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Decomposing Global Yield Curve Co-Movement*

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Abstract

This paper explains the co-movement of global yield curve dynamics using a Bayesian hierarchical factor model augmented with macro fundamentals. Our novel modeling approach reveals the relative importance of global shocks through two transmission channels: the policy and risk channels. Global inflation is the most important traditional macro fundamentals for international yields and operates through a policy channel. Economic uncertainty and sentiment are also important in driving global yield co-movements, through a risk channel.

Keywords: Global Yield Curves, Co-Movement, Transmission Channels, Global Fundamentals, Sentiment, Economic Uncertainty, Bayesian Factor Model.
JEL Classification Codes: C11; C32; E43; F3; G12; G15.

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1 Introduction

Factor models are widely used to analyze the term structure from a domestic and, increasingly, from an international perspective.[1] Co-movement of international bond yield curves has been highlighted by the ground breaking work of Diebold, Li and Yue (2008) and Jotikasthira, Le and Lundblad (2015). An important issue addressed by these papers is what macro fundamentals drive common movements in international yields. Diebold, Li and Yue (2008) indicate that the first latent factor in US, UK, German and Japanese yields is correlated with G7 inflation and the second latent factor in yields is correlated with G7 economic growth. Jotikasthira, Le and Lundblad (2015) combine latent factors and macro fundamentals to explain yields, suggesting global inflation and the US level factor explain over 70% of UK and German yields, although they concede their two-step estimation procedure may be less efficient than a one-step estimation approach.

Given that yield curves co-move internationally, how much of the variance in global bond yield co-movement is driven by global factors? Are global factors more readily identifiable as macro fundamentals or latent information? Moreover, is it worthwhile extending the potential economic explanations for global yields beyond inflation and economic growth? To answer these questions we introduce an innovative econometric approach to model international bond yield co-movement and determinants.

Our approach is innovative to the global bond yield literature for the following reasons. Firstly, to analyze the global term structures of seven advanced economies, we extend the dynamic hierarchical factor model of Moench, Ng and Potter (2013) and explicitly incorporate global macro variables. We consequently identify latent factors in an international yield curve model in an efficient one-step estimation procedure with the help of global macro fundamentals. This allows us to be agnostic on whether macro fundamentals, la-

[1] See Bernanke and Boivin (2003), Kose, Otrok and Whiteman (2003) and Moench, Ng and Potter (2013) for earlier factor models.

tent information or both drive yields internationally, letting the data speak for itself.[2] Secondly, and as alluded to by Piazzesi and Schneider (2007), it may be important to consider an extended group of potential determinants to explain yield movements. We extend the set of potential economic explanations beyond the conventional inflation and economic growth. In particular, we assess the relevance of sentiment and economic uncertainty for yields, consistent with the work of Kumar and Lee (2006), Bansal and Shaliastovich (2010), Benhabib and Wang (2015) and Bloom (2014).

Our results parallel and extend the work on the determinants of international yield co-movement from Diebold, Li and Yue (2008) and Jotikasthira, Le and Lundblad (2015). We find that two global yield factors can explain, on average, more than 60% of bond yields' variance across seven countries, and country-specific components contribute to most of the remaining variance. Global inflation and a global Level factor explain over 70% of the co-movement. A new finding is that alternative determinants like sentiment and economic uncertainty matter for the movements of the global Level factor, which is strongly supported by regression results.

In the spirit of Wright (2011) and Jotikasthira, Le and Lundblad (2015), we perform a simple but informative decomposition of two transmission channels. These channels contribute to co-movement in the global term structures of interest rates. The first channel is a central bank *policy channel*, reflects the expectation of future short rates and has a strong link to global inflation. Our empirical evidence highlights that the importance of the policy channel is increased in the global financial crisis, potentially because of a strong policy reaction during this period. The second channel is a *risk compensation channel*, reflects yield term premia and can be mostly explained by the global Level factor.[3] Our

[2] Our hierarchical model has three levels. At the global level, we allow global macroeconomic fundamentals to interact with global bond factors. At a lower level, national bond factors are driven by global bond factors and country-specific components. At the lowest level, the term structure of each country is driven by national bond factors and idiosyncratic noise.

[3] Jotikasthira, Le and Lundblad (2015) emphasize the risk channel significantly impacts US, UK and

results imply that the global risk premia is mainly driven by sentiment and economic uncertainty in the last 20 years.

Our proposed model has several salient features. The model-implied global short rate expectations do not violate the zero lower bound (ZLB) without imposing any hard restrictions, as we employ a plausible identification strategy for the underlying interest rate dynamics. Our results are robust to alternative specifications. With the augmentation of global macro fundamentals, our results are not sensitive to the number of yield factors specified in our model. We extend Moench, Ng and Potter (2013) by introducing an unrestricted covariance matrix in the global dynamics, which allows us to better incorporate time-series information while maintaining a good cross-sectional fit. This extension also helps reconcile the *macro spanning condition* with a phenomenon in the global interest rate data.[4]

The structure of the paper is as follows. In Section 2 we introduce the model and describe estimation and identification. In Section 3 we describe the data and present preliminary analysis. In Section 4 we report our core empirical results. In particular, we decompose the yield co-movements into two transmission channels and highlight the roles of global inflation and global Level factor. We assess the relationship between the global Level factor and measures of sentiment and economic uncertainty. In Section 5 we perform robustness checks by testing whether the results are sensitive to the macro spanning condition. In Section 6 we conclude and summarize the implications of this analysis.

German yields, using the data before the 2008 global financial crisis.

[4]See Joslin, Priebsch and Singleton (2014) and Bauer and Rudebusch (2015) for detailed discussion about the *macro spanning condition*. We test the robustness of our results to this condition and discuss the implication in our context in the online appendix.

2 The Yield Curve Model

To analyze global bond yields, a factor methodology is expedient. The framework can model bond yields across countries, using both macro and latent factors. In the spirit of multi-level factor models, Kose, Otrok and Whiteman (2003) propose a dynamic model to study international business cycle co-movements, and Moench, Ng and Potter (2013) propose a hierarchical model to study the US housing market.[5] We adopt the hierarchical factor structure, because there are fewer parameters to estimate and greater efficiency. A low dimension of factors are used in this structure, making it attractive for bond yield modeling. Moreover, we extend Kose, Otrok and Whiteman (2003) and Moench, Ng and Potter (2013), and allow the dynamic factors to interact with each other at the global level.

Extending the bond yield factor structure of Joslin, Priebsch and Singleton (2014) and Coroneo, Giannone and Modugno (2015), our model for global bond yields X_{ibt} can be written as:[6]

$$X_{ibt} = \Lambda_{ib}^{F} F_{bt} + e_{ibt}^{X}, \tag{2.1}$$

$$F_{bt} = \Lambda_{b}^{G} G_{t} + e_{bt}^{F}, \tag{2.2}$$

$$\begin{bmatrix} G_t \\ M_t \end{bmatrix} = \psi^{G} \begin{bmatrix} G_{t-1} \\ M_{t-1} \end{bmatrix} + u_t, \tag{2.3}$$

in which the subscript i indicates the maturities of bond yields, the subscript b indicates

[5] Kose, Otrok and Whiteman (2003) identify regional factors that are uncorrelated with the global factors, while Moench, Ng and Potter (2013) aim to find the global factors driving the regional factors. In fact, two frameworks are compatible and Moench, Ng and Potter (2013) can be considered nested in Kose, Otrok and Whiteman (2003).

[6] The nominal short rate follows a Taylor-type rule in our parsimonious model structure without an explicit imposition of the *macro spanning condition*. Interested readers can consult the online appendix for the economic implications. We test the robustness of our results to an alternative specification in Section 5.

the countries and the subscript t indicates periods of time. In the above, Λ_{ib}^F, Λ_b^G and ψ^G are model parameters, and e_{ibt}^X, e_{bt}^F and u_t are error terms. Note that each element in e_{ibt}^X and e_{bt}^F follows a first order autoregressive process, and we do not assume homoskedastic innovations for these error terms; the covariance matrix of u_t is unrestricted. In the country-level Equation (2.1), X_{ibt} represent the bond yield of country b at maturity i, and F_{bt} are the latent yield factors of country b. In Equation (2.2), G_t are the latent global yield factors that drive the national yield factors F_{bt}. Finally Equation (2.3) describes the interactions between the yield factors and the global macro fundamentals M_t using a Vector Autoregression (VAR).[7]

After some algebra, our system can be rewritten as a simple equation showing that bond yield variance is driven by three levels of innovations:

$$X_{ibt} = f_{ibt}^G(G_{t-1}, M_{t-1}, u_t) + f_{bt}^F(e_{bt}^F) + e_{ibt}^X, \tag{2.4}$$

where f^G and f^F are linear functions which can be mapped from the coefficients of our model. In our model, we include four global macro variables extracted from national data: monetary policy rate, inflation, real activity and financial conditions, such that M_t is a 4×1 vector. The former three are standard macro fundamentals in term structure modeling, see for example, Ang and Piazzesi (2003). Additionally, we include financial conditions because liquidity and credit risk measures are suggested by Dewachter and Iania (2012).

A key feature of our model is to augment the VAR system of global yield factors with global macro factors M_t. By extending the 'Dynamic Hierarchical Factor Model' proposed by Moench, Ng and Potter (2013), the proposed model captures the interdependencies among global macro variables and pricing factors. The dynamics of the global factors are characterized by an unrestricted Factor Augmented Vector Autoregressive (FAVAR)

[7]When referring to global macro fundamentals, 'fundamental' and 'factor' are used interchangeably in this paper.

model. Factor augmentation has various advantages as suggested by Bernanke and Boivin (2003), and it is also of importance in the context of this paper. Global macro factors are incorporated to provide an economic interpretation of yield movements and exploit the underlying dynamics. Moreover, incorporating the information drawn from a large set of variables is helpful to negate the potential non-fundamentalness of the VAR, as suggested by Fernández-Villaverde and Rubio-Ramírez (2007) and Leeper, Walker and Yang (2013). The extended version of the hierarchical model is denoted as 'Fundamentals-Augmented Hierarchical Factor Model' (FHF). Technical details of our FHF are summarized in the online appendix.

The model proposed in this paper has a similar structure to Diebold, Li and Yue (2008) but contrasts in that we consider both latent and macro fundamentals in a one-step approach. A one-step Bayesian technique provides more accurate estimates for the following reasons. Diebold, Rudebusch and Aruoba (2006) and Pooter (2007) provide evidence that a one-step approach produces more effective estimates. Two-step estimation introduces bias if it does not fully consider the dynamics of the factors at a higher level. As shown in the previous literature, directly introducing macro fundamentals can provide a meaningful narrative which delineates the macro shocks that drive global term structures. Our hierarchical one-step framework allows us to jointly estimate the global yield factors and country-specific components, and hence builds upon the contribution of Bauer and Diez de los Rios (2012), Abbritti et al. (2013) and Jotikasthira, Le and Lundblad (2015). Identification schemes of structural shocks can be directly introduced in this one-step approach and posterior coverage intervals are readably available, without running additional regressions that can potentially introduce bias.

2.1 Identification

To identify the global factors, a standard approach is the Principal Component method, but this only considers cross-sectional properties and hence may not fully reveal underlying time-series dynamics or structural shocks. In this paper therefore we use an alternative identification scheme and specify a factor structure. While Moench, Ng and Potter (2013) use zero restrictions which are of a statistical nature, we impose restrictions implied by the dynamic Nelson-Siegel (NS) term-structure model. In other words, the loadings of country-level factors are exactly the same as in Diebold, Li and Yue (2008). The NS identification scheme is popular in term structure modeling, and we choose this scheme to fix ideas.[8]

We closely follow Diebold, Li and Yue (2008) to impose cross-sectional restrictions and only specify two global factors, as Bauer and Hamilton (2015) suggest that only the Level and the Slope factors are robust predictors of excess bond returns. This specification is echoed by the study of Jotikasthira, Le and Lundblad (2015). Moench (2012) and Abbritti et al. (2013) posit that an additional factor (Curvature) is helpful in revealing the term premium dynamics. Indeed, without macro information, the term premium dynamics in our sample varies substantially if the Curvature factor is added. Reassuringly adding global macro fundamentals ensures term premium dynamics are not sensitive the number of factors, allowing a more parsimonious parameterization.[9] This is due to the nature of our identification strategy that the identified factors incorporate the time-series information of global macro fundamentals. These fundamentals are weakly identified from bond yields, but are helpful in characterizing the global dynamics when included in our

[8] The details of the restrictions can be found in the online appendix. The two schemes, Diebold, Li and Yue (2008) and Moench, Ng and Potter (2013), share similar results. In fact, the identified factors from two schemes are nearly identical subject to rotations. For more information regarding factor identification we refer the reader to Bai and Wang (2015).

[9] The results are qualitatively and quantitatively similar with a resonantly small number of factors (≤ 4), and are available upon request.

system.

We estimate our model using a Bayesian estimation technique, specifically, Gibbs sampling. Following Moench, Ng and Potter (2013), we assume that the prior distribution for all factor loadings coefficients is Gaussian, and the prior distribution for the univariate variance parameters is a scaled inverse chi-square distribution.[10] These conjugate priors simplify the estimation problem, both mathematically and computationally. For the FAVAR of global dynamics we use uninformative priors, see Koop and Korobilis (2009). In the Gibbs sampling, we begin with $50,000$ burn in draws and then save every 50th of the remaining $50,000$ draws. These 1000 draws are used to compute posterior means and standard deviations of the factors, as well as the posterior coverage intervals in the following sections.

We identify global structural shocks using the Cholesky decomposition. The ordering of our global VAR system is the following: economic growth, inflation, the policy rate, financial condition index (FCI), Level and Slope factors. The ordering of the first three variables is standard in the related literature, for example Christiano, Eichenbaum and Evans (2005). These three are followed by FCI, Level and Slope, such that these fast-moving variables can react to the contemporaneous macro shocks of the first three variables. The Level and Slope are placed the lowest in the ordering because Hubrich et al. (2013) argue that the bond yields react immediately to policy change and liquidity conditions, but the monetary policy only react to asset price movements if there are prolonged. It is worth noting that we do not find a significant difference using alternative orderings.

[10] The specified prior distributions are $N(0,1)$ and Scale-inv-$\chi^2(0.4, 0.1^2)$ for loading and variance parameters, respectively.

2.2 Decomposing Transmission Channels

Based on our hierarchical model structure, we employ a novel scheme to decompose the variance of long rates driven by global factors into two channels, in the spirit of Wright (2011) and Jotikasthira, Le and Lundblad (2015). The first channel is the influence on the current short rate and expected future short rates. The current short rate and future short rate expectations are closely connected to monetary policy, so we regard this channel as the *policy channel*. The movements in the policy channel are in line with the Expectation Hypothesis. The second channel is the *risk compensation channel*, and this accounts for bond market risk compensation at longer maturities. Risk compensation is frequently call the 'term premia', which is the difference between the actual long yield and the Expectation Hypothesis consistent long yield.

More formally, we denote $y_t(\tau)$ as the global-driven yield at time t for a bond of τ-period maturity, i.e. $y_t(\tau) = f^G_{\tau bt}(G_{t-1}, M_{t-1}, u_t)$. Our decomposition can be described by

$$y_t(\tau) = y_t(\tau)^{EH} + TP_t(\tau). \tag{2.5}$$

The first term in the above equation is the Expectations Hypothesis (EH) consistent bond yield, which is equal to the average of expected short yields $E_t y_{t+i}(1)$. $y_t(\tau)^{EH}$ is given by:

$$y_t(\tau)^{EH} = \frac{1}{\tau}\sum_{i=0}^{\tau-1} E_t y_{t+i}(1) = f^{EH}(\mu_t), \tag{2.6}$$

where f^{EH} is a linear function, and μ_t collects the identified global structural shocks. The time-varying term premium is therefore,

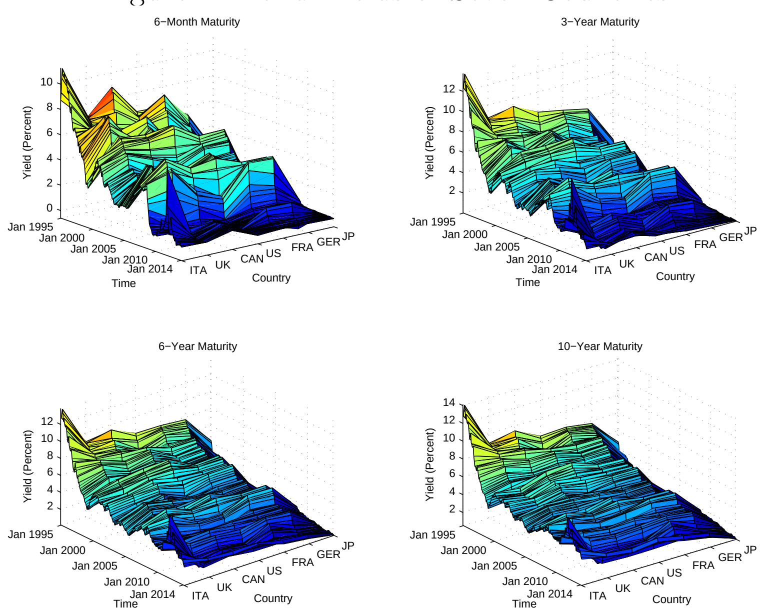
$$TP_t(\tau) = y_t(\tau) - y_t(\tau)^{EH} = f^{TP}(\mu_t), \tag{2.7}$$

where f^{TP} is a linear function. In summary, the policy channel determines expected short rates while the risk compensation channel accounts for movements in the term premia. See the online appendix for technical details.

3 Data Description and Preliminary Evidence

We obtain monthly bond yield data from Bloomberg for seven advanced countries: Canada, France, Germany, Italy, Japan, the UK and the US. The empirical analysis focuses on government yields of eleven maturities: 3, 6, 12, 24, 36, 48, 60, 72, 84, 96 and 120 months. Figure 1 shows the dynamics of bond yield at four maturities across all seven countries. All four maturities trend down from the beginning of the sample period, with the shorter rates displaying greater variance across time and countries.

Figure 1: Bond Yields of Seven Countries



Notes:
1. The above charts plot the bond yields for the seven countries in the sample. The sample includes Italy (ITA), Canada (CAN), France (FRA), Germany (GER), Japan (JP), the UK and the US, spanning from Dec. 1994 to Mar. 2014.
2. From top left clock-wise we have bond yields of maturities 6 months, 3 year, 10 years and 6 years. More information about the data is provided in the online appendix.

Our empirical model uses macroeconomic variables from Bloomberg, and indicators of financial condition from St. Louis Federal Reserve Economic Data (FRED). We construct four global macro factors using a list of macro fundamentals among the seven countries, and the fundamentals include inflation (CPI), Industrial Production (IP) and the change in monetary policy rates (PR). We also use a large number of country-specific series of Financial Condition Index (FCI) to construct a global FCI. The full sample of monthly data is from December 1994 to March 2014. The details about the data are described in the Data Appendix C.

Before we implement our one-step estimation, the global macro factors M_t are extracted from country-specific macro series. There are four categories of country-specific macro series: economic growth, inflation, change in policy rate and Financial Condition Index (FCI). We employ a new approach proposed by Koop and Korobilis (2014) to extract the global macro indicators from country-specific series.[11]

In Table 1 we report summary statistics for bond yields at representative maturities. All yields increase with maturity, suggesting positive term spreads. Volatility generally decreases with maturity. All seven countries' yields are highly persistent, with first-order autocorrelation greater than 0.95. Japanese average yields are the lowest, usually below two percent and are less persistent when compared to other yields.

[11]Nevertheless, our main results are robust to the measure of global macro factors using Stock and Watson (2002) or the measure from the OECD database. Koop and Korobilis (2014) is preferred as the explanatory power of the factors for bond yields is stronger. The online appendix displays the estimated macro factors used to augment our proposed model.

Table 1: Descriptive Statistics of Bond Yields

Country	Maturity	Mean	Std. Dev.	Min.	Max.	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
US	3	2.82	2.27	0.01	6.39	0.99	0.74	0.26
	12	3.04	2.26	0.10	7.20	0.98	0.76	0.30
	60	3.92	1.88	0.59	8.03	0.97	0.76	0.42
	120	4.57	1.45	1.54	8.00	0.97	0.72	0.43
UK	3	3.91	2.32	0.28	7.50	0.99	0.77	0.47
	12	4.00	2.36	0.12	7.45	0.99	0.77	0.48
	60	4.51	2.00	0.58	8.94	0.98	0.75	0.43
	120	4.85	1.66	1.57	8.90	0.97	0.72	0.29
Germany	3	2.49	1.52	0.00	5.14	0.98	0.66	0.27
	12	2.63	1.53	0.01	5.82	0.98	0.64	0.25
	60	3.48	1.55	0.33	7.47	0.97	0.67	0.35
	120	4.17	1.46	1.22	7.69	0.97	0.70	0.35
France	3	2.63	1.68	0.01	7.93	0.98	0.56	0.21
	12	2.76	1.66	0.02	7.04	0.97	0.58	0.22
	60	3.67	1.49	0.69	7.87	0.96	0.61	0.29
	120	4.42	1.32	1.85	8.14	0.96	0.63	0.30
Italy	3	3.44	2.58	0.28	11.00	0.98	0.63	0.24
	12	3.73	2.50	0.60	11.74	0.98	0.57	0.17
	60	4.90	2.39	1.95	14.01	0.96	0.51	0.11
	120	5.61	2.22	3.42	14.14	0.97	0.54	0.09
Canada	3	3.10	1.91	0.21	8.88	0.96	0.59	0.28
	12	3.36	1.90	0.49	8.88	0.97	0.64	0.33
	60	4.23	1.80	1.19	9.40	0.97	0.74	0.45
	120	4.75	1.69	1.72	9.48	0.97	0.74	0.41
Japan	3	0.25	0.34	0.00	2.24	0.89	0.28	0.07
	12	0.31	0.37	0.01	2.48	0.89	0.39	0.07
	60	0.91	0.66	0.13	4.07	0.92	0.57	0.17
	120	1.66	0.77	0.55	4.79	0.94	0.60	0.18

Notes: This table presents descriptive statistics for monthly yields at different maturities across seven countries. The sample period is 1994:12–2014:03. We use the following abbreviations. **Std. Dev.**: Standard Deviation; **Min.**: Minimum; **Max.**: Maximum; $\boldsymbol{\hat{\rho}(k)}$: Sample Autocorrelation for Lag k.

3.1 Variance Decomposition of Model Hierarchies

As mentioned above, we identify two latent pricing factors for each country, which can account for the majority of bond yield variance. The global Level factor in our model drives the national Level factors. Similarly the global Slope drives national Slope factors. Table 2 displays the importance of the global innovations ($Share_G$), country-specific innovations ($Share_F$) and idiosyncratic noise ($Share_X$) from Eq. (2.3), (2.2) and (2.1) respectively, relative to the total variation in the data of each country. It is clear that the global factors explain the vast majority of country yields: $Share_G$ is greater than 0.6 for almost all countries.[12] Consequently, this characteristic leads us to believe the co-movement of international bond yields is generally very strong and dominates national or idiosyncratic movements. The evidence is consistent with the importance of the global factors found in Diebold, Li and Yue (2008) and Jotikasthira, Le and Lundblad (2015). As the global factors account for a large proportion of the information in national term structures, we are interested in the dynamics of the two global factors, Level and Slope, and seek to provide sensible economic interpretations for the factors in this study.

Table 2: Decomposition of Variance of Hierarchies

Country	Posterior Mean (Std. Dev.)		
	Global $Share_G$	Country $Share_F$	Idiosyncratic $Share_X$
US	0.75(0.07)	0.24(0.07)	0.01(0.00)
UK	0.85(0.05)	0.13(0.04)	0.02(0.01)
Germany	0.74(0.07)	0.22(0.06)	0.04(0.01)
France	0.76(0.07)	0.22(0.06)	0.02(0.00)
Italy	0.36(0.10)	0.63(0.10)	0.01(0.00)
Canada	0.71(0.07)	0.27(0.07)	0.02(0.00)
Japan	0.68(0.08)	0.30(0.07)	0.03(0.01)

Notes: This table summarizes the decomposition of variance for the three-level hierarchical model of bond yields. It displays the importance for yields of the global ($Share_G$), country-specific ($Share_F$) components and idiosyncratic noise ($Share_X$) using Eq. (2.3), (2.2) and (2.1), respectively. The quantities are averaged over all maturities. Parentheses $(\cdot)$ contain the posterior standard deviation of shares in a specific block. Std. Dev. denotes the posterior standard deviation of the posterior mean.

Although global factors clearly dominate yields, national factors remain important. The variance explained by country-specific components (i.e. $Share_F$) is non-trivial and more than two standard deviations from zero. This in turn implies, that the sum of $Share_G$ of global factors and $Share_F$ of country-specific components account for $96-99\%$

[12]The exception is Italy, potentially as those yields bear higher sovereign and hence country-specific risks. Our results also suggest that the variance accounted by the global yield curve gradually increases with yield maturity, see the online appendix.

of bond variation across all countries.[13] The idiosyncratic noise is largely irrelevant and our model is doing a good job modeling yield co-movement. It is consistent with the early evidence of Litterman and Scheinkman (1991) that two or three factors can capture most of the variation in bond yields.[14] Having identified significant co-movement in yields using a latent factor approach, we now seek to reconcile this result with macro fundamentals.

4 Empirical Results

4.1 Decomposition of Structural Shocks

We begin our results with a decomposition of short yields. In Section 3.1, we show that the global yield factors account for the majority of the variance of bond yields.[15] To evaluate the relative importance of global macro fundamentals and latent factors in driving the co-movement in short rates, we further decompose the 120-month forecast error variance (FEV) of global shocks. As shown in Table 3, the global Level factor is the most important global factor and accounts for more than 40% of the variance. The Level factor anchors the level of global yield curves, which can be affected by various sources.[16] Among all fundamentals, CPI accounts for a significant fraction of bond yield co-movement at 3-month maturity, contributing up to 25% of FEV of co-movement. These two global factors account for around 70% of the variance across seven countries, which parallels the finding in Jotikasthira, Le and Lundblad (2015).

[13]In other words, the sum equals to the share of variance of national yield factors. Note there is a clear distinction between national factors and country-specific components. Country-specific components are the movements in national factors that are not driven by global factors.

[14]In the online appendix, we present auxiliary analyses about global and country-level factor dynamics.

[15]There are important co-movements of yields, although the co-movements are primarily at the long end of the yield curve according to Byrne, Fazio and Fiess (2012) and Jotikasthira, Le and Lundblad (2015). Jotikasthira, Le and Lundblad (2015) suggest it is due to the uncoupling of short-term policy rates in different countries.

[16]For example, Christensen and Rudebusch (2012) indicate that quantitative easing causes declines in government bond yields. We will explore the economic content of global latent factors in Section 4.2.

Table 3: Short Rate Variance Explained by Global Factors

Country	Posterior Mean (Standard Deviation)					
	IP	CPI	PR	FCI	Level	Slope
US	0.02(0.02)	0.22(0.12)	0.03(0.03)	0.08(0.05)	0.47(0.15)	0.17(0.07)
UK	0.02(0.02)	0.24(0.12)	0.03(0.03)	0.09(0.05)	0.42(0.14)	0.20(0.08)
Germany	0.02(0.02)	0.25(0.12)	0.03(0.03)	0.09(0.05)	0.40(0.14)	0.20(0.08)
France	0.02(0.02)	0.24(0.12)	0.03(0.03)	0.09(0.05)	0.42(0.15)	0.20(0.08)
Italy	0.02(0.02)	0.22(0.12)	0.03(0.03)	0.09(0.05)	0.46(0.15)	0.18(0.07)
Canada	0.02(0.02)	0.22(0.12)	0.03(0.03)	0.08(0.05)	0.46(0.15)	0.18(0.08)
Japan	0.02(0.02)	0.14(0.11)	0.02(0.02)	0.06(0.04)	0.66(0.15)	0.10(0.07)

Notes: 1. This table summarizes the posterior mean of the decomposition of 120-month forecast error variance of 3-month short rates driven by innovations of global yield and macro factors. In each parenthesis $(\cdot)$ the posterior standard deviation of shares in a specific block is calculated from our draws, see Section 2. Larger Standard Deviation means higher uncertainty in the estimates, but should be interpreted with caution since they may not have normal distributions.
2. IP, CPI, PR, FCI, Level and Slope denote the variance shares of shocks to global fundamentals at different yield maturities. The quantities are the same for seven countries. The global fundamentals include the Industrial Production growth rate (YoY), inflation, change in policy rate (YoY), FCI, global Level and global Slope, respectively. The shares in each row sum up to 1.
3. We employ Cholesky decomposition to identify the shocks using the following ordering: IP, CPI, PR, FCI, Level and Slope. The details can be found in the online appendix.

4.1.1 Policy and Risk Compensation Channels

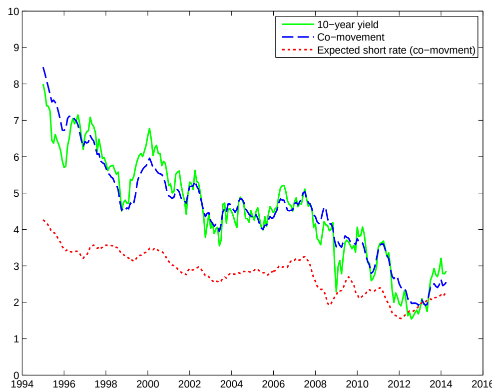
Country-specific policy rates are decided by national monetary authorities who may have different policy targets or be faced with idiosyncratic shocks. However, our empirical evidence suggests that monetary policies can be coordinated to respond to global inflation or other commonalities. Would this result be propagated to long yields? Maturities at the long end of the yield curve can be explained by either a policy channel or a risk channel, which we have discussed in the methodology section.

Figure 2 shows that US 10-year long yields are strongly driven by global co-movement. Moreover, it is highly unlikely that global-driven short rate expectations violate the zero lower bound (ZLB).[17] In the figure, the increase at the long end of the expected short

[17] In reality, violations of the zero lower bound are possible as some global rates have been negative recently.

rates is related to market expectations about the policy liftoff from the ZLB, as suggested in Bauer and Rudebusch (2016). As shown in Figure 3, long yields are similarly driven by global co-movement in other G7 countries, but Italian long yields diverge from the co-movement because of the 2010-2014 sovereign debt crisis.

Figure 2: US 10-Year Bond Yields and Co-Movement



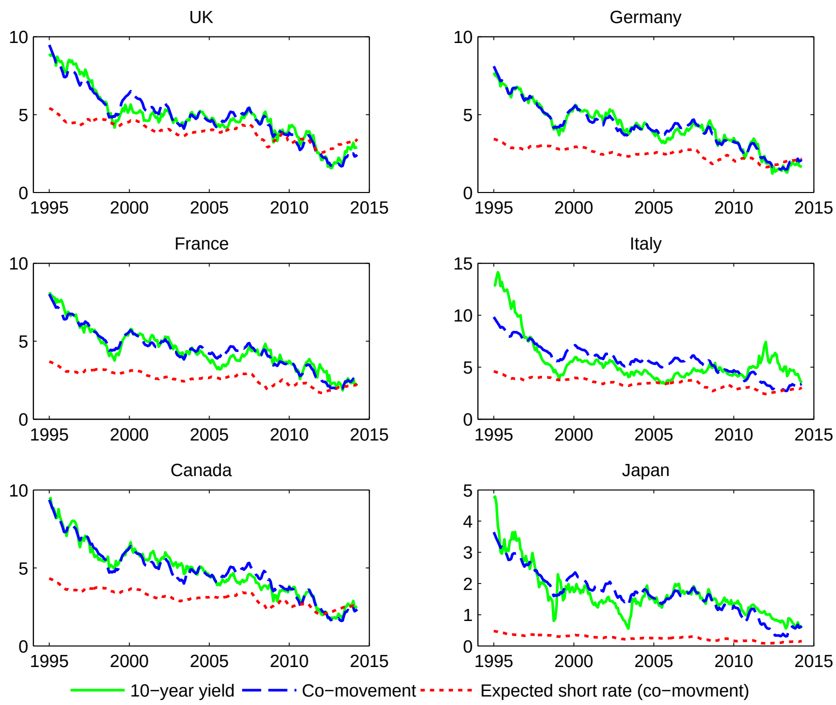
Notes: This figure shows in percentage units the observed US 10-year bond yields, and the global-driven yield movements and expected short rates implied by the model. The 10-year nominal yields are plotted by the solid line. The dashed line plots the portion of yields driven by global factors (co-movement), and the dotted line is the expected short rate part in the co-movement.

We firstly assess the relative importance of the policy and risk channels in driving the co-movement in long yields. Table 4 displays the proportions of long yield variance driven by global factors which are due to these channels.[18] Firstly, we find that long bond co-movements are largely driven by the risk compensation channel. For all seven countries' long rates, this risk channel accounts for more than 53% of the total variance of commonalities. The relative importance of the risk compensation channel is in line with the results in Jotikasthira, Le and Lundblad (2015).[19] Indeed for Japan, the risk compensation channel accounts for 96% of long bond movements. The extended zero interest-rate policy

[18]We focus upon 10-year yields since other yields present similar results.

[19]Jotikasthira, Le and Lundblad (2015) indicate the risk compensation channel accounts for around 80% and 42% for the US and Germany, respectively. We include the financial crisis period in our sample so we have a decreased share for the US and an increased share for Germany.

Figure 3: 10-Year Bond Yields and Co-Movements



Notes: This figure shows in percentage units the observed 10-year bond yields, and the global-driven yield movements and expected short rates implied by the model of G7 countries except US. The 10-year nominal yields are plotted by the solid line. The dashed line plots the portion of yields driven by global factors (co-movement), and the dotted line is the expected short rate part in the co-movement.

in Japan potentially compresses the response of short rate expectations when faced with global shocks.

Table 4: Decomposition of Long Yield Variance through Two Channels

Country	Channel	Fraction	Posterior Mean (Std. Dev.)					
			IP	CPI	PR	FCI	Level	Slope
US	Policy	47%	0.02	0.30	0.04	0.05	0.49	0.10
			(0.02)	(0.19)	(0.04)	(0.05)	(0.23)	(0.11)
	Risk Compensation	53%	0.02	0.11	0.04	0.07	0.70	0.06
			(0.02)	(0.08)	(0.02)	(0.05)	(0.14)	(0.06)
UK	Policy	39%	0.02	0.30	0.04	0.05	0.49	0.10
			(0.02)	(0.19)	(0.04)	(0.05)	(0.23)	(0.11)
	Risk Compensation	61%	0.02	0.10	0.04	0.07	0.70	0.06
			(0.02)	(0.08)	(0.02)	(0.05)	(0.13)	(0.06)
Germany	Policy	23%	0.02	0.32	0.04	0.05	0.47	0.10
			(0.02)	(0.19)	(0.04)	(0.05)	(0.24)	(0.11)
	Risk Compensation	77%	0.02	0.12	0.04	0.08	0.68	0.06
			(0.02)	(0.09)	(0.02)	(0.05)	(0.14)	(0.06)
France	Policy	33%	0.02	0.32	0.04	0.05	0.46	0.10
			(0.02)	(0.19)	(0.04)	(0.05)	(0.24)	(0.11)
	Risk Compensation	67%	0.02	0.10	0.04	0.07	0.70	0.06
			(0.02)	(0.08)	(0.02)	(0.05)	(0.13)	(0.06)
Italy	Policy	29%	0.02	0.23	0.03	0.04	0.57	0.10
			(0.02)	(0.18)	(0.04)	(0.05)	(0.22)	(0.11)
	Risk Compensation	71%	0.02	0.18	0.04	0.05	0.22	0.11
			(0.02)	(0.08)	(0.02)	(0.04)	(0.12)	(0.06)
Canada	Policy	27%	0.02	0.32	0.04	0.05	0.47	0.10
			(0.02)	(0.19)	(0.04)	(0.05)	(0.24)	(0.11)
	Risk Compensation	73%	0.02	0.13	0.05	0.09	0.66	0.06
			(0.02)	(0.10)	(0.03)	(0.06)	(0.15)	(0.06)
Japan	Policy	4%	0.02	0.30	0.04	0.05	0.50	0.10
			(0.02)	(0.19)	(0.04)	(0.05)	(0.23)	(0.11)
	Risk Compensation	96%	0.02	0.14	0.05	0.09	0.63	0.06
			(0.02)	(0.11)	(0.03)	(0.06)	(0.17)	(0.06)

Notes: 1. This table summarizes the decomposition of 120-month forecast error variance of the 10-year bond yields driven by innovations of factors through two channels: the policy and the risk premium channels. In each parenthesis $(\cdot)$ the posterior standard deviation of shares in a specific block is calculated from our draws, see Section 2. Larger standard deviation means higher uncertainty in the estimates, but should be interpreted with caution since they may not have normal distributions.
2. IP, CPI, PR, FCI, Level and Slope denote the variance shares of shocks to global fundamentals at different maturities in the country-level block. The global fundamentals include the Industrial Production growth rate (YoY), inflation, change in policy rate (YoY), global Level, global Slope and FCI, respectively. The shares in each row sum up to 1.
3. We employ Cholesky decomposition to identify the shocks using the following ordering: IP, CPI, PR, FCI, Level and Slope. The details can be found in the online appendix.

Moreover, we find that global inflation and the global Level are still the main driver of long yields through each of the channels. The two factors together explain more than two-thirds of the co-movement in long yields. Lastly, while future short rate expectations are significantly driven by global inflation, term premia is primarily driven by the global Level. Global inflation plays an important role through the policy channel since central banks use inflation targeting. Even when the policy rule is constrained near the zero lower bound, there can still be adjustment to global inflation shocks through changes in expectations of future short rates.

4.2 What is Behind the Global Yield Factors?

The global Level factor is one of the most important factors driving global yield co-movement. However, its economic implications and meaning are not well understood. In this section, we go one step further and explore the economic content of global Level and Slope factors, purged of the contemporaneous correlation with macro fundamentals.

We appeal to two possible explanations that are well documented in the macro-finance literature. The first explanation corresponds to the sentiment-based theory favored by Kumar and Lee (2006), Bansal and Shaliastovich (2010) and Benhabib and Wang (2015). As suggested in Ludvigson (2004), the consumer confidence index is a widely used measure of investor sentiment. We obtain leading indicator aggregates of G7 from the OECD database as proxies of global sentiment, which include the composite leading indicator, business confidence index and consumer confidence index. Our second explanation is that asset prices can be driven by economic uncertainty, see Bloom (2014) for a comprehensive review. We use the US and Europe economic policy uncertainty indicators constructed by Baker, Bloom and Davis (2013) as the measure of economic uncertainty.

Table 5 reports regression results on the determinants of global co-movements. The regression of global Level factor on global macro factors used in this paper shows only a relatively smaller portion of variance is driven by macro factors (i.e. around 20%), which is consistent with our previous findings. Adding sentiment and/or economic uncertainty measures greatly increases the explanatory power, and the adjusted R^2 is increased by more than 50%. With respect to the global Slope factor, macro information and the sentiment measures can significantly increase the adjusted R^2, although the Slope factor is relatively less important in driving yield movements.

Table 5: Co-Movement Regressions

	CLI^{G7}	BCI^{G7}	CCI^{G7}	PU^{US}	PU^{EU}	M+Constant	$adjR^2$
Level						*	20.14%
	-0.35(0.08)	-0.07(0.09)	0.89(0.05)			*	65.22%
				-0.01(0.00)	-0.01(0.00)	*	58.74%
	-0.48(-0.01)	-0.07(-0.01)	0.47(0.08)	0.08(0.00)	0.06(0.00)	*	72.96%
Slope						*	42.28%
	0.60(0.05)	-0.42(0.06)	-0.03(0.03)			*	60.72%
				0.00(0.00)	0.00(0.00)	*	42.72%
	0.62(0.00)	-0.43(0.00)	0.02(0.05)	0.06(0.00)	0.05(0.00)	*	60.80%
TP	-0.17(0.00)	-0.12(0.00)	0.27(0.04)	0.04(0.00)	0.03(0.00)	*	93.76%
y^E	-0.41(-0.01)	-0.04(0.00)	0.36(0.06)	0.07(0.00)	0.05(0.00)	*	83.47%

Notes: This table summarizes the regressions of global Level and Slope factors, and the US 10-year term premia (TP) and long-term short rate expectations y^E, on global macro variables, leading indicators and/or policy uncertainty indicators. M collects global macro variables used in our models. The leading indicators are G7 aggregates from the OECD database, where CLI, BCI and CCI are the composite leading indicator, business confidence index and consumer confidence index, respectively. Policy uncertainty indicators include the US policy uncertainty index PU^{US} and the Europe policy uncertainty index PU^{EU}, which are calculated by Baker, Bloom and Davis (2013). The sample is from 1994:12 to 2014:03 at monthly frequency. The standard errors are given in parentheses $(\cdot)$ and the Adjusted R^2 are reported.

In Table 5 we also report the regressions of the global-driven movements of the US 10-year bond through two channels. The results for other countries or at other maturities are very similar, as the global-driven movements of all countries are linear functions of global factors. All measures of sentiment and economic uncertainty are highly significant. With the strikingly high explanatory power for the movements through both channels, we are reassured that latent information unexplained by macro fundamentals is indeed closely related to investor sentiment and economic uncertainty. This important finding parallels the fast-growing literature with the consideration of sentiment or economic uncertainty.

The co-movement in global yield curves can be almost exclusively characterized by the information of macro variables, sentiment and economic uncertainty. This evidence implies that central banks' policy functions go beyond national macro fundamentals, and the economic agent's view about global risk incorporates the information of investor sentiment and economic uncertainty.

4.3 What Do We Learn from the Global Financial Crisis?

During the global financial crisis, global short rates were very low, reaching the zero lower bound in many countries. Consequently, the variation in short rates was relatively small, and a rule-based monetary policy tends to be constrained. As shown in Figure 3 and Table 4, Japan is not likely to adjust expectations of future short rates possibly because of the prolonged zero interest-rate policy.

Does zero interest-rate policy during the financial crisis affect the transmission mechanisms of long yields in the other countries? Specifically, were short rate expectations less impacted by global shocks, similar to the case of Japan? To this end, we consider an economic agent, who has full knowledge about the economic system but is uncertain about the volatility of global shocks. Therefore, this agent is like an Bayesian econometrician and relies on Bayesian updating. This follows Orlik and Veldkamp (2014) and they interpret the resulting changes in the variance estimates as uncertainty shocks. This argument is echoed by the recent work of Creal and Wu (2014) in the context of yield curves, where they find that the biggest impact of volatility changes has occurred in the US since the global financial crisis. However, whether these changes affect the transmission channels of global yields is a remaining question.

The covariance matrix of our economic agent follows an inverse-Wishart distribution. At time t, the Bayesian agent needs to update the estimate of the covariance matrix based on conditional information:

$$\Sigma^{G^*}|X_t, X_{t-1}, ..., X_1 \sim W^{-1}(n_t\Sigma_t^X + \Sigma_0^{G^*}, n_t + \nu_0), \tag{4.1}$$

where Σ_t^X is the sample covariance estimate, n_t is the sample size, $\Sigma_0^{G^*}$ and ν_0 are prior parameters. The posterior mean estimator of the covariance matrix is given by

$$E[\Sigma^{G^*}|X_t, X_{t-1}, ..., X_1] = \frac{n_t\Sigma_t^X + \Sigma_0^{G^*}}{n_t + \nu_0 - p - 1}, \tag{4.2}$$

where p is the size of the covariance matrix. Following Koop and Korobilis (2013), this estimator can be approximated by an exponentially weighted moving average approach:

$$E[\Sigma^{G^*}|X_t, \Sigma_{t-1}^{G^*}] \approx \delta^G u_t u_t' + (1-\delta^G)\Sigma_{t-1}^F, \tag{4.3}$$

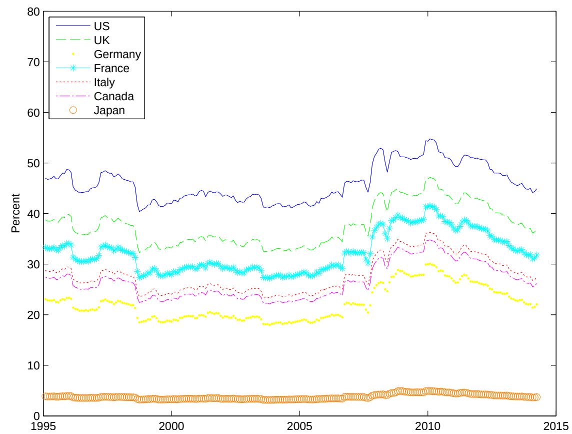
where u_t is a vector of forecast errors, $\Sigma_{t-1}^{G^*}$ is the covariance estimate at time $t-1$, and δ^G is a sufficiently small scalar called 'forgetting factor'. The forgetting factor is used to discount previous information and allows us to accurately estimate the conditional covariance matrix.[20] This updating process suggests that the economic agent learns from new information and then adjusts his or her expectations when facing structural shocks.

Using the conditional estimates of covariance, we reveal the time-varying importance of the policy channel for 10-year bonds in Figure 4. The fractions accounted by the policy channel are trending together for all countries. We do not observe rapid changes in the fractions for each countries during the sample period. In the zero lower bound episode, the relative importance of the policy channel has a detectable increase for all countries except Japan, suggesting global co-movement is caused more by global shocks to short rate expectations. Contrary to the case in Japan, the zero interest-rate policy for the

[20]We set $\delta^G = 0.05$ such that the mean of estimates of the conditional covariance matrix over the sample period matches the unconditional estimate, see Koop and Korobilis (2013) for details about the forgetting factor.

other countries does not hamper the power of global shocks in influencing the economic agent's short rate expectations.

Figure 4: Time-Varying Importance of the Policy Channel

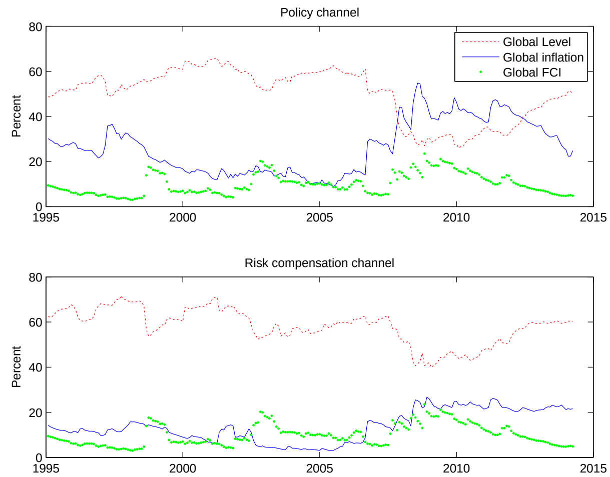


Notes: The figure displays the fraction of long yield variance accounted by the policy channel for seven countries over time.

Although the empirical evidence does not reveal compelling changes in the relative importance of the two transmission channels during the financial crisis, the relative importance of global shocks through each channel varies substantially. Take the US for example, Figure 5 sets out the most important three global shocks to 10-year yields through the policy and the risk channels.[21] Global inflation and global FCI have become more important through both channels since the global financial crisis. In particular, global inflation explains more than 40% of the variance through the policy channel during the financial crisis. In contrast, the global Level dominates throughout the whole sample period through the risk compensation channel.

[21]For the other countries we have quantitatively and qualitatively similar results.

Figure 5: Time-Varying Importance of Global Shocks (US)



Notes: The figure displays the fractions of US long yields' variance accounted by shocks to global Level, global inflation and global FCI through the policy and the risk compensation channels over time.

5 Robustness

As we have discussed, a salient feature of our approach is that we introduce a flexible identification scheme robust to alternative model specifications, in particular, the macro spanning restrictions. Although we employ a seemingly unspanned setup for parsimony, our results are in fact not sensitive to the alternative setup. If macro information is truly spanned by bond yields, then our identified factors are naturally close to rotations of macro factors and hence can satisfactorily span the macros.

To validate the above argument, we proceed with a robustness check by allowing global macro factors to be pricing factors. Equation (2.1) now becomes

$$X_{ibt} = \Lambda^F_{ib} F_{bt} + \Lambda^M_{ib} M_t + e^X_{ibt}.$$

We then examine to what extent the *macro spanning condition* affects bond yields, as macro factors now have direct influence. Table 6 provides a quantitative evaluation of structural shocks in a spanned setup. Not surprisingly, spanned and unspanned setups give qualitatively indistinguishable and quantitatively similar results because of the reasons we discuss in the online appendix.

Table 6: Decomposition of Long Yield Variance through Two Channels

Country	Channel	Fraction	Posterior Mean (Std. Dev.)					
			IP	CPI	PR	FCI	Level	Slope
US	Policy	44%	0.02	0.31	0.04	0.05	0.48	0.10
			(0.02)	(0.19)	(0.04)	(0.05)	(0.24)	(0.11)
	Risk Compensation	56%	0.02	0.15	0.06	0.09	0.61	0.06
			(0.02)	(0.11)	(0.03)	(0.06)	(0.17)	(0.06)
UK	Policy	37%	0.02	0.32	0.04	0.05	0.46	0.10
			(0.02)	(0.19)	(0.04)	(0.05)	(0.24)	(0.11)
	Risk Compensation	63%	0.02	0.15	0.05	0.09	0.63	0.07
			(0.01)	(0.10)	(0.03)	(0.06)	(0.16)	(0.06)
Germany	Policy	16%	0.02	0.36	0.05	0.05	0.42	0.10
			(0.02)	(0.19)	(0.05)	(0.06)	(0.23)	(0.10)
	Risk Compensation	84%	0.02	0.10	0.05	0.07	0.69	0.07
			(0.02)	(0.07)	(0.02)	(0.04)	(0.12)	(0.05)
France	Policy	34%	0.02	0.30	0.04	0.05	0.49	0.10
			(0.02)	(0.19)	(0.04)	(0.05)	(0.23)	(0.11)
	Risk Compensation	66%	0.02	0.11	0.05	0.07	0.69	0.06
			(0.02)	(0.09)	(0.02)	(0.05)	(0.14)	(0.06)
Italy	Policy	29%	0.02	0.32	0.04	0.05	0.46	0.10
			(0.02)	(0.19)	(0.04)	(0.05)	(0.24)	(0.11)
	Risk Compensation	71%	0.02	0.19	0.04	0.05	0.24	0.11
			(0.02)	(0.09)	(0.02)	(0.05)	(0.14)	(0.06)
Canada	Policy	24%	0.02	0.31	0.04	0.05	0.49	0.09
			(0.02)	(0.19)	(0.04)	(0.05)	(0.23)	(0.10)
	Risk Compensation	76%	0.02	0.11	0.04	0.07	0.70	0.06
			(0.01)	(0.08)	(0.02)	(0.05)	(0.13)	(0.06)
Japan	Policy	4%	0.02	0.32	0.04	0.05	0.47	0.10
			(0.02)	(0.19)	(0.04)	(0.05)	(0.24)	(0.11)
	Risk Compensation	96%	0.02	0.09	0.04	0.05	0.73	0.08
			(0.02)	(0.08)	(0.02)	(0.04)	(0.12)	(0.07)

Notes: 1. This table summarizes the decomposition of 120-month forecast error variance of the 10-year bond yields driven by innovations of factors through two channels: the policy and the risk premium channels. The *macro spanning condition* is imposed. In each parenthesis $(\cdot)$ the posterior standard deviation of shares in a specific block is calculated from our draws, see Section 2. Larger standard deviation means higher uncertainty in the estimates, but should be interpreted with caution since they may not have normal distributions.
2. IP, CPI, PR, FCI, Level and Slope denote the variance shares of shocks to global fundamentals at different maturities in the country-level block. The global fundamentals include the Industrial Production growth rate (YoY), inflation, change in policy rate (YoY), global Level, global Slope and FCI, respectively. The shares in each row sum up to 1.
3. We employ Cholesky decomposition to identify the shocks using the following ordering: IP, CPI, PR, FCI, Level and Slope. The details can be found in the online appendix.

6 Conclusion

We propose a fundamentals-augmented hierarchical factor model to jointly identify global and national Level and Slope factors augmented with global fundamentals: inflation, real activity, changes in policy rate and financial conditions. Co-movement accounts for on average two thirds of variability in global bond yields. Global inflation and Level shocks explain global yield co-movement, through a policy channel and a risk compensation channel. Shocks to global inflation play an important role through the policy channel, especially during the financial crisis, while shocks to the global Level factor matter through the risk channel. Moreover, we find that the information in the global Level factor can be largely explained by measures of sentiment and economic uncertainty.

There are many possible avenues for future work. Country-specific components account for nonnegligible variance of bond yields, which are related to 'spillover effects' and are potentially caused by divergence in monetary policies. It would be interesting to specifically evaluate to what extent spillovers across different countries contribute to bond yield movements. Motivated by our findings in this paper, it is desirable to propose a structural model with the consideration of sentiment and economic uncertainty to explain global transmissions.

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Online Appendix: Not for Publication

Appendix A Discussion about Model Specification

A.1 Macro-Spanning Condition

Our approach is also related to the setup of Joslin, Priebsch and Singleton (2014) and Coroneo, Giannone and Modugno (2015), who impose knife-edge restrictions on the loadings of bond yields, so the macro factors cannot be inverted from yields. They denote this setting as *Unspanned Macro Risks* and argue that it is a more realistic assumption. By definition, if there exist Unspanned Macro Risks, macro factors do not directly or contemporaneously impact yields and they influence current yields only through their correlation with the yield factors.[22]

To test whether macro variables can be spanned by bond yields in our sample period, we regress inflation and industrial production on principal components (PCs) of bond yields. Table A.1 shows macro variables are weakly spanned by PCs, which parallels the finding in Bauer and Rudebusch (2015) that macro variables may not be spanned by lower-order PCs. This is because the principal component method only considers cross-section variance, see Stock and Watson (2002), and Bauer and Rudebusch (2015) suggest high-order PCs that are useful in spanning macro factors are likely to be contaminated by measurement errors.

Table A.1: Economic Measure Regressions on Bond Yield Factors

	CPI			IP		
	2 PCs	3 PCs	5 PCs	2 PCs	3 PCs	5 PCs
Global	**8.24%**	8.56%	28.63%	**7.20%**	17.05%	16.62%
US	9.88%	13.26%	38.84%	18.37%	22.07%	27.16%
UK	3.12%	2.69%	18.94%	23.12%	23.22%	56.08%
JP	-0.50%	0.07%	3.75%	3.91%	4.47%	9.28%
GER	13.03%	12.72%	32.48%	8.99%	8.63%	19.05%
FRA	2.14%	2.30%	6.74%	0.41%	0.79%	16.37%
CAN	18.37%	19.46%	38.25%	22.08%	22.15%	31.73%
ITA	17.33%	28.69%	29.66%	9.22%	8.86%	28.14%

Notes: This table reports the Adjusted R^2 of regressions in which CPI inflation and industrial production growth rate (year on year) are regressed on different numbers of principal components (PCs) of bond yields. The sample is from 1994:12 to 2014:03 at monthly frequency. The global variables are G7 aggregates from OECD database.

[22] Joslin, Priebsch and Singleton (2014) suggest that the fully spanned assumption, i.e. the macro factors can be inverted as linear combinations of yields, is often questioned and might be counterfactual. We test the robustness of our hierarchical factor model to the unspanned restriction of Joslin, Priebsch and Singleton (2014), and the results are available upon request.

Therefore, we adopt the unspanned restrictions advocated by the data for parsimony, and Bauer and Rudebusch (2015) suggest that spanned and unspanned models deliver similar results. It is worth highlighting the robustness of Moench, Ng and Potter (2013): Unlike principal components, this method identifies factors by allowing for not only cross-sectional variance but also time series properties. This also means, even in the extreme case that unspanned restrictions are not necessary, the identified factors will cater to the true dynamics and hence mitigate specification errors. The potential loss caused the parsimonious unspanned setup, if any, should be economically insignificant.

Note that unspanned restrictions do not violate Taylor-type policy rules. To see this, we write down the restrictions about macro variables M_t following Bauer and Rudebusch (2015):

$$M_t = \gamma_0 + \gamma_P P_t^L + OM_t,$$

where OM_t captures the orthogonal macroeconomic variation not captured by lower-order PCs P_t^L. For convenience, assuming M_t, P_t have the same dimension and γ_P is invertible,[23] then the short rate r_t is a linear function of PCs and hence a linear function of M_t:

$$r_t = \beta P_t^L = C(\gamma_0, \gamma_P, OM_t) + \beta\gamma_p^{-1} M_t,$$

where C is a function of $(\gamma_0, \gamma_P, OM_t)$. It is clear the short rate is a linear function of macro variables.

In contrast, given the *macro spanning condition* that M_t is fully spanned, i.e. $OM_t = 0$, using macro factors only can fit the bond yields very well. If there are substantial fitting errors when we use macro factors only, we may need to reconsider the validity of this condition or incorporate latent factors.

The *macro spanning condition* should not be confused with the issue whether bond yields are significantly driven by macro factors. That is, even we assume macro factors are fully spanned by bond yields, macro factors do not necessarily have higher explanatory power for yields, especially when macro factors are weakly spanned by a low dimension of PCs as in Table A.1. The *macro spanning condition* is only about whether bond factors include all information of macro variables that can be used to estimate term premia, and term premia is always a linear function of macro factors in a macro-finance model, no matter these factors are spanned or not.[24]

A separate but related questions is, how much of the variance of bond yields can be explained by macro factors and why, which is what we are trying to answer in this paper. Bauer and Rudebusch (2015) explicitly indicate that 'spanned and unspanned models have identical implications for projections of macro variables on yield factors'. Following their argument, our results are considered robust with the identification strategy proposed

[23]Nevertheless, the result does not depend on the dimension of M_t or P_t^L.

[24]Macro spanning, by construction, means macro factors are a subset of pricing factors, and therefore pricing factors have all information of macro factors. This intuition has been discussed formally in Duffee (2013).

by Moench, Ng and Potter (2013), since the pricing factors are identified allowing for time-series information of global macro fundamentals and can satisfactorily capture cross-sectional information. Moreover, our extension with unrestricted covariance matrix of global dynamics ensures the identification of global structural shocks is not sensitive to the ambiguity about the *macro spanning condition*.

A.2 Cross-Sectional Restrictions

In this paper, we do not impose no-arbitrage constraints in our model as the constraints are silent about identifying the latent factors and shocks. In other words, latent factors are not identified with no-arbitrage constraints alone. Duffee (2013) suggests Nelson-Siegel restrictions are nearly equivalent to no-arbitrage in characterizing the cross section of interest rate term structure. Joslin, Le and Singleton (2013) show that Gaussian no-arbitrage macro-finance models are close to factor-VAR models when risk premia dynamics are not constrained. Duffee (2014) also indicates that the no-arbitrage restrictions are unimportant if a model aims to pin down physical dynamics. Since our focus here is not on the structure of risk premia dynamics, we choose to impose no such restrictions to avoid potential misspecification. The potential drawback of no-arbitrage models is that it imposes very strong restrictions on the dynamics of risk prices, in order to 1) ensure no-arbitrage consumption and 2) identify the model with flat likelihood. Kim and Singleton (2012) and Jotikasthira, Le and Lundblad (2015) indicate the no-arbitrage framework may generate implausibly term premiums in the financial crisis. Instead, we impose Nelson-Siegel restrictions here, which provide a parsimonious structure and satisfactory performance in cross-sectional fittings of term structure.

Appendix B Econometric Methods

In this paper we propose a novel approach which extends the hierarchical factor model of Moench, Ng and Potter (2013) by augmenting the model with macro factors. We apply the NS restrictions similar to Diebold, Li and Yue (2008) for the yield factor identification. The estimation of our model is in one step, which should provide more accurate estimates when compared to other multi-step estimations. We call the new model 'Fundamentals-Augmented Hierarchical Factor Model' (FHF).

Our proposed hierarchical model has three levels of factor dynamics, but we only focus on the global level that is augmented with global macro factors. At the global level, the dynamics of the global yield factors can be regarded as an unrestricted Factor-Augmented Vector Autoregressive (FAVAR) system. We conduct the analysis in two steps. The first step is to extract the latent global yield factors, using the proposed 'Fundamentals-Augmented Hierarchical Dynamic Factor Model'. The second step is to directly use the estimation results of FAVAR at the global level to identify the shocks of interest.

B.1 Fundamentals-Augmented Hierarchical Factor Model

To extract the latent factors, a principal component method is commonly utilized. Bai and Ng (2006) have shown that the estimated factors from the principal components method can be treated as though they are observed, if $\sqrt{T}/N \to \infty$ as $T, N \to \infty$. However, the method of principal components is not well suited for the present analysis, because the number of series available[25] is much smaller than the large dimensions that the principal component method typically requires. Accordingly, the FHF is proposed to extract the latent global factors.

B.1.1 A Three-Level Hierarchical Factor Model

Following the framework developed by Moench, Ng and Potter (2013), a three-level model is considered here. Level one is the national level, which describes how national yield factors drive the yields at different maturities. Level two is the global-national level, illustrating how the global yield factors govern the national yield factors. Level three displays the autoregressive dynamics of the global factors.

Firstly, we treat a block (identified as b) as one of the seven countries, so $b = 1, 2, ..., B$ where $B = 7$. At the national level, the bond yield data for a specific country are stacked in the vector X_{bt}, and the dynamic representation is given by

$$X_{b,t} = \Lambda_b^F F_{b,t} + e_{b,t}^X, \tag{B.1}$$

where $X_{b,t}$ is an $N_b \times 1$ vector of yields of country b at different maturities, $F_{b,t}$ is a $k_b \times 1$ vector of latent common yield factors at national level, Λ_b^F is an $N_b \times k_b$ coefficient matrix

[25] There are only seven countries so $N = 7$.

and $e^X_{b,t}$ is the vector of idiosyncratic components. Note that in our model $N_b = 11$ and $k_b = 2$ for $b = 1, 2, ..., B$; in other words, for each country, we use yield data of 11 different maturities and assume that 2 factors can explain most of the yield variance.

Stacking up $F_{b,t}$ across seven countries produces a $K^F \times 1$ vector F_t.[26] At the global-national level, it is assumed that

$$F_t = \Lambda^G G_t + e^F_t, \tag{B.2}$$

where K^G global common factors are collected into the vector G_t, Λ^G is a $K^F \times K^G$ coefficient matrix and e^F_t are country-specific components at the global-national level.

The dynamics of the global factors G_t are described at level three:

$$G_t = \Psi^G G_{t-1} + \epsilon^G_t, \tag{B.3}$$

where Ψ^G is the coefficient matrix and the innovations $\epsilon^G_t \sim N\Big(\mathbf{0}, \Sigma^G\Big)$[27].

The model is completed by specifying the dynamics of idiosyncratic and country-specific components $e^X_{b,t}$ and e^F_t.

$$e^X_{b,t} = \Psi^X_b e^X_{b,t-1} + \epsilon^X_{b,t}, \tag{B.4}$$

$$e^F_t = \Psi^F e^F_{t-1} + \epsilon^F_t, \tag{B.5}$$

where Ψ^X_b is an $N_b \times N_b$ diagonal coefficient matrix, Ψ^F is a $K^F \times K^F$ diagonal coefficient matrix, the innovations $\epsilon^X_{b,t} \sim N\Big(\mathbf{0}, \Sigma^X_b\Big)$ and $\epsilon^F_t \sim N\Big(\mathbf{0}, \Sigma^F\Big)$.[28]

B.1.2 An Extension with Macro Factor Augmentation

Assuming at level three, i.e. the level that describes the global factor dynamics, the factor dynamics are augmented with Macro information. So the Equation (B.3) can be rewritten as

$$G^*_t = \begin{bmatrix} G_t \\ M_t \end{bmatrix} = \psi^G \begin{bmatrix} G_{t-1} \\ M_{t-1} \end{bmatrix} + u_t, \tag{B.6}$$

$$u_t \sim N\Big(\mathbf{0}, \Sigma^{G^*}\Big),$$

where Σ^{G^*} is the variance-covariance matrix of u_t and is unconstrained. Σ^{G^*} needs not be a diagonal matrix in our extension. The evolution of the global factors displayed here uses only one lag here for simplicity; in practice, more lags can be used to estimate the

[26] $K^F = \sum_{b=1}^B k_b$ and $F_t = \Big(F_{1,t} \quad F_{2,t} \quad ... \quad F_{B,t}\Big)'$

[27] $\Sigma^G = \text{diag}\big((\sigma^G_1)^2, ..., (\sigma^G_{K^G})^2\big)$.

[28] $\Sigma^X_b = \text{diag}\big((\sigma^X_{b,1})^2, ..., (\sigma^X_{b,N_b})^2\big)$ and $\Sigma^F = \text{diag}\big((\sigma^F_1)^2, ..., (\sigma^F_{K^F})^2\big)$.

factor dynamics. The Equation (B.6) is indeed a factor-augmented vector autoregressive (FAVAR) system. The estimates from this system will be used for the identification of shocks for the structural analysis.

B.1.3 Estimation via Gibbs Sampling

Before we proceed with the estimation scheme, the parameters needed to be estimated are summarized for better illustration. Collect $\{\Lambda_1^F, ..., \Lambda_B^F\}$ and Λ^G into $\mathbf{\Lambda}$, $\{\Psi_1^X, ..., \Psi_B^X\}$, Ψ^F and ψ^G into $\mathbf{\Psi}$, and $\{\Sigma_1^X, ..., \Sigma_B^X\}$, Σ^F, Σ^{G^*} into $\mathbf{\Sigma}$. To sum up, the parameters we need to estimate are $\mathbf{\Lambda}$, $\mathbf{\Psi}$ and $\mathbf{\Sigma}$.

A Bayesian method, i.e., Markov Chain Monte Carlo (MCMC), is used to estimate the model. A simple extension of the algorithm in Carter and Kohn (1994) is proposed here. Based on the observed values of M_t, and the initial values of $\{F_{b,t}\}$ and G_t from the method of principal components, for each iteration we construct the Gibbs sampler in the following steps:

1. Draw G_t, conditional on F_t, $\mathbf{\Lambda}$, $\mathbf{\Psi}$ and $\mathbf{\Sigma}$.
2. Draw ψ^G, conditional on Σ^{G^*}, G_t and M_t.
3. Draw Σ^{G^*}, conditional on ψ^G, G_t and M_t.
4. Draw Λ^G, conditional on G_t and F_t.
5. For each b, draw $F_{b,t}$, conditional on $\mathbf{\Lambda}$, $\mathbf{\Psi}$, $\mathbf{\Sigma}$ and G_t.
6. For each b, draw b_{th} elements of Ψ^F and Σ^F, conditional on G_t and F_t.
7. For each b, draw the Λ_b^F, Ψ_b^X and Σ_b^X, conditional on F_t and $X_{b,t}$.

Similar to Diebold, Li and Yue (2008) and Moench, Ng and Potter (2013), the elements of $\mathbf{\Lambda}$ and $\mathbf{\Psi}$ are set to have normal priors, and $\mathbf{\Sigma}$ follow inverse gamma priors. Given the conjugacy, the posterior distributions are not difficult to compute. Regarding the factors G_t and F_t, we follow Carter and Kohn (1994) and Kim and Nelson (1999) to run the Kalman filter forward to obtain the estimates in period T and then proceed backward to generate draws for $t = T-1, ..., 1$. It is worth noting that, if we impose hard restrictions on Λ^G and Λ_b^F, then there is no need to draw these parameters in the above Gibbs sampling.

B.2 Nelson-Siegel Restrictions

Following Diebold, Li and Yue (2008), we can use two factors to summarize most of the information in the term structure of interest rates. As we show in the Section 3.1, two factors have accounted for around 99% of the bond yield variance across all countries.

The below Equation (B.7) describes how restrictions are imposed; the restrictions used in our hierarchical factor model are in fact fixing the loading of the factors. Let $y_t\left(\tau\right)$

denote yields at maturity τ, then the factor model for a single country we use is of the form

$$y_t(\tau) = L_t^{NS} + \frac{1-e^{-\tau\lambda}}{\tau\lambda} S_t^{NS} + \varepsilon_t(\tau), \tag{B.7}$$

where L_t^{NS} is the "Level" factor, S_t^{NS} is the "Slope" factor, and ε_t is the error term. Additionally, λ in the exponential functions controls the shapes of loadings for the NS factors; following Diebold and Li (2006) and Bianchi, Mumtaz and Surico (2009), we set $\lambda = 0.0609$.[29]

The interpretations of Nelson-Siegel factors are of empirical significance. The Nelson-Siegel Level factor L_t^{NS} is identified as the factor that is loaded evenly by the yields of all maturities. The Slope factor S_t^{NS} denotes the spread between the yields of a short- and a long-term bond, and its movements are captured by putting more weights on the yields with shorter maturities.

The following Figure B.1 depicts the shapes of the loadings of the NS factors. In our model estimation, we fixed the Λ_b^F in Equation (B.1) by the NS loadings. We further set the Λ^G in Equation (B.2) to a diagonal matrix to identify the global factors, and the intuition behind is that the country-level Level (Slope) factor is only driven by the global Level (Slope) factor.

B.3 Decomposition of Variance Driven by Global Factors

Recall Equation (B.6) that describes the dynamics of the global factors G_t^* at level three in Section B.1:

$$G_t^* = \psi^G G_{t-1}^* + u_t,$$

We can rewrite this as an implied Wold MA(∞) representation:

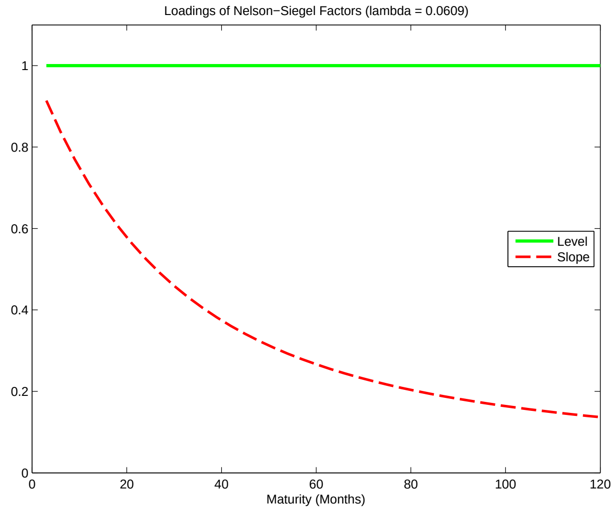
$$G_t^* = \sum_{i=0}^{\infty} \psi_i \mu_{t-i}, \tag{B.8}$$

where μ_t are the orthogonal innovations and Cholesky decomposition is needed to take into account the contemporaneous correlation of the shocks.

With simple algebra, we can write the bond yield co-movements driven by the global

[29] Alternatively, we can select the value of λ from a grid of reasonable values by comparing the goodness of fit. However, if we do not specify the factor dynamics and fit the Nelson-Siegel model in a static way, the selection may not be optimal. Also we choose a single value of λ for all the countries, as Nelson and Siegel (1987) indicate that there is little gain in practice by fitting λ individually. Therefore, we set $\lambda = 0.0609$ to fix the ideas because 1) this value is the mostly used in the related literature so revealing the dynamics the associate latent factors is more desirable, and 2) using this value we have a relatively better fit of the 'global short rate factor'. To ensure the robustness, we also try a grid of reasonable values; we find the results are qualitatively similar and hence our findings are robust to the selection of λ.

Figure B.1: Loadings of Nelson-Siegel Factors



Notes: The solid green line and red dashed line are the loadings for Level and Slope factors, respectively ($\lambda = 0.0609$). The horizontal axis shows the maturities of bonds, and the unit is month.

factors X_t^G as the following equation:

$$X_t^G = B \sum_{i=0}^{\infty} \psi_i \mu_{t-i}, \tag{B.9}$$

where B can be mapped from the loadings Λ^F (in Equation B.1) and Λ^G (in Equation B.2). The impulse response at time $t + h$ is therefore:

$$X_{t+h}^G = B \sum_{i=0}^{\infty} \psi_i \mu_{t+h-i}. \tag{B.10}$$

It is easy to have the error of the optimal h-step ahead forecast at time t:

$$X_{t+h}^G - \hat{X}_{t+h|t}^G = B \sum_{i=0}^{h-1} \psi_i \mu_{t+h-i}. \tag{B.11}$$

The mean squared error of X_{t+h}^G is given by

$$\text{MSE}(X_{t+h}^G) = \text{diag}\Big(B(\sum_{i=0}^{h-1} \psi_i \psi_i') B'\Big). \tag{B.12}$$

Therefore, the contribution of the kth factor to the MSE of the h-step ahead forecast of the yield at the jth maturity is

$$\Omega_{jk,h} = \sum_{i=0}^{h-1} R_{jk,i}^2 / \mathrm{MSE}(X_{t+h}^G), \tag{B.13}$$

where $R_{jk,i}$ is the element in row j, column k of $R_i = B\psi_i$.

B.3.1 Decomposition of Policy Channel and Risk Compensation Channel

The policy channel is consistent with the Expectation Hypothesis (EH). The EH consistent long yield is given by

$$y_t(\tau)^{EH} = \frac{1}{\tau}\sum_{i=0}^{\tau-1} \mathrm{E}_t y_{t+i}(1), \tag{B.14}$$

where $y_t(\tau)$ is the element of yield data X_t at maturity τ. That is to say, the EH consistent long yield is equal to the average of expected short yields $\mathrm{E}_t y_{t+i}(1)$. If we only focus on the part driven by global factors, then after some iterations, the above equation can be written as

$$y_t(\tau)^{EH} = \frac{1}{\tau}B(I + \psi^G + \psi^{G^2} + ... + \psi^{G^{\tau-1}})\sum_{i=0}^{\infty}\psi_i\mu_{t-i}. \tag{B.15}$$

The term premia (risk compensation channel) is given by

$$TP_t(\tau) = y_t(\tau) - y_t(\tau)^{EH}. \tag{B.16}$$

In other words, the term premia is the difference between the long yield and the EH consistent long yield. We can use similar transformations as in Equations (B.10) and (B.13) to compute the impulse response and variance decomposition of the above two channels.

Appendix C Data Appendix

Table C.2: List of Financial Condition Indexes

Series ID	**Description**
STLFSI	St. Louis Fed Financial Stress Index [1]
KCFSI	Kansas City Financial Stress Index [1]
ANFCI	Chicago Fed Adjusted National Financial Conditions Index [1]
CFSI	Cleveland Financial Stress Index [1]
VIX	CBOE S&P Volatility Index [1]
BFCIUS	Bloomberg United States Financial Conditions Index [1]
BFCIEU	Bloomberg Euro-Zone Financial Conditions Index [1]
GFSI	BofA Merrill Lynch Global Financial Stress Index [1]
EASSF	Euro Area Systemic Stress Indicator Financial Intermediary [1]
WJF	Westpac Japan Financial Stress Index [1]
GSF	Goldman Sachs Financial Index [1]
BCF	Bank of Canada Financial Conditions Index [1]

Notes:
1. In square brackets [·] we have a code for data transformations used in this data set: [1] means original series is used. The series are not seasonally adjusted.
2. Data are attained from Bloomberg, spanning from Jan. 1990 to Mar. 2014. The data may be unbalanced. The first five series can also be attained from St. Louis Federal Reserve Economic Data (http://research.stlouisfed.org/).

Table C.3: List of Yields

Series ID	**Description**
ITA	Italy Sovereign (IYC 40) Zero Coupon Yields [1]
CAN	Canada Sovereign (IYC 7) Zero Coupon Yields [1]
FRA	France Sovereign (IYC 14) Zero Coupon Yields [1]
GER	German Sovereign (IYC 16) Zero Coupon Yields [1]
JP	Japan Sovereign (IYC 18) Zero Coupon Yields [1]
UK	United Kingdom (IYC 22) Zero Coupon Yields [1]
US	Treasury Actives (IYC 25) Zero Coupon Yields [1]

Notes:
1. In square brackets [·] we have a code for data transformations used in this data set: [1] means original series is used. The series are not seasonally adjusted.
2. Data are attained from Bloomberg, spanning from Dec. 1994 to Mar. 2014. The yields are of the following 11 maturities: 3 months, 6 months, 1 year, 2 years, 3 years, 4 years, 5 years, 6 years, 7 years, 8 years and 10 years.
3. The zero-coupon yields are calculated step-by-step using the discount factors that are derived from standard bootstrapping, given the set of coupon bonds, bills, swaps or a combination of these instruments. A minimum of four instruments at different tenors are required for each yield curve. The bootstrapping is similar to the Unsmoothed Fama-Bliss method, see Fama and Bliss (1987).

Table C.4: List of Real Activity Indicators

Series ID	**Description**
IMFIPUS	IMF US Industrial Production SA [5]
IMFIPUK	IMF UK Industrial Production SA [5]
IMFIPJP	IMF Japan Industrial Production SA [5]
IMFIPGER	IMF Germany Industrial Production SA [5]
IMFIPFR	IMF France Industrial Production SA [5]
IMFIPITA	IMF Italy Industrial Production SA [5]
IMFIPCAN	IMF Canada Industrial Production SA [5]

Notes:
1. In square brackets [·] we have a code for data transformations used in this data set: [5] means log first-order difference (annually growth rate) is used.
2. Data are attained from Bloomberg, spanning from Jan. 1990 to Mar. 2014. The data may be unbalanced.

Table C.5: List of CPI and Policy Rates

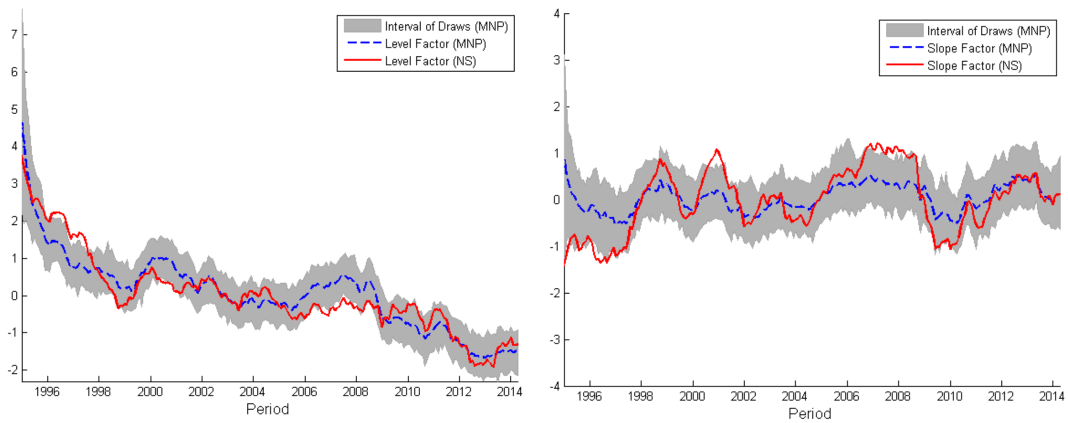
Series ID	Description
IMFCPIUS	IMF US CPI % Change in Percent per Annu [1]
IMFCPIUK	IMF UK CPI % Change in Percent per Annu [1]
IMFCPIJP	IMF Japan CPI % Change in Percent per Annu [1]
IMFCPIGER	IMF Germany CPI % Change in Percent per Annu [1]
IMFCPIFR	IMF France CPI % Change in Percent per Annu [1]
IMFCPIITA	IMF Italy CPI % Change in Percent per Annu [1]
IMFCPICAN	IMF Canada CPI % Change in Percent per Annu [1]
IMFFUNDUS	IMF US Federal Funds Rate in Percent per Annu [5]
IMFFUNDUK	IMF UK Bank of England Official Bank Rate [5]
IMFFUNDJP	IMF Japan Official Rate in Percent per Annu [5]
IMFFUNDCAN	IMF Canada Official Rate in Percent per Annu [5]
IMFFUNDEU	IMF Euro Area Official Rate in Percent per Annu [5]

Notes:
1. In square brackets [·] we have a code for data transformations used in this data set: [1] means original series is used. The series are all seasonally adjusted; [5] means log first-order difference (annually) is used.
2. Data are attained from Bloomberg, spanning from Jan. 1990 to Mar. 2014. The data may be unbalanced.

Appendix D Additional Results

D.1 Comparison of Factor Indentification Schemes

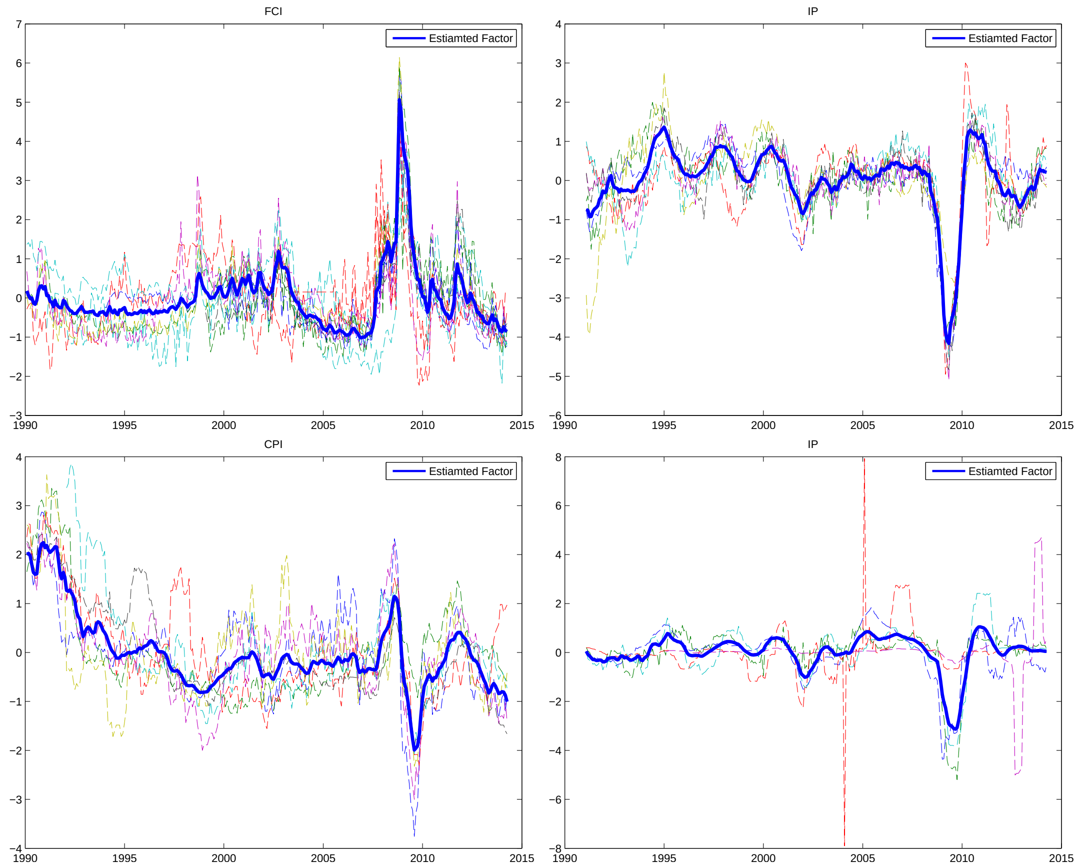
Figure D.2: Identified Factors from Different Schemes (MNP vs. NS)



Notes:
1. In the above two charts, the factors identified by the scheme of Moench, Ng and Potter (2013) are plotted against the factors identified by the NS scheme of Diebold, Li and Yue (2008). To better serve the comparison purpose, the factors are extracted from a less complicated system without a macro factor augmentation.
2. The upper chart shows the Level factors, while the lower chart displays the Slope factor. The dashed blue lines are the median values of MNP identified factors and the gray areas cover all the draws from the MCMC estimation. The solid red lines are the median values of NS identified factors.

D.2 Global Macro Factors

Figure D.3: Estimated Global Macro Factors



Notes:
1. In the above charts, the thick blue lines are the global macro factors, which are estimated using the method proposed by Koop and Korobilis (2014). The Matlab code can be obtained in website https://sites.google.com/site/dimitriskorobilis/matlab/. The other thin lines with different colors are the standardized series for the estimation.
2. From top left clock-wise we have global factors of financial condition indexes, real activity, policy rates and inflation. The data used for the factor estimation are described in Appendix C, spanning from Jan. 1990 to Mar. 2014.

Table D.6: Correlations between the National Series and Global Factors

FCI	STLFSI	KCFSI	ANFCI	CFSI	VIX	BFCIUS	BFCIEU	GFSI	EASSF	WJF	GSF	BCF
Correlation	0.945	0.952	0.568	0.695	0.845	0.935	0.848	0.866	0.701	0.528	0.671	0.814

IP	IMFIPUS	IMFIPUK	IMFIPJP	IMFIPGER	IMFIPFR	IMFIPITA	IMFIPCAN
Correlation	0.899	0.889	0.767	0.831	0.940	0.946	0.731

CPI	IMFCPIUS	IMFCPIUK	IMFCPIJP	IMFCPIGER	IMFCPIFR	IMFCPIITA	IMFCPICAN
Correlation	0.805	0.810	0.761	0.525	0.887	0.891	0.739

PR	IMFFUNDUS	IMFFUNDUK	IMFFUNDJP	IMFFUNDCAN	IMFFUNDEU
Correlation	0.844	0.911	0.330	0.914	0.073

Notes: This table summarizes the correlations between the national macro series in Data Appendix and the global macro factors shown in Figure D.3, for four categories: Financial Condition Index, Industrial Production growth rate, CPI and the change (YoY) of policy rate.

D.3 Co-Movement in Yields

D.3.1 Factor Dynamics

In this section, we depict the dynamics of the global yield factors estimated from our proposed 'Fundamentals-Augmented Hierarchical Factor Model'. As mentioned before, we extract two national yield factors that account for more than 96% of the variance of the term structure. We now focus on the global yield factors, as these factors typically drive the national Level and Slope factors. Firstly, we calculate the arithmetic sum of the global Level and Slope factors to evaluate the effect on the global short rate co-movement. This sum is denoted as the *global short rate factor*, and reflects the global co-movement in short rates across countries.[30] From the left panel of Figure D.4, we can see the global short rate factor is strongly correlated with the first principal component of short rates across the seven advanced economies, also implying our model successfully captures the global co-movement of the short rates.[31] One feature of the movements of the global short rate factor is that it falls sharply after the Global Financial Crisis, consistent with a global expansion in monetary policy.

It is straightforward to decompose the global short rate factor into the global Level and Slope. The movements of these two factors are shown in the right panel of Figure D.4, in which we also highlight some distinct historical events: January 1999 and the start of the euro area, US recessions in 2001 and 2008 as defined by NBER and the European sovereign debt crisis. As we have already discussed, Level and Slope factors control the shape of the term structure, which can be informative in revealing useful macroeconomic information. For example, before 1999 there is a downward trend for the Level factor and an upward trend for the Slope factor, which means the global term structure is moving down and flattening.[32] This phenomenon indicates a moderation in global term structure, possibly caused by greater integration.[33] We can observe two clear trends abstracting from temporary disturbances in the factors. Firstly, the downward-trending global Level seems to relate to the decreasing inflation level in the period of the Great Moderation, as suggested by Evans and Marshall (2007) and Koopman, Mallee and Van der Wel (2010).

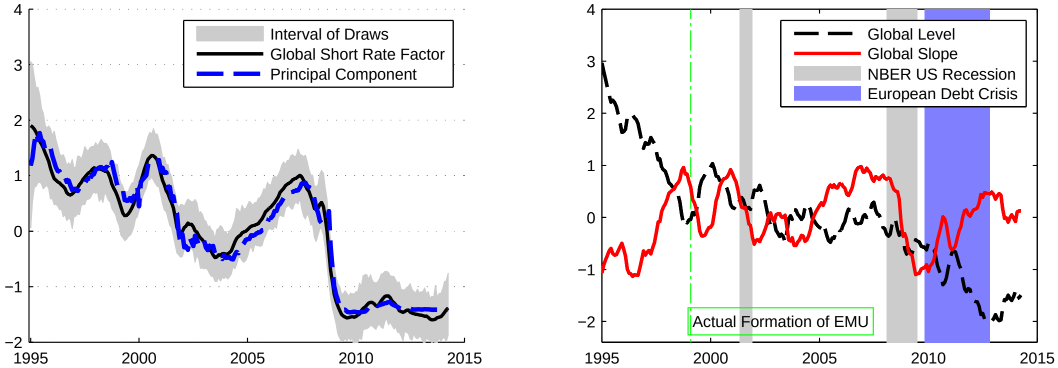
[30] By NS restrictions, for a bond at very short maturity, we have the equation that *short rate* $= \beta_1 L_t^{NS} + \beta_2 S_t^{NS}$, where the loadings equal to one, i.e. $\beta_1 = \beta_2 = 1$. Therefore, the short rate is directly driven by the sum of two factors in our model construction, see Appendix B.2 for details.

[31] Note that there is a smaller proportion of bond yield movements in country level that are not captured by the global yield factors. We find that these country-specific movements in national yield factors can be largely explained by the divergence of monetary policy in different countries. The results are consistent with the findings in Jotikasthira, Le and Lundblad (2015), but not shown here as we focus on the global co-movement.

[32] An increase in the level factor is consistent with higher yields on average. An increase in the slope factor is consistent with a flatter yield curve. In an extreme case, if two factor are moving in opposite directions but with the same magnitude, then the short rates stay still and long rates are driven by the changes in the Level factor.

[33] The strong negative correlation between the Level and Slop disappears after 1999 and reappears after the financial crisis.

Figure D.4: Global Short Rate Factor and the Decomposition



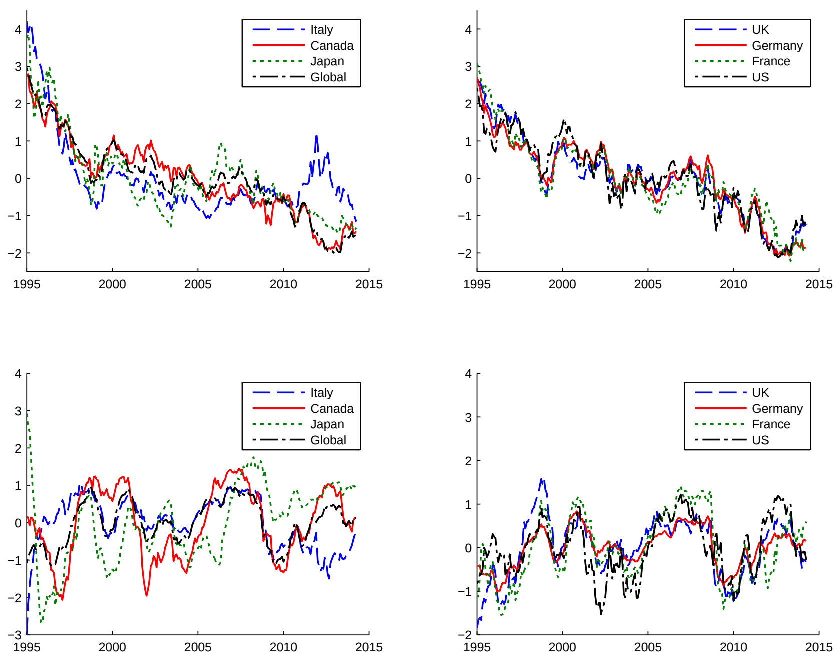
Notes: 1. The left panel shows the global short rate factor (i.e. an arithmetic sum of extracted global Level and Slope factors) and the first principal component of the national short-run policy rates (dashed line). The first principal component of national policy rates, which accounts for more than 84% of total variance of national policy rates. The gray areas cover all the draws of the global short rate factor (i.e. Level + Slope) from our model, and the solid black line is the median value of the draws. Data standardization implies yields can fall below zero.
2. The right panel shows the decomposition of the median of the global short rate factor. We decompose the short rate factor into the global Level (dashed line) and the global Slope (solid red line). In general, the Level factor controls the level of the term structure whereas the Slope factor controls the slope of the term structure. The shaded areas cover some major recession periods in the US and Europe.

Secondly, the Slope factor is declining during US recessions, suggesting it is related to real economic activity, as indicated in Kurmann and Otrok (2013).

D.3.2 Commonality of Level and Slope

We firstly plot our identified Level and Slope factors in Figure D.5, respectively, in order to evaluate the commonalities in country-level yield factors. The Slope factors are relatively less persistent than the Level factors. From the figures it is evident that a strong co-movement in Level factor dynamics exists, but some also exists for the Slope. We also calculate the communality statistics for all countries in Table D.7 to better quantify matters. That is we calculate the proportion of national level or slope factor explained by the global equivalent. This indicates that the commonality in Level factor dynamics is stronger but co-movement remains in the Slope. Generally, we find significant co-movement among Germany, France, Canada, UK and US. In contrast, the Level and Slope factors of Italy are relatively more divorced from the global factors, consistent with Table 2 above; the Japanese Slope factor is much less common among all Slope factors as the communality statistic is nearly zero. The above findings are reassuringly in line with the results in Diebold, Li and Yue (2008).

Figure D.5: Estimated Global and National Factors



Notes: The upper panels show the median values of global Level and Slope factors and the national Level factors of Italy, Canada and Japan. The lower panels show the median values of the national Level and Slope factors of the UK, Germany, France and the US.

Table D.7: Communality Table of Level and Slope

Level		Slope	
Country	Communality	Country	Communality
Italy	0.45	Italy	0.24
Canada	0.94	Canada	0.35
France	0.94	France	0.67
Germany	0.94	Germany	0.91
Japan	0.80	Japan	0.04
UK	0.98	UK	0.77
US	0.90	US	0.51
Average	0.85	**Average**	0.50

Notes: This table summarizes for all countries the communality statistics of global Level and Slope factors for national Level and Slope factors. For example, the communality for a given country is interpreted as the proportion of the variation in the national Level factor explained by the global Level factor. Likewise for the Slope communality.

D.4 Variance Decomposition across Maturities

Table D.8: Decomposition of Variance (US)

Maturity (Month)	Posterior Mean (Standard Deviation)		
	$Share_G$	$Share_F$	$Share_X$
3	0.65(0.08)	0.32(0.08)	0.02(0.01)
6	0.68(0.08)	0.32(0.08)	0.01(0.00)
12	0.71(0.08)	0.29(0.08)	0.00(0.00)
24	0.74(0.07)	0.26(0.07)	0.01(0.00)
36	0.76(0.07)	0.24(0.07)	0.01(0.00)
48	0.77(0.07)	0.22(0.07)	0.01(0.00)
60	0.78(0.07)	0.22(0.06)	0.00(0.00)
72	0.79(0.06)	0.21(0.06)	0.00(0.00)
84	0.79(0.06)	0.21(0.06)	0.00(0.00)
96	0.79(0.06)	0.21(0.06)	0.01(0.00)
120	0.78(0.07)	0.20(0.06)	0.03(0.01)

Notes: This table summarizes the decomposition of variance for the three-level hierarchical model of US bond yields. $share_G$, $share_F$ and $share_Z$ denote the variance shares at different maturities in the country-level block of shocks ϵ_G, ϵ_F and ϵ_X, respectively. In each parenthesis $(\cdot)$ the posterior standard deviation of shares in a specific block is calculated from our draws, see Section 2. Larger standard deviation means higher uncertainty in the estimates, but should be interpreted with caution since they may not have normal distributions.

Table D.9: Decomposition of Variance

Maturity	UK			Germany			France		
(Month)	$Share_G$	$Share_F$	$Share_X$	$Share_G$	$Share_F$	$Share_X$	$Share_G$	$Share_F$	$Share_X$
3	0.80(0.06)	0.20(0.06)	0.01(0.00)	0.70(0.08)	0.23(0.06)	0.07(0.02)	0.66(0.08)	0.27(0.07)	0.07(0.02)
6	0.81(0.06)	0.19(0.06)	0.00(0.00)	0.71(0.08)	0.23(0.06)	0.06(0.02)	0.70(0.08)	0.28(0.07)	0.03(0.01)
12	0.83(0.05)	0.17(0.05)	0.01(0.00)	0.72(0.08)	0.23(0.07)	0.05(0.02)	0.73(0.07)	0.27(0.07)	0.00(0.00)
24	0.84(0.05)	0.14(0.05)	0.01(0.00)	0.75(0.08)	0.23(0.07)	0.02(0.01)	0.75(0.07)	0.24(0.07)	0.01(0.00)
36	0.86(0.05)	0.13(0.04)	0.01(0.00)	0.76(0.07)	0.23(0.07)	0.01(0.00)	0.77(0.07)	0.22(0.06)	0.02(0.00)
48	0.88(0.04)	0.12(0.04)	0.00(0.00)	0.77(0.07)	0.23(0.07)	0.00(0.00)	0.78(0.06)	0.21(0.06)	0.01(0.00)
60	0.89(0.04)	0.11(0.04)	0.00(0.00)	0.77(0.07)	0.22(0.07)	0.01(0.00)	0.79(0.06)	0.20(0.06)	0.00(0.00)
72	0.89(0.04)	0.11(0.04)	0.00(0.00)	0.76(0.07)	0.21(0.07)	0.02(0.01)	0.80(0.06)	0.20(0.06)	0.00(0.00)
84	0.88(0.04)	0.10(0.04)	0.01(0.00)	0.75(0.08)	0.21(0.06)	0.04(0.01)	0.80(0.06)	0.20(0.06)	0.00(0.00)
96	0.86(0.04)	0.10(0.03)	0.04(0.01)	0.73(0.08)	0.20(0.06)	0.06(0.02)	0.80(0.06)	0.19(0.06)	0.01(0.00)
120	0.80(0.06)	0.09(0.03)	0.11(0.03)	0.71(0.08)	0.19(0.06)	0.10(0.03)	0.78(0.06)	0.18(0.05)	0.04(0.01)

Notes: This table summarizes the decomposition of variance for the three-level hierarchical model of bond yields. For each country, $share_G$, $share_F$ and $share_Z$ denote the variance shares at different maturities in the country-level block of shocks ϵ_G, ϵ_F and ϵ_X, respectively. In each parenthesis $(\cdot)$ the posterior standard deviation of shares in a specific block is calculated.

Table D.10: Decomposition of Variance (Continued)

Maturity	Italy			Canada			Japan		
(Month)	$Share_G$	$Share_F$	$Share_X$	$Share_G$	$Share_F$	$Share_X$	$Share_G$	$Share_F$	$Share_X$
3	0.31(0.09)	0.66(0.09)	0.03(0.01)	0.52(0.10)	0.36(0.08)	0.12(0.03)	0.5(0.10)	0.44(0.09)	0.06(0.01)
6	0.32(0.10)	0.67(0.09)	0.01(0.00)	0.57(0.09)	0.36(0.08)	0.07(0.02)	0.54(0.10)	0.43(0.09)	0.03(0.01)
12	0.34(0.10)	0.66(0.10)	0.00(0.00)	0.63(0.09)	0.35(0.08)	0.02(0.01)	0.60(0.09)	0.39(0.09)	0.02(0.00)
24	0.35(0.10)	0.64(0.10)	0.00(0.00)	0.70(0.08)	0.30(0.08)	0.00(0.00)	0.65(0.08)	0.31(0.08)	0.04(0.01)
36	0.36(0.10)	0.63(0.10)	0.00(0.00)	0.74(0.07)	0.26(0.07)	0.00(0.00)	0.69(0.08)	0.28(0.07)	0.03(0.01)
48	0.37(0.10)	0.62(0.10)	0.00(0.00)	0.76(0.07)	0.24(0.07)	0.00(0.00)	0.72(0.07)	0.26(0.07)	0.02(0.00)
60	0.38(0.10)	0.62(0.10)	0.00(0.00)	0.77(0.07)	0.23(0.07)	0.00(0.00)	0.75(0.07)	0.25(0.07)	0.01(0.00)
72	0.38(0.10)	0.61(0.10)	0.00(0.00)	0.78(0.06)	0.22(0.06)	0.00(0.00)	0.76(0.07)	0.24(0.07)	0.00(0.00)
84	0.39(0.10)	0.61(0.10)	0.01(0.00)	0.79(0.06)	0.21(0.06)	0.00(0.00)	0.76(0.07)	0.23(0.06)	0.01(0.00)
96	0.39(0.10)	0.60(0.10)	0.01(0.00)	0.79(0.06)	0.21(0.06)	0.00(0.00)	0.75(0.07)	0.22(0.06)	0.02(0.01)
120	0.39(0.10)	0.59(0.10)	0.02(0.00)	0.79(0.06)	0.20(0.06)	0.01(0.00)	0.73(0.07)	0.21(0.06)	0.06(0.02)

Notes: This table summarizes the decomposition of variance for the three-level hierarchical model of bond yields. For each country, $share_G$, $share_F$ and $share_Z$ denote the variance shares at different maturities in the country-level block of shocks ϵ_G, ϵ_F and ϵ_X, respectively. In each parenthesis $(\cdot)$ the posterior standard deviation of shares in a specific block is calculated.