Efficient pricing of discrete arithmetic Asian options under mean reversion and jumps based on Fourier-cosine expansions

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Abstract

We propose an efficient pricing method for arithmetic Asian options based on Fourier-cosine expansions. In particular, we allow for mean reversion and jumps in the underlying price dynamics. There is an extensive body of empirical evidence in the current literature that points to the existence and prominence of such anomalies in the prices of certain asset classes, such as commodities. Our efficient pricing method is derived for the discretely monitored versions of the European-style arithmetic Asian options. The analytical solutions obtained from our Fourier-cosine expansions are compared to the benchmark fast Fourier transform based pricing for the examination of its accuracy and computational efficiency.

Keywords:
Arithmetic Asian options, Fourier-cosine expansions, Fast Fourier transform, Mean reverting process, Jump diffusion

1. Introduction

A topic of ongoing interest is the long standing hard problem of pricing arithmetic Asian options. The payoffs of these path-dependent exotics are based on the arithmetic average of the underlying prices monitored at fixed dates prior to maturity. The monitoring dates used to measure the arithmetic averages may also be taken at different frequencies, such as daily, weekly or monthly. Unlike its closely related geometric type, the prices of the more commonly traded arithmetic Asian options must be approximated numerically. This is mainly due to the absence of an analytically tractable solution for the distribution of the sum of log normally distributed random variables.

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Asian options, introduced in 1987, are now widely traded in the commodities market as a hedging tool. For instance, various delivery companies in the gas market utilise Asian options to their advantage under risk management (see Eydeland and Wolyniec (2003)). The popularity of Asian options arises mainly from its averaging effect, which is able to reduce possible risk of market manipulation in the price of the underlying at maturity. In addition, since averages move in a more stable way in comparison to individual prices, the volatility inherent in the underlying price is reduced as a result. Further information on Asian options with its history and evolution may also be found in Boyle and Boyle (2001) and Marena et al. (2014).

It has been well documented that the prices of certain asset classes, such as commodities, show evidence of mean reversions and jumps. Hence, the pricing of options within these asset classes has also become an important focus in the field of quantitative finance. For example, due to the impact of relative prices on the supply of both copper and oil, prices tend to fluctuate randomly around some equilibrium level (see Schwartz (1997)). In addition to the above mentioned commodities, Bessembinder et al. (1995) provides strong evidence supporting mean reversion in nine commodity markets, while Casassus and Collin-Dufresne (2005) reveals the existence of such anomaly in the precious metals market. Apart from commodities, however, evident motivating the patterns of mean reversion has also been found in exchange rates and, interestingly, certain stock prices as well (see Jorion and Sweeney (1996) and Chaudhuri and Wu (2003)).

In addition to the mean reverting property, jumps in the underlying price dynamic is another prominent feature. For instance, Jorion (1988) examined the prices of stock market indices and exchange rates for discontinuities, while Geman and Roncoroni (2006) and Seifert and Uhrig-Homburg (2007) conducted investigations to provide empirical evidence of jumps in the power market. In addition, Geman and Roncoroni (2006) found further evidence in support of mean reversion. Further empirical evidence in support of jumps in commodity prices may also be found in the current literature (see Deng (2000), Hilliard and Reis (1999), and Schmitz et al. (2014)).

Combining the fundamental ideas above, we propose an efficient pricing method for discrete arithmetic Asian options under a pricing dynamic which exhibits both jumps and mean reversion. Essentially, the model is a jump-diffusion extension of the one utilised by Fusai et al. (2008) (as proposed by Chung and Wong (2014)). Apart from accuracy in the pricing, computational efficiency is also of equal importance, if not more, particularly, for high frequency traders. Such notion brings about the non-trivial problem of finding a reasonable trade-off between accuracy and efficiency in the pricing methods. As a result, efficient pricing methods of exotic options have also gained much interest from both practitioners and academics alike.

In option pricing, the valuation of complex contracts requires efficient numerical methods. The conditional expectation of the option payoff under the risk-neutral measure can be bridged with the solution of a partial differential equation through the well-known Feynman-Kac theorem. It then follows that various numerical pricing techniques, including numerical integration, can be developed. These numerical integration techniques rely on the transformation into the Fourier domain, which is particularly useful especially since the density function of many relevant underlying price process, required for the integration in the original domain, is not known. However, its Fourier transform, the characteristic function often is. It then follows that the fast Fourier transform (FFT) method, introduced by Carr and Madan (1999) and Dempster and Hong (2002), may be applied to calculate the option price efficiently. However, Fang and Oosterlee (2008) proposed a novel pricing method, the Fourier-cosine expansions (COS method), as an alternative to the FFT. Such method could further improve the speed in the pricing.

In this paper, we propose to price discrete arithmetic Asian options under the assumption of mean reversion and jumps with the COS method. We show through numerical analysis that the COS method is indeed more efficient than the benchmark FFT, used by Chung and Wong (2014). It was also shown in Chung and Wong (2014) that the FFT is superior to the commonly implemented Monte Carlo simulation.

The remainder of this paper is organised as follows. In section 2, we explore the proposed diffusion model for the underlying price dynamics with mean reversion and jumps. The joint characteristic function between the arithmetic average of the asset prices and its terminal value is also derived. Section 3 briefly introduced the COS method and the procedures to price the Asian option in question. We present a set of numerical results and analysis in Section 4 to evaluate the accuracy and efficiency of the COS method benchmarked to the FFT. The sensitivity of the COS prices to the underlying parameters are then analysed. We conclude the paper in Section 5.

2. Price process with mean reversion and jumps

2.1. Model specification

Let $(\Omega, \mathcal{F}, \mathbb{Q})$ be a probability space on which a Brownian motion process W_t and a Poisson process N_t, with intensity $\lambda > 0$, is defined for $0 \leq t \leq T$. Furthermore, we assume independence between the Brownian motion and Poisson process. Suppose $\mathbb{Q}$ is the risk neutral measure under which the price process is governed by the following dynamics:

$$dS_t = \kappa\Big(\theta - \frac{\mu\lambda}{\kappa} - S_{t_-}\Big)dt + \sigma\sqrt{S_{t_-}}dW_t + JdN_t, \tag{1}$$

where $J \sim Exp(\mu)$ and $N_t \sim Poi(\lambda t)$.

The model proposed here is an extension of the Fusai et al. (2008) model, whereby a jump component has been added to the original spot price process, which is defined as a square root process driven by a Brownian motion. The jump size J and its arrival rate N_t are independent, and are modelled with an exponential distribution and a Poisson process, respectively. More specifically, the proposed price process is a CIR model with an exponential jump extension. Further justifications for the specific choice of jump dynamics can be found in Hoepfner (2009) and Beliaeva and Nawalkha (2012), with the latter suggesting the non-existence of an analytical solution under lognormal jumps.

The use of the CIR as a base model gives rise to two main advantages in terms of Asian option pricing. Firstly, since we are interested in the average price of the underlying, the existence of the characteristic function for $\int_0^T r_t dt$ in the CIR model, used widely in the modelling of interest rates, helps simplify the problem at hand. Secondly, instead of a log price, by choosing suitable parameters according to the Feller condition, we can model the stock price directly under the CIR model while maintaining its positivity. Such positivity is consistent even after jumps are added, as the jump sizes are modelled using an exponential distribution, which is always positive.

Our aim is to price an Asian option at initial time 0 that matures at terminal time T. The underlying price will be recorded at some regular time interval to allow for the discretely monitored Asian option in question. We split the pricing interval $[0, T]$ into $n + 1$ sets of $\triangle$-spaced monitoring dates. Hence, we have dates $0, \triangle, ..., n\triangle = T$. Such a setup allows for the computation of an analytical price for the Asian option with payoff depending on arithmetic average $A_n = \sum_{j=0}^{n} \omega_j S_{j\triangle}$ and the terminal price $S_{n\triangle}$, where ω_j is the weight assigned to price $S_{j\triangle}$ and $\sum_{j=0}^{n} \omega_j = 1$. It is worthwhile mentioning that the weights assigned to the underlying price at different time intervals need not be equal. Table 1 summarises the various type of options that may be priced under our model assumption.

Table 1: Payoff functions of various options

Option type	Payoff function
Fixed strike Asian call	$\max\{A_n - K, 0\}$
Fixed strike Asian put	$\max\{K - A_n, 0\}$
Floating strike Asian call	$\max\{S_{n\triangle} - A_n - K, 0\}$
Floating strike Asian put	$\max\{K + A_n - S_{n\triangle}, 0\}$
European call	$\max\{S_{n\triangle} - K, 0\}$
European put	$\max\{K - S_{n\triangle}, 0\}$

2.2. Derivation of joint characteristic function

The joint characteristic function between $S_{n\triangle}$ and A_n is required for us to price the Asian options analytically. We first determine the characteristic function of $S_{t+\triangle}$ and proceed to derive the joint characteristic function of the pair $S_{n\triangle}$ and A_n.

The characteristic function of $S_{t+\triangle}$ can be defined as $f^{\varphi}(t, S_t) \equiv \mathbb{E}_t^{\mathbb{Q}}[e^{ivS_{t+\triangle}}]$ with parameter set $\varphi = \{\kappa; \mu; \lambda; \theta; \sigma\}$.[1] The generalised Feynman-Kac theorem (see Duffie et al. (2000) and Cont and Tankov (1975)) implies that f^{φ} solves

[1]From here, we drop the $\mathbb{Q}$ for notational convenience

the following partial integro-differential equation (PIDE):

$$\frac{\partial f^{\varphi}}{\partial \tau} + \kappa\Big(\theta - \frac{\mu\lambda}{\kappa} - S_u\Big)\frac{\partial f^{\varphi}}{\partial S_u} + \frac{1}{2}\sigma^2 S_u \frac{\partial^2 f^{\varphi}}{\partial S_u^2} \quad + \\ \lambda \int_{-\infty}^{\infty} [f^{\varphi}(u, S_u + J) \quad - \quad f^{\varphi}(u, S_u) q(J)] dJ = 0,$$

with boundary condition $f^{\varphi}(t + \triangle, S_{t+\triangle}) = e^{i\upsilon S_{t+\triangle}}$, where $u \in [t, t + \triangle]$ and $\tau = t + \triangle - u$, and $q(J)$ is the distribution of J. Coefficients, $\kappa(\theta - \frac{\mu\lambda}{\kappa} - S_u)$ and σ, of the mean reverting asset price process (1) are both affine in nature. It follows that the solution to (2) is of exponential affine form $f^{\varphi}(u, S_u) = e^{-\alpha^{\varphi}(\tau;\upsilon)S_u - \beta^{\varphi}(\tau;\upsilon)}$. Substituting into (2) above, and matching the characteristic function of the exponential distribution governing the jumps, we obtain $\mathbb{E}(e^{-\alpha^{\varphi}(\tau;\upsilon)}) = \frac{1}{1+\mu\alpha^{\varphi}(\tau;\upsilon)}$. Further simplification will allow us to obtain the following ODE (with differentiations taken with respect to τ):

$$\alpha^{\varphi\prime}(\tau;\upsilon) + \kappa\alpha^{\varphi}(\tau;\upsilon) + \frac{1}{2}\sigma^2[\alpha^{\varphi}(\tau;\upsilon)]^2 = 0 \tag{2}$$

$$\beta^{\varphi\prime}(\tau;\upsilon) - \kappa\Big(\theta - \frac{\mu\lambda}{\kappa}\Big)\alpha^{\varphi}(\tau;\upsilon) + \lambda\Big(\frac{1}{1 + \mu\alpha^{\varphi}(\tau;\upsilon)} - 1\Big) = 0 \tag{3}$$

with initial conditions $\alpha^{\varphi}(0;\upsilon) = -i\upsilon$ and $\beta^{\varphi}(0;\upsilon) = 0$.

Solving for $\alpha^{\varphi}(\triangle;\upsilon)$ from the Bernoulli equation, and $\beta^{\upsilon}(\triangle;\upsilon)$ through integration, we obtain the following:

$$\Psi^{\varphi}(\upsilon) = \mathbb{E}_t(e^{i\upsilon S_{t+\triangle}}) = e^{-\alpha^{\varphi}(\triangle;\upsilon)S_t - \beta^{\varphi}(\triangle;\upsilon)}, \tag{4}$$

where

$$\alpha^{\varphi}(\triangle;\upsilon) = \frac{\frac{\sigma^2}{2\kappa}(e^{\kappa\triangle} - 1) - \frac{e^{\kappa\triangle}}{\upsilon} i}{\frac{\sigma^4}{4\kappa^2}(e^{\kappa\triangle} - 1)^2 + \frac{e^{2\kappa\triangle}}{\upsilon^2}} \tag{5}$$

$$\beta^{\varphi}(\triangle;\upsilon) = \kappa\Big(\theta - \frac{\mu\lambda}{\kappa}\Big)\int_0^{\triangle} \alpha^{\varphi}(\tau;\upsilon)d\tau - \lambda\int_0^{\triangle}\Big(\frac{1}{1 + \mu\alpha^{\varphi}(\tau;\upsilon)} - 1\Big)d\tau. \tag{6}$$

The joint characteristic function between $S_{n\triangle}$ and A_n can be derived by utilising (4) and repeating the law of iterated expectation. Hence, following the methodology as outlined in Chung and Wong (2014), we have the joint characteristic function between $S_{n\triangle}$ and $\sum_{j=0}^{n}\omega_j S_{j\triangle}$ under price dynamics (1):

$$\begin{aligned}\Psi^{\varphi}_{A_n}(\phi;\gamma) &= \mathbb{E}_0\Big(e^{i\phi S_{n\triangle} + i\gamma\sum_{j=0}^{n}\omega_j S_{j\triangle}}\Big) \\ &= e^{i\Gamma_0^{\varphi}(\triangle;\phi,\gamma)S_0 - \sum_{j=0}^{n-1}\beta^{\varphi}(\triangle;\Gamma^{\varphi}_{j+1}(\triangle;\phi,\gamma))}\end{aligned} \tag{7}$$

where $\Gamma_j^{\varphi}(\triangle;\phi,\gamma)$ satisfies the following recursive equation:

$$\Gamma_j^{\varphi}(\triangle;\phi,\gamma) = i\Gamma^{\varphi}(\triangle;\Gamma^{\varphi}_{j+1}(\triangle;\phi,\gamma)) + \gamma\omega_j, \tag{8}$$

for $j = n-1, n-2, ..., 0$, and starting value $\Gamma_n^{\varphi}(\triangle;\phi,\gamma) = \phi + \gamma\omega_n$.

3. Asian option pricing with Fourier-cosine expansions

The pricing of options under the COS method, as with all numerical integration techniques, follows from the discounted expected payoff approach under the risk-neutral measure $\mathbb{Q}$:

$$v(x,t) = e^{-r(T-t)}\mathbb{E}[v(y,T)|x] = e^{-r(T-t)}\int_{\mathbb{R}} v(y,T)g(y|x)dy, \tag{9}$$

where $v(x,t)$ denotes the option value at time t, and r is the interest rate. x and y are state variables at time t and expiration date T, respectively. Typically, the option's payoff function, v, is known, but its transitional density function $g(y|x)$ is not. Fang and Oosterlee (2008) proposed an approximation of the transition probability, based on (9), with a truncated domain $[a;b]$ by a truncated Fourier-cosine series expansion with N terms, based on the conditional characteristic function, i.e.:

$$g(y|x) \approx \frac{2}{b-a}\sum_{h=0}^{N-1}{}' Re\Big\{\psi\Big(\frac{h\pi}{b-a};x\Big)e^{-ih\pi\frac{a}{b-a}}\Big\}\cos\Big(h\pi\frac{y-a}{b-a}\Big), \tag{10}$$

where $\psi(\nu;x)$ is the conditional characteristic function of $g(y|x)$, and a, b denotes the integration range in the original domain. $\sum'$ indicates that the first term of the summation is multiplied by a weight of one-half. The integration range $[a,b]$ may also be determined by making use of the cumulants, such that the error of the approximation is within some tolerance level Tol, i.e. $|\int_{\mathbb{R}} g(y|x)dy - \int_a^b g(y|x)dy| < Tol$ (see Fang and Oosterlee (2008)). Finally, replacing the conditional density function in (9) with its approximation (10), and interchanging the summation and integration, we obtain the COS formula to price an option with payoff $v(x,t)$:

$$\hat{v}(x,t) = e^{-r(T-t)}\sum_{h=0}^{N-1}{}' Re\Big\{\psi\Big(\frac{h\pi}{b-a};x\Big)e^{-ih\pi\frac{a}{b-a}}\Big\}V_h \tag{11}$$

where $\hat{v}(x,t)$ is the approximation of the option value at time t, and

$$V_h := \frac{2}{b-a}\int_a^b v(y,T)\cos\Big(h\pi\frac{y-a}{b-a}\Big)dy \tag{12}$$

are the Fourier-cosine coefficients of payoff $v(y,T)$.

Having derived the joint characteristic function between $S_{n\triangle}$ and A_n, we use the COS method to price Asian options. First, consider the contingent claim $v(y,T) = \max(\rho y - k, 0)$ at time T, where $k = \rho K$, $\rho = +1$ for calls and $\rho = -1$ for puts. This setup allows for both fixed strike Asian options $\Big(y = \sum_{j=0}^n \omega_j S_{j\triangle}\Big)$ and floating strike Asian options $\Big(y = S_{n\triangle} - \sum_{j=0}^n \omega_j S_{\omega j}\Big)$, as well as for a plain vanilla option $\Big(y = S_{n\triangle}\Big)$. Under the assumption of risk neutrality, with $g_{\rho y}(u)$ as the density of ρy, the arbitrage free price of our option at initial time 0 is:

$$v(y,T;k;\rho) = e^{-r(T-t)} \int_{-\infty}^{\infty} \max(u-k,0) g_{\rho y}(u) du. \tag{13}$$

Through expression (13), and substituting the joint characteristic function derived in (7) into (11), we arrive at the COS method for pricing the various arithmetic Asian options mentioned in table 1, where:

$$V_h = \begin{cases} \frac{2}{b-a}\Big(\Pi_{1,h}(K,b) - K\Pi_{2,h}(K,b)\Big) & \text{for calls,} \\ \frac{2}{b-a}\Big(K\Pi_{2,h}(a,K) - \Pi_{1,h}(a,K)\Big) & \text{for puts,} \end{cases} \tag{14}$$

where $\Pi_{1,h}$ and $\Pi_{2,h}$ are from the mathematical results below.

Proposition 1. *The cosine series coefficients, $\Pi_{1,h}$, of a function $\mathcal{H}(y) = y$ on $[x_1, x_2] \subset [a,b]$ given by,*

$$\Pi_{1,h}(x_1,x_2) := \int_{x_1}^{x_2} y \cos\left(h\pi \frac{y-a}{b-a}\right) dy, \tag{15}$$

and the cosine series coefficients, $\Pi_{2,h}$, of another function $\mathcal{H}(y) = 1$ on $[x_1, x_2] \subset [a,b]$ given by,

$$\Pi_{2,h}(x_1,x_2) := \int_{x_1}^{x_2} \cos\left(h\pi \frac{y-a}{b-a}\right) dy, \tag{16}$$

are both known analytically.

Proof. A straightforward calculation shows that

$$\begin{aligned} \Pi_{1,k}(x_1,x_2) &= \frac{1}{\left(\frac{h\pi}{b-a}\right)^2}\Big[\cos\left(h\pi\frac{x_2-a}{b-a}\right) - \cos\left(h\pi\frac{x_1-a}{b-a}\right) \\ &+ \frac{h\pi}{b-a}\sin\left(h\pi\frac{x_2-a}{b-a}\right)x_2 - \frac{h\pi}{b-a}\sin\left(h\pi\frac{x_1-a}{b-a}\right)\Big] \end{aligned}$$

and

$$\Pi_{2,k}(x_1,x_2) = \begin{cases} \left[\sin\left(h\pi\frac{x_2-a}{b-a}\right) - \sin\left(h\pi\frac{x_1-a}{b-a}\right)\right]\frac{b-a}{h\pi}, & \text{if } h \neq 0. \\ (x_1 - x_2), & \text{otherwise.} \end{cases}$$

4. Numerical Results

In this section, a variety of numerical analyses are performed to test the performance of the COS method against its alternative competitor, the FFT method, as a benchmark. In addition to Chung and Wong (2014), which concluded that the FFT outperforms Monte Carlo simulations in terms of both pricing accuracy and efficiency, we further show that the COS method is more efficient, and does not compromise the pricing accuracy.

4.1. Truncation range for COS method

The error analysis of the COS method, presented in Fang and Oosterlee (2008), has shown that over a well-specified truncation range for the integration in (12), the overall error converges either exponentially or algebraically, depending on whether the density function belongs to $\mathbb{C}^\infty([a,b] \in \mathbb{R})$ or has a discontinuity in one of its derivatives, respectively. Such a truncation range, $[a,b]$, may be determined by making use of the n-th cumulant, c_n, of $y = \sum_{j=0}^{n} \omega_j S_{j\triangle}$ (for fixed strike) or $y = S_{n\triangle} - \sum_{j=0}^{n} \omega_j S_{\omega j}$ (for floating strike), as proposed in Fang and Oosterlee (2008):

$$[a,b] := \left[c_1 - L\sqrt{c_2 + \sqrt{c_4}}, \quad c_1 + L\sqrt{c_2 + \sqrt{c_4}}\ \right] \quad \text{with } L = 10. \tag{17}$$

Readers are referred to Fang and Oosterlee (2008) and Fang and Oosterlee (2009) for detailed discussions on the choices of c_n and L.

4.2. Comparison of COS method against FFT

Our main focus is the performance comparison between the proposed COS method and that of the FFT. Constant parameters are utilised in our models to ease the demonstration. In particular, we make use of the same constant parameters as specified in Chung and Wong (2014). These parameter values are summarised in table 2. In addition, our comparison will be performed on both fixed and floating strike Asian options, for the different frequencies of monitoring dates n, where $n = 4, 12, 26, 52$ and 252. These dates correspond to quarterly, monthly, biweekly, weekly and daily monitoring setups. The resulting relative price differences between the COS method and FFT are shown in figure 1. Our numerical result shows a relative pricing difference (or error) in the order

Table 2: Parameter values for the numerical analyses

$S_0 = 1$	$\kappa = 0.3$
$\theta = 1.05$	$\sigma = 0.7$
$\lambda = 5$	$\mu = 0.1$
$T = 1$	$r = 0.04$
$\omega_j = \frac{1}{n+1}$	

between 0.006% and 0.02% for the fixed strike Asian options, and between 0.02% and 0.2% for the floating strike, indicating a negligible difference between the two approaches. These results, together with that of Chung and Wong (2014), suggest a high pricing accuracy for both the FFT and COS method in pricing arithmetic Asian options.

In terms of pricing efficiency, the COS method dominates that of the FFT. Using a computer equipped with a 3.5GHz quad-core Intel Core i7-4850HQ processor, the COS method takes only between 0.01-1s (ranging between $\frac{1}{15}$ and $\frac{1}{2}$ of the time required by the FFT method) to obtain the option prices, depending on the choice of monitoring dates and integration grid sizes, N.

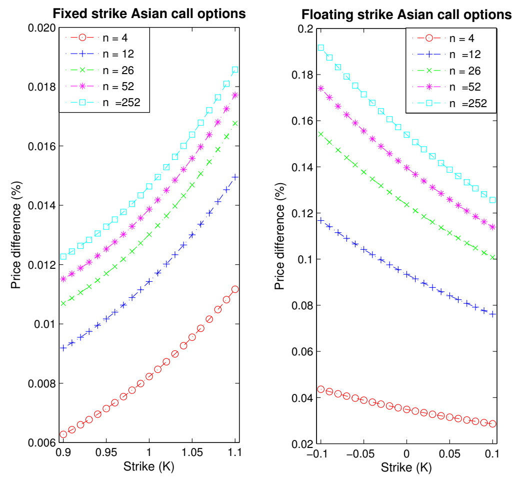

Figure 1: Price difference between COS and FFT methods for Fixed and Floating strike Asian calls

In Table 3, the cpu time and relative error information, comparing the COS and the FFT method, are presented for the pricing of Asian options. For this particular example, we price for fixed strike arithmetic Asian options, with weekly monitoring dates ($n = 52$), and grid sizes ranging from $N = 128$ to $N = 1024$. The COS method uses significantly less cpu time to obtain the option prices, while at the same time, produces equal level of accuracy to that of the FFT (evidence from the negligible absolute relative pricing errors).

Table 3: cpu time differences and relative error between COS method and FFT

n = 52	N	128	256	512	1024
COS	sec	0.0244	0.0431	0.0524	0.1207
FFT	sec	0.2458	0.2551	0.2712	0.2954
abs. rel. err.		1.6754e-05	1.6840e-05	1.6838e-05	1.6838e-05

Table 4 displays the cpu time comparison and the relative error information between the COS and FFT methods in calculating Asian option prices. In this example, we calculate for fixed strike Asian options across the different monitoring dates, ranging from quarterly ($n = 4$) to daily ($n = 252$), and grid size $N = 4096$. The COS method once again proves to be superior to the alternative FFT method in terms of efficiency for all monitoring dates. However, the efficiency improvement is of a decreasing rate as we increase the monitoring frequency. Such patterns are not dissimilar to the results of Fang and Oosterlee (2008), whereby the COS method's rate of efficiency improvements was shown to decrease as the number of grid sizes, N, is increased.

Table 4: cpu time differences and relative error between COS method and FFT

	n	4	12	26	52	252
COS	sec	0.0396	0.0669	0.1063	0.1648	0.7042
FFT	sec	0.2745	0.3135	0.3675	0.4625	1.3861
abs. rel. err.		9.7365e-06	1.3956e-05	1.5865e-05	1.6838e-05	1.7689e-05

4.3. Price sensitivity to changes in model parameters

Apart from our comparison on the pricing efficiency and accuracy, we evaluate the effect of parameter value changes on the Asian option price computed from the proposed COS method. This falls particularly in line with the analyses performed by Chung and Wong (2014). Inclusion of such analyses also provide further robust evidence on the stability of the COS method in comparison to the FFT if time-dependent parameters were advocated. The three parameters observed are (i) the jump intensity, (ii) the mean level, and (iii) the asset volatility of the proposed commodity price dynamic (1).

We plot the Asian call option prices against different values of the three parameters mentioned above. Parameter values in table 2 are used as a base case and altered within a specified range to find different Asian option prices. The resulting prices are calculated using the COS method, with $K = S_0$ for fixed strike Asian options and $K = 0$ for the floating. Prices against each changing parameter are then plotted in Figures 2-4.

From Figure 2, it is clear that both fixed and floating Asian call option prices are increasing functions of jump intensity, λ. Such result may be deemed valid as an increase in jump intensity also introduces more variability into the underlying asset price dynamic, which in turn increases the value of the options. Long term mean levels should also have a positive relationship with call option prices, as greater long term mean levels implies asset prices will tend to remain at a higher level. Such notion is evidence in Figure 3, which shows a higher Asian call option prices for greater long term mean levels (the opposite will hold for puts). Finally, Figure 4 confirms the trivial notion of a positive relationship between volatility and option prices. The greater the volatility the more the variability there is in the asset price, and thus the greater the option value.

It is also worthwhile emphasising that the above results are consistent with the FFT case presented in Chung and Wong (2014), further reinforcing the stability in the pricing accuracy of the COS method to that of the alternative FFT. When the resulting COS prices are compared to the FFT as a benchmark, the relative errors (or price differences) were also found to be negligible (not dissimilar to that of Figure 1).

Finally, in Figure 5 we present the rate of convergence of the COS Asian option prices when monitoring frequencies are increased. Both fixed and floating strike Asian option prices tend to converge or stabilise for weekly monitoring frequencies and above.

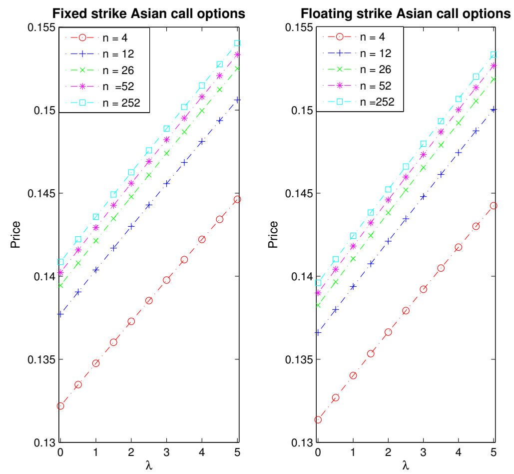

Figure 2: Asian option price against jump intensity under COS method

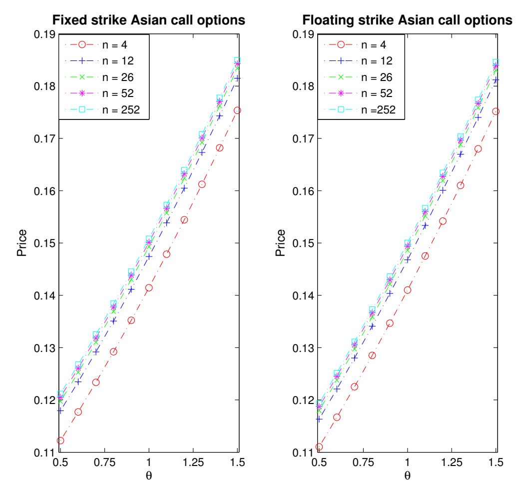

Figure 3: Asian option price against mean levels under COS method

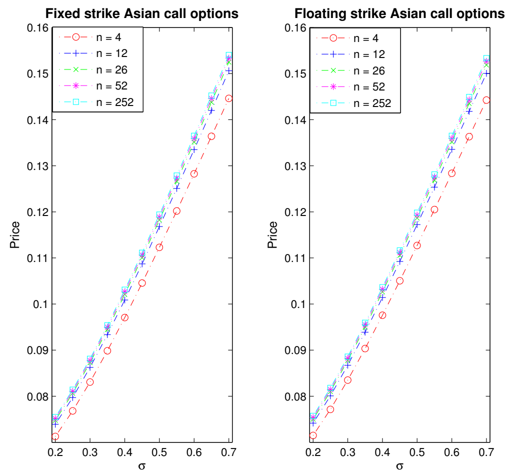

Figure 4: Asian option price against asset volatility under COS method

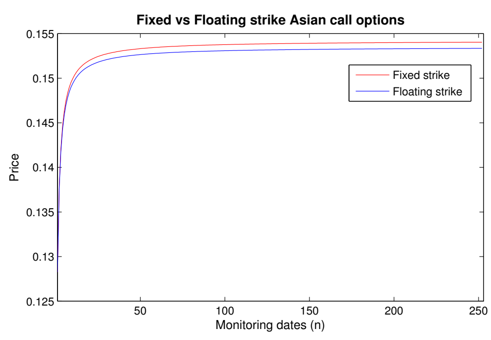

Figure 5: Asian option price under COS method for different monitoring dates

5. Conclusion

In this paper, we proposed the pricing of arithmetic Asian options with the Fourier-cosine method. In particular, we assume a mean reverting jump diffusion process in modelling the underlying commodity price dynamics. Our main focus lies in the investigation of the efficiency and accuracy of the COS method in comparison to the widely accepted FFT. The COS method were shown through our numerical analyses to be more efficient than the benchmark FFT, while producing an equal level of accuracy. Such results are also of particular significance to high frequency traders in search of a better tradeoff between pricing accuracy and efficiency, and a superior method to that of the currently preferred FFT.

To further demonstrate the stability of the COS method, investigations on the price sensitivity to different underlying parameters were conducted. The results presented in this paper further support the use of jumps in the price dynamic, and the inclusion of time-varying mean level and asset volatility. In addition, it demonstrated the stability of the COS method in comparison to the alternative FFT when underlying parameters vary. Further work may include the investigation of COS method pricing of early exercise Asian options of the arithmetic type, in particular, with mean reversion and jumps inherent in the underlying price dynamics.

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