Collateral, liquidity and debt sustainability*

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Abstract

We study Markov-perfect optimal fiscal policy in an economy with financial frictions and sovereign default in the form endogenously determined haircuts on outstanding debt. Government bonds facilitate tax smoothing, but also provide collateral and liquidity services that mitigate financial frictions. There exists a debt Laffer curve, which induces the government to issue bonds to a point where marginal debt has negative welfare effects. Debt positions in the order of magnitude of annual output remain sustainable despite the option to default. When default happens, liquidity on the bond market is impaired, which can trigger extended periods of recurrent haircuts.

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Short title: Debt sustainability

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The sustainability of public debt has become a serious concern to investors and policy-makers during the recent financial crisis. Fears of sovereign default and the associated rising borrowing costs have forced several European countries into severe fiscal austerity measures. In the United States, concerns about the sustainability of public debt have featured prominently in the recent debate on the *fiscal cliff*. They are also prevalent in Japan, which faces the highest debt-to-GDP ratio among OECD countries.

The countries referred to above are developed economies where a substantial fraction of government debt is held domestically. The recurrent concerns about their debt sustainability indicate that the incentives of governments to default on debt held by domestic residents are not yet well understood. This is partly due to the scarcity of recent historical default episodes in advanced economies, but also due to the scarcity of relevant theoretical quantitative studies. Indeed, the literature on sovereign default has mainly studied the sustainability of *external* debt in developing economies – the empirically relevant case before the crisis.[1] The (normative) fiscal policy literature, in turn, has focused on the determination of taxes and domestic government debt in environments with and without commitment, yet abstracting from the possibility of outright government default.[2] In the present paper we address this shortcoming by introducing *fractional default* as a policy instrument into a closed-economy model of optimal discretionary fiscal policy.[3] We examine optimal default policies and their interaction with government spending, tax and debt policies. We are particularly interested in (i) the determination of the

[1]Prominent examples include Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008).

[2]Important papers are Barro (1979), Lucas and Stokey (1983), Aiyagari, Marcet, Sargent, and Seppala (2002), Klein, Krusell, and Rios-Rull (2008).

[3]Our approach of modelling domestically-held government debt subject to the the risk of fractional haircuts differentiates our work from most previous research on sovereign default. Both features have empirical support. See e.g. Reinhart and Rogoff (2011) on the importance of domestic government debt, and Cruces and Trebesch (2013) on the incidence of fractional haircuts in sovereign debt restructuring.

long-run level of government debt and its sustainability under the option to default, (ii) the optimal size of haircuts, and (iii) the dynamics of debt during default episodes.

Our core framework is the classic closed-economy model of Lucas and Stokey (1983) augmented with endogenous government spending as in Debortoli and Nunes (2013). We extend this framework in several dimensions. First, we introduce financial frictions into the economy's production sector: Firms must finance their wage bill in advance using collateralised loans; and the scale of profitable investment projects is limited by entrepreneurs' access to external finance. This generates a role for government debt as *collateral* and as *private liquidity* (cf. Holmstrom and Tirole, 1998). Second, in line with the open-economy literature on sovereign debt (e.g. Arellano, 2008), we introduce a fixed reputational cost of defaulting by assuming temporary disruptions of the market for government debt following a default event. Finally, we consider aggregate uncertainty in the form of a disaster shock to labour productivity. Starting from this extended environment, our paper provides a coherent theory establishing financial frictions as a single force driving both the accumulation of public debt and its sustainability.

We find that the steady state level of debt in our model is strictly positive and sizeable (84% of output in our baseline calibration), unlike in earlier models that predict negative or zero long-run debt (e.g. Aiyagari, Marcet, Sargent, and Seppala, 2002).[4] This property is owed to the role of government bonds in mitigating financial frictions. Notably, the banking sector's demand for collateral is fully satiated in the steady state, that is, at the margin debt accumulation is driven only by the trade-off between the liquidity demand for government bonds and the tax

[4]Alternative theories that rationalise positive long-run debt rest on political frictions (Persson and Svensson, 1989; Alesina and Tabellini, 1990), monetary-fiscal interactions (Ellison and Rankin, 2007; Diaz-Gimenez, Giovannetti, Marimon, and Teles, 2008; Martin, 2009) or simply the government's relative impatience as in the literature on external sovereign debt and default.

distortions associated with a higher debt burden. We further find that the government's debt policy is subject to a *Laffer curve*. The issuance of an extra unit of debt reduces collateral and liquidity premia and thus raises the interest rate costs of the entire inframarginal quantity of debt. When debt is scarce this effect is particularly strong and issuing more debt actually generates less revenue for the government. These dynamics on the 'bad' side of the debt Laffer curve are the flip side of the positive welfare effects of marginal debt: The economy is on the 'bad' ('good') side of the debt Laffer curve precisely when marginal debt has positive (negative) welfare effects. This induces the government to accumulate sufficient debt to escape the 'bad' side of the debt Laffer curve, that is, to build up debt to a point beyond the welfare-maximizing level.

The government's optimal haircut policy takes into account the repercussions for financial intermediation resulting from a (partial) default on government bonds. Endogenous costs of default arise due to the post-default scarcity of debt and are thus proportional to the size of the imposed haircut. While the government is not forced to run a balanced budget during a default episode, we assume that public debt ceases to be tradeable on secondary markets. This reputational cost of default has a fixed-cost character. The government balances the fixed and variable costs of default against the additional tax distortions under full repayment. Since the latter are increasing in debt, there is a maximum sustainable level of debt, a *fiscal limit*. For debt levels exceeding the fiscal limit the discretionary government exercises its default option. In our calibrated economy the fiscal limit is in the order of magnitude of annual output and the optimal haircut amounts to approximately 45%.

Finally, we examine the dynamics of debt and the government's repayment policy during a default episode. We show that equilibrium default occurs in the form of a series of recurrent haircuts: As long as the market for government debt remains disrupted in the aftermath of an initial default, the government repays its debt only partially. Only when the secondary market trading of public debt is resumed, the government 'graduates' from default and the economy converges back towards its long-run steady state.

Our work is related to a number of recent papers that study the determination of public debt under optimal discretionary fiscal policy, though without default. In a model without capital and with exogenous government expenditure, Krusell, Martin, and Rios-Rull (2006) uncover a multiplicity of steady states that depend on initial conditions and are thus similar to those under full commitment. Considering endogenous government expenditure instead, Debortoli and Nunes (2013) establish convergence to zero long-run debt as a robust outcome driven by the government's interest rate manipulation motive. Our model nests their economy as a special case and inherits a generalised interest rate manipulation motive. Moreover, we consider outright default as part of the optimal policy mix and study the sustainability of public debt.

This focus on debt sustainability is also central to the vast literature on external sovereign debt and default. There, debt is held externally, fiscal policy is largely absent, governments decide about default in a discretionary fashion, and costs of default are exogenous. Notable recent exceptions include the studies by Cuadra, Sanchez, and Sapriza (2010) who examine the role of fiscal policy, Mendoza and Yue (2012) who assess business cycle implications in an environment with endogenous default costs, and Adam and Grill (2012) who analyze optimal sovereign default as the solution to a Ramsey plan. Moreover, Yue (2010) and Arellano, Mateos-

Planas, and Rios-Rull (2013) have recently analyzed aspects of partial default emerging under debt renegotiation or as an expensive alternative to new borrowing in the context of models with exogenous costs of default.

Strategic default on domestic government debt has recently been studied by D'Erasmo and Mendoza (2012), Juessen and Schabert (2012), Sosa-Padilla (2012) and Pouzo (2013). However, different from our paper, D'Erasmo and Mendoza (2012) focus on redistributive implications. Juessen and Schabert (2012) consider a setup with risk-neutral agents and exogenous default costs. Similar to our approach, Sosa-Padilla (2012) invokes a working capital constraint to generate endogenous default costs, but the government's default decision is binary and debt is again priced by risk-neutral agents. Pouzo (2013) proceeds under the assumption that the government can commit to its tax policy but not to the repayment of outstanding debt; different from our model, default triggers a temporary breakdown of the primary bond market, but debt continues to be traded on secondary markets and hence retains a positive valuation in anticipation of a future recovery of the primary market.

Finally, the consideration of the role of public debt and endogenous default costs in the presence of financial frictions connects our paper to models with incomplete markets in the tradition of Aiyagari (1994). Woodford (1990), Holmstrom and Tirole (1998) and Aiyagari and McGrattan (1998) show how public debt can help to relax financial constraints. Angeletos, Collard, Dellas, and Diba (2013) explore the implications for optimal fiscal policy under commitment. Given our no-commitment approach, we uncover the strategic manipulation of bond prices and the option to default as additional motives facing policy-makers. Brutti (2011) and Gennaioli, Martin, and Rossi (2014) study sovereign default in three-period economies where sovereign default destroys firms' ability to insure against idiosyncratic shocks or the balance sheets of domestic banks, respectively. They find that financial frictions can render sizeable government debt positions sustainable even in the absence of reputational costs of default. Our paper re-examines these findings in a fully dynamic environment which allows to analyze the determination of long-run debt and shows that reputational fixed costs of default are critical to render ergodic debt positions sustainable under fractional default. Notably, this result obtains despite the essential role of public debt for production in our model.

The rest of this paper is organised as follows. In Section 1 we lay out our model economy. In Section 2 we examine Markov-perfect optimal fiscal policy while maintaining the assumption of commitment to full debt repayment. In Section 3 we introduce the option of fractional default. In Section 4 we study the quantitative implications of optimal fiscal policies in a calibrated economy. In Section 5 we analyze the robustness of these implications to variations in the degree of the endogenous and reputational costs of default, while Section 6 examines the implications of aggregate uncertainty. In Section 7 we conclude.

1 The Environment

The economy is populated by households, firms and a government. There is a single non-storable output good, which is either consumed by households or transformed at a unitary rate into a public good by the government. The government can commit to fully repay outstanding debt, but it lacks inter-temporal commitment to its choices for the income tax rate, public spending and debt issuances.[5]

Households. There is a continuum of measure one of identical, infinitely-lived households. The preferences of a representative household $j \in [0,1]$ are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t^j, 1 - n_t^j, g_t), \tag{1}$$

where $\beta \in (0,1)$ is a time discount factor, c_t^j and n_t^j denote consumption and labour effort of household j, and g_t denotes the level of public good provision. The period utility function $u(\cdot)$ is assumed to be additively separable in its three arguments and twice continuously differentiable, with partial derivatives $u_c > 0$, $u_{cc} < 0$, $u_l > 0$, $u_{ll} \leq 0$, $u_g > 0$ and $u_{gg} \leq 0$.

Each household is composed of three types of members: workers, bankers and entrepreneurs.[6] Workers supply labour to competitive firms; the other agents either become bankers or get access to an entrepreneurial investment technology. The assignment to these two activities is stochastic; an individual agent becomes banker with probability $1-\theta$ and entrepreneur with probability θ, respectively. Household j enters period t with a stock of b_t^j government bonds. Initially, all bonds are held by bankers and entrepreneurs, each of them holding the same amount b_t^j. Then, the household members separate and individuals learn their type (banker or entrepreneur) before the government's policy decisions are announced.

Workers and firms. Workers supply their labour services n_t, taking the wage rate w_t as given. Firms are perfectly competitive and have access to a production technology that trans-

[5]The assumption of comittment to full debt repayment will be relaxed in Section 3. Section 6 will introduce aggregate uncertainty.

[6]Each household comprises a continuum $[0,1]$ of workers and a continuum $[0,1]$ of agents who become either bankers or entrepreneurs.

forms labour into consumption goods at a unitary rate. Specifically, the technology allows the representative firm to produce

$$y_t^1 = \tilde{n}_t, \tag{2}$$

where $\tilde{n}_t$ denotes labour hired by the firm. Production is subject to a moral hazard problem which, in the absence of monitoring, makes it impossible for firms to pledge funds to workers and outside creditors. Firms must therefore finance their wage bill in advance, and they can do so using intra-period loans from financial intermediaries (bankers).

Bankers. Bankers act as delegated monitors. In order to meet firms' demand for working capital, they issue deposits contracts, d_t, to outside creditors. However, although banks have a greater capacity to pledge funds to outside creditors, they are also subject to moral hazard. They can therefore only issue deposits if they are able to post collateral to cover at least a fraction $\xi^c \in (0, 1)$ of the amount issued.[7] Government bonds are the sole source of collateral available to bankers, such that the collateral constraint facing a representative banker from household j is given by

$$d_t^j \leq \frac{b_t^j}{\xi^c}, \tag{3}$$

where d_t^j denotes the deposits issued. Note that the banking sector is competitive, and hence working capital loans do not carry a positive interest rate unless the supply of loans is depressed by the bankers' availability of collateral. Aggregating across firms and bankers, equilibrium in

[7] We assume an environment of impersonal market interactions so that household members cannot avoid financial frictions internally; outside creditors are thus best thought of as workers from households other than the banker's own (cf. Gertler and Karadi, 2011).

the bank-intermediated market for working capital loans implies that the economy's aggregate wage bill is constrained by[8]

$$w_t \tilde{n}_t \leq \frac{(1-\theta)b_t}{\xi^c}. \tag{4}$$

Entrepreneurs. Entrepreneurs have access to a profitable investment technology. Specifically, they can invest in projects that deliver a gross return $R > 1$ per unit of investment (both in consumption goods). Denoting by X_t^j the investment scale of the representative entrepreneur from household j, the investment technology is characterised by

$$y_t^{j,2} = RX_t^j. \tag{5}$$

Similar to the operation of banks, there is a moral hazard problem that limits entrepreneurs' access to external finance. As a consequence, internal investment, x_t^j, is necessary to attract external funds, e_t^j. External funds take the form of intra-period loans from workers and bankers that pay zero interest as there is no discounting within the period. To raise the consumption goods required for internal investment, entrepreneurs sell their liquid assets (government bonds) on the secondary market; hence, $x_t^j = z_t b_t^j$, where z_t denotes the bond's market price.[9] They

[8]Notice from (4) that ξ^c can be interpreted as a compound parameter reflecting both the collateral constraint facing bankers and the working capital constraint facing firms. In particular, (4) can be rewritten as $\xi^w w_t \tilde{n}_t \leq (1-\theta)b_t/\xi^d$ with $\xi^c = \xi^w \xi^d$. This highlights that our model is observationally equivalent to an alternative specification where firms need to finance in advance only a fraction ξ^w of their wage bill.

[9]We assume that the secondary market for government debt is large enough to absorb the supply of bonds from entrepreneurs. Formally, $w_t n_t + (1-w_t)\tilde{n}_t \geq \theta z_t b_t$, where variables without superscript denote economy-wide aggregates. This condition is satisfied in all our numerical experiments.

then augment their internal funds by acquiring external funds subject to the constraint

$$e_t^j \leq \frac{x_t^j}{\xi^l}, \tag{6}$$

where $\xi^l \in (0,1)$. Constraint (6) is always binding when $R > 1$, resulting in an investment scale of $X_t^j = (1+\xi^l)/(\xi^l) z_t b_t^j$ per entrepreneur.

Aggregation. After production in the competitive and entrepreneurial sector has taken place, workers, bankers and entrepreneurs transfer their earnings back to the household. Consumption-savings decisions are then made at the household level; hence there is perfect consumption insurance within households. Aggregating over household members, the total income of household j in period t is the sum of the wage income earned by workers, profits in the competitive sector (given free entry of firms, these profits arise whenever collateral is scarce and then completely accrue to the bankers), and entrepreneurs' net return from investment,

$$I_t^j = w_t n_t^j + (1-w_t)\tilde{n}_t^j + \theta(R-1)\frac{1+\xi^l}{\xi^l} z_t b_t^j. \tag{7}$$

Note that (7) does not include income from maturing government debt b_t^j. The household's budget constraint is given by

$$c_t^j + q_t b_{t+1}^j \leq (1-\tau_t) I_t^j + b_t^j, \tag{8}$$

where τ_t is a proportional income tax and q_t denotes the price of a newly issued government bond that promises one unit of wealth in the beginning of $t+1$.

The government. The government is benevolent and maximises the utility (1) of the representative household. Its policy tools are the income tax τ_t, the level of public good provision g_t, and the issuance of new debt B_{t+1}. The government's budget constraint is given by

$$g_t + B_t \leq \tau_t I_t + q_t B_{t+1}. \tag{9}$$

The government cannot commit to a fixed policy path over time. It can, however, make credible policy announcements within a given time period. This timing structure implies that the government is a Stackelberg leader vis-à-vis the private sector. Table 1 summarises the timing of events in any given period t.

Table 1: *Timing of Events in Period t*

1.	The household endows each of its bankers and entrepreneurs with b_t government bonds.
2.	The household members separate and individual types (banker or entrepreneur) are realised.
3.	The government announces its policies (τ_t, g_t, B_{t+1}).
4.	Bankers issue deposits, d_t, subject to collateral constraint (3) and make working capital loans to firms. Firms hire labour, $\tilde{n}_t$, subject to constraint (4). They produce $y_t^1 = \tilde{n}_t$ consumption goods.
5.	Entrepreneurs sell their government bonds to raise internal funds, x_t, and raise external funds, e_t, from workers and bankers subject to external finance constraint (6). They invest into projects of scale $X_t = x_t + e_t$, which return $y_t^2 = RX_t$ consumption goods.
6.	The government collects income taxes, $\tau_t I_t$, transforms g_t units of the consumption good into a public good, repays the maturing debt B_t and issues new debt B_{t+1} at price q_t. Households consume c_t and purchase newly issued government debt, b_{t+1}.

2 Markov-perfect Optimal Fiscal Policy without Default

Under lack of commitment, the government in a given time period can choose policy variables for that period but it cannot control policy variables for the future. To characterise the optimal policies we adopt a *primal approach*. Accordingly, the incumbent government directly chooses consumption c, labour n, and debt issuance b' for the current period, taking as given the policy rules $\{\hat{c}, \hat{n}, \hat{b}\}$ employed by future governments, and subject to the requirement that its choices are consistent with a private-sector equilibrium.

Private-sector equilibrium. Households in our model are atomistic and take prices $(w_t, z_t, q_t)_{t=0}^{\infty}$ and policies $(\tau_t, g_t, B_{t+1})_{t=0}^{\infty}$ as given. They choose consumption, labour supply, labour demand, and savings to maximise their objective function (1). Since there is no discounting within the time period, the secondary market price of a government bond must equal its repayment rate, that is, $z_t = 1$ in all periods. Adopting recursive notation and dropping the superscript j, the optimization problem faced by the representative household reads

$$\begin{aligned}\tilde{V}(b; w, q, \tau, g) \quad = \quad & \max_{c,n,\tilde{n},b'} \min_{\lambda,\mu} u(c, 1-n, g) + \beta \tilde{V}(b'; w', q', \tau', g') \\ & -\lambda\Big(c + qb' - (1-\tau)\Big\{wn + (1-w)\tilde{n} + \theta(R-1)\frac{1+\xi^l}{\xi^l}b\Big\} - b\Big) \\ & -\mu\Big(w\tilde{n} - \frac{(1-\theta)b}{\xi^c}\Big).\end{aligned}$$

From the first-order conditions associated with this problem, it is straightforward to show that the households' policies in a private-sector equilibrium satisfy the Euler equation

$$q \quad = \quad \beta \frac{u_c'}{u_c}(1 + \pi' + \phi') \tag{10}$$

and the budget constraint

$$u_c c + \beta u_c'(1 + \pi' + \phi')b' = \frac{u_l}{w}n + u_c(1+\pi)b, \tag{11}$$

where

$$\pi = \theta(1-\tau)(R-1)\frac{1+\xi^l}{\xi^l}, \quad (12)$$

$$\phi = (1-\theta)(1-\tau)\frac{(1-w)}{w\xi^c}, \quad (13)$$

denote the liquidity and the collateral premia on government bonds, respectively. The Euler equation (10) thus highlights the three roles played by government bonds in our model: (i) they allow households to shift consumption over time; (ii) they provide liquidity and hence allow households to increase entrepreneurial investment; and (iii) they are a source of collateral to bankers.

The optimal fiscal policy problem. Inspection of the private-sector equilibrium conditions shows that the aggregate state vector in our model consists of only one variable, b. The policy rules $\{\hat{c}, \hat{n}, \hat{b}\}$ are thus of the form $c = \hat{c}(b)$, $n = \hat{n}(b)$, and $b' = \hat{b}(b)$. Via households' optimal consumption-leisure choice and equations (12) and (13) these rules further imply decision rules for the tax rate, $\hat{\tau}(b)$, the liquidity premium, $\hat{\pi}(b)$, and the collateral premium, $\hat{\phi}(b)$, respectively. Plugging these functions into Euler equation (10), we can write the bond pricing function as

$$Q(u_c, b') = \beta\frac{u_c(\hat{c}(b'))}{u_c}\left(1+\hat{\pi}(b')+\hat{\phi}(b')\right). \quad (14)$$

As households have a finite intertemporal elasticity of substitution, the bond price depends on the current and future marginal utility of consumption. This entails an *interest rate manipulation motive* for the government, which seeks to sustain bond prices via a debt policy that induces expectations of increased future marginal utility as well as liquidity and collateral premia. Note also that the wage rate falls below labour productivity if firms' access to working capital loans is strictly constrained by the bankers' pledgeable collateral (cf. equation (13)). This allows us to write the wage rate as a function

$$\omega(b,n) = \begin{cases} 1 & \text{if } (1-\theta)b > \xi^c n \\ \frac{(1-\theta)b}{\xi^c n} & \text{otherwise.} \end{cases} \tag{15}$$

Using the aggregate resource constraint to substitute for public consumption in the household utility function (1), and the private-sector optimality condition $(u_l)/(u_c) = (1-\tau)w$ to eliminate taxes from (12) and (13), the discretionary government's optimization problem under commitment to full debt repayment is given by

$$\begin{aligned} V(b) &= \max_{c,n,b'} \min_{\gamma} u(c, 1-n, n+rb-c) + \beta V(b') \\ &\quad + \gamma \left(u_c c + u_c Q(u_c, b')b' - \frac{u_l}{\omega(b,n)} n - u_c\{1+\pi(b,c,n)\}b \right), \end{aligned} \tag{16}$$

where $r = \theta(R-1)\frac{1+\xi^l}{\xi^l}$ denotes the net return to entrepreneurial investment, γ is a non-negative Lagrangian multiplier and $V(b')$ is the continuation value function.

Definition 1. *A Markov-perfect equilibrium is a set of policy functions $\mathcal{P} = \{\hat{c}, \hat{n}, \hat{b}\}$, a value function V and a bond pricing function Q such that:*

(i) given the value function V and the bond pricing function Q, the policy functions $\{\hat{c}, \hat{n}, \hat{b}\}$ solve the government's optimization problem (16);

(ii) given the policy functions $\{\hat{c}, \hat{n}, \hat{b}\}$, the bond pricing function Q satisfies (14);

(iii) given the policy functions $\{\hat{c}, \hat{n}, \hat{b}\}$, the value function satisfies the Bellman equation

$$V(b) = u(\hat{c}(b), 1 - \hat{n}(b), \hat{n}(b) + rb - \hat{c}(b)) + \beta V(\hat{b}(b)).$$

The following proposition characterises the optimal debt policy in a Markov-perfect equilibrium.[10]

Proposition 1. *In a Markov-perfect equilibrium, the optimal debt policy satisfies the generalised Euler equation*

$$\gamma' \left(u_c'(1+\pi') - \frac{u_l'}{\omega(b',n')^2}\omega_1(b',n')\{n' + rb'\} \right) - u_g' r = \gamma u_c'(1 + \pi' + \phi')\left(1 + \varepsilon_{b'}^q\right), \qquad (17)$$

where $\varepsilon_{b'}^q = \frac{Q_2(u_c,b')b'}{Q(u_c,b')}$ denotes the elasticity of the bond price q with respect to changes in debt issuance b'.

The generalised Euler equation equates the marginal cost of entering the next period with a higher stock of outstanding debt to the marginal benefit of relaxing implementability constraint (11) via issuing additional debt. Whenever government bonds provide liquidity and collateral services, (17) dictates convergence to a positive level of long-run debt.[11] The role of government debt in mitigating financial frictions also suggests that the accumulation of moderate levels of debt may have positive welfare effects.

[10] We assume differentiability of policy and value functions, mainly to build intuition based on the government's generalised Euler equation. From equation (15), it is clear that there emerges a local non-differentiability at the point where the collateral constraint (4) becomes non-binding. Apart from this point, first-order conditions remain useful to characterise optimal government policy.

[11] The Web Appendix establishes this formally and provides some discussion of the government's interest rate manipulation motive encapsulated in condition (17).

Proposition 2. *The accumulation of moderate levels of debt has positive welfare effects if government bonds serve as collateral, $\xi^c > 0$, or provide liquidity, $\xi^l > 0$, with a sufficiently high return to investment in the entrepreneurial sector,*

$$r > \frac{u_c}{u_l}\left(\frac{1-\frac{u_l}{u_g}}{\frac{u_l}{u_g}-\frac{u_{ll}}{u_l}n}\right). \tag{18}$$

Without financial frictions, i.e., if $\xi^c = 0$ and $r = 0$, welfare is monotonically decreasing in debt.

For the government's value function to be increasing in debt, the marginal benefit from relaxing the collateral constraint and from increased liquidity must exceed the marginal cost from increased taxation. The marginal benefit is generically (weakly) decreasing in the level of debt, while the marginal cost due to increased tax distortions is strictly increasing in debt. Accordingly, the value function in our model has an inverted U-shape.

Tightly linked to the shape of the value function is the emergence of a *debt Laffer curve*, i.e., a situation where – locally – a marginal increase in the quantity of debt issued is associated with a reduction in the revenue for the government from that operation. When this happens, we say the economy is on the 'bad' side of the debt Laffer curve.

Proposition 3. *The optimal debt policy is subject to a Laffer curve: When marginal debt has positive (negative) welfare effects, the economy is on the 'bad' ('good') side of the debt Laffer curve.*

Since the issuance of debt helps to relax financial frictions, an extra unit of debt tends to reduce collateral and liquidity premia and hence the price for all inframarginal units of debt. A debt

Laffer curve emerges whenever bond prices fall strongly in the amount of debt issued, i.e., when $\varepsilon^q_{b'} < -1$. But this happens precisely when debt is scarce – and therefore valuable – so that collateral and liquidity premia are both relevant and highly sensitive to the amount of debt.[12] It is thus precisely the beneficial effect of the government's debt policy via the relaxation of financial constraints that induces a Laffer curve.

3 Markov-perfect Optimal Fiscal Policy with Default

We now introduce the option of fractional default, i.e., the government can decide in a discretionary manner on the fraction $\rho \in [0, 1]$ of outstanding debt it repays. At the same time, and in line with the sovereign default literature, we introduce reputational costs of default. Following a default, the government is temporarily excluded from the primary bond market, and outstanding bonds can no longer be traded on the secondary market. Since entrepreneurs' access to external funds crucially relies on their ability to sell their bonds on the secondary market, government default thus precludes entrepreneurial production. The duration of the market exclusion is stochastic; with a constant probability α an excluded government can re-access the bond market in the next period. However, we assume that during the bond market exclusion the government can still issue debt in the form of loans. Loans serve as collateral but are not tradeable on secondary markets and hence not a source of liquidity.[13] The loss in

[12] The debt Laffer curve facing the government thus has a U-shape (not the inverted U-shape familiar from other contexts); compare Figure 3 below.

[13] Consistent with this assumption is the empirical evidence presented in Bai, Julliard, and Yuan (2012). These authors analyze Eurozone sovereign bond markets in the period 2006-2012 and find that secondary market liquidity has been significantly reduced during the recent crisis, with markets basically drying up in countries that received a bailout (Greece and Portugal). Their data is compiled from both international and domestic interdealer markets and therefore a good indicator for the secondary market trading volume at both levels, and in particular for the loss in liquidity relevant to domestic agents. Relatedly, Arellano, Mateos-Planas, and Rios-Rull (2013) document that countries continue to borrow when they are in arrears. The complete exclusion also

liquidity raises the government's borrowing cost during a default episode. Finally, note that the costs via reduced collateral depend on the size of the implemented haircut, whereas the repercussions of market exclusion are of a fixed-cost nature.

The optimal fiscal policy problem under fractional default. It is convenient to cast the incumbent government's optimal policy problem under the option to default as a two-stage decision problem. The government first decides whether or not to repay the entirety of its outstanding debt. Conditional on this decision, the government then chooses its relevant policy instruments. Define $V^o(b)$ as the value function for a government that has the option to default and starts the current period with b outstanding bonds. This value function satisfies

$$V^o(b) = \max\{V^{nd}(b), V^d(b)\}, \tag{19}$$

where $V^{nd}(b)$ is the value conditional on full repayment ($\rho = 1$) and $V^d(b)$ is the value conditional on default ($\rho < 1$). The no-default value function is the solution to

$$\begin{aligned} V^{nd}(b) \quad = \quad & \max_{c,n,b'} \min_{\gamma} u(c, 1-n, n+rb-c) + \beta V^o(b') \\ & + \gamma \left(u_c c + u_c Q^b(u_c, b')b' - \frac{u_l}{\omega(b,n)} n - u_c\{1 + \pi(b,c,n)\}b \right), \end{aligned} \tag{20}$$

where $\omega(b, n)$ and $Q^b(u_c, b')$ are the pricing functions for labour and newly issued bonds, re-

from the primary market for debt considered in the literature (cf. Arellano, 2008) thus has the counterfactual implication of zero outstanding debt following a default. Introducing a primary market for loans is one way to address this concern.

spectively. The government's value function under default is given by

$$V^d(b) = \max_{\rho \in [0,1]} \tilde{V}^d(\rho b), \tag{21}$$

where

$$\tilde{V}^d(\rho b) \quad = \quad \max_{c,n,\ell'} \min_{\gamma} u(c, 1-n, n-c) + \beta W^o(\ell') + \gamma \left(u_c c + u_c Q^\ell(u_c, \ell')\ell' - \frac{u_l}{\omega(\rho b, n)} n - u_c \rho b \right)$$

is the value function conditional on a given repayment rate $\rho < 1$, and ℓ' and $Q^\ell(u_c, \ell')$ denote newly issued loans and the underlying pricing function, respectively. This formulation makes clear that what ultimately matters for allocations and welfare is the *effective state* ρb. Since this state can be regulated via the repayment policy $\rho \in [0,1]$, the default value function $V^d(b)$ is necessarily *non-decreasing* over the entire state space. Specifically, $V^d(b)$ is increasing whenever the optimal policy prescribes full debt repayment, and constant whenever it prescribes partial default. Finally, $W^o(\ell)$ is the value function of a government that starts the period with ℓ outstanding loans,

$$W^o(\ell) = \alpha \max\{W_a^{nd}(\ell), W^d(\ell)\} + (1-\alpha) \max\{W_e^{nd}(\ell), W^d(\ell)\}, \tag{22}$$

where $W_a^{nd}(\ell)$ is the value function conditional on full repayment of a government that regains access to the bond market in the beginning of the period, $W_e^{nd}(\ell)$ is the no-default value function of a government that remains excluded from the bond market, and $W^d(\ell)$ is the value function conditional on default. These value functions satisfy[14]

$$\begin{aligned}
W_a^{nd}(\ell) &= \max_{c,n,b'}\min_{\gamma} u(c,1-n,n-c)+\beta V^o(b') \\
&\quad +\gamma\left(u_c c+u_c Q^b(u_c,b')b'-\frac{u_l}{\omega(\ell,n)}n-u_c\ell\right), && (23)\\
W_e^{nd}(\ell) &= \max_{c,n,\ell'}\min_{\gamma} u(c,1-n,n-c)+\beta W^o(\ell') \\
&\quad +\gamma\left(u_c c+u_c Q^\ell(u_c,\ell')\ell'-\frac{u_l}{\omega(\ell,n)}n-u_c\ell\right), && (24)\\
W^d(\ell) &= \max_{\rho\in[0,1]} \tilde{W}^d(\rho\ell), && (25)
\end{aligned}$$

where $\tilde{W}^d(\rho\ell)$ denotes the value function conditional on a given repayment rate ρ on loans,

$$\tilde{W}^d(\rho\ell)=\max_{c,n,\ell'}\min_{\gamma} u(c,1-n,n-c)+\beta W^o(\ell')+\gamma\left(u_c c+u_c Q^\ell(u_c,\ell')\ell'-\frac{u_l}{\omega(\rho\ell,n)}n-u_c\rho\ell\right).$$

The following proposition characterises the optimal default policy when the government has access to the bond market.

Proposition 4. *The government's optimal default policy is of a fiscal limit type, i.e., the government optimally defaults if and only if its inherited stock of bonds exceeds a threshold level* $\bar{b}$. *The optimal repayment policy takes the form*

$$\hat{\rho}(b)=\begin{cases} 1 & \text{if } b\le\bar{b} \\ \underline{b}/b & \text{if } b>\bar{b}\end{cases}$$

[14]It is not necessary to index $W^d(\ell)$ by a or e, since default precludes the current government's option of immediate bond market access. Moreover, because default hampers the liquidity of maturing bonds, the value of defaulting is independent of whether outstanding liabilities are in the form of bonds or loans, that is, $V^d(x)=W^d(x)$ and $\tilde{V}^d(x)=\tilde{W}^d(x)$, where x denotes the (effective) amount of outstanding liabilities. Finally, note also that $W_e^{nd}(\ell)=\tilde{W}^d(\rho\ell)$ for $\rho=1$. Accordingly, $W_e^{nd}(\ell)=W^d(\ell)$ whenever the optimal policy prescribes full debt repayment.

where $\underline{b}$ is the lowest level of (effective) debt that maximises post-default welfare, $\underline{b} = \arg\max_b \tilde{V}^d(b)$. At $\underline{b}$ the collateral constraint is strictly binding.

Equivalent results apply also for the case where the government is excluded from the bond market and outstanding debt is in the form of loans.[15] The intuition behind Proposition 4 is readily seen (compare also Figure 2 below). The value function conditional on default, $V^d(b)$, is non-decreasing over the entire state space and constant whenever the optimal policy prescribes $\rho < 1$. Denote this constant level of welfare by $\bar{V}^d$. The value function conditional on full repayment, $V^{nd}(b)$, has an inverted U-shape. Hence there exists a unique default threshold $\bar{b}$, implicitly defined via $V^{nd}(\bar{b}) = \bar{V}^d$.[16] For higher levels of debt, $b > \bar{b}$, the optimal haircut reduces effective debt to the lowest level that maximises post-default welfare. This post-default level of effective debt $\underline{b}$ necessarily induces a strictly binding collateral constraint. Intuitively, this is because default destroys the liquidity value of maturing debt. The optimal default policy, which trades off the marginal benefits from reduced taxation with the marginal costs from reduced collateral, therefore has to reduce the available collateral below its satiation level. A government exercising its default option will thus find it optimal to make the post-default level of debt so scarce that financial intermediation is hampered.

It is interesting to contrast the optimal repayment policy characterised in Proposition 4 with the endogenous debt recovery function arising in Yue's (2010) model of debt renegotiation. In both models a default episode results in a unique level $\underline{b}$ of post-default debt, which implies that the repayment rate of defaulted debt decreases with the pre-default level of debt. However, Yue

[15] The optimal default policy for loans is formally characterised in the Web Appendix, which also provides a formal definition of the Markov-perfect equilibrium under the option to default.

[16] Along its upward-sloping segment the no-default value function V^{nd} strictly dominates the default value function V^d. Default reduces the (valuable) amount of effective debt and additionally subjects the economy to the costs from the secondary market collapse. Default can thus not be optimal.

(2010) studies an endowment economy with exogenous default costs where default is initially complete, but subsequent debt renegotiation results in an endogeous recovery rate; new net lending becomes available once the renegotiated debt is fully repaid. By contrast, in our model default is partial in the first place and disciplined via the endogenous costs for production from lost collateral and liquidity; the duration of secondary market exclusion is stochastic, but borrowing on the primary market remains possible.

Based on Proposition 4 we can formally examine the sustainability of optimal fiscal policy in the face of strategic default incentives.

Definition 2. *Given a state space $\mathbb{B}$ and a region $\tilde{\mathbb{B}} \subset \mathbb{B}$, a Markov-perfect optimal fiscal policy $\mathcal{P}$ is sustainable over $\tilde{\mathbb{B}}$ if*

(i) the incumbent government finds it optimal to employ the policy $\mathcal{P}$ and to fully honor inherited debt for all $b \in \tilde{\mathbb{B}}$ when it perceives that all future governments will employ the policy $\mathcal{P}$ and fully honor inherited debt in $\tilde{\mathbb{B}}$; and

(ii) the debt policy $\hat{b} \in \mathcal{P}$ is ergodic, i.e., it satisfies $\hat{b}(b) \in \tilde{\mathbb{B}}$ for all $b \in \tilde{\mathbb{B}}$.

Two comments are in order. First, if the steady state debt level under commitment to full repayment lies in a sustainable region $\tilde{\mathbb{B}}$, then the Markov-perfect optimal fiscal policy under the option to default coincides with the Markov-perfect optimal fiscal policy under commitment to full repayment over this region. Second, under our concept of sustainability, default does not occur in equilibrium if the initial level of debt is in a sustainable region. For the case when there are no reputational default costs, we obtain the following result.

Proposition 5. *Absent reputational costs of default, no Markov-perfect optimal fiscal policy is sustainable over regions on the 'good' side of the debt Laffer curve.*

To understand the underlying intuition, notice that the value function $V^{nd}(b)$ inherits the properties discussed in Proposition 3: It is *increasing* in debt on the 'bad' side of the Laffer curve and *decreasing* on the 'good' side. Denote by $\bar{V}$ the maximum of $V^{nd}(b)$ and by b^{**} the level of debt at which the maximum is attained, i.e., $V^{nd}(b^{**}) = \bar{V}$. It is then straightforward to see that the default value function in the absence of reputational costs satisfies $V^d(b) = V^{nd}(b)$ for all $b \leq b^{**}$ and $V^d(b) = \bar{V}$ for all $b > b^{**}$. Hence the government defaults for all debt levels on the 'good' side of the Laffer curve. Reputational costs of default are therefore critically needed to support debt positions that are consistent with conventional pricing of public debt ($\varepsilon^q_{b'} > -1$). Notice also that this result is obtained despite our assumptions which assign an important role to public debt as a source of collateral and private liquidity.

4 A Calibrated Economy

We now examine the key quantitative properties of Markov-perfect optimal fiscal policies in a calibrated economy. We consider an instantaneous utility function u that is additively separable and allows for curvature in all its arguments,

$$u(c, 1-n, g) = (1-\omega_g)\left(\omega_c \frac{c^{1-\sigma_c}-1}{1-\sigma_c} + (1-\omega_c)\frac{(1-n)^{1-\sigma_l}-1}{1-\sigma_l}\right) + \omega_g \frac{g^{1-\sigma_g}-1}{1-\sigma_g}, \qquad (26)$$

where ω_c and ω_g denote preference weights on private and public consumption, and σ_c, σ_l and σ_g are (inverse) elasticities. We target data at annual frequency and select parameter values as follows. The three elasticities σ_c, σ_l and σ_g are each set to the value 2, which is in the middle of the parameter range typically considered in the macroeconomic literature. The preference weights are chosen such that, in the model's steady state, $g^*/c^* = 0.25$ and $n^* = 0.3$; the resulting values are $\omega_c = 0.15$ and $\omega_g = 0.015$.[17]

Our choice of the parameters regulating the importance of financial frictions is meant to be suggestive. In Section 5 below we will examine the robustness of our quantitative findings to alternative parameterizations. The collateral parameter is set to $\xi^c = 0.4$, corresponding to a debt-to-equity ratio of 2.5; this matches the relevant statistic for financial corporations in the U.S. in 2012 (OECD, 2014).[18] The parameters R, θ and ξ^l matter jointly, as determinants of the return to entrepreneurial investment, $r = \theta(R-1)(1+\xi^l)/(\xi^l)$. The individual parameter values are selected in line with evidence from the Survey of Consumer Finances (SCF). As discussed in Moskowitz and Vissing-Jorgensen (2002), the SCF reports a median of the distribution of capital gains in private business investment of roughly 7%. This motivates our choice of $r = 0.07$. We set $\theta = 0.25$ as a compromise between the population share (8%) of entrepreneurs and the fraction of wealth (33%) controlled by them (cf. Covas and Fujita, 2011). For simplicity, we set $\xi^l = \xi^c$, implying $R = 1 + (r\xi^l)/(\theta\{1 + \xi^l\}) = 1.08$. We choose the discount factor $\beta = 0.92$ to

[17]Given the utility specification in (26), our model nests the economy of Debortoli and Nunes (2013) as a special case, which allows us to assess the effects of collateral, liquidity and default against a well-defined benchmark. Notice therefore that our parameterization for σ_g implies that government expenditure is relatively unresponsive to changes in debt and would result in a positive steady state level of debt also in Debortoli and Nunes (2013). The Web Appendix provides some robustness analysis regarding the steady state effects of variations in σ_g.

[18]Financial corporations are all private and public entities engaged in financial activities, such as monetary institutions, financial intermediaries, insurance companies and pension funds. We work with this broad concept of financial intermediation – as opposed to a narrow concept based only on banks – in order to capture the importance of government debt for the operation also of non-bank financial corporations. Looking at the banking sector only would imply a leverage ratio of 5.2 ($\xi^c \approx 0.19$). The statistics for the U.K. in 2012 are a debt-to-equity ratio for financial corporations of 8 and a leverage ratio in the banking industry of 12; this would correspond to a collateral parameter of $\xi^c \approx 0.13$ or $\xi^c \approx 0.08$, respectively. See Table 4 below for key outcomes under the alternative values $\xi^c = 0.2$ and $\xi^c = 0.1$.

match an annual risk-free real interest rate of about 3% in the presence of a steady state with a satiated demand for collateral but a positive liquidity premium.[19] Finally, we set $\alpha = 0.5$ which implies that, on average, the bond market is impaired during the default period and the two following periods. This duration of two years is consistent with the empirical evidence reported by Bai, Julliard, and Yuan (2012), and it is also broadly in line with estimates and calibrations reported in the sovereign debt literature (cf. Cruces and Trebesch, 2013; Arellano, 2008; Cuadra, Sanchez, and Sapriza, 2010). Our parameter choices are summarised in Table 2. For given parameters, we solve the model numerically using dynamic programming techniques.[20]

Table 2: *Parameter Values*

Parameter	Value	Description
σ_c	2	elasticity of private consumption
σ_g	2	elasticity of public consumption
σ_l	2	elasticity of leisure
ω_c	0.15	weight of consumption (priv.+publ.) vs. leisure
ω_g	0.015	weight of public vs. private consumption
ξ^c	0.4	inverse of bank debt-to-equity ratio
r	0.07	return to entrepreneurial investment
θ	0.25	share of entrepreneurs
ξ^l	0.4	inverse of entrepreneurial debt-to-equity ratio
R	1.08	entrepreneurial investment technology
β	0.92	discount factor
α	0.5	market reaccess probability

Steady state. The steady state values of key endogenous variables are presented in Table 3. In line with our calibration target, steady state output in the competitive sector is roughly equal to $y^{1^*} = 0.3$. Value added in the entrepreneurial sector is significantly smaller, $y^{2^*} = 0.019$, such that total output is given by $y^* = 0.3220$. Private and public consumption amount to 80%

[19]The bond pricing function (14), evaluated at steady state, then implies $q^* = \beta(1 + \pi^*) = \beta(1 + (1 - \tau^*)r)$.
[20]The Web Appendix provides further details on our computational algorithm.

and 20% of total output, respectively ($c^* = 0.2578$, $g^* = 0.0642$). The steady state level of debt is positive and sizeable, $b^* = 0.2712$, which corresponds to a debt-to-GDP ratio equal to 84%. Steady state debt is fully honored, $\rho = 1$. The bond price $q^* = 0.97$ implies an annual interest rate close to our calibration target of 3% and the tax rate is empirically plausible, $\tau^* = 22.5\%$. The wage rate is equal to labour productivity, $w^* = 1$, i.e., there is endogenous accumulation of debt beyond the satiation point for collateral demand.

Table 3: *Steady State Values*

Variable	Steady state
y^1	0.3030
y^2	0.0190
y	0.3220
c	0.2578
g	0.0642
b	0.2712
b/y	0.8422
q	0.9699
τ	0.2248
w	1.0000
ρ	1.0000

Policy functions. The optimal policy functions are displayed in Figure 1.[21] These functions display kinks in the region of the state space where the collateral constraint kicks in as well as discontinuities at the fiscal limit. Note that the state space $\mathbb{B} = [b_{min}, b_{max}]$ can be partitioned into four regions that differ significantly in how optimal policies react to variations in the inherited debt level. In the first region, $\mathbb{B}^1 = [b_{min}, b_1)$, debt is so scarce that the collateral constraint is strictly binding. In the second region, $\mathbb{B}^2 = [b_1, b_2)$, the collateral constraint is non-binding under optimal policies, but its existence nevertheless affects the government's

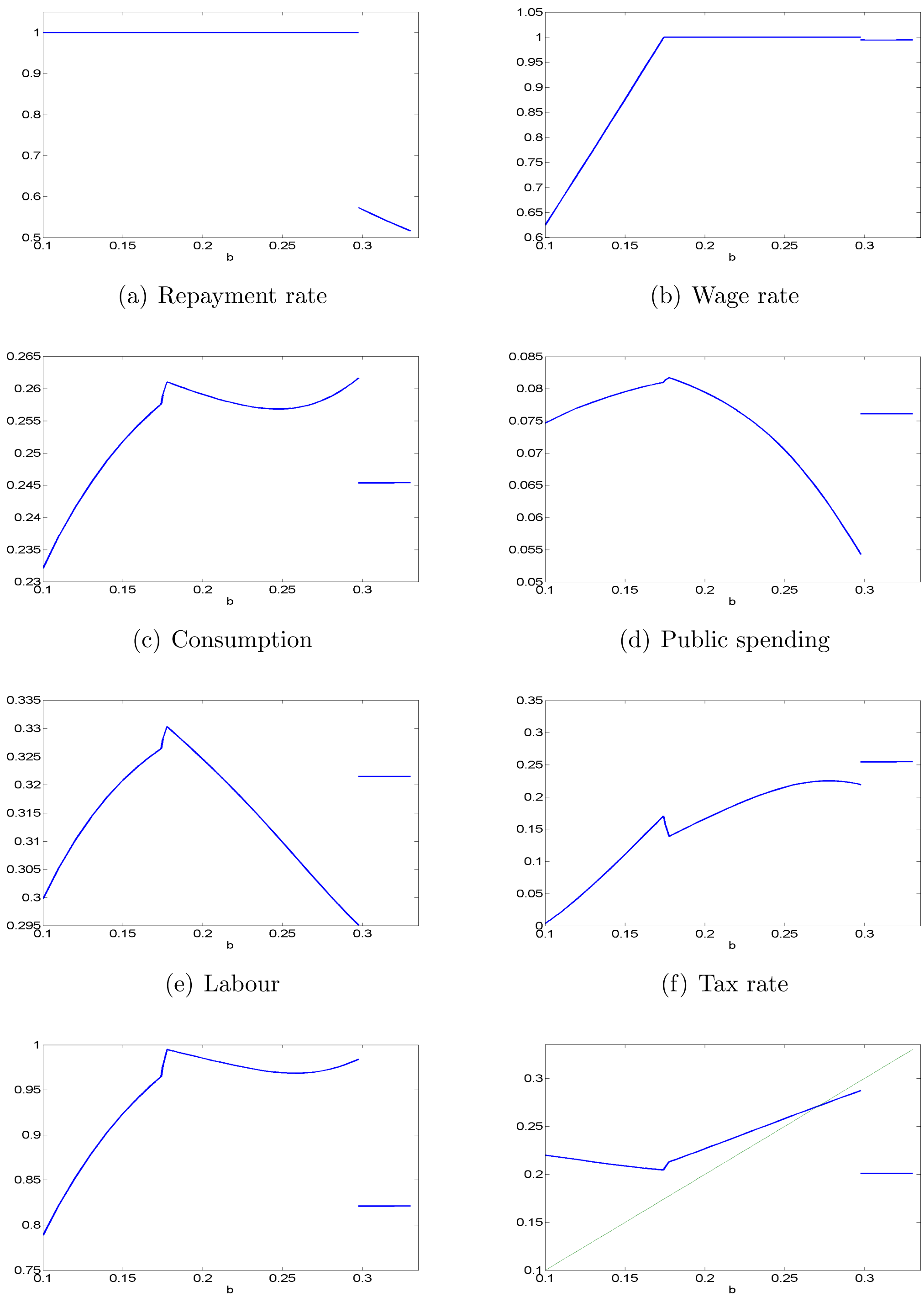
[21] We restrict attention to the case when outstanding debt is in the form of bonds. Plots of the policy functions for the case when outstanding debt is in the form of loans are available in the Web Appendix.

optimal policy trade-offs. In the third region, $\mathbb{B}^3 = [b_2, b_3]$, the collateral constraint has no distortionary effects on optimal policies. Finally, whereas the government fully repays debt in regions $\mathbb{B}^1$, $\mathbb{B}^2$ and $\mathbb{B}^3$, it fractionally defaults in region $\mathbb{B}^4 = (b_3, b_{max}]$. In our calibrated economy, $b_1 = 0.1745$, $b_2 = 0.1800$ and $b_3 = 0.2975$, which corresponds to approximately 54%, 56% and 92% of steady state output, respectively. At the fiscal limit b_3, the haircut on debt amounts to about 40%.

Welfare. Figure 2 shows the value functions of the government under the option to default. The top panel contrasts the government's value function conditional on no default ($V^{nd}(b)$) and on default ($V^d(b)$) when bond markets are fully operational. Under full repayment, the government's value function has an inverted U-shape. Under partial default, it is monotonically increasing for low levels of debt and constant from $\underline{b} = 0.1705$ onwards. The two value functions intersect at the fiscal limit $\bar{b} = b_3 = 0.2975$, which corresponds to about 92% of steady state output.

The bottom panel of Figure 2 contrasts the government's value functions when outstanding debt is in the form of loans, as formalised in equation (22). The no-default value functions $W_a^{nd}(\ell)$ and $W_e^{nd}(\ell)$ have an inverted U-shape. The value function conditional on default actually coincides with its counterpart when maturing debt is in the form of bonds, $W^d(\ell) = V^d(b)$ for $\ell = b$; this is because default hampers the liquidity of maturing bonds. In analogy to the case of fully operational bond markets, there thus emerge two further default thresholds for maturing loans. Conditional on regaining market access, the government fully honors its debt up to the point where $W_a^{nd}(\ell)$ and $W^d(\ell)$ intersect, which corresponds to $\bar{\ell}_a = 0.2630$ in our calibrated economy. On the other hand, $W_e^{nd}(\ell) \leq W^d(\ell)$ globally, and the inequality becomes

Figure 1: *Policy Functions*



(a) Repayment rate

(b) Wage rate

(c) Consumption

(d) Public spending

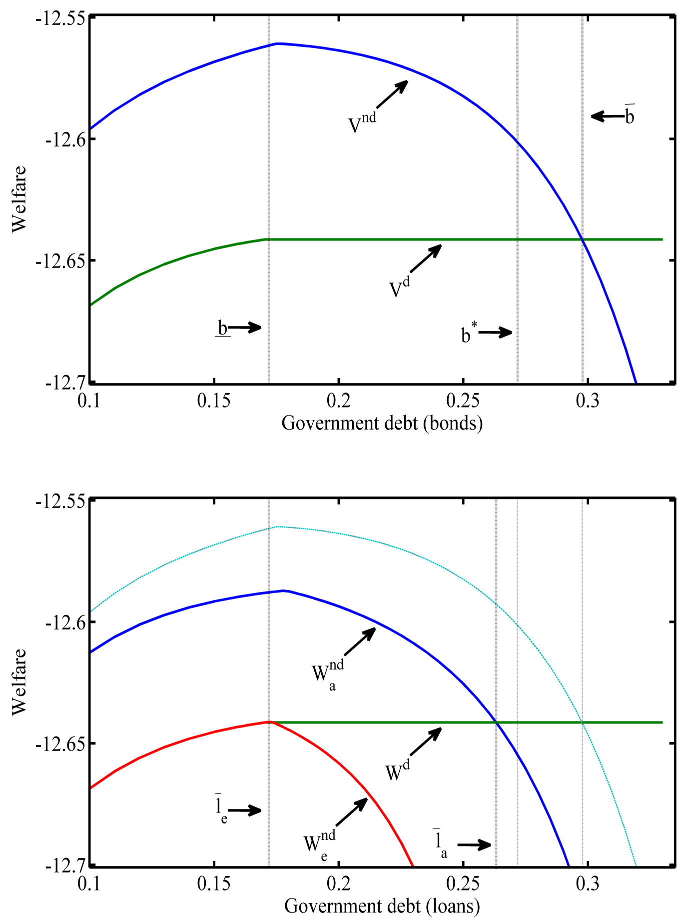
(e) Labour

(f) Tax rate

(g) Price of new debt

(h) Debt issuance

Figure 2: *Value Functions*

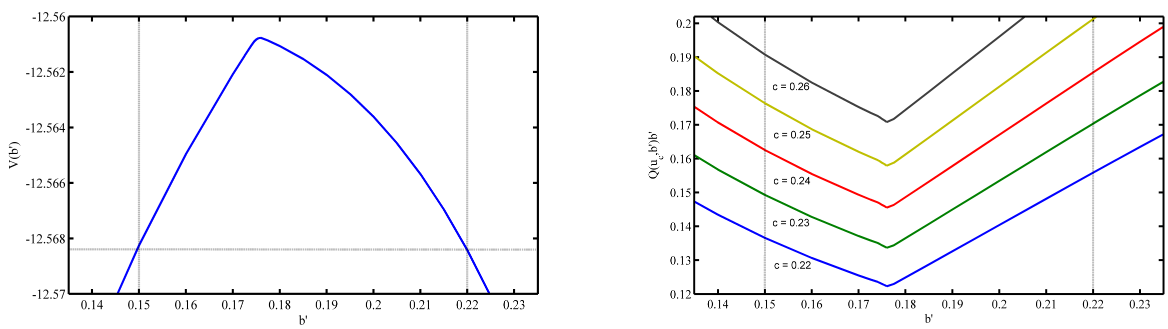


strict when $\ell > \underline{b}$; when excluded from the bond market, the government thus fully honors its maturing loans up to this threshold, $\bar{\ell}_e = \underline{b} = 0.1705$.

The debt Laffer curve. Inspection of the debt policy function $\hat{b}$ displayed in Figure 1 shows that, independent of the initial debt stock, the government always issues an amount of debt that is sufficient to ensure a non-binding collateral constraint in the future. To understand the intuition behind this finding, first note that, as prescribed by Proposition 2, the social welfare function has an inverted U-shape. Specifically, the welfare function is initially upward-sloping in region $\mathbb{B}^1$, where the collateral constraint is strictly binding, and later downward-sloping.

Given this shape, for every relevant choice $b' \in \mathbb{B}^1$ there hence exists an alternative choice $\tilde{b}' > b'$ such that $V^{nd}(\tilde{b}') = V^{nd}(b')$. Since b' and $\tilde{b}'$ deliver the same continuation payoff to the government, a necessary condition for b' to be optimal is to generate a higher current revenue from debt creation compared to $\tilde{b}'$. Formally, given the optimal choice of current consumption, $c = \hat{c}(b)$, the debt issuance b' can be an optimal choice only if $Q(u_c, b')b' > Q(u_c, \tilde{b}')\tilde{b}'$. Figure 3

Figure 3: *Welfare and the Debt Laffer Curve*



(a) Value function

(b) Revenue from debt issuance

shows that this is not the case in our calibrated economy. In particular, the figure shows that debt choices $b' \in \mathbb{B}^1$ generate a *lower* current revenue than the corresponding choices $\tilde{b}' \in \mathbb{B}^3$.[22] This pattern reflects the *debt Laffer curve* discussed in Proposition 3. Facing declining bond prices associated with suboptimal, low choices of $b' \in \mathbb{B}^1$, the government responds by an aggressive debt policy in order to escape the Laffer curve region. An important corollary to this observation is that, at least in the context of our quantitative results, the no-sustainability result of Proposition 5 can be further generalised. Specifically, regions on the 'bad' side of the debt Laffer curve are not ergodic so that Markov-perfect optimal fiscal policy fails to be

[22] As an example, consider an inherited level of debt $b = 0.1$ and the two alternative debt choices $b' = 0.15$ and $\tilde{b}' = 0.22$. These two choices deliver the exact same continuation welfare level $V^{nd}(0.15) = V^{nd}(0.22) = -12.5685$. Yet, the current revenue from issuing $\tilde{b}' = 0.22$ exceeds the current revenue from issuing $b' = 0.15$ for all possible values of c, including the optimal one at $c \approx 0.23$.

sustainable also here.

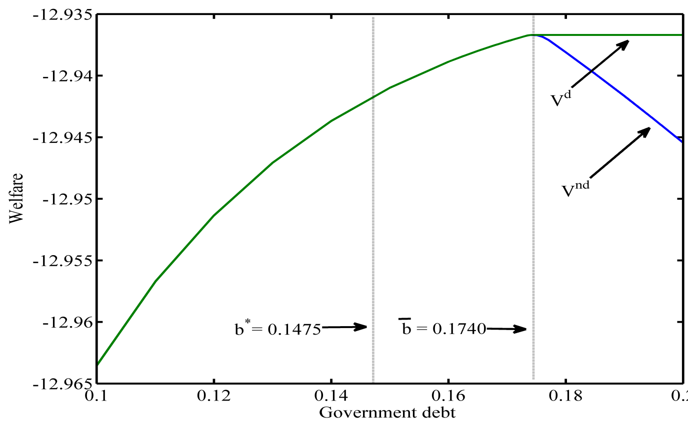
5 Default Costs and Debt Sustainability

The interaction of both reputational (fixed) and contemporaneous (variable) default costs is critical for our model to jointly generate empirically plausible statistics for steady state debt, the default threshold and the haircut imposed at this threshold. In order to further highlight this point, but also to examine the robustness of our quantitative results to alternative parameterizations, we now examine the contribution of each default cost channel to the determination of long-run debt and its sustainability.

Liquidity. The liquidity role of government debt is essential to generate a steady state with government liabilities in excess of the level satiating the economy's collateral constraint. To see this, note that our calibration, particularly that of ξ^c, implies a demand for collateral in the order of 50% of output. For higher levels of debt, the economy's collateral constraint is slack, which leaves the government facing a trade-off between the liquidity services of increased debt and the associated tax distortions. Figure 4 depicts the value functions $V^{nd}(b)$ and $V^d(b)$ for the case where public debt is not needed as a source of liquidity ($R = 1$).[23] The steady state emerges at $b^* = 0.1475$, and the fiscal limit at $\bar{b} = 0.1740$, which coincides with the level of debt that just satiates the economy's demand for collateral. Both points are located at significantly lower debt-to-GDP ratios than their counterparts from our benchmark calibration. Another important takeaway from Figure 4 is that the steady state is located in the upward-sloping

[23]Notice that this scenario makes bonds and loans equivalent so that there are no costs from a secondary market freeze.

Figure 4: *Value Function without Liquidity Role*



segment of $V^{nd}(b)$ and therefore sustainable. The reason behind this is that the collateral constraint is strictly binding at b^*. Clearly, debt positions $b > \bar{b}$ are not sustainable and cannot constitute a steady state. However, also at $\bar{b}$, where the collateral premium on government debt just vanishes, the government's interest manipulation motive induces a decumulation of debt.[24] In the complete absence of a liquidity role, the model rationalises debt convergence to only moderate levels. The first three columns of Table 4 consider the effects of variations in the importance of liquidity demand around the benchmark $R = 1.08$. The findings are as expected: An increase in R increases overall output as well as the share of entrepreneurial production in it, pushes both steady state debt and the fiscal limit to a higher level, but decreases the repayment rate at the fiscal limit. Quantitatively, however, these effects are relatively small, and throughout the demand for collateral remains satiated at steady state.

Collateral. The collateral role of government debt is essential to generate empirically plausible haircuts in the case of default. This property is illustrated in Figure 5, which plots the value functions, along with the steady state and the fiscal limit, for the case where public debt plays

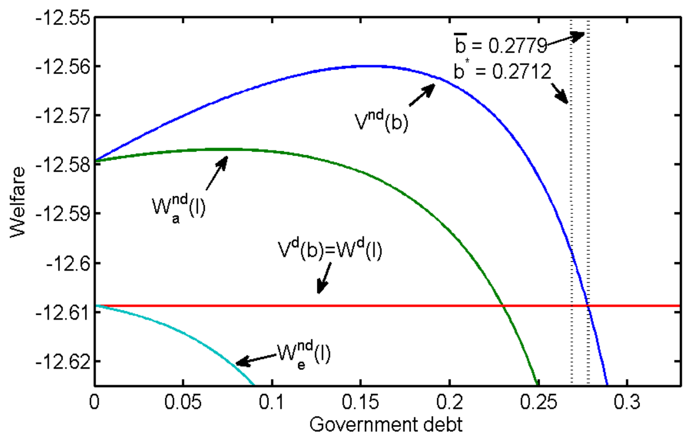
[24]Steady states with a strictly binding collateral constraint are obtained also under parameterizations in the neighborhood of $R = 1$. Liquidity benefits must thus be sufficiently strong to induce debt accumulation to a point where the demand for collateral is satiated.

Table 4: *Steady State and Fiscal Limit under Varying Liquidity and Collateral Roles*

Variable	$R = 1.06$ $\xi^c = 0.4$	$R = 1.08$ $\xi^c = 0.4$	$R = 1.10$ $\xi^c = 0.4$	$R = 1.08$ $\xi^c = 0.2$	$R = 1.08$ $\xi^c = 0.1$	$R = 1.08$ $\xi^c = 0$
y^1	0.3057	0.3030	0.3003	0.3003	0.3003	0.3030
y^2	0.0137	0.0190	0.0247	0.0190	0.0190	0.0190
y	0.3194	0.3220	0.3250	0.3220	0.3220	0.3220
b	0.2590	0.2712	0.2831	0.2712	0.2712	0.2712
b/y	0.8109	0.8422	0.8711	0.8422	0.8422	0.8422
$\bar{b}$	0.2682	0.2975	0.3223	0.2852	0.2808	0.2779
$\bar{b}/y$	0.8397	0.9239	0.9917	0.8857	0.8720	0.8630
$\rho(\bar{b})$	0.6283	0.5731	0.5337	0.3156	0.1567	0
$\underline{b}$	0.1685	0.1705	0.1720	0.0900	0.0440	0

no role as collateral ($\xi^c = 0$). Note that, absent a demand for collateral, the value function

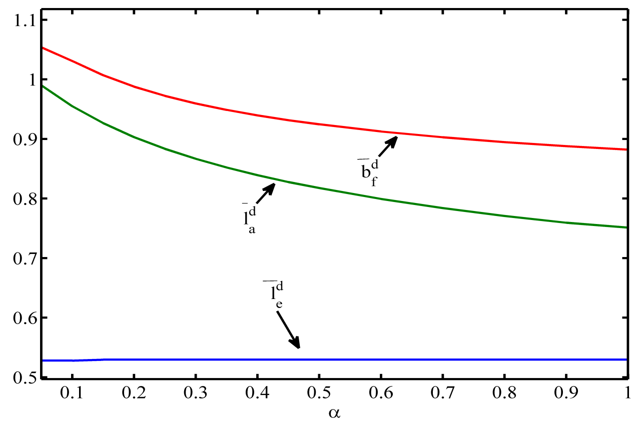
Figure 5: *Value Function without Collateral Role*



under default $V^d(b)$ is flat such that, conditional on default occurring, the government imposes a 100% haircut ($\rho = 0$). The absence of a collateral role actually leaves the steady state level of debt unaffected at $b^* = 0.2712$, and the same is true for the other steady state variables. On the other hand, the collateral role has quantitatively relevant implications for the optimal haircut and by consequence for the fiscal limit itself, which is now at $\bar{b} = 0.2779$. This reflects a feedback effect: Conditional on defaulting, the government's optimal policy in the absence of a collateral role is to completely wipe out its liabilities; this implies a more advantageous default value function and thus a tighter fiscal limit. The fourth and fifth column of Table 4 assess the effects of varying the collateral parameter between our baseline of $\xi^c = 0.4$ and the scenario of $\xi^c = 0$ just discussed. As discussed above, the collateral constraint remains non-binding in steady state for a wide set of parameterizations around our benchmark of $R = 1.08$. Under a non-binding collateral constraint, in turn, the long-run level of debt is completely invariant to ξ^c and instead determined via the trade-off between liquidity provision and tax distortions. In consequence, all steady state outcomes remain unchanged, and also the quantitative effect on the fiscal limit is minor. However, with the repayment rate $\rho(\bar{b})$ falling from about 57% to 32%, 16% and ultimately 0%, the reduced demand for collateral has important effects on the optimal haircut imposed. Throughout, the effective post-default level of debt $\underline{b}$ is tight enough to induce a binding collateral constraint (cf. Proposition 4).

Market exclusion. For sufficiently high levels of debt the liquidity value of government bonds is dominated by the associated tax distortions, resulting in a downward-sloping value function, $V_b^{nd}(b) < 0$. However, since default via *fractional repayment* of maturing debt amounts to rescaling the effective level of debt, any level of debt such that $V_b^{nd}(b) < 0$ is not sustainable, unless there is some additional fixed cost of defaulting. The loss in liquidity due to the government's exclusion from the bond market is therefore critically needed in order to sustain sizeable debt positions. Figure 6 illustrates how the three default thresholds $\{\bar{b}, \bar{\ell}_a, \bar{\ell}_e\}$ depend on the market re-access probability α. We observe that debt-to-GDP ratios in the order of magnitude of 100% are sustainable under quite moderate average exclusion durations. The default thresholds $\bar{b}$ and $\bar{\ell}_a$ are both monotonically decreasing in α. This reflects that a higher probability of market re-access lowers the expected cost of the bond market exclusion triggered by default;

Figure 6: *Default Thresholds as a Fraction of Steady State Output*



accordingly, the maximum sustainable level of debt is reduced. Quantitatively, however, an increase in α above our benchmark of $\alpha = 0.5$ has only relatively minor consequences for the fiscal limit: expressed as a fraction of steady state output, it changes from 92% for $\alpha = 0.5$ to 89% for $\alpha = 0.9$. Finally, the default threshold $\bar{\ell}_e$ is independent of α because the government is already excluded from the bond market and thus incurs only the contemporaneous costs due to the reduction in pledgeable collateral. At the threshold $\bar{\ell}_e$ these costs exactly balance the benefits of default due to reduced tax distortions. As also the benefits are independent of the re-access probability, so is the default threshold $\bar{\ell}_e$.

Labour supply. The elasticity of labour supply has been identified as an important determinant of endogenous default costs in the recent literature on external sovereign debt (Mendoza and Yue, 2012).[25] Table 5 examines the implications of variations in σ_l, the inverse of the Frisch elasticity of labour supply, on the determination of long-run debt and its sustainability. As seen, the steady state levels of output and debt are monotonically decreasing in σ_l, and so is the fiscal limit. However, when normalised by steady state output to remove the level

[25]The comparison with this literature is complicated as our specification of preferences does not eliminate wealth effects on labour supply.

Table 5: *Steady State and Fiscal Limit under Varying Labour Supply Elasticity*

Variable	$\sigma_l = 1$	$\sigma_l = 2$	$\sigma_l = 3$
y^1	0.3525	0.3030	0.2697
y^2	0.0199	0.0190	0.0182
y	0.3724	0.3220	0.2879
b	0.2840	0.2712	0.2597
b/y	0.7626	0.8422	0.9020
$\bar{b}$	0.3017	0.2975	0.2904
$\bar{b}/y$	0.8102	0.9239	1.0087
$\rho(\bar{b})$	0.6497	0.5731	0.5303
$\underline{b}$	0.1960	0.1705	0.1540

effect induced by the underlying change in preferences, both debt statistics are increasing in σ_l. A lower labour supply elasticity supports enhanced debt sustainability because it implies a lower tolerance towards fluctuations in hours worked and hence increased costs of default. Since default induces increased labour supply (owing to wealth effects) and the post-default labour allocation is increasing in the level of debt, the same argument also explains why $\rho(\bar{b})$, the optimal repayment rate at the fiscal limit, is decreasing in σ_l.

6 Aggregate Uncertainty

Within the non-stochastic environment studied so far, default never occurs in equilibrium provided the government's initial level of liabilities does not exceed the fiscal limit. To address this concern, we now introduce aggregate uncertainty. Specifically, labour productivity, which was normalised to $\bar{A} = 1$ in the deterministic verion of the model, is now assumed to be subject to an i.i.d. shock, that is, $A_t = \bar{A} - \varepsilon_t$ where $\varepsilon_t = 0$ with probability $1 - p$ and $\varepsilon_t = 0.2$ with (small) probability p. The negative shock is thus a large but rare event. As the probability or the magnitude of this negative event converge to zero, the setup with aggregate uncertainty degenerates to the deterministic model. Accordingly, we define the risky steady state as the allocation 'where agents choose to stay at a given date if they expect future risk and if the realization of of shocks is zero at this date' (Coeurdacier, Rey, and Winant, 2011, p.398). Conditional on the negative shock, the value of debt as collateral and liquidity is actually reduced sufficiently to induce default. Aggregate uncertainty therefore has an immediate implication on the pricing of government debt and on the government's optimal debt policy.

The values of endogenous variables in the risky steady state are displayed in Table 6, which for convenience restates also their deterministic counterparts from Table 3.[26] The steady state

Table 6: *Steady State Values*

Variable	Deterministic ststst.	Risky stst. ($p = 0.01$)	Risky stst. ($p = 0.03$)
y^1	0.3030	0.3035	0.3042
y^2	0.0190	0.0188	0.0186
y	0.3220	0.3223	0.3228
c	0.2578	0.2575	0.2572
g	0.0642	0.0648	0.0656
b	0.2712	0.2688	0.2652
b/y	0.8422	0.8340	0.8214
q	0.9699	0.9665	0.9606
τ	0.2248	0.2252	0.2259
w	1.0000	1.0000	1.0000
ρ	1.0000	1.0000	1.0000

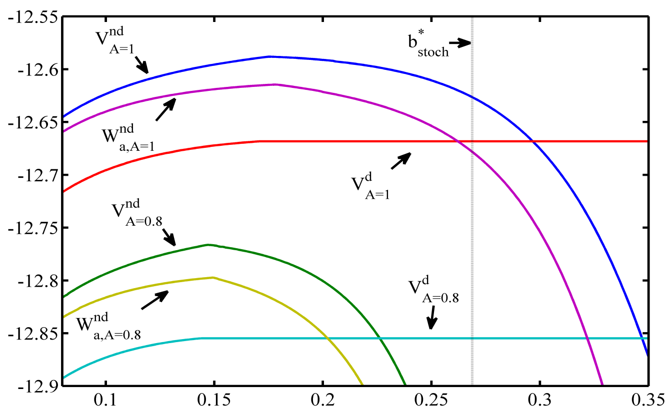
effects of low-probability default risk have the expected sign but are quantitatively small. In particular, steady state debt remains sustainable and is only mildly reduced, from 84.2% of output to 83.4% when $p = 0.01$ and to 82.1% when $p = 0.03$. This reduction is the precautionary response to default risk, which also implies that interest rates on bonds are higher. However, the increase amounts to only less than half a percentage point when $p = 0.01$ and to about one

[26]In the Web Appendix, we provide an equivalent table detailing the simulation means for the same endogenous variables.

percentage point when $p = 0.03$. These premia reflect the haircut and loss of liquidity, but also the increased collateral value and marginal utility from consumption in the event of default, which occurs with probability p.

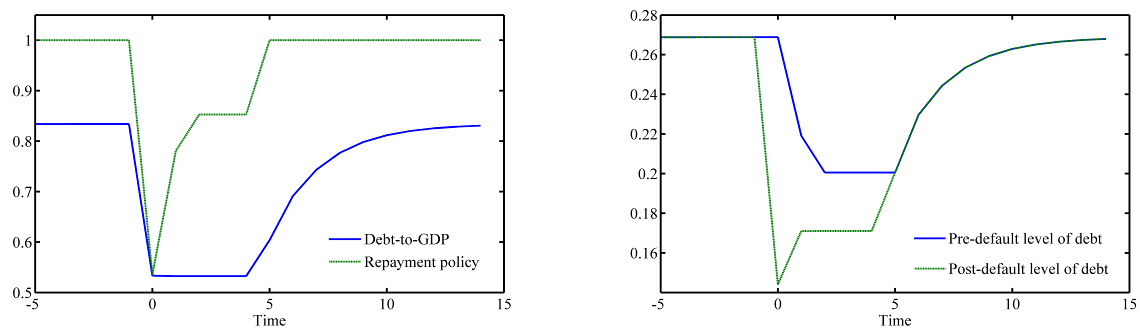
Figure 7 provides further insights into the effects of aggregate uncertainty when $p = 0.01$. It plots two families of value functions indexed by the realization of the aggregate shock. The

Figure 7: *State-dependent Value Functions*



plots confirm that the risky steady state level of debt b^*_{stoch} remains sustainable as long as $\varepsilon = 0$, whereas the negative shock leads the discretionary government to prefer default. Figure 8 traces the dynamics following a default event (again for $p = 0.01$). We assume that, in period 0, the economy with initial debt at b^*_{stoch} is hit by the negative shock $\varepsilon = 0.2$. In response, the government repays only a fraction $\rho_0 \approx 0.54$ of outstanding debt. The implied haircut is slightly larger than its deterministic counterpart because of the adverse productivity shock. Default triggers the government's temporary exclusion from the bond market. Underlying the dynamics presented in Figure 8 is a scenario where labour productivity immediately recovers to $\bar{A} = 1$, while the bond market exclusion spell lasts five periods. Thus, public debt has no liquidity value for an extended period. This has two consequences. First, as the costs of default remain muted,

Figure 8: *Post-default Dynamics*



(a) Repayment policy and debt-to-GDP (b) Pre- and post-default levels of debt

we observe a pattern of *recurrent defaults* (compare the value functions plotted in Figure 7). Second, loans to the government trade only at depressed prices, reflecting the combined effect of correctly anticipated default and their failure to provide liquidity benefits. The transition dynamics are driven by the government's effort to smooth consumption in the face of the recovery in labour productivity. In detail, period 0 is followed by a two-period transition, where the government issues a lower and decreasing profile of new liabilities, $\ell_1 > \ell_2 = \ell_3 \ldots$, which are repaid at an incomplete but increasing rate ($\rho_1 \approx 0.78$ and $\rho_2 \approx 0.84 = \rho_3 \ldots$). The initial default brings effective debt down from 84% to 54% of steady state output; it then remains constant at this level until the government gets access to the bond market again. Once this is the case, the government 'graduates' from default, i.e., public debt returns to its stochastic steady state level without further defaults occurring.

In sum, the introduction of aggregate uncertainty renders default an equilibrium event and implies determinate and recurring haircuts. The prediction of possibly protracted debt resolution is an aspect linking our theory to the literature on sovereign debt restructuring through bargaining (cf. Yue, 2010). Otherwise, our main findings from the deterministic setup regarding bond pricing, steady state, fiscal limit and partial default remain robust to the introduction of aggregate uncertainty.

7 Conclusion

This paper has provided a quantitative framework to study the joint determinants of government debt and its sustainability in a closed economy subject to financial frictions. Fiscal policy is distortionary and implemented under lack of commitment, which may extend also to the repayment of maturing government debt. Since debt is held domestically, it is valued as an instrument to smooth consumption, but also as a source of collateral and liquidity. This gives rise to endogenous default costs whose magnitude varies along with the size of the haircut on outstanding debt. The existence of a debt Laffer curve induces the government to issue bonds to a point where marginal debt has negative welfare effects. The model can thus rationalise substantial steady state debt. When default triggers the government's temporary exclusion from the bond market, debt positions in the order of magnitude of annual output remain sustainable in the face of the option to default. Equilibrium default occurs in response to adverse productivity shocks and takes the form of a fractional repayment rescaling the government's effective liabilities. The model also predicts the possibility of extended periods of recurrent defaults by governments without access to the bond market.

A Proofs

Proof of Proposition 1. Recall the definition of $Q(u_c, b')$ via the bond pricing function (14),

$$Q(u_c, b') = \beta \frac{u_c(b')}{u_c}\left(1 + \hat{\pi}(b') + \hat{\phi}(b')\right),$$

and the associated partial derivatives,

$$\begin{aligned} Q_1(u_c, b') &= -\beta \frac{u_c(b')}{(u_c)^2}\left(1 + \hat{\pi}(b') + \hat{\phi}(b')\right), \\ Q_2(u_c, b') &= \beta \frac{u_c(b')}{u_c}\left(\frac{u_{cc}(b')}{u_c(b')}\hat{c}_b(b')\left\{1 + \hat{\pi}(b') + \hat{\phi}(b')\right\} + \left\{\hat{\pi}_b(b') + \hat{\phi}_b(b')\right\}\right). \end{aligned}$$

Similarly, from the definition of $\omega(b, n)$ in (15),

$$\omega(b, n) = \begin{cases} 1 & \text{if } (1-\theta)b > \xi^c n \\ \frac{(1-\theta)b}{\xi^c n} & \text{otherwise.} \end{cases}$$

This expression makes clear that $\omega(b, n)$ has a kink but remains continuous at the point where the collateral constraint (4) becomes non-binding. This property is inherited by the other relevant policy functions as well. Moreover, $\omega(b, n)$ and the other policy functions are differentiable everywhere except at this point. With a slight abuse of notation, we thus obtain the partial derivatives $\omega_1(b, n) = \omega_2(b, n) = 0$ when $\omega(b, n) = 1$, and otherwise

$$\begin{aligned} \omega_1(b, n) &= \frac{(1-\theta)}{\xi^c n}, \\ \omega_2(b, n) &= -\frac{(1-\theta)b}{\xi^c n^2}. \end{aligned}$$

Apart from the non-differentiability at the point where the collateral constraint becomes non-binding, optimal government behavior under commitment to full debt repayment can be characterised in terms of first-order conditions. Given the additive separability of (1), these conditions are given by

$$\begin{aligned}
0 &= u_c(1+\gamma)+\gamma u_{cc}(c-(1+\pi)b)+\gamma(u_{cc}Q(u_c,b')b'+u_cQ_1(u_c,b')u_{cc}b')-\gamma u_c\pi_c b-u_g \\
&= u_c(1+\gamma)+\gamma u_{cc}(c-(1+\pi)b)-\gamma u_c\pi_c b-u_g, \\
0 &= u_l(1+\gamma\frac{1}{\omega(b,n)})-\gamma u_{ll}\frac{1}{\omega(b,n)}n+\gamma u_c\pi_n b-u_g-\gamma u_l\frac{1}{\omega(b,n)^2}n\omega_2(b,n), \\
0 &= \beta V_b(b')+\gamma(u_cQ_2(u_c,b')b'+u_cQ(u_c,b')) \\
&= \beta V_b(b')+\gamma\beta u_c(b')\left(1+\hat{\pi}(b')+\hat{\phi}(b')\right)(1+\varepsilon^q_{b'}),
\end{aligned}$$

where $\varepsilon^q_{b'}=\frac{Q_2(u_c,b')b'}{Q(u_c,b')}=\frac{u_{cc}(b')}{u_c(b')}\hat{c}_b(b')b'+\frac{\left(\hat{\pi}_b(b')+\hat{\phi}_b(b')\right)b'}{\left(1+\hat{\pi}(b')+\hat{\phi}(b')\right)}$. The envelope condition for b is

$$\begin{aligned}
V_b(b) &= -\gamma\left(u_c(1+\pi)-\frac{u_l n}{\omega(b,n)^2}\omega_1(b,n)+u_c\pi_b b\right)+u_g r \\
&= -\gamma\left(u_c(1+\pi)-\frac{u_l}{\omega(b,n)^2}\omega_1(b,n)\{n+rb\}\right)+u_g r.
\end{aligned}$$

Substitution into the first-order condition with respect to b' yields the generalised Euler equation (17),

$$\gamma'\left(u_c'(1+\pi')-\frac{u_l'}{\omega(b',n')^2}\omega_1(b',n')\{n'+rb'\}\right)-u_g'r=\gamma u_c'(1+\pi'+\phi')(1+\varepsilon^q_{b'}).$$

□

Proof of Proposition 2. When $\xi^c>0$, government debt is essential for production due to its collateral role. Thus, by construction, in the neighborhood of $b = 0$ welfare is increasing in debt. When $\xi^c = 0$ but $r > 0$, $\omega_1(b, n) = 0$ and the envelope condition for b is

$$V_b(b) = -\gamma u_c(1 + \pi) + u_g r = -\gamma u_c \left(1 + \frac{u_l}{u_c} r\right) + u_g r,$$

where the second equality follows from $\pi = \frac{u_l}{u_c} r$. The first-order condition with respect to n implies

$$u_g - u_l = \gamma \left(u_l - u_{ll} n + u_c \pi_n b\right) = \gamma \left(u_l - u_{ll} \left\{n + rb\right\}\right),$$

where the second equality follows from $\pi_n = -\frac{u_{ll}}{u_l}\pi = -\frac{u_{ll}}{u_c} r$. Solving for γ yields

$$\gamma = \frac{u_g - u_l}{u_l - u_{ll}\left(n + rb\right)} > 0.$$

Substituting into the envelope condition and evaluating at $b = 0$,

$$V_b(0) = -\frac{u_g - u_l}{u_l - u_{ll} n} u_c \left(1 + \frac{u_l}{u_c} r\right) + u_g r.$$

It follows that $V_b(0) > 0$ if and only if

$$\left(\frac{(u_l)^2 - u_g u_{ll} n}{u_l - u_{ll} n}\right) r \quad > \quad \frac{u_g - u_l}{u_l - u_{ll} n} u_c,$$

or equivalently,

$$r > \frac{u_g u_c - u_l u_c}{(u_l)^2 - u_g u_{ll} n} = \frac{\frac{u_c}{u_l} - \frac{u_c}{u_g}}{\frac{u_l}{u_g} - \frac{u_{ll}}{u_l} n} = \frac{u_c}{u_l} \frac{1 - \frac{u_l}{u_g}}{\frac{u_l}{u_g} - \frac{u_{ll}}{u_l} n}.$$

Finally, absent financial frictions, that is, when $\xi^c = 0$ and $r = 0$, the envelope condition for b is unambiguously negative, $V_b(b) = -\gamma u_c < 0$. □

Proof of Proposition 3. The marginal revenue from issuing additional debt b' is given by $\frac{dQ(u_c,b')b'}{db'} = Q(u_c, b') \left(1 + \frac{Q_2(u_c,b')b'}{Q(u_c,b')}\right) = Q(u_c, b')\,(1 + \varepsilon^q_{b'})$. Accordingly, there is a debt Laffer curve whenever $\varepsilon^q_{b'} < -1$. From the generalised Euler equation (17), $-V_b(b') = \gamma u'_c(1 + \pi' + \phi')\,(1 + \varepsilon^q_{b'})$. Since $\gamma u'_c(1 + \pi' + \phi') > 0$, it follows that $V_b(b') > 0$ if and only if $\varepsilon^q_{b'} < -1$. □

Proof of Proposition 4. Problem (21) shows that, by choosing ρ, the government can effectively regulate the state ρb in the value function $\tilde{V}^d(\rho b)$, subject to the constraint $\rho \in [0, 1]$. Accordingly, as ρ is chosen optimally, the value function $V^d(b)$ is *non-decreasing* over the entire state space. To see this formally, note that the first-order condition for ρ associated with problem (21) implies

$$\gamma \left(\frac{u_l n}{\omega(\rho b, n)^2} \omega_1(\rho b, n) b - u_c b \right) \geq 0, \tag{A.1}$$

with equality in case of an interior solution (recall our earlier comments in the proof of Proposition 1 concerning the differentiability of $\omega(b, n)$). But then the envelope condition associated with $\tilde{V}^d(\rho b)$ implies

$$\tilde{V}_b^d(\rho b) = \gamma \left(\frac{u_l n}{\omega(\rho b, n)^2} \omega_1(\rho b, n)\rho - u_c \rho \right) = \gamma \frac{\rho}{b} \left(\frac{u_l n}{\omega(\rho b, n)^2} \omega_1(\rho b, n) b - u_c b \right) \geq 0, \qquad \text{(A.2)}$$

where the weak inequality follows from (A.1), that is, under the optimal repayment policy associated with problem (21). It thus follows that $V^d(b)$ is non-decreasing.

Returning to (A.1), since $u_c b > 0$, it follows that an interior solution can only arise when $\omega_1(\rho b, n) > 0$. The same argument also implies that $\omega_1(\rho b, n) > 0$ is a necessary condition for a corner solution at $\rho = 1$.[27] Intuitively, $\omega_1(\rho b, n) > 0$ is optimal because default destroys the liquidity value of maturing debt. The optimal default policy is thus left with balancing the marginal benefits from reduced taxation with the marginal costs from reduced collateral. The optimal repayment policy conditional on default, $\tilde{\rho}^d(b)$, therefore ensures that the collateral constraint is strictly binding. Given $\omega_1(\rho b, n) > 0$, the envelope condition (A.2) implies

$$\tilde{V}_b^d(b) = \gamma \left(\frac{u_l n}{\omega(\rho b, n)^2} \omega_1(\rho b, n)\rho - u_c \rho \right) = \gamma \left(\frac{u_l n}{\omega(\rho b, n)} \frac{1}{b} - u_c \rho \right).$$

This expression is monotonically decreasing in b and ρ. Given some ρ, there is thus a unique b such that $\tilde{V}_b^d(b) = 0$. Let $\underline{b}$ denote the level of debt such that $\tilde{V}_b^d(\underline{b}) = 0$ when $\rho = 1$. When $\rho = 1$ and $b < \underline{b}$, $\tilde{V}^d(b)$ is increasing; a corner solution at full repayment, $\tilde{\rho}^d(b) = 1$, is thus indeed an optimizing choice, and $V^d(b)$ is increasing. Conversely, when $\rho = 1$ and $b > \underline{b}$, $\tilde{V}^d(b)$ is decreasing, which contradicts (A.2); the optimal repayment policy conditional on default is thus adjusted to an interior solution $\tilde{\rho}^d(b) < 1$, and $V^d(b)$ is flat. Finally, when $b = \underline{b}$, full

[27] A corner solution at $\rho = 0$ can never occur because debt is essential for production.

repayment, $\tilde{\rho}^d(b) = 1$, is optimal.

Taking stock, when $b < \underline{b}$, $V^d(b)$ is strictly increasing. Moreover, at $\underline{b}$ the collateral constraint is strictly binding. When $b < \underline{b}$, the government always finds it optimal to fully repay its maturing bonds, $\tilde{\rho}^d(b) = 1$. However, due to the market exclusion costs of default, it follows that $V^{nd}(b) > V^d(b) = \tilde{V}^d(b)$ for all $b < \underline{b}$. By contrast, for any level of debt $b > \underline{b}$ such that the government finds it optimal to default, $V^d(b)$ is constant, i.e., the value conditional on default is independent from initial debt. Denote this value by $\bar{V}^d$. Moreover, under the premise that the no-default value function $V^{nd}(b)$ is monotonically decreasing for large levels of debt and hence of an inverse U-shape, there exists a unique level of debt, $\bar{b} > \underline{b}$, such that $V^{nd}(\bar{b}) = \bar{V}^d$. By the same argument, $V^{nd}(b) \geq \bar{V}^d$ for $b \leq \bar{b}$, and $V^{nd}(b) < \bar{V}^d$ for $b > \bar{b}$. Accordingly, the government fully repays its outstanding bonds up to the threshold level $\bar{b}$ and partially defaults if inherited debt exceeds this threshold. This is the optimal (unconditional) repayment policy associated with problem (19); denote it by $\hat{\rho}(b)$.

In order to explicitly characterise the optimal (unconditional) repayment policy $\hat{\rho}(b)$, recall first that $V^{nd}(b) \geq V^d(b)$ when $b \leq \bar{b}$; hence, $\hat{\rho}(b) = 1$ for all $b \leq \bar{b}$. Conversely, when $b > \bar{b}$, $V^{nd}(b) < V^d(b)$ and, since $\bar{b} > \underline{b}$, $\hat{\rho}(b) = \tilde{\rho}^d(b) < 1$. But this implies that, for $b \geq \underline{b}$, condition (A.1) holds at equality and $\omega_1(\rho b, n) > 0$. By the household's optimal consumption-leisure choice $\frac{u_l}{u_c} = (1-\tau)\omega(\rho b, n)$, so that condition (A.1) implies

$$\rho b = \frac{\frac{u_l}{u_c} n}{\omega(\rho b, n)} = (1-\tau)n. \tag{A.3}$$

But for interior solutions $\hat{\rho}(b) = \tilde{\rho}^d(b) < 1$, $V^d(b) = \bar{V}^d$ is constant; that is, b does not matter for allocations and welfare, and $(1-\tau(b))n(b)$ is constant. It thus follows that the right-hand side in (A.3) is constant, implying that $\hat{\rho}(b)b$ must be constant and equal to $\underline{b}$ for all b that induce an interior solution for ρ. Since $\rho \leq 1$, we thus have $\hat{\rho}(b) = \underline{b}/b$ for $b > \underline{b}$. □

Proof of Proposition 5. For the policy $\mathcal{P}$ to be sustainable over some region $\tilde{\mathbb{B}} \subset \mathbb{B}$, the incumbent government must find it optimal to employ the policy $\mathcal{P}$ and to fully honor inherited debt for all $b \in \tilde{\mathbb{B}}$ when it perceives all future governments to employ the policy $\mathcal{P}$ and to fully honor inherited debt in $\tilde{\mathbb{B}}$. Absent reputational costs, the incumbent government's optimal policy problem is given by

$$\max_{c,n,b',\rho} \min_{\gamma} u(c, 1-n, n + r\rho b - c) + \beta V(b') + \gamma \left(u_c c + u_c Q(u_c, b')b' - \frac{u_l}{\omega(\rho b, n)} n - u_c\{1+\pi\}\rho b \right).$$

This problem can be decomposed into two stages: First, the government decides on the haircut on outstanding debt; second, given the haircut, it chooses the remaining policy instruments. By construction, the solution to the government's second stage problem is given by the policy functions $\mathcal{P}$ and the corresponding value function $V(b)$. The first stage choice of the optimal haircut $\hat{\rho}(b)$, in turn, is the solution to

$$\max_{\rho \in [0,1]} V(\rho b).$$

Thus, for any region $\tilde{\mathbb{B}}$, it is optimal to fully repay debt over the entire region $\tilde{\mathbb{B}}$ if and only if the value function is non-decreasing over the entire region $\tilde{\mathbb{B}}$. However, from Proposition 3, if the value function is non-decreasing in initial debt, this implies that the economy cannot be on the 'good' side of the debt Laffer curve. □

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