9, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting)" (2005, p. 67). There is no mention of fact retrieval or that students should work towards the most efficient solution.

Concerns about solution efficiency were raised in a recent conversation one of us had with a practicing middle-school teacher. The teacher described participating in a professional development session with high-school teachers who told her that they evaluate students on the efficiency as well as the accuracy of their solutions. This teacher was surprised because she had always encouraged her students to choose the strategy they were most comfortable using (e.g., repeated subtraction, extrapolating from known facts), rather than encouraging them to choose the most efficient solution. A widespread emphasis on using multiple strategies rather than on efficiency may be one of the reasons that students struggle when they are faced with applying their numerical skills in required coursework in high school or university.

These observations are consistent with experiences we have had teaching psychology students in statistics courses, where their lack of competence and discomfort with simple arithmetic impeded their understanding of the advanced principles and procedures they were expected to learn. Declining fluency among university students is not unique to Canada, unfortunately. Similar declines have been observed among students in Northern Ireland over the same time frame (Mulhern & Wylie, 2004). In 1992, a sample of university students in Northern Ireland obtained a mean score of 53% on a test of mathematical reasoning that used six components - calculation, algebraic reasoning, graphical interpretation, proportionality and ratio, probability and sampling, and estimation. This relatively low level of performance was concerning enough, but by 2002, the mean score of a comparable sample had declined to 41%. A larger sample of students from across the United Kingdom completed the test in 2003 and performed similarly, obtaining a mean score of 43% (Mulhern & Wylie, 2005). In 2013, psychology, nursing and medical students in Northern Ireland achieved scores of 46%, 45% and 73% respectively on the same mathematical reasoning test (Thompson, Wylie, Mulhern, & Hanna, 2015). Notably, students' calculation performance on a subset of items from this test showed marked deterioration across the three decades (cf. Greer & Semrau, 1984). Thus, many students have incomplete knowledge of fundamental skills related to number, procedural knowledge of calculation, and related mathematical reasoning skills, and the situation seems unlikely to change.

What are the implications of weak numeracy skills for students' success in higher education?

Given the fundamental role of calculation, algebraic reasoning, graphical interpretation, proportionality and ratio, probability and sampling, and estimation in the statistical and research methods used in the social sciences, we contend that students who do not have the requisite skills to efficiently and effectively retrieve simple facts and execute basic procedures will be disadvantaged when they are expected to use these skills in the service of conceptual understanding and problem solving within their disciplines. Accordingly, higher education teachers of courses such as statistics, which require these skills, need to recognize that many students do not have adequate mathematical knowledge and skills to benefit from class instruction unless additional support is provided. Although one solution is to provide instruction in basic mathematical processes on university entry, a better solution is for students to obtain these skills before reaching university. Clearly, students who begin their university careers with weak or incomplete number knowledge and skill are vulnerable to poor performance on more advanced topics.

How and when should students master numerical skills?

Our work has shown that the efficiency of numeracy skills has declined substantially among university students in the last 25 years. Our experience teaching psychology, statistics, and cognitive science to students in a range of disciplines alerted us to this decline, and our research on basic cognitive processes in arithmetic demonstrated the extent of the problem. We argue that early attention to learning basic numerical knowledge is important in preparing students for the technologically complex future, in helping them achieve success in higher education, and in allowing them to make sound financial and health decisions. Fluency is achievable by the majority of individuals but only if it is integrated into a comprehensive program of mathematical competency.

These issues have recently been debated in the Canadian media and we stress that fluency is only one component of a well-rounded mathematical curriculum. When we speak about this issue to parents, educators, and members of the public, we use the example of word reading to draw an analogy to mathematics. If children or adults struggle to identify individual words while they are reading, they will have limited success understanding sentences or paragraphs; similarly, having to labor over basic number knowledge will inhibit their ability to learn higher-level skills. Consistent with this view, research has shown that basic fluency with number patterns, arithmetic facts, calculation procedures, and fundamental principles is an important precursor to the acquisition of algebra and other more complex skills (Siegler et al., 2012).

Fluency in numeracy skills is obtained by combining clear instruction with sufficient practice. Practice is a fundamental aspect of skill across many areas of achievement: We expect hockey players to practice passing, tennis players to practice serving, and musicians to practice scales and intervals. Furthermore, competency with basic skills builds confidence, as shown by the success of tutoring programs that focus on training fundamental skills. Learning simple arithmetic facts and principles does not have to mean painful “drill and kill” (Boaler, 2015). Well-designed instructional programs can facilitate mastery (see Solomon & Mighton, this issue) and will support continued skill development when children also have regular practice that is integrated into ongoing procedural and conceptual learning. We believe that appropriate instruction and opportunity to gain mastery of numerical skills should occur within the educational curriculum, rather than being considered the primary responsibility of parents or tutors. Numerical fluency is an important skill that young adults need so that they are fully prepared for the educational and life challenges of the information-based economy.

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