QAA

Subject Benchmark Statement

Mathematics, Statistics and Operational Research: Draft for Consultation

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Contents

How can I use this document?

This document is a Subject Benchmark Statement for mathematics, statistics and operational research that defines what can be expected of a graduate in the subject, in terms of what they might know, do and understand at the end of their studies.

You may want to read this document if you are:

- involved in the design, delivery and review of programmes of study in mathematics, statistics and operational research or related subjects
- a prospective student thinking about studying mathematics, statistics and operational research, or a current student of the subject, to find out what may be involved
- an employer, to find out about the knowledge and skills generally expected of a graduate in mathematics, statistics and operational research.

Explanations of unfamiliar terms used in this Subject Benchmark Statement can be found in the Quality Assurance Agency for Higher Education's (QAA's) glossary.[1]

[1] The QAA glossary is available at: www.qaa.ac.uk/about-us/glossary.

About Subject Benchmark Statements

Subject Benchmark Statements form part of the UK Quality Code for Higher Education (the Quality Code) which sets out the Expectations that all providers of UK higher education reviewed by QAA are required to meet.[2] They are a component of Part A: Setting and Maintaining Academic Standards, which includes the Expectation that higher education providers 'consider and take account of relevant Subject Benchmark Statements' in order to secure threshold academic standards.[3]

Subject Benchmark Statements describe the nature of study and the academic standards expected of graduates in specific subject areas, and in respect of particular qualifications. They provide a picture of what graduates in a particular subject might reasonably be expected to know, do and understand at the end of their programme of study.

Subject Benchmark Statements are used as reference points in the design, delivery and review of academic programmes. They provide general guidance for articulating the learning outcomes associated with the programme but are not intended to represent a national curriculum in a subject or to prescribe set approaches to teaching, learning or assessment. Instead, they allow for flexibility and innovation in programme design within a framework agreed by the subject community. Further guidance about programme design, development and approval, learning and teaching, assessment of students, and programme monitoring and review is available in Part B: Assuring and Enhancing Academic Quality of the Quality Code in the following Chapters:[4]

- *Chapter B1: Programme Design, Development and Approval*
- *Chapter B3: Learning and Teaching*
- *Chapter B6: Assessment of Students and the Recognition of Prior Learning*
- *Chapter B8: Programme Monitoring and Review*.

For some subject areas, higher education providers may need to consider other reference points in addition to the Subject Benchmark Statement in designing, delivering and reviewing programmes. These may include requirements set out by professional, statutory and regulatory bodies, national occupational standards and industry or employer expectations. In such cases, the Subject Benchmark Statement may provide additional guidance around academic standards not covered by these requirements.[5] The relationship between academic and professional or regulatory requirements is made clear within individual statements, but it is the responsibility of individual higher education providers to decide how they use this information. The responsibility for academic standards remains with the higher education provider who awards the degree.

Subject Benchmark Statements are written and maintained by subject specialists drawn from and acting on behalf of the subject community. The process is facilitated by QAA. In order to ensure the continuing currency of Subject Benchmark Statements, QAA initiates regular reviews of their content, five years after first publication, and every seven years subsequently.

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[2] The Quality Code, available at www.qaa.ac.uk/assuring-standards-and-quality/the-quality-code, aligns with the *Standards and Guidelines for Quality Assurance in the European Higher Education Area*, available at: www.enqa.eu/wp-content/uploads/2013/06/ESG_3edition-2.pdf.
[3] Part A: Setting and Maintaining Academic Standards, available at: www.qaa.ac.uk/assuring-standards-and-quality/the-quality-code/quality-code-part-a
[4] Individual Chapters are available at: www.qaa.ac.uk/assuring-standards-and-quality/the-quality-code/quality-code-part-b.
[5] See further Part A: Setting and Maintaining Academic Standards, available at: www.qaa.ac.uk/assuring-standards-and-quality/the-quality-code/quality-code-part-a.

Relationship to legislation

Higher education providers are responsible for meeting the requirements of legislation and any other regulatory requirements placed upon them, for example by funding bodies. The Quality Code does not interpret legislation nor does it incorporate statutory or regulatory requirements. Sources of information about other requirements and examples of guidance and good practice are signposted within the Subject Benchmark Statement where appropriate. Higher education providers are responsible for how they use these resources.[6]

Equality and diversity

The Quality Code embeds consideration of equality and diversity matters throughout. Promoting equality involves treating everyone with equal dignity and worth, while also raising aspirations and supporting achievement for people with diverse requirements, entitlements and backgrounds. An inclusive environment for learning anticipates the varied requirements of learners, and aims to ensure that all students have equal access to educational opportunities. Higher education providers, staff and students all have a role in, and responsibility for, promoting equality.

Equality of opportunity involves enabling access for people who have differing individual requirements as well as eliminating arbitrary and unnecessary barriers to learning. In addition, disabled students and non-disabled students are offered learning opportunities that are equally accessible to them, by means of inclusive design wherever possible and by means of reasonable individual adjustments wherever necessary.

[6] See further the *UK Quality Code for Higher Education: General Introduction*, available at: www.qaa.ac.uk/publications/information-and-guidance/publication?PubID=181.

About this Subject Benchmark Statement

This Subject Benchmark Statement refers to bachelor's degrees and integrated master's programmes in mathematics, statistics and operational research, including MMath type programmes.[7]

This version of the statement forms its third edition, following initial publication of the Subject Benchmark Statement in 2002 and review and revision in 2007.[8] The annex covering programmes of the MMath type was published in 2009 and is now incorporated in this statement.

Note on alignment with higher education sector coding systems

Programmes of study which use this Subject Benchmark Statement as a reference point are generally classified under the following codes in the Joint Academic Coding System (JACS).[9]

G100 (Mathematics); G110 (Pure mathematics); G120 (Applied mathematics); G121 (Mechanics (mathematical)); G130 (Mathematical methods); G140 (Numerical analysis); G150 (Mathematical modelling); G160 (Engineering/industrial mathematics); G170 (Computational mathematics); G190 (Mathematics not elsewhere classified); G200 (Operational research); G290 (Operational research not elsewhere classified); G300 (Statistics); G310 (Applied statistics); G311 (Medical statistics); G320 (Probability); G330 (Stochastic processes); G340 (Statistical modelling); G350 (Mathematical statistics); G390 (Statistics not elsewhere classified); G900 (Others in mathematical sciences).

Summary of changes from the previous Subject Benchmark Statement (2007)

This revised benchmark statement sets out academic standards for bachelor's and integrated master's degrees in mathematics, statistics and operational research including MMath programmes. As the annex to mathematics, statistics and operational research Subject Benchmark Statement covering MMath type programmes, published in 2009, was designed for use alongside the benchmark statement, rather than as a free-standing document, this review provided an opportunity to combine the documents for ease of use. Therefore all sections of this Subject Benchmark Statement are relevant to the design of an integrated master's programme.

Public awareness of the importance of mathematics and its continuing development is higher than at any time in the past, with public engagement programmes such as television shows, science festivals and popular lectures making a strong impact. The growing influence of the subject area has been considered during the review process and it is recognised that this benchmark statement may be used in conjunction with other statements, with the aim of furthering graduate employability and developing the skills and knowledge required for entry into the diverse range of careers which are now available.

[7] Bachelor's degrees are at level 6 in *The Framework for Higher Education Qualifications in England, Wales and Northern Ireland* (2008) and level 10 in the *Scottish Credit and Qualifications Framework* (2001), and master's degrees are at level 7 and level 11 respectively.
[8] Further information is available in the *Recognition scheme for Subject Benchmark Statements*, available at: www.qaa.ac.uk/publications/information-and-guidance/publication?PubID=190.
[9] Further information about JACS is available at: www.hesa.ac.uk/content/view/1776/649/.

1 Introduction

1.1 Mathematics, statistics and operational research are traditionally considered as comprising the mathematical sciences. There is a considerable amount of overlap and inter-reliance amongst these three subjects.

1.2 Although mathematics has a particularly long heritage, dating back to early civilisations, it is important to appreciate that all branches of mathematics, statistics and operational research are dynamic, vibrant subjects which permeate most areas of modern society. For example, internet search engines, mobile communications, efficient transportation of goods and trialling of new medical treatments all fundamentally rely on applications of mathematics, statistics and operational research.

1.3 In recent years, the influence of mathematics, statistics and operational research on other subjects has grown significantly. The use of mathematics, statistics and operational research in subjects such as physics, engineering, psychology and management is well established; however, much research in subjects such as biology, economics, political science and sociology is now also heavily reliant on mathematics, statistics and operational research applications.

1.4 Although mathematics, statistics and operational research is a very broad grouping of subjects, it is possible to deal with the entire subject area within a single benchmark statement, provided that users of this document bear in mind that there is very wide variability among the programmes that come within its scope. One key area of variability is in respect of what might be termed the 'style' of the programme. Some programmes are concerned more with the underlying theory of the subject and the way in which this establishes general propositions leading to methods and techniques which can then be applied to other areas of the subject. Other programmes are more concerned with understanding and applying mathematical results, methods and techniques to many parts of the overall subject area. For convenience, these different 'styles' will be referred to, respectively, as 'theory-based programmes' and 'practice-based programmes'.

1.5 While there are a few programmes that are entirely theory-based or entirely practice-based, most have elements of both approaches and there is a complete spectrum of programmes covering the range between the two extremes. It is possible for programmes with the same title to have very different emphases; it is the curriculum of a programme (rather than its title) that makes clear its position within the spectrum. It is important to note that all of these different emphases are valuable and one should not be viewed as of higher status than another.

1.6 Mathematics, statistics and operational research subjects have a cumulative nature, and the repeated revisiting of topics leading to progressively deeper understanding of the underlying principles is a prominent feature of most programmes. Consequently, learning outcomes set at programme level are considered particularly important.

1.7 Mathematics, statistics and operational research programmes have been sufficiently flexible to adapt quickly to innovations; good examples are the burgeoning interest in the areas of financial mathematics and big data. Many providers have developed new modules, and sometimes even whole new programmes, in response to such developments. This practice is to be encouraged and is indicative of the dynamic and evolving nature of mathematics, statistics and operational research.

Explanation of terms

1.8 This benchmark statement uses the term 'programmes' to mean entire schemes of study followed by learners leading to a qualification. The term 'modules' is used to mean the individual units of study that make up a programme. Some programmes are highly structured, with a closely laid down progression of modules to be taken at particular points of which many, except perhaps towards the end of the programme, are compulsory. Other programmes allow learners much more freedom to assemble a collection of modules that has a coherence relevant to an individual's preferences but which may be very different from the modules studied by another learner on the same programme.

Mathematics

1.9 Mathematics is a major intellectual subject in its own right, with a pedigree that extends back through various cultures, including the ancient Greeks and even earlier civilisations. It has its roots in the systematic development of methods to solve practical problems in areas such as surveying, mechanical construction and commerce. The subject evolved with the realisation that such methods, when stripped of the details of the particular situation, had a wide range of applications and highlighted the essential common characteristics of many different problems. Thus, generalisation and abstraction became important features of the subject. Today mathematics is a subject in which strict logical deductions are used to draw conclusions that follow with certainty from a given set of assumptions. These assumptions may be abstractions of fundamental concepts such as number, shape or symmetry, or they may be simplified models of real-world systems. While the mathematics of earlier times still remains relevant, it is now only a tiny part of an ever-expanding and dynamic subject.

1.10 The abstract study of mathematics is in itself an intellectual pursuit of value, opening up a world that contains both excitement and beauty. But the subject never loses its contact with the real world. Understanding of the world is facilitated by identifying and codifying patterns. This abstraction allows mathematicians to find deeper relationships within the patterns than could otherwise have been found from observation or unaided reasoning. This then enables common solutions to be found to problems that would otherwise have seemed unrelated. As an example, consider the topic of group theory. This subject grew out of a desire to understand and codify symmetry in the world around us, from physical shapes such as cubes and wheels, to symmetry found within numbers. Today group theory is at the heart of many different fields, such as cryptography, quantum mechanics, crystallography and image processing.

1.11 The breadth of the applicability of mathematics is immense. Mathematics has always been fundamentally important in science and technology, but in recent decades there has been an explosive growth in the use of mathematics in all areas of modern life. Mathematics is integral to how the internet works: it is used in efficient ranking of search results, in the security of electronic financial transactions and in the compression of images and videos. Mobile phones rely on mathematics to provide network coverage, to use global positioning systems and to perform fast calculations on tiny processors. Mathematics underpins medical technology such as body scanners, and is used in epidemiology to predict and manage disease outbreaks. It is used to model climate change, to design robots, to control space probes, to better understand the growth of cancer tumours and to manage financial risk.

Statistics

1.12 Statistics is the science of drawing conclusions from data. At its heart is probability theory, a branch of mathematics that is used to model uncertainty and quantify risk. However, statistics also has many other aspects, including the design of processes to collect data, methods for describing and visualising data that will reveal patterns within it, and ways of communicating conclusions to non-experts.

1.13 From the eighteenth and nineteenth centuries onwards, the results of applying statistical methods to data have had profound consequences for society. For example, modern medicine depends upon epidemiological studies and clinical trials that employ statistics in an essential way. Today's computing and communication technologies are resulting in massively larger amounts of data being collected. New statistical and probabilistic methods are being developed to understand the underlying processes producing such data, to extract information, and to predict future outcomes.

Operational research

1.14 Operational research is a more recent subject, beginning during the twentieth century. Many of its origins are to be found in the organisation of activities during the Second World War. The subject ranges from complex optimisation procedures with significant mathematical underpinning to non-mathematical but academically rigorous problem-structuring methods. It finds important applications throughout industry, business and commerce, in government, health and social services and the armed forces. For example, in healthcare, operational research has been used to determine the optimal number of beds and nurses in intensive care units; transportation applications include finding the best routes for delivery of goods to widespread locations; supermarkets use operational research in determining their inventory levels; the military employs operational research techniques in strategic analysis of deployment of personnel and equipment. The subject area of analytics has become increasingly associated with operational research in recent years and operational research has become one of the key quantitative management tools of modern times.

1.15 Operational research is perhaps less well focused as a subject than either mathematics or statistics. Model-building is crucial to operational research, and this aspect is firmly rooted in both mathematics and statistics. However, the problem-solving and decision-making aspects of the subject have a broader base and may, additionally or alternatively, draw on a range of non-mathematical subjects. For example, behavioural science and sociology might be particularly important and because of this heritage, a relatively high proportion of programmes in the operational research area are practice-based.

1.16 Although the name 'operational research' is generally well understood, a number of providers use other titles for programmes in this area, such as 'management science'. Titles of this sort often indicate very practice-based programmes, perhaps with little mathematical content. Such programmes, by virtue of their design, might not fall entirely within the subject benchmark for mathematics, statistics and operational research.

Relationships with other subjects

1.17 The very applicability of mathematics, statistics and operational research links it, to varying levels, with a huge swathe of subjects, including: the natural sciences (physics, astronomy, chemistry, biology, environmental science), all branches of engineering, computer science and informatics, economics, accountancy, actuarial science and finance as well as others in which quantitative methods are of use, such as psychology and social science. These connections may frequently be as strong as, or even stronger than, relationships within mathematics, statistics and operational research. One aspect of the depth of these connections is that experts in mathematics, statistics and operational research often become recognised experts in these other subjects. In terms of programme design, some programmes in these areas are sufficiently dependent on mathematics, statistics and operational research that this Subject Benchmark Statement is of relevance.

Career opportunities for mathematics, statistics and operational research graduates

1.18 Graduates of mathematics, statistics and operational research programmes have an extremely wide choice of careers available to them. Employers greatly value the intellectual ability, rigour, logical thinking and abstract reasoning that graduates acquire, their familiarity with numerical and symbolic thinking, and the analytic approach to problem solving that is their hallmark. These skills, when developed alongside more generic skills (such as communication and team-working skills) make mathematics, statistics and operational research graduates highly employable. The skills of mathematics, statistics and operational research graduates are explored in more detail in section 3.

1.19 Careers are readily available in areas where explicit mathematics, statistics and operational research skills are to the fore. For example, mathematics, statistics and operational research graduates work at the heart of the financial services sector and the ICT, pharmaceutical and aerospace industries. They work in the private and public sectors applying their skills to areas such as digital security, risk management, large scale decision-making and the analysis of data. The more traditional sectors of teaching and academic research remain vitally important too. The skills developed within mathematics, statistics and operational research programmes also provide numerous opportunities for graduates to find employment which is less directly related to their subject domain.

1.20 The learned societies and professional bodies in mathematics, statistics and operational research promote careers in the mathematical sciences in various ways. An extensive source of information is available to students and graduates of mathematics, statistics and operational research programmes via the website www.mathscareers.org.uk.

2 Nature and extent of mathematics, statistics and operational research

The cumulative nature of mathematics, statistics and operational research

2.1 The subjects included in mathematics, statistics and operational research are largely cumulative: what can be learned depends heavily on previously learned material, often in quite a detailed way. This applies to mathematics, statistics and operational research more than to many other subjects. Mathematics, statistics and operational research programmes are designed to allow for this with topics sequenced so that prerequisite knowledge is always taken into account. In pure mathematics in particular, the study of most advanced topics is difficult or impossible, unless appropriate, corresponding, elementary and intermediate areas have been covered. However, knowledge of applications may often be developed in parallel with underlying theory and in more practice-based programmes it may be that the understanding of a method is developed alongside its use.

2.2 The cumulative nature of mathematics, statistics and operational research strongly influences the starting point of any programme, and as a consequence, the level a graduate from the programme may be expected to attain. Different programmes are designed with different entry standards. It is right that this should be so, as the population of learners who might wish to follow mathematics, statistics and operational research programmes is itself very diverse. Different levels of knowledge, skill and ability are catered for across the subject areas as a whole.

The nature of mathematics, statistics and operational research programmes

2.3 Diversity of provision arises from the variation in breadth and depth between programmes and in respect of their position on the spectrum of style between theory-based and practice-based, and also in respect of the choices available within programmes. In some programmes, certain branches of the subject are developed in considerable depth; in others, the work advances on a broad front covering a quite wide area. Graduates from strongly theory-based programmes acquire different subject-specific knowledge and understanding from those who study the same topics in practice-based programmes. In programmes which feature some part of mathematics, statistics and operational research allied with another subject, emphasis is often given to subject knowledge which is particularly apposite to the other subject, taking any overlap of coverage into account.

2.4 An important further source of diversity is, in many cases, the influence of the research and professional interests of academic staff. While undergraduate programmes in mathematics, statistics and operational research are not generally expected to reach the frontiers of knowledge, it is a stimulating experience for a learner to be taught a subject by someone who is an active researcher or professional in the field. The choice of material presented in mathematics, statistics and operational research programmes, while mainly determined by its educational value, is nevertheless often influenced in detail by the research and professional interests of the academic staff. Naturally these are not the major driving factors behind the programme design, but they are of importance in providing a learning experience that is a vibrant and stimulating intellectual adventure.

2.5 Despite the diversity, general characteristics of all mathematics, statistics and operational research programmes may be identified. All programmes aim to provide learners with a solid understanding of basic knowledge and ideas, and a mastery of some areas.

Whether the programme is more theory-based or more practice-based, there is emphasis on the inculcation of precise understanding and the use of rigorous methods. This comes through formulating and solving challenging problems, either within mathematics, statistics and operational research itself or in other application areas.

2.6 Single honours mathematics programmes often include some modules in statistics and operational research, although these may not be compulsory. Programmes which focus on statistics or operational research include some mathematics, although the amount varies significantly depending on the focus and style of the programme.

2.7 There are many mathematics, statistics and operational research programmes that combine the study of two or all three of the separate subjects. For example, many programmes include pure mathematics, applied mathematics and statistics. The title of a programme may not make this immediately clear. Such a programme might be entitled 'mathematics' as an indication of a broad approach. At other providers this title might indicate a programme very firmly focused on theory-based mathematics.

2.8 In some programmes, learners have the opportunity to study modules outside their main subjects. Such modules can be important in mathematics, statistics and operational research programmes not only to support future employment but also because problems suitable for mathematical analysis may arise in them.

2.9 Several higher education providers offer programmes where learners spend a year, or shorter periods, in a supervised professional placement in industry, business or commerce.

2.10 Programmes are available where the principal study is within mathematics, statistics and operational research, but additionally there is a specialism in a foreign language where learners may spend up to a year abroad, studying in the language of that country.

Programmes focusing on mathematics

2.11 Programmes focusing on mathematics generally involve abstract mathematics, logical argument, applications of mathematics, mathematical modelling and problem solving, frequently facilitated by the use of mathematical software. Rigorous mathematical proof is also important, particularly in more theory-based programmes. Many programmes also contain probability theory and the fundamentals of statistics.

2.12 Applications of mathematical theories, methods and techniques are explored, either in other areas of mathematics or in distinct application areas or, in many cases, in both. Such areas may include continuum mechanics, statistics, operational research, physics, astronomy, chemistry, biology, engineering, finance, economics, actuarial science and many others. Modules covering these applications may be taught by the subject department, in which case they may or may not be the same modules that are taught by specialists in those areas. In many cases, however, the teaching is done within the mathematics department. A common example here is mathematical physics, a subject in which a significant proportion of the researchers in the UK are located in mathematics departments. Modules taught by mathematics departments tend to be different in nature and style from those on the same topics taught by another department, although they need not be.

Programmes focusing on statistics

2.13 Programmes focusing on statistics generally involve many of the areas referred to in paragraph 2.11. They also generally involve understanding and managing variability through the science of data investigation, formulating probability-based models in order to make inferences from samples, statistical theory, applications of statistics in other subject areas and communicating the results of statistical investigations. Data analysis, particularly through the use of statistical software, is universal.

2.14 Programmes in statistics feature problems taken from many application areas. These include biology, chemistry, medicine, pharmaceuticals, engineering, geography, archaeology, environmental science, actuarial science, economics, management, law and many others. Separate modules in these areas may also be available; such modules are often taught by the respective subject departments.

Programmes focusing on operational research

2.15 Programmes focusing on operational research may appear under a number of different titles, and only in some cases does the phrase 'operational research' appear in the title. Examples of other titles include business decision methods, business systems modelling and management science. Particularly in respect of the last of these, it may not be clear from the title alone whether or not it is a mathematics, statistics and operational research programme, but the curriculum clarifies this.

2.16 Programmes in operational research are inherently concerned with the processes of modelling complex and often ill-determined problems. Interpretation of the results in the original context is especially important. Mathematical and non-mathematical aspects of the modelling may both be important. In respect of the mathematical aspects, the techniques of mathematical programming are likely to be central, including linear, non-linear, integer and dynamic programming. Discrete mathematics and some of the areas of statistics referred to in paragraph 2.13 are also likely to be involved.

2.17 Programmes in operational research, like those in statistics, feature problems taken from many application areas. These include manufacturing, corporate and strategic planning, transportation, distribution, logistics and location and many others. Separate modules in these areas may also be available and are often taught by the respective subject department.

Integrated master's programmes

2.18 Integrated master's programmes are designed primarily to increase the depth, rather than the breadth, of study and better prepare students for postgraduate research studies or employment, as a substantial project is usually included.

2.19 Mathematics-led integrated master's programmes are also available with a focus on particular areas of application such as climate science, biology, business or finance.

Joint programmes

2.20 Aspects of mathematics, statistics and operational research may be found in joint programmes with many other subjects and the mathematics, statistics and operational research benchmark statement is a reference point in these cases, in so far as it can be applied to programmes in which the mathematics, statistics and operational research content is only a proportion of the whole. The mathematics, statistics and operational research benchmark statement is relevant, in a similar way, where learners assemble a programme of study from a wide selection of modules including some in mathematics, statistics and operational research subjects.

Mathematics, statistics and operational research subjects in other programmes

2.21 Mathematics, statistics and operational research subjects are unique in the extent to which they necessarily occur in programmes in other areas. For example, mathematics has a key role throughout engineering and the physical sciences, statistics is widely used in the experimental and social sciences and both statistics and operational research are important in management. The style and presentation of mathematics, statistics and operational research subjects in these other programmes is generally different from that in a mathematics, statistics and operational research programme. In many cases, the mathematics, statistics and operational research subject is taught by a mathematics, statistics and operational research department, but in others it is taught by the department in which the programme is based. In general, the mathematics, statistics and operational research Subject Benchmark Statement is unlikely to apply to mathematics, statistics and operational research teaching of this kind, although it is invariably important that the other programmes pay due attention to the place of mathematics, statistics and operational research within them.

Professional body accreditation of mathematics, statistics and operational research programmes

2.22 The Institute of Mathematics and Its Applications may accredit relevant programmes in mathematics, statistics and operational research that meet their requirements and graduates can apply for Chartered Mathematician status.

2.23 Accreditation may be awarded by the Royal Statistical Society to programmes in statistics that meet its criteria. Graduates from such accredited programmes are automatically eligible, on application, for the professional award of Graduate Statistician. Graduate Statisticians with appropriate professional experience may be eligible for the professional award of Chartered Statistician.

2.24 The OR Society does not currently accredit programmes.

2.25 The Institute of Mathematics and Its Applications, the Royal Statistical Society and the OR Society all operate individual professional recognition schemes based on either the nature of the individual's education, their work experience or a combination of both.

3 Knowledge, understanding and skills

Introduction

3.1 As discussed in section 2, the subject area of mathematics, statistics and operational research covered by this benchmark statement is very wide. Therefore the knowledge and skills that may be expected of graduates in the area are correspondingly wide ranging.

3.2 Graduates have knowledge and skills that are specific to areas within mathematics, statistics and operational research. In this statement, such knowledge and skills are referred to as subject-specific. At the higher levels of study, this knowledge and these skills naturally vary between graduates because of the different areas of the subject(s) that different students pursue. This diversity, which is a natural feature of the mathematics, statistics and operational research subject area, is to be welcomed, and must not be restricted in any way. Furthermore, it is dynamic and evolving, as programmes develop to encompass new areas of study. It is, however, possible to discern subject-specific knowledge and skills that are demonstrated by all mathematics, statistics and operational research graduates.

3.3 Although most of the foundations of knowledge and skills in mathematics, statistics and operational research are generally laid in the earlier parts of programmes, this is not exclusively the case. Equally, the earlier stages are not necessarily exclusively concerned with laying foundations; in many programmes it may be entirely proper for more advanced work or for work on applications to start at an early stage, provided always that any necessary prerequisite knowledge is in place.

3.4 No attempt has been made to construct a comprehensive listing of subject-specific knowledge for all graduates from programmes covered by the mathematics, statistics and operational research Subject Benchmark Statement. Such a listing would be far too prescriptive, may well force unnecessary and undesirable modifications in some existing programmes, and would confer no positive benefits.

3.5 Graduates from the mathematics, statistics and operational research area are expected to also have highly developed skills of a more general kind. Obvious examples are that they should be highly numerate and that most graduates are thoroughly at home with applications in computing and modelling.

Subject-specific knowledge and understanding

General principles

3.6 All graduates have knowledge and understanding of mathematical methods and techniques appropriate to their main field of study, and from a range of other areas of mathematics, statistics and operational research. In addition, most graduates have met at least one major area of application of their subject in which it is used in a serious manner and is essential for proper understanding. Different types and levels of understanding are developed in the graduates according to the focus of the programme.

Methods and techniques

3.7 All graduates have some knowledge and understanding of, and the ability to use, mathematical methods and techniques appropriate to their programme. Common ground for all programmes includes calculus and linear algebra. All programmes cover methods and techniques that pertain to a range of areas of mathematics, developed in depth according to their own requirements. As examples, graduates from programmes in operational research may have considerable knowledge of constrained optimisation and its application to allocating scarce resources, or of modelling different decision-making processes; whereas graduates from programmes concentrating on applications of mathematics in physics or engineering may have correspondingly deep knowledge of methods for dealing with differential equations. These examples have been deliberately chosen as being fairly far apart in the spectrum of mathematics, statistics and operational research programmes, but it is to be emphasised that the methods and techniques covered in them are not mutually exclusive.

Areas of mathematics, statistics and operational research

3.8 Graduates from theory-based programmes have knowledge and understanding of results from a range of major areas of mathematics, statistics and operational research. Examples of possible areas are: algebra, analysis, geometry, number theory, differential equations, continuum mechanics and mathematical physics, but there are many others. This knowledge and understanding supports the knowledge and understanding of mathematical methods and techniques, by providing a firmly developed mathematical context.

3.9 Graduates from practice-based programmes may also have knowledge of results from a range of areas of mathematics, statistics and operational research, but the knowledge is generally designed to support the understanding of models and how and when they can be applied, rather than (though this may be covered as well) providing a deep understanding of the mathematical derivation of the models.

3.10 Graduates of programmes in statistics have knowledge of core areas of mathematics and a range of major areas in statistics. Examples include exploratory data analysis, inference, likelihood, linear models, stochastic processes and time series. Graduates are able to use a statistical package for data analysis.

3.11 Graduates of programmes in operational research have experience of a wide range of applications. These may include examples from fields such as healthcare, transportation, logistics, strategic planning, manufacturing and retail distribution. It is often the case that specialised modules in these areas are available, sometimes being taught by staff from the respective subject departments.

Mathematical thinking and logical processes

3.12 Graduates have an understanding of the importance of assumptions and an awareness of where they are used and of possible consequences of their violation. This includes an appreciation of the distinction between the roles of validity of assumptions and validity of arguments.

3.13 Graduates also appreciate the power of generalisation and abstraction in developing mathematical theories or methods to use in problem solving. Theory-based programmes may tend to emphasise the role of logical mathematical argument and deductive reasoning, often including formal processes of mathematical proof; practice-based programmes may tend to emphasise understanding and use of structured mathematical or analytical approaches to problem solving.

3.14 Knowledge and understanding under this heading informs and underpins many other activities that may appear in various programmes, such as axiomatic approaches to advanced pure mathematics or the general role of modelling.

Numerical mathematics and mathematical computing

3.15 All graduates have knowledge and understanding, at the level required for their programmes, of some processes and pitfalls of mathematical approximation.

3.16 All graduates have some knowledge and understanding of mathematical computing, with direct experience of specialist software and/or of programming, with an awareness of the appropriateness of the software for the problems being addressed and, when feasible, the nature of the algorithms on which it is based.

Modelling

3.17 Modelling is the process of constructing a mathematical representation of a process or structure. All graduates are expected to have knowledge and understanding of this process. Generally the problems come from at least one application area, but they may also come from other areas within mathematics, statistics and operational research.

3.18 All graduates of practice-based programmes and many from theory-based programmes have knowledge and understanding of a range of modelling techniques and their conditions and limitations, and of the need to validate and revise models. Graduates also know how to use models to analyse, and as far as possible solve, the underlying problem or to consider a range of scenarios resulting from modifications to it, as well as how to interpret the results of these analyses.

Depth of study

3.19 All graduates have knowledge and understanding developed to higher levels in particular areas. The higher-level content of programmes is often reflected in the title of the programme. For example, graduates from programmes with titles involving statistics have substantial knowledge and understanding of the essential theory of statistical inference and of many applications of statistics. Programmes with titles such as mathematics may range quite widely over several branches of the subject, but nevertheless graduates from such programmes have treated some topics in depth. Integrated master's programmes generally include aspects of mathematics, statistics and operational research in greater depth than bachelor's programmes.

Subject-specific skills

3.20 Mathematics, statistics and operational research graduates have subject-specific skills developed in the context of a very broad range of activities. These skills have been developed to a sufficiently high level to be used after graduating, whether it be in the solution of new problems arising in professional work or in higher academic study, including multidisciplinary work involving mathematics.

3.21 Many of the subject-specific skills to be expected of all mathematics, statistics and operational research graduates are directly related to the fundamental nature of mathematics, statistics and operational research as a problem-based subject area, whether the problems arise within mathematics, statistics and operational research itself or come from distinct application areas. Thus, graduates are able to:

- demonstrate knowledge of key mathematical concepts and topics, both explicitly and by applying them to the solution of problems
- comprehend problems, abstract the essentials of problems and formulate them mathematically and in symbolic form, so as to facilitate their analysis and solution,

and grasp how mathematical processes may be applied to them; including where appropriate an understanding that this might give only a partial solution

- select and apply appropriate mathematical processes
- construct and develop logical mathematical arguments with clear identification of assumptions and conclusions
- use computers as an aid to mathematical study and for acquiring any further information
- present their mathematical arguments, using appropriate notation, and the conclusions from them with accuracy and clarity.

3.22 Graduates from programmes focused on particular branches of mathematics, statistics and operational research have other subject-specific skills that are relevant to those particular branches. An exhaustive list of such skills is not helpful, but as examples, graduates from programmes focusing on:

- pure mathematics have skills relating particularly to logical argument and solving problems in generality, and facility with abstraction, including the rigorous development of formal theories
- applied mathematics have skills relating particularly to formulating real-world problems in mathematical terms, solving the resulting equations analytically or numerically, and giving contextual interpretations of the solutions
- statistics have skills relating particularly to the design and conduct of experimental and observational studies and the analysis of data resulting from them
- operational research have skills relating particularly to the formulation of complex problems of optimisation and the interpretation of the solutions in the original contexts of the problems.

General skills

3.23 Graduates from the mathematics, statistics and operational research area acquire many general skills honed by their experiences of studying mathematics, statistics and operational research subjects. All these are essentially problem-solving subjects, whether the problems arise within mathematics, statistics and operational research itself or come from areas of application. Thus the graduates' experiences are embedded in a general ethos of numeracy and of analytical approaches to problem solving. In addition, an important part of most mathematics, statistics and operational research programmes is to take theoretical knowledge gained in one area and apply it elsewhere. The field of application is often a significant topic of study in its own right, but the crucial aspect of the process is the cultivation of the general skill of transferring expertise from one context to another.

3.24 A number of general skills are to be expected of all mathematics, statistics and operational research graduates, although in some cases they are likely to be developed more in graduates from some programmes than in others. All of these competencies enhance the general employability of mathematics, statistics and operational research graduates and include:

- study skills, particularly including the ability to learn independently
- the ability to work independently with patience and persistence, pursuing the solution of a problem to its conclusion
- skills of time management and organisation
- adaptability, in particular displaying readiness to address new problems from new areas
- the ability to transfer knowledge from one context to another, to assess problems logically and to approach them analytically

- highly developed skills of numeracy, including being thoroughly comfortable with numerate concepts and arguments in all stages of work
- information technology skills and the ability to obtain information from a variety of sources, always taking care that these sources are referred to appropriately
- communication skills, generally including the ability to work in teams, to contribute to discussions, to write coherently and to communicate results clearly
- knowledge of ethical issues, where appropriate, including the need for sensitivity in handling data of a personal nature.

3.25 In summary, the mathematics, statistics and operational research graduate attributes include an enhancement of many general skills (such as numeracy, IT skills, critical understanding and assessment of complex problems, and the ability to identify and analyse problems leading to formulation of solutions) as well as subject-specific skills (such as mathematical modelling, data analysis and numerical methods).

4 Teaching, learning and assessment

Introduction

4.1 Mathematics, statistics and operational research subjects are characterised by the need for a high degree of conceptual and abstract thinking within the learning process. While this poses considerable challenges to the learner, it also offers correspondingly great rewards. It is often intellectually very demanding to achieve understanding at the necessary degree of abstraction; yet, achieving deep understanding of complex ideas is a more than adequate reward for the effort required. To master a new concept is a satisfying experience in itself and gives the learner who has achieved it the confidence to take on similar challenges in the future. It often has direct practical benefits as well because, once a concept is understood, its particular instances are easily learned.

4.2 Mathematics, statistics and operational research graduates are rightly seen as possessing considerable skill in abstract reasoning, logical deduction and problem solving, and for this reason they find employment in a great variety of careers and professions, as discussed in section 1. Teaching and learning methods assist the development of these skills, by encouraging not merely the capacity for abstract reasoning, but also the learners' capacities for independent and self-motivated learning, problem-solving skills, communication, team-working and reflection.

4.3 A consequence of the cumulative nature of mathematics, statistics and operational research is that, to a greater extent than for many subjects, deeper learning of topics occurs as the material is re-encountered either in conscious later revision for delayed assessments, or as a consequence of later modules revisiting the material at a higher level or using it as an ingredient for more advanced work. For this reason it is important that assessment is considered from a programme perspective and end of module assessments, if they are used, are not regarded as a definitive measure of learners' final mastery.

4.4 A variety of learning, teaching and assessment methods are in use. These may differ markedly depending on the style of the programme, the subject matter, the level and progression route of the learners and the resources which are available.

4.5 It is entirely practicable for mathematics, statistics and operational research programmes to be delivered through distance learning. As with all programmes it is necessary for mechanisms to be in place to support students in learning and assessment.

4.6 As in all subjects, mathematics, statistics and operational research teaching has developed, following changes in wider educational practice. This situation is continually evolving and departments continue to respond to changing circumstances. Innovation in learning, aimed at improvement, is to be welcomed, with the outcomes critically evaluated and disseminated.

Teaching and learning

4.7 Mathematics, statistics and operational research programmes encourage students to take responsibility for their own learning; foster knowledge of and enthusiasm for the subject; stimulate engagement and active participation in the learning process; and develop resilience and the ability to communicate information about the subject in written and oral form. They encourage learning in depth, and also encourage students to reflect on and take responsibility for their own learning and to develop their academic self confidence.

4.8 Learners within mathematics, statistics and operational research are likely to meet a range of learning activities including, importantly, regular work on practice exercises and problem solving, as this is key to achieving good understanding. Programmes generally include lectures and smaller group teaching, unless based on distance learning where alternate formats may occur. The detailed nature of these, and of any other types of activities that are used, are determined appropriately for particular subject areas within mathematics, statistics and operational research, bearing in mind the style of the programme. Some providers may choose to develop some skills by activities such as placements or ancillary modules.

4.9 E-learning in various forms is widely used in mathematics, statistics and operational research programmes. The range of online resources to support teaching and learning continues to expand and the use of technology and specialist software to carry out technical mathematics, statistics and operational research work has become extremely widespread and is to be expected in mathematics, statistics and operational research programmes. Examples of this include the use of general computer algebra systems and specialised programs for particular areas of algebra; the use of sophisticated programs for advanced numerical analysis and numerical solution of equations; the use of statistical packages for data analysis and model building; and the use of mathematical programming software for formulating and solving operational research problems. This is in addition to the use of standard spreadsheets, graphics systems and specialised systems for mathematical typesetting.

4.10 Providers may choose to adopt a range of learning and teaching methods. As part of the nature of the subject area, learners benefit from seeing arguments developed by tutors in 'real time', with appropriate emphases. It is common to present extended arguments and this is likely to require considerable space for display of the tutor's material. Teaching spaces have appropriate facilities that allow both the development of extended mathematical arguments (requiring space) and effective projection equipment. In particular, it is possible to display digital material such as output from specialist software at the same time as mathematical/statistical arguments that are being developed. Traditional board-based lectures continue to be widely used, often augmented by more interactive teaching approaches. Lectures may be supported by problem classes, tutorials, workshops and seminars. Further methods that providers may use include, for example, group work, peer assessment and projects. Whatever methods are chosen, they are directed at providing engaging and stimulating learning experiences for students.

Assessment

4.11 As with learning activities, students undergo a range of methods of assessment, depending on the knowledge and skill being assessed. It is expected that assessment is carried out according to context and purpose, recognising that learners may exhibit different aptitudes in different forms of assessment. E-assessment finds widespread application, particularly for formative work.

4.12 The assessment of mathematics, statistics and operational research subjects is not necessarily restricted to the assessment of mathematical knowledge and understanding alone. Many programmes also assess other attributes, for example the ability to use mathematical ideas in the context of an application, the ability to carry out an extended statistical investigation or the ability to communicate effectively in written and oral form within the context of mathematics, statistics and operational research and more generally, or the ability to work effectively as part of a team.

4.13 Setting assessment tasks in mathematics, statistics and operational research subjects requires a great deal of professional judgement. Questions in written examinations are developed with great care to be appropriate in level and content.

Marking of mathematics, statistics and operational research assessments often requires considerable professional judgement and is not, as is often thought, entirely deterministic. For example, where a learner describes a model-building process or discusses the results of an analysis of some data, professional judgement of the worth of the learner's response is an inherent and essential part of the assessment process. In addition, professional judgement is required in awarding appropriate grades to solutions that are flawed but not wholly incorrect, or which use the correct method but with only partial success.

4.14 For many mathematics, statistics and operational research assessments, the variation in marks achieved may be greater than in other subject areas. Particularly in written examinations and tests, perfect or near-perfect solutions meriting very high marks arise more frequently than in most other subjects. On the other hand, the problem-solving nature of mathematics, statistics and operational research can result in some learners having difficulty even starting some questions.

4.15 It is also an inherent characteristic of the subject that an individual student's performance may vary greatly over different modules. Some students, of course, do uniformly well in all modules. However, even the best students frequently find some particular area(s) of mathematics, statistics and operational research difficult to grasp, and this may lead to quite low marks in a profile that is clearly of overall excellence with consistently very good performances elsewhere. In such a case, a department’s internal and external examiners might well judge the student to be of high quality overall.

4.16 In similar vein, some students may present profiles where a number of modules are failed, perhaps only narrowly, whereas other modules are passed at a comfortable if moderate level. The examiners might well judge that such a student, considered overall, has demonstrated a reasonable knowledge of the subject and that there is positive achievement worthy of reward.

4.17 For these reasons, overall assessment of mathematics, statistics and operational research students often relies on averaging or preponderance systems. In such systems, an overall view is taken of a student's achievements. A level of success on each individual module that is commensurate with the overall performance might not be required, and it is entirely acceptable that this should be so. Global assessment regulations might be less applicable in mathematics, statistics and operational research than elsewhere and could seriously infringe professional judgement.

5 Benchmark standards

Introduction

5.1 The subject area covered by the mathematics, statistics and operational research benchmark statement is very wide, and so the standards that may be expected of graduates in the area can only be specified in a fairly general way.

5.2 Benchmark standards for mathematics, statistics and operational research are defined at threshold levels of competence for both a bachelor's degree with honours and an integrated master's degree. It is intended that students meet these standards in an overall sense, not necessarily in respect of each and every statement listed.

5.3 For mathematics, statistics and operational research, standards above the threshold are demonstrated through the depth of the student's understanding of concepts or techniques, the breadth of the student's knowledge, the amount of support and guidance the student requires to undertake an extended task, the complexity of the problems that the student can solve or model, the student's ability to construct and present a reasoned argument or proof and how far the student can progress through it, and the facility with which the student performs calculations or manipulations.

Benchmark standards for honours degrees

5.4 A graduate who has reached the bachelor's degree with honours threshold level should be able to demonstrate:

i. a reasonable understanding of the basic body of knowledge for the programme of study, normally including calculus and linear algebra
ii. a reasonable level of skill in calculation and manipulation within this basic body of knowledge and some capability to solve problems formulated within it
iii. application of core concepts and principles in well defined contexts, showing judgement in the selection and application of tools and techniques
iv. an understanding of logical arguments, identifying the assumptions made and the conclusions drawn
v. a familiarity with the notion of mathematical modelling and a reasonable level of skill in comprehending problems, formulating them mathematically and obtaining solutions by appropriate methods
vi. an ability to communicate straightforward arguments and conclusions reasonably accurately and clearly
vii. competent use of appropriate computer technology in mathematics
viii. the ability to manage their own learning and make use of appropriate resources.

Benchmark standards for master's degrees

5.5 A graduate who has reached the integrated master's degree threshold level should be able to demonstrate:

i. a good understanding of the main body of knowledge for the programme of study including some advanced topics
ii. a very good level of skill in calculation and manipulation of the material within this body of knowledge, and be capable of solving complex problems formulated within it
iii. application of a range of concepts and principles in loosely defined contexts, showing good judgment in the selection and application of tools and techniques

iv. a high level of capability in developing and evaluating logical arguments
v. familiarity with the notion of mathematical modelling, and ability to abstract the essentials of problems, formulating them mathematically, obtaining solutions by appropriate methods and interpreting these solutions
vi. confident communication of arguments and effective and accurate conveyance of conclusions
vii. effective use of appropriate computer technology in mathematics
viii. the ability to work competently and independently, to be aware of own strengths and to understand when help is needed
ix. competence in planning and developing an advanced project themed in mathematics, statistics and operational research.

Appendix: Membership of the benchmarking and review groups for the Subject Benchmark Statement for mathematics, statistics and operational research

Membership of the review group for the Subject Benchmark Statement for mathematics, statistics and operational research (2014)

Chair

Professor Duncan Lawson (Chair)	Newman University

Higher education provider representatives

Professor David Arrowsmith	Queen Mary University of London
Dr Toby Bailey	University of Edinburgh
Professor Jeff Griffiths	Cardiff University
Dr Mary McAlinden	Oxford Brookes University, Higher Education Academy
Professor Andrew Osbaldestin	University of Portsmouth
Professor Alice Rogers	King's College London
Professor Charles Taylor	Leeds University
Dr Jon Warren	Warwick University

Employer representative

Dr Mark Gittoes	HEFCE

Student reader

Justine Edwards	University of South Wales

QAA officer

Brigitte Stockton	Quality Assurance Agency for Higher Education

Membership of the benchmarking group for the annex to the Subject Benchmark Statement for mathematics, statistics and operational research (2009)

Details provided below are as published in the intial publication of the annex.

Professor Adrian Bowman	Royal Statistical Society
Professor Peter Giblin	Heads of Departments of Mathematical Sciences in the UK
Mr Gerald Goodall	Royal Statistical Society
Mr Michael Grove	Higher Education Academy Subject Centre for mathematics, statistics and operational research
Dr Stuart Johns	Operational Research Society
Professor Duncan Lawson (Chair)	Higher Education Academy Subject Centre for mathematics, statistics and operational research
Dr Niall MacKay	London Mathematical Society
Dr David Salinger	Heads of Departments of Mathematical Sciences in the UK
Professor Nigel Steele	Institute of Mathematics and its Applications
Dr Brian Stewart	London Mathematical Society
Dr David Stirling	Institute of Mathematics and its Applications

Membership of the review group for the Subject Benchmark Statement for mathematics, statistics and operational research (2007)

Details provided below are as published in the second edition of the Subject Benchmark Statement.

Professor Peter Giblin	Heads of Departments of Mathematical Sciences in the UK
Gerald Goodall	Royal Statistical Society
Michael Grove	Higher Education Academy Subject Centre for mathematics, statistics and operational research
Dr Stuart Johns	Operational Research Society
Professor Duncan Lawson (chair)	Higher Education Academy Subject Centre for mathematics, statistics and operational research
Dr Niall MacKay	London Mathematical Society
Professor Nigel Steele	Institute of Mathematics and its Applications

Membership of the original benchmark statement group for mathematics, statistics and operational research (2002)

Details below are as published in the original Subject Benchmark Statement for mathematics, statistics and operational research.

Professor Rob Archbold	University of Aberdeen
Professor Russell Cheng	University of Southampton
Professor Neville Davies	The Nottingham Trent University
Dr John Erdos	King's College London
Dr Judy Goldfinch	Napier University
Mr Gerald Goodall	The Royal Statistical Society
Mr Tony Palmer	De Montfort University
Professor Chris Robson (Chair)	University of Leeds
Dr Stephen Ryrie	University of the West of England, Bristol
Professor Peter Saunders	King's College London
Dr Stephen Siklos	University of Cambridge
Professor Joan Walsh	University of Manchester (retired)

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Southgate House, Southgate Street, Gloucester GL1 1UB

Tel: 01452 557 000
Email: enquiries@qaa.ac.uk
Website: www.qaa.ac.uk

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