“A Random Matrix Approach to Portfolio Management and Financial Networks”

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A Thesis submitted for the degree of Doctor of Philosophy

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2016

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Acknowledgements

I would like to thank the Comisión Nacional de Investigación Científica y Tecnológica (CONICYT) for supporting this research under the Becas Chile Scholarship Scheme. I would also like to thank Maria Papageorgiou and my family for all their support.

Finally, I would like to offer special thanks to my supervisor, Professor Sheri Markose, for her guidance and encouragement over these years.

Many thanks,

Nicolas A. Eterović

Abstract

This thesis is an application of Random Matrix Theory (RMT) to portfolio management and financial networks. From a portfolio management perspective, we apply the RMT approach to clean measurement noise from correlation matrices constructed for large portfolios of stocks of the FTSE 100. We apply this methodology to a number of correlation estimators, i.e., the sample correlation matrix, the Constant Conditional Correlation Model (CCC) of Bollerslev (1990), the Dynamic Conditional Correlation (DCC) Model of Engle (2002) and the Regime-Switching Beta CAPM Correlation Model, based on Ang and Bekaert (2004). For these estimators, we find that the RMT-filtering delivers portfolios with the lowest realised risk, the best prediction accuracy and also the highest cumulated returns and Sharpe Ratios. The gains from using the RMT-filtering, in terms of cumulated wealth, range from 65%, for the sample correlation matrix to 30%, for the regime-dependent correlation estimator. In the case of regime switching CAPM models, we find that the regime switching correlation matrices, in the high volatility regime are found to be a good filter which makes further RMT-filtering to be redundant. This establishes the validity of using regime sensitive portfolio management to deal with asymmetric asset correlations during high and low volatility regimes.

From a financial network perspective, we assess the stability of a global banking network built from bilateral exposures of 18 BIS reporting banking systems to net debtor countries. For this, we applied the eigen-pair method of Markose (2012), which is based on the work of May (1972, 1974) for random networks. We use a stability condition based on the maximum eigenvalue (λ_{max}) of a matrix of net bilateral exposures relative to equity capital as a systemic risk index (SRI). We provide evidence of the early warning capabilities of λ_{max}, when this surpasses a prespecified threshold. We use the right and left associated eigenvectors as a gauge for systemic importance and systemic vulnerability, respectively. The λ_{max} SRI was found to be superior in terms of early warning when compared to the standard SRIs based on market price data, viz. the DCC-MES of Acharya et al. (2010), the SRISK of Acharya et al. (2012) and the DCC-ΔCoVaR of Adrian and Brunnermeier (2011).

Chapter 1

Introduction

1.1 Motivation

Over the last decades, financial research and practitioners have made use of tools borrowed from other fields in an effort to maximise their returns while minimising the risks of their investments. Random Matrix Theory (RMT) is an example of this. Originally developed by Wigner (1955) in the field of Statistical Physics to explain the behaviour of energy levels of complex nuclei, it has been recently applied to clean noise from financial correlations. RMT seeks to compare the eigenvalue distribution of the empirical correlation matrix with the eigenvalue distribution of a random matrix, generated from random *i.i.d* returns. The distribution of the random correlation matrix acts as a "null hypothesis", allowing a separation of noise and information in the empirical correlation matrix. The RMT approach has been used with great success in a number of studies to clean noise from correlation matrices, delivering better portfolio analytics (see, Laloux et al., 1999, 2000; Plerou et al., 1999, 2000a,b, 2001, 2002). Applications of the RMT to a financial context have been mainly focused on the sample correlation matrix of stock market returns, but this represents only the starting point of a large family of correlation estimators that are used in academia and in the financial industry to describe the behaviour of financial correlations. In this thesis, we seek to extend the RMT approach to different correlation estimators and markets, and assess its performance using a wide battery of portfolio analytics. In addition, we also use some results of RMT to assess the stability of financial networks in a cross-border setting.

From a portfolio management perspective, we review research using RMT, which shows how the eigenvalue density of the sample correlation matrix can be fitted relatively well by the density of a correlation matrix calculated from random returns, with the exception of some large eigenvalues that deviate. This suggests that these eigenvalues carry economically relevant information, whereas the rest can be regarded as "noisy". In Laloux et al. (1999) and Plerou et al. (2002) filtering techniques aimed at eliminating these "noisy" eigenvalues have been successfully proved to be beneficial in reducing both, the realised risk of optimal portfolios, and to correctly predict this realised risk.[1] In a similar vein, some results based on RMT have been very successful in reflecting the growing instability of financial networks, with great early warning capabilities (see, Markose et al., 2015).

In this thesis, we assess the benefits of the RMT-filtering using a wide battery of tests and correlation estimators. We test how the RMT-filtering can be used to reduce the realised risk of minimum variance portfolios and also its implications for portfolio returns and risk-adjusted measures. We also assess the dynamic stability of financial networks and show under what conditions these are said to be stable. Here dynamic stability is understood as the ability of the network to remain in equilibrium in face of a small perturbation.

1.2 Objectives

The main objectives of this thesis are summarised as follows:

1. Following the work of Laloux et al. (1999) and Plerou et al. (2002), we recreate some of their standard results, by applying the RMT approach to large portfolios of stocks from the FTSE 100. We test the performance of the different filters used in the literature and determine under which conditions the RMT-filtering will be more beneficial, in terms of realised risk reduction, prediction accuracy and risk-adjusted measures of portfolio returns.

2. We extend the use of the RMT-filtering to other correlation estimators, which may also be afflicted by noise. Examples of these estimators are the Constant Conditional Correlation

[1]By realised risk we refer to the standard deviation of portfolio returns over the investment period.

model (CCC) of Bollerslev (1990), the Dynamic Conditional Correlation (DCC) model of Engle (2002) and the Regime-Switching Beta-CAPM correlation estimator, based on Ang and Bekaert (2004). At least for the last two estimators, we are the first to apply the RMT-filtering to large correlation matrices, with great improvements in realised risk estimates.

3. As far as we are concerned, we are the first to combine the RMT approach with a Vector Autoregressive Analysis (VAR) to explore the macroeconomic determinants of the Chilean stock market. Here the main objective is to build portfolios with different degrees of exposure to the market portfolio as predicted by the RMT approach and then explore their macroeconomic determinants. Here we make a clear distinction between macroeconomic factors that are local from those that are external.

4. Finally, using some RMT results for the stability of financial networks, we assess the dynamic stability of a global banking network built from bilateral exposures of national banking systems to debtor countries. Here we apply the eigen-pair method of Markose (2012), who based on the work of May (1972, 1974) for random networks, develops a stability condition based on the maximum eigenvalue (λ_{max}) of a matrix of net bilateral exposures relative to equity capital. Here we provide further evidence of the early warning capabilities of λ_{max}, when this surpasses a prespecified threshold. We also use the right and left associated eigenvectors as a gauge for systemic importance and systemic vulnerability, respectively. The early warning capabilities of these metrics are then compared with recent systemic risk indices (SRIs) based on market price-based data, namely, the DCC-MES of Acharya et al. (2010), the SRISK of Acharya et al. (2012) and the DCC-ΔCoVaR of Adrian and Brunnermeier (2011). We also provide a thorough analysis for these metrics, in terms of their significance and what are their main determinants.

1.3 Thesis Outline

This thesis has been organised as follows. In Chapter 2 we review the mainstream literature on applications of RMT for improving realised risk estimates (Section 2.2), while in Section 2.3 we introduce the seminal results of May (1972, 1974) for network stability. Section 2.4 reviews some results from cross-border contagion and introduces the increasing role of cross-border financial networks for analysing systemic risk. In Chapter 3 we provide more detail on how the RMT works (Section 3.2) and also about the different filtering methods used in the literature (Section 3.3). In Section 3.4 we detail the test methodology that has been applied. In particular, for the portfolio analysis, we divide the analysis in an in-sample analysis (Section 3.4.1) and an out-of-sample analysis (Section 3.4.2). Section 3.5, introduces the eigen-pair method of Markose (2012) applied to a cross-border setting.

The results of this thesis have been divided into five chapters:

Chapter 4 provides a spectral analysis of correlations by using some standard RMT tools. Section 4.2 provides an interpretation of the meaning of eigenvalues in a financial context, while in Section 4.3 we introduce the concept of *inverse participation ratio* to identify the number of significant stocks in the associated eigenvectors. Section 4.4 and Section 4.5 examine the time-evolution of eigenvalues and eigenvectors and show how the behaviour of eigenvalues is able to describe some stylised facts of financial markets, such as correlation asymmetries and collectivity during market drawdowns.

Chapter 5 provides an application of RMT-filtering to a FTSE 100 portfolio by assessing the amount of noise in the system, as mesured by the RMT. We then examine the effect of RMT-filtering on realised risk estimates. We compare the effects of filtering on in-sample risk by using bootstrapping techniques (Section 5.4). We then assess the filtering method out-of-sample, by using the optimal estimation windows obtained in the in-sample analysis. Here we focus on assessing the performance of the RMT-filtering on a wide range of metrics, from realised risk to risk adjusted measures of returns and diversification measures.

Chapter 6 applies the RMT-filtering to regime-dependent correlation matrices. In Section 6.3 we provide a spectral analysis of correlation matrices under two different regimes (e.g., high and normal volatility regimes) and underline the main differences in the applicability of the RMT-filtering. In Section 6.4 we introduce the correlation estimators that will be used in the analysis, while in Section 6.5 and Section 6.6, these are assessed in and out-of-sample using a wide battery of tests.

Chapter 7 is an application of RMT to the Chilean Stock Market. Although many results are repeated from previous chapters, one of the main contributions is the analysis of Section 7.3.3, where we apply the RMT-filtering to some standard correlation estimators, namely, the Constant Conditional Correlation model (CCC) of Bollerslev (1990) and the Dynamic Conditional Correlation (DCC) model of Engle (2002). Section 7.4 uses the RMT approach and explores the macroeconomic determinants of the Chilean Stock Market.

Chapter 8 provides a network analysis of the Core Global Banking System Network (CGBSN) for 18 BIS reporting countries. Here we introduce our systemic risk indices (SRIs) in a cross-border setting (Section 8.2) and provide an analysis of the behaviour of the network over time (Section 8.4), while in Sections 8.5-8.6 we assess the early warning capabilities of the different SRIs. Section 8.6.3 explores the determinants of market price-based SRIs.

In Chapter 9 we summarise all findings. We provide a discussion of our conclusions and also discuss some lines of future research. Finally, in Appendices A-B-C, we report some tables corresponding to Chapters 5-7-8, respectively.

Chapter 2

Literature Review

2.1 Introduction

Markowitz (1959) Portfolio Theory is one of the most relevant topics in quantitative finance. It basically seeks to determine the optimal portfolio weights which either maximise return for a given level of risk, or minimise risk for a given return. In its different flavours, Markowitz Portfolio Theory relies upon a series of assumptions and is constructed based on the first and second sample moments of financial asset returns. While the first moments may be estimated by calculating the sample mean returns or by some univariate econometric specification, the estimation of the second moments, i.e., the covariance matrix of returns, can be more difficult to estimate. This is particularly true, when the length of the time-series, T is not very large compared to the number of assets, N. For instance, for a time-series of length T, in order to build a portfolio of N assets, we would need to estimate $(N^2+N)/2$ covariances from NT returns. This results in estimation noise, since for a given number of assets, we have few records available to estimate all the parameters. This latter would suggest the use of a longer time-series. However, financial correlations are not stable over time (i.e., non-stationary), therefore data should not be too old, in order to better capture current market conditions.

Thus, the problem boils down to find ways to estimate financial correlation matrices over relatively short time horizons, but at the same time dealing with the estimation noise arising from the choice of a short time-series. A possible solution to this shortcoming has been found in the field of Econophysics, by analysing the spectral properties (i.e., eigenvalues) of the sample correlation matrix. For instance, Laloux et al. (1999) and Plerou et al. (1999) find that the eigenvalue distribution of the sample correlation matrix can be fitted relatively well by the spectrum of a random correlation matrix, constructed from *i.i.d* time-series returns. This suggests that most of the informational content in the sample correlation matrix is mainly dominated by measurement noise. Conversely, *deviations from this random counterpart could indicate relevant economic information.* The Econophysics literature has employed these insights using Random Matrix Theory (RMT), a technique borrowed from Statistical Physics, which allows a separation of noise and information and has been used to obtain only statistically significant correlation estimates. These correlations have been found to be stable over time and have been employed in a number of studies in the framework of Markowitz Portfolio Theory. Filtered correlation estimates using only statistically significant eigenvalues as predicted by the RMT approach, resulted in better correlation forecasts than unfiltered estimates (Plerou et al., 1999, 2000a,b, 2001, 2002; Gopikrishnan et al., 2001).

The use of Random Matrix Theory (RMT) has not been limited to correlation matrices. It has also been used to assess the dynamical stability of financial networks. Dynamical stability of networks takes into account the time evolution of the network. In dynamical systems, a system is said to be stable, if small perturbations from an equilibrium decay with time. Keeping this in mind, May (1972) derives a stability condition for complete random matrices, where the dynamical stability of the network will depend on the maximum eigenvalue of the weighted adjacency matrix. While his results were based on random networks, which are by definition, unstructured. Recent studies have shown the applicability of these results to structured networks, i.e., small-world networks, which are more similar to real-world networks (Sinha, 2005).

In the rest of this chapter we review the key literature which supports the argument of this thesis. We first introduce the role of Random Matrix Theory (RMT) in physics and its applications to a financial context. We then review the role of RMT in improving Markowitz portfolio optimisation. Of central importance to this thesis is the work carried out by three independent groups: Laloux et al. (1999, 2000), Plerou et al. (1999, 2000a,b, 2001, 2002) and Pafka and Kondor (2003, 2004); Pafka et al. (2004). We also discuss the applications of RMT to assess the stability of financial networks, in particular the work carried out by May (1972, 1974), in the field of theoretical biology and Markose (2012); Markose et al. (2012) on the stability of financial networks in the OTC derivative market.

2.2 Applications of Random Matrix Theory to Portfolio Management

Applications of Random Matrix Theory (RMT) to portfolio management started by the end of 1990s, with the work of three independent groups of researchers; Laloux et al. (1999, 2000), Plerou et al. (1999, 2000a,b, 2001, 2002) and Pafka and Kondor (2003, 2004); Pafka et al. (2004).

2.2.1 Laloux et al. (1999, 2000)

They studied the eigenvalue distribution of a sample correlation matrix for $N = 406$ assets of the S&P 500, based on daily normalised returns during 1991-1996, for a total of $T = 1309$ days. They find that the largest eigenvalue λ_1 is 25 times larger than the predicted value by the RMT approach, λ_{max}.This means that the largest eigenvalue and its associated eigenvector reflect the "market" itself, as the associated eigenvector carries roughly equal components on all of the N stocks. Regarding the remainder eigenvalues, the authors find that 94% of them fall in the region predicted by the RMT. Thus, less than 6% of the eigenvectors, which are responsible for the 26% of the total volatility, would seem to carry some information.[1] This suggests that most of the eigenvalues of the sample correlation matrix are dominated by noise.

As there is a closed-form solution for the theoretical bounds of the eigenvalue spectrum of a random matrix, "information" can be separated from "noise" and this can be used in practice to improve the estimates of realised risk of Markowitz portfolios.[2] Keeping this in mind, the authors designed a "filter" for the correlation matrix, by keeping only the statistically significant

[1] The percentage of the variation explained by the deviating eigenvalues can be calculated as the ratio between the sum of the deviating eigenvalues and the total number of assets N (recalling that the total sum of eigenvalues is equal to N).

[2] The theoretical bound for the eigenvalue distribution of a random matrix, under certain conditions, it will be given by the bounds of the Marčenko-Pastur Distribution. This will provides us with a cut-off point to choose the number of significant eigenvalues in the sample correlation matrix. This will be developed further in the coming sections.

eigenvalues predicted by the RMT.[3] The "filtered" correlation matrix is then used to construct optimal portfolios. For doing this, they split their sample in two subperiods. They estimate the *filtered* correlation matrix using the first subperiod and construct a family of optimal portfolios and the so-called "efficient frontiers". In order to isolate the effects of noise in the correlation matrix from any additional source of noise in the estimation of returns, they use *realised* returns.[4] Likewise, using *realised* returns they calculate a measure of *realised* risk by constructing portfolios using the second subperiod. They find that the use of the sample correlation matrix tends to underestimate *realised* risk by a factor of 3. When using the *filtered* correlation matrix this factor is reduced to 1.5.

2.2.2 Plerou et al. (1999, 2000a,b, 2002); Gopikrishnan et al. (2001)

In addition to their eigenvalue density, symmetric random matrices have universal statistical properties. For example, the distribution of spacings between neighbouring eigenvalues is the same for all real symmetric random matrices. Real symmetric random matrices whose elements are independently drawn from a Gaussian distribution constitute the Gaussian Orthogonal Ensemble (GOE) and are, with respect to their statistical properties, representative for all real symmetric random matrices.[5] In Plerou et al. (1999) they study whether the sample correlation matrix exhibits these universal statistical properties predicted by Random Matrix Theory. For that purpose, they use two datasets: (i) 30-min data on 1000 records of U.S. publicly traded companies over the 2-year period 1994-1995 and (ii) one-day prices fluctuations of 422 U.S. stocks for the 35-year period 1962-96. In line with Laloux et al. (1999), they find that the 98% of the eigenvalues of the sample matrix fall within the theoretical eigenvalue density. In order to test whether these eigenvalues are genuinely random, they test for the universal properties such as eigenvalue spacings and correlations and find that the Nearest-Neighbour distribution as well as long range spectral correlations show good agreement with the universal predictions of the GOE (Plerou et al., 1999, 2002). This in turn verifies that the bulk of the eigenvalue spectrum of

[3] That is, only keeping the eigenvalues that deviate from the theoretical edge of the spectrum.

[4] By doing this, they assume perfect forecast on the future average returns. While this would not make sense in that if we knew future returns we would not need to make any forecast, it is a plausible way to assess the effects of noise filtering on the ability to forecast *realised* risk.

[5] The study of these properties is important as the eigenvalue pdf alone does not prove the randomness and lack of information on the eigenvalue spectrum.

the sample correlation matrix does not contain economically relevant information. In a similar vein, when analysing the eigenvectors associated to noisy eigenvalues, they find that these can be described by a normal distribution. Also, the eigenvectors associated to the eigenvalues located at the lower edge of the theoretical spectrum were found to be localised, that is, they were dominated by very few large components (Plerou et al., 1999, 2002). In Gopikrishnan et al. (2001), the eigenvectors associated with the twelve largest eigenvalues were found to have an economic interpretation; the eigenvector associated with the largest eigenvalue was found to describe correlations reflecting the market index, while the remainder ones describing correlations between large capitalisation companies, industry sectors or shared business in certain geographical areas (Gopikrishnan et al., 2001; Plerou et al., 2002). These statistically significant correlations were found to be stable in time and therefore suitable for forecasting purposes.

Similar to Laloux et al. (2000), the authors use these notions to construct optimal portfolios. For doing so, they split the sample 1994-1995 into two one-year periods. For the first period, they calculate the sample correlation matrix C_{1994} and use the returns of the second period to construct a family of optimal portfolios and obtain the *predicted* risk, Ω_{1994}.[6] For this family of portfolios, they also compute the *realised* risk in 1995, Ω_{1995}, but using C_{1995}. Since the meaningful information in C is contained in the deviating eigenvectors, they construct a "filtered" correlation matrix, C^f_{1994}, with a similar method to the one of Laloux et al. (2000), by keeping only the statistically significant eigenvalues and construct a family of optimal portfolios. They find that:

- When using the sample correlation matrix, the gap between predicted and realised risk is significantly large, by a factor of 2.7.
- When using the filtered correlation matrix, using only the statistically significant eigenvalues as predicted by the RMT, this gap is reduced to 1.25. This suggests that the portfolios constructed using the filtered correlation matrix are significantly more stable in time.

We next summarise the work of Pafka and Kondor (2003, 2004); Pafka et al. (2004) who built on the above research, introduce a slightly different approach for estimating the correlation

[6]Predicted risk is defined as the standard deviation of portfolio returns using either the sample correlation matrix or its unfiltered counterpart. It is therefore not a proper forecast, but forward validation to verify the time stability of correlation coefficients

matrix: Exponentially Weighted Moving Average (EWMA) correlation matrices combined with RMT.

2.2.3 Pafka and Kondor (2003, 2004); Pafka et al. (2004)

The work carried out by Pafka and Kondor (2003, 2004) and Pafka et al. (2004) is with no doubt illuminating, since they were the first to apply the RMT-filtering on estimated correlation matrices, i.e., exponentially weighted moving average correlation matrices (EWMA).

In Pafka and Kondor (2003, 2004), they studied the effects of noise on portfolio risk using simulation results. Non-random covariance matrices were built, and then they introduced noise to these. The true (non-random) risk was assumed to be known, in order to calculate minimum variance portfolios. The advantage of this approach is that is not necessary to have the forecasted returns as inputs and therefore the only noise in the system could be attributed to the covariance matrix.

One of their main findings is that the effects of noise differ depending on whether we derive portfolio weights from a portfolio optimisation framework or we are measuring the risk of a portfolio with a risk fixed weights (i.e, not derived from a portfolio optimisation problem). In the first case, noise has an important effect, as weights are also depending on the noisy covariance matrix, whereas in the second case, it becomes of second order, typically between 5-15%.[7] This is so, because investors do not really care about the composition of their portfolios, but instead the overall risk of their portfolios. In the presence of noise, weights can be substantially different from the true optimum, but not necessarily the overall risk. Based on this reasoning, the authors conclude that in practice, the risk of optimal portfolios, calculated with the noisy covariance matrices should be reconciled.

These latter results, partly clashed with the findings of Laloux et al. (1999, 2000), Plerou et al. (1999, 2000a,b, 2002) and Gopikrishnan et al. (2001) and it was further argued that the large discrepancies between the “predicted” and “realised” risk obtained in these studies were attributed to low values of $Q = T/N$, whereas for large values of T/N this effect becomes much smaller. However, as pointed out by Daly et al. (2008), although the simulation results

[7] That is, the difference between the “true” risk of the minimum variance portfolio determined in the presence of noise and the portfolio risk calculated from the “true” covariance matrix.

of Pafka and Kondor (2003, 2004) were indicative, there were a number of aspects that made the approach less than satisfactory for assessing the realised risk of medium term investments. The use of generated covariance matrices is very simplistic and therefore fails to account for the full market structure. This is so, because markets change over time (are heteroskedastic) introducing additional sources of error when using real data and definitively not accounted in generated data. In addition to this, the time scale used in these studies is much shorter than what they used in their simulations. The simulations also only considered the effect of noise at each level of $Q = T/N$. They did not take into account the possibility that the optimal value of Q may be quite different in the presence of noise than without. Finally, all the studies shown so far only considered the case of equally weighted covariances, where each observation contributes equally to the estimates of the covariance matrix. As pointed out by Litterman and Winkelmann (1998), volatilities and correlations tend to vary over time, and thus the older returns are, the less relevant they are for revealing the covariance structure at a given time. This is why, the use of decay weights became a widely spread practice for risk management. There are many ways to apply decaying weights to covariance matrices, while the most popular are the ones based on the likelihood function.

In later work, Pafka et al. (2004) extended the RMT approach to Riskmetrics (Zangari, 1996) type financial forecast. They derived a covariance estimator to take account the heteroskedasticity of financial returns, by using an exponentially weighted moving average (EWMA) and then applied the RMT approach to filter the noise of the covariance matrix. They calculated the spectrum of a large exponentially weighted random matrix, whose upper bound needed to be known to apply the filter. The spectrum is valid in a limiting case, as the number of assets becomes infinite, and the decay factor tends to unity. They found good agreement between that limiting case and the one with a finite number of assets and a realistic decay factor. From this spectrum they were able to find its upper bound and developed RMT filters, in the same context as in the equally weighted case. Using this bound, they kept the eigenvalues above the bound and the rest were filtered. They show using empirical data that this approach outperformed in a portfolio optimisation context over both, the method of exponentially weighted moving alone and the equally weighted standard RMT-filtering.

Using bootstrapping techniques, Pafka et al. (2004) tested six methods for forecasting realised risk, based on equally and exponentially weighted forecasts. The unfiltered forecasts were compared to those filtered using RMT, and those filtered by retaining only the largest eigenvalue. Their results can be summarised as follows:

- The RMT filtered exponentially weighted forecasts resulted in the lowest realised risk from the six methods. The unfiltered forecasts were found to perform better when a low number of assets were used, or alternatively when using a larger T (for equal weights) or when the decay factor approached to one (for exponential). The largest eigenvalue filter was the least successful method for both weightings.

- In agreement with previous simulations, Pafka and Kondor (2003, 2004) found that the benefits of filtering were reduced, for fixed N, as the value of T increased. A similar effect was noted for exponential weights, where increasing the decay factor was broadly found to reduce the effect of filtering. In the presence of filtering, however, the best parameter values reduced the amount of data used, compared to the unfiltered case.

- In general and assuming the existence of sufficient data, the improvements seen after filtering were of the same order and even below, the simulated effect of noise (Pafka and Kondor, 2003, 2004). These authors also found that the decay factors, which produced the least risky portfolios, were higher than the range suggested by Riskmetrics (0.94) and further concluded that the unfiltered Riskmetrics recommended forecast was unsuitable for their portfolio optimisation problem, more than doubling portfolio risk compared to the best filtered result.

2.2.4 Other similar studies

Emerging markets have also been studied. For example, studies applying the RMT approach to emerging markets, namely, Nilantha et al. (2007) for Sri Lanka and Kulkarni and Deo (2007); Pan and Sinha (2007) for India, Medina and Mansilla (2008) for Mexico, find good agreement with the universal properties of the RMT, with fewer deviations. In particular, Pan and Sinha (2007), using 201 stocks of the National Stock Exchange (NSE), found fewer deviations from the bulk distribution than those found for the NYSE. The authors argue that this was due to the relative weakness of intra-sector interactions between stocks, compared to the market mode. Based on this, they suggested that the emergence of an internal structure of multiple groups of strongly correlated components is an indication of market development.

Wilcox and Gebbie (2004, 2007) analysed the Johannesburg Stock Exchange (JSE). In this case, they had to deal with difficulties of missing data and illiquid stocks. In the presence of such data, they still found good agreement with the RMT. For all data cleaning methods tested, the authors reported that most eigenvalues agreed with the RMT, while there were a small number of large deviating values. They also noted that the choice of data cleaning method affected the results, with some introducing more noise than others. The portfolios considered here were also large, containing between 250 and 350 shares.

While these studies have been very successful in establishing the broad applicability of the RMT approach to a number of different economic contexts and financial markets, most of the existing literature using RMT to forecast realised risk has focused on relatively tranquil times in financial markets. One of the few recent studies focusing on more turbulent periods is that of Sandoval et al. (2012), who used the RMT approach to clean the correlation matrix of Brazilian stocks contained in the BOVESPA Index. They use this procedure to construct portfolios of stocks based on the Markowitz framework to build better predictions of future risk based on past data. They do this for years of low and high volatility for the Brazilian market during the years 2004-2010. They find that the *filtered* correlation matrix using the RMT approach is certainly better than the *unfiltered* one, but it fails to provide a better forecast during high volatility periods.

There have also been some work focusing on the performance of the filtering method itself. This is the case of Daly et al. (2010) who study the effects of using different RMT-based filters on realised risk estimates. They find that the filtering method matters, with some filters providing more stability than others. They further developed a filter that maximises the eigenvalue spacings in the trace of the correlation matrix (Krzanowski stability-based filter) and find that their filter lowers realised risk, on average, by up to 6.7%.

2.3 Applications of Random Matrix Theory to Network Stability

Networks change over time, examples of this, are the spread of diseases along with patterns of human travel, people make new friends and more nodes are added to the social network. In complex networks the node dynamics can be very different, for example, neural networks might be featured by very complicated non-linear, even chaotic dynamics, while in SRI-models of disease, spreading nodes can take only one of three possible states (susceptible, recovered and infected). In both of these cases, the individual node dynamics is influenced by the wiring of the network. Stability, which is a dynamical property, is determined by the topology of the network.

Mathematical models have been developed to analyse dynamic stability in the presence of perturbances. Perturbances can be different, so as stability, in whose case we can distinguish between *static stability* and *dynamic stability.* By static stability, we refer to the ability of the network to retain its connectivity properties after nodes have been removed.[8] In contrast, dynamical stability, takes into account the time-evolution of the network. In dynamical systems, a system is said to be stable, if small perturbations from an equilibrium decay with time.

In the seminal work of May (1972, 1974), the Wigner condition of eigenvalues for complete random matrices was extended to sparse random networks. He was the first to state that the stability of a dynamical network system will depend on the size of the maximum eigenvalues of the weighted adjacency matrix of the network. May (1974) derived the maximum eigenvalue of the network, denoted by λ_{max}, in terms of three network parameters;

$$\lambda_{max} = \sqrt{Np\sigma^2}, \tag{2.1}$$

where p, is the probability of connectivity, N the total number of nodes and σ, is the standard deviation of node strength. May (1974) stated that network instability follows when $\sqrt{Np}\sigma > 1$. Then, an initially small perturbation will gradually grow with time and drive the system away from its equilibrium state. This led to the so-called May-Wigner Stability Theorem, which states

[8]The difference between random and intentional removal can be quite drastic: in scale-free network structure that describes, for instance, the internet, a random removal of nodes has only a minor effect on connectivity and average path length, whereas a targeted attack on the most highly connected nodes can quickly disconnect the network into separate clusters (Boccaletti et al., 2006).

that increasing the complexity of a network inevitably leads to its destabilisation, such that a small perturbation will be able to disrupt the entire system. This theorem poses a trade-off between heterogeneity in node strength, σ and connectivity p, in order for the network to remain stable. In a non-zero mean random matrix, highly connected networks can remain stable only if they are homogeneous in node strength, viz. σ should be very small.

May's suggestion that increasing network complexity leads to decrease in stability was supported by numerical simulations, but it ran counter to the empirically established conventional wisdom that biodiversity promotes ecosystem stability. Similarly, in the economics literature, Allen and Gale (2000) gave rise to a mistaken view that only follows in the case of homogenous graphs, that is, increasing connectivity monotonically increases system stability in the context of diversification of counterparty risk.

The criticism to the May-Wigner Stability Theorem, is that is only valid for random networks and cannot be applied to real-world networks, which presumably are structured. Most networks occurring in nature are not random. Some studies suggest that introducing a hierarchical organization (by partitioning the adjacency matrix of the network into blocks) can increase the stability of a network under certain conditions.

Small-world networks allow the possibility of having different kinds of structures in a network. These networks have the global properties of a random network (short average path length between the elements) while at the local level they resemble regular networks with a high degree of clustering among neighbours.

In Sinha (2005) is examined how the introduction of small-world topological structure into networks affect their stability. His main results indicate that, regardless of the network structure, the parameter values at which the stability-instability transition occurs with increasing complexity is identical to that predicted by the May-Wigner criteria, but the nature of this transition changes with the topology. In this context, the May-Wigner criteria are valid for analysing system stability of small world networks.

As we will show, for analysing the stability of networks, under certain conditions, the explicit dynamics at the nodes can be ignored and the dynamic stability is governed by the maximum eigenvalue of the linear stability (adjacency) matrix. In the following chapters, we will explain how this condition is extended by Markose (2012) to the context of a global banking network constructed from bilateral exposures of national banking systems to net debtor countries, relative to their equity capital. From this, we will derive the maximum eigenvalue of the stability matrix, which will give us some insights into the stability of the global banking network over time and also the possibility of early warning signals.

2.4 Financial Contagion in a Cross-Border Setting and the Role of Financial Networks

Since the emerging markets crises in Asia, Russia and Brazil in the late 1990s, there has been increasing interest in the study of cross-border contagion and spillovers that can spread a financial crisis from one country to another (Claessens and Forbes, 2001). Calvo and Reinhart (1996) distinguish between fundamental-based contagion (spillover), which arises when the infected country is linked to others via trade or finance, and “pure” contagion, which arises when common shocks and all channels for potential interconnection are either not present or have been controlled for. There are at least three main strands of research exploring cross-border contagion. The first studies the co-movement of asset prices and tests whether a change in asset prices in country A has some effect on asset prices in country B, using a number of econometric techniques (Baig and Goldfajn, 1999; Bae et al., 2003; Corsetti et al., 2002; Forbes and Rigobon, 2001).

A second strand of the literature investigates the existing financial and trade links by exploring the channels through which contagion could take place (Dornbusch et al., 2000; Kruger et al., 2000). For example, in the literature of early warning signals (EWS), Catão and Milesi-Ferretti (2014) suggest the ratio of net foreign liabilities to GDP as being informative of an eventual economic crisis in a cross-border setting. They find that when this ratio exceeds the threshold of 50% in absolute terms and 20% of country specific historical mean, it becomes a good signal for external crisis prediction. Gourinchas and Obstfeld (2012) using data from 1973 to 2010, find that the rapid increase in leverage and sharp appreciation of the currency are two factors that emerge consistently as the most robust and significant predictors of financial crises. Bruno and Shin (2015a,b) build a model of international banking systems and identify bank leverage cycle as a main determinant of the transmission of financial conditions across borders through banking.

They identify that local currency appreciation together with higher leverage in the banking sector as main determinants of financial instability and develop a model that links exchange rate and financial stability.

A third strand of the literature, the network-based one, which is more recent, has focused on the role of international banks in transmitting financial shocks across borders.Von Peter (2007) and Minoiu and Reyes (2013) and Hattori and Suda (2007) analyse the topological properties of the global banking network using BIS Locational Banking Statistics spanning over 30 years. They study the evolution of connectivity and density of transactions over the period 1978-2010 and find that the global banking system is much more connected during 2000s, when compared to previous decades, while the main trend is a procyclical path with the global capital flows. Von Peter (2007) provides centrality measures of reporting countries to the 200 or so peripheral countries. He calculates a banking centre measure, or "global hub" and measures of both unweighted node degree, closeness and betweenness, as well as weighted centrality measures ("intermediation", based on weighted node degrees) and "prestige" (based on a weighted centrality indicator). His results place the UK and US as the main international banking centres that can explain the market share dominance in attracting foreign deposits. Network statistics based on BIS data have also been used to develop early warning signals for cross-border financial crisis. Minoiu et al. (2013) find that network statistics improve the performance of EWS compared to more traditional macroeconomic variables.

So far, there are few studies testing contagion risk and the stability of cross-border flows. Degryse et al. (2010) is one of the first to analyse the systemic importance of countries such as the US with large global liabilities using the BIS Consolidated Banking Statistics for 17 reporting countries and carry out Furfine (2003) style contagion exercise for national banking systems from country specific defaults over the period 1999-2006. They find that high level of liabilities coming from the UK and US can pose a huge threat for the rest of the global financial system. Another study is that of Castrén and Rancan (2014), who introduced the notion of *Global Macro-Networks*, which is a combination of cross-border exposures of banking systems of countries with the flow of funds between sectors within countries. The authors quantify the magnitude of contagion losses stemming from the failure of key debtor countries and the sectors within them with a metric they call *Loss Multiplier*, which they use as a measure of systemic risk. Despite, the seminal nature of their concept of a global macro-net, as contagion losses are greater after the onset of a crisis than before, the Castrén and Rancan (2014) *Loss Multiplier* SRI gives no early warning and peaks well after the crisis, Markose et al. (2015). However, there is yet no consensus on a financial network-based systemic risk index.

As pointed out by Markose (2013) an approach based on cross-border liabilities relative to equity capital of national banking systems for analysing systemic risk allows a more complete picture of the role of leverage in destabilising the global financial system. The concept of tipping points have been used in the context of financial contagion and domino effects from the failure of a trigger bank or from negative shocks, concepts made popular by Furfine (2003). However, few have acknowledged that such instability propagation in a network is a property of dynamical systems and for that reason needs to be studied by using spectral or eigenvalue methods as seminally pointed out by May (1974, 1972). Markose (2012) uses the May approach and adapts the epidemiology framework in Wang et al. (2003) to show how a weighted graph of financial liabilities can provide an appropriate dynamic characterisation of a financial system. Stability conditions are governed by a "cure" rate, defined as a threshold of equity capital that provides a permissible buffer against contagion losses, and "infection" rates, arising from the exposures of financial participants to their counterparties. This provides the basis for the so-called eigen-pair method which simultaneously determines the maximum eigenvalue of the network of liabilities (adjusted for equity capital), to indicate the stability of the overall system, along with eigenvector centrality measures. This will be developed in further detail in Chapter 3, Section 3.5.

Chapter 3

Background and Methodologies

3.1 Introduction

In this chapter we define the tools that will be used across the chapters of this thesis. We begin by explaining how the Random Matrix Theory-based filtering (henceforth, RMT-filtering) works and how it can be used in a portfolios optimisation context. We describe how the separation between information and noise takes places in the eigenvalue spectrum for which there is a theoretical benchmark given by the Marčenko-Pastur Distribution (Section 3.2.1). This allows us to compare the properties of the sample correlation matrix with those of a random matrix and identify noisy eigenvalues. This is done by using the theoretical bounds of the Marčenko-Pastur Distribution, which provides us with a cut-off point, above which we can keep informationally relevant eigenvalues and discard the noisy ones. A detailed discussion on the different filtering methods is given in Section 3.3.

Once the filtered and unfiltered correlation estimators are obtained, these are used as inputs for portfolio optimisation as described in Section 3.4, where we divide our analysis in an in-sample analysis (Section 3.4.1) and out-of-sample analysis (Section 3.4.2). In both cases, we analyse the performance of the RMT-filtering on *Minimum Variance Portfolios.* In Section 3.4.2, we also provide a detailed description of the performance metrics used in all the chapter of this thesis. In particular, this methodology will be used in three chapters of this thesis: Chapter 5 analyses the performance of the RMT-filtering on the sample correlation matrix for portfolios constructed from 77 stock in the FTSE 100 Index; Chapter 6 applies the RMT-filtering to regime-switching correlations for portfolios using the same 77 stocks and; Chapter 7, is a novel application of the RMT-filtering, where we analyse the macro determinants of the Chilean Stock Market (Eterovic and Eterovic, 2013), by using only the statistically significant principal components as predicted by the RMT approach.

Finally, in Section 3.5 we introduce the eigen-pair method pioneered by Markose (2012) for financial networks. Here we are particularly interested in analysing the Core Global Banking Network constructed from the BIS Consolidated Banking Statistics for 18 national banking systems. This is then combined with data on the value of equity capital, from Bankscope, to build the *Stability Matrix*, which represents the net exposures of national banking systems relative to their equity capital. The maximum eigenvalue of the stability matrix (henceforth, λ_{max}) will give us some insights into the stability of the Core Global Banking Network over time. Therefore, λ_{max} can be regarded as a "*systemic risk index*" (SRI). In addition, the eigen-pair method delivers the associated right and left eigenvectors, reflecting rankings for "*systemic importance*" and "*systemic vulnerability*", respectively. This approach is used in Chapter 8.

3.2 Random Matrix Theory (RMT)

Random Matrix Theory (RMT) was originally conceived during the 1950s by Eugene Wigner in the field of mathematical physics. Wigner was seeking to describe the general properties of energy levels of excited states of heavy nuclei as measured in nuclear reactions (Wigner, 1958). Such a complex nuclear system is represented by an Hermitian operator H (called the *Hamiltonian*) living in an infinite-dimensional Hilbert space governed by physical laws. Unfortunately, for every specific case, H is not known and even if it were known, it would be extremely complicated to write down and even if we could write it down, no computer would be able to solve its eigenequation, $Hv = \lambda v$ (the so-called Schrödinger Equation of the physical system), where λ and v are the eigenvalue-eigenvector corresponding to H.

Faced with these constraints, Wigner argued that one should instead regard a specific Hamiltonian H as behaving like a large random matrix that is a member of a large class (or ensemble) of Hamiltonians, all of which would have the similar universal properties as the H in question (Wigner, 1955).[1] Under this setting, the energy levels (represented by the eigenvalues of H) of the physical system could then be approximated by the eigenvalues of a large random matrix.

Since the 1960s, Wigner and his colleagues, including Freeman Dyson and Madan Lal Mehta, have worked on Random Matrix Theory (RMT) and developed it to the point that it became a very powerful tool in mathematical physics (Mehta, 1991). During the last decades, more interest has been paid to RMT. One of the most important discoveries in RMT was its connection to quantum chaos, which led to a Random Matrix Theory of quantum transport. The RMT approach has since become a major tool in many fields, ranging from number theory and combinatorics, wireless communications to multivariate statistical analysis and principal components analysis. A common element shared in these kind of situations is that RMT has been used as an indirect method for solving complicated problems arising from physical or mathematical systems.

Using these insights, Laloux et al. (1999, 2000), Plerou et al. (1999, 2000a,b, 2001, 2002) and Gopikrishnan et al. (2001) have applied the RMT approach to a financial context. In finance, RMT predictions represent an average over all possible interactions between assets. In this context, deviations from the universal predictions of RMT identify system-specific, non-random properties of the system considered, providing clues about the significant interactions.

The understanding of the stability of correlations between different stocks has been an ongoing quest in the last decades, not only because of a better understanding of the economy as a complex dynamical system, but also for practical reasons such as asset allocation and risk forecast. As opposed to most physical systems, where correlations between subunits can be related to basic interactions, in the case of financial markets, the underlying interactions are not known.

In order to quantify the cross-correlations between financial assets, we first calculate the price returns of stocks over a period of time Δt:

$$G_i(t) = lnS_i(t + \Delta t) - lnS_i(t), \tag{3.1}$$

where $S_i(t)$ denotes the price of stock i. As different stocks have different volatility levels, we

[1] One of Wigner's most famous results is the *Semicircle Law*, for a more detailed explanation, see Appendix ??.

proceed to define the normalised returns:

$$g_i = \frac{G_i(t) - < G_i >}{\sigma_i}, \tag{3.2}$$

where $\sigma_i = \sqrt{< G_i^2 > - < G_i >^2}$ is the standard deviation of G_i and $\langle \cdot \rangle$ denotes a time average over the period considered. We then compute the equal-time cross-correlation matrix C with elements:

$$C_{ij} =< g_i(t) g_j(t) > . \tag{3.3}$$

By construction, the elements C_{ij} are bounded to the domain $-1 \leq C_{ij} \leq 1$, where $C_{ij} = 1$ corresponds to perfect correlations, $C_{ij} = -1$ corresponds to perfect anti-correlations, and $C_{ij} = 0$ corresponds to uncorrelated pairs of stocks.

As pointed out by Plerou et al. (1999, 2000a, 2002), when estimating correlation matrices we have the following dilemma: (i) if we use a long time-series of returns to calculate the correlation matrix, we will have that as market conditions change over time, the correlations between any pair of stocks may not be stationary, which lead us to use a short time-series, but (ii) if we use a short time-series, the finite length of the time-series available to estimate correlations introduces "measurement noise".

This poses the following question: How can we identify from C_{ij} those stocks that remained significantly correlated (on average) over the period studied? To answer this question, the statistics of C have to be analysed against the "null hypothesis" of a random matrix, that is, a correlation matrix constructed from mutually uncorrelated time-series. If the properties of C agree with those of a random correlation matrix, then it follows that the contents of the empirically measured C are random. Conversely, deviations of the properties of C from those of a random correlation matrix convey information about "genuine" correlations (Plerou et al., 2002).

3.2.1 The Marčenko-Pastur Distribution (MP)

While in the previous section we defined the way the empirical correlation matrix C is constructed, we turn now to define its random counterpart:

$$R = \frac{1}{T} A A^T, \tag{3.4}$$

where R has the same dimension of C and A is a TxN matrix containing random $i.i.d$ normal returns with zero mean and unit variance. By construction R belongs to the type of matrices often referred to as Wishart Matrices in multivariate statistics. The analysis of Wishart matrices is carried out on an eigenvalue basis, through the following decomposition:

$$R = V \Lambda V^{-1}, \tag{3.5}$$

where the columns of V are represented by each eigenvector and the diagonal matrix Λ contains each eigenvalue associated to each eigenvector. An interesting feature of these random matrices is that their properties are known. In particular, in the limit $N \to \infty$, $T \to \infty$ such that $Q = T/N > 1$ is fixed, the distribution $P_{rm}(\lambda)$ of eigenvalues λ of the random correlation matrix R is given by:

$$P_{rm} = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda}. \tag{3.6}$$

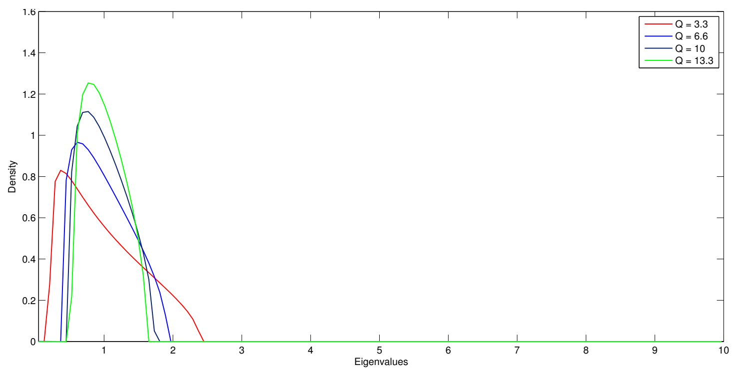


Figure 3.1: The Marčenko-Pastur Distribution for different values of Q. When $T \to \infty$, $Q \to \infty$, the noise band shrinks to zero.

This distribution is known as the Marčenko-Pastur Distribution. In the limiting case, eigenvalues λ are bounded in the interval $\lambda_- \leq \lambda_i \leq \lambda_+$ (minimum and maximum eigenvalues of R) which are given by:

$$\lambda_\pm = 1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}}. \tag{3.7}$$

In Figure 3.1, we can see how the distribution behaves under different values of Q. In particular, by examining Equation 3.7 we can see that in the extreme case where $T \to \infty$ and keeping the number of assets N constant, the gap between the largest and smallest eigenvalues predicted by the RMT shrinks to zero. We then compare the eigenvalue distribution $P_c(\lambda)$ of C with $P_{rm}(\lambda)$. Plerou et al. (1999) and Laloux et al. (1999) find the presence of a well-defined "bulk" of eigenvalues of C that falls within the bounds $\lambda_- \leq \lambda_i \leq \lambda_+$ for $P_{rm}(\lambda)$. This in turn suggests that the contents of C that agree with $P_{rm}(\lambda)$ are mostly random except for the values that deviate. These latter eigenvalues would therefore be the ones that contain economically relevant information.

The distinction between information and noise, with the help of the bounds of the Marčenko-Pastur Distribution in the eigenvalue spectrum, can then be used to "filter" the sample correlation matrix. This "filtered" correlation matrix should provide more stable correlations than the standard sample correlation matrix. In what follows, we review some methods used to filter the correlation matrix. While they all share in common the use of the eigenvalues that deviate from the Marčenko-Pastur Distribution, they differ in the replacing values at the noisy part of the spectrum.

3.3 The RMT-Filtering

There are many methods for filtering the correlation matrix based on the predictions of the RMT. We will focus on the two most popular methods. These methods are basically based on replacing the eigenvalues inside of the spectrum of the sample correlation matrix and at the same time maintaining its trace. For this purpose, the eigenvalues inside of the spectrum are chosen to be the ones that are less or equal than the maximum eigenvalue predicted by the RMT.[2] In both filtering methods, only the deviating eigenvalues in the upper bound are considered. The reason why the smallest deviating eigenvalues are not considered in these filters is that while large eigenvalues are *clearly* separated from the MP bounds, the same does not always applies for the smallest deviating eigenvalues. In general, small eigenvalues can be found outside of the lower edge of the spectrum, which is consistent with the fact that the length of the time-series T and the number of assets N are finite. In addition to this, by simple inspection of the eigenvectors associated to the largest eigenvalues, *clear* non-randomness, and stability over time has been verified, while this has not been verified in the eigenvectors associated to the smallest deviating eigenvalues. In what follows, we discuss these two methods.

3.3.1 Laloux et al. (2000)'s Filter

The filtering method proposed by Laloux et al. (2000) takes the set, $\Lambda = \{\lambda_i\}_{i=1}^{n}$, of eigenvalues of some NxN matrix, C, and the corresponding eigenvectors V and defines a subset

$$\Lambda_{noisy} = \{\lambda \in \Lambda : \lambda \leq \lambda_+\}, \tag{3.8}$$

with the noisy eigenvalues, where λ_+ is the maximum eigenvalue predicted by RMT. Then the filtered eigenvalues are defined as

$$\Lambda_{filtered} = \Lambda_{new} \cup (\Lambda - \Lambda_{noisy}), \tag{3.9}$$

where

$$\Lambda - \Lambda_{noisy} = \{\lambda \in \Lambda : \lambda \notin \Lambda_{noisy}\}, \tag{3.10}$$

are the eigenvalues assumed to contain information and

$$\Lambda_{new} = \{\lambda_i : \lambda_i = \bar{\Lambda}_{noisy} \forall i = 1, ..., n\}, \tag{3.11}$$

[2]That is, the upper bound of the Marčenko-Pastur Distribution, i.e., $\lambda_{\pm} = 1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}}$.

where n is the number of elements in Λ_{noisy} and $\bar{\Lambda}_{noisy}$ is the mean of all elements of Λ_{noisy}. Put it differently, the noisy eigenvalues are all replaced by their means. These filtered eigenvalues $\Lambda_{filtered}$ are then combined, through the eigendecomposition theorem,[3] with the original eigenvectors, V, to construct a filtered matrix

$$C_{filtered} = V\Lambda_{filtered}V^{-1}, \tag{3.12}$$

where $\Lambda_{filtered}$ is a matrix with $\Lambda_{filtered}$ on the main diagonal and zeroes elsewhere. By replacing the noisy eigenvalues by their mean noisy eigenvalue implies that the trace of $C_{filtered}$ is equal to the trace of C.

3.3.2 Plerou et al. (2002)'s Filter

As developed in Plerou et al. (2002), this method is the same as Laloux et al. (2000), but differs in that the noisy eigenvalues are all replaced by zeroes. Then, they build the *filtered* correlation matrix $C_{filtered}$, by setting its main diagonal elements equal to 1 to preserve the trace of the original matrix C to prevent system distortion.

3.4 Methodology for Portfolio Analysis

Having defined the different RMT-filtering methods, we use the filtered and unfiltered correlation estimators as inputs in our portfolio optimisation framework. For this, we first make a very strong assumption on individual return volatilities, that is, we assume that these are known as we are mainly focused on correlations, which are the main purpose of this thesis. In this context, individual volatilities are calculated over the investment horizon. Therefore, any source of variation in our estimates comes strictly from the improvements in correlations. In addition, we focus on *minimum variance portfolios*. As pointed out by Rosenow (2008), the volatility prediction for *minimum variance* portfolios is a more powerful test for assessing the performance of the RMT-filtering, as it not only gauges the prediction of the average covariance, but of the whole covariance structure: the covariance matrix is first used to calculate the portfolio weights,

[3] Let C be a square matrix and let V be a matrix of its eigenvectors. If V is a square matrix then $C = V\Lambda V^{-1}$ where Λ is a diagonal matrix containing the associated eigenvalues on the main diagonal.

and then used to estimate the variance of that portfolio. Another advantage of estimating *minimum variance portfolios* is that their weights only depend on the return variances and covariances, but not on the expected returns. This means that we can assess the use of the RMT approach in filtering noise due to measurement error isolated from any other additional source of noise that could arise in case we had to perform a forecast of returns.

Put it differently, on each test date, we estimate the minimum variance portfolio by choosing the weights that minimise the total expected risk. The predicted portfolio volatility at time t is given by:

$$\Omega^2_{t,predicted} = \sum_{i,j=1}^{N} w_{i,t} w_{j,t} \Sigma_{ij,t}, \tag{3.13}$$

where, $w_{i,t}$ are the weights of capital invested in stock i at time t. Regarding the minimum variance portfolio weights:

$$w_{i,t} = \frac{\sum_{j=1}^{N} \Sigma^{-1}_{ij,t}}{\sum_{k,l=1}^{N} \Sigma^{-1}_{k,l}}, \tag{3.14}$$

which minimises the variance under the constraint that the total invested money is equal to one. Regarding our measure of realised risk, we define:

$$\Omega^2_{t,realised} = \sum_{i,j=1}^{N} w_{i,t} w_{j,t} (< g_{i,t} g_{i,t} > - < g_{i,t} >< g_{j,t} >), \tag{3.15}$$

where the "$< \cdot >$", represents the average of the time-series returns over the investment horizon, which consist in the ex-post volatility over the evaluation period (i.e. an investment horizon $t^f = 20$ days, 1-month). As pointed out by Pafka and Kondor (2003, 2004), this realised measure of risk can be considered a good proxy for the true portfolio risk, which is always unknown.

3.4.1 In-sample Methodology

Following Pafka et al. (2004) and Daly et al. (2008), the test methodology for evaluating the in-sample performance of the RMT-filtering is a bootstrap approach. This analysis consists in taking bootstrapped samples, together with the mean across these samples. That is, for a given value of N (in this case, $N = 77$), we randomly select a test date t. Everything up to t is used for estimation and everything afterwards as realised, future information. We repeat this random selection 1000 times, with replacement, and calculate the mean, across all bootstrapped samples, of the realised forecast of minimum variance portfolios, calculated using the forecasted covariance matrix. The forecasted matrices consist on the *filtered* and the standard sample correlation matrices. We measure the performance of the RMT-filtering by calculating the ratio γ, which is the ratio between the *realised* portfolio risk and the *predicted* portfolio risk, using the *filtered* and sample correlation matrices. More formally,

$$\gamma = \frac{\Omega_{t,realised}}{\Omega_{t,predicted}} = \sqrt{\frac{\sum_{i,j=1}^{N} w_{i,t} w_{j,t} \Sigma_{ij,t}^{realised}}{\sum_{i,j=1}^{N} w_{i,t} w_{j,t} \Sigma_{ij,t}^{predicted}}}. \tag{3.16}$$

This measure will give us some insights into how close our predictions are from the realised ones. This ratio will be calculated as an average over many test dates across 1000 bootstrapped samples. This method is in contrast to the one used by Laloux et al. (1999, 2000) and Plerou et al. (2000a, 2001); Gopikrishnan et al. (2001); Rosenow et al. (2002); Plerou et al. (1999) in that in these studies use a single test date in the middle of the sample. In addition, these studies were focused on measuring the performance of the RMT-filtering for all the portfolios along the efficient frontier, rather than on *minimum variance portfolios.*

3.4.2 Out-of-sample Methodology

As we mentioned in Equation 3.7 of Section 3.2.1, the bounds of the Marčenko-Pastur Distribution are highly dependent on $Q = T/N$, which is the ratio between the length of the time-series and the total number of assets. We also argued that for a fixed N when T increases, the noise band shrinks to zero. This means that realised risk estimates are highly dependent on the ratio Q, with higher values of Q being less noisy than low values. However, as we mentioned in Section 3.2, the use of longer time-series is problematic, as these are nonstationary, which calls for the use of shorter time-series that are afflicted by measurement noise and this needs to be cleaned.

Our out-of-sample methodology consists in using the optimal value of Q obtained in our in-sample results, which for a given number of assets N, is mainly dependent on the length of the time-series T and use this optimal T to estimate our minimum variance portfolios.

As opposed to the in-sample testing where we use 1000 randomly sample dates, in the out-of-sample testing we are assessing each available date only once. Using these insights, we forward validate as follow: For a given test date t, for a given filtering method f, we use the estimation window T and estimate the ratio γ associated with this forecasting method. It is worthwhile noting that for this analysis we only consider portfolios invested before $t - t_F$, or on that date, where t_F stands for the investment period (20 days). This happens, because on day t, we do not have knowledge of investments made after $t - t_F$, since conceptually they are still invested.

Keeping this in mind, we assess the out-of-sample performance of the different correlation estimators, by comparing the average ratio γ, introduced in the in-sample section. This ratio will provide us with some insights into how close the *predicted* risk is to the *realised* risk for the different correlation estimators. We also assess the performance of the different strategies in their ability to produce portfolios with the lowest average *realised risk*. In addition, we calculate the Mean Squared Error (MSE) as a gauge for the volatility of our risk forecast.[4] More formally, these performance measures are defined as follows:

$$\gamma = \frac{\Omega_{t,realised}}{\Omega_{t,predicted}} = \sqrt{\frac{\sum_{i,j=1}^{N} w_{i,t} w_{j,t} \Sigma_{ij,t}^{realised}}{\sum_{i,j=1}^{N} w_{i,t} w_{j,t} \Sigma_{ij,t}^{predicted}}}, \tag{3.17}$$

$$MSE = \sum_{t=1}^{T} \frac{(\Omega_{t,real} - \Omega_{t,predicted})^2}{T}. \tag{3.18}$$

As opposed to most studies in the RMT literature, we are the first in analysing the implications of using the RMT-filtering on portfolio returns. For doing this, we calculate the cumulated wealth at the end of the out-of-sample period, which is defined as the compounded returns constructed for the different strategies (i.e. *weights*) using the *realised* returns over the investment horizon (Equation 3.19). We also judge the out-of-sample performance of our portfolios by computing risk-adjusted measures of returns, such as the *Sharpe Ratio* (Equation 3.20), *Sortino Ratio* (Equation 3.21) and *Omega Ratio* (Equation 3.22). These performance metrics are defined

[4]For the purpose of the thesis, the MSE is used to gauge the volatility of the volatility forecast. However, throughout the thesis, we'd rather use the term "risk" rather than volatility, this is why, we refer to the volatility of the risk forecast.

as follows:

$$\Pi_{t=1}^{T}(1+r_t) \tag{3.19}$$

$$SP = \frac{E(r_t - r_t^f)}{Var(r_t - r_t^f)}, \tag{3.20}$$

$$Sortino = \frac{R_p - MAR}{\sqrt{\frac{1}{T}\sum_{t=0,R_P<MAR}^{T}(R_{P,t} - MAR)^2}} \tag{3.21}$$

$$\Omega(L) = \frac{\int_L^b (1 - F(x))dx}{\int_a^L F(x)dx} \tag{3.22}$$

The *Sharpe Ratio* (Equation 3.20) is calculated as the ratio between expected excess returns of a portfolio and its standard deviation.[5] This ratio will provides us with insights into how well the returns for the different strategies compensate investors for the risk taken. The *Sortino Ratio* is an enhanced version of the Sharpe Ratio (Equation 3.21), which only takes into account downside volatility by separating volatility in the upside from volatility in the downside by setting a Minimum Acceptable Return (MAR), which in this case we set at 0.5%.[6]

In Equation 3.22, the Omega ratio as defined by Shadwick and Keating (2002) is a non-parametric method, since it does not rely upon calculating the moments of a distribution of returns for determining the ranking of each hedge fund versus a threshold level. The ratio considers returns below and above a specific loss threshold return level. By doing this, it places a threshold return on the unit distribution of returns (the return PDF with an area of one underneath) which have been accumulated by summing (taking the integral) from the left hand side to form the CDF, $F(x)$. The returns below a certain threshold are considered as losses, and

[5] Excess returns are defined as the difference between the expected portfolio returns and the risk-free asset, which in this case is the 1-month Libor.

[6] One of the main disadvantages of using the Sharpe Ratio is that it uses volatility as a measure of risk for the entire time-series returns. Consequently, both large upswing and large downswing values are penalised and they translate into higher volatility and lower Sharpe Ratio. The idea behind this measure is that investors do not want their upside risk to be penalised. In this context, by setting a Minimum Acceptable Return (MAR) we split returns into two categories, i.e. those returns greater or equal than MAR (upside deviations) from those returns less than MAR (downside deviations). Thus, the Sortino Ratio only uses a measure of downside returns in the calculation of its denominator, which means that the higher the Sortino Ratio is, the better the manager is at controlling downside returns while not being penalised by upside returns (Sortino and Price, 1994).

the returns above this threshold are considered as profits. The Omega ratio is non-parametric and reflects all moments of the distribution so a fund with a high excess kurtosis will be ranked lower than one with no excess kurtosis assuming both funds have the same return, volatility and skewness.

Finally, following Pantaleo et al. (2011) we assess the performance of our portfolios by measuring their degree of diversification across stocks in the portfolio. This will allow us to disentangle the differences between the different strategies from an asset allocation perspective. Formally, these measures are defined as follows:

$$N_{eff} = \frac{1}{\sum_i^N w_i^2}, \tag{3.23}$$

$$N_p = \arg\min_l \sum_{i=1}^{l} \geq p \sum_{i=1}^{N} |w_i|, \tag{3.24}$$

$$w^-/w^+ = Mean\left(\frac{\sum_i^N |w_i^-|}{\sum_i^N w_i^+}\right), \tag{3.25}$$

Equation 3.23 stands for effective portfolio diversification, where N_{eff} is equal to 1 when all wealth is invested in one stock, whereas it is N when the wealth is equally divided among stocks, i.e., $w_i = 1/N$. When *short-sale* is not permitted, N_{eff} has a clear meaning. In contrast, when *short-sale* is permitted there might be some ambiguity in the interpretation. This is why in Equation 3.24, we consider the absolute value of the weights and we compute the smallest number of stocks for which the sum of absolute weights is larger than a given percentage p of the sum of the absolute value of all weights. For our analysis, we consider $p = 0.9$, thus N_{90} will be the minimum number of stocks in the portfolio, such that their absolute weight cumulate to 90% of the total of asset absolute weights. We are also interested in how extreme portfolio weights are when *short-sale* is permitted. For doing this, we analyse the ratio between the absolute sum of positive and negative weights, which is defined in Equation 3.25.

3.4.3 Summary of Performance Measures

It is worthwhile noting that for the In-sample analysis of Section 3.4.1, we are mainly interested in the ratio γ, while in the Out-of-sample analysis, we provide a more complete battery of tests to assess the performance of our correlation estimators in a portfolio context. This does not mean, however, that we use the same performance metrics for all the correlation estimators used in this thesis. For example, measures of portfolio diversification are more relevant to assess the performance of portfolios using the sample correlation matrix and its RMT-filtered version in Chapter 5 than for assessing the performance of the regime-switching correlation matrix and its RMT-filtered version in Chapter 6. For this latter, we focus on the ability of the correlation estimators to deal with downside volatility, hence we use the Sortino and Omega ratios. In all chapters, we test the ability of the RMT-filtered correlation estimators to produce (i) portfolios with the *lowest* realised risk and (ii) to correctly predict realised risk. In Table 3.1 we provide a summary for the performance measures described above.

Out-of-Sample Metrics	Chapter 5	Chapter 6	Chapter 7
γ	Section 5.5.1	Section 6.6.1	Section 7.3.3
MSE	Section 5.5.1	Section 6.6.1	Section 7.3.3
$\Omega_{realised}$	Section 5.5.1	Section 6.6.1	Section 7.3.3
$\Omega_{predicted}$	Section 5.5.1	Section 6.6.1	Section 7.3.3
N_{eff}	Section 5.5.3	-	-
N_p	Section 5.5.3	-	-
w^-/w^+	Section 5.5.3	-	-
$\Pi_{t=1}^{T}(1+r_t)$	Section 5.5.2	Section 6.6.2	-
Sharpe Ratio	Section 5.5.2	Section 6.6.2	-
Sortino Ratio	-	Section 6.6.2	-
Omega Ratio	-	Section 6.6.2	-

Table 3.1: Summary of performance metrics used across the chapters.

3.5 The Eigen-Pair Method for Network Stability

Before introducing the approach to assess the stability of financial networks, it is worthwhile mentioning some of the challenges faced in the financial network literature. As pointed out by Castrén and Rancan (2014), despite the evident usefulness of network tools in modelling interconnections, the financial applications are still very limited. A reason for this, is that network representation requires very granular data on counterparty exposures, which is rarely available, at least from public sources. In order to deal with this shortcoming, previous research has often been based on estimated linkages. Facing these constraints, most of the pre 2007 literature simulated financial network models assuming that they were random graphs or used the Entropy method for network formation which aims to maximize the homogeneity of financial flows between FI and its counterparties. Many have discussed (Mistrulli, 2011; Cont et al., 2013) why networks produced by the entropy method or as random graphs are not suited to characterize real world financial networks. In a recent paper Solorzano-Margain et al. (2013) based on the extensive bilateral data on liablities and exposures of FIs in the mexican financial system, financial contagion arising from the unexpected failure of an FI on others s found to be more widespread than from results obtained from calibrated financial network models mostly based on maximum entropy algorithm surveyed in Upper (2011). This means that in order to avoid model risk from calibration algorithms, structural bilateral balance sheet and off-balance sheet data based network models are needed to study systemic risk from financial interconnections. In this context, we limit our analysis to *observed* cross-border exposures and ignore *within* country interlinkages, which are not observable and would be subject to mismeasurement in case of being estimated by the methods mentioned above.

We introduce the eigen-pair method of Markose (2012, 2013) based on the spectral analysis of cross-border exposures of national banking systems relative to their equity capital. In contrast to a portfolio generalisation of risk as in the case of market price-based systemic risk indices (henceforth, SRIs), the spectral method involves the power iteration algorithm for the matrix representing bilateral exposures, which characterises the long range impact of individual node failure on counterparties of counterparties in the chain of indebtedness.

The main focus here is on the *Core of the Global Banking System Network* (CGBSN) (18 BIS reporting countries). We first define a network by two sets: a set of N nodes (vertices), E and a set of links between the nodes (edges).

The CGBS network is a directed weighted network, that is, there is only one non-reciprocal link, where the direction of a link represents obligations that all sectors of a country at the start of an arrow have towards the banking system of another country at the end of the arrow. The links are weighted by the amount due. In this context, the weighted matrix of the CGBS network is defined as:

$$X = \begin{bmatrix} 0 & x_{12} & \cdots & x_{1j} & \cdots & x_{1N} \\ x_{21} & 0 & \cdots & x_{2j} & \cdots & x_{2N} \\ \vdots & \vdots & 0 & \cdots & \cdots & \cdots \\ x_{i1} & \vdots & \cdots & 0 & \cdots & x_{iN} \\ \vdots & \vdots & \cdots & \cdots & 0 & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{Nj} & \cdots & 0 \end{bmatrix}. \tag{3.26}$$

Here X is an NxN matrix, and x_{ij} are the gross liabilities of country i to banking system of country j. Likewise, the transpose of x_{ij}, i.e., x_{ji}, denotes the gross liabilities of country j to banking system of country i. Notice that $x_{ii} = 0$. The sum of an i-th row $\sum_{j=1}^{N} x_{ij}$ represents the total payable amount of a country i to all j countries' banking systems. The sum of an j-th column $\sum_{i=1}^{N} x_{ij}$ represents the total receivable amount of the banking system of a country j.

The causal direction of contagion comes from the default of a net debtor country i to the net creditor national banking system j. Specifically, this occurs if country i owes more to the banking system of country j, than what country j owes to the banking system of country i relative to j's equity capital. The default of country i will have an impact on the solvency of country j's banking system and on its non-banking sectors. This can be expressed as $\theta_{ij} = \frac{(x_{ij}-x_{ji})^{+}}{C_{j0}}$, where the numerator takes only positive values and is equal to zero if the net exposure is negative $(x_{ij} - x_{ji}) < 0$. The denominator C_{j0}, is initial capital of country j. This matrix Θ is referred to as the stability matrix:

$$\Theta = \begin{bmatrix} 0 & \frac{(x_{12}-x_{21})^{+}}{C_{20}} & \cdots & 0 & \cdots & \frac{(x_{1N}-x_{N1})^{+}}{C_{N0}} \\ 0 & 0 & \cdots & \frac{(x_{2j}-x_{j2})^{+}}{C_{j0}} & \cdots & \frac{(x_{2N}-x_{N2})^{+}}{C_{N0}} \\ \vdots & \vdots & 0 & \cdots & \cdots & \cdots \\ \frac{(x_{i1}-x_{1i})^{+}}{C_{10}} & \vdots & \cdots & 0 & \cdots & \frac{(x_{iN}-x_{Ni})^{+}}{C_{N0}} \\ \vdots & \vdots & \cdots & \cdots & 0 & \vdots \\ 0 & \frac{(x_{N2}-x_{2N})^{+}}{C_{20}} & \cdots & \frac{(x_{Nj}-x_{jN})^{+}}{C_{j0}} & \cdots & 0 \end{bmatrix}, \tag{3.27}$$

Here, we will discuss the importance of matrix Θ in describing the dynamics of the cascade failure of a trigger debtor country on the rest of the system. In the first instance, a failure of a net debtor country or a subset thereof, can lead to failures in the national banking systems that are directly exposed to it. Unlike Degryse et al. (2010), we will focus on the contagion propagated solely from the failure of national banking systems on their liabilities to other national banking systems. For this the Θ matrix will be scaled each quarter by the proportion of cross-border banking system liabilities to total cross-border liabilities of both banking and non-banking sector liabilities. The rationale behind the scaling is that the BIS Consolidated Banking Statistics does not report interbank data, but exposures of national banking systems to debtor countries. This is why we need to apply this transformation to approximate interbank data. As a fixed scalar is used to scale the Θ matrix at each quarter, this will directly scale the maximum eigenvalue that we will produce as the SRI for the cross-border banking financial network. Hence, in what follows the notation for the Θ matrix will remain unchanged. The overall dynamics for the rate of failure of banking system of country i at $q+1$, is given by:

$$u_{i,q+1} = (1-\rho)u_{i,q} + \sum_j \frac{(x_{j,i} - x_{i,j})^+}{C_{i0}} u_{j,q} = (1-\rho)u_{i,q} + \sum_j \theta_{j,i} u_{j,q}, \tag{3.28}$$

$$= (1-\rho)\left(1 - \frac{C_{i,q}}{C_{i,0}}\right) + \sum_j \theta_{j,i} u_{j,q}. \tag{3.29}$$

Here, in the first term of Equation 3.28, ρ can be seen as the capital threshold for banking system losses and it can be considered as the rate of cure in the epidemic literature. Hence, $(1-\rho)$ provides the local rate of failure of a banking system stemming from a lack of its own capital buffer. The failure rate depends on $u_{iN} = 1 - C_{i,q}/C_{i,0}$, for $q > 0$. Here, $C_{i,q}$ is the remaining capital at q relative to initial capital at $q = 0$. The second term, $\sum_j \theta_{j,i} u_{j,q}$, adds up to the infection rates sustained from counterparties weighted net liabilities.[7] In matrix notation,

[7] We will briefly compare the spectral method for contagion with the popular Furfine (2003) contagion algorithm. In the latter, it is assumed that a country (originally a financial institution) can spread contagion only if the losses it faces due to the failure of counterparties exceeds a given threshold of its capital buffer. In Equation 3.28, the latter condition of failure implies that $u_{j,q} = 1$, and otherwise, $u_{j,q} = 0$ for i's counterparties and if i is deemed to have failed at $q+1$, $u_{iq+1,j} = 1$ and i will trigger a wave of contagion at $q+1$. In the spectral method, there is no threshold condition for failure and contagion losses are propagated at the rate of $u_{i,q} > 0$.

the dynamics of failure of national baking systems is given by:

$$U_{q+1} = [\Theta' + (1-\rho)I]\, U_q, \tag{3.30}$$

where Θ' is the transpose of matrix in Equation 8.1 with each element $\theta'_{ij} = \theta_{ji}$ and I is the identity matrix. The system stability of Equation 3.30 is evaluated on the basis of the power iteration of the initial matrix $Q = [\Theta' + (1-\rho)I]$, where U_q takes the form:

$$U_q = [\Theta' + (1-\rho)I]^q\, U_0 = Q^q U_0, \tag{3.31}$$

where U_0 with elements $(u_{10}, u_{20}, ..., u_{N0})$ gives the initial failure rate of a national banking system. For instance, when $U_0 = (u_{10}, u_{20}, ..., u_{N0}) = (1, u_{20}, ..., u_{N0})$, this means that the trigger banking system that fails at initial date 0 is banking system 1, and the non-failed national banking systems have a zero but low initial failure rate of $u_{i0} = 1/C_{i0}$. Moving forward, the ratio of capital at q to initial capital, C_{iq}/C_{i0}, gives the probability of survival for a non-failed banking system and hence at the $q+1$th iteration, the rate of failure of a non-failed bank is given by $u_{iq+1} = (1 - C_{iq+1}/C_{i0})$. Based on the work of May (1974, 1972), Markose (2012) shows that the stability of the network system is determined by the maximum eigenvalue of the initial matrix $Q = [\Theta' + (1-\rho)I]$ when it satisfies the following conditions:

$$\lambda_{max}(Q) = (1-\rho) + \lambda_{max}(\Theta) < 1, \tag{3.32}$$

Hence,

$$\lambda_{max}(\Theta) < \rho. \tag{3.33}$$

Here ρ is the homogeneous capital loss threshold. The threshold ρ can be regarded as a percentage of banking system equity capital to buffer losses. In order to capture the effects of failure of net debtor banking systems on a net creditor banking systems, λ_{max} is rescaled by the proportion of bank-to-bank cross-border flows over the total amount of cross-border flows (i.e., bank-to-bank plus bank-to-nonbank), as reported by Bruno and Shin (2015a).[8] If this condition

[8] In Appendix C.4, we derive an absolute total equity capital loss threshold T_C which is equivalent to the

is violated, then any negative shock, in the absence of outside interventions, can propagate through the networked system as a whole and potentially cause capital losses proportionate to $\lambda_{max}(\Theta)$. The specific estimates of capital losses, when the stability condition in Equation 8.2 is violated depends on the left eigenvectors. Further, note $\lambda_{max}(\Theta)$ where Θ has been scaled by a scalar for each quarter to reflect the proportion of cross border banking sector liabilities in the BIS data to the total of all sectors, is a simple linear adjustment. The matrix Θ has two sets of eigenvectors: right (V^R) and left (V^L) eigenvector. The right eigenvector V^R gives the rank order of systemic importance, while the left eigenvector V^L gives the rank order of the countries that are vulnerable. The right eigenvector centrality assigns a relative centrality score to all nodes in the network based on the principle that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes. This means that the importance of a node is not ranked based on the size of the exposure, but instead based on how important a banking system's neighbours are. In other words, a node is important because is connected to other important nodes. Likewise, the left eigenvector centrality, which is a reflection of the right eigenvector centrality, provides the impact of the exposures of each banking system to others. Under the same principle, a node is vulnerable because is connected to highly vulnerable nodes. Since the i-th node's centrality is proportional to the inverse of the maximum eigenvalue λ_{max}^{-1} and the centrality measure of all its neighbours (Newman, 2010), then denoting the right eigenvector centrality of i-th node as the i-th element of the right eigenvector (V_i^R), we define:

$$V_i^R = \lambda_{max}(\Theta)^{-1} \sum_{j=1}^{N} \theta_{ij} V_j^R, \tag{3.34}$$

Using matrix notation, we obtain the eigenvalue equation:

$$\Theta V^R = \lambda_{max}(\Theta) V^R. \tag{3.35}$$

Similarly, for $\lambda_{max}(\Theta)$, there is another associated eigenvector V^L, which by definition of the

empirical average total equity capital to risk weighted assets of the cross border banks for all the 18 countries when the regulatory ratio for this is 10%.

eigenvalue equation we get:

$$V^L\Theta = \Theta' V^L = \lambda_{max}(\Theta)V^L. \tag{3.36}$$

The matrix Θ is a non-negative matrix with real entries, hence $\lambda_{max}(\Theta)$ is a real positive number. The Perron-Frobenius theorem guarantees non-negative eigenvectors V^R and V^L under the assumption that Θ represents the irreducible network.[9] Using the power iteration algorithm in Equation 3.31, it can be shown that in the steady state, the potential capital loss for banking system of country i can be estimated as the product of $\lambda_{max}(\Theta)$ and country i's left eigenvector centrality while using the infinity norm, $v^L_{i,\infty}$:

$$u_{i,\#} \cong \lambda_{max}(\Theta')v^L_{i,\infty}. \tag{3.37}$$

Note that when using the infinity norm, the left eigenvector centrality of the most vulnerable country is equal to 1.[10] This implies that the maximum percentage potential capital loss of the most vulnerable country is given by $\lambda_{max}(\Theta)$. Likewise, the most systemically important country can potentially bring about $\lambda_{max}(\Theta)\%$ of equity capital for the system as a whole. The proof of Equation 4.1 can be found in Appendix C.1.

[9] For any randomly selected pairs of nodes (i, j) in an irreducible network, there is a path between them, viz. Θ is strongly connected.

[10] For this analysis, it is important to make sure that the right and left eigenvectors associated with the largest eigenvalue are given using the infinity norm. The infinity norm of a vector x denoted as, $||x||_\infty$, is the largest number in the vector. Hence, the highest ranked country, in terms of its eigenvector centrality an index of 1. There is a simple conversion from the eigenvector produced using the Euclidean norm to one using the infinity norm (see, Markose, 2012).

Chapter 4

Eigenvalue Analysis for the FTSE 100

4.1 Introduction

As we described in Section 3.2, the RMT approach allows us to identify the optimal number of eigenvalues in the sample correlation matrix. In general, the leading eigenvalues are the ones expected to carry most of the economically relevant information about market correlations and therefore, these can be used for forecasting purposes by using the filters defined in Section 3.3.

In addition to this, an eigenvalue analysis also has the property to deliver some very interesting results about market behaviour. For example, Plerou et al. (2001) show that the time evolution of the largest eigenvalue reflects the degree of collective behaviour of stocks. Song et al. (2011) finds that the time evolution of the largest eigenvalue is related to the average correlation coefficient of the sample correlation matrix. They also find that the time evolution of the second largest eigenvalue and its associated eigenvector was closely related to certain market events, such as financial crises and currency crashes. The analysis of the information content of eigenvalues can also be used to identify the presence of clusters characterised by economic sectors of stocks with similar correlation structures (see Coronnello et al., 2005).

Using these insights, we provide an eigenvalue analysis for the FTSE 100. For doing this, in Section 4.2 we first investigate the spectral properties of the sample correlation matrix and contrast it with those of a random matrix. We also provide a financial interpretation of the deviating eigenvalues and their associated eigenvectors. In Section 4.3 we introduce the notion of *Inverse Participation Ratio*, which allows us to identify the number of significant stocks that belong to each eigenvector. In Section 4.4 we analyse the time evolution of the largest eigenvalue and eigenvectors and show how sensitive market correlations are to common shocks arising in financial markets. Finally, in Section 4.5 we show how the behaviour of eigenvalues can be useful to illustrate some stylised facts of financial correlations, i.e., asymmetries and participation of financial assets.

4.2 A Spectral Analysis: The RMT Approach in Practice

Here we provide a spectral analysis of the sample correlation matrix for 78 stocks contained in the FTSE 100, using daily data for the year 2007.[1] We therefore have $T = 260$ and $N = 78$, which implies $Q = T/N = 3.33$.

We then construct the correlation matrix C^{2007} based on the average time-series returns and normalised variance equal to 1. Using Equation 3.7, we can obtain the upper and lower bounds of the theoretical Marčenko-Pastur (MP) distribution. In this case, we have $\lambda_+ = 2.395$ and $\lambda_- = 0.2045$.

Similarly, we generate a Wishart correlation matrix R composed by cross-correlations of mutually independent returns (bounded by the aforementioned edges).[2] We then plot the Marčenko-Pastur (henceforth, MP) density of the eigenvalues for 10000 trials and compare it with the eigenvalue density of the sample correlation matrix C^{2007}. The results of this exercise are presented in Figure 4.1.

As expected, the "bulk" of the eigenvalues lies within the bounds of the MP distribution. However, there are obvious deviations from the RMT predictions. The largest eigenvalue is well above the maximum predicted by the RMT predictions ($\lambda_+ = 2.395$). This value as well

[1] Throughout this section, we will be using these 78 stocks for our estimation period and out-of-sample forecast for the years 2000-2012. The selection of these stocks was based on two criteria: (i) their availability over the entire sample, which allows to make reliable comparisons and (ii) were the most frequently traded.

[2] $R = \frac{A'A}{T}$, where A is a TxN matrix of $i.i.d$ normal returns.

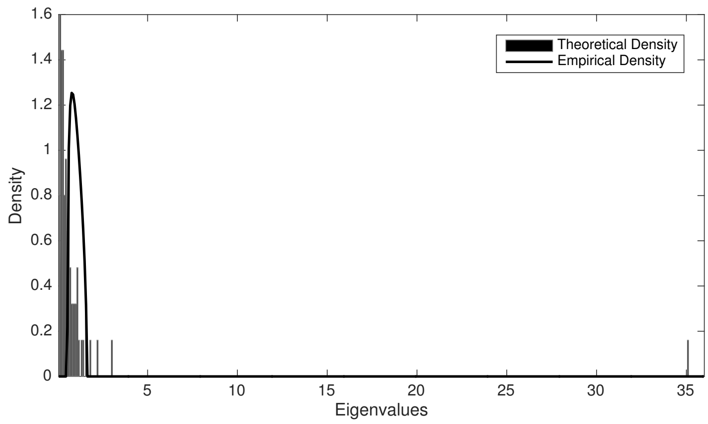


Figure 4.1: The Marčenko-Pastur Distribution vs. The Sample Eigenvalue Density of C^{2007}. We can see two clear deviations from the upper edge of the spectrum.

as the remaining deviating eigenvalues, are the ones we expect to carry economically relevant information, while those within the distribution bounds can be considered just as noise.

For the year 2007, there are two large deviations from the upper bound predicted by the RMT, $\lambda_1 = 35.09$ and $\lambda_2 = 3.043$. We also find two clear deviations in the lower bound, $\lambda_{78} = 0.0247$ and $\lambda_{77} = 0.0562$. From a portfolio management perspective, the list of eigenvalues can be seen as different levels of risk of a certain portfolio. Likewise, the eigenvector components associated with each eigenvalue can be regarded as the weights of different stocks in a certain portfolio, where some stocks are *long* while others are *short* (Plerou et al., 2002). In order to better clarify this point, in Figure 4.2, we plot the eigenvector components corresponding to the two largest eigenvalues.

In Figure 4.2(a), we observe that the eigenvector components associated with the largest eigenvalue are all positive and contribute in a similar way to the eigenvectors. This suggests a strong collective behaviour of stocks. Typically, it has been found in the literature that the largest eigenvalue, λ_1, represents the risk of the market portfolio. Similarly, the eigenvector components associated with the second largest eigenvalue (panel(b)) have been found to be represented by companies with the largest market capitalisations (Gopikrishnan et al., 2001).

In stock markets, the largest eigenvalue corresponds to the risk of a portfolio whose weights are the same for each asset. There is no neutrality in this portfolio: the only bet is whether the

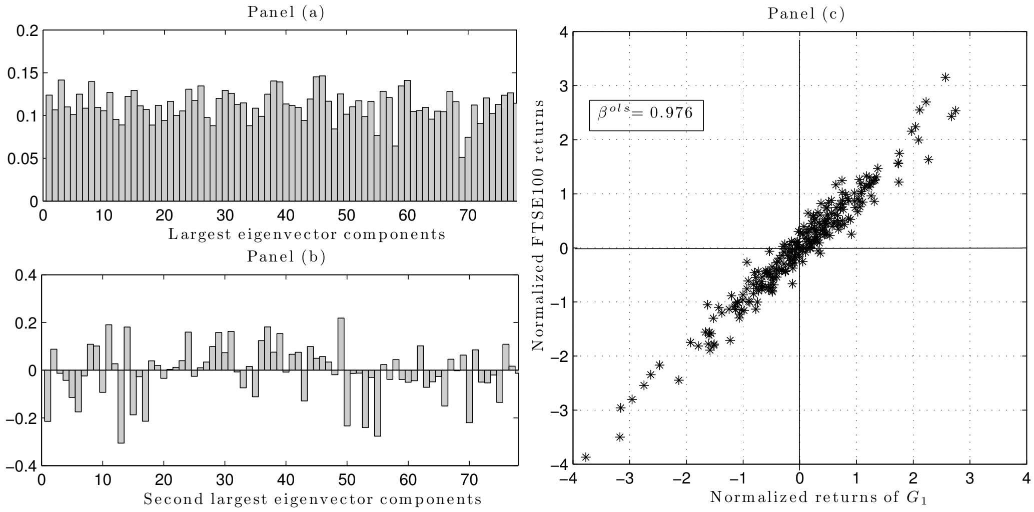


Figure 4.2: Eigenvector Components of the Two Largest Eigenvalues: Year 2007.

market as a whole goes up or down, this is why the risk is large. Conversely, if two stocks move very tightly together (e.g., Coca-Cola and Pepsi), then buying one and selling the other leads to a portfolio that hardly moves, being only sensitive to events that strongly differentiate the two companies (Bouchaud and Potters, 2009).

For example, if we construct a time-series returns using the weights contained in the largest eigenvector u^1, using the following expression:

$$G_t^1 = \sum_{j=1}^{n} u_j^1 G_j(t). \tag{4.1}$$

This basically defines the returns on the portfolio defined by u^1.[3] We then proceed to normalise these returns and run an OLS regression against the normalised returns of the market index G_t^{FTSE}.

We plot the results of this regression in Figure 4.2(c). We can see that there is a clear linear and positive relationship between the returns G_t^1 and $G_t^{FTSE100}$, with a strong partial correlation coefficient $< G_t^{FTSE100}, G_t^1 >= 0.9761$. From this, we can therefore conclude that the largest eigenvalue corresponds to the risk of the market mode. We repeat the same exercise for the

[3]In multivariate statistical analysis, equation 4.1 defines the first principal component.

smallest eigenvector, u^{78}. We plot the correlation coefficients in Figure 4.3(c).

From a portfolio perspective, the smallest eigenvector determines the least risky portfolio one can build. This can be corroborated by the nearly zero beta coefficient of the regression against the market index. This implies that the portfolio constructed from the smallest eigenvalues is almost totally immune to the systematic risk emanating from the collective behaviour of stocks represented by the largest eigenvalue.

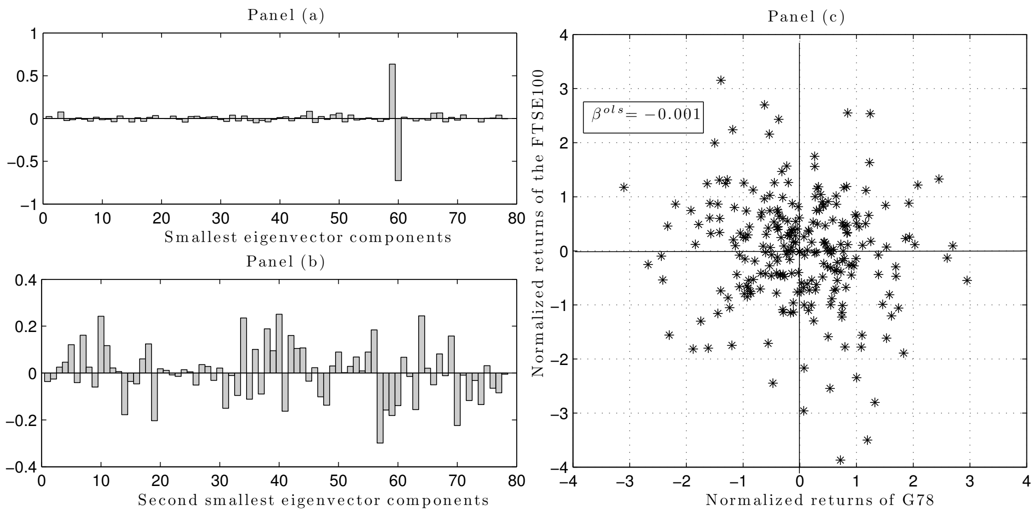


Figure 4.3: Eigenvector Components of the Two Smallest Eigenvalues: Year 2007.

Figure 4.3(a,b) plots the eigenvector components corresponding to the smallest deviating eigenvalues, λ_{78} and λ_{77}. Here we can see that the eigenvector corresponding to the smallest eigenvalue exhibits clear preference for some pairs of stocks with the highest correlations. Finally, we plot the eigenvector components of two eigenvalues inside the bounds of the MP distribution in Figure **??**.

As we mentioned above, the eigenvalues inside of the bulk of the MP distribution are assumed to be random and therefore should not carry economically relevant information. This appears to be the case, as the eigenvector components in both cases, do not seem to show clear preferences for any stocks. Overall, our results are in line with the findings of most of the existing literature on the subject (Laloux et al., 1999, 2000; Plerou et al., 1999, 2000a,b; Gopikrishnan et al., 2001; Plerou et al., 2001, 2002; Bouchaud and Potters, 2009). In the next section, we look deeper into the significant eigenvalues and eigenvectors to identify the individual stocks that participate in

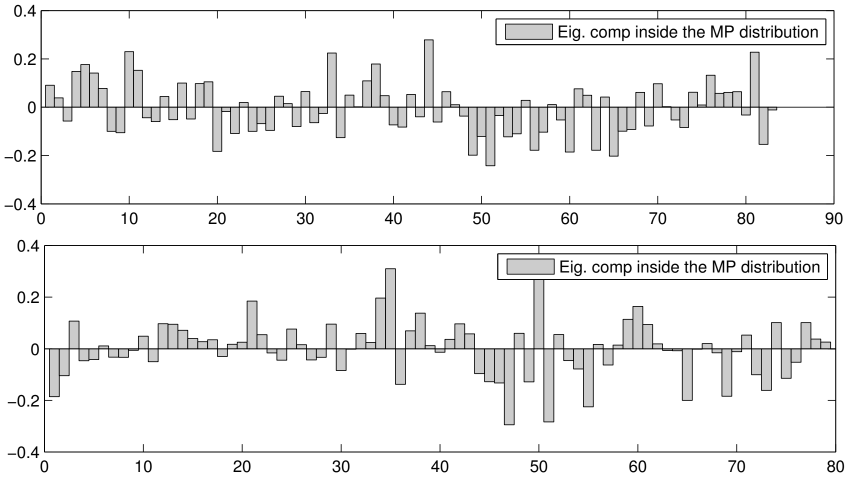


Figure 4.4: Eigenvector Components Associated with Two Noisy Eigenvalues: Year 2007. Their components do not exhibit a clear preference for some stocks over others.

each of them.

4.3 The Number of Significant Components in the Deviating Eigenvectors

Since the eigenvalues that are close to the bounds of the MP distribution are more likely to be noisy, we quantify the number of components that participate significantly in each eigenvector. For doing this, we use the notion of *inverse participation ratio* (IPR), often applied in localisation theory (Guhr et al., 1998).[4] The IPR of eigenvector u^k is defined as

$$I^k = \sum_{l=1}^{N} [u_l^k]^4, \tag{4.2}$$

where u_l^k, $l = 1, .., 78$ are the components of eigenvector u^k. The meaning of I^k can be illus-

[4]In physics, the *Inverse Participation Ratio* (IPR) is a simple way to quantify how many states (eigenvectors) a particle (company) is distributed over when there is some intrisic uncertainty about where the particle is located. In quantum mechanics, the source of uncertainty arises because a particle can exist in a quantum superposition of states over many locations. In this context, the IPR provides a sensitive count for how many atoms a bond is delocalised over. This result also applies in cases where the probability distribution is not evenly distributed. More formally, the IPR can be expressed as follows: $IPR = 1/\sum_i^n p_i^2$, where p_i is the probability that the particle is in state i. For a quantum system, $p_i = |\psi_i|^2$, where ψ_i is the amplitude of the wave function in state i.

trated by two limiting cases: (i) a vector with identical components, $u_l^k = 1/\sqrt{N}$ has $I^k = 1/N$, whereas (ii) a vector with one component $u_1^k = 1$ and a remainder zero has $I^k = 1$. Thus, the IPR quantifies the reciprocal of the number of eigenvector components that contribute significantly. In our case, for 2007, we have that the eigenvector associated to the largest eigenvalue has $IPR1 = 0.0140$, which implies $PR1 = 71.46 \approx 71$ significant stocks in the largest eigenvector, which is almost the total number of stocks used.

The significant firms in the largest eigenvector are presented in Table A.1a in Appendix A. By inspecting the largest eigenvector we can see that it has very strong components in the Life Insurance sector, Financial Services, Banking and Real Estate Investments Trust and Mining to some extent. In general, the largest eigenvector would seem to include most sectors in the FTSE 100 index.

Regarding the rest of the deviating eigenvectors, the interpretation of their meaning is not as straightforward as in the former case. This is so, because the largest eigenvalue is of order of magnitude larger than the others, hence constraining the remaining $N - 1$ since $TrC = N$. Therefore, in order to avoid the effect of the largest eigenvalue, we first need to remove it from the rest. For doing this, we regress, $G^1(t)$, calculated in Section 4.2, against each return and compute the residuals $\epsilon_i(t)$.[5] Then, we calculate again the correlation matrix using $\epsilon_i(t)$ and compute the eigenvectors u^k of C. The number of significant participants in the second largest eigenvector for the year 2007, can be found in Table A.1b in Appendix A.

In this case we have $PR2 \approx 32.21$ significant components with strong emphasis in the Real Estate Investment Trust, Banking, Life Insurance, Mining and Oil & Gas sectors. Also, the removal of the first component allowed to identify a third and fourth deviating eigenvalues in the upper bound, $PR3 \approx 34$ and $PR4 \approx 35$ with strong weights on the Banking, Pharmaceutical & Biotechnology and Banking sectors.

Finally, the eigenvectors corresponding to the smallest eigenvalues, each has $PR78 \approx 3$ and $PR77 \approx 15$ significant participants. We find these contain as significant participants, pairs of stocks with the largest correlations of the sample. For example, the two largest components of u^{78} correspond to stocks belonging to the same multinational, SDR and SRDC (Schroders Plc)

[5] Alternatively, we could have used the returns on the FTSE 100 index and the results would not change stemming from the high correlation between both of an order of 0.9761.

with $C_{i,j} = 0.8989$, the largest correlation of the sample. Typically, it has been found in the literature that the components in the smallest eigenvectors represent a very small number of stocks with large correlations.[6] In particular, in each of these eigenvectors the relative sign of the two largest components is *negative*. That is, pairs of stocks with correlation coefficients well above the average $< C_{i,j} >$ effectively "decouple" from the rest of the stocks.

These results are enlightening from an asset allocation perspective, where the use of the RMT approach allows us to identify groups of stocks that can be used to contruct portfolios with different degrees of exposure to the systematic risk embeded in the market index.

4.4 The Evolution of the Largest Eigenvalues and Eigenvectors Over Time

In the last section, we studied the number of significant components in the eigenvectors associated to the deviating eigenvalues predicted by the RMT. Now we ask whether these eigenvector components are stable over time and whether these are sensitive to common shocks. As we showed in the last section, each of the deviating eigenvectors defined "eigenportfolios" of correlated clusters of stocks. In this context, by assessing the stability of eigenvectors, we want to verify whether these clusters of stocks remain correlated over time and also whether these clusters participate collectively when faced to common shocks.

To answer these questions, we first examine the time evolution of the two largest eigenvectors calculated on an daily rolling window of length 100 days starting from year 2000. In Figure 4.5, we show a colour representation where we grouped the eigenvector components in sectoral clusters of companies and their time evolution. Vertical lines represent common shocks affecting all sectors at a given point in time, as they represent a colour change across sectors, while horizontal colour lines represent whether eigenvector components remained stable over time. In Figure 4.5(a), we observe that the components of the eigenvector associated to the largest eigenvalue would seem to be mostly positive indicating the presence of a common factor driving the behaviour of most stocks. This common factor has highly positive components in the Mining, Banking, Food and

[6] See, Laloux et al. (1999, 2000); Plerou et al. (2000a); Gopikrishnan et al. (2001); Plerou et al. (2002); Bouchaud and Potters (2009).

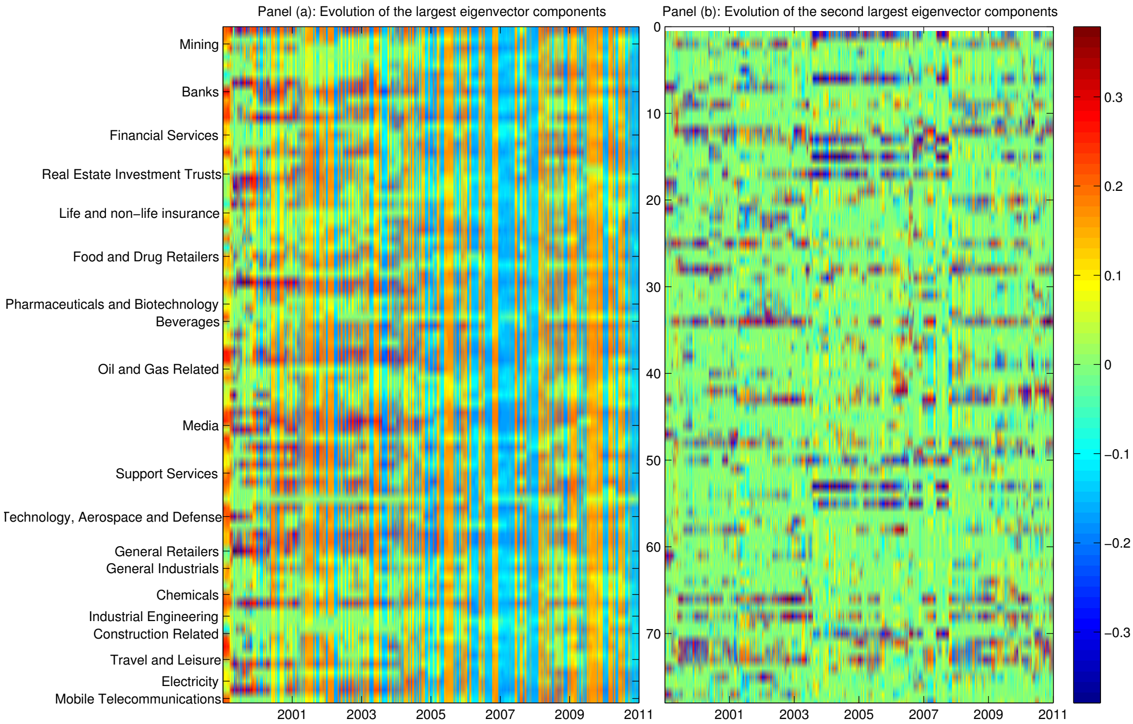


Figure 4.5: Time Evolution of the Eigenvector Components.

Drug retail and General Retail sectors. The vertical lines, represent common shocks that affect most sectors at a given time. All in all, it would seem that most sectors were strongly affected by the current crisis, particularly during the years 2008-2010. Regarding the eigenvector components associated to the second largest eigenvalue (Figure 4.5(b)), these have a more complex structure and it would not seem to carry meaningful information, except during periods of stress in financial markets, especially in the Banking, Financial Services and Support Services sectors. This is in line with the findings of Song et al. (2011).

In sum, the components of the largest eigenvalue seem to reflect common global and domestic factors affecting the FTSE index. In order to fully understand the direct relationship between macroeconomic determinants and the evolution of the eigenvalues a proper regression analysis is needed. We leave this interesting endeavour for Chapter 7.

4.5 Asymmetries and Company Participation in Financial Correlations with RMT Tools

The RMT approach can be useful to describe some stylised facts of financial correlations, such as correlation asymmetries and co-movement of stocks under different volatility periods. It has been often argued that financial assets are more correlated during drawdowns than during drawups, suggesting that the benefits of financial diversification may not be as strong as we believe (Ang and Chen, 2002).

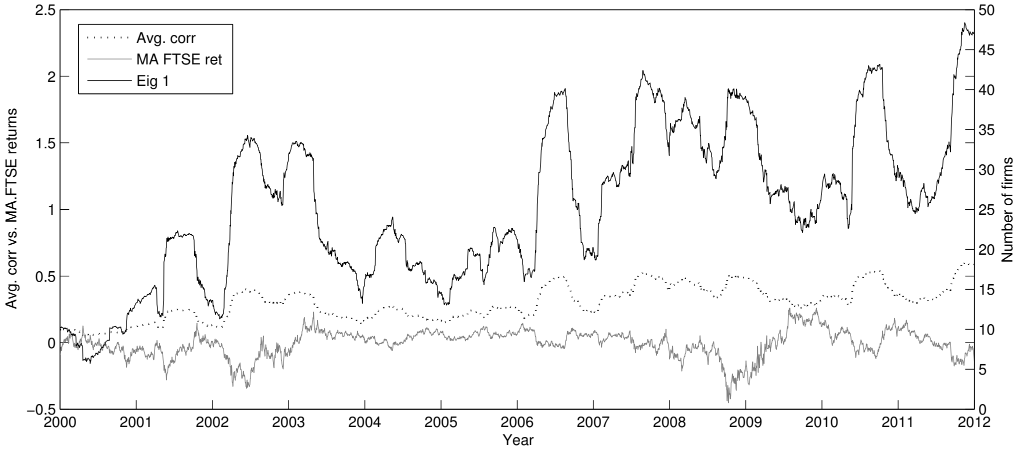


Figure 4.6: Time evolution of the Largest Eigenvalue (Left y-axis) vs. Average Correlation Coefficient and the FTSE Returns (Right y-axis).

A straightforward way to verify this is to examine the evolution of the largest eigenvalue calculated on a daily rolling window of length 100 days and compare its evolution with the average correlation coefficient calculated with the same length. This is done in Figure 4.6. Here we observe that in times when the average correlation coefficient is high, the largest eigenvalue follows the same dynamics (Rosenow et al., 2003). As we showed in Section 4.2, eigenvalues represent the risk of the portfolio returns defined by their associated eigenvectors (i.e., principal components) and as the largest eigenvalue reflects the market, which means that during drawdowns the market has become riskier.

Another very interesting phenomenon is the degree of participation of financial assets under different volatility periods. That is, we do not only find that correlations increase in high volatil-

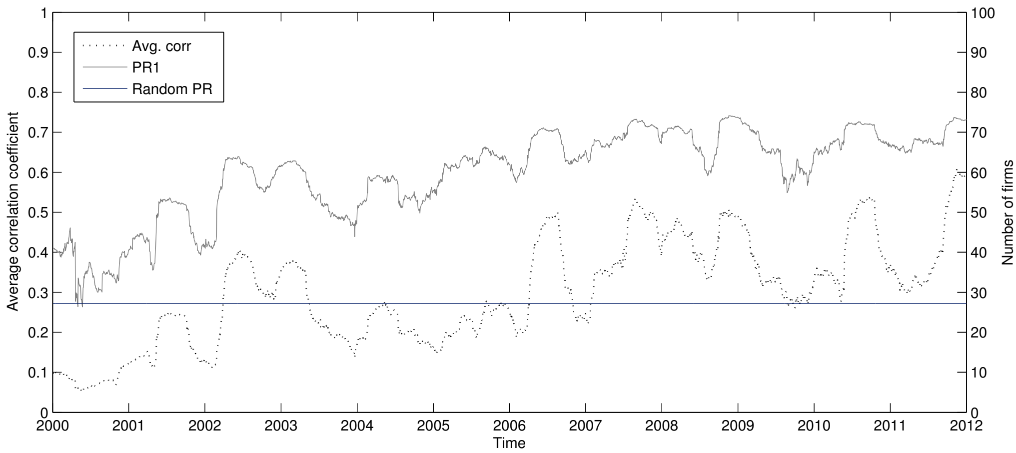


Figure 4.7: Time evolution of the Participation Ratio (PR) Associated to Largest Eigenvector of C vs. the PR for Random Returns (Right y-axis) and the Average Correlation Coefficient (Left y-axis).

ity periods, but also there are more companies participating significantly in the eigenvectors associated to the largest eigenvalue (Fenn et al., 2011). In order to verify this, we plot the time evolution of the *Participation Ratios* (PR), introduced in Section 4.3, calculated over a daily rolling window of length 100 days. This is done in Figure 4.7. In addition, we plot the average correlation coefficient calculated on a rolling window of the same length.

Here we observe that the PR of the largest eigenvalue increased considerably from 2000 to 2012, and there was a sharp increase between 2006 and 2008. This sharp rise of the PR implies that many assets were highly correlated during the ensuing financial crisis. In order to test the significance of the PR for the largest eigenvalue, we compare it to the corresponding PR for random returns. Figure 4.7 shows that between 2006 and 2010, the PR of the observed returns was significantly larger than the expected for random returns, which emphasises the larger number of different assets that were correlated during this period.

4.6 Summary of main findings

In the present chapter we have shown the benefits of the RMT approach to describe market correlations. Our findings can be summarised as follows:

1. The information content of the deviating eigenvalues/eigenvectors predicted by the RMT has the potential to identify groups or clusters of highly correlated stocks. These results are very appealing from an asset allocation perspective, since the analysis of the eigenvector components can be of great help to build portfolios with decreasing levels of risk.

2. By examining the evolution of the largest eigenvalue and its associated eigenvector, we find that there are clusters of correlated stocks that remain stable over time. We also find that these clusters behave in a similar way in face to common shocks affecting the stock market. This is particularly true during periods of crisis.

3. The evolution of the largest eigenvalue is able to reproduce most stylised facts of financial correlations, such as the asymmetric behaviour of correlations under different volatility periods. During market drawdowns, that is, when average correlations increase and market returns are low, the largest eigenvalue peaks the highest.

4. The use of RMT tools can also be very useful to identify the number of stocks that participate significantly under different volatility periods, which is not easy reveal by using standard financial tools. That is, by using the participation ratios introduced in section 4.3, we find that average correlations do not only increase, but also that stocks that were not significantly correlated under periods of low volatility, in face of a downturn, their correlations will become significant. We interpret this as co-movements between assets in face of a shock, rather than as correlations between assets *per se*.

All in all, we show that the use of Random Matrix Theory (RMT) can be a very powerful tool of analysis by providing valuable informations about market behaviour, which could be used by institutional investors to take their investment decisions or by policymaking institutions seeking to monitor financial markets by examining very few indicators.

In what follows, we apply these insights into a portfolio optimisation context and show how the RMT-filtering can be used to improve the forecast of the sample correlation matrix in Chapter 5.

Chapter 5

Portfolio Analysis for the FTSE 100: RMT-Filtering

5.1 Introduction

In this chapter we apply the concepts introduced in the previous sections and consider the case of a portfolio manager trading stocks of the FTSE 100 and assess whether the RMT-filtering can be used to reduce portfolio risk. For doing this, in Section 5.2 we first illustrate the benefits of noise filtering by replicating the standard methodology used in the literature (Laloux et al., 2000; Rosenow et al., 2002). Following these studies, we split the sample into two subperiods of one year and apply the RMT-filtering to the sample correlation matrix of the first subperiod and construct a family of optimal portfolios. Then, for the same family of optimal portfolios we compute the *realised risk*, by using next year's sample correlation matrix and then compare the percentage deviation between the *predicted* risk and the *realised* one. This quantity is then compared with the one obtained using the sample correlation matrix. In order to capture different volatility periods, we do this for the whole sample considered 2000-2012. For this setting we find that the RMT-filtering has the potential to provide the best *realised* estimates for the whole sample. These results are more pronounced when *short-sale* is permitted.

As pointed out by Pafka and Kondor (2003, 2004) and Daly et al. (2008), the effects of noise on risk estimates are strongly dependent on the ratio $Q = T/N$, where T is the length of the time-series and N is the total number of assets. This is why in Section 5.3 we review some work dealing with the sensitivity of noise to the ratio Q. We then apply these insights into a more realistic environment, that is, by applying the different strategies as if we were an active portfolio manager for different values of Q.

In Section 5.4 and Section 5.5, results are separated into in-sample and out-of-sample, respectively. For the in-sample analysis, we assess the effects of noise filtering on *realised* portfolio risk for different values of Q by using *boostrapping* techniques. Here we find that the RMT-filtering performs the best for low values of the ratio Q, which is consistent with the findings of Pafka and Kondor (2003) and Daly et al. (2008). For the out-of-sample testing, we use a battery of tests to assess the performance of the RMT-filtering, ranging from the ratio γ (Section 3.4.1) and Mean Squared Error (MSE) to return measures, such as the cumulated wealth at the end of the out-of-sample period and risk-adjusted return measures such as the *Sharpe Ratios* for the different strategies.

Here we find that the RMT-filtering has the potential to produce portfolios with the lowest *realised* risk and also to correctly predict it. However, consistent with the in-sample results, these benefits are highly dependent on the ratio $Q = T/N$ and the existence/absence of *short-sale*, with the RMT-filtering performing the best for low values of Q, when *short-sale* is permitted. When *short-sale* is not permitted, similar to Pantaleo et al. (2011), we find that the *filtered* correlation matrix delivers results comparable to those of the sample correlation matrix. We therefore recommend the use of the RMT-filtering, only for low values of Q and when *short-sale* is permitted. In Section 5.5.3, we provide an analysis of the weights for the different strategies, by using different measures of diversification in order to provide a possible explanation for this outperformance of the RMT-filtering. In Section 5.5.4, we gauge the robustness of our findings by splitting the sample in two different volatility periods. Finally, in Section 5.6 we provide conclusions and recommendations.

5.2 Illustration of Noise Filtering to Improve Portfolio Risk Estimates

The underlying randomness contained in the "bulk" of eigenvalues seen in the previous sections has important implications for optimal portfolio selection. In the context of a Markowitz Portfolio Theory (MPT), the effect of noise has a strong weight on the smallest eigenvectors, which are precisely the ones that determine the least risky portfolios. This is why the Random Matrix Approach allows reconciling the mean-variance approach of Markowitz as a tool for providing better risk predictions. Consider a portfolio $\Pi(t)$ of stocks with prices S_i. The return on $\Pi(t)$ is given by

$$\Phi = \sum_{i=1}^{N} w_i G_i, \tag{5.1}$$

where $G_i(t)$ is the return on stock i and w_i is the fraction of wealth invested in stock i. The fractions w_i are normalised such that $\sum_{i=1}^{N} w_i = 1$. The risk of holding the portfolio $\Pi(t)$ can be quantified by the variance

$$\Omega^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i C_{ij} w_j \sigma_i \sigma_j, \tag{5.2}$$

where σ_i is the standard deviation (average volatility) of G_i, and C_{ij} are the elements of the sample correlation matrix C. In order to find an optimal portfolio, we must minimise Ω^2 under the constraint that the return on the portfolio is some fixed value Φ. In addition, we also have the constraint that $\sum_{i=1}^{N} w_i = 1$. The resolution of this minimisation can be implemented by using two Lagrange multipliers, which leads to a system of linear equations for w_i, which can then be solved. The family of optimal portfolios can therefore be represented as a plot of the return Φ as a function of risk Ω^2, which is the *Efficient Frontier*.

To illustrate the effect of randomness of C on the selected optimal portfolios, we carry out our analysis for the years 2006-2007. We first calculate the correlation matrix C^{2006} using 260 daily returns and use the returns G_i^{2007} of the next period. We then construct a family of 100 optimal portfolios, and plot Φ as a function of the predicted risk Ω_p^2 for 2007. For this family of portfolios, we also compute the realised risk Ω_r^2 during 2007 using C^{2007}. We do this in order to compare the performance of the RMT approach isolated from any additional source of noise that could arise, for example, in case we had to make a forecast of returns.

Since the meaningful information in C is contained in the deviating eigenvectors (whose eigenvalues are outside of the RMT bounds), we use the filtering proposed by Plerou et al. (2002) (see, Section 3.3). We then repeat the above approach for finding the optimal portfolio using the *filtered* correlation matrix, C_f^{2006} instead of the *unfiltered*, C^{2006}. We plot the efficient frontiers for both methods, for *filtered* and *unfiltered* matrices, in Figure 5.1.

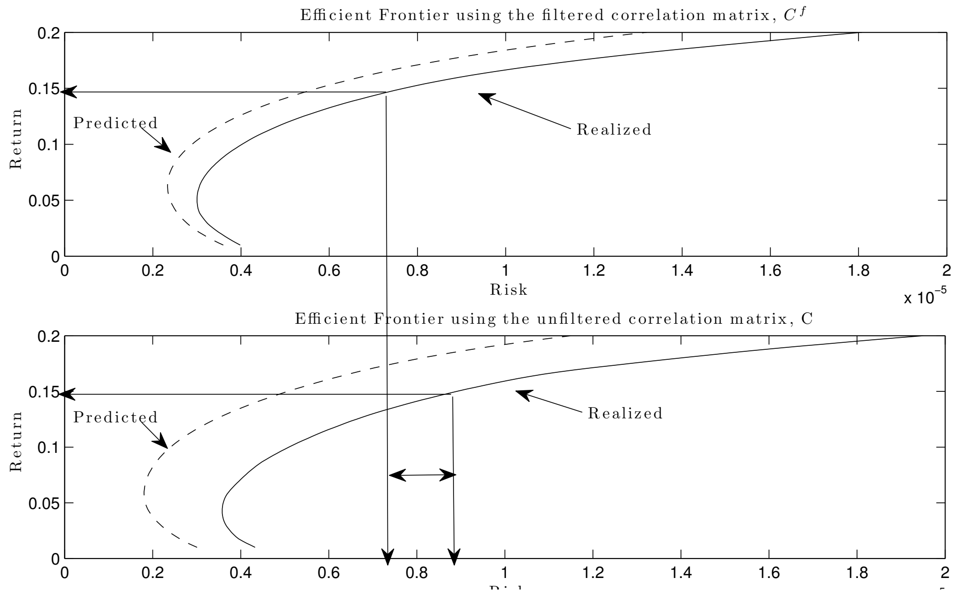


Figure 5.1: Efficient Frontier during 2007. Top panel shows the difference between *predicted* and *realised* risk, for the same family of optimal portfolios using the *filtered* correlation matrix, C_f. Bottom panel shows the difference between *predicted* and *realised* risk, but using the *unfiltered* correlation matrix, C. As it can be seen, the predicted risk is closer to the realised one when we use the *filtered* correlation matrix. Short-sale permitted.

A first observation is that the *filtered* correlation matrix, for a given level of return, achieves a lower *realised* risk than when using the *unfiltered* correlation matrix. A second observation is regarding the quality of prediction. That is, how close our risk predictions are from the realised ones. For illustrating this, we measure the relative performance of the *filtered* portfolios vs. the *unfiltered* ones by calculating the difference between both curves with the Mean Average

Percentage Error (MAPE), which is defined as:

$$MAPE = \frac{1}{n}\sum_{i=1}^{N}\left|\frac{\Omega_i^2 r - \Omega_i^2 p}{\Omega_i^2 p}\right|. \tag{5.3}$$

Here the *unfiltered* correlation matrix underestimates realised risk by 88%, while the *filtered* one only underestimates realised risk by 59%. There is certainly a sharp contrast between the use of the *filtered* and *unfiltered* matrices, with the former performing far better than the latter in forecasting portfolio risk.

In light of these results, an interesting question would be to assess how the *filtered* correlation matrix performs relative to the *unfiltered* one during different volatility regimes in the stock market. For doing this, we apply the same methodology to consecutive one-year periods from 2000-2001 to 2010-2011 in Table 5.1. In columns (1)-(2) we compare the MAPE using the *filtered* and *unfiltered* correlation matrices, restricting “short sale”, while in columns (3)-(4) we allow for “short sale”. Finally, in columns (5) and (6) we present the volatility of the FTSE 100 calculated as the mean of the absolute value of returns for the year used for estimation and the VIX index as a measure of international market volatility.

Here we observe that when “short sale” is not permitted, there are no major differences between the portfolios built using the *filtered* correlation matrix and those using the *unfiltered* matrix. Moreover, the overall improvement calculated as the average difference is *negative.* This result could cast doubts on the effectiveness of the RMT-filtering in predicting realised risk during periods of changing volatility.

However, these results are reversed when we allow for “short sale”. Here we observe that the *filtered* correlation matrix is far superior to the *unfiltered* one in all cases. “Short sale” involves selling assets that are borrowed in expectation of a fall in the asset’s price. In case the price declines, the investor covers his position by buying an equivalent number of assets at the new lower price and returns to the lender the assets that were borrowed. In mathematical terms, when we allow for “short sale”, portfolio weights are allowed to be negative, and investors can continuously sell low return assets and reinvest in higher yield assets and generate an infinite expected return. Conversely, the investor could “short sell” a high yield asset and reinvest the

	No short sale		Short sale			
	Unfiltered	Filtered	Unfiltered	Filtered	Vol. FTSE 100	VIX
Prediction-Realised	(1)	(2)	(3)	(4)	(5)	(6)
2000-2001	0.227	0.242	0.407	**0.099**	0.902	23.3
2001-2002	0.121	**0.108**	0.300	**0.114**	1.005	25.7
2002-2003	0.024	0.035	0.485	**0.151**	1.233	27.2
2003-2004	0.058	0.064	0.748	**0.525**	0.860	21.9
2004-2005	0.166	0.213	0.385	**0.346**	0.490	15.4
2005-2006	0.198	**0.175**	0.223	**0.095**	0.421	12.7
2006-2007	0.161	**0.142**	0.883	**0.588**	0.580	12.7
2007-2008	0.058	0.070	0.459	**0.331**	0.794	17.4
2008-2009	0.042	**0.040**	0.988	**0.579**	1.582	32.5
2009-2010	0.126	**0.117**	0.739	**0.486**	1.068	31.4
2010-2011	0.092	**0.083**	0.362	**0.254**	0.788	31.4
Average	0.115	0.117	0.543	0.324	0.883	22.0
Improvement		-0.012		0.403		

Table 5.1: Mean Average Percentage Error (MAPE) for each year of the sample. The MAPE reflects the difference between *predicted* and *realised* risk for a given family of optimal portfolios along the efficient frontier. Best results are marked in bold face.

proceeds into a low yield asset, therefore achieving an infinite negative return.[1] In this context, "short selling" would seem to expand the opportunity set for investment decisions. The use of the RMT-filtering allows to exploit these opportunities to construct portfolios that are robust to high volatility periods.[2]

These results are different from those of Sandoval et al. (2012) for the Brazilian Stock Market, who based on a number of agreement measures between *predicted* and *realised* risk, i.e., MAPE, Mean Squared Error (MSE), the angle between risk vectors, Simple Distance and the Kullback-Leibler distance measure, concluded that the RMT-filtering did not produce better results than the *unfiltered* approach, in particular during high volatility periods. They also find that the RMT-filtering combined with the single-index model significantly improves risk estimates for most of the measures analysed. Our results differ from his in that when allowing for "short sale" the RMT-filtering performs unequivocally better than when using the standard sample correlation matrix. In fact, as pointed out by Jagannathan (2003) and DeMiguel et al. (2009), when we impose "short sale" constraints and minimise the portfolio variance, this is equivalent

[1]Evidently, a rational investor will never do this and this is why only the upward sloping part of the efficient frontier is considered.

[2]While short-selling may not be allowed in certain markets and during specific times in financial markets (e.g., crises), it is still possible to circumvent this by using derivative positions.

to shrinking the extreme elements of the covariance matrix, which is also a way to deal with the estimation error (noise) that arises when we estimate large covariance matrices, making the effects of the RMT-filtering redundant. This also could be a possible reason why the RMT-filtering does not further improve *realised* risk estimates when "short-sale" is not permitted.

5.3 Sensitivity of the RMT Approach to the Data Size $Q = T/N$

As we mentioned in Section 2.2.3, the effects of noise are very sensitive to the ratio $Q = T/N$, where T is the length of the time-series and N is the number of assets. Based on simulated data, Pafka and Kondor (2003) find that for small values of Q, of the order of 1.66, noise has an important effect on risk estimates, while for large values of Q, around 5, the error due to noise drops to acceptable levels. In this context, the large discrepancies found between *predicted* and *realised* risk by Laloux et al. (1999, 2000) and Plerou et al. (2000a, 2002) could be attributed to low values of Q.

They further investigate this possibility by estimating the magnitude of this effect for values of N and T similar to the ones used in Plerou et al. (2002) and Laloux et al. (2000). In Laloux et al. (2000) daily returns on $N = 406$ stocks belonging to the S&P 500 covering the period 1991-1996 ($T = 1309$ daily observations) were used to calculate the correlation matrix. As pointed out by Pafka and Kondor (2003), for this portfolio size and this length of the time-series, the risk of the optimal portfolio in the presence of noise is about 20% larger than without. Similarly, for Laloux et al. (2000) 30-min returns on the largest $N = 1000$ U.S. stocks over the two-year period 1994-1995 ($T = 6448$ observations for each series) were used and the ratio between the *predicted* and *realised* risk was about 1.09, that is, the decrease in efficiency due to noise is only around 9%. These findings suggest that the impact of noise on the portfolio optimisation problem would not seem to be as dramatic as we think.

In Pafka et al. (2004), they assess the performance of a number of covariance matrix estimators by using bootstrapping techniques. For doing this, they split the sample into an estimation period (using past information) and an evaluation period (using future information). They calculate different correlation matrix estimators (i.e. equally weighted and exponentially weighted compared to their "filtered" and "unfiltered" versions) using the first period and then used them to construct "optimal" portfolios. They evaluate the performance of these estimators based on the standard deviation of the corresponding portfolio returns in the second period. In order to reduce the error that may arise from the use of a single sample, they perform their evaluation on a large number of bootstrapped samples from a larger dataset of daily stock returns. They compared the performance of their estimators for different values of N and T_2. The criteria used for comparison is the ex-post volatility (i.e. the volatility in the "investment period") of the minimum variance portfolios constructed by using the estimators based on ex-ante return data (i.e. before the investment period). The volatility measures were obtained by averaging over a larger number of bootstrapped samples.

For a sample size of $N = 100$ and T_2 (investment period of one month), they find that the RMT-filtering performs the best for both uniform and exponential weighting. For the case of uniform weighting, they also suggest that the best choice for the length of the time window T_1 is around 250, (i.e. one year of daily data). They also find that the parameter used for exponential weighting commonly used by practitioners ($\alpha = 0.94$) is completely inappropriate for portfolio optimisation with a large number of assets. This approach was also used later on by Daly et al. (2008) to assess the performance of some RMT-filters, with similar findings to those of Pafka et al. (2004).

As we mentioned in Section 3.2, when predicting future correlations from past correlations, one has to balance two competing effects: the width $\lambda_+ - \lambda_- = 4\sqrt{N/T}$ of the noise part of the spectrum decreases when long time-series are used. This means that the longer the time-series, the effects of noise tend to decrease. This can be illustrated in Figure 5.2, where we plot the percentage of deviating eigenvalues as a function of T. Here we observe that as T increases there are more eigenvalues deviating from the noise part of the spectrum of the MP distribution, indicating that the effects of noise would tend to decrease with the sample size. This suggest that long time-series over many years should be analysed. However, as correlations are not stable over time, the estimation period should not be too long in order to correctly describe the dynamics of correlations (Rosenow et al., 2003).

In the next section, we show the implications of using different sample size on portfolio risk

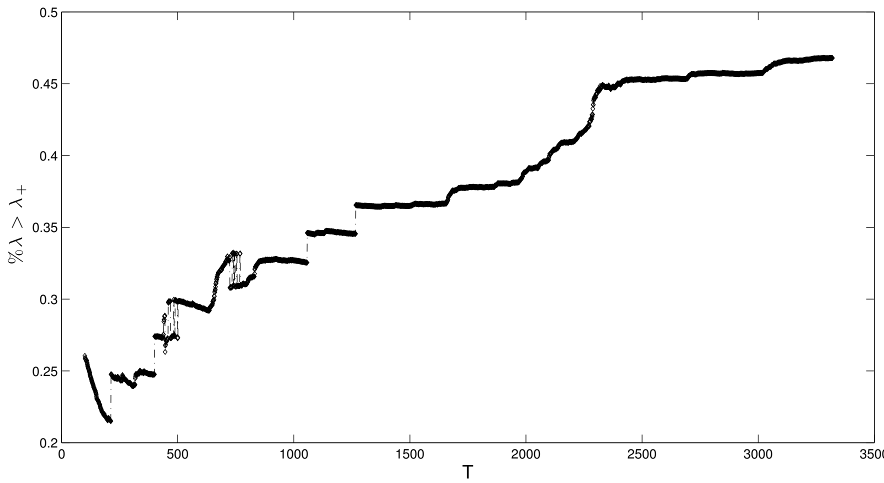


Figure 5.2: Percentage of Deviating Eigenvalues as a Function of T. The larger T gets, the smaller the noise band of the MP spectrum and the more eigenvalues deviate from the upper edge λ_+.

estimates and provide some insights on under what conditions the RMT-filtering will perform the best for a set of stock returns of the FTSE 100.

5.4 In-Sample Analysis

As described in Section 3.4.1, the in-sample methodology consists in a boostrap approach. For this, we use the first 500 observations of the sample, running from 04/01/00 until 30/11/01, for 77 stocks included in the FTSE 100. Here, for a given number of assets N, we randomly select a test date t and estimate the portfolio up to t and everything afterwards is used as realised, future information (usually, a forecast horizon of 20 days). We repeat this random selection 1000 times, with replacement, and calculate the mean across all bootstrapped samples of the ratio γ, defined in Section 3.4.1, which is the ratio between realised and predicted risk. The ratio γ has been widely used in the econophysics literature (see, Pafka et al., 2004; Pafka and Kondor, 2004; Daly et al., 2008, 2010) to assess the performance of the RMT-filtering in forecasting *realised* risk. Similarly, in the econometrics literature, this ratio has also been used to assess the performance of multivariate volatility models (see, Engle and Sheppard, 2001; Engle, 2002).[3] We estimate our portfolios using the sample correlation estimator (henceforth, C^{SAMP}) and the RMT-filtering correlation estimator, using the method of (Plerou et al., 2002) (henceforth, $C^{SAMP-RMT1}$).

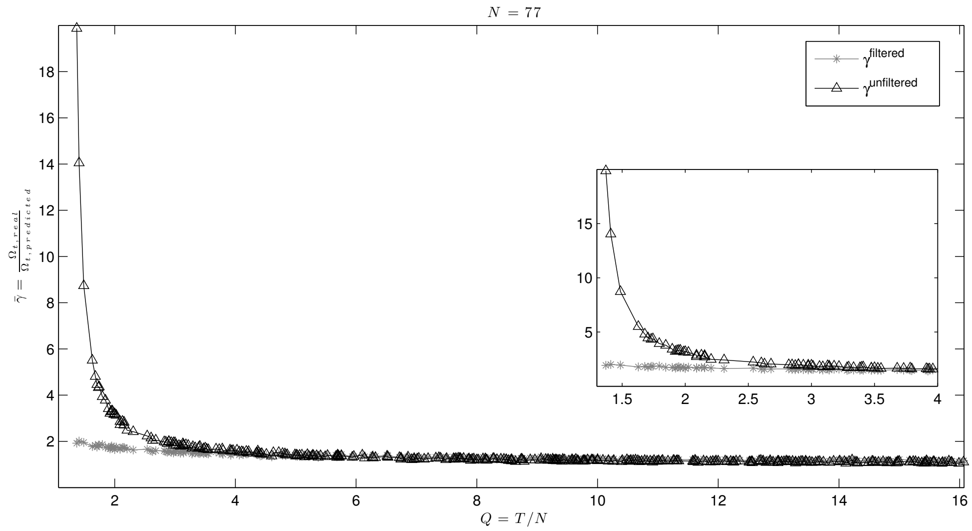


Figure 5.3: Mean Bootstrapped (in-sample) for the ratio γ of *realised* to *predicted* risk, for the *filtered* approach, $\bar{\gamma}^f = \frac{\Omega_{fr}}{\Omega_f}$, compared to the *unfiltered* approach, $\bar{\gamma}^u = \frac{\Omega_{ur}}{\Omega_u}$, as a function of $Q = T/N$. Notice that filtering is most effective for low values of Q. *Short-sale* allowed.

In Figure 5.3 we plot the mean bootstrapped results for an evaluation period of 20 days, allowing for *short-sale*.[4] Here we observe that the RMT-filtering has great potential to predict *realised* risk for low values of $Q = T/N$. In particular, the forecasting ability of the *filtered* correlation matrix would seem to perform the best when Q ranges between $1.6 \leq Q \leq 3.2$. Taking into account that we are estimating our portfolios for a fixed number of assets ($N = 77$), the optimal length of the time-series used would lie between $123 \leq T \leq 250$, that is, between 6 and 12 months ($\approx$ 1 year of data). These results are consistent with those found by Pafka et al. (2004) ($\approx$ 250 days) and Daly et al. (2008) ($\approx$ 250 days), for 100 large-cap stocks of the S&P 500.

These results are encouraging, since we find that for a given value of Q, the RMT-filtering has

[3] This ratio is very similar to the Mean Absolute Percentage Error (MAPE), introduced in Section 5.2, which measures how far the *predicted* risk is from the *realised* risk.

[4] When allowing for *short-sale* we limit our weights to be no bigger than 100% on any asset and to "go short" on 150%.

the potential to achieve more accurate estimates of the *realised* risk of minimum risk portfolios, when compared to the *unfiltered* correlation matrix based on historical information. The sample correlation matrix would seem to perform better for values of Q greater than 4 (see, inset, in Figure 5.3). These values are similar as those suggested by Pafka and Kondor (2003) who find that for values of Q greater 5, the effects of noise drop to acceptable levels.

In the next section we assess the out-of-sample performance of the RMT-filtering in predicting realised portfolio risk, by using a battery of tests and different estimation windows. We also investigate whether RMT-filtered correlations are able to deliver greater cumulated portfolio returns at the end of the out-of-sample period than the sample correlation matrix.

5.5 Out-of-Sample Analysis

In this section we provide an out-of-sample analysis that will allow us to assess how each forecasting method would have performed in practice in case of being used. One of the main disadvantages of the bootstrapping approach is that it tends to favour models with more parameters, since the best parameter values are assessed after testing.

For the out-of-sample testing we split the sample into two subperiods. The first subperiod, that we used in the previous section for the in-sample analysis covers the period from 04/01/00 until 30/11/01 ($\approx$ 500 observations), while the second subperiod is the evaluation period, which starts in 04/12/01 until 25/09/12 ($\approx$ 2820 observations).

As explained in Section 3.4.2, for every test date t', we consider portfolios that were invested before $t' - t^f$, or on that date, where t^f is the investment period (in this case, 20 business days). That is, we estimate the portfolio variance at a given month and compare it to the *realised* variance on that month, which is calculated as the sample covariance of the *realised* returns over the investment horizon ($t^f = 20$ days). Subsequent retraining is done on a monthly basis.

As we showed in the in-sample analysis and as observed by Pafka and Kondor (2003), the effects of noise would seem to strongly depend on the ratio $Q = T/N$, with the RMT-filtering performing the best for small values of Q while for larger values the effects of noise are almost negligible. To allow for this possibility, we estimate our models using different time windows, based on three estimation windows; 125 days ($\approx$ 6 months), 250 days ($\approx$ 1 year) and 500 days ($\approx$

2 years). All these models use the first subperiod (01/04/00 until 12/31/02 $\approx$ 500 observations) as initial training period.

In contrast to most studies in the literature, we estimate our portfolios with *short-sale* constraints. We believe this provides a more realistic setting as *short-sale* is not always allowed in financial markets stemming from the increase in risk it brings to portfolios. Alternatively, we also allow for *short-sale*, however, none of the estimated portfolios ever took an extreme position.[5]

Based on the above, we estimate our portfolios using the following correlation estimators:

- Sample Correlation Matrix: C^{SAMP}.
- Filtered Sample Correlation Matrix Keeping the Largest Eigenvalue: C^{SAMP1}.
- Filtered Sample Correlation Matrix - Method 1: $C^{SAMP-RMT1}$ (Plerou et al., 2002).
- Filtered Sample Correlation Matrix - Method 2: $C^{SAMP-RMT2}$ (Laloux et al., 2000)

Here we compare and contrast the performance of the sample correlation matrix, C^{SAMP}, with their RMT-filtered versions, using both methods described in Section 3.3. We also filter the sample correlation matrix by keeping only the largest eigenvalue, as we want to assess the effectiveness of the RMT-filtering in selecting the optimal number of eigenvalues.

5.5.1 Realised Risk Estimates

In Table 5.2, we report our *realised* risk estimates, using the performance metrics defined in Section 3.4.2 with *short-sale* constraints. Here we observe that there are no major improvements of using the RMT-filtering on the sample correlation matrix. For the best of the cases, we obtain a slightly better prediction and lower volatility of prediction, measured by the MSE. For instance, when using an estimation window of 125, using (Plerou et al., 2002)'s method, $C^{SAMP-RMT1} - 125$, delivers a γ ratio of 1.067 and MSE of 0.084, compared to 1.084 and 0.115, respectively, when using $C^{SAMP} - 125$. As we move to longer estimation windows, the benefits of filtering become even more negligible.

[5]Similar to the in-sample analysis, when allowing for *short-sale* we limit our weights to be no bigger than 100% on any asset and to "go short" on a 150%.

Model	No short-sale				Short-sale			
	Ω_{est}	Ω_{real}	MSE $\cdot$ 10^{-2}	$\bar{\gamma}$	Ω_{est}	Ω_{real}	MSE	$\bar{\gamma}$
$C^{SAMP} - 125$	0.102	0.114	0.115	1.084	0.045	0.142	0.012	2.976
$C^{SAMP1} - 125$	0.098	0.114	0.115	1.123	0.059	0.113	0.005	1.729
$C^{SAMP-RMT1} - 125$	0.102	0.113	0.084	1.067	0.066	0.102	0.002	1.452
$C^{SAMP-RMT2} - 125$	0.096	0.115	0.119	1.160	0.065	0.101	0.002	1.466
$C^{SAMP} - 250$	0.106	0.112	0.068	1.019	0.063	0.108	0.003	1.625
$C^{SAMP1} - 250$	0.101	0.112	0.077	1.060	0.060	0.113	0.005	1.699
$C^{SAMP-RMT1} - 250$	0.106	0.111	0.062	1.008	0.069	0.101	0.002	1.353
$C^{SAMP-RMT2} - 250$	0.101	0.113	0.079	1.084	0.068	0.099	0.001	1.361
$C^{SAMP} - 500$	0.106	0.111	0.057	0.994	0.071	0.100	0.001	1.334
$C^{SAMP1} - 500$	0.102	0.111	0.065	1.042	0.061	0.114	0.005	1.677
$C^{SAMP-RMT1} - 500$	0.107	0.110	0.053	0.983	0.073	0.100	0.001	1.287
$C^{SAMP-RMT2} - 500$	0.103	0.111	0.063	1.034	0.072	0.098	0.001	1.277

Table 5.2: Out-of-sample estimates of the variance of *minimum variance portfolios.* Notice that the *filtered* correlation matrices provide the *lowest* average *realised risk* and the best *realised risk* forecast in the absence of *short-sale* constraints. When these constraints are in place, results are not conclusive.

Table 5.2 (Columns, 5-6) presents results allowing for a *short-sale* strategy.[6] Here observe a more substantial difference between the *filtered* correlation matrix and the *unfiltered* one. In general, when comparing the results for different estimation windows for risk forecast, we find that risk predictions using the RMT-filtering are significantly better for short estimation windows, but these results worsen as we increase the length of the time-series. For instance, when using an estimation window of 125 days, the predicted risk, using $C^{SAMP-RMT1} - 125$, underestimates realised risk by 45.2%, while when using the sample correlation matrix, $C^{SAMP} - 125$, we underestimate realised risk by 197.6%. Under this scenario the difference in the quality of prediction is striking. These results are robust across estimation windows, but the difference in performance of the RMT-filtering seems to be narrowed as we increase the estimation window. Another observation is that the portfolios built using only one eigenvalue in the RMT-filtering, C^{SAMP1}, do not generate better predictions than the ones selecting the optimal number of eigenvalues as predicted by the RMT to include in the filter, namely, $C^{SAMP-RMT1}$ and $C^{SAMP-RMT2}$. This means that the remaining eigenvalues that deviate from the Marčenko-Pastur Distribution do contain economically relevant information, which can be used to build better risk forecast

[6]We are aware that this may not be realistic as *short-sale* is not always allowed in financial markets, but we acknowledge that reality must lie between these two extreme cases.

of realised risk of minimum variance portfolios. In general, portfolios builts using the *filtered* correlation matrices deliver lower realised risk estimates, lower volatility of prediction (measured by their MSE) and better realised risk forecast.

5.5.2 Risk-Adjusted Returns Estimates

Another interesting question is about whether the *filtered* correlation matrices deliver greater cumulated portfolio returns for a given level of risk. For this, we calculate cumulated portfolio returns at the end of the out-of-sample period. This performance measure is calculated by applying the different strategies (i.e. *weights*) and using the ex-post returns, which are calculated as the average returns over the investment horizon. We then compute the compounded returns until the end of the out-of-sample period, starting with an initial £1 investment.

In Figures 5.4a-5.4c-5.4e we plot cumulated returns, for different estimation windows and strategies (i.e. using the *filtered* vs. *unfiltered* correlation matrices), with *short sale* constraints. A first observation is that there are no major differences in cumulated returns between the *filtered* and *unfiltered* correlation matrices. The filtered correlation matrix, using only the largest eigenvalue in the filter, C^{SAMP1}, would seem to perform better than the rest. These differences become almost negligible as we increase the estimation window.

In contrast, when *short-sale* is allowed in Figures 5.4b-5.4d-5.4f, we find significant differences in cumulated returns. For instance, when using the *filtered* correlation matrix with Plerou et al. (2002)'s method, $C^{SAMP-RMT1} - 125$, we obtain $\pounds 3.22$, whereas when using the sample correlation matrix we obtain $\pounds 1.90$. This is particularly true for short estimation windows. For example, when using an estimation window of 6 months ($\approx$ 125 days) we obtain the highest cumulated portfolio return ($\approx$ £3.22). Conversely, when using an estimation window of 2 years ($\approx$ 500 days), the *filtered* correlation matrix still delivers a higher cumulated return ($\pounds 2.66$-2.76 for both methods) when compared to the sample correlation matrix ($\pounds 2.03$) but the difference seems to be shrunk. We also find differences between both RMT-based filtering methods, with the filtering method of Plerou et al. (2002), $C^{SAMP-F1}$, performing slightly better for shorter estimation windows than the method of Laloux et al. (2000), $C^{SAMP-F2}$, which performs better for longer estimation windows. A similar observation can be noticed for the realised risk estimates

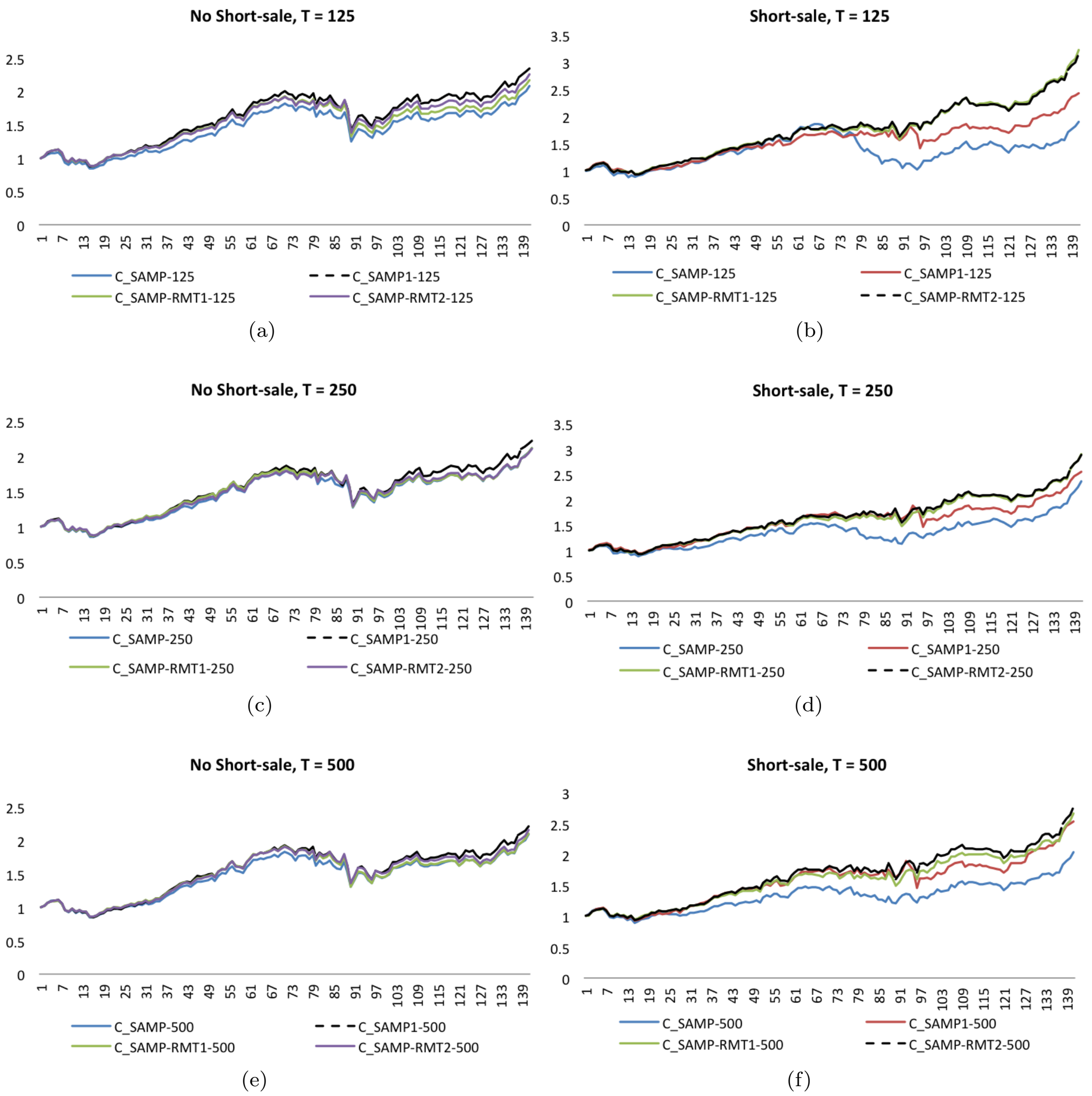


Figure 5.4: Cumulated Wealth at the end of the out-of-sample period for different estimation windows, with and without *short-sale*.

of Section 5.5.1. In general, in terms of cumulated wealth, the benefits seen after filtering, amount to 65%.

	No short-sale		Short-sale	
Model	Cum. Ret	Sharpe Ratio	Cum. Ret	Sharpe Ratio
$C^{SAMP}-125$	2.075	0.192	1.906	0.217
$C^{SAMP1}-125$	2.344	0.180	2.435	0.309
$C^{SAMP-F1}-125$	2.167	0.172	3.223	0.320
$C^{SAMP-F2}-125$	2.252	0.180	3.155	0.309
$C^{SAMP}-250$	2.119	0.185	2.356	0.229
$C^{SAMP1}-250$	2.226	0.169	2.545	0.283
$C^{SAMP-F1}-250$	2.115	0.170	2.898	0.283
$C^{SAMP-F2}-250$	2.104	0.169	2.888	0.283
$C^{SAMP}-500$	2.091	0.184	2.038	0.227
$C^{SAMP1}-500$	2.218	0.173	2.540	0.261
$C^{SAMP-F1}-500$	2.103	0.167	2.666	0.255
$C^{SAMP-F2}-500$	2.157	0.173	2.767	0.261

Table 5.3: Cumulated Wealth at the end of the out-of-sample period and Sharpe Ratios using the *filtered* and *unfiltered* correlation matrices for different estimation windows. The *filtered* correlation matrices provide the best end of the period return and the highest Sharpe Ratio in most cases.

Table 6.3 provides the cumulated returns and Sharpe Ratios for the different strategies. The *Sharpe Ratio* is calculated as the ratio between expected excess return of a portfolio and its standard deviation. Excess returns are defined as the difference between the expected portfolio returns and the risk-free asset, which in this case is the 1-month Libor. This ratio will provides us some insights into how well the returns of the different strategies compensate investors for the risk taken. That is, when comparing different strategies (i.e., *filtered* v.s *unfiltered* correlations for different estimation windows), the one with the highest *Sharpe Ratio* provides the best return for the same risk (or, equivalently, the same return for lower risk). Here we see that the portfolios built using the *filtered* correlation matrices, not only provide the highest cumulated returns but also the highest *Sharpe Ratios.* This is particularly true when *short sale* is permitted. Conversely, when *short sale* is not permitted, portfolios built using the sample correlation matrix outperform the RMT-filtered ones.

5.5.3 Are RMT-Filtered Portfolios More Diversified?

Now we investigate how can be explained the success of the RMT-filtering in prividing the best realised risk estimates of minimum variance portfolios as well as the highest cumulated returns and Sharpe Ratios. For doing this, in Table 5.4 we look deeper into the weights for the different strategies by using the measures introduced in Section 3.4.2, i.e., the effective portfolio diversification, N_{eff} (Equation 3.23), N_{90} (Equation 3.24) and the ratio between negative and positive weights, w^-/w^+, when allowing for *short-sale* (Equation 3.25).

	No short-sale		Short-sale		
	N_{eff}	N_{90}	N_{eff}	N_{90}	w^-/w^+
Model	(1)	(2)	(3)	(4)	(5)
$C^{SAMP} - 125$	7.41 (3.47)	8.12 (3.49)	2.43 (1.09)	46.81 (2.42)	0.605
$C^{SAMP1} - 125$	8.63 (4.07)	9.54 (4.13)	10.77 (3.51)	45.37 (2.62)	0.281
$C^{SAMP-F1} - 125$	7.90 (3.68)	8.78 (3.90)	8.55 (2.94)	44.57 (2.73)	0.319
$C^{SAMP-F2} - 125$	7.50 (3.74)	8.23 (4.07)	8.45 (3.11)	44.21 (3.14)	0.296
$C^{SAMP} - 250$	7.18 (3.29)	8.00 (3.23)	4.29 (1.64)	45.41 (2.67)	0.479
$C^{SAMP1} - 250$	8.18 (3.59)	9.12 (3.53)	10.11 (2.82)	45.56 (2.66)	0.290
$C^{SAMP-F1} - 250$	7.42 (3.41)	8.34 (3.29)	7.75 (2.63)	44.70 (3.03)	0.336
$C^{SAMP-F2} - 250$	7.06 (3.45)	7.82 (3.57)	7.75 (2.87)	44.29 (3.66)	0.312
$C^{SAMP} - 500$	7.37 (3.35)	8.29 (3.54)	5.38 (2.27)	45.09 (2.74)	0.420
$C^{SAMP1} - 500$	8.18 (3.56)	9.31 (3.75)	9.84 (2.81)	45.72 (2.91)	0.294
$C^{SAMP-F1} - 500$	7.28 (3.39)	8.19 (3.68)	7.20 (2.56)	44.81 (2.93)	0.345
$C^{SAMP-F2} - 500$	7.12 (3.45)	7.92 (3.80)	7.41 (2.75)	44.49 (3.32)	0.322

Table 5.4: Average Effective Portfolio Diversification, N_{eff} and N_{90}, for different time windows, using the *filtered* and *unfiltered* correlation matrices, with/without *short-sale*. Standard errors in parentheses.

Recall from Section 3.4.2 that when *short-sale* is allowed, the interpretation of N_{eff} can be dubious, hence we should look at N_{90} instead of N_{eff}. When *short-sale* is not permitted, we find that there are no major differences in the level of diversification in portfolio weights. In fact, this is consistent with the findings of the previous sections, where we did not find major differences between the portfolios using the *filtered* correlation matrices and those using the sample correlation matrix. When *short-sale* is allowed, in Column (4), we observe that the sample correlation matrix produces portfolios which are, on average, slightly more diversified than when using the *filtered* correlation matrix. However, the quantity N_{eff}, in Column 3, is also very informative, as a high N_{eff} implies a relatively small $\sum_i^N w_i^2$, which indicates that the weights produced by the *filtered* correlation matrices, on average, take less extreme positions than when using the sample correlation matrix. In column (5), we also quantify the amount of *short-selling*, measured as the ratio between the total sum of negative v.s positive weights and verify that the portfolios built using the sample correlation matrix suffer from an overexposure to *short-selling*. This is in line with previous studies assessing the effects of different spectral correlation estimators (see, Pantaleo et al., 2011). However, as opposed to this study, we do not find significant differences in effective diversification for the different correlation estimators with/without *short sale*.

5.5.4 Robustness Test to the Recent Economic Crisis: Sub-sample Estimates

In the previous section we showed the out-of-sample benefits of using the RMT-filtering for the whole sample. However, these results could be biased as the benefits from the RMT-filtering may have kicked in at the beginning of the sample (which was a relatively low volatility period) and then vanished as we approach the last 4 years of the sample (which was a high volatility period). This is why we ask whether these results are robust to the recent and ongoing economic crisis, which are the moments where good investment strategies are needed the most. For doing this, we split the sample into two sub-periods; the first sub-period, namely, the pre-crisis period, starts in November 2002 and ends in September 2006, while the second sub-period from September 2006 until July 2010, covers the start of the Subprime Crisis and the subsequent U.S. Banking Crisis, which then spread to the rest of the world, affecting mainly European countries, included the UK. We therefore classify this period as a high volatility period, where the VIX index, which is a good proxy for international volatility rose from an average of 16.4 (2003-2006) to 25.4 (2007-2010). Likewise, in the UK, stock market volatility, measured as the monthly standard deviation of the returns of the FTSE 100 rose from 3.88% (2003-2006) to 7.06% (2007-2010).

Thus we should expect market correlations to behave differently under these two scenarios. Specifically, we expect correlations to be stable during the first sub-period (2003-2006), whereas we expect correlations to be higher during the second sub-period (2007-2010). This has important implications from an asset allocation perspective, where the high co-movement between assets during market downturns can generate great losses to investors holding these assets in their portfolios. Thus, the aim of this section is to examine how effective the RMT-filtering would have been in case of being used at the beginning of these two sub-periods.

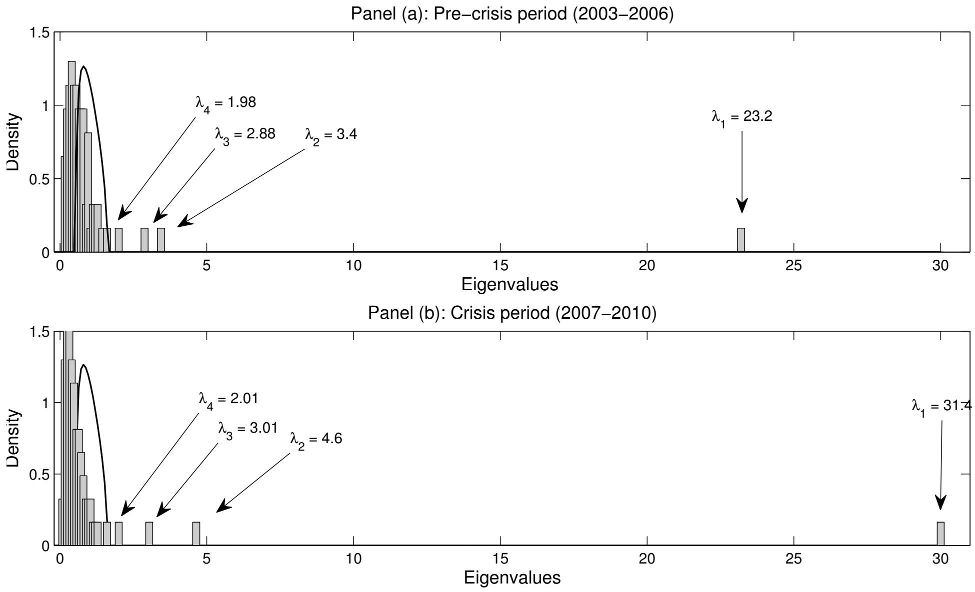


Figure 5.5: Panel (a): Eigenvalue Distribution for the Pre-crisis Period (2003-2006). Panel (b): Eigenvalue Distribution for the Crisis period (2007-2010). For a given value of $Q = 13.54$.

As we saw in Chapter 4, we can also see the difference between these two periods by inspecting their eigenvalue distributions. Recall from Section 4.2 that the largest eigenvalue reflects the risk of the market portfolio. Therefore, we should expect the largest eigenvalue to be greater during the second period (2007-2010) than during the first period (2003-2006). In Figure 5.5, we plot the eigenvalue distribution of the sample correlation matrix for both periods. In addition, we plot the theoretical eigenvalue density (the Marčenko-Pastur Distribution) for a given value of Q (13.54). A first observation is that the largest eigenvalue is bigger in magnitude during the years 2007-2010 ($\lambda_1^{2007-2010} = 31.4$) than during the years 2003-2006 ($\lambda_1^{2003-2006} = 23.2$), reflecting the higher market risk faced during the economic crisis. A second observation relates to the amount of the total variance explained by the deviating eigenvalues. For example, during the pre-crisis period, the percentage of the total variance explained by the deviating eigenvalues amounts to 40.6%, whereas for the crisis period this quantity amounts to 52%. While this could be attributed to the increase of the largest eigenvalue, it is worth noting that the remainder deviating eigenvalues also increase in magnitude when transitioning from one period to the other. This basically evidences the release of new useful information from the noisy part of the Marčenko-Pastur spectrum, which is incorporated in the deviating eigenvalues during the crisis period.

In order to verify whether this new information is translated into better portfolio estimates, we proceed to re-estimate our models as we had been trading at the beginning of each of these sub-periods. For doing this, we take the first 250 days of each sub-period in order to have enough observations for the evaluation period. We then assess the benefits of the RMT-filtering using the same performance measures introduced in Section 3.4.2. In Table 5.5, we show the *realised* risk estimates for both sub-periods under *short-sale* constraints. Here we find that most results are maintained. In particular, when *short-sale* is not permitted, the *filtered* correlation matrices do not perform any different to those using the sample correlation matrix. In many cases, it is preferable to use the sample correlation estimator with no filtering.

	Pre-crisis: Years 2003-2006				Crisis: Years 2007-2010			
Model	Ω_{est} x 10^{-1} (1)	Ω_{real} x 10^{-1} (2)	MSE x 10^{-2} (3)	$\bar{\gamma}$ (4)	Ω_{est} (5)	Ω_{real} (6)	MSE x 10^{-2} (7)	$\bar{\gamma}$ (8)
$C^{SAMP}-125$	0.647	0.809	0.662	0.834	0.157	0.169	0.210	0.981
$C^{SAMP1}-125$	0.627	0.802	0.663	0.817	0.149	0.172	0.320	0.906
$C^{SAMP-F1}-125$	0.659	0.789	0.546	0.873	0.159	0.168	0.190	0.994
$C^{SAMP-F2}-125$	0.629	0.799	0.674	0.824	0.153	0.171	0.270	0.938
$C^{SAMP}-250$	0.684	0.787	0.433	0.908	0.164	0.166	0.110	1.038
$C^{SAMP1}-250$	0.660	0.785	0.499	0.881	0.156	0.169	0.150	0.964
$C^{SAMP-F1}-250$	0.692	0.780	0.406	0.928	0.165	0.165	0.110	1.051
$C^{SAMP-F2}-250$	0.662	0.793	0.490	0.869	0.160	0.167	0.140	1.009

Table 5.5: Out-of-Sample Estimates of the *realised* risk of *minimum variance portfolios* for different estimation windows and different volatility periods. Short-sale constraints. Notice that there are no major differences from using the *filtered* correlation matrices or the sample correlation matrix.

When we permit *short-sale* (Table 5.5) we observe that the portfolios built using the *filtered* correlation matrices deliver the best results for all performance measures analysed. Here the effects of the RMT-filtering are strong in magnitude and therefore, more likely to be statistically significant. In this case, both filtering methods deliver very similar results in terms of *realised* risk estimates, volatility of prediction and prediction accuracy, measured by the ratio γ.

In Table 5.7, we analyse cumulated returns for both volatility period, allowing/constraining

Model	Pre-crisis: Years 2003-2006				Crisis: Years 2007-2010			
	Ω_{est} x 10^{-1}	Ω_{real} x 10^{-1}	MSE x 10^{-5}	$\bar{\gamma}$	Ω_{est} x 10^{-1}	Ω_{real} x 10^{-1}	MSE x 10^{-2}	$\bar{\gamma}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$C^{SAMP} - 125$	0.343	1.017	5.230	0.347	0.640	2.001	2.490	0.339
$C^{SAMP1} - 125$	0.458	0.780	1.340	0.614	0.793	1.776	1.490	0.490
$C^{SAMP-F1} - 125$	0.498	0.751	0.929	0.691	0.951	1.491	0.047	0.682
$C^{SAMP-F2} - 125$	0.493	0.741	0.906	0.691	0.952	1.492	0.046	0.677
$C^{SAMP} - 250$	0.472	0.800	1.400	0.605	0.867	1.543	0.640	0.585
$C^{SAMP1} - 250$	0.477	0.782	1.200	0.632	0.814	1.797	1.450	0.499
$C^{SAMP-F1} - 250$	0.521	0.753	0.742	0.714	0.981	1.470	0.040	0.721
$C^{SAMP-F2} - 250$	0.516	0.741	0.723	0.719	0.989	1.457	0.036	0.727

Table 5.6: Out-of-Sample Estimates of the *realised* risk of *minimum variance portfolios* for different estimation windows and different volatility periods. Short-sale permitted. Notice that the *filtered* correlation matrix provides the *lowest* average *realised risk* and the best *realised risk* predictions in all cases.

short-selling. Here we find that when *short-sale* is not permitted, the sample correlation matrix outperforms the *filtered* correlation matrices in the *low* volatility period. In high volatility periods, at least the filtered correlation matrix with the method of Laloux et al. (2000), outperforms the sample correlation matrix in both, cumulated returns and Sharpe ratios. When we permit *short-sale*, we find that during the pre-crisis period cumulated returns and Sharpe ratios are very similar for all correlation estimators. The main difference between *filtered* and sample correlation estimators kicks in when we analyse these quantities during the crisis period. Here we find that the portfolios built using the sample correlation matrix deliver significantly poorer cumulated returns and Sharpe ratios when compared with the *filtered* correlation matrices. For instance, $C^{SAMP} - 125$, delivers cumulated returns of £1.35 and Sharpe Ratio of 0.194, whereas $C^{SAMP-F1} - 125$ yields £1.63 and a Sharpe Ratio of 0.387. Taking into account, that these portfolios could be worth millions of pounds, these differences are striking.

This is undoubtly a very relevant result, since we have just shown that the benefits of the RMT-filtering come from its ability to track sudden changes in volatility, delivering the best *realised* risk estimates, the highest Sharpe ratios and cumulated returns. This is particularly true when *short-sale* is permitted. Only in this case, the RMT-filtering probes to be robust to the existence of different volatility periods in financial markets.

Following the analysis of Section 5.5.3, we investigate the portfolio weights under both volatil-

Model	Pre-crisis: Years 2003-2006				Crisis: Years 2007-2010			
	No Short-sale		Short-sale		No Short-sale		Short-sale	
	Cum. Ret	Sharpe Ratio	Cum. Ret	Sharpe Ratio	Cum. Ret	Sharpe Ratio	Cum. Ret	Sharpe Ratio
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$C^{SAMP} - 125$	1.594	0.632	1.484	0.562	1.197	0.100	1.353	0.194
$C^{SAMP1} - 125$	1.651	0.596	1.532	0.571	1.129	0.177	1.317	0.362
$C^{SAMP-F1} - 125$	1.604	0.604	1.512	0.560	1.185	0.141	1.633	0.385
$C^{SAMP-F2} - 125$	1.614	0.596	1.531	0.571	1.245	0.177	1.599	0.362
$C^{SAMP} - 250$	1.562	0.630	1.335	0.573	1.215	0.116	1.639	0.194
$C^{SAMP1} - 250$	1.624	0.536	1.547	0.471	1.160	0.173	1.315	0.412
$C^{SAMP-F1} - 250$	1.529	0.546	1.391	0.435	1.202	0.148	1.769	0.471
$C^{SAMP-F2} - 250$	1.536	0.536	1.432	0.471	1.246	0.173	1.671	0.412

Table 5.7: Cumulated Wealth at the end of the out-of-sample period and Sharpe ratios using the *filtered* and sample correlation matrices for different window lengths and different volatility periods. Notice that the *filtered* correlation matrix provides the best end of the period return and the highest Sharpe Ratios in most cases.

ity periods. In Table 5.8, in Columns (2) and (4), we show the measures of portfolio diversification, N_{90} and the average of the absolute sum of negative/positive weights, w^-/w^+. Similar to Section 5.5.3, we observe no major differences between the degree of diversification for the portfolios using the *filtered* correlation matrices and those using the sample correlation matrix. Likewise, when analysing different volatility periods, the magnitude of these quantities does not appear to change when transitioning from one period to the other. Therefore, the success of the RMT-filtering does not come from the fact that invests in more stocks than the standard correlation matrix.

Model	Pre-crisis: Years 2003-2006		Crisis: Years 2007-2010	
	N_{90}	w^-/w^+	N_{90}	w^-/w^+
	(1)	(2)	(3)	(4)
$C^{SAMP} - 125$	46.10 (2.07)	0.565	46.68 (2.20)	0.643
$C^{SAMP1} - 125$	45.73 (2.39)	0.259	45.52 (2.70)	0.300
$C^{SAMP-F1} - 125$	44.65 (3.08)	0.288	44.73 (3.09)	0.372
$C^{SAMP-F2} - 125$	44.31 (3.03)	0.266	44.94 (3.47)	0.354
$C^{SAMP} - 250$	45.42 (2.42)	0.437	46.02 (2.47)	0.543
$C^{SAMP1} - 250$	45.44 (2.56)	0.274	45.86 (2.59)	0.312
$C^{SAMP-F1} - 250$	45.34 (3.03)	0.306	44.68 (3.58)	0.387
$C^{SAMP-F2} - 250$	45.26 (2.94)	0.281	44.84 (3.66)	0.371

Table 5.8: Portfolio Diversification, N_{90}, for different time windows, using the *filtered* and sample correlation matrices, under different volatility periods. Standard errors in parentheses.

The correlation estimators, however, do differ in their degree of exposure to *short-selling*. In Columns (2) and (4), we find that the portfolios using the sample correlation matrix increase their exposure to *short-selling* during the crisis. While a similar phenomenon happens with the portfolios built using the *filtered* correlation matrices, the difference in magnitude is sizeable. For instance, for $C^{SAMP-F1}-125$, the ratio is $w^{-}/w^{+}=0.373$, when compared to 0.644, when using $C^{SAMP}-125$. Moreover, by observing cumulated returns and Sharpe Ratios during the crisis, we find that the portfolios built using the sample correlation matrix did not only exhibit an overexposure to *short-selling*, but also made the wrong bet by investing in artificially low risk assets and therefore dampening Sharpe ratios.

5.6 Concluding Remarks

In the present chapter we have investigated the benefits of the RMT-filtering on the sample correlation matrix, constructed from daily data on 77 stocks contained in the FTSE 100, covering the period 04/01/00 until 25/09/12. We estimated a number of *minimum variance* portfolios, using the sample correlation matrix, and the RMT-based filtered correlation matrices, using the methods of Plerou et al. (2002) and Laloux et al. (2000) as described in Section 3.3. These portfolios were estimated using different estimation windows and allowing/restricting *short-sale*. Our main findings can be summarised as follows:

- We find that the RMT-filtering delivers the lowest *realised* risk, the lowest volatility of prediction and the best prediction accuracy when compared with the sample correlation estimator. This is particularly true in the absence of *short-sale* constraints. Under these conditions, the effects of the RMT-filtering, translated in an improvement of 65% in terms of cumulated wealth, for the best result.

- When *short-sale* constraints are in place, results are ambiguous and we therefore recommend the use of the sample correlation estimator.

- We also find that the RMT-filtering performs the best for short estimation windows, of the order of 6 months to 1 year of data (e.g., $1.6 \leq Q \leq 3.2$). This is consistent with what it has been found in the literature (see, Pafka et al., 2004; Daly et al., 2008). For higher values of Q (e.g., $3.2 < Q < 6.5$) these benefits would appear to taper, as the *filtered* correlation matrix converges to the sample correlation matrix, as the sample size increases.

- These results are robust to both, low and high volatility periods in financial markets. In the former case, we find that there are no major differences between using the sample and filtered correlation matrices. In the latter case, we find that the RMT-filtered correlation estimators outperform the sample correlation estimator in both cumulated returns and Sharpe Ratios.

These results are not surprising since for a given value of Q, the RMT approach selects different numbers of eigenvalues to include in the filter, depending on the volatility period faced during the estimation period (see, Figure 5.6). For example, from June 2007 until present (the Crisis period), the RMT-filtering selected more often different number of eigenvalues to include in the filter (a sudden jump from 1 to 3). This evidences the release of new information from the noisy part of the spectrum that is incorporated in the *filtered* correlation matrix when facing market downturns.

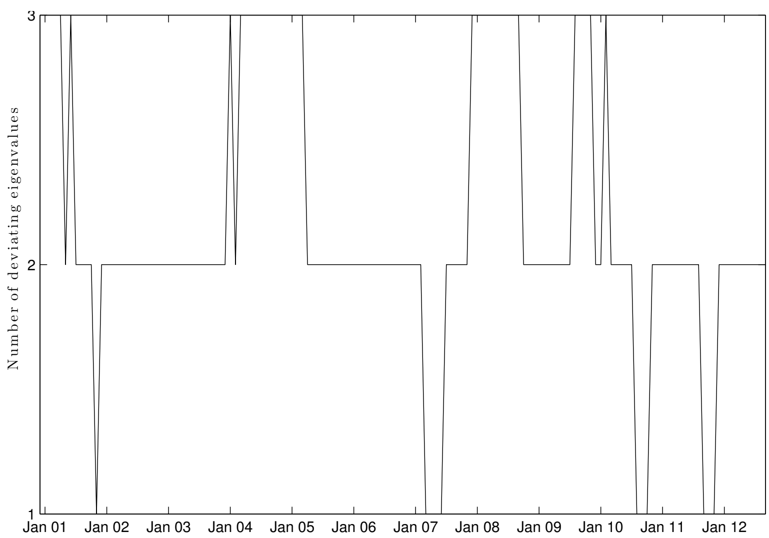


Figure 5.6: Number of Deviating Eigenvalues included in the RMT-filtering over the out-of-sample period. For a given value of Q, there are eigenvalues entering and exiting the Marčenko-Pastur distribution.

This is in contrast to the sample correlation matrix, which treated all eigenvalues as equally significant in both periods. We therefore stress the importance of distinguishing the noise coming from the use of a short time-series and the noise introduced by the nonstationarity of the data. In the latter case, we have just shown that the RMT-filtering is also able to deal with this problem. We therefore recommend the use of the RMT-filtering under two conditions; (i) when using a short time window, because of its effectiveness in tracking sudden changes in volatility and (ii) when allowing for *short-sale*, as it allows taking full advantage of *short-selling* positions to overcome high volatility periods. In this context, *short-selling* can be a very powerful strategy when using the right correlation estimator.

We also suggest a possible explanation for the success of the RMT-filtering. It would seem that the portfolios built using the sample correlation matrix tend to take, on average, more extreme positions and are more overexposed to *short-selling* than those built using the *filtered* correlation matrix. These results are robust to both volatility periods.

In Pantaleo et al. (2011), the performance of nine covariance estimators was analysed, for 100 highly capitalised stocks of the NYSE. The covariance estimators were based on those aimed to shrink the covariance matrix (*Shrinkage Estimators*), or by creating clusters of stocks (*Agglomerative Hierarchical Clustering Estimators*) and others based on the spectral properties of the correlation matrix (*Random Matrix-based filtering*). They assessed the performance of these models by analysing their *realised* risk, for different estimation windows, with and without *short sale*. They find that when *short-sale* is allowed, their estimators are able to produce portfolios which are significantly less risky than those using the sample correlation matrix (i.e.*unfiltered*). They also find that the portfolios using the sample correlation matrix had a greater exposure to *short-selling* than those using the *filtered* correlation matrix. However, when *short-sale* is not allowed, they find that the *realised* risk of the portfolios built using the sample correlation matrix are comparable to those of the other estimators, but they are significantly less diversified. Our results are similar in that, for short estimation periods, we find significant differences between the portfolios built using the *filtered* and sample correlation matrices, which is particularly true when *short-sale* is allowed. However, we do not find major differences in the degree of diversification for the different correlation estimators and time windows used, except in the amount of overexposure to *short-selling*, which may shed some lights about the possible causes for the larger risk found in the portfolios built using the sample correlation matrix.

Chapter 6

A Comparative Study: Regime Switching Models and RMT-Filtering

6.1 Introduction

Since the seminal work of Longin and Solnik (2001), who formally established the statistical significance of an assymetric correlation phenomenon, large amount of research has been devoted to the study of this asymmetric behaviour in financial markets. These studies, have been motivated by the idea that correlations of asset returns are higher during market downturns than during boom market conditions. An early strand of literature has made use of time-varying volatility models, such as GARCH-M models with asymmetry to capture this phenomenon (Engle and Kroner, 1995; Bekaert and Harvey, 1997; DeSantis et al., 1999; Bekaert and Wu, 2000), Jump Models (Das and Uppal, 2004), where returns are drawn from a bivariate normal to produce larger downward correlations.

A second strand of the literature on the asymmetric behaviour of correlations makes use of Regime-Switching (RS) models (Hamilton, 1989). The first of this kind is the RS-Normal Models, which mix two different bivariate normal distributions allowing returns to switch between a regime with lower conditional means, higher volatility, higher correlations and another depicting normal market conditions of higher positive means, lower volatilities and lower asset correlations. This model was used by Ang and Bekaert (2002) to examine international asset allocation under higher correlations with downside moves in country returns. They estimate this model for equity returns from the US, Germany and the UK and find that international equity returns would seem to be characterised by two regimes: "a normal regime and a bear market regime, where stock market returns are, on average, lower and much more volatile than in normal times. Importantly, in the bear market regime, correlations between various returns are higher than in the normal regime" (Ang and Bekaert, 2002). The authors find that this model captures a large part of the asymmetric behaviour of international equity returns of developed countries. A second version of these models is the RS-GARCH model, which combines elements of the switching behaviour of pure RS-Normal Models with the volatility persistence of GARCH processes (Engle and Kroner, 1995; Gray, 1996).

The existence of asymmetries in financial correlations is of common knowledge for most researchers and those in the financial industry. However, one of the major difficulties found by researchers has been the inability to correctly predict sudden changes in correlations. In Ang and Chen (2002) the performance of the aforementioned models was examined. Of these, they find the Regime-Switching Normal Model to be the most successful in matching the magnitude of empirical correlation asymmetries. The authors argue that while Regime-Switching Models are quite successful in capturing asymmetries in financial correlations, there is still a significant amount of asymmetries left unexplained. Hence, in the present study we ask whether the RMT-filtering approach for asset correlations can improve the performance of minimum variance portfolios based on the RS-approach introduced by Ang and Chen (2002).

The purpose of this chapter is to investigate the statistical properties of regime-dependent correlation matrices constructed from stock returns in the FTSE 100 using Random Matrix Theory (RMT) and analyse the implications of this for the optimal portfolio weights in standard Markowitz's Portfolio Theory and also for Hamilton's two state regime sensitive portfolio optimisation. As we saw in the previous sections, the underlying argument behind the use of the RMT-filtering is that when we estimate correlation matrices, we usually have too little observations compared to the number of parameters in the correlation matrix. This results in estimation noise, which in turn affects correlation estimates and subsequently, portfolio estimates. We expect regime-dependent correlation matrices to be afflicted by a similar phenomenon. In this context, regime-dependent portfolio weights could be improved to yield better risk forecasts and higher portfolio returns by the same reasoning.

In this chapter, we estimate *filtered* and *unfiltered* versions of regime-dependent correlation matrices, where regimes are driven by the market index and construct a number of portfolio metrics. We report the following findings; (i) the RMT-based filtering improves the performance of regime-dependent correlation matrices, in terms of better *realised* risk estimates and risk-adjusted measures of returns. However, the source of the improvement comes from filtering correlations in Regime 1 (low volatility regime). This happens because there is good agreement between the eigenvalue distribution in Regime 1 and the Marčenko-Pastur Distribution. This allows us to distinguish signals from noise in the eigenvalue spectrum for Regime 1. While this is true for correlation in Regime 1, correlations for Regime 2 (high volatility regime) are far from normal, which means that, when contrasted with the Marčenko-Pastur, there is significantly more information, which makes filtering redundant; (ii) these results hold for short estimation windows and when *short-sale* is permitted; (iii) we also find that the *filtered* sample correlation matrix can be as competitive as any regime-dependent estimator, in terms of *realised* risk estimates and risk-adjusted measures of returns; (iv) finally, from an asset allocation perspective, we find that the *filtered* regime-dependent correlation estimators deliver more diversified portfolio weights and also place less resources on Financials than their *unfiltered* counterparts.

The rest of the chapter is organised as follows: Section 6.2 provides a brief introduction to Regime-Switching Models and introduce a parsimonious version of the CAPM Model with regime shifts (Section 6.2.1) and an illustration of the model (Section 6.2.2). Section 6.3 provides an spectral analysis of the regime-dependent correlation matrices. Section 6.4 defines the correlation estimators used in our analysis. In Section 6.5, we report our in-sample results by constructing *realised* risk forecasts for minimum variance portfolios using a bootstrap approach. Section 6.6 reports the out-of-sample results of our correlation estimators in terms of both, their ability to correctly predict *realised* risk of minimum variance portfolios (Section 6.6.1) and also in their ability to produce portfolios with the highest cumulated returns and risk-adjusted measures of returns (Section 6.6.2). Section 6.7 studies the implications of our analysis from an asset allocation perspective. Finally, in Section 6.8 we provide conclusions and recommendations.

6.2 Regime-Switching Models

As we mentioned in the introduction, the existence of different "regimes" in financial markets has great implications from an asset allocation perspective. This is so, because the optimal portfolios during bear markets are substantially different from those of a normal market. As pointed out by Ang and Bekaert (2002), the benefits of financial diversification dominate the costs of ignoring the regimes, but a Regime-Switching strategy will out-perform static strategies out-of-sample. As reported by Hess (2006), Regime-Switching provides valuable timing signals for portfolio rebalancing. He finds that investors should apply more aggressive portfolio strategies during market downturns, because these are precisely the moments in which there is great commonality between assets delivering great losses to investors. This calls for a model that, on the one hand, can be able to describe the co-movement between assets and, on the other hand, can capture asymmetries in correlations, especially during market downturns. In Ang and Bekaert (2002), the Regime-Switching CAPM model was introduced to model asymmetric correlations in international portfolios. This model was found to provide portfolio decisions based on bull/bear regimes and asymmetric correlations with inspiring investment insights. In the next section, we introduce this specification to model regime-dependent correlations and investigate the spectral properties of regime-dependent correlation matrices with the help of RMT tools.

6.2.1 A Parsimonious RS-Model: The Beta-CAPM

In order to illustrate how the Beta-CAPM works, we assume that the market, which is given by the returns of the FTSE 100, is the one that switches between regimes and drives all asset returns. In this context, market returns (in excess of the 1-month Libor rate) are defined as:

$$y_t^{ftse} = u_t^{ftse} + \sigma^{ftse}(s_t)\epsilon_t^{ftse}, \tag{6.1}$$

where u_t^{ftse} is the market expected return and $\sigma^{ftse}(s_t)\epsilon_t^{ftse}$ is the conditional volatility. In this model, these two values are allowed to change depending on the realisation of the regime, s_t. The "switch" between the two regimes is governed by a transition probability matrix, characterised by two transition probabilities:

$$P = p(s_t = 1/s_{t-1} = 1), \tag{6.2}$$

$$Q = p(s_t = 2/s_{t-1} = 2). \tag{6.3}$$

If the portfolio manager knows the regime, the expected excess returns for the market index next period will be either:

$$e_1^{ftse} = Pu^{ftse}(s_{t+1} = 1) + (1 - P)u^{ftse}(s_{t+1} = 2), \tag{6.4}$$

$$e_2^{ftse} = (1 - Q)u^{ftse}(s_{t+1} = 1) + Qu^{ftse}(s_{t+1} = 2). \tag{6.5}$$

In practice, we cannot be certain about which regime we are in and therefore we need to infer it from the data available at each point in time. We do this by constructing the regime probability, which is the probability that tomorrow's regime is the first regime given current and past information. In this simple model, the information set consists simply on the market returns data.

The model embeds time-varying volatility for the market return, which consists of two components. For example, if the regime today is the first regime, the conditional variance for the market excess return is given by:

$$\begin{aligned}\Sigma_1^{ftse} = P(\sigma^{ftse}(s_{t+1}))^2 + (1 - P)(\sigma^{ftse}(s_{t+1} = 2))^2 + \\ + P(1 - P)[u^{ftse}(s_{t+1} = 2) - u^{ftse}(s_{t+1} = 1)]^2,\end{aligned} \tag{6.6}$$

$$\Sigma_2^{ftse} = (1-Q)(\sigma^{ftse}(s_{t+1}))^2 + Q(\sigma^{ftse}(s_{t+1}=2))^2 + \\ +Q(1-Q)[u^{ftse}(s_{t+1}=2) - u^{ftse}(s_{t+1}=1)]^2. \tag{6.7}$$

The first component in these equations is simply a weighted average of the conditional variances in the two regimes; the second component is a jump component that arises because the conditional mean is different across regimes. We model the individual asset excess returns, y^j_{t+1}, using a CAPM-inspired beta model:

$$y^j_{t+1} = \alpha^i + \beta^j u^{ftse}(s_{t+1}) + \beta^j \sigma^{ftse}(s_{t+1})\epsilon^{ftse}_{t+1} + \bar{\sigma}^j \epsilon^j_{t+1} \tag{6.8}$$

$$y^j_{t+1} = \alpha^j + \beta^j y^{ftse}_{t+1} + \bar{\sigma}^j \epsilon^j_{t+1} \tag{6.9}$$

In Equation 6.9, individual returns are modeled as a function of the market index and an idiosyncratic volatility term. This model is very parsimonious and only requires the estimation of the index market process, a constant term α, one beta and an idiosyncratic volatility per stock. With regime switches, this simple model captures time-variation in expected returns, volatility and correlations, all driven by the market index regime.

In order to derive the expected returns and covariance matrix for stock returns, we introduce further notation. We first define the covariance matrix, conditional on period t, denoted by $\Sigma_i = \Sigma(s_t = i)$ and the vector of excess returns, $e_i = e(s_t = i)$. Given that the mean of the market index switches between states, the expected excess returns of stock i are given by $\alpha_j + \beta^j e_i^{ftse}$ for the current regime i, where e_i^{ftse} are given by Equations 6.6 and 6.7. In matrix notation, let

$$\alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix} \text{ and } \beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix}.$$

Hence, the expected return vector is given by:

$$e_i = \alpha + \beta e_i^{ftse}, \qquad i = 1, 2. \tag{6.10}$$

In Equation 6.9, expected returns differ across individual stocks only through their different betas with respect to the market index, which is regime dependent. Regarding the regime-dependent covariance matrix, this has three components (Equation 6.11). The first is an idiosyncratic part, captured by the matrix V, which is a matrix that captures variances and covariances between the error terms for each asset. By doing this, we are assuming that there are other sources of variation between assets, intra-industry correlations not necessarily driven by the market index. However, systematic risk is driven by the variance of the market index and the betas as in any factor model. As the market index next period depends on the realisation of the regime, we have two possible covariance matrices for the unexpected returns next period:

$$\Omega_i = (\beta\beta')(\sigma^{ftse}(s_{t+1} = i))^2 + V, \qquad i = 1, 2. \tag{6.11}$$

Finally, the actual covariance matrix takes into account the regime structure, in that it depends on the realisation of the current regime and it adds a jump component to the conditional covariance matrix, which arises because the conditional means transition between regimes. Consequently, the conditional covariance matrices can be written as:

$$\Sigma_1 = P\Omega_1 + (1 - P)\Omega_2 + P(1 - P)(e_1 - e_2)(e_1 - e_2)', \tag{6.12}$$

$$\Sigma_2 = (1 - Q)\Omega_1 + Q\Omega_2 + Q(1 - Q)(e_1 - e_2)(e_1 - e_2)'. \tag{6.13}$$

It is straightforward to show that this model structure implies that the correlations implied by Ω_1, the normal regime, will be lower than the correlations implied by Ω_2, the high volatility regime. This model is estimated in the following steps:

1. We first estimate a Seemingly Unrelated Regression (SUR) Model for stock returns with respect to the market excess returns and obtain the α's, β's coefficient and V.[1] These

[1]The SUR model, initially proposed by Zellner (1962), is a generalisation of a linear regression model that

coefficients are therefore not regime-dependent, as the model does not allow for asset-specific regimes.

2. Estimate the market index equation by Maximum Likelihood (MLE) and obtain the unconditional parameters $(e_1^{ftse}, e_2^{ftse}, \sigma_1^{ftse}, \sigma_2^{ftse}, P, Q)$.

3. We plug the values obtained in (1) and (2), in the equations shown above and we obtain the regime-dependent covariance matrices.

In what follows, we apply these steps to our data and obtain the regime-dependent covariance matrices and analyse their properties.

6.2.2 Data Analysis: The Existence of Different Regimes in the FTSE 100 Index

We use the returns of the FTSE 100 Index, covering the period from January 2000 until October 2012. Keeping this in mind, we estimate a two-state Regime Switching Model for the market index as described in Equation 6.1. Likewise, we estimate the SUR parameters for each individual stock of our sample and build the regime-dependent covariance matrices of Equations 6.11 and 6.12.[2] The results of this exercise are presented in Table 6.1.

	Regime 1		Regime 2		Transition Pbs.	
	μ_1	σ_1	μ_2	σ_2	P	Q
Estimates	0.81	15.77	-2.02	39.75	0.99	0.98
Std. Errors	(0.003)	(0.000)	(0.012)	(0.006)	(0.02)	(0.01)

Table 6.1: Regime-Switching Estimation for the FTSE Index. All parameters are monthly and expressed in percentages, except for the transition probabilities P and Q. Whole sample estimates.

These results provide clear evidence of the existence of an asymmetric behaviour in mean (as $\mu_1 > \mu_2$) and variance (as $\sigma_1^2 < \sigma_2^2$) for the FTSE 100 Index. All the coefficients are statistically

consists of several regression equations, each having its own dependent variable and potentially different sets of exogenous explanatory variables. A special case of this model is when the error terms are assumed to be correlated across equations. This is estimated by FGLS with a specific form of the variance-covariance matrix and is far more efficient than estimating each equation separately by OLS.

[2]In order to make these new results comparable to those of Chapter 5, we use the same 77 stocks used in the previous analysis.

significant at 1%. Assuming that the behaviour of individual stocks is driven by the market index, we should expect this to induce asymmetries in correlation matrices. The expected returns and standard deviation in Regime 1 are 0.81% and 15.77% per month, respectively. While in Regime 2, these quantities are -2.02% and 39.75% per month. In this case, standard deviations are signficantly higher than those found in previous studies (for example, Ang and Bekaert (2002)). Having said this, it is worthwhile noting that these differences are not puzzling since we are using daily instead of monthly data, hence we should expect to see higher standard deviations when using data that is sampled more frequently. Nonetheless, this does not change the main message: "Regime 1 is much more stable than Regime 2, with less volatility and positive returns, in contrast to Regime 2, which is considerably more volatile and exhibits *negative* returns".

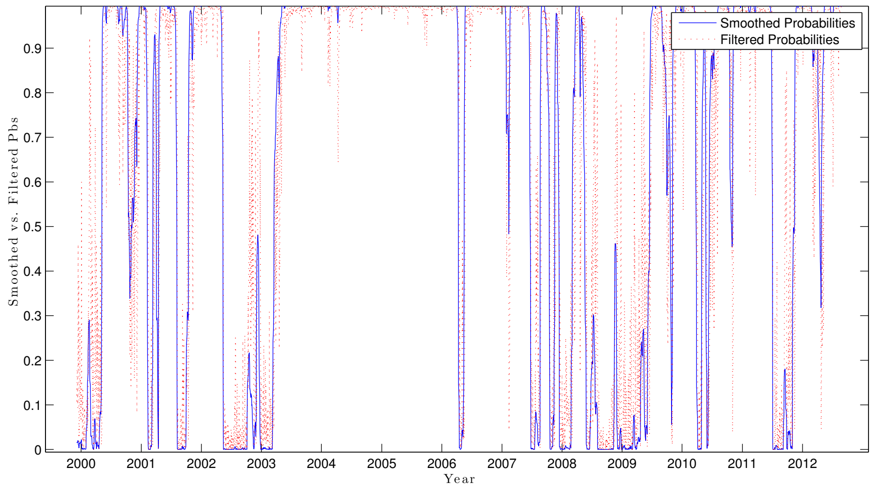


Figure 6.1: Smoothed and Filtered Probabilities for Regime 1 - Whole Sample Estimates (2000-2012).

We can also see that the transition probabilities P and Q, show that both regimes are highly persistent. For example, Regime 1 is more persistent than Regime 2. In this case, there is 99% of probability that the market stays in Regime 1 next period given that it is currently in Regime 1, while there is a 98% of staying in Regime 2 next period given that the process is already in Regime 2. While these probabilities would appear to be similar, they yield very different results when calculating the duration of regimes. For example, for Regime 1, the expected duration is approximately $1/(1-P) = 105.6$ days ($\approx$ 5 months), while the duration of Regime 2 is approximately $1/(1-Q) = 45.09$ days ($\approx$ 2 months).[3] We also report the regime probabilities, namely, the filtered and smoothed probabilities. The former is the probability that the regime next period is the low volatility period regime given current information, while the latter is the probability that the regime in the next period is the low volatility period given all of the information available in the data sample, which is calculated backwards by using the filtered and forecasting probabilities.

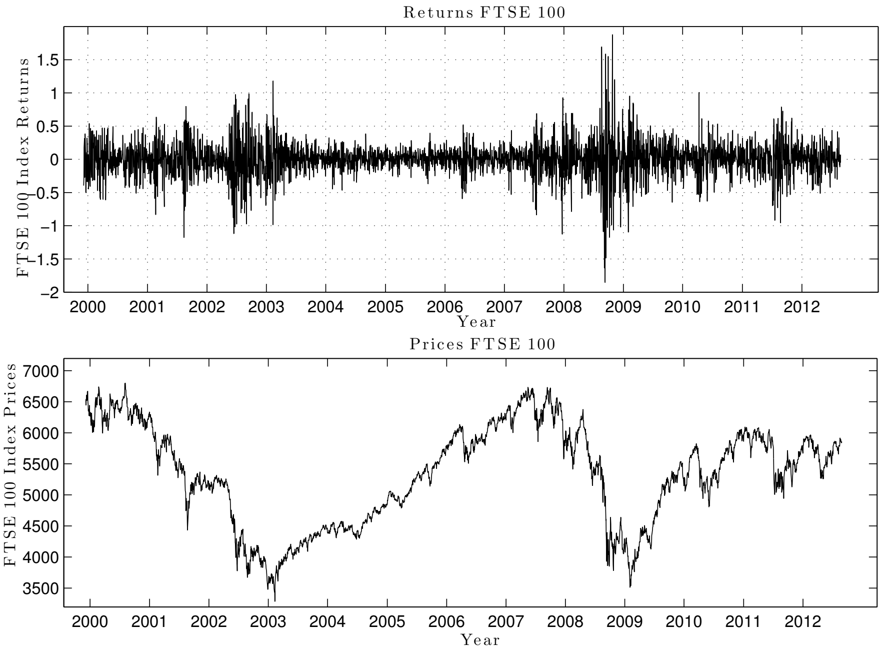


Figure 6.2: FTSE 100 Returns and Index - Whole Sample Estimates.

In Figure 6.1, we show the probabilities of being in Regime 1. For instance, if the filtered probability is greater than 0.50, we are in Regime 1, otherwise we are in Regime 2. From this figure, it is easy to see that there were many times where these probabilities have dipped down from the cut-off of 0.50, hence showing greater probability of being in Regime 2. Keeping this in mind, by looking at the filtered probabilities, and ignoring short-lived regime changes, we can identify approximately 4 regimes shifts during the entire sample; A more volatile regime in 2000-2003 followed by a quiet regime in 2003-2007, another volatile period in 2007-2009, which

[3]The expected duration of Regime 1 is calculated as: $\sum_{k=1}^{\infty} kp_1^{k-1}(1-p_1) = 1/(1-p_1)$; (see, Hamilton, 1989).

coincides with the Global Financial Crisis (GFC) and a post-crisis period in 2010-2012.[4]

It is worthwhile mentioning the reason why we do not use monthly data, which is that given the total number of stock ($N = 77$), we would need at least 6 years of data to estimate the models and we would be left with very few observations to assess the performance of the models out-of-sample. Even if we could obtain longer data, the FTSE 100 index has changed its composition over the years, which means that our sample would be significantly reduced. Recall that we are also interested in assessing the performance of the RMT-filter under different values of the ratio $Q = T/N$. The use of daily data also leads us to make the strong assumption that daily regime probabilities on a given day will remain constant over the evaluation period, which in our case is 20 days. This latter would not seem to be problematic since regimes are highly persistent.

In Figure 6.2, we can see the returns of the FTSE 100 Index and below the FTSE 100 Index prices. By comparing these quantities, with those depicted in Figure 6.1, we can see that when the market is in Regime 2, the FTSE 100 Index returns are mostly *negative*. This is particularly true, during the 3rd quarter of 2007. Conversely, large returns are associated with a more stable regime, such as Regime 1.

Why is this important? The importance of this is that financial correlations exhibit a similar pattern to that of the market. For example, in Figure 6.3, we plot the distribution of the sample correlation matrix for the entire sample on a daily rolling window of 250 days. Here we observe that, in general, correlations are faily positive and that in periods of high volatility, such as during the Subprime Crisis and the crisis that followed, there was a shift in the distribution towards the right, that is, higher skewness, as opposed to more tranquil times, where the distribution is more centered and also with a lower excess of kurtosis. In the next section, we will explore the spectral properties of the regime-dependent correlation matrices, with the help of RMT tools. As we will show, similar conclusions can be derived from the analysis that follows.

[4]Recall that for our analysis we are using daily data. Therefore, we should expect that the use of daily data will tend to bring more volatility in regime shifts. For this reason, we consider only longer periods of time for the process to be in a certain regime.

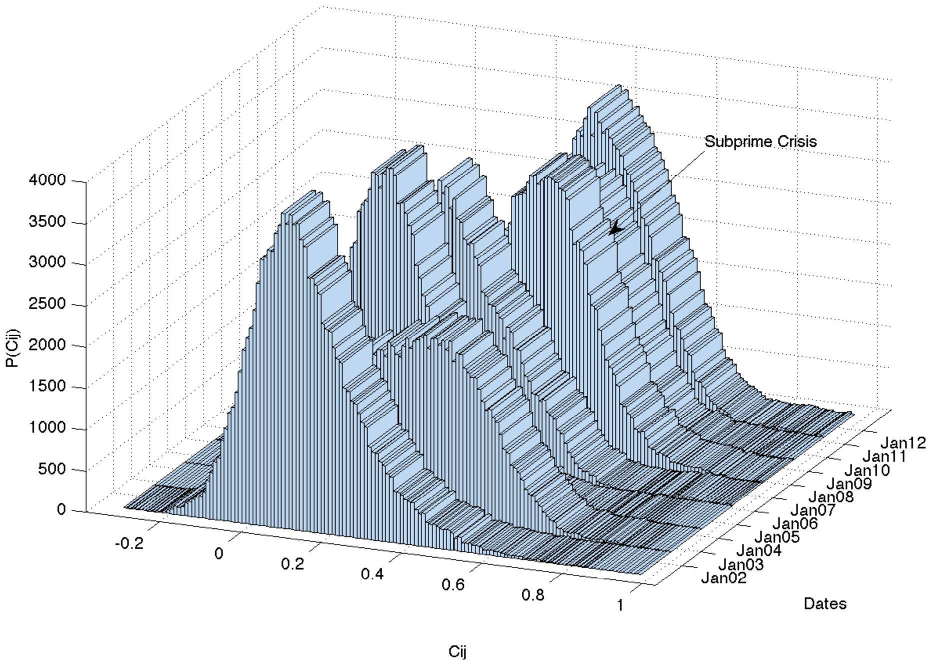


Figure 6.3: Distribution of the Sample Correlation matrix estimated on a daily rolling window of length 250 days. Note that the distribution shifts to the right post year 2008.

6.3 Spectral Properties of Regime-Dependent Correlation Matrices: Combining RMT with MRS

In this section, we analyse the spectral properties of regime-dependent correlation matrices, constructed as described in Section 6.2.1. In Figure 6.4 we plot the eigenvalue distributions of the regime-dependent correlation matrices against the Marčenko-Pastur Distribution (MP) for our in-sample period, which uses the first 500 observations. Panel (a) presents the eigenvalue distribution for Regime 1. Here we observe that the largest eigenvalue is of the order of $\lambda_1 = 9.8$, explaining a 12.7% of the total variance of the data. There are three large deviations from the theoretical upper limit, $\lambda_+ = 1.33$. In contrast, the eigenvalue distribution for Regime 2 (Panel (b)), exhibits one large deviation at the upper edge of the spectrum, $\lambda_1 = 24$, which is considerably larger in magnitude compared to Regime 1, capturing 31.1% of the total variance of the data. A second observation is that there are more eigenvalues lying outside at the lower bound of the MP distribution. This property of eigenvalues is known as "eigenvalue repulsion".

By eigenvalue repulsion we refer to the dynamics of the small eigenvalues are contrary to those of the largest eigenvalue.[5] For instance, a decrease in the largest eigenvalue, with a corresponding increase in the small eigenvalues, implies a redistribution of the correlation structure across more dimensions of the vector space spanned by the correlation matrix. Therefore, additional eigenvalues are needed to explain the correlation structure of the data. Conversely, when the correlation structure is dominated by a small number of factors (e.g, the "single-factor model" of stock returns), the number of eigenvalues needed to describe the correlation structure in the data is reduced. This means that fewer eigenvalues are needed to describe the correlation structure of "drawdowns" than that of "draw-ups" (Conlon et al., 2009).

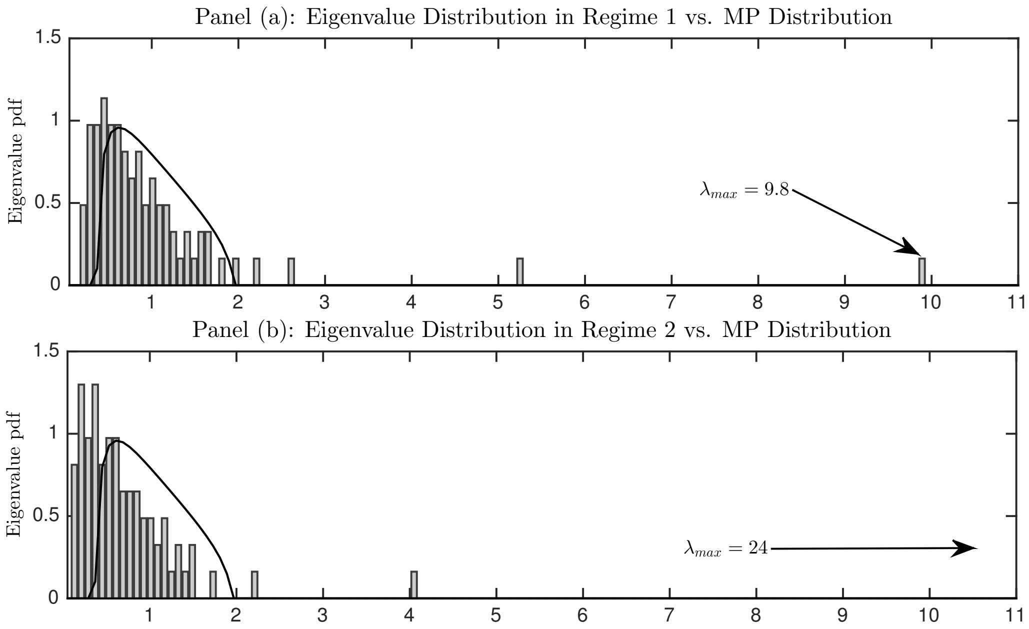


Figure 6.4: Eigenvalue Distributions by Regime vs. the Marčenko-Pastur Distribution.

A Kolgomorov-Smirnov test cannot reject the null that the eigenvalue distribution for Regime 1 belongs to the Marčenko-Pastur (MP) Distribution at 10% significance. However, the test rejects the null for the eigenvalue distribution for Regime 2.[6] This means that filtering the corre-

[5] This is a consequence of the fact that the trace of the correlation matrix must remain constant under transformations and any change in the largest eigenvalue must be reflected by a change in one or more of the other eigenvalues.

[6] The Kolgomorov-Smirnov test, is a nonparametric test of the equality of continuous, one-dimensional probability distributions that can be used to compare a sample with a reference probability distribution. It basically quantifies the distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution.

lation matrix in Regime 2 would be redundant, as there is already enough information outside of the bounds of the MP distribution. This suggests that Regime Switching correlations might be benefited from using the RMT approach. More especifically, these benefits would come from filtering correlations in Regime 1, where there is a close-form solution for the eigenvalue spectrum, which is given by the Marčenko-Pastur Distribution. In what follows, we explore the implications of RMT-filtering on regime-dependent correlation matrices in a portfolio optimisation framework.

6.4 RMT-Based Correlation Estimators

Regarding our correlation estimators, we introduce new notation and propose nine estimators:

- Sample Correlation Matrix: C^{SAMP}.
- Filtered Sample Correlation Matrix - Method 1: $C^{SAMP-RMT1}$ (Plerou et al., 2002).
- Filtered Sample Correlation Matrix - Method 2: $C^{SAMP-RMT2}$ (Laloux et al., 2000)
- CAPM-based Correlation Matrix: C^{CAPM}.
- Filtered CAPM-based Correlation Matrix - Method 1: $C^{CAPM-RMT1}$.
- Filtered CAPM-based Correlation Matrix - Method 2: $C^{CAPM-RMT2}$.
- Regime Switching Correlation Matrix: C^{RS}.
- Regime Switching Correlation Matrix - Method 1: $C^{RS-RMT1}$.
- Regime Switching Correlation Matrix - Method 2: $C^{RS-RMT2}$.

The first three estimators are those used in Chapter 5, where we estimated the effects of the RMT-filtering on the sample correlation matrix. The fourth and the seventh estimators are the CAPM-based estimator and the Regime Switching estimator (MS), respectively and their filtered versions. In a portfolio optimisation context, the RS correlation estimator works as follows: First, we estimate the regime-dependent correlation matrices based on the model described in Section 6.2.1. Second, we take the *filtered* probabilities at the end of the in-sample period and compare it to a cut-off point of 0.5. If the *filtered* probability is greater than 0.5, then we classify next period as Regime 1, otherwise we classify it as Regime 2. From this decision rule, we use either of the regime-dependent correlation matrices as input for portfolio optimisation.

6.5 In-Sample Analysis

For the in-sample testing, we use the first 500 observations of our sample and follow the methodology introduced in Section 3.4.1. This analysis consisted in taking bootstrapped samples, together with the mean across these samples. That is, for a given number of stocks, we randomly select a test date t and estimate everything up to t and everything afterwards is regarded as realised future information.[7] We repeat this exercise 1000 times, with replacement and calculate the mean, across all bootstrapped samples of the statistic γ, which is the ratio between the realised portfolio risk and the predicted risk of *minimum variance portfolios.* This metric will gauge how close our predictions are from the realised ones. For the purpose of this chapter, we will test the in-sample performance of three correlation estimators; the *unfiltered* regime-switching correlation estimator (C^{MS}), the *filtered* regime-switching correlation estimator, using the method of Plerou et al. (2002) ($C^{RS-RMT1}$) and the *filtered* regime-switching correlation estimator, using the method of Laloux et al. (2000) ($C^{RS-RMT2}$). For these cases, the RMT-filtering is only applied to the low volatility regime (Regime 1).[8]

The results of this exercise are depicted in Figure 6.5.[9] A first observation is that the RMT-filtering delivers much better realised risk estimates than its *unfiltered* version. This is especially true for short estimation windows. However, in contrast to the case when the sample correlation matrix is filtered, both the *filtered* and *unfiltered* regime-dependent correlation matrices require in general more observations to deliver better risk estimates. This happens because longer estimation windows allow for a better identification of regimes, whereas short estimation windows will tend to make the wrong bet when estimating the filtered probabilities. In general, the benefits of filtering would seem to persist over the in-sample period, for values of Q, within the interval

[7]For this particular case, we leave an horizon window of 20 business days.

[8]It is worthwhile mentioning that the present study is mainly focused on correlation modeling, therefore, individual variances are assumed to be known and are the same across correlation estimators.

[9]For the present analysis, we allow for *short-sale* by limiting weights to be no bigger than 100% on any asset and to "go short" on a 150%.

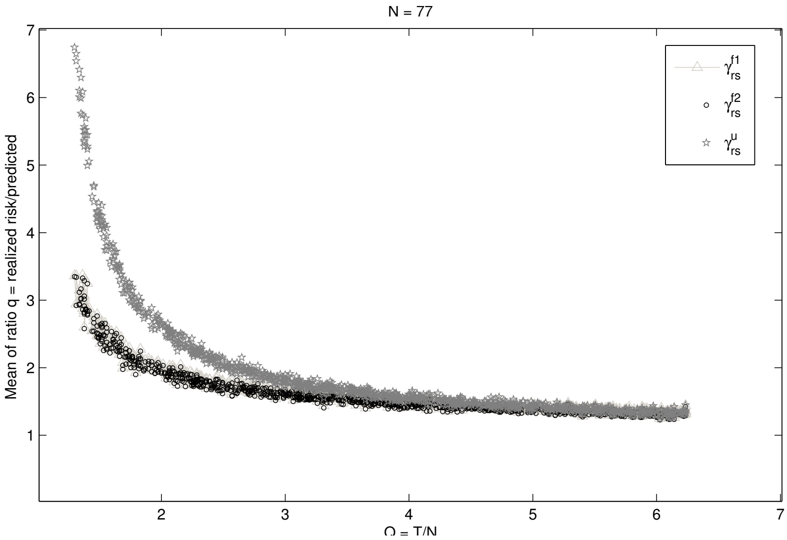


Figure 6.5: Mean Bootstrapped (in-sample) for the ratio of *realised* to *predicted* risk, for the *filtered* regime-dependent correlation matrices, $\bar{\gamma}^f = \frac{\Omega_{fr}}{\Omega_f}$, compared to the *unfiltered* approach, $\bar{\gamma}^u = \frac{\Omega_{ur}}{\Omega_u}$, as a function of $Q = T/N$. Notice that filtering is most effective for low values of Q.

$[1.30, 5]$. For values above 5, we still get some improvements, but these are not sizeable. A second observation is that both filtering methods deliver similar realised risk estimates. While differences are not noticeable with the naked eye, the filtering method 2 (Laloux et al., 2000) delivers slightly better realised risk estimates.

6.6 Out-of-Sample Analysis

For the out-of-sample analysis, we follow the methodology described in Section 3.4.2. Similar to Chapter 5, we split the sample in two subperiods. The first subperiod, that we used in the previous section for the in-sample analysis covers the period from 04/01/00 until 30/11/01 ($\approx$ 500 observations), while the second subperiod is the evaluation period, which starts in 04/12/01 until 25/09/12 ($\approx$ 2820 observations). As explained in Section 3.4.2, for every test date t', we consider portfolios that were invested before $t' - t^f$, or on that date, where t^f is the investment period (in this case, 20 business days). That is, we estimate the portfolio variance at a given month and compare it to the *realised* variance on that month, which is calculated as the sample covariance of the *realised* returns over the investment horizon ($t^f = 20$ days).

As we saw in Section 6.5, the optimal value of Q falls within the interval $Q = T/N \in [1.3, 5]$, with the RMT-filtering performing the best for low values of Q, whereas for larger values, these benefits would tend to taper. An estimation window of 250 days, given a fixed number of assets, $N = 77$, is consistent with a value of $Q = 3.24$, which is exactly in the middle of the aforementioned interval, while an estimation window of 500 days is consistent with $Q = 6.49$, well outside this interval. We expect the model to favour short over long estimation windows. As opposed to most studies in the literature, we estimate our models with *short-sale* constraints. We think that this provides a more realistic setting as *short-sale* is not always allowed in financial markets stemming from the increase in risk that brings to portfolios. Alternatively, we also allow for *short-sale*, however, none of the estimated portfolios ever took an extreme position.[10] All these models use the first subperiod (01/04/00 until 12/31/02 $\approx$ 500 observations) as initial training period. Subsequent retraining is done on a monthly basis.

6.6.1 Realised Risk Estimates

In Table 6.2, we report our *realised* risk estimates, using the performance metrics defined in Section 3.4.2 with *short-sale* constraints (Columns 2-5). Here we observe that there are no major improvements in realised risk estimates by using the RMT-filtering on regime-dependent correlation matrices. In most cases, estimates are only slightly better when applying the RMT-filter. This is consistent with the findings of Chapter 5, where we reported that there were no significant gains from filtering the sample correlation matrix when *short-sale* constraints are in place.

When *short-sale* is allowed (Columns 6-9) we observe that there are great improvements in realised risk estimates. In general, the *filtered* regime-dependent correlation matrices deliver the best realised risk estimates, the lowest volatility of prediction (MSE) and also the best prediction accuracy. For instance, when using $C^{RS-F1-250}$, we obtain a ratio $\bar{\gamma} = 1.44$ and MSE of 0.21, when compared to the *unfiltered* regime-dependent correlation matrix, C^{RS-250}, which yields $\bar{\gamma} = 1.64$ and MSE of 0.27. Similar to the case of the sample correlation matrix, as we increase the estimation window to 500, the benefits of filtering become less evident. An interesting

[10]We refer to extreme positions to those portfolios that place all the weight on one or very few stocks. Also, similar to the in-sample analysis, when allowing for *short-sale* we limit our weights to be no bigger than 100% on any asset and to “go short” on a 150%.

Model	No short-sale				Short-sale			
	Ω_{est}	Ω_{real}	MSE x 10^{-2}	$\bar{\gamma}$	Ω_{est}	Ω_{real}	MSE x 10^{-2}	$\bar{\gamma}$
$C^{SAMP-250}$	0.106	0.112	0.068	1.055	0.063	0.108	0.310	1.689
$C^{SAMP-F1-250}$	0.106	0.111	0.062	1.042	0.069	0.101	0.200	1.407
$C^{SAMP-F2-250}$	0.101	0.113	0.079	1.121	0.068	0.099	0.190	1.408
$C^{CAPM-250}$	0.106	0.112	0.068	1.054	0.063	0.108	0.310	1.690
$C^{CAPM-F1-250}$	0.106	0.111	0.061	1.042	0.069	0.101	0.200	1.408
$C^{CAPM-F2-250}$	0.109	0.112	0.068	1.016	0.085	0.098	0.069	1.131
C^{RS-250}	0.106	0.112	0.055	1.069	0.064	0.106	0.270	1.640
$C^{RS-F1-250}$	0.107	0.111	0.050	1.052	0.069	0.101	0.210	1.441
$C^{RS-F2-250}$	0.104	0.112	0.060	1.100	0.068	0.100	0.200	1.446
$C^{SAMP-500}$	0.106	0.111	0.057	1.028	0.071	0.100	0.150	1.390
$C^{SAMP-F1-500}$	0.107	0.110	0.053	1.016	0.073	0.100	0.160	1.339
$C^{SAMP-F2-500}$	0.103	0.111	0.063	1.070	0.072	0.098	0.140	1.324
$C^{CAPM-500}$	0.106	0.111	0.057	1.028	0.071	0.100	0.150	1.390
$C^{CAPM-F1-500}$	0.108	0.110	0.052	1.016	0.073	0.100	0.160	1.339
$C^{CAPM-F2-500}$	0.111	0.111	0.060	0.989	0.088	0.097	0.058	1.090
C^{RS-500}	0.105	0.110	0.052	1.063	0.071	0.099	0.150	1.369
$C^{RS-F1-500}$	0.106	0.110	0.490	1.049	0.073	0.097	0.130	1.309
$C^{RS-F2-500}$	0.104	0.110	0.567	1.081	0.072	0.097	0.130	1.309

Table 6.2: Out-of-sample Estimates of the realised risk of *minimum variance portfolios.* Notice that the *filtered* correlation matrices provide the *lowest* average *realised risk* and the best *realised risk* forecast in all cases, when *short-sale* is allowed.

observation is that the *filtered* versions of the sample and CAPM-based correlation estimators can be as competitive as the *filtered* RS-based correlation estimators. An example of this, is that $C^{SAMP-RMT1-250}$ delivers a ratio $\bar{\gamma} = 1.40$ very close to 1.44, delivered by $C^{RS-RMT1-250}$. This suggests that the benefits of the RMT-filtering are larger for the sample correlation matrix than for the regime-dependent correlation estimators.

6.6.2 Risk-Adjusted Returns Estimates

In Table 6.3 we report results on the out-of-sample performance of the models in terms of their cumulated wealth and risk-adjusted measures. These portfolio strategies are associated to the risk performance measures calculated above. Here we observe that, when *short-sale* is not permitted, the *filtered* regime-dependent correlation estimators deliver relatively similar cumulated returns and only slightly better risk adjusted measures than their *unfiltered* counterparts.

In contrast, when we allow for *short-sale* we observe that the *filtered* RS-based correlation estimators deliver the highest cumulated returns (£3.32-£3.31) and the highest Sharpe ratio

	No short-sale				Short-sale			
Model	Cum.Ret	Sharpe	Sortino	Omega	Cum.Ret	Sharpe	Sortino	Omega
$C^{SAMP} - 250$	2.119	0.172	0.147	1.333	2.356	0.209	0.211	1.439
$C^{SAMP-RMT1} - 250$	2.115	0.170	0.143	1.335	2.899	0.285	0.327	1.716
$C^{SAMP-RMT2} - 250$	2.104	0.169	0.144	1.335	2.888	0.283	0.320	1.715
$C^{CAPM} - 250$	2.118	0.172	0.147	1.333	2.356	0.209	0.211	1.439
$C^{CAPM-RMT1} - 250$	2.115	0.170	0.143	1.335	2.905	0.286	0.328	1.719
$C^{CAPM-RMT2} - 250$	2.170	0.180	0.154	1.358	2.775	0.275	0.296	1.671
$C^{RS} - 250$	2.054	0.163	0.132	1.314	2.546	0.230	0.238	1.512
$C^{RS-RMT1} - 250$	2.078	0.166	0.136	1.322	3.321	0.319	0.401	1.882
$C^{RS-RMT2} - 250$	2.041	0.163	0.131	1.311	3.317	0.320	0.396	1.881
$C^{SAMP} - 500$	2.091	0.171	0.144	1.323	2.038	0.185	0.161	1.332
$C^{SAMP-RMT1} - 500$	2.103	0.167	0.140	1.327	2.666	0.255	0.276	1.604
$C^{SAMP-RMT2} - 500$	2.157	0.173	0.150	1.344	2.767	0.261	0.288	1.634
$C^{CAPM} - 500$	2.090	0.171	0.144	1.323	2.038	0.185	0.161	1.332
$C^{CAPM-RMT1} - 500$	2.103	0.167	0.140	1.327	2.666	0.255	0.276	1.604
$C^{CAPM-RMT2} - 500$	2.214	0.182	0.159	1.366	2.763	0.265	0.283	1.638
$C^{RS} - 500$	2.023	0.161	0.129	1.297	2.135	0.194	0.174	1.369
$C^{RS-RMT1} - 500$	2.038	0.162	0.132	1.304	2.559	0.252	0.271	1.569
$C^{RS-RMT2} - 500$	2.031	0.162	0.130	1.300	2.630	0.259	0.280	1.597

Table 6.3: Cumulated Wealth at the end of the out-of-sample period and Sharpe Ratios using the *filtered* and *unfiltered* correlation matrices for different window lengths. We estimate the filtered versions of the sample correlation matrix, the CAPM-Based correlation matrix and the Regime Switching versions for *Minimum Variance Portfolios*.

(0.310-0.320), when compared to any of the other strategy. This represents an improvement of 30% in total cumulated wealth, for the RS-based correlation estimator and 23%-17.7% for the sample and CAPM-based correlation estimators, respectively. Taking into account that these portfolios could be worth millions of pounds, these improvements are sizeable. We also find that the *filtered* portfolios, in general, exhibit lower downside volatility (e.g., less "bad volatility", higher Sortino Ratio) and most of this volatility is in the upside (e.g., more "good volatility", higher Omega Ratio). In this case, the *filtered* RS-based correlation estimator is the clear winner by delivering strategies with the lowest downside volatility, measured by its Sortino ratio. These results are robust across correlation estimators and different filtering methods. In particular, the models filtered using the method of Plerou et al. (2002) (Method 2), tend to perform better than the method used in Laloux et al. (2000) (Method 1).

In Figure 6.6, we plot the cumulated wealth delivered by our correlation models. Here we can see that the *filtered* RS-based correlation estimators deliver the highest cumulated wealth compared to any other *unfiltered* strategy.

6.7 Implications for Asset Allocation

As we have shown so far, the *filtered* correlation estimators deliver the highest cumulated returns and risk-adjusted measures. In this section, we ask what are the industry sectors privileged by our correlation estimators. This is with no doubts a very important question, since we are mainly interested in understanding how the previous analysis can be translated into an asset allocation context. For doing this, we group portfolio weights delivered in each optimisation round by industry sectors; Basic Materials (BM), Industrials (IND), Financials (FIN), Consumer Goods (C.G), Consumer Services (C.SS), Health (HE), Oil & Gas (O&G), Telecommunications (TEL) and Technology (TECH).[11]

In Table 6.4, we provide the average weights delivered by our correlation estimators over the out-of-sample period (from 01/02/03 until 10/16/12), using an estimation window of 250

[11]We use the Industry Classification Benchmark (ICB), which is an industry classification taxonomy launched by Dow Jones and FTSE in 2005 and now owned solely by FTSE International. It is used to segregate markets into sectors within the macroeconomy. The ICB uses a system of 10 industries, partitioned into 19 supersectors, which are further divided into 41 sectors, which then contain 114 subsectors. In the present analysis, we use the most general, which comprises 10 industries.

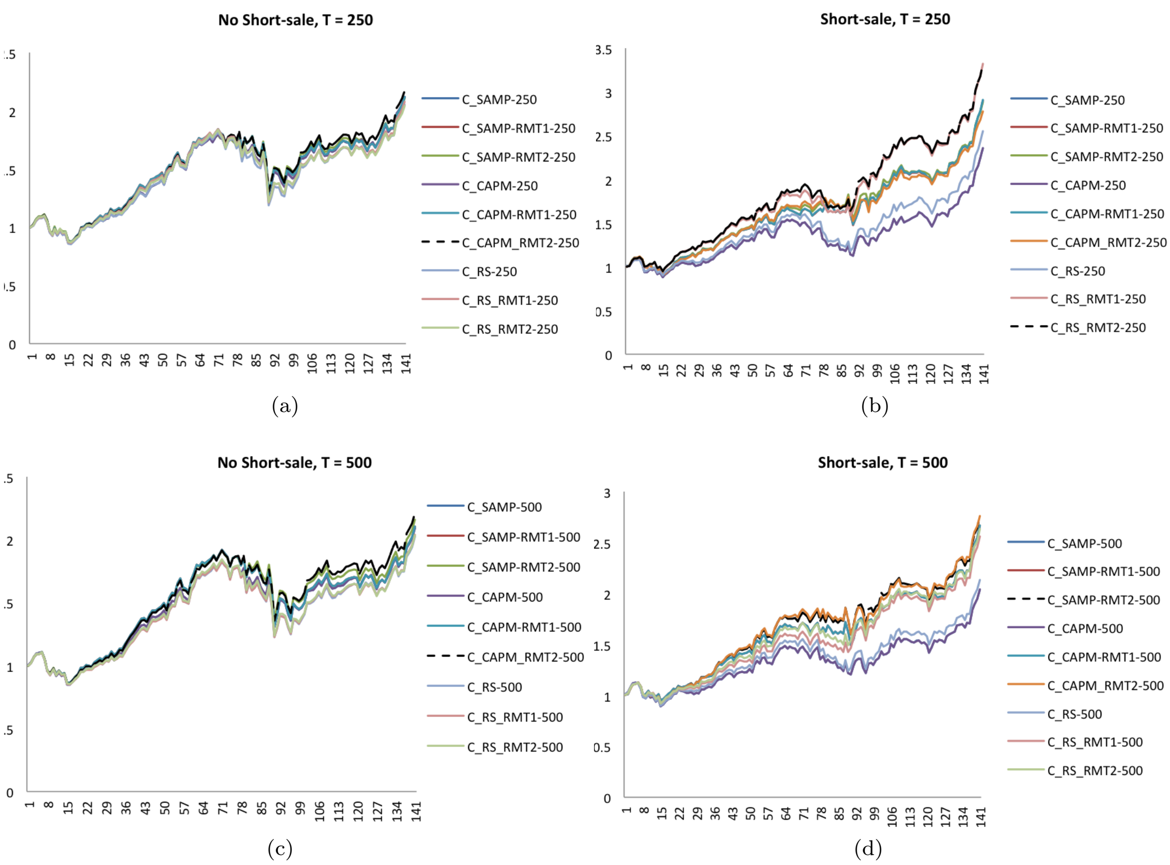


Figure 6.6: Cumulated Wealth at the end of the out-of-sample period for different estimation windows, with and without *short-sale*.

Model	BM	IND	FIN	C.G	C.SS	HE	O&G	UT	TEL	TECH
C^{SAMP}	0.019	0.189	0.164	0.095	0.261	0.071	0.059	0.111	0.025	0.006
$C^{SAMP-RMT1}$	0.006	0.198	0.077	0.115	0.298	0.082	0.073	0.128	0.012	0.011
$C^{SAMP-RMT2}$	0.016	0.201	0.059	0.118	0.316	0.091	0.060	0.122	0.010	0.009
C^{CAPM}	0.019	0.189	0.164	0.095	0.261	0.071	0.059	0.111	0.025	0.006
$C^{CAPM-RMT1}$	0.006	0.197	0.077	0.116	0.298	0.082	0.073	0.129	0.012	0.011
$C^{CAPM-RMT2}$	0.012	0.182	0.059	0.129	0.308	0.092	0.057	0.140	0.013	0.008
C^{RS}	0.021	0.188	0.168	0.092	0.263	0.068	0.056	0.113	0.025	0.006
C^{RS1}	0.024	0.183	0.158	0.101	0.268	0.076	0.057	0.108	0.021	0.004
C^{RS2}	0.015	0.198	0.191	0.073	0.251	0.050	0.053	0.125	0.034	0.012
$C^{RS-RMT1}$	0.015	0.190	0.126	0.097	0.275	0.076	0.070	0.119	0.019	0.012
$C^{RS1-RMT1}$	0.016	0.186	0.098	0.108	0.285	0.088	0.077	0.116	0.012	0.013
$C^{RS-RMT2}$	0.020	0.192	0.115	0.100	0.288	0.082	0.061	0.115	0.018	0.010
$C^{RS1-RMT2}$	0.022	0.189	0.081	0.112	0.305	0.096	0.064	0.110	0.012	0.009

Table 6.4: Average Composition of *minimum variance* portfolios for all correlation estimators, using an estimation window of length 250. *Short-sale* allowed.

days (*short-sale* permitted). Here we observe that portfolios weights are relatively similar across estimators. In general, portfolios weights are mainly concentrated in Consumer Services (C.SS), followed by Industrial (IND) and Financials (FIN). Regarding the *filtered* correlation estimators, these tend to place less emphasis on financials and distribute more evenly the weights across industries. In the same line, the *filtered* sample correlation matrix would seem to apply a similar strategy to the regime-dependent correlation matrices, by investing less in Financials and placing more resources on Utilities and Consumer Goods.

6.8 Concluding Remarks

In the present study, we apply Random Matrix Theory-based filtering to regime-dependent correlation estimators as of Ang and Bekaert (2004), for a large dimensional system, using daily records of 77 stocks of the FTSE 100, covering the period 2000-2012. To these models, we applied the RMT-filtering using two methods, the one of Plerou et al. (2002) (Method 1) and the one of Laloux et al. (2000) (Method 2) and then built *minimum variance* portfolios in the spirit of Markowitz Portfolio Theory (MPT). We also assess the performance of these models based on the cumulated wealth at the end of the period and risk-adjusted measures of returns. Our findings can be summarised as follows:

- In terms of applicability of the filter in a Regime Switching context, we find that the spectrum in Regime 1 (low volatility regime), can be fitted relatively well by the Marčenko-Pastur Distribution, which allows us to distinguish signals from noise in the correlation matrix for Regime 1. In this context, filtering is only applied to the correlation matrix of Regime 1. Recall that the Marčenko-Pastur Distribution is based on the assumption of normality of returns. While this may be true for Regime 1, returns for Regime 2 are far from being normal.

- In terms of cumulated returns, we find that the RMT-filtering enhances the performance of portfolio returns in 30% for the regime-dependent correlations and 23%-17.7%, for the CAPM-based and sample correlation estimators, respectively. In addition, the *filtered* regime-dependent correlation estimators are able to deal better with downside risk than their *unfiltered* version, with more volatility in the upside than in the downside.

- We also find that the RMT-based filtering works best for short estimation windows and when *short-sale* is permitted.

- These results are robust to both filtering methods. In particular, the method of Plerou et al. (2002) (Method 1) delivers better cumulated returns than the method of Laloux et al. (2000) (Method 2) across estimators.

- Finally, from an asset allocation perspective, we find that the success of the *filtered* regime-dependent correlation estimators comes from their ability to deliver more diversified portfolios and also by placing less resources on Financials. This finding is common to all *filtered* correlation estimators.

To conclude, the present study puts in evidence the importance of filtering methods in bringing greater stability in the eigenvalue spectrum for the case of regime-dependent correlation models. We have shown that even when we use estimated correlation matrices, rather than historical correlation matrices, the RMT-filtering can still enhance the performance of our portfolios.

Chapter 7

An Application of RMT to an Emerging Market

Published as: "Separating the Wheat from the Chaff: Understanding Portfolio Returns in an Emerging Market"(2013) (With Dalibor S. Eterovic). Emerging Markets Review, vol. 16, issue C, pages 145-169.

7.1 Introduction

The present chapter applies some results of Random Matrix Theory (RMT) to the Chilean Stock Market. Two questions frame our investigation. First, we ask if all the return correlations across the Chilean stock market are equally statistically significant. Second, we ask which are the main macroeconomic drivers affecting the Chilean stock market returns. To answer these questions, we use Random Matrix Theory (RMT) to study the daily returns of 83 Chilean stocks that are part of the IPSA and IGPA indices from January 2000 to January 2011. The RMT helps us to separate the wheat from the chaff in the correlation matrix. Using Markowitz's Portfolio Theory (MPT), we compare the efficiency of portfolios constructed under RMT with others constructed under the standard approach which considers all covariances in the correlation matrix equally significant. Finally, we use a Vector Autorregresion approach (VAR) to determine the impact of a set of macroeconomic and financial variables on the optimal portfolios derived from our significant eigenvalues.

We focus on the Chilean stock market as we think provides a good case study for other emerging markets due to a number of reasons. First, the Chilean equity market is one of the most developed within the emerging market world with a market capitalisation of 120% of its GDP.

Other markets such as the Brazilian one have much lower market capitalisations as percentage of their GDP (58%) while Mexico has a market capitalisation of 39% of GDP. Second, Chile is a small open economy with almost no restrictions to the access of international investors. Therefore, both idiosyncratic and global factors are likely to be important determinants of the stock market returns. Assessing the relative importance of domestic versus international factors in explaining domestic market volatility is key for developing public policy and market regulation in other emerging economies that are following the path of financial liberalisation.

Our main findings can be summarised as follows: First, applying Random Matrix Theory to a portfolio composed of Chilean equities improves its efficiency compared to a portfolio constructed using a standard MPT approach by at least 48%.[1] Second, using VAR analysis, we identify global risk aversion as the main macroeconomic determinant of the Chilean equity market returns, followed in importance by shocks to the monthly rate of inflation and the country's monetary policy rate. Third, it is possible to diversify away some of the market portfolio risk by adding positions on the portfolios constructed by the second and third largest eigenvalues. Fourth, the three smallest eigenvalues produce portfolio returns that are mostly uncorrelated with macroeconomic shocks. These portfolios are also uncorrelated with the market portfolio. Finally, we show that the insights provided by the RMT approach can help us to improve some existing models of the MV-GARCH literature, with significant improvements in *realised* risk predictions.

This chapter contributes to the academic literature in two important ways. First, by applying RMT to the Chilean stock market we provide further evidence of the benefits of using this technique in building efficient portfolios. Second, to the best of our knowledge we are one of the first to explore the relationship between macroeconomic variables and portfolio returns constructed using statistically significant correlations. The rest of the chapter is organised as follows. In Section 7.2, we provide an eigenvalue analysis of the Chilean Stock Market, while in Section 7.3, we apply the RMT notions in a Markowitz's portfolios framework to improve portfolio risk estimates. In Section 7.4, we look for the macroeconomic and financial drivers of stock returns. Finally, in Section 7.5, we conclude and propose lines for future research.

[1] Throughout this chapter, efficiency is measured by the Mean Percentage Error, which measures how close the risk of our portfolios are from the "true" realised risk. This is an average result over the sample analysed in the most restrictive scenario, that is, without allowing for a "short-sale" strategy.

7.2 Eigenvalue Analysis for the Chilean Stock Market

Here we apply the same analysis of Chapter 4 to 83 records of Chilean stocks, using daily data for the year 2005.[2] We therefore have $T = 260$ and $N = 83$, which implies $Q = T/N = 3.132$.

We then construct the correlation matrix C^{2005} based on the average time-series returns and normalised variance equal to 1. Using Equation 3.7 from Chapter 3, we obtain the upper and lower bounds of the theoretical Marčenko-Pastur (MP) distribution. In this case, we have $\lambda_+ = 2.4492$ and $\lambda_- = 0.1892$.

Similarly, we generate a Wishart correlation matrix R composed by cross-correlations of mutually independent returns. We then plot the Marčenko-Pastur density of the eigenvalues for 10000 trials and compare it with the eigenvalue density of the sample correlation matrix C^{2005}. The results of this exercise are presented in figure 7.1.

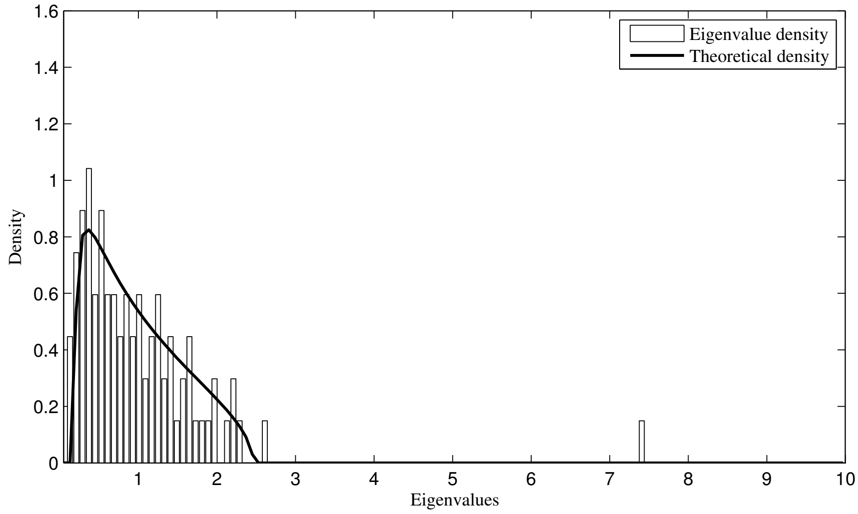


Figure 7.1: Eigenvalue Distribution of the Sample Correlation Matrix vs. The Marčenko-Pastur Distribution. Year 2005

As expected, the majority of the eigenvalues lie within the bounds of the MP distribution. However, there are obvious deviations from the RMT predictions. The largest eigenvalue is well above the maximum predicted by the RMT ($\lambda_+ = 2.4492$). This value as well as the remaining deviating eigenvalues, are the ones that are expected to carry economically relevant information,

[2]Thoughout the chapter, we will be using these 83 stocks for every year of the sample (2000-2010). The selection of these stocks was based on two criteria: (i) their availability over the entire sample, which allows to make reliable comparisons and (ii) were the most frequently traded.

while those within the distribution bounds can be considered just noise.

For 2005, there are two large deviations from the upper bound predicted by the RMT, $\lambda_1 = 7.4196$ and $\lambda_2 = 2.5942$. We also find three clear deviations in the lower bound of the MP distribution, $\lambda_{83} = 0.1419$, $\lambda_{82} = 0.1543$ and $\lambda_{81} = 0.1572$. As we mentioned in Section 4.2, from a portfolio management perspective, the list of eigenvalues can be seen as different levels of risk of a certain portfolio. Likewise, the eigenvector components associated to each eigenvalues can be regarded as the weights of the different stocks in a certain portfolio. In figure 7.2 (panel (a) and (b)), we plot the components corresponding to the eigenvector 1 and 2 from the two largest eigenvalues.

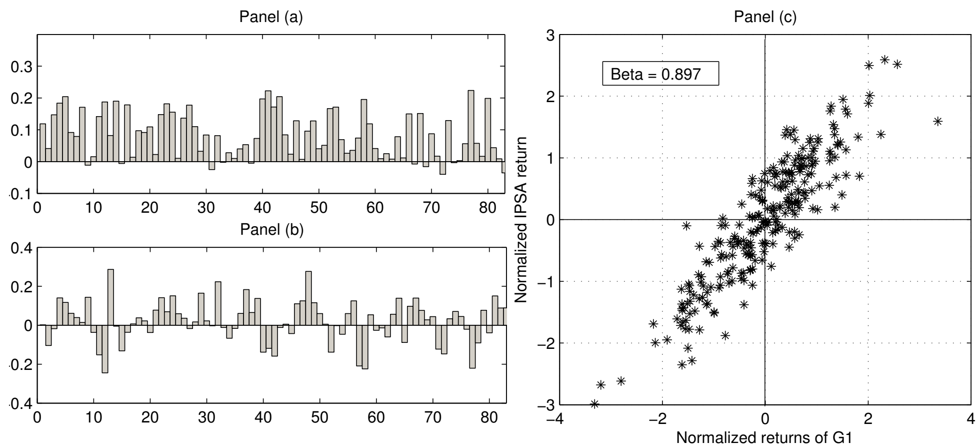


Figure 7.2: Eigenvector Components of the Two Largest Eigenvalues: Year 2005.

In figure 7.2 we observe that most of these components are positive and contribute in a similar way to the eigenvectors. This suggests a strong collective behaviour of stocks. From a portfolio perspective, eigenvector components can be seen as the weights of the different stocks in a certain portfolio, where some stocks are *long* while others are *short*. In this line, the eigenvalue λ_1 represents the risk of the market portfolio. In general, large eigenvalues correspond to a mix of risky assets.

In stock markets, the largest eigenvalue corresponds to the risk of a portfolio whose weights are the same for each assets. There is no diversification on this portfolio: the only bet is whether the market as a whole goes up or down, this is why the risk is large. Conversely, if two stocks move very tightly together (e.g., Coca-Cola and Pepsi), then buying one and selling the other leads to a portfolio that hardly moves, being only sensitive to events that strongly differentiate the two companies (Bouchaud and Potters, 2009). In order to prove this latter point, we construct a time-series using the weights contained in the largest eigenvector u^1 using Equation 4.1, G_t^1,[3] which defines the principal components associated to the largest eigenvalue. This principal component is then normalised and we run an OLS regression against the normalised returns of the market index G_t^{ipsa}, represented by the Indice de Precios Selectivo de Acciones (IPSA), which is the market index that measures the price variations of the 40 most traded stocks.[4]

We plot the results of this regression in figure 7.2 (panel (c)). We observe that there is a clear linear and positive relationship between the returns G_t^1 and G_t^{ipsa}, with a strong correlation coefficient $< G_t^{ipsa} G_t^1 >= 0.8977$. From this, we can therefore conclude that the largest eigenvalue corresponds to the risk of the market mode. We repeat the same exercise for the smallest eigenvector u^{83}. We plot the correlation coefficients in figure 7.3.

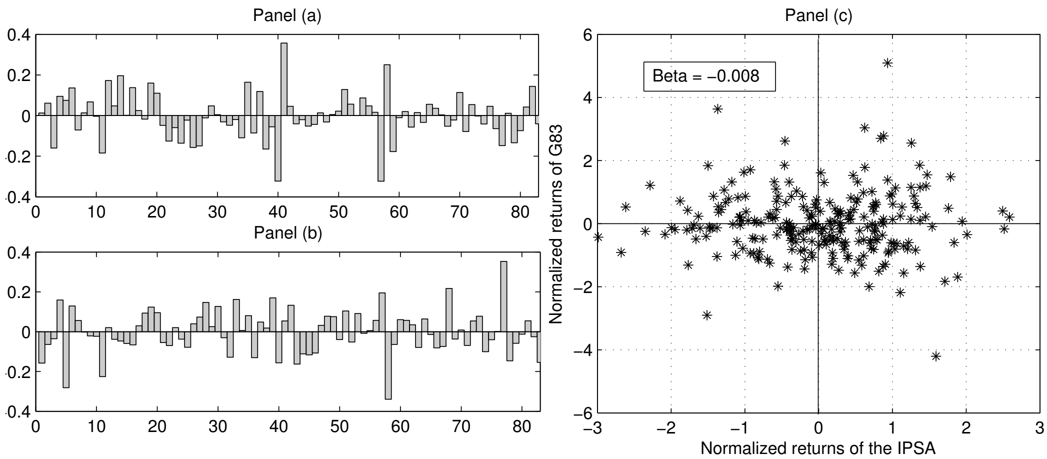


Figure 7.3: Eigenvector Components of the Two Smallest Eigenvalues: Year 2005.

From a portfolio perspective, the smallest eigenvalue determines the least risky portfolio one can build. This can be corroborated by the nearly zero beta coefficient of the regression against the market index. This in turn implies that the portfolio constructed from the smallest eigenvalues is almost totally immune to the systematic risk emanating from the collective behaviour of stocks represented by the largest eigenvalue.

Figure 7.3 plots the eigenvector components corresponding to the smallest eigenvalues, λ_{83}

[3] $G_t^1 = \sum_{j=1}^{n} u_j^1 G_j(t)$.

[4] Due to their high correlation, using the IGPA would not change the results.

and λ_{82}. Here we can see that the eigenvector corresponding to the smallest eigenvalue exhibits clear preference for some pairs of stocks with the highest correlations. Finally, we plot the eigenvector components of two eigenvalues inside the bounds of the MP distribution in figure 7.4.

As we mentioned above, the eigenvalues inside of the bulk of the MP distribution are assumed to be random and therefore should carry no economically relevant information. This appears to be the case, as the eigenvector components for both cases, do not seem to show major preferences for any stocks. Overall, our results are in line with the findings of most of the existing literature on the subject (Laloux et al., 1999, 2000; Plerou et al., 1999, 2000a,b; Gopikrishnan et al., 2001; Plerou et al., 2001, 2002; Bouchaud and Potters, 2009). In the next section, we look deeper into the significant eigenvalues and identify the individual stocks that participate in each of them.

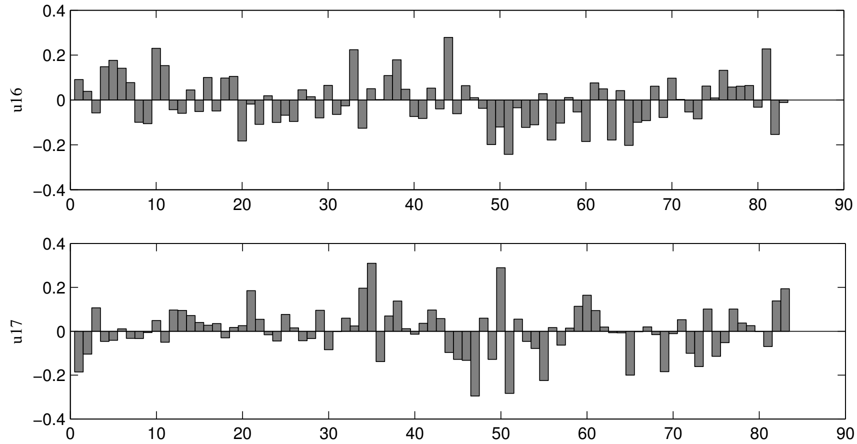


Figure 7.4: Eigenvector Components of Two Eigenvalues Inside of the Bulk: Year 2005.

The Number of Significant Components in the Deviating Eigenvectors

As in Chapter 4, we quantify the number of components that participate significantly in each eigenvector by using the *Inverse Participation Ratio* (IPR), which quantifies the reciprocal of the number of eigenvector components that contribute significantly (see, Section 7.2).

In this case, we have that the eigenvector associated to the largest eigenvalue has a $IPR1 = 0.0287$, which implies that we have $PR1 = 34.86 \approx 35$ significant stocks in the largest eigenvector. This is a very important result, which together with the high correlation of the first principal component with the market returns of IPSA index, is basically telling us that most of the volatility of the IPSA-IGPA index is driven by these 35 stocks. The significant firms in the largest eigenvector are presented in Table B in Appendix B of this chapter. By inspecting the largest eigenvector we can see that it has very strong components in the mining sector, with companies such as SQM-A, Oro Blanco and CalicheraA; energy sector, in companies such as Enersis and Endesa, as well as in the retail sector represented by Falabella. Also very important is the contribution of the food and beverage industry, represented by firms such as Andina A and B, CCU, San Pedro and Conchatoro. In general, the largest eigenvector appears to include most sectors of the Chilean stock market.

Regarding the rest of the deviating eigenvectors, the interpretation of their meaning is not as straightforward as in the former case. This is so because the largest eigenvalue is of order of magnitude larger than the others, hence constraining the remaining $N-1$ since $TrC = N$. Therefore, in order to avoid the effect of the largest eigenvalue, we first need to remove it from the rest. For doing this, we regress, $G^1(t)$, calculated in Section 7.2, against each return and compute the residuals $\epsilon_i(t)$. Then, we calculate again the correlation matrix using $\epsilon_i(t)$ and compute the eigenvectors u^k of C. The number of significant participants in the second largest eigenvector for the year 2005, can be found in Table B.2b. In this case we have $PR2 \approx$ 15 significant components with strong emphasis in the mining sector with companies such as CalicheraA, Oro Blanco and SQM-A/B, in the energy and combustibles sector, with companies like ECL, Colbun and COPEC as well as in the forest and agro-industrial sector with companies like CMPC, IANSA and Campos. Also, the removal of the first component allowed to identify a third deviating eigenvalue in the upper bound, with $PR3 \approx 28$, with again strong weights on the mining, energy and banking and investment sectors.

Finally, the smallest eigenvectors, each has $PR83 \approx 24$ and $PR82 \approx 24$ significant participants. Recall that these eigenvectors determine the least risky portfolios of stocks one can build when using the Markowitz Portfolio Theory (MPT). Typically, it has been found in the literature that the components in the smallest eigenvectors represent a very small number of stocks with large correlations.[5] As we can see in Table B.2b, this is not the case for the Chilean stock market. For the smallest eigenvector, u^{83}, although, we do find strong negative correlations between some

[5] See, Laloux et al. (1999, 2000); Plerou et al. (2000a); Gopikrishnan et al. (2001); Plerou et al. (2002); Bouchaud and Potters (2009).

pairs of stocks, instead of finding a small portfolio we rather find a quite diversified portfolio of stocks composed by a mix of sectors, with strong preferences for stocks in the food and beverage industry, metal-mechanic industry, investment and real estate and to a lower extent mining. The portfolio composed by these stocks has a correlation with the IPSA, $< G_{83}^{2005} G^{ipsa} >= -0.0081$. Regarding the second smallest eigenvalue (not shown), u^{82}, we notice it has strong weights again in the food and beverage, mining, energy sectors and to a lower extend on public services and private pension funds (AFPs). Therefore, the results for the Chilean equity market are different to what has been found in the literature for the U.S, where the portfolios defined by the smallest eigenvector contain very few stocks with very large negative correlations. A similar phenomenon has nonetheless been found in recent literature on emerging markets (Pan and Sinha, 2007), where there is a tendency for relatively weak intra-sector interactions between stocks. In this context, the emergence of an internal structure of multiple groups of strongly correlated components would be sign of market development.

7.3 Portfolio Analysis for the Chilean Stock Market

The underlying randomness contained in the "bulk" of eigenvalues seen in Section 7.2 has important implications in optimal portfolio selection. In the context of a Markowitz's optimal portfolio theory, the effect of noise has a strong weight on the smallest eigenvalues, which are precisely those that determine the least risky portfolios. This is why the Random Matrix Approach allows reconciling the mean-variance approach of Markowitz as a tool for providing better risk predictions.

7.3.1 Noise Filtering Using the Sample Correlation Matrix

To find the effect of randomness of C on the selected optimal portfolio, we carry out our analysis for the years 2005-2006. We first calculate the sample correlation matrix C_{2005} and using the returns G_i for 2006, we construct a family of 100 optimal portfolios, and plot Φ as a function of the predicted risk Ω_p^2 for 2006. For this family of portfolios, we also compute the realised risk Ω_r^2 during 2006 using C_{2006}. We do this in order to compare the performance of the RMT approach isolated from any additional source of noise that could arise, for example, in case we had to make a forecast of returns.

Since the meaningful information in C is contained in the deviating eigenvectors (whose eigenvalues are outside of the RMT bounds), we use the filtering approach carried out in the study of Plerou et al. (2002). They construct the filtered matrix C^f, by retaining only the deviating eigenvectors, using the eigenvector decomposition $C = V\Lambda V^{-1}$, where V contains the eigenvectors and Λ is a diagonal matrix containing the eigenvalues. Plerou et al. (2002) construct the filtered matrix C^f, by setting a new diagonal matrix Λ^f, with elements $\Lambda^f_{ii} = (0, .., 0, 0, ., 0, \lambda_2, \lambda_1)$. Then they transform Λ^f back to the basis of C, resulting in the *filtered* sample correlation matrix C^f.[6]. In addition, they set the diagonal elements $C^f_{ii} = 1$, to preserve $Tr(C) = Tr(C^f) = N$. We use this filtering technique and we repeat the above approach for finding the optimal portfolio using C^f instead of C. We plot the efficient frontiers for both approaches, for the *filtered* and *unfiltered* matrices, in figure 7.5.[7]

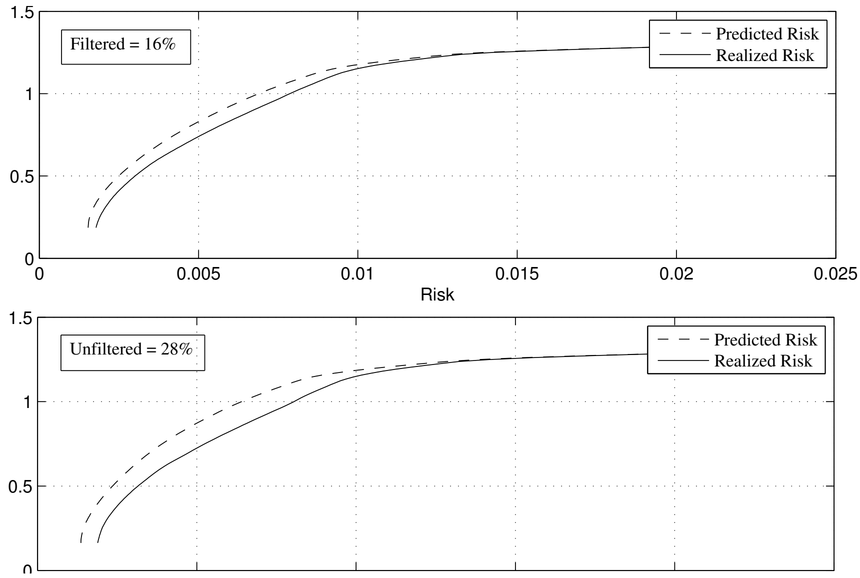


Figure 7.5: Efficient Frontier during 2006.

[6]The reason why the smallest deviating eigenvalues are not considered in these filters is that while large eigenvalues are *clearly* separated from the RMT bounds, the same does not always applies for the smallest deviating eigenvalues. In general, small eigenvalues can be found outside of the lower edge of the spectrum, which is consistent with the fact that the length of the time-series T and the number of assets N are finite. In addition to this, by simple inspection of the eigenvectors associated to the largest eigenvalues, *clear* non-randomness, and stability over time has been verified, while this has not been verified in the eigenvectors associated to the smallest deviating eigenvalues.

[7]For this example, we also allow for "short-sale". Although, "short-selling" is not always possible as increases the risk of the portfolio.

In order to measure the relative performance of the *filtered* portfolios vs. the *unfiltered* ones, we calculate the difference between both curves by using the Mean Average Percentage Error (MAPE), defined as:

$$MAPE = \frac{1}{n}\sum_{i=1}^{N}\left|\frac{\Omega_i^2 r - \Omega_i^2 p}{\Omega_i^2 p}\right| \tag{7.1}$$

In this case, we have that the *unfiltered* approach, which uses the sample correlation matrix, underestimates realised risk in 28%, while when using the RMT approach it only underestimates realised risk in 16%. There is certainly a sharp contrast between the use of the *filtered* and *unfiltered* matrices, with the *filtered* matrix performing far better than the *unfiltered* matrix for forecasting portfolio risk.

In this context, also an interesting question would be to assess how the *filtered* approach performs relative to the *unfiltered* one, during different volatility regimes in the stock market. To do this, we apply the methodologies presented in Chapter 5, Section 5.2 to consecutive one-year periods from 2000-2001 to 2009-2010 in Table 7.1. In columns (1)-(2) we compare the MAPE using the *filtered* and sample correlation matrix, not allowing for "short sale", while in columns (3)-(4) we compare again both approaches, but this time allowing for "short-sale". Finally, in columns (5) and (6) we present the volatility of the IPSA calculated as the mean of the absolute value of returns for the *predicted* year and the VIX index as a measure of international market volatility.

By comparing columns (1) and (2) we observe that in most cases, the *filtered* correlation matrix delivers better realised risk estimates, with only one exception; when using the correlation matrix of 2007 to predict the realised risk of 2008. During 2007 the burst of the housing bubble in the U.S led to the start of a crisis whose zenith was reached in 2008. Notice that "short-sale" constraints are a very extreme assumption. The reason for this is that "short-sale" constraints can be easily circumvented with the use of derivative positions, allowing better flexibility.

The results for 2007-2008 are reversed when we allow for "short- sale". From columns (3) and (4), we can see that the *filtered* approach delivers better realised risk estimates in all cases. Under this setting, "short-selling" would seem to provide more flexibility to overcome high volatility periods in financial markets, which are precisely the times where good investment decisions are

	No short sale		Short sale			
	Unfiltered	Filtered	Unfiltered	Filtered	Vol. IPSA	VIX
Previous-Predicted	(1)	(2)	(3)	(4)	(5)	(6)
2000-2001	0.307	0.100	0.345	0.066	0.745	23.3
2001-2002	0.029	0.027	0.344	0.096	0.615	25.7
2002-2003	0.238	0.189	0.458	0.206	0.708	27.2
2003-2004	0.255	0.220	0.735	0.375	0.695	21.9
2004-2005	0.058	0.019	0.428	0.108	0.567	15.4
2005-2006	0.288	0.163	0.412	0.179	0.581	12.7
2006-2007	0.333	0.276	0.487	0.169	0.545	12.7
2007-2008	0.073	0.130	0.367	0.173	0.888	17.4
2008-2009	0.127	0.041	0.367	0.192	1.168	32.5
2009-2010	0.052	0.023	0.379	0.186	0.754	31.4
Average	0.176	0.119	0.439	0.175	0.727	22.07
Improvement	0.481		3.71			

Table 7.1: Mean Average Percentage Error for each year of the sample.

needed the most.

Our results are different from those of Sandoval et al. (2012) for the Brazilian stock market, who based on a number of agreement measures between *predicted* and *realised* risk, i.e., MAPE, Mean Squared Error (MSE), the angle between risk vectors, Simple Distance and the Kullback-Leibler distance measure, conclude that the RMT-filtering did not produce better results than the *unfiltered* approach, in particular during high volatility periods. They also find that the RMT-filtering combined with the single-index model significantly improves risk estimates for most of the measures analysed. We only find this phenomenon in one case in our sample; when using the correlation matrix of 2007 to predict the one of 2008, with “short-sale” constraints. Conversely, when “short-sale” is allowed, the *filtered* approach outperforms the *unfiltered* one in all cases.

In the next section, we will try to separate out diversifiable from non-diversifiable risk with the use of a simple one-factor model, to extract the market effect and assess the performance of the RMT approach for the filtered and sample correlation matrices.

7.3.2 Improving our Results by Getting Rid of the Non-diversifiable Risk

Now we extend our previous results by removing the effect of the market portfolio from our time-series returns and verify how the RMT-filtering performs under this new scenario. In the previous analysis we were not able to fully distinguish between diversifiable and non-diversifiable risk. Diversifiable risk is directly related to correlations between stocks. While non-diversifiable risk is mainly driven by market momentum and/or breaking news that may affect all stocks at the same time and inducing new correlations that could not be distinguished from the real correlations between stocks. This is particularly true, during drawdowns. To remove the market effect, we run an OLS regression of the form $G_i(t) = \alpha_i + \beta_i G(t)^{IPSA} + \epsilon_i(t)$, where $G_i(t)$ are the individual returns and $G(t)^{IPSA}$ are the returns of the IPSA index, retrieve the residuals $e_{i,t} = G_{i,t} - \beta G_t^{IPSA}$ of that regression and build again our correlation matrix. This new correlation matrix would be to some extent immune from the market effect. Recent emerging markets literature has made use of this approach to distinguish between co-movements in financial assets from "pure contagion" (Bunda et al., 2009).[8]

In figure 7.6, we plot the volatility of the IPSA index, measured as the absolute value of its returns, together with the volatility of a Telecommunication Company (ENTEL), while in the third plot we include the volatility of ENTEL after removing the IPSA index. We can see that most peaks in the time-series are now smoother after removing the effects of the market. Table 7.2 presents the performance of the RMT approach, for each year of the sample when the market effect is removed. As we can see by observing columns (7) and (8), the *filtered* approach outperforms the *unfiltered* one in every year with the exception of 2007-2008. As in the case without removing the non-diversifiable risk, when we allow for "short-sale", the *filtered* approach performs unequivocally better than the *unfiltered* one. It would seem that the removal of the market mode allow us to improve our risk estimates compared to the risk estimates of the previous section. Overall, these results are encouraging since it tells us that we can use the RMT approach also during periods of high volatility, when good investment strategies are most needed.

[8]Alternatively, we could have used the time-series built from the largest eigenvector, with similar results given the high correlation between the largest eigenvector returns and the IPSA-IGPA indices.

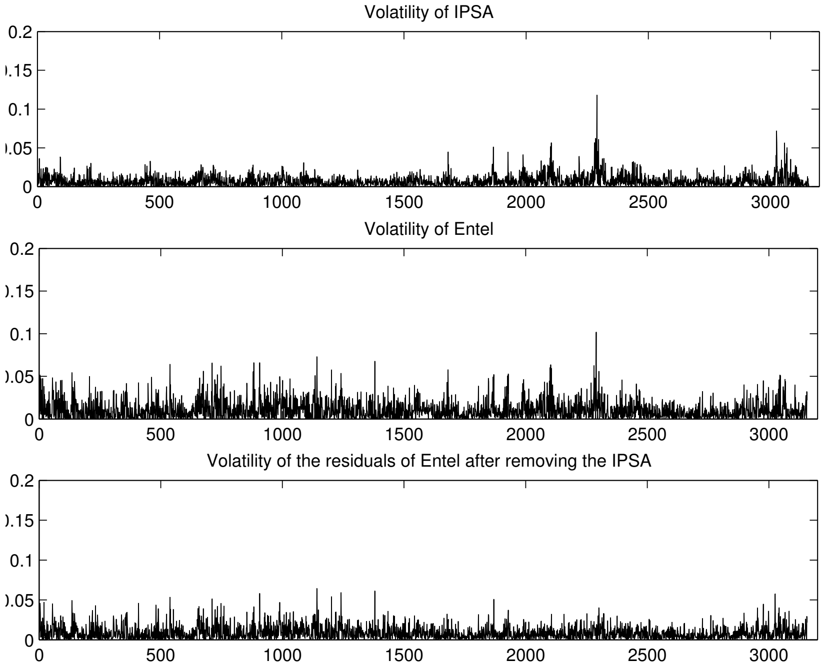


Figure 7.6: Volatility After Removing the Market Mode.

	No short sale		Short sale			
	Unfiltered	Filtered	Unfiltered	Filtered	Vol. IPSA	VIX
Previous-Predicted	(1)	(2)	(3)	(4)	(5)	(6)
2000-2001	0.319	0.083	0.347	0.076	0.745	23.3
2001-2002	0.031	0.012	0.364	0.089	0.615	25.7
2002-2003	0.229	0.145	0.454	0.201	0.708	27.2
2003-2004	0.240	0.219	0.706	0.391	0.695	21.9
2004-2005	0.049	0.026	0.422	0.131	0.567	15.4
2005-2006	0.308	0.090	0.450	0.154	0.581	12.7
2006-2007	0.420	0.241	0.527	0.210	0.545	12.7
2007-2008	0.105	0.184	0.443	0.227	0.888	17.4
2008-2009	0.109	0.061	0.377	0.040	1.168	32.5
2009-2010	0.091	0.017	0.418	0.139	0.754	31.4
Average	0.190	0.108	0.451	0.166	0.727	22.07
Improvement	0.763		3.71			

Table 7.2: Mean Average Percentage Error for each year of the sample without the market mode.

This is particularly true, when investors can exploit a full opportunity set, giving room for a "short-sale" strategy. However, "short-sale" is not always possible in financial markets. Despite this potential constraint, the use of the RMT approach could be used for medium/long term investment decisions, as oppose to active strategies that try to "beat the market" second by second, implying greater chances of large losses. This is quite evident when observing the outstanding performance of the strategy suggested by the RMT approach in horizons greater than one year.

7.3.3 Improving Standard Correlation Models with RMT Tools

As we have shown so far, the RMT-filtering can be very useful to improve the prediction of the *realised* risk of a family of efficient portfolios. The aim of this section is to introduce some existing models of the MV-GARCH literature and verify whether these models can be improved with the use of RMT-filtering. In this context, the use of Random Matrix Theory can be useful to overcome the so-called "curse of dimensionality", a problem which is very common in high dimensional systems.

We propose two MV-GARCH models, which combine elements of the MV-GARCH literature with the possibility of using an optimal number p of principal components or factors predicted by the RMT analysis of the correlation matrix. The first model is the *Conditional Constant Correlation (CCC) model* of Bollerslev (1990). In this model only the parameters of the N univariate GARCH processes have to be estimated simultaneously, whereas the time-constant correlation matrix is the unconditional correlation matrix of GARCH residuals. The covariance matrix in the CCC models is described by $\sum_{ij,t} = C_{i,j}\sigma_{i,t}\sigma_{j,t}$. The second model is the *Dynamic Conditional Correlation (DCC) model* of Engle (2002). This model is similar to the CCC model but allows the correlation matrix to be time-varying. To the best of our knowledge only the CCC model has been tested in a RMT context with significant improvements in *realised* risk predictions (Varga-Haszonits and Kondor, 2007; Rosenow, 2008).

To estimate conditional correlations and show the benefits of noise filtering, we estimate the realised risk *minimum variance* portfolios, according to four different specifications:

1. Model I: CCC(p) model, using $p = p^{RMT}, N$ principal components, where p^{RMT} is the

number of principal components predicted by the RMT and N is the full set of principal components.

2. Model II: DCC(p)-w model, using $p = p^{RMT}, N$ principal components, where p^{RMT} is the number of principal components predicted by the RMT and N is the full set of principal components.

3. Model III: Factor-GARCH model, using $p = 1, 2$ principal components.[9]

4. Model IV: RiskMetrics covariance estimator of (Zangari, 1996), $H_t = 0.06\epsilon_t\epsilon_t' + 0.94H_{t-1}$.

In both, the CCC model and the DCC model, the RMT filter is carried out over the standardised covariance matrix of GARCH residuals at every time step.

We use the same daily data set covering the period 2000-2011 to estimate these models. In order to make sure a MV-GARCH model is the correct specification we perform the ARCH test of Engle (1982) on individual time-series and we end up with a subsample of 40 firms for our analysis. Unlike the previous analysis, we rather focus on predicting the realised risk of *minimum variance portfolios* than on the portfolios along the efficient frontier. In order to compare and contrast the performance of the models out-of-sample we use *forward validation*. This method considers every test date t' and for each one uses data prior to the test date ($t = t' - F$) to optimise any model parameter. This enables the comparison between *filtered* and *unfiltered* models and also provides some insights of their stability over time, as well as for model selection (Hjorth and Hjort, 1982). This approach has also been used in the Econophysics literature to evaluate the performance of different filtering approaches (Daly et al., 2008, 2010). In order to capture the time-varying nature of correlations, the DCC model is estimated over a daily rolling window of length 2-years, while the rest of the models are estimated on a rolling window of length 1-year.[10]

The *forward validation* is performed over a period of 2547 days (covering the period 2002-2011), 520 (years 2000-2001) days of which were used as the initial trading period. Subsequent

[9]This model is similar to the O-GARCH model proposed by Alexander (2002). However, it differs from the former in that we keep residual variances and therefore will tend to work well for weakly correlated time-series, as stocks, where the O-GARCH may suffer from identification problems.

[10]The difference in the window length in the DCC model stems from the fact that over relatively short horizons, correlations will tend to be more stable, in whose case, the DCC model will tend to be very similar to the CCC model. This is why we estimate the DCC model using a wider window in order to capture some of this time-variation in correlations. In either case, we verified that the benefits of the RMT-filtering remain robust in both cases.

trading was done daily. This approach has the appeal of simulating a real trading environment giving some insights on how the model would have performed in the past in case of being used.

In this context, for each test date t' the predicted volatility of minimum variance portfolios will be given by

$$\Omega^2_{t,predicted} = \sum_{i,j=1}^{N} w_{i,t} w_{j,t} H_{ij,t} \tag{7.2}$$

where $H_{ij,t}$ is the *predicted* covariance matrix estimated under the aforementioned models and $\{w_i\}_{i=1}^{N}$ are the weights of the minimum variance portfolio. On the other hand, the *realised* risk of the minimum variance portfolio is defined by

$$\Omega^2_{t,realised} = \sum_{i,j=1}^{N} w_{i,t} w_{j,t} (< g_{i,t} g_{i,t} > - < g_{i,t} >< g_{j,t} >), \tag{7.3}$$

where the expectation values are taken over $F = 60$ daily returns *preceding* the test date t'.[11] Although we do not impose "short-sale" constraints, none of the portfolio weights ever took an extreme position over the sample. We calculate the average predicted portfolio variance, the average realised variance, the mean square error (MSE) of the prediction and the MAPE. The results of the estimation of these models can be found in table 7.3.

Model	Ω^2_{est} x 10^{-4}	Ω^2_{real} x 10^{-4}	MSE x 10^{-8}	MAPE
$CCC - (p^{RMT})$	0.103	0.295	0.114	0.522
$CCC - (N)$	0.097	0.323	0.139	0.578
$DCC - (p^{RMT})$	0.129	0.279	0.086	0.532
$DCC - (N)$	0.125	0.290	0.118	0.581
$Factor - GARCH(1)$	0.043	0.450	0.401	0.830
$Factor - GARCH(2)$	0.043	0.459	0.426	0.838
RiskMetrics	0.025	0.559	0.757	0.931

Table 7.3: Estimation of the Realised Risk of Minimum Variance Portfolios with MV-GARCH Models.

We assess the performance of *minimum variance portfolios* first based on their ability to produce portfolios with a low average *realised* variance and second and more importantly, by

[11] This is because on day t', we do not fully know the outcome of investments made after $t = t' - F$, since conceptually they are still being invested.

their ability to correctly predict this variance. Regarding the first criterium of assessment we find that by far the best results are obtained by the $CCC - (p^{RMT})$ and the $DCC - (p^{RMT})$ models. Their *realised* variance is significantly smaller compared to their $Factor - GARCH$ versions and also smaller than their *unfiltered* counterparts. The models using the RMT-filtering also produced portfolios with the lowest mean squared error (MSE) and mean average percentage error between 52% and 54%. These results also stress the shortcomings of using the Exponentially Weighted Moving Average (EWMA) models, which, despite their simplicity, can lead to a serious underestimation of the underlying risks of minimum variance portfolios.

Overall we find two main tendencies: for good covariance prediction, it is necessary to describe the time dependence of the correlation strength correctly, which can be done by choosing a relatively short estimation period for correlations. Second, a short estimation period for correlations induces noise, which needs to be removed by filtering. The clear winner of the comparison are the $CCC - (p^{RMT})$ and the $DCC - (p^{RMT})$ models. These two models have an excellent prediction accuracy for minimum variance portfolios. These results are encouraging since we have just shown that the use of RMT-filtering can be very helpful to improve some existing models of the MV-GARCH literature.

7.4 Macroeconomic Determinants of the Chilean Stock Market Returns

In the previous section, we applied the RMT-filtering to the Chilean stock market and obtained optimal portfolios. These portfolios showed improved efficiency compared to a benchmark of standard Markowitz portfolios. Now we turn to the macroeconomic and financial determinants of the Chilean stock market returns. The question we try to answer here is which are the main macroeconomic and financial factors affecting the Chilean stock market returns? To answer this question we rely on Vector Autorregresion analysis (VAR). Our main dependent variables are the monthly returns of optimal portfolios constructed using the 3 largest and the 3 smallest statistically significant eigenvalues with as predicted by the RMT for the whole sample.[12] We

[12] Monthly portfolio returns are constructed based on the same analysis of Section 7.2, but using the significant eigenvectors calculated for the whole sample. This results in 6 deviating eigenvalues; 3 deviating on the upper

denominate the portfolio returns as G_t^1, G_t^2 and G_t^3 for the returns of the portfolios constructed using the first, second and third largest eigenvalues and G_t^{81}, G_t^{82} and G_t^{83} for the returns of the portfolios constructed using the first, second and third smallest eigenvalues.

To account for macroeconomic factors affecting the returns, we include 3 domestic macroeconomic variables in our estimations: the seasonally adjusted monthly change of industrial production (x_t^{IMACEC}), the seasonally adjusted monthly inflation (π_t) and the monetary policy rate (r_t).[13] Representing more than 60% of Chilean exports, copper prices are likely to be one of the main channels through which international shocks translate to the Chilean economy. We include a variable with the monthly change of the price of copper (P_t^{copper}) to account for this effect. Finally, to account for international financial factors we include the VIX. This variable is commonly used as a gauge for international stock market volatility and investors risk aversion.

We estimate an unrestricted VAR with the following structure:

$$VAR(L) : \left(VIX, P_t^{copper}, x_t^{IMACEC}, \pi_t, r_t, G_t^1, G_t^2, G_t^3, G_t^{81}, G_t^{82}, G_t^{83}\right) \tag{7.4}$$

Producing impulse responses from this econometric model requires an identification strategy. To deal with this issue, we follow Pesaran and Shin (1998) and use a generalised impulse response function for an unrestricted VAR. This identification method has the advantage over the Choleski decomposition in that it produces impulse responses, which are invariant to the ordering of the variables in the system.

We run a battery of tests to get the optimal number of lags for our VAR (Table 7.4). The majority of the tests recommend the use of 1 lag in the regressions.

Since we are mainly interested in the effects of macroeconomic and financial variables on the returns of our portfolios we focus on this set of impulse responses. Figure 7.7 presents the impulse responses for the VAR(1).

The impulse response functions suggest that shocks to the macroeconomic variables have important and varied effects on the portfolio returns:

- VIX: A positive shock to the VIX (an increase of the international volatility) has a

edge and 3 deviating eigenvalues on the lower edge of the spectrum. We constructed returns based on their corresponding eigenvectors.

[13] We treated the seasonality of time-series using the X12 methodology for seasonally adjusted time-series.

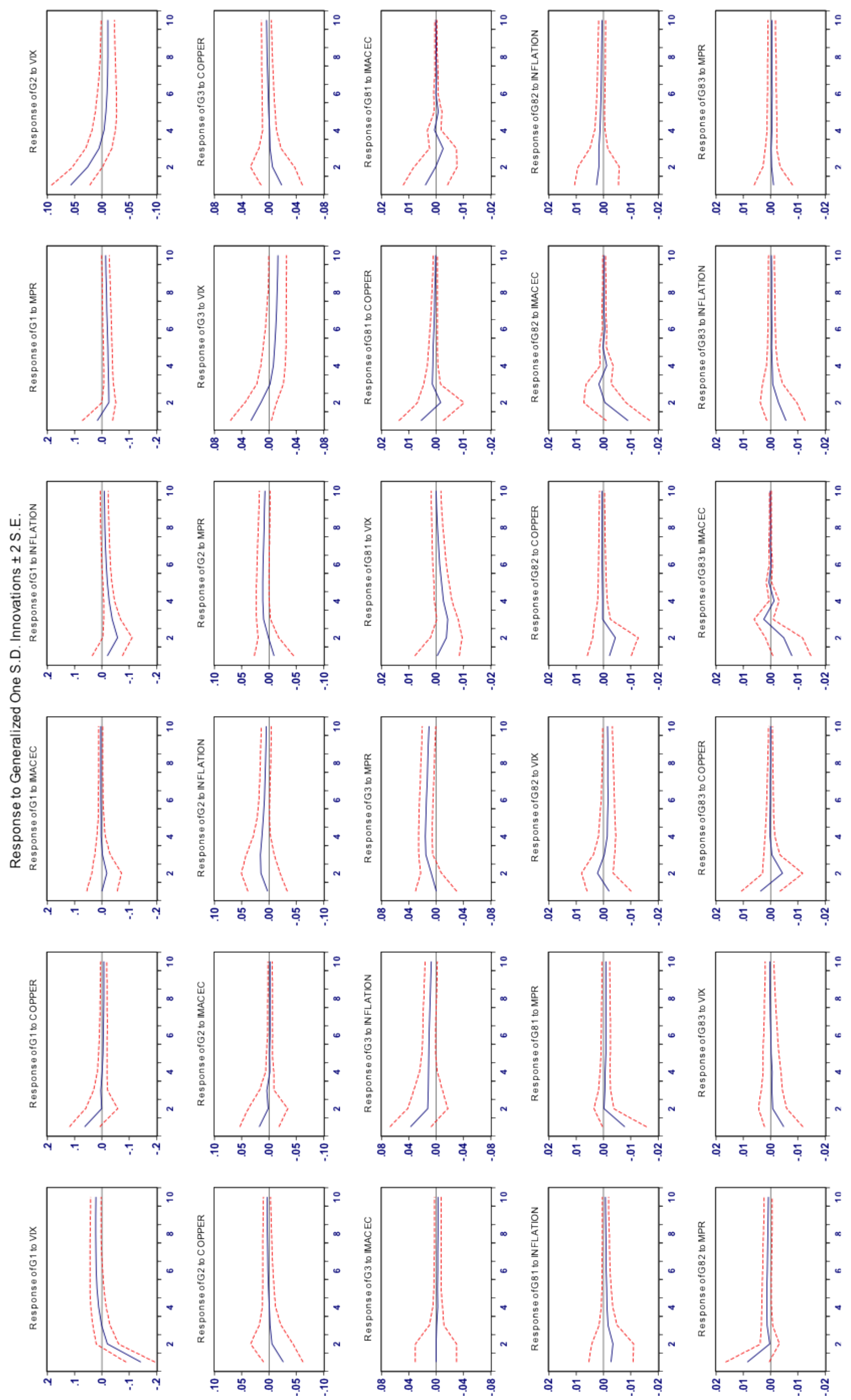


Figure 7.7: Generalized Impulse Responses

Lag	LR-test	LR-Mod	FPE	AIC	SC	HQ
0	-641.522	NA	1.94e-09	11.154	11.413	11.259
1	-265.279	675.309	2.49e-11*	6.791*	9.907*	8.056*
2	-177.902	140.400	4.64e-11	7.365	13.338	9.790
3	-69.583	153.682*	6.58e-11	7.582	16.412	11.167
4	28.544	120.773	1.27e-10	7.973	19.659	12.718
5	150.565	127.235	2.02e-10	7.956	22.498	13.860
6	296.965	125.128	2.90e-10	7.521	24.921	14.585

Table 7.4: VAR Lag Order Selection Criteria Notes: *indicates lag order selected by the criterion; LR: sequential modified LR test statistic (each test at 5% level); FPE: Final prediction error; FPE: Final prediction error; AIC: Akaike information criterion; SC: Schwarz information criterion; HQ: Hannan-Quinn information criterion; Endogenous variables: G_t^1, G_t^2, G_t^3, G_t^{83}, G_t^{82}, G_t^{81}, x_t^{IMACEC}, π_t, r_t, P_t^{copper} and VIX. Sample: 2000M10 2011M1.

transitory negative effect on the returns of the portfolio constructed using the first largest eigenvalue and a transitory positive effect on the returns of the portfolio constructed with the second largest eigenvalue. The negative impact on returns happens during the first and second months for G_t^1. The positive effects on G_t^1 also happen during the first and second months. On the other hand, unexpected shocks to the VIX have no effects on the portfolio returns constructed with the smallest eigenvalues.

- P_t^{copper}: A shock to the monthly change of the price of copper has a positive effect on G_t^1 but no effect on the remaining portfolios.
- x_t^{IMACEC}: A positive shock to the economic dynamism measured as the monthly change of x_t^{IMACEC}, does not have any impact on the returns of the portfolios constructed with the 3 largest eigenvalues nor the returns of the portfolio constructed with the third smallest eigenvalue. However, it has a transitory negative impact on the returns of the portfolios constructed with the first and second smallest eigenvalues. These effects occur during the 1st month of the shock.
- π_t: A positive shock on the seasonally adjusted monthly inflation has a transitory negative impact on the returns of the portfolio constructed with the largest eigenvalue and a transitory positive one on the returns of the portfolio constructed with the third largest eigenvalue. On the other hand, inflation shocks would not seem to have an impact on G_t^{81}, G_t^{82} and G_t^{83}.

- r_t: A positive shock to the monetary policy rate (an unexpected increase of the r_t) has a transitory negative impact on the returns of the portfolio constructed with the largest eigenvalue and a transitory positive one on the returns of the portfolio constructed with the third largest and the second smallest eigenvalues.

To better understand the relative importance of each macroeconomic shock on the portfolio returns we carry out a Variance Decomposition Analysis. To do this, we need a different identification strategy for the residuals. We opted for the Choleski decomposition with the ordering presented in Equation 7.4. The international volatility (VIX) is the most exogenous variable, followed by the copper price (P_t^{copper}). Then, we included the domestic macroeconomic variables beginning with the x_t^{IMACEC}, π_t and r_t. Finally, we include our monthly portfolio returns ordered from the one constructed by largest eigenvalue to the portfolio return constructed by the smallest eigenvalue. Table 7.5 presents the results of this exercise.

Our results have important implications for risk diversification across Chilean equities. First, the main macroeconomic determinant of the market portfolio volatility (G_t^1) is the international volatility (VIX). This variable explains more than 20% of the market portfolio returns variance after 12 months. Domestic inflation followed by the monetary policy rate (r_t) are also important macroeconomic determinants explaining 6.3% and 2.5% of the market portfolio returns variance after 12 months, respectively. Second, it would be possible to diversify away some of the market portfolio risk by adding positions on the portfolios constructed with the second and third largest eigenvalues. The VIX explains 10% of the returns variance of G_t^2 and 5.7% of the return variance of G_t^3. But, the impact is of different sign than for the market portfolio returns. Third, as expected, the three smallest eigenvalues produce portfolio returns that are mostly uncorrelated with macroeconomic shocks. These portfolios are also uncorrelated with the market portfolio.[14]

In sum, the market portfolio (which is very close to the IPSA-IPGA indices) is mostly affected by international shocks and therefore should be used by investors willing to increase their

[14]It should be noted, however, that the use of these portfolios might be problematic for at least two reasons. First, since they are very close to the bounds of the MP distribution, in whose case they may not be significant and their existence may be due to the fact that N and T are finite in practice. Second, if eigenvalues are very close to zero (as it would be the case if they were affected by noise), the covariance will be almost singular which means that its invertibility will be at stake distorting portfolio weights and consequently, risk estimates. This is why, in practice, we rather focus on the eigenvalues that *clearly* deviate from the MP distribution, that is, we focus on the largest ones. The main results of the VAR analysis are robust to the exclusion of the three smallest eigenvalues from the specification.

G^1	$S.E.$	VIX	P_t^{copper}	x_t^{IMACEC}	π_t	r_t	G_t^1	G_t^2	G_t^3	G_t^{81}	G_t^{82}	G_t^{83}
3 Months	0.323	19.653	0.163	0.450	4.334	1.006	71.455	0.117	0.762	1.280	0.256	0.518
		(6.191)	(1.269)	(1.568)	(3.015)	(1.357)	(7.213)	(1.267)	(1.55761)	(1.592)	(1.400)	(1.815)
12 Months	0.336	21.375	0.198	0.436	6.323	2.529	65.805	0.132	0.858	1.369	0.491	0.478
		(5.735)	(1.225)	(1.483)	(3.986)	(1.978)	(7.052)	(1.207)	(1.581)	(1.483)	(1.340)	(1.628)
G^2	$S.E.$	VIX	P_t^{copper}	x_t^{IMACEC}	π_t	r_t	G_t^1	G_t^2	G_t^3	G_t^{81}	G_t^{82}	G_t^{83}
3 Months	0.213	8.799	0.100	0.317	0.909	0.328	38.605	50.031	0.214	0.652	0.001	0.039
		(4.958)	(1.704)	(1.505)	(1.986)	(1.381)	(6.027)	(7.077)	(2.079)	(1.257)	(0.972)	(1.145)
12 Months	0.218	10.169	0.131	0.317	2.321	1.404	36.810	47.622	0.307	0.728	0.147	0.038
		(5.208)	(1.647)	(1.443)	(2.964)	(1.739)	(5.749)	(7.046)	(2.129)	(1.195)	(0.948)	(1.099)
G_t^3	$S.E.$	VIX	P_t^{copper}	x_t^{IMACEC}	π_t	r_t	G_t^1	G_t^2	G_t^3	G_t^{81}	G_t^{82}	G_t^{83}
3 Months	0.177	2.694	0.287	0.031	4.341	0.878	15.459	0.781	75.204	0.273	0.040	0.006
		(3.085)	(1.610)	(1.435)	(3.849)	(0.968)	(5.496)	(1.676)	(6.429)	(1.307)	(1.068)	(0.993)
12 Months	0.186	5.684	0.401	0.072	6.568	3.690	13.961	0.730	68.072	0.468	0.335	0.015
		(4.791)	(1.500)	(1.347)	(4.720)	(1.943)	(5.047)	(1.489)	(7.243)	(1.191)	(1.170)	(1.019)
G_t^{81}	$S.E.$	VIX	P_t^{copper}	x_t^{IMACEC}	π_t	r_t	G_t^1	G_t^2	G_t^3	G_t^{81}	G_t^{82}	G_t^{83}
3 Months	0.046	1.477	2.117	0.943	0.797	1.998	1.594	1.524	1.060	87.205	0.019	1.259
		(2.320)	(2.712)	(2.091)	(1.964)	(2.541)	(2.222)	(2.638)	(1.767)	(6.044)	(1.010)	(2.112)
12 Months	0.046	2.084	2.120	0.987	1.041	2.1487	1.620	1.531	1.060	86.122	0.033	1.249
		(4.003)	(2.651)	(2.145)	(2.205)	(2.596)	(2.174)	(2.576)	(1.727)	(6.820)	(1.024)	(2.072)
G_t^{82}	$S.E.$	VIX	P_t^{copper}	x_t^{IMACEC}	π_t	r_t	G_t^1	G_t^2	G_t^3	G_t^{81}	G_t^{82}	G_t^{83}
3 Months	0.045	0.452	1.196	3.895	0.573	2.718	2.504	1.852	1.346	3.222	81.552	0.684
	(1.405)	(2.423)	(3.143)	(2.123)	(2.163)	(2.859)	(2.782)	(2.449)	(2.436)	(6.853)	(1.866)	
12 Months	0.046	1.397	1.179	3.884	0.9735	2.997	2.468	1.824	1.377	3.212	80.011	0.671
		(2.301)	(2.344)	(3.121)	(2.232)	(2.164)	(2.751)	(2.710)	(2.417)	(2.342)	(7.143)	(1.822)
G^{83}	$S.E.$	VIX	P_t^{copper}	x_t^{IMACEC}	π_t	r_t	G_t^1	G_t^2	G_t^3	G_t^{81}	G_t^{82}	G_t^{83}
3 Months	0.040	1.473	1.870	5.160	1.985	0.016	2.150	2.098	1.932	1.858	0.410	81.043
		(1.984)	(2.142)	(3.955)	(2.331)	(0.888)	(2.399)	(2.073)	(2.498)	(2.402)	(1.630)	(6.743)
12 Months	0.040	1.521	1.869	5.259	2.067	0.105	2.150	2.093	1.932	1.856	0.433	80.710
		(2.420)	(2.096)	(3.937)	(2.369)	(1.029)	(2.363)	(2.018)	(2.463)	(2.355)	(1.586)	(6.936)

Table 7.5: Variance Decomposition of Portfolio Returns.

exposure to global risks or when they are expecting a decrease in international volatility. The portfolios constructed with the second and third largest eigenvalues, on the other hand, could be used by investors to increase exposure to global risks but when they are expecting an increase on global volatility (due to their positive correlation with the VIX). Finally, as theoretically expected, the portfolios constructed using the smallest eigenvalues are uncorrelated to macroeconomic variables and global risk aversion.

7.5 Concluding Remarks

In this chapter we apply Random Matrix Theory (RMT) to study the correlations of 83 Chilean stocks that are part of the IPSA and IGPA indices during the period 2000 to 2011. We find that using RMT to identify statistically significant correlations within our sample of stocks significantly improves the efficiency of a family of Markowitz Portfolios.

Moreover, by using Vector Autoregressive analysis we identify the global risk aversion as the main driver of the market returns followed in importance by shocks to the monthly rate of inflation and the country's monetary policy rate. Using the statistically significant eigenvalues we can construct portfolios, which are uncorrelated to macroeconomic and financial factors such as global risk aversion.

Despite the recent success of RMT in financial applications, challenges remain. Recent evidence has suggested that meaningful correlations can be measured in the bulk of the MP eigenvalue spectrum (Burda et al., 2004; Burda and Jurkiewicz, 2004; Malevergne and Sornette, 2004; Kwapièn et al., 2006). One possible reason for this is the fact that we are assuming that our returns are normally distributed and under this premise we carry out our comparisons with the sample correlation matrix. In reality, returns in financial markets are not normally distributed and exhibit a number of features, such as fat tails, volatility clustering and non-stationarity. Recent literature has been improving these matters by developing extensions of the MP distribution to account for these phenomena (Potters et al., 2005; Bouchaud and Potters, 2009). However, still one of the main difficulties has been to generate predictions during critical periods, where there is a high collectivity between stocks, especially during drawdowns. In this context, an interesting line of future research is to extend this approach focusing on turbulent periods in financial markets. In the present chapter, we have tried to deal with this by combining the RMT approach with some standard models of the MV-GARCH literature in order to capture to some extent the short run dynamics of financial correlations with significant improvements in correlation estimates. Another interesting line of future research would be extending our analysis to regional stocks such as the MILA or the MSCI LATAM. Also, it would be interesting to apply RMT to cross assets within the Chilean financial market including the return of equities, fixed income and currencies. This would give us a more complete picture of the factors that account for the volatility of the Chilean financial market.

All in all, the results obtained for the Chilean case provide important evidence of the broad applicability of the RMT approach to equity returns in emerging markets portfolios. The Chilean Stock Market is characterised by the existence of large institutional investors such as private pension funds that maintain important structural positions on this market. Applying RMT to their portfolio optimisation could be useful in diversifying some of the systematic risk of the Chilean market.

Chapter 8

Early Warning and Systemic Risk in Core Global Banking: Financial Network and Market Price-based Methods

8.1 Introduction

Having started in the US as the bursting of a housing bubble, the 2007 Global Financial Crisis (GFC) that spread globally has underscored the importance of banking system stability in terms of cross-border propagation of contagion. The role of cross-border exposures in the dynamics of the GFC with European banks having a surprisingly large exposure to US securitized assets (Allen et al., 2011), which led to a 50% equity capital loss (valued at $1.6 trillion), has led many to conclude that global financial interconnectedness is a major vehicle for systemic risk and macroeconomic instability (Soramäki et al., 2007; Degryse et al., 2010; Castrén and Rancan, 2014). The tax payer bailout of the financial systems of the US, UK and of some Euro-zone countries in 2007-9 period of the GFC costing over $14 trillion, Alessandri and Haldane (2009), has intensified efforts by regulators and academics to identify, avert and manage the large-scale breakdown of financial intermediation due to the domino effects of insolvency and illiquidity.

While there is no universally agreed definition, systemic risk is defined as the "disruption to the flow of financial services that is caused by the impairment of all or parts of the financial system and has the potential to have serious negative spillovers to the real economy".[1] This motivates a second question, which is how can we quantify systemic risk and identify those financial institutions that are systemically important by posing a major threat to the financial system?[2]

The development of systemic risk indices (henceforth, SRIs) has taken two main approaches. These are market price oriented statistical SRIs and the financial network approach championed by Haldane (2009) based on the interconnectedness of assets and liabilities of individual economic agents or sectors. Due to the ease of publicly available market price data on FIs, and the popularity of generalizing from the Basel II individual risk measures such as Value at Risk and Expected Shortfall to a portfolio approach, the majority of SRI analytics have taken this route. Of particular interest here are the purely market price-based SRI, namely, the *Marginal Expected Shortfall* (MES) of Acharya et al. (2010), the $\Delta CoVaR$ of Adrian and Brunnermeier (2011) and an hybrid SRISK developed by Acharya et al. (2012) and Brownlees and Engle (2012), which combines balance-sheet data on leverage and equity with market price-based data.

However, these market price-based SRIs have not been free from criticism. One of the main problems is the absence of any early warning signals and they tend to be "coincident" or "near-coincident" indicators of financial instability (Arsov et al., 2013). This happens because most mainstream risk management models suffer from the so-called "paradox of volatility", a term coined by Borio and Drehmann (2011) and adopted by others (Brunnermeier and Sannikov, 2015). That is, volatility of asset returns, which is a key measure of risk tends to be underestimated during asset price booms when systemic risk from leverage is building up on the balance sheets of banks and non-bank sectors.[3] This was presaged by Minsky (1986) who claimed that asset

[1] IMF-BIS-FSB (2009) and IMF (2011).

[2] By SIFIs, we refer to those financial institutions whose distress or disorderly failure, stemming from their size, complexity and systemic interconnectedness, would lead to a massive disruption to the rest of the financial system and, ultimately, the economic activity (Financial Stability Board, 2011).

[3] The volatility paradox can be seen in publicly available volatility indexes, such as the VIX and the V-FTSE in that they are extremely low during asset price booms and are at a local minimum just before the market crashes (see Markose, 2013).

bubbles mask the growing financial system fragility as the enhanced market values for assets and the perception of low risk from volatility measures encourage procyclical excessive growth of leverage of banking systems with a commensurate growth of both their exposures and liabilities within interbank and with non-bank sectors. Sooner or later, this may result in system collapse.[4] Borio et al. (2001) give an extensive discussion on the general problem of procyclicality and underestimation of market price-based SRIs. Benoit et al. (2013) find that some market price-based SRIs represent *systematic* risk rather than *systemic* risk and tend to covary with the market index, which makes them highly procyclical. They also find that the ΔCoVaR can be explained by its own VaR in the time-series domain, which means that forecasting systemic risk, using ΔCoVaR is not different from forecasting VaR of an individual institution. SRISK, though well determined in the cross-section by the leverage of individual banks or of national banking systems, it loses information when aggregated into an index form. This calls for new methods for systemic risk estimation that can capture the growing risk from chains of indebtedness between FIs and also other economic sectors, both nationally and in a cross-border setting.

In the context of highly integrated international financial markets, the cross-border exposure of banks has become one of the major catalysts for the global financial crisis (Haldane, 2009; Allen et al., 2011; Minoiu and Reyes, 2013; Castrén and Rancan, 2014). As pointed out by Degryse et al. (2010), shocks that impact the proper functioning of this densely interconnected network of activities involving cross-border banking may affect not only banks themselves but also the economies they operate in. This is why, it is vital to investigate the topological structure of global banking and its vulnerability to liabilities of debtor countries.

Following Minsky (1986), we hold the view that bilateral balance sheet-based interlinkages are essential to assess the fragility of the financial system. The important point about the Minsky thesis of financial busts from leverage (see, Schularick and Taylor, 2012) is that the growth of leverage and ensuing financial fragility of the system does not happen overnight. In view of this and the above discussion, some have recognized the pitfalls of relying on market price based SRIs that underestimate the build up of systemic risk of failure from growing bilateral obligations relative to the buffers of their counterparties. Hence, it is sensible to include early warning of

[4]The idea of procyclicality of leverage is now acknowledged to be a major source of systemic risk and is well articulated in Adrian and Shin (2010, 2011a).

impending financial crisis as an important component of SRIs in order that regulators are not blindsided by low volatility measures in asset returns.[5] Regulatory authorities mandating the reporting of bilateral financial obligations of relevant financial agents may become *de riguer*, Markose (2013), to directly to assess the instability of the financial system. Alternatively, as in Brunnermeier and Cheridito (2013) two stage reduced form macro-models which will explicitly work to overcome the volatility paradox in their new Syst Risk index or some other model that inverts the low volatility in asset returns to proxy leverage (see, Bruno and Shin, 2015b,a) have been proposed. As bilateral balance sheet information is typically not publicly available, the BIS Consolidated Banking Statistics is an exception in a cross-border setting between debtor countries and exposed banking systems.

The objective of this chapter is to compare and contrast the market price-based SRIs for the global banking system with a network based SRI. For the latter, we follow the eigen-pair method developed by Markose (2012) and Markose et al. (2012). We estimate a SRI based on a spectral measure of stability of financial networks constructed from a matrix of cross-border exposures of national banking systems. As the spectral approach describes the stability of dynamical systems, we give an appropriate dynamical characterization of the rates of failure of national banking systems when losses exceed a given regulatory capital threshold. The network based SRI is the maximum eigenvalue of this stability matrix and tipping points are identified when the latter is greater than a prespecified capital loss threshold. We also use the associated right and left eigenvectors of this matrix of national banking system exposures relative to equity capital, to gauge systemic importance and vulnerability of debtor countries and national banking systems, respectively. For this analysis, we use data from the Consolidated Banking Statistics reported by the BIS and banks' equity capital from Bankscope for the period 2005Q4-2013Q4 for 18 core country banking systems. Note that only the equity capital of banks with cross-border exposures in each reporting country is taken from Bankscope.

For the market price-based SRIs, namely, the DCC-MES, DCC-ΔCoVaR and SRISK, we use the MSCI Financials corresponding with the 18 BIS reporting national banking systems and the MSCI World as proxy for the market index. This chapter is amongst the first comprehensive

[5]However, interestingly, early warning has not been included in the list of desirable properties of so called theoretical multivariate measures of risk that involve several interconnected components (see, Armenti et al., 2015).

applications of these measures in a cross-border setting.

Our findings covering the period 2005Q4-2013Q4 can be summarised as follows: First, we find that the SRI based on the maximum eigenvalue constructed from cross-border financial network exposures peaks way before the start of the financial crisis in September 2007. Our eigen-pair SRI remarkably estimates the 60% loss of equity capital that was realised in 2009 for the global banking system from the most systemically important debtor country identified as the US. Second, we are also able to identify the increasing vulnerability of the Belgian (2008Q3) and Portuguese (2013Q4) banking systems before the bankruptcies of their major banks.[6] Third, we find that market price-based SRIs tend to be highly contemporaneous with standard market risk measures, such as the VIX/VSTOXX, making them inversely related to asset price booms. That is, market price-based SRIs are found to peak with the crisis or even after it. Finally, we corroborate the Benoit et al. (2013) findings that the market price-based SRIs, namely, DCC-MES, ΔCoVaR and SRISK, are determined by systematic risk measures. While the eigenpair SRI uses a capital loss threhold to identify tipping points, the lack of similar thresholds in the market price-based SRIs is found to be the add to the difficulty of monitoring the onset of potential crises.

The chapter is organised as follows: Section 8.2 briefly describes the eigen-pair method developed by Markose (2012) and Markose et al. (2012). We also derive the market price-based SRIs in a cross-border setting. Section 8.3 describes the data and estimation methodology used in our analysis. In Section 8.4 we provide a network analysis of cross-border exposures in the core global banking network. Section 8.5 gives the results on cross-border banking network stability, contrasted with market price-based SRIs. Section 8.6 analyses rankings of systemic importance and vulnerability over time for different groups of countries using both methods. Finally, in Section 8.7 we report conclusions and future work.

8.2 Systemic Risk Indices (SRIs) in a Cross-Border Setting

In this section we provide a brief review of the eigen-pair method of Markose (2012), which was further developed in Section 3.5. We also review some popular SRIs based on publicly available

[6]The Greek case and a fuller discussion of the Eurozone Crisis is given in Markose et al. (2015).

market price-based data. As pointed out by Benoit et al. (2013), these measures are constantly monitored by many central banks around the world.

8.2.1 Eigen-Pair Method based on Network Analysis

As we described in Section 3.5, the eigen-pair method of Markose (2012, 2013) is based on the spectral analysis of cross-border exposures of national banking systems relative to their equity capital. Here the main focus is the *Core of the Global Banking System Network* (CGBSN) (18 BIS reporting countries), which is a directed weighted network, where the direction of a link represents obligations that all sectors of a country at the start of an arrow have towards the banking system of another country at the end of the arrow. These obligations are then netted and divided by their respective equity capital to yield the stability matrix Θ:

$$\Theta = \begin{bmatrix} 0 & \frac{(x_{12}-x_{21})^+}{C_{20}} & \cdots & 0 & \cdots & \frac{(x_{1N}-x_{N1})^+}{C_{N0}} \\ 0 & 0 & \cdots & \frac{(x_{2j}-x_{j2})^+}{C_{j0}} & \cdots & \frac{(x_{2N}-x_{N2})^+}{C_{N0}} \\ \vdots & \vdots & 0 & \cdots & \cdots & \cdots \\ \frac{(x_{i1}-x_{1i})^+}{C_{10}} & \vdots & \cdots & 0 & \cdots & \frac{(x_{iN}-x_{Ni})^+}{C_{N0}} \\ \vdots & \vdots & \cdots & \cdots & 0 & \vdots \\ 0 & \frac{(x_{N2}-x_{2N})^+}{C_{20}} & \cdots & \frac{(x_{Nj}-x_{jN})^+}{C_{j0}} & \cdots & 0 \end{bmatrix}, \tag{8.1}$$

As we mentioned in Section 3.5, Markose (2012), based on the work of May (1974, 1972) shows that the stability of the network system is determined by the maximum eigenvalue of the stability matrix:

$$\lambda_{max}(\Theta) < \rho. \tag{8.2}$$

Where ρ is the homogeneous capital loss threshold. The threshold ρ can be regarded as a percentage of banking system equity capital to buffer losses. If this condition is violated, then any negative shock, in the absence of outside interventions, can propagate through the networked system as a whole and potentially cause capital losses proportionate to $\lambda_{max}(\Theta)$.

In addition, Θ has two sets of eigenvectors: right (V^R) and left (V^L) eigenvector. The right eigenvector V^R gives the rank order of systemic importance, while the left eigenvector V^L gives the rank order of the countries that are vulnerable. These rank orders of systemically important financial systems, will be compared to those using market price-based data as described below.

8.2.2 Marginal Expected Shortfall (MES)

In our cross-border setting, the MES represents the marginal contribution of the financial system of country i to global systemic risk, measured by the Expected Shortfall (ES) of the global system, proxied by the MSCI World Index. The MES was initially proposed by Acharya et al. (2010) and later on extended in a dynamic conditional correlation (DCC) version by Brownlees and Engle (2012). The *Expected Shortfall* (ES) at the $\alpha\%$ level is the expected return in the worst $\alpha\%$ of the cases. This could be extended to the general case, where returns exceed a given threshold C. In formal terms, denoting the MSCI World returns at time t by $r_{m,t}$ and the country MSCI Financial returns by $r_{i,t}$, the conditional ES of the system can be defined as:

$$ES_{mt}(C) = E_{t-1}(r_m|r_{mt} < C) = \sum_{i=1}^{N} w_{it} E_{t-1}(r_{it}|r_{it} < C). \qquad (8.3)$$

Here, $w_{i,t}$ are the weights of the country MSCI Financials. In this context, the MES corresponds to the partial derivative of the system ES with respect to the weight of country i in the global financial system:

$$MES_{it}(C) = \frac{\partial ES_{mt}(C)}{\partial w_{it}} = E_{t-1}(r_{it}|r_{it} < C). \qquad (8.4)$$

The higher a country's MES, the higher the individual contribution of the country to the risk of the global system. In general, when modelling the distribution of financial returns, the critical issues are dynamic volatility and the modelling of asymmetries. It is well documented that asset return distributions are skewed and fat-tailed. Moreover, the volatility of asset returns is not constant, it is mean-reverting and tends to cluster. Another important stylised fact of asset returns volatility is that a large negative price shock increases volatility by much more than a positive price shock of the same magnitude, which is also known as "leverage-effect". To address at least the dynamics of volatility, we follow Brownlees and Engle (2012) and Benoit et al. (2013) and assume that the returns of the world market and national banking system i at time t, follow a bivariate GARCH(1,1) process for the demeaned returns:

$$R_t = H_t^{1/2} V_t. \tag{8.5}$$

Here $R_t' = (r_{mt}, r_{it})$ denotes a return vector of the world market and the MSCI Financials, respectively. The random vector $V_t' = (\epsilon_{mt}, \xi_{it})$ are the associated standardised innovations and are assumed to be $i.i.d$ with the following first moments: $E(V_t) = 0$ and $E(V_t V_t') = I_2$, a two-dimensional identity matrix. The H_t matrix denotes the conditional variance-covariance matrix:

$$H_t = \begin{pmatrix} \sigma_{mt}^2 & \sigma_{it}\sigma_{mt}\rho_{it} \\ \sigma_{it}\sigma_{mt}\rho_{it} & \sigma_{it}^2 \end{pmatrix}. \tag{8.6}$$

Here σ_{it} and σ_{mt}, denote the conditional standard deviations and ρ_{it} the conditional correlation between the world market and the country financial returns. Based on Equations 8.5 and 8.6, Brownlees and Engle (2012) show that the MES can be expressed as:

$$MES_{it}(C) = \sigma_{it}\rho_{it}E_{t-1}\left(\epsilon_{mt}|\epsilon_{mt} < \frac{C}{\sigma_{mt}}\right) + \sigma_{it}\sqrt{1-\rho_{it}^2}E_{t-1}\left(\xi_{mt}|\epsilon_{mt} < \frac{C}{\sigma_{mt}}\right). \tag{8.7}$$

Then, the MES can be expressed as a function of the country's return volatility, its correlation with the global index return, and the comovement of the tail distribution. As demonstrated by Benoit et al. (2013), the $i.i.d$ assumption on $\xi_{i,t}$ and $\epsilon_{m,t}$ implies that the conditional expectation $E_{t-1}\left(\xi_{mt}|\epsilon_{mt} < \frac{C}{\sigma_{mt}}\right)$ is null. It is worthwhile mentioning that the assumption that $\xi_{i,t}$ and $\epsilon_{m,t}$ are $i.i.d$ is a very strong assumption. In fact, there are a number of reasons to believe that extreme values of these disturbances could occur at the same time for systemically risky national banking systems. That is, when the world market is in its tail, the national banking system's disturbances may be even further in the tail if there is serious risk of default. [7] Thus, the MES

[7] While there are many ways to model this dependence between the disturbance terms, many studies have relied on the use of Kernel functions where additional assumptions are needed, making results highly dependent on these assumptions. Faced to these potential shortcomings, we limit our analysis to the most simplistic case.

can be expressed as:

$$MES_{it}(\alpha) = \beta_{it} ES_{mt}(\alpha). \tag{8.8}$$

Under these assumptions, ranking systemically important financial systems is equivalent to sorting financial systems based on their betas (Benoit et al., 2013). This SRI is estimated using daily data using the Dynamic Conditional Correlation Model of Engle and Sheppard (2001), from which the time-varying beta is obtained (Equations (8.5) and (8.6)). In order to compare the MES with the network-based SRI based on λ_{max}, we calculate the quarterly MES by averaging the daily MES within each quarter. To obtain our broad measure of MES, quarterly MES are averaged across countries, as done by Acharya and Stefen (2012) to calculate country MES.[8]

8.2.3 SRISK

The SRISK measure was developed by Acharya et al. (2012) and Brownlees and Engle (2012). It is an extension of the MES by considering both the liabilities and the size of the financial system. In a cross-country setting, the SRISK corresponds to the expected shortfall of the financial returns of a given country, conditional on a crisis affecting the global market (e.g., the MSCI World). The country with the largest shortfall is assumed to be the largest contributor to the crisis and can be, therefore, regarded as the most systemically important. Following Acharya et al. (2012), the SRISK is defined as:

$$SRISK_{i,t} = MAX\left[0; \phi(D_{i,t} + (1 - LRMES_{i,t})W_{i,t}) - (1 - LRMES_{i,t})W_{it}\right]. \tag{8.9}$$

Here ϕ is the prudential capital ratio, which is assumed to be 8%, D_{it} is the book value of total liabilities of country i's banking system, and W_{it} is the market capitalisation or market value of equity of the national banking system. The $LRMES_{i,t}$ is the long-run marginal expected shortfall of country i. It measures the sensitivity of the country's equity return to the downturn of the world market in the case of a financial crash. Acharya et al. (2012) and Brownlees and Engle (2012) define a market crash as a hypothetical 40% semiannual world market decline. LRMES is particularly difficult to estimate because it corresponds to an extremely rare event. In fact,

[8] Alternatively, we could have used the maximum within each quarter, although our main results do not change.

there have been only three episodes of this magnitude over the last century (1929, 2000 and 2008). In addition, the LRMES is typically not available in closed form for this class of dynamic models. We follow Brownlees and Engle (2012) and implement a simulation based procedure to obtain exact LRMES predictions. This procedure consists of simulating a random sample of the h-period country and market (arithmetic) returns conditional on the information set available on day T. The LRMES for day T is then calculated using the Monte Carlo average of the simulated returns,

$$LRMES_{i,T} = \frac{\sum_{s=1}^{S} R_{i,T+1:T+h} I\{R_{m,T+1:T+h} < C\}}{\sum_{s=1}^{S} I\{R_{m,T+1:T+h} < C\}}, \tag{8.10}$$

where C is the semiannual 40% drawdown and S is the number of runs of simulations, which in this case is $S = 50,000$. $R_{i,T+1:T+h}$ are cumulated returns over a six months period. The details of the simulation algorithm are provided in C.2. If we define leverage as $L_{i,t} = (D_{i,t} + W_{i,t})/W_{i,t}$, SRISK becomes:

$$SRISK_{i,t} = MAX\left[0; (\phi L_{i,t} - 1 + (1-\phi) LRMES_{i,t}) W_{i,t}\right]. \tag{8.11}$$

Notice that the SRISK is a weighted average of leverage-1 and the LRMES. In order to allow comparison with the rest of the SRIs, we scale the SRISK by the country's equity. Hence, throughout the analysis, we use:

$$SRISK^{\#}_{i,t} = \frac{SRISK_{i,t}}{W_{i,t}}. \tag{8.12}$$

In order to compare and contrast the $SRISK^{\#}$ with λ_{max}, we perform this simulation procedure for each quarter using data for that particular quarter and construct the quarterly $SRISK^{\#}_{i,t}$ measure for every country i. To calculate the overall index, we follow Acharya et al. (2010) and we take the average of the quarterly $SRISK^{\#}$ across countries.

8.2.4 ΔCoVaR

This measure was developed by (Adrian and Brunnermeier, 2011) and is based on the concept of Value-at-Risk, denoted as VaR(α), which is the maximum loss within the α%-confidence interval. Then, the CoVaR corresponds to the VaR of the MSCI World Index returns obtained conditionally on some event $C(r_{it})$ observed for country i:

$$Pr\left(r_{mt} \leq CoVaR_t^{m|C(r_{it})}|C(r_{it})\right) = \alpha \tag{8.13}$$

The ΔCoVaR of country i is then defined as the difference between the VaR of the MSCI World returns conditional on this country's financial system being in distress and the VaR of the MSCI World returns conditional on the financial system of country i being in its median state[9]:

$$\Delta CoVaR_{it}(\alpha) = CoVaR_t^{m|r_{it}=VaR_{it}(\alpha)} - CoVaR_t^{m|r_{it}=Median(r_{it})}. \tag{8.14}$$

We follow Benoit et al. (2013) and derive the DCC variant of ΔCoVaR as a function of the conditional correlations, volatilities, and VaR:

$$\Delta CoVaR_{it}(\alpha) = \gamma_{it}[VaR_{it}(\alpha) - VaR_{it}(0.5)]. \tag{8.15}$$

Here $\gamma_{it} = \rho_{it}\sigma_{mt}/\sigma_{it}$ with $\rho_{i,t}$ denoting the conditional correlation between the returns of the MSCI Financial returns of country i and the MSCI World returns. If we assume that the marginal distribution of the returns is symmetric around zero, ΔCoVaR is strictly proportional to VaR[10]:

$$\Delta CoVaR_{it}(\alpha) = \gamma_{it} VaR_{it}(\alpha). \tag{8.16}$$

This means that while ΔCoVaR-based rankings and VaR-based rankings of systemic risk may differ in the cross-section (that is, at a given point in time t), in the time-series, that is, for a given financial system i, ΔCoVaR is proportional to VaR. As a consequence, as noted by Benoit et al. (2013), forecasting the future evolution of the contribution of a country's financial system i to global systemic risk is equivalent to forecast its risk in isolation. We compute the quarterly ΔCoVaR by taking the average within each quarter. In a similar fashion, the overall index is calculated by taking the average of the quarterly ΔCoVaR across countries.

[9] Following Adrian and Brunnermeier (2011), we focus on the event of $r_{i,t} = VaR^i_\alpha$.

[10] A similar exercise carried out without assuming that the marginal distribution of returns is symmetric and results do not change the main conclusions of the chapter

8.3 Data Description

In this section we describe the data used to construct the network-based SRI and also the aggregated financial indices used to construct the market price-based SRIs.

8.3.1 BIS Consolidated Banking Statistics and Bankscope Data

The Core Global Banking System Network (CGBSN) is based on *Consolidated Banking Statistics*, Table 9D of the BIS, which gives foreign claims of reporting banks on an ultima risk basis. The information on banking system equity for each country is taken from Bankscope database.

The sample of BIS Consolidated Banking Statistics consists of 18 reporting countries and spans quarterly from 4th quarter of 2005 until 4th quarter 2013.[11] This data provides information on positions of reporting countries' banking systems via-a-vis counterparts located outside of the reporting countries (BIS, 2013). We consider only the core counterparts that correspond to the same reporting countries.

The second ingredient needed to construct the CGBS network is information about the amount of equity, which serves as a buffer for losses incurred by banks, is reported by Bankscope.[12] Total bank equity should be understood in terms of a balance sheet equation as total assets less total liabilities. As data comes disaggregated for individual institutions, the total equity for given country's banking system is calculated as a sum of cross-border banks headquartered in the country (excluding central banks). This is treated as a proxy for the amount of equity capital in the banking system of the country.

Bankscope data on total bank equity is reported on a annual basis, so data is taken for the end of the year, that is, for the fourth quarter of a given year. In order to obtain data for every quarter over the sample, the missing quarters are filled with the help of linear interpolation.[13] A similar approach is followed for the data on national banking system liabilities used to compute

[11] The countries in the sample are Australia, Austria, Belgium, France, Germany, Greece, India, Ireland, Italy, Japan, Netherlands, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom and United States.

[12] Data for "Total equity" is defined as the sum of 'Common equity + Non-controlling interest + Securities revaluation reserves + Foreign Exchange Revaluation Reserves + Other Revaluation reserves". As quoted from the user guide of Bankscope available at: https://bankscope.bvdinfo.com. Total equity has the code 11840 in the Bankscope database.

[13] This was the method preferred to interpolate the value of equity as polynomial interpolation tends to generate extremes when the degree of the polynomial is large. We also interpolated the value of equity using cubic spline and results are very similar.

the SRISK measure of systemic risk.

8.3.2 MSCI Financials Data

In order to estimate the DCC-MES, DCC-ΔCoVaR and SRISK SRIs, we use the individual MSCI Financials for the 18 countries and the MSCI World as a proxy for the market return that is faced by the financial system of each country in the sample. The MSCI World Financial Index captures large and mid cap representation across 23 Developed Markets (DM).[14] We use daily returns of MSCI Financials for each country during the period 03/10/2005 until 31/12/2013. This data has been retrieved from Bloomberg and it has been adjusted for bank holidays.[15]

8.4 Network Analysis of the Core Global Banking System

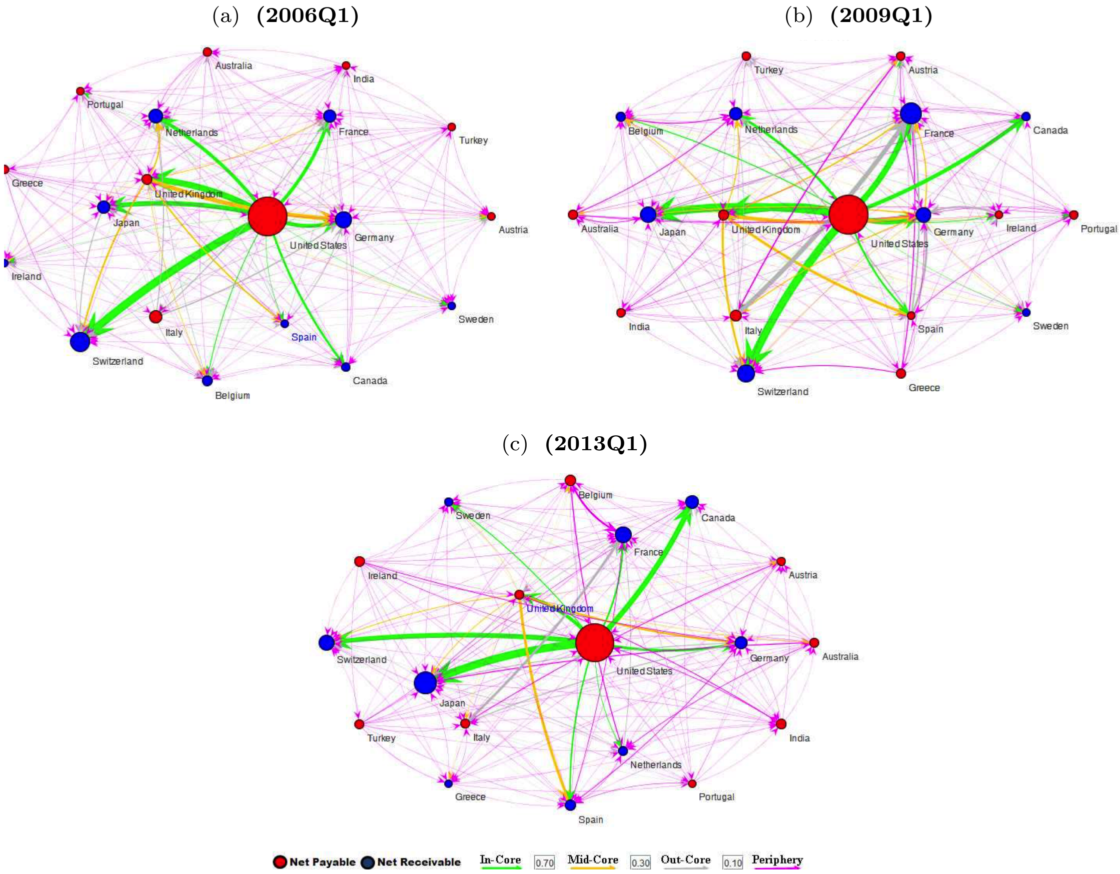
Based on the methodology introduced in Section 3.5, we explore the evolution of the network topology of the core global banking system. For this analysis, we have chosen three specific periods: Figure 8.1 (Panel (a)) represents the pre-crisis period; Panel (b) depicts the midst of the crisis, while Panel (c), shows the network topology after the crisis (2013Q1). These figures represent the banking systems of 18 countries, with net cross-border exposures. The size of the node is proportional to the total netted position. Blue nodes are net lenders, while red nodes are net borrowers. The links are weighted and the thicker the edge, the larger the cross-border claim of a node at the end of an arrow on the originating node.

Before the crisis the US is followed by Italy and UK in net payables, strongly borrowing from Switzerland, the UK, Germany and Japan. After the crisis, Italy and the UK are no longer among the top net borrowers and are overtaken by Belgium, followed by Ireland, India and Turkey. As we can see, Belgium started as a net lender and after the Eurozone crisis became a net borrower. In fact, the Belgian banking system suffered huge losses during 2008 when Fortis group was sold to BNP Paribas and in 2011 Dexia group was dismantled.

[14] DM countries include: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong kong, Ireland, Israel, Italy, Japan, the Netherlands, Norway, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United States and United Kingdom.

[15] We reckon the fact that the MSCI Financials also includes non-bank financial institutions. Nonethess, the banking sector comprises nearly 45% of the index and it can be therefore considered as a good proxy for the performance of the banking sector.

Figure 8.1: CGBS Network Topology. Direction of arrows goes from borrowing country to lending banking system.



Note: Net payables are coded in red. Net receivables are coded in blue. Authors calculations.

During 2006Q1, the largest net lender is Switzerland followed by Germany and the Netherlands. During the crisis France becomes the most exposed to foreign debt, while Switzerland, Japan and Germany occupy the second, third and fourth position, respectively. In 2013, the largest net lender is Japan, followed by Switzerland, France and Germany. Also Canada becomes one of the top net lenders.

Among the GIIPS the largest borrower is always Italy. Spain is the only banking system from GIIPS countries that started as a net lender, became a borrower during the crisis and then returned to a net lender position in 2013Q1 (with net receivables even higher than the Netherlands). Ireland started as a net lender and became gradually more indebted in 2013. On the other hand, Greece held net debt of $156bn before the crisis, in 2009 its debt increased significantly to \$228bn, but during 2013 became a small net lender with \$13n in net receivables.

An interesting observation is that the GIIPS countries did not increase their overall borrowing from 2006Q1 to 2009Q1, but instead they changed the composition of their lenders. In fact, during 2009Q1, GIIPS countries strongly borrowed from Non-GIIPS Eurozone countries, as in the case of the strong borrowing of Italy from France and Spain and Ireland from Germany. This effect vanishes as we approach 2013Q1, where the non-GIIPS Eurozone countries decreased their overall lending and their exposure to GIIPS countries. Finally, the US is the largest borrower in all three periods. It owes a huge amount payable to Switzerland and the UK. During the financial crisis, the American banking system increased its debt to Japan and France, but during 2013Q1 it decreased its borrowing from European economies and Canada and Japan became their main lenders.

The tiered layout in these graphs is constructed according to the gross borrowing of the country. The range of gross borrowing of all countries is taken as a ratio of each country's gross payables divided by that of the country with the highest borrowing. Countries that are ranked in the top 70 percentile of this ratio constitute the inner core. The mid-core is constructed from countries between the 70 and 30 percentile and the out-core between 30 and 10 percentile. Countries with the lowest ratio belong to the periphery. The links are colour coded based on their tier membership of the originating node.

The only country in the inner tier is always the US. We can see that it owes huge amount payable to Switzerland and the UK. With the rise of the GFC the American banking system increased its debts to Japan and France, but in 2013 the exposures of European economies become smaller and increase in Japanese and Canadian exposure can be observed.

The mid-core tier is populated initially only by the United Kingdom, joined by Germany just during the crisis in 2009. The highest weight of the British out-degree has a link between the UK and Germany equal to \$551.6bn in 2006 and gradually becoming lower (\$312.9bn in 2009 and \$218.7bn in 2013), eventually being overtaken by the debt to Spain in 2013 (\$304.4bn). The outer-core banking systems are mostly France, Germany, Japan, the Netherlands and from the GIIPS: Italy and Spain. One can observe that during the GFC (2009) and after the Eurozone Crisis (2013) Italy holds a significant debt towards France. All remaining countries belong to the periphery of the Core Global Banking System network.

It can be seen that the network topology becomes more tiered during the financial crisis and after the crisis when more banking systems move towards periphery. Nevertheless, every banking system within the Core Global Banking System can be of a potential threat due to the highly interconnected structure of the network, as the sovereign crisis started in a peripheral country and created waves throughout the system (Kalbaska and Gatkowski, 2012; Arghyrou and Kontonikas, 2012).

8.5 Spectral and Market Price-Based SRIs

The level of instability of the Core Global Banking System Network (CGBSN), reflected by the maximum eigenvalue (λ_{max}) of matrix Θ is depicted in Figure 8.2 as a dark grey line (right axis). In order to capture the effects of failure of net debtor banking systems on a net creditor banking systems, we rescale the index by the proportion of bank-to-bank cross-border flows over the total amount of cross-border flows (i.e., bank-to-bank plus bank-to-nonbank), as reported by Bruno and Shin (2015a). As we can see, $\lambda_{max}(\Theta)$ (right axis), jumps from a value of 0.28 in 2006Q4 to over 0.57 in 2007Q1 and approaching 0.6 in 2008Q4, reflecting the unsustainable liabilities of debtor countries relative to capital in the exposed national banking systems. This suggests that λ_{max} provides a reasonable early warning for the crisis that was about to come, with the collapse of two Bear Stearns hedge funds followed by the collapse of two BNP Paribas hedge funds in 2007Q3 that were highly exposed to Subprime Mortgage Backed Securities. What is remarkable is the estimation of a 57% loss of capital in the system as indicated by $\lambda_{max}(\Theta)$ by 2007Q1 that occurs during the course of the crisis (see Allen et al., 2011). After the financial crisis unraveled, λ_{max} falls in 2009 to 0.49 and remains at a high level, between 0.49 and 0.54, until the end of 2011. If we assume that a maximum eigenvalue of over 30%, exceeds the capital equity ratio that banks consider as desirable, system instability is still a persistent problem even until the last quarter of 2013.[16] The index also exhibits some spikes during the first Greek bailout in 2010Q2 and the US Debt Ceiling crisis in 2011Q3, which caused huge uncertainty in the financial markets.

[16]Markose (Reserve Bank of India, 2011) finds that the Basel III capital ratio 6% for risk weighted assets typically implies capital ratio of 30% for total assets. Thus, a value of $\rho = 0.3$ can be considered a proxy for capital adequacy ratios of banking systems.

Maximum Eigenvalue ($\lambda_{max}(\Theta)$) of the CGBSN vs. DCC-ΔCoVaR, DCC-MES and SRISK during the period (2005Q4-2013Q4).

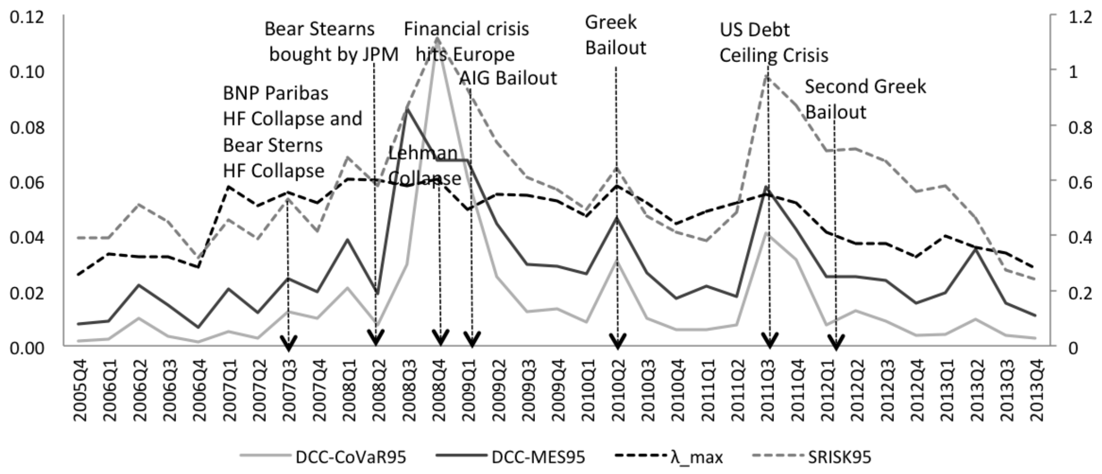


Figure 8.2: Note: For quarters between 2011Q4-2012Q4 the Maximum Eigenvalue, λ_{max} (right axis), is calculated for CGBS network without Greece, when it was virtually bankrupt with negative equity capital in banking system.

We compare the above results with market price-based SRIs, i.e., the DCC-MES, DCC-ΔCoVaR and the SRISK, which are calculated for every quarter, using data for that particular quarter. The country indices have been averaged to obtain the aggregated index for the whole financial system (left axis). Here we can see that while SRISK starts picking up in 2007Q2, the DCC-MES and the ΔCoVaR exhibit at large increase with the Lehman collapse during 2008Q3. During 2008Q4, the indices increase even further as the credit crunch hits Europe's banking sector as the European banking and insurance giant Fortis is partly nationalised to ensure its survival in the Netherlands during the last four months of 2008, causing a daily downturn of 129% (of log returns or 72% of ordinary returns) in the Dutch banking index the 14^{th} October 2008. These measures also seem to coincide with subsequent event in financial markets, such as the Sovereign Greek Debt Crisis (2010Q2) and the US Debt Ceiling Crisis (2011Q3). The SRISK, which is an hybrid SRI by combining leverage together with market price data, does not provide any early warning for the sample analysed. A possible explanation for this is given by Benoit et al. (2013), who argue that the SRISK tends to depend on leverage during calm periods, while during crisis periods tends to depend on the beta of the country, as correlations inscrease and equity drops. In this particular case, the beta effect would seem to predominate over the leverage effect. Therefore, for the sample analysed, the SRISK mainly behaves as a purely market price-based SRI. For further analysis, see the Appendix of this chapter.

We explore the degree of procyclicality of these indices by computing the R^2's from a regression of the VIX index against the systemic risk measures at different time lags to gauge their explanatory power.[17] As documented by Adrian and Shin (2011b), standard risk management risk measures, such as the VaR of financial equity returns, fluctuate procyclically with measures of risk such as the implied volatility of options on banks' traded shares. This means, that financial intermediaries are shedding risks and withdrawing credit precisely when the financial system is under stress, serving to amplify the downturn. In Table 8.1, we report contemporaneous and lagged regression R^2 coefficients (for up to 2 lags) between the different systemic risk measures and the VIX (Column 1). In addition, taking into account that most countries in our sample are European, we also report the R^2's with the VSTOXX (Column 2).[18] Finally, in Column 3, we report the R^2's of a regression of the TED spread on the systemic risk measures. The TED spread is the difference between the 3-month T-bill and the rate at which banks lend to each other on a 3-month period (measured by the Libor) and it is often used as a good proxy for interbank credit risk and perceived health of the banking system.[19]

We calculate the R^2's based on 50,000 bootstrapped samples. We report the standard errors in parentheses. The results of this exercise are presented in Table 8.1. Here we can see that the market price-based SRIs are contemporaneous with the VIX/VSTOXX, with R^2's of over 70% at time t, which tends to decrease as the number of lags increases. The *maximum eigenvalue* exhibits the lowest contemporaneous explanatory power (40%), which then increases slightly and remains statistically significant after two lags. Market price-based SRIs tend to decrease their explanatory power with time lags and in some cases, they even lose significance, as the case of the DCC-ΔCoVaR. Another interesting finding can be seen in Column 3, where λ_{max} exhibits a low but significant explanatory power with respect to the TED spread, thus reflecting the

[17]The VIX is the symbol for the Chicago Board Options Exchange Volatility Index. Often used as a good proxy for international risk aversion. A high value of the VIX corresponds to a more volatile market.

[18]Which is the European counterpart of the VIX, based on the EURO STOXX 50 realtime options prices and is designed to reflect the market expectations of near-term up to long-term volatility.

[19]When the Ted spread goes up, the interbank default risk is considered to be higher and when the spread decreases, the interbank default risk is considered to be lower. When there is a downturn in the economy, banks suspect that some banks may encounter problems. However, they do not know which banks, so they restrict interbank lending, resulting in higher Ted spreads and lower liquidity in the interbank market, which ultimately produces lower credit availability for consumers and corporates.

	VIX	VSTOXX	TED Spread
Systemic Risk Measures	(1)	(2)	(3)
λ_t^{max}	**0.401*****	0.338***	0.265***
	(0.126)	(0.121)	(0.095)
λ_{t-1}^{max}	0.444***	0.343***	0.288***
	(0.118)	(0.132)	(0.101)
λ_{t-2}^{max}	0.436***	0.373***	0.245**
	(0.124)	(0.139)	(0.112)
$DCC-MES_t$	**0.820*****	0.828***	0.273*
	(0.060)	(0.056)	(0.184)
$DCC-MES_{t-1}$	0.370***	0.333*	0.036
	(0.194)	(0.193)	(0.060)
$DCC-MES_{t-2}$	0.147*	0.147*	0.033
	(0.136)	(0.129)	(0.049)
$DCC-\Delta CoVaR_t$	**0.784*****	0.720***	0.152*
	(0.063)	(0.078)	(0.091)
$DCC-\Delta CoVaR_{t-1}$	0.294**	0.265**	0.026
	(0.162)	(0.147)	(0.044)
$DCC-\Delta CoVaR_{t-2}$	0.058	0.074	0.025
	(0.060)	(0.067)	(0.030)
$SRISK_t$	**0.716*****	0.807***	0.147*
	(0.1031)	(0.064)	(0.097)
$SRISK_{t-1}$	0.274**	0.290**	0.023
	(0.149)	(0.155)	(0.038)
$SRISK_{t-2}$	0.061	0.082	0.042
	(0.069)	(0.086)	(0.051)

Table 8.1: Bootstrapped R^2's from a regression of the VIX, VSTOXX and TED spread (taken as the last day of every quarter) on the SRIs. DCC-MES, DCC-ΔCoVaR and SRISK are calculated at a 5% tail. Standard Errors in Parentheses. * Significance at 10%; ** Significance at 5% and *** Significance at 1%. Own Calculations.

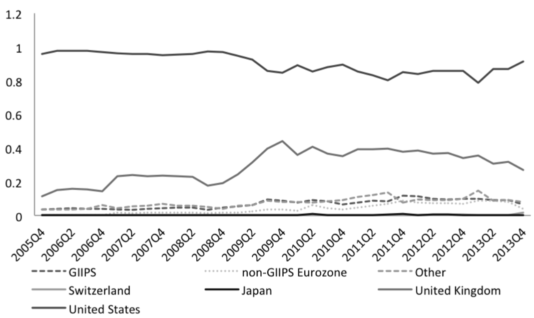
potential tightening of credit conditions in the interbank market with some time in advance. Market price-based SRIs are not able to reflect this phenomenon, not even contemporaneously.

8.6 Systemic Risk Importance and Vulnerability in the Global Banking System

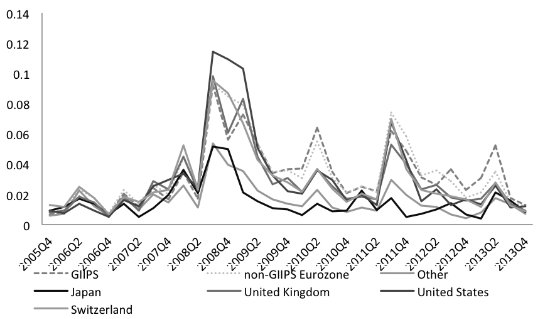
As we mentioned above, the eigen-pair method introduced by Markose (2012) not only delivers an index for systemic stability, based on the maximum eigenvalue (λ_{max}) of the stability matrix (Θ), but also delivers simultaneously its associated right (V_R) and left (V_L) eigenvectors. The right eigenvector centrality delivers the rank order of the systemically important countries whose default on their obligations can cause damage to banking systems that are exposed to them. The left eigenvector delivers the rank order of those banking systems that are vulnerable to contagion. It has been found that the rank order based on the right eigenvector centrality (V_R) is a good proxy for the countries that will bring about the largest losses by their default to creditor banking systems, whereas the left eigenvector centrality (V_L) ranks order those banking systems which are vulnerable to default by debtor countries.

Figure 8.3: Rankings of Systemic Importance for Core Europe, the US, the UK, Japan, Switzerland and average for other countries (2005Q4-2013Q4)

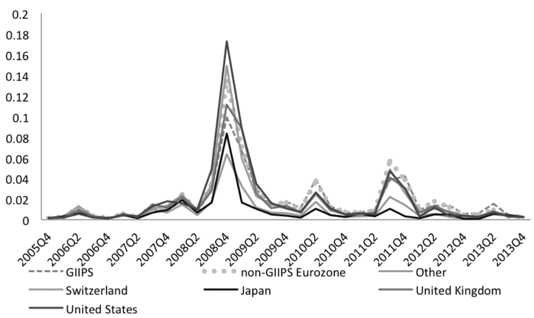
(a) RIGHT EIGENVECTOR CENTRALITY (V_R)



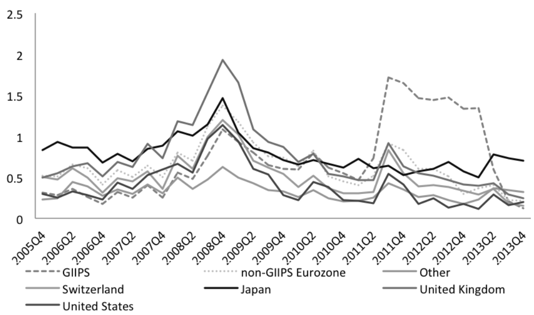
(b) DCC-MES



(c) DCC-ΔCoVaR



(d) SRISK



Note: Estimations based on Right Eigenvector Centrality (V_R), DCC-MES, DCC-CoVaR and SRISK. For quarters (2011Q4-2012Q4) Greece is removed due to negative equity; GIIPS: Portugal, Italy, Ireland, Greece, Spain; Eurozone non-GIIPS: Germany, France, the Netherlands, Belgium, Austria; Other: Australia, Canada, India, Sweden.

The *Right Eigenvector Centrality* (V_R) for the major economies and blocks in our sample is depicted in Figure 8.3 (Panel (a)). Here we verify the large systemic threat posed by the US. In fact, this can be verified in the whole Core Global Banking System Network which is populated by American liabilities. Thus, it should not be surprising that the failure of Bear Stearns in 2007 and Lehman Brothers collapse in 2008 triggered contagion waves throughout the global financial system. The *Right Eigenvector Centrality* (V_R) at that time was at levels above 0.9. The index fell to around 0.8 during the Eurozone Sovereign Crisis, with a visible tweak due to the Greek bankruptcy and bailout, but returned to almost 0.9 in 2013Q4. The United Kingdom is ranked as the second most important country. The British index increased twofold in the aftermath of the Subprime Financial Crisis and due to its closeness to the Eurozone, but did not return to its pre-crisis levels.

The rest of the countries, including non-GIIPS Eurozone countries (Germany and France), are of marginal systemic importance when compared with the aforementioned financial systems, which can give us some insights into how much damage would be caused, in case of financial default of the US or the UK. These two financial systems are a clear example of the "too interconnected to fail" phenomenon proposed by Markose et al. (2012).

An interesting point, regarding the GIIPS countries, is that their systemic importance was higher than that of the non-GIIPS Eurozone countries until the last quarter of 2011. A possible explanation for this phenomenon is that most outstanding debt of the GIIPS were restructured after the Eurozone Sovereign Crisis and the main burden was taken by the rest of the Eurozone, leading to an increase in the systemic threat posed by the non-GIIPS Eurozone countries. It is also noteworthy that Japan and Switzerland are the main lenders in the CGBSN and are ranked the lowest when it comes to systemic importance.

Regarding the market price-based rankings of systemic importance (Panels (b,c,d)), these tend to indicate the systemic importance of the US at the start of the crisis. The DCC-MES (Panel (b)) ranks the US as most systemically important at the start of the crisis in 2007Q3, but then is overtaken by Switzerland and Japan, during 2008Q1 and 2008Q2, respectively. Then picks up again during 2008Q3, with the Lehman collapse, to fall again during 2009Q1, when the crisis was already set in Europe. Here aside the four main systemic contributors, namely, the US, Switzerland, the UK and Japan, the GIIPS systemic importance start picking up during 2008Q3, when Ireland falls into recession, but then is overtaken by the non-GIIPS Eurozone countries in 2008Q4, when most of their banks start facing difficulties.

A point that is worthwhile noting here is that most country groups reach their first peak in 2008Q3, which coincides with the Lehman Brothers collapse in the US. In Switzerland, UBS having troubles due to Subprime-related investments[20] and the credit crunch reaching the UK, with housing prices falling more than a 10%. The only exception is Japan, which attains its highest in 2008Q4, which coincides with the effects of the crisis spreading into Asia.[21]

The DCC-ΔCoVaR (Figure 8.3, Panel (c)) reports a similar story. Here we observe that all groups of countries reach their highest point during 2008Q4. During this quarter, the non-GIIPS Eurozone countries rank as third, after the US and Switzerland, reflecting the fact that the crisis mainly affected the European Core countries and then spread to the periphery. The US ranks first until 2009Q3, but then is overtaken by the GIIPS and non-GIIPS Eurozone countries. Finally the SRISK (Panel (d)) mainly ranks the UK as the most systemically important country, followed by Japan, non-GIIPS countries, Switzerland and the US for the period 2008Q4. An interesting observation here is that the importance of the non-GIIPS Eurozone countries is always above of that of the GIIPS, except during the period 2011Q3-2013Q1. Here Greece, reports *negative*, therefore driving the average of GIIPS countries. This period also comprehends the second Greek bailout in 2012Q1.

While the market price-based SRIs can give us some insights on what are the potential losses coming from one country to the rest of the system, only ΔCoVaR give us a ranking for the countries that are most susceptible to suffer from these potential losses. We allow for this possibility in Figure 8.4, where we plot the *Left Eigenvector Centrality* (V_L, Panel (a)), while in Panel (b), we plot the vulnerability rankings delivered by the DCC-ΔCoVaR for the same group of countries.[22] In Figure 8.4, Panel (a), we can see a different story as the one shown by Figure 8.3. Now Switzerland is the most systemically vulnerable financial system in the CGBSN. Japan was ranked the third most vulnerable financial system, after non-GIIPS Eurozone block, but then surprisingly, after 2012 increased its vulnerability to 0.28 and moved to the second position in the ranking.

In terms of vulnerability, the non-GIIPS Eurozone countries are the second most fragile in the CGBSN, slightly surpassed by Japan in 2012-2013. At the same time, the evolution of the GIIPS's *Left Eigenvector Centrality* reveals how their financial systems were damaged during the

[20] UBS eventually received a capital injection from the government in October 2008.

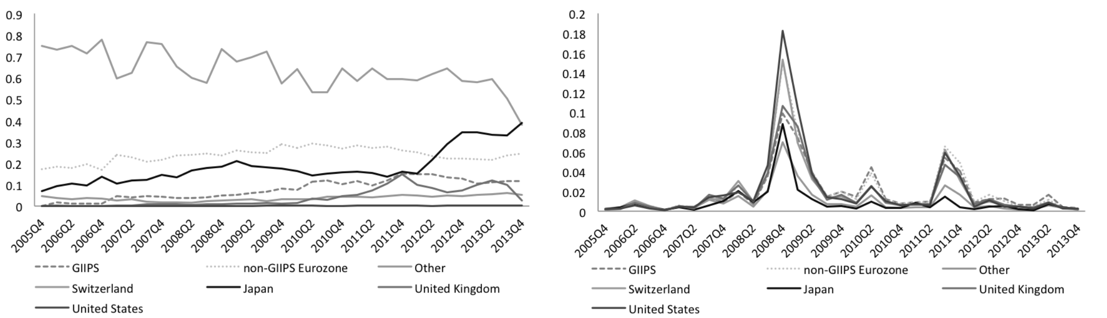
[21] In October 2008, Japan's stock index, the Nikkei, fell 10%, its biggest drop in 20 years.

[22] Vulnerability, in a ΔCoVaR context, is defined as the VaR of the country, conditional on the VaR of the market (Adrian and Brunnermeier, 2011).

crisis. Starting with a relatively low value, the index experienced a jump in 2007Q1 and then again in 2010Q2 when the Sovereign Crisis hit the Eurozone. The high vulnerability of the non-GIIPS Eurozone countries is the result of high lending of these banking systems, especially to the GIIPS, which became more vulnerable with the equity capital eroded by the losses incurred during the Eurozone crisis.

Figure 8.4: Rankings of Systemic Vulnerability for the US, the UK, Japan, Switzerland and averages for GIIPS, non-GIIPS Eurozone and other countries (2005Q4-2013Q4)

(a) LEFT EIGENVECTOR CENTRALITY (V_L) (b) DCC-ΔCoVaR



Note: For quarters (2011Q4-2012Q4) Greece is removed due to negative equity.

The impact of the Sovereign Crisis in the Eurozone can also be seen in the case of the United Kingdom. Its vulnerability increased considerably between 2010Q1 and 2011Q4, when it reached its maximum. A possible explanation for this could be the UK acting as an intermediary for refinancing the GIIPS financial systems, borrowing money from financial systems outside of the Eurozone and lending them to the GIIPS. The British *Left Eigenvector Centrality* only improved in the last quarter of 2013. Finally, the US's *Left Eigenvector Centrality* is extremely low at all times, while the CGBSN is only able to capture the cross-border banking exposures, there are other possible contagion channels, with which the American financial system would probably be more vulnerable. This latter, underlines the need of more granular mapping of current financial interconnections to analyse the propagation of contagion in financial systems (Markose, 2013).

The DCC-ΔCoVaR ranks the US as the most vulnerable, followed by Switzerland and the non-GIIPS Eurozone countries during 2008Q4. The UK picks up during 2009Q1 but then its vulnerability decreases. After 2009Q3, the highest rank of vulnerability is mainly dominated by the non-GIIPS Eurozone countries and the GIIPS.

8.6.1 Systemic Importance and Vulnerability of Core Eurozone Countries

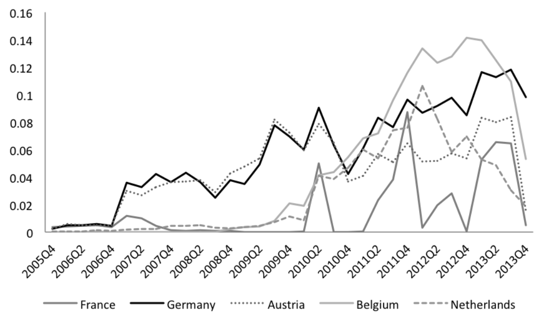
While we have made the distinction that a systemically important financial system is not necessarily the most vulnerable, a very dangerous situation could arise when a country of high systemic importance is at the same time, increasingly vulnerable. In this context, a small perturbation can harm the financial system, which in turn can cause a huge damage to the global network. A clear example of this instability was the rise of the UK's vulnerability (the second most systemically important country based on the CGBSN). In Figure 8.5, we plot the SRIs of the non-GIIPS Eurozone countries. We are particularly interested in the largest economies of the Eurozone: France and Germany. Panel (a) shows the evolution of the *Right Eigenvector Centrality* (V_R) for these economies. Here we can see that the systemic importance of all these countries was minimal before the crisis unraveled. The *Right Eigenvector Centrality* for these countries experienced an early increase in 2007Q1, especially in the case of Germany and Austria. The importance of these two financial systems increased steadily throughout and even after the US financial crisis, whereas the Dutch and Belgian indices grew just before the European Crisis. French potential to trigger contagion losses increased considerably in the midst of the Sovereign Crisis. This trend seems to be disrupted between 2011Q4 and 2012Q4.[23] Furthermore, France became the most vulnerable banking system of the non-GIIPS Eurozone countries after 2009, only surpassed by the Netherlands in the last quarter of 2013. By 2013, Germany was the most systemically important country of the group, while France and the Netherlands posed the least threat of triggering significant contagion in the CGBSN.

The DCC-MES and DCC-ΔCoVaR based rankings remain relatively low and homogeneous during the US Financial crisis and all countries start picking up during 2008Q2, reaching their maximum during 2008Q4, with the exception of Belgium, which peaks in 2008Q3. After that, they only react in the face of critical market event, such as the Greek bailout during 2010Q2, the growing uncertainty over Greece staying in the Eurozone (2011Q4) and the second Greek bailout (2012Q1). Here the Netherlands is ranked as first from 2008Q3 and then remains mostly in that position until 2011Q4, overtaken by France, only during 2011Q4 (Panel (b)). This highest peak

[23]This is not due to the removal of bankrupted Greece from the sample, as experiments conducted without Greece in the whole period exhibit no change in the French systemic importance index.

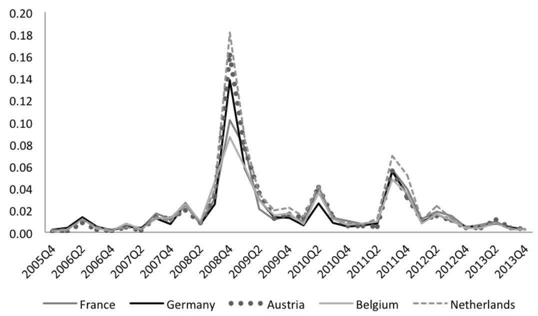
Figure 8.5: Rankings of Systemic Importance for Core Eurozone Countries (2005Q4-2013Q4)

(a) RIGHT EIGENVECTOR CENTRALITY (V_R)

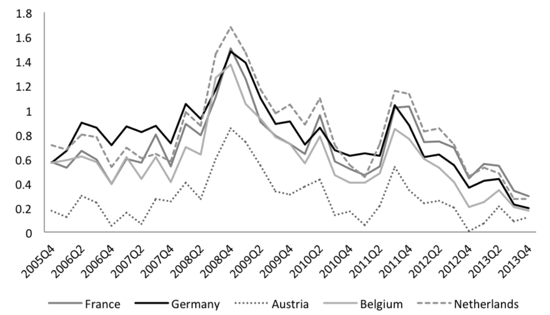


(b) DCC-MES

(c) DCC-ΔCoVaR



(d) SRISK



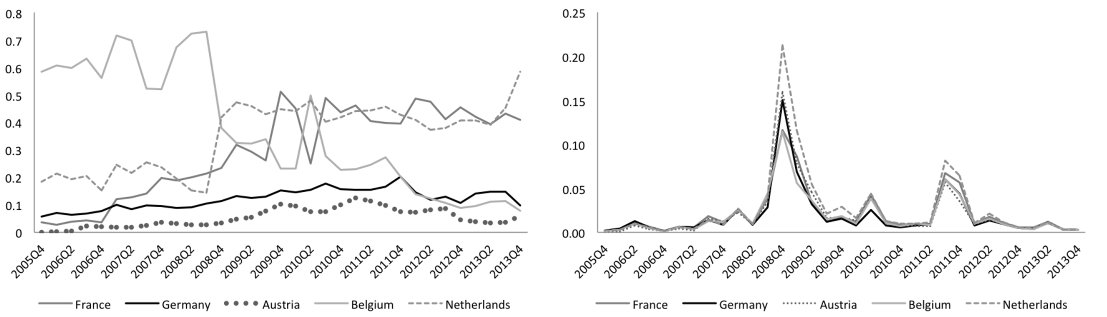
For quarters (2011Q4-2012Q4) Greece is removed due to negative equity.

during 2008Q4 coincides with the Dutch government providing a capital injection of EUR 10 billion to ING Group during October 2008. The SRISK ranks Germany as the most systemically important until 2008Q2, overtaken by the Netherlands in 2008Q3, which leads for most of the sample. Then, France takes the lead in the last two quarters.

Indeed, the recent financial crisis reached its peak in the Netherlands in the last four months of 2008. During this period, two financial companies, Fortis NV and ABN AMRO, were experiencing great difficulties. In fact, prior to its collapse in 2008, Fortis was the largest financial services company in Belgium, named as Fortis SA/NV, and in the Netherlands, named as Fortis NV. As it can seen from Figure 8.6 (Panel (a)), the Belgian banking system was the most vulnerable before 2008Q3. On the 13th of October 2008, the Belgian government designed and approved a mechanism to compensate certain categories of Fortis shareholders. In the same month, the Dutch government became involved and bailed-out the bank.

Figure 8.6: Rankings of Systemic Vulnerability for Core Eurozone Countries (2005Q4-2013Q4)

(a) LEFT EIGENVECTOR CENTRALITY (V_L) (b) DCC-ΔCoVaR



Note: For quarters (2011Q4-2012Q4) Greece is removed due to negative equity. Source: Own calculations.

In fact, this event is reflected in Panel (a), where the *Left Eigenvector Centrality* of Belgium drops almost to the half (from 0.73 to 0.41), while the Dutch index more than doubles (from 0.14 to 041, if we also add the capital injection granted to ING over the same month).[24] A similar phenomenon is observed for the DCC-MES in Figure 8.5b, Panel (b). Finally, on December 2008, the Belgian government announces that will proceed with the sale of Fortis assets to BNP Paribas SA, which in turn had an effect on French vulnerability during 2009. As it can be seen in Figure 8.6, Panel (a), when the *Left Eigenvector Centrality Index* for Belgium falls, it increases for France. While, the DCC-ΔCoVaR-based ranking is able to identify the vulnerability of Belgium and Netherlands during 2008Q4, they are unable to capture the growing instability of the Belgian banking system, whereas the *Left Eigenvector Centrality* does so way before the crisis.

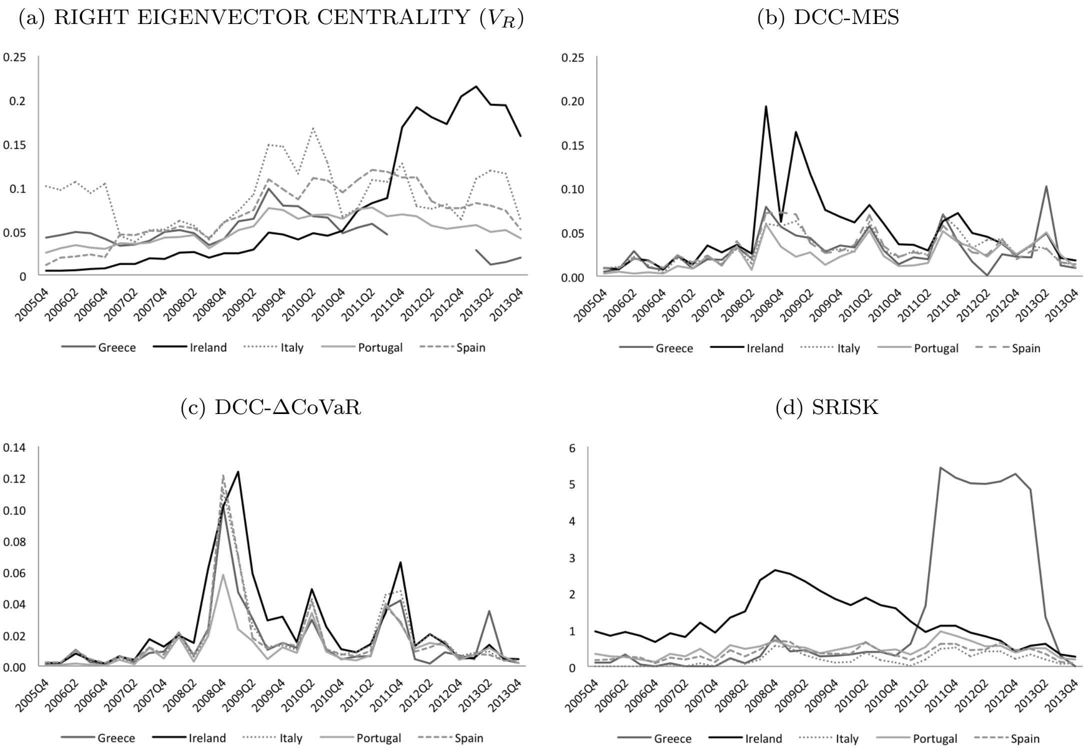
8.6.2 Systemic Importance and Vulnerability of Eurozone Periphery Countries

As we showed in the previous section, the rise of systemic importance coupled with an increase in systemic vulnerabilities of non-GIIPS Eurozone countries during the financial crisis of 2008

[24] Another Dutch bank which was involved with changes in the management, ownership, and financial conditions was the ABN AMRO. This bank was acquired by a banking consortium consisting of the Royal Bank of Scotland (RBS) Group, Santander Group and Fortis. On the 13th of October 2008, the British government declared a bail-out package for the financial system. This resulted in a total state ownership in RBS of 58%. Following the collapse of Fortis and nationalization of RBS, the ABN AMRO Bank was nationalized by the Dutch government. Consequently, the Dutch banking index return declines extremely on the 14th of October 2008.

and the Sovereign Crisis, provides evidence of the building-up of instability in the high-income Eurozone countries at that time. Figure 8.7 illustrates the SRIs for the GIIPS. In Panel (a) we plot the *Right Eigenvector Centrality* (V_R), which shows that, on average, the systemic importance of the GIIPS was higher than that of the non-GIIPS Eurozone countries. If in addition to this, we look at the *Left Eigenvector Centrality* (V_L) of Figure 8.8 (Panel (a)), we can see precisely the opposite. In this context, the GIIPS posed more systemic threat and were less vulnerable than non-GIIPS Eurozone countries.

Figure 8.7: Rankings of Systemic Importance for Eurozone Periphery Countries (2005Q4-2013Q4)



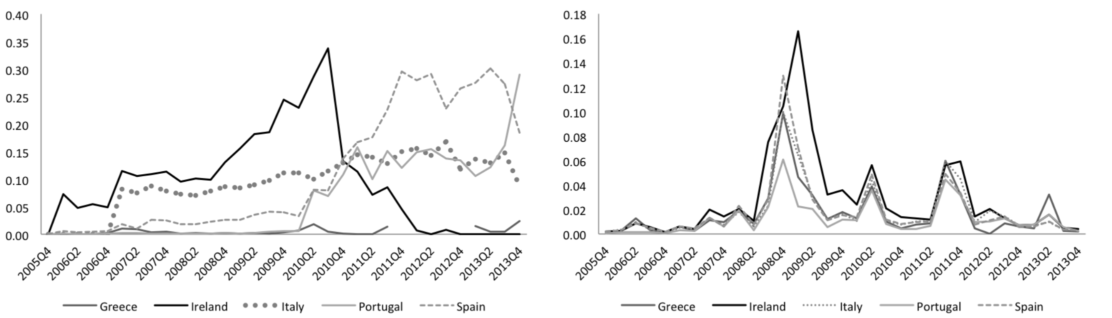
Note: For quarters (2011Q4-2012Q4) Greece is removed due to negative equity. Source: Own calculations.

As we showed in Section 8.4, GIIPS were heavily indebted to the Eurozone countries before the Sovereign Crisis. The strong cross-border flows between the GIIPS and non-GIIPS Eurozone countries could have been the channel of systemic risk spread, which led to the creation of the European Financial Stability Facility (EFSF) after the €110 bn bailout of Greece in May 2010, help from the EFSF for Ireland (November 2010), Portugal (May 2011) and again Greece in February 2012 (Arghyrou and Kontonikas, 2012).

With more than 400% growth of cross-border banking claims of countries such as Ireland or Spain between 2003 and 2008 (Bruno and Shin, 2014), it is not surprising the increase in the *Right Eigenvector Centrality* of GIIPS countries until the second half of 2009 as depicted in Figure 8.7 (Panel (a)). Before the Sovereign Crisis in 2010, the most systemically important country of the GIIPS was Italy followed closely by Spain and Greece. By 2010Q2, all GIIPS countries but Ireland, had a higher systemic importance ranking than Germany. The *Right Eigenvector Centrality Index* rose sharply for Ireland by the end of 2011 and during 2012, when it started raising money from financial markets. By the end of 2013, the most systemically important countries were Ireland, Italy, Spain, Portugal and Greece. Market price-based SRIs tend to agree that Ireland is the most systemically important before and during the crisis, followed by Greece and Spain. Notice that the SRISK for Greece jumps sharply between 2012Q2 and 2013Q1, reflecting the increasing instability of the Greek banking system.[25]

Figure 8.8: Rankings of Systemic Vulnerability for Eurozone Periphery Countries (2005Q4-2013Q4)

(a) LEFT EIGENVECTOR CENTRALITY (V_L) (b) DCC-ΔCoVaR



Note: For quarters (2011Q4-2012Q4) Greece is removed due to negative equity. Source: Own calculations.

When we analyse the systemic vulnerability of GIIPS in Figure 8.8 (Panel (a)) it can be noticed that Ireland became very vulnerable before the financial crisis of 2008 but with the bailout in the last quarter of 2010 its *Left Eigenvector Centrality* fell to very low levels. In fact,

[25]It is worthwhile noting that this sharp increase is due to the fact that Greece reports negative equity during this period, which we have assumed to be slightly positive.

this event can also be seen in market price-based SRIs in Figure 8.7, where Ireland ranks as one of the most systemically important of all the GIIPS, as well as one of the most vulnerable in Figure 8.8 (Panel (b)). Spain became the most vulnerable country among the GIIPS due to the Sovereign Crisis. Italian vulnerability increased already in 2007 and remain in a steady rise until the end of 2012. Something that is worthwhile noting is the recent vulnerability of the Portuguese banking system during the second half of 2013 (Figure 8.8, Panel (b)). This eventually led to the collapse of the major Portuguese Banco Espirito Santo in August 2014, generating fears of a second round of contagion in the Eurozone. The eigen-pair method shows that Portugal is becoming the most vulnerable banking system among the GIIPS, event which is neither anticipated by any of the other SRIs presented above.

8.6.3 What Explains Market Price-Based SRIs?

In this section we ask what are the possible reasons for the divergence in rankings of systemically important banking systems. For answering this question, we follow Benoit et al. (2013) and calculate the R^2's from a time-series and cross-sectional regression of the SRIs presented above and some of the ingredients used to construct them, namely, the beta of the country's banking system, total book value liabilities, value of equity and Value at Risk.[26] As pointed out by Benoit et al. (2013), under certain conditions, the DCC-MES and beta tend to identify the same systemically important financial institutions. This latter begs the question of why not rank institutions/countries by the betas in the first place? Another concern, is that it leads to the confusion between *systemic* and *systematic* risk.[27] In addition, as we have shown throughout this chapter, most of these market price-based SRIs tend to increase during market downturns, which makes them to be highly procyclical. Indeed, this can be seen in Table 8.2, where the R^2's shows an almost perfect cross-sectional correlation between the DCC-MES and beta, where we can confirm this finding. That is, rankings of SIFIs with DCC-MES are equivalent to rank countries with the highest betas.

Although the SRISK is a function of beta, it does not seem to be much sensitive to beta, in

[26]When we estimate cross-sectional regressions, we regress cross-country SRIs against cross-country betas at each t. In contrast, in the time-series domain, we regress individual country SRIs against individual country beta *for that particular country.* In both cases, the R^2s are retrieved and subsequently averaged.

[27]*Systematic risk* is already accounted for in the banking regulation since the 1996 Amendment of the Basel Accord as regulatory capital depends on the banks's market risk VaR.

	Time-Series					Cross-Section				
MES	EQ	LTQ	LVG	BETA	VaR	EQ	LTQ	LVG	BETA	VaR
avg	0.039	0.082	0.181	0.631	0.508	0.037	0.036	0.018	**1.000**	0.387
std	0.045	0.113	0.152	0.350	0.274	0.047	0.048	0.018	**0.000**	0.266
max	0.183	0.479	0.491	1.000	0.874	0.183	0.201	0.588	**1.000**	0.852
min	0.000	0.000	0.000	0.107	0.035	0.000	0.000	0.000	**1.000**	0.000
ΔCoVaR	EQ	LTQ	LVG	BETA	VaR	EQ	LTQ	LVG	BETA	VaR
avg	0.023	0.048	0.190	0.064	**0.802**	0.031	0.026	0.167	0.859	0.341
std	0.022	0.089	0.159	0.088	**0.247**	0.043	0.035	0.169	0.077	0.228
max	0.085	0.483	0.468	0.319	**0.985**	0.195	0.154	0.578	0.966	0.869
min	0.000	0.000	0.000	0.000	**0.057**	0.000	0.000	0.002	0.597	0.011
SRISK	EQ	LTQ	LVG	BETA	VaR	EQ	LTQ	LVG	BETA	VaR
avg	0.197	0.196	**0.589**	0.202	0.588	0.029	0.081	**0.791**	0.191	0.366
std	0.170	0.175	**0.235**	0.145	0.225	0.032	0.080	**0.214**	0.195	0.322
max	0.641	0.735	**0.992**	0.572	0.873	0.171	0.308	**0.996**	0.556	0.980
min	0.012	0.002	**0.075**	0.010	0.012	0.000	0.001	**0.065**	0.001	0.001

Table 8.2: R^2 statistics (average, minimum, maximum and standard deviation) obtained from a regression of a systemic risk measure (MES, ΔCoVaR and SRISK, on firm characteristics, namely, equity capital (EQ), liabilities (LTQ), leverage (LVG), beta and VaR. We report two types of regressions: time-series regressions for each of the 18 countries and cross-sectional regressions for the 33 data points available (2005Q4-2013Q4). Bold figures indicate the explanatory variable with the highest R^2's. Authors calculations.

b

both the cross-section and time-series, with an R^2 of 0.191 and 0.201, respectively. However, the SRISK would seem to be explained by leverage (LVG), with a cross-sectional R^2 of 0.791. Cross-sectional correlations are very important in this case, as these are the ones that provides with the rankings at a given point in time. For this particular case, we have that ranking countries based on SRISK would tend to coincide with ranking countries based on their total leverage.

We find that the DCC-ΔCoVaR is explained in a 80% in the time-series by its VaR. A high correlation in the time-series domain means that while rankings may differ, for a given country, DCC-ΔCoVaR is proportional to its VaR. This means that forecasting the future evolution of the contribution of a country to systemic risk is equivalent to forecast its risk in isolation.This provides evidence that most SRIs can be summarised by using standard risk measures. While we have made use of a very limited sample, our findings are in line with those studies using larger samples (Benoit et al., 2013).

8.7 Concluding Remarks

The present study illustrates the use of financial network approach as an important tool for assessing the stability of financial systems. Using Consolidated cross-border banking data from the BIS and aggregated equity data from Bankscope for 18 countries, we apply the eigen-pair method developed by Markose (2012) and Markose et al. (2012) to estimate systemic risk and identify systemically important countries and countries with vulnerable banking systems.

The main contribution of this research is that it is one of the first to provide a cross-border comparison of systemic risk indices based on market price-based data and the network approach to cross-border liabilities. As opposed to most of SRIs in the literature, the eigen-pair method based on bilateral netted exposures relative to equity capital of cross-border banks in national banking systems, makes a clear distinction between systemic importance and systemic vulnerability. Standard market price-based SRIs lack any threshold above which one can identify early turning points in system stability. In contrast, the eigen-pair method provides a close-form metric through the condition $\lambda_{max}(\Theta) < \rho$, where ρ is the regulatory capital buffer, obtained at length in Appendix C.4. In particular, the use of this metric in a network setting has been proved to have valuable early warning capabilities. In fact, the maximum eigenvalue of the stability matrix Θ, peaks way before the Subprime crisis started. While we are aware of the fact there is some statistical uncertainty due to the short sample length used in our analysis, $\lambda_{max}(\Theta)$ seems to indicates the 60% loss in equity capital witnessed during the 2007 Crisis (see, Figure 8.2 and Appendix C.4). Moreover, the *Left Eigenvector Centrality* was able to highlight the increasing vulnerability of the Portuguese banking system more than half a year before the collapse of one of its major banks and also indicate problems in the Belgian banking system way before its major financial group had to be bailed out.

The above results are compared with standard market price-based SRIs, namely, the DCC-MES of Acharya et al. (2010), the SRISK of Acharya et al. (2012) and the DCC-ΔCoVaR of Adrian and Brunnermeier (2011), using data on the MSCI Financials for the 18 countries of our sample. These models have been deliberately estimated in their simplest form, avoiding state variables as in the case of DCC-ΔCoVaR and without estimating their predicted values.[28]

[28]Indeed, many of these authors have used bank balance sheet characteristics such as book leverage, profitability (ROA), non-performing loans, whole-sale funding and mortgage loans as regressors to obtain forward measures

All this with the idea to assess whether these indicators are able to reflect any risk build up before a crisis unravels.[29] Our main finding is that these measures exhibit an extremely high contemporaneous correlation with market sentiment measures such as the VIX/VSTOXX, which makes them highly procyclical, with little early warning capabilities. Likewise, most of them tend to match perfectly market events and only during those times are able to identify systemically important financial systems. In this context, these measure are useful only in the cross-sectional domain, that is, to identify systemically important financial systems at a point in time. In general, rankings based on these measures tend to agree on the dominance of the US and the UK at the start of the crisis and also are able to reflect the interplay between Non-GIIPS Eurozone and GIIPS countries. They also reflect the systemic importance of Ireland at the start of the crisis, as well as the Netherlands during 2008Q4. However, they are not able to pick up the increasing vulnerability of the Belgian banking system before the crisis nor the vulnerability of the Portuguese banking system at the end of 2013. Surprisingly, the SRISK, which combines leverage with market price-based data is not able to capture the increasing instability of the countries' banking system. A possible explanation for this, is that while leverage is increasing in the run up of the crisis, the reliance on the market through the LRMES nullifies this effect, preventing the SRISK to provide any early warning.

Following Benoit et al. (2013), we assess whether market price-based SRIs can be rationalised by standard risk measures, such as beta, VaR and liabilities. First, we find that the DCC-MES can be well explained by country betas in the cross-section, which is not any different to ranking countries by their betas. Second, we find that the ΔCoVaR can be well explained by its VaR in the time-series domain, which means that forecasting ΔCoVaR is not any different than forecasting the risk of the country's banking system in isolation with its VaR. Finally, we find that the SRISK is highly explained by leverage. These findings are in line with those using larger samples of financial institutions (Benoit et al., 2013).

The present study outlines the importance of the informational content of cross-border balance sheet data in a network setting for assessing global financial stability. Future research

of systemic risk (Idier et al., 2013).

[29] This is what Paul McCulley famously coined the term often known as the "Minsky Moment", after Hyman Minsky, to describe the point at which markets lose confidence in the sustainability of a process of ever greater amounts of leverage provided against increasingly over-valued collateral. See McCulley (2008).

should widen the scope of this methodology by including exposures of sectors within countries and between sectors from different countries, as in the Castrén and Rancan (2014) framework. This would require the use of more sectoral data on cross-border exposures of banking systems.

Chapter 9

Conclusions and Future Work

9.1 Goals of the Thesis

In this section, we summarise the goals of this thesis:

- We showed the benefits of using the RMT approach to financial correlations of stocks for the FTSE 100. We tested the performance of the different filters used in the literature and determine under which conditions the RMT-filtering will be more beneficial, using a wide range of different tests, in terms of realised risk reduction, prediction accuracy and risk-adjusted measures of portfolio returns.
- We extended the RMT-filtering to other correlation estimators, which may also be afflicted by noise. Examples of these estimators are the Constant Conditional Correlation model (CCC) of Bollerslev (1990), the Dynamic Conditional Correlation (DCC) model of Engle (2002) and the Regime-Switching Beta-CAPM correlation estimator, based on Ang and Bekaert (2004). At least for the last two estimators, we were the first to apply the RMT-filtering to large correlation matrices, with great improvements in realised risk estimates.
- We were the first to combine the RMT approach with a Vector Autoregressive approach (VAR) to explore the macroeconomic determinants of the Chilean stock market. Here the main goal was to build portfolios with different degrees of exposure to the systematic risk embedded in the market portfolio with the help of RMT tools and then explore their

macroeconomic determinants, by making a clear distinction between macroeconomic factors that are local from those that are external.

- We used some RMT results for the stability of financial networks and assessed the dynamic stability of a global banking network built from bilateral exposures of country banking systems to debtor countries. Here we applied the eigen-pair method of Markose (2012), who based on the work of May (1972, 1974) for random networks, develops an stability condition based on the maximum eigenvalue λ_{max} of a matrix of net bilateral exposures relative to equity capital. Here we provided further evidence of the early warning capabilities of λ_{max}, when this surpasses a prespecified threshold. We also used the right and left associated eigenvectors as a gauge for systemic importance and systemic vulnerability, respectively. These metrics were then contrasted with recent systemic risk indices (SRIs) based on market price-based data, namely, the DCC-MES of Acharya et al. (2010), the SRISK of Acharya et al. (2012) and the DCC-ΔCoVaR of Adrian and Brunnermeier (2011) in terms of their early warning capabilities. We also provided a thorough analysis for these metrics, in terms of their significance and their main determinants.

9.2 Concluding Remarks

In this section we review the main conclusions of this thesis from the point of view of an institutional investor considering implementing the RMT-filtering to clean noise from correlation matrices and also from a central banker's perspective seeking to monitor financial institutions based on a network approach.

9.2.1 Sample Correlation Matrix

In this thesis we examined the application of two RMT filters to build portfolios of stocks in the FTSE 100 and also the Chilean stock market. We studied the effect on realised risk forecast of minimum variance portfolios using both, an in-sample and an out-of-sample analysis. Our results are in good agreement with previous studies (Pafka et al., 2004; Daly et al., 2008, 2010) in that the RMT-filtering is able to improve the forecast of the realised risk of minimum variance portfolios.

When filtering the sample correlation matrix we find that the RMT-filtering delivers the lowest *realised* risk, the lowest volatility of prediction and the best prediction accuracy when compared to the sample correlation estimator. This is particularly true in the absence of *short-sale* constraints. We also find that the RMT-filtering performs the best for short estimation windows, of the order of 6 months to 1 year of data (e.g., $1.6 \leq Q \leq 3.2$). This is consistent with what it has been found in the literature (see, Pafka et al., 2004; Daly et al., 2008). For higher values of Q (e.g., $3.2 < Q < 6.5$) these benefits would seem to be diminished, as the *filtered* correlation matrix converges to the sample correlation matrix, when the sample size increases. These results are robust to both, low and high volatility periods in financial markets. In the former case, we find that there are no major differences between using the sample and filtered correlation matrices. In the latter case, we find that the RMT-filtered correlation estimators outperform the sample correlation estimator in both cumulated returns and Sharpe Ratios. We also suggest a possible explanation for the success of the RMT-filtering. It would seem that the portfolios built using the sample correlation matrix tend to take, on average, more extreme positions and are more overexposed to *short-selling* than those built using the *filtered* correlation matrix. These results are robust to both volatility periods.

9.2.2 Other Correlation Estimators

In terms of applicability of the RMT-filtering in a Regime Switching context, we find that the spectrum in Regime 1 (low volatility regime), can be fitted relatively well by the Marčenko-Pastur Distribution, which allows us to distinguish signals from noise in the correlation matrix for Regime 1. In this context, filtering is only applied to the correlation matrix of Regime 1. Recall that the Marčenko-Pastur Distribution is based on the assumption of normality of returns. While this may be true for Regime 1, returns for Regime 2 are far from being normal. In terms of cumulated returns, we find that the RMT-filtering enhances the performance of portfolio returns in 30% for the regime-dependent correlations and 23%-17.7%, for the CAPM-based and sample correlation estimators, respectively. In addition, the *filtered* regime-dependent correlation estimators are able to deal better with downside risk than their *unfiltered* version, with more volatility in the upside than in the downside. We also find that the RMT-based filtering works better for short estimation windows and when *short-sale* is permitted. These results are robust to both filtering methods. In particular, the method of Plerou et al. (2002) (Method 1) delivers better cumulated returns than the method of Laloux et al. (2000) (Method 2) across estimators. Finally, from an asset allocation perspective, we find that the success of the *filtered* regime-dependent correlation estimators comes from their ability to deliver more diversified portfolios and also by placing less resources on Financials. This finding is common to all *filtered* correlation estimators.

Regarding the conditional correlation estimators, we find that by far the best results are obtained by the filtered versions $CCC-(p^{RMT})$ and the $DCC-(p^{RMT})$ models. Their *realised* variance is significantly smaller compared to their $Factor-GARCH$ versions and also smaller than their *unfiltered* counterparts. The models using the RMT-filtering also produced portfolios with the lowest mean squared error (MSE) and mean average percentage error between 52% and 54%. These results also stress the shortcomings of using the Exponentially Weighted Moving Average (EWMA) models, which, despite their simplicity, can lead to a serious underestimation of the underlying risks of minimum variance portfolios.

9.2.3 Combining RMT with VAR Analysis

In our study for the Chilean Stock Market. We use the statistically significant principal components as predicted by the RMT approach and we are able to construct portfolios with different degrees of exposure to the market portfolio. By contrasting this to a number of local and external macroeconomic variables, we compute impulse responses that allow us to identify the global risk aversion as the main driver of the market returns, followed in importance by shocks to the monthly rate of inflation and the country's monetary policy rate. With the use of statistically significant eigenvalues we are also able to construct portfolios, which are uncorrelated to macroeconomic shocks.

9.2.4 Systemic Risk in a Cross-Border Setting and the Role of Financial Networks

In this study we provide a cross-border comparison of systemic risk indices based on market price-based data and the network approach to cross-border liabilities. As opposed to most of SRIs in the literature, the eigen-pair method based on bilateral netted exposures relative to equity capital of national banking systems, makes a clear distinction between systemic importance and systemic vulnerability. Standard market price-based SRIs lack any threshold above which one can identify early turning points in system stability as a whole. In contrast, the eigen-pair method provides a close-form metric through the condition $\lambda_{max}(\Theta) < \rho$, where ρ is the regulatory capital buffer. In particular, the use of this metric in a network setting has been proved to have valuable early warning capabilities. In fact, the maximum eigenvalue of the stability matrix Θ, peaks way before the Subprime crisis started. A remarkable finding is that $\lambda_{max}(\Theta)$ for the networked system indicates the 60% loss in equity capital witnessed during the 2007 Crisis. Moreover, the *Left Eigenvector Centrality* was able to highlight the increasing vulnerability of the Portuguese banking system more than half a year before the collapse of one of its major banks and indicate problems in the Belgian banking system way before its major financial group had to be bailed out.

9.3 Summary

In the majority of portfolio metrics analysed in this thesis, we have shown the benefits of the RMT-filtering in producing portfolios with the lowest realised risk and also to correctly predicted it. In the same vein, the associated portfolios were also found to deliver more diversified portfolios, with greater cumulated returns and Sharpe Ratios. These portfolios were also found to deal better with downside risk.

We also showed that the RMT-filtering can also be used to improve the performance of *estimated* correlation matrices, as the case of the Constant Conditional Correlation (CCC) model of Bollerslev (1990), the Dynamic Conditional Correlation (DCC) models of Engle (2002) and the Regime-Switching Beta-CAPM correlation estimator, based on Ang and Bekaert (2004).

These results hold in the absence of *short-sale* constraints and when using relatively short estimation windows (usually, 6 months to 1 year of data), as in this case the RMT-filtering performs the best.

Finally, we showed how the stability condition seminally derived in May (1972, 1974) for random networks and later on extended by Markose (2012) can be used to build a systemic risk index with great early warning on the growing instability of cross-border banking flows. As opposed to market price-based SRIs, the eigen-pair method delivers a threshold for which the stability of the networked system can be assessed. This outlines the importance of the informational content of cross-border balance sheet data in a network setting for assessing global financial stability.

9.4 Future Work

A number of interesting areas of research came along with the development of this thesis. From a theoretical perspective, for example, recent evidence suggests that meaningful correlations can be measured in the bulk of the MP eigenvalue spectrum (Burda et al., 2004; Burda and Jurkiewicz, 2004; Malevergne and Sornette, 2004; Kwapièn et al., 2006). This happens because we are assuming that returns are normally distributed, which is not the case in real life. Indeed, returns in financial markets are not normally distributed and exhibit a number of features, such as fat tails, volatility clustering and non-stationarity. Recent literature has been improving these matters by developing extensions of the MP distribution to account for these phenomena (Potters et al., 2005; Bouchaud and Potters, 2009). In this context, an interesting line of future research could be focused on extending the RMT approach to more fat-tailed distributions.

In a more practical setting, it would be interesting to study the correspondence between the stocks privileged by the RMT-filtering with those of a Fama and French (1992) factor model. To see whether these correspond to low price to earnings stocks (value stocks), high market capitalisation stocks (size stocks) and high divident stocks (growth stocks). This would provide us with some insights into how noisy correlations are in these different segments.

Finally, it would be an interesting line of research the analysis of correlation-based networks in the spirit of Song et al. (2011). For example, centrality measures based on these networks could be used to identify stocks that are less central in the network to design optimal trading strategies (Pozzi et al., 2013).

Appendix A

Tables & Figures from Chapter 5

Company	Abbreviation	Classification	Sign
PRU LN Equity	PRU	Life Insurance	+
OML LN Equity	OML	Life Insurance	+
ADN LN Equity	ADN	Financial Services	+
SDRC LN Equity	SDRC	Financial Services	+
LGEN LN Equity	LGEN	Life Insurance	+
AV/ LN Equity	AV	Life Insurance	+
LLOY LN Equity	LLOY	Banks	+
SDR LN Equity	SDR	Financial Services	+
EMG LN Equity	EMG	Financial Services	+
CSCG LN Equity	CSCG	Real Estate Investments Trusts	+
BLT LN Equity	BLT	Mining	+
HSBA LN Equity	HSBA	Banks	+
RSA LN Equity	RSA	Nonlife Insurance	+
STAN LN Equity	STAN	Banks	+
BARC LN Equity	BARC	Banks	+
WPP LN Equity	WPP	Media	+
IAG LN Equity	IAG	Travel and Leisure	+
RBS LN Equity	RBS	Banks	+
ANTO LN Equity	ANTO	Mining	+
LAND LN Equity	LAND	Real Estate Investments Trusts	+
AAL LN Equity	AAL	Mining	+
WOS LN Equity	WOS	Support Services	+
BLND LN Equity	BLND	Real Estate Investments Trusts	+
SAB LN Equity	SAB	Beverages	+
UU/ LN Equity	UU	Gas, Water and Multiutilities	+
HMSO LN Equity	HMSO	Real Estate Investments Trusts	+
NXT LN Equity	NXT	General Retailers	+
DGE LN Equity	DGE	Beverages	+
REX LN Equity	REX	General Industrials	+
PSON LN Equity	PSON	Media	+
RR/ LN Equity	RR	Aerospace and Defense	+
CPI LN Equity	CPI	Support Services	+
SVT LN Equity	SVT	Gas, Water and Multiutilities	+
IMI LN Equity	IMI	Industrial Engineering	+
WTB LN Equity	WTB	Travel & Leisure	+
MGGT LN Equity	MGGT	Aerospace and Defense	+
WEIR LN Equity	WEIR	Industrial Engineering	+
IAP LN Equity	IAP	Financial Services	+
TSCO LN Equity	TSCO	Food & Drug Retailers	+
BT/A LN Equity	BTA	Fixed Line Telecommunications	+
MKS LN Equity	MKS	General Retailers	+
BNZL LN Equity	BNZL	Support Services	+
REL LN Equity	REL	Media	+
AGK LN Equity	AGK	Support Services	+
AZN LN Equity	AZN	Pharmaceuticals & Biotechnology	+
SMIN LN Equity	SMIN	General Industrials	+
MRW LN Equity	MRW	Food and Drug Retailers	+
ARM LN Equity	ARM	Technology Hardware and Equipment	+
JMAT LN Equity	JMAT	Chemicals	+
BP/ LN Equity	BP	Oil & Gas Producers	+
ABF LN Equity	ABF	Food Producers	+
SHP LN Equity	SHP	Pharmaceuticals and Biotechnology	+
BA/ LN Equity	BA	Aerospace and Defense	+
CRH LN Equity	CRH	Construction and Materials	+
SRP LN Equity	SRP	Support Services	+
SGE LN Equity	SGE	Software & Computer Services	+
SSE LN Equity	SSE	Electricity	+
VOD LN Equity	VOD	Mobile Telecommunications	+
RDSB LN Equity	RDSB	Oil & Gas Producers	+
AMEC LN Equity	AMEC	Oil Equipment & Services	+
CRDA LN Equity	CRDA	Chemicals	+
KGF LN Equity	KGF	General Retailers	+
RIO LN Equity	RIO	Mining	+
GKN LN Equity	GKN	Automobile & Parts	+
SN/ LN Equity	SN	Health Care Equipment and Services	+
BATS LN Equity	BATS	Tobacco	+
CNA LN Equity	CNA	Gas, Water & Multiutilities	+
NG/ LN Equity	NG	Gas, Water & Multiutilities	+
ULVR LN Equity	ULVR	Food Producers	+
BG/ LN Equity	BG	Oil & Gas Producers	+
BSY LN Equity	BSY	Media	+

(a) Largest Eigenvector Components - Year 2007.

Company	Abbreviation	Classification	Sign
CSCG LN Equity	CSCG	Real Estate Investments Trusts	+
LAND LN Equity	LAND	Real Estate Investments Trusts	+
LLOY LN Equity	LLOY	Banks	+
HMSO LN Equity	HMSO	Real Estate Investments Trusts	+
RBS LN Equity	RBS	Banks	+
BLND LN Equity	BLND	Real Estate Investments Trusts	+
AV/ LN Equity	AV	Life Insurance	+
BARC LN Equity	BARC	Banks	+
IAG LN Equity	IAG	Travel & Leisure	+
PRU LN Equity	PRU	Life Insurance	+
LGEN LN Equity	LGEN	Life Insurance	+
OML LN Equity	OML	Life Insurance	+
WOS LN Equity	WOS	Support Services	+
NXT LN Equity	NXT	General Retailers	+
KGF LN Equity	KGF	General Retailers	+
ABF LN Equity	ABF	Food Producers	+
TSCO LN Equity	TSCO	Food & Drug Retailers	+
SHP LN Equity	SHP	Pharmaceuticals & Biotechnology	+
ADN LN Equity	ADN	Financial Services	+
MKS LN Equity	MKS	General Retailers	+
EMG LN Equity	EMG	Financial Services	+
RSA LN Equity	RSA	Nonlife Insurance	+
CRDA LN Equity	CRDA	Chemicals	+
AZN LN Equity	AZN	Pharmaceuticals & Biotechnology	+
RRS LN Equity	RRS	Mining	-
ANTO LN Equity	ANTO	Mining	-
BLT LN Equity	BLT	Mining	-
AAL LN Equity	AAL	Mining	-
RIO LN Equity	RIO	Mining	-
BP/ LN Equity	BP	Oil & Gas Producers	-
BG/ LN Equity	BG	Oil & Gas Producers	-
RDSB LN Equity	RDSB	Oil & Gas Producers	-

(b) Second Largest Eigenvector Components - Year 2007.

Company	Abbreviation	Classification	Sign
BARC LN Equity	BARC	Banks	+
RBS LN Equity	RBS	Banks	+
HSBA LN Equity	HSBA	Banks	+
GSK LN Equity	GSK	Pharmaceuticals & Biotechnology	+
AZN LN Equity	AZN	Pharmaceuticals & Biotechnology	+
LLOY LN Equity	LLOY	Banks	+
BATS LN Equity	BATS	Tobacco	+
KGF LN Equity	KGF	General Retailers	-
SMIN LN Equity	SMIN	General Industrials	-
JMAT LN Equity	JMAT	Chemicals	-
ARM LN Equity	ARM	Technology Hardware & Equipment	-
CPI LN Equity	CPI	Support Services	-
NXT LN Equity	NXT	General Retailers	-
CRDA LN Equity	CRDA	Chemicals	-
WTB LN Equity	WTB	Travel & Leisure	-
BA/ LN Equity	BA	Aerospace and Defense	-
IAP LN Equity	IAP	Financial Services	-
TLW LN Equity	TLW	Oil & Gas Producers	-
REX LN Equity	REX	General Industrials	-
BNZL LN Equity	BNZL	Support Services	-
AGK LN Equity	AGK	Support Services	-
RRS LN Equity	RRS	Mining	-
BLT LN Equity	BLT	Mining	-
ADN LN Equity	ADN	Financial Services	-
MGGT LN Equity	MGGT	Aerospace and Defense	-
RIO LN Equity	RIO	Mining	-
SRP LN Equity	SRP	Support Services	-
IMI LN Equity	IMI	Industrial Engineering	-
AMEC LN Equity	AMEC	Oil Equipment & Services	-
SDRC LN Equity	SDRC	Financial Services	-
WEIR LN Equity	WEIR	Industrial Engineering	-
ANTO LN Equity	ANTO	Mining	-
SDR LN Equity	SDR	Financial Services	-
AAL LN Equity	AAL	Mining	-

(c) Third Largest Eigenvector Components - Year 2007.

Company	Abbreviation	Classification	Sign
ANTO LN Equity	ANTO	Mining	+
AAL LN Equity	AAL	Mining	+
BLT LN Equity	BLT	Mining	+
RBS LN Equity	RBS	Banks	+
RIO LN Equity	RIO	Mining	+
BARC LN Equity	BARC	Banks	+
CRH LN Equity	CRH	Construction & Materials	+
NXT LN Equity	NXT	General Retailers	+
LGEN LN Equity	LGEN	Life Insurance	+
KGF LN Equity	KGF	General Retailers	+
LLOY LN Equity	LLOY	Banks	+
GKN LN Equity	GKN	Automobile & Parts	+
WOS LN Equity	WOS	Support Services	+
ABF LN Equity	ABF	Food Producers	-
GSK LN Equity	GSK	Pharmaceuticals & Biotechnology	-
SAB LN Equity	SAB	Beverages	-
SN/ LN Equity	SN	Health Care Equipment and Services	-
CNA LN Equity	CNA	Gas, Water & Multiutilities	-
IAP LN Equity	IAP	Financial Services	-
AZN LN Equity	AZN	Pharmaceuticals & Biotechnology	-
MGGT LN Equity	MGGT	Aerospace and Defense	-
TSCO LN Equity	TSCO	Food & Drug Retailers	-
BT/A LN Equity	BT/A	Fixed Line Telecommunications	-
BA/ LN Equity	BA	Aerospace and Defense	-
UU/ LN Equity	UU	Gas, Water & Multiutilities	-
MRW LN Equity	MRW	Food & Drug Retailers	-
ULVR LN Equity	ULVR	Food Producers	-
RB/ LN Equity	RB	Household Goods & Home Construction	-
VOD LN Equity	VOD	Mobile Telecommunications	-
IMT LN Equity	IMT	Tobacco	-
SVT LN Equity	SVT	Gas, Water & Multiutilities	-
DGE LN Equity	DGE	Beverages	-
SSE LN Equity	SSE	Electricity	-
BATS LN Equity	BATS	Tobacco	-
NG/ LN Equity	NG	Gas, Water & Multiutilities	-

(d) Fourth Largest Eigenvector Components - Year 2007.

Company	Abbreviation	Classification	Sign
SDRC LN Equity	SDRC	Financial Services	+
VOD LN Equity	VOD	Mobile Telecommunications	+
SDR LN Equity	SDR	Financial Services	-

(e) Smallest Eigenvector Components - Year 2007.

Company	Abbreviation	Classification	Sign
AAL LN Equity	AAL	Mining	+
RDSB LN Equity	RDSB	Oil & Gas Producers	+
VOD LN Equity	VOD	Mobile Telecommunications	+
RIO LN Equity	RIO	Mining	+
RBS LN Equity	RBS	Banks	+
GSK LN Equity	GSK	Pharmaceuticals & Biotechnology	+
BARC LN Equity	BARC	Banks	+
HSBA LN Equity	HSBA	Banks	+
BP/ LN Equity	BP	Oil & Gas Producers	+
SDR LN Equity	SDR	Financial Services	+
BG/ LN Equity	BG	Oil & Gas Producers	+
TSCO LN Equity	TSCO	Food & Drug Retailers	+
BLT LN Equity	BLT	Mining	+
SDRC LN Equity	SDRC	Financial Services	-
HMSO LN Equity	HMSO	Real Estate Investments Trusts	-

(f) Second Smallest Eigenvector Components - Year 2007.

Table A.1: Significant Participating Firms in Each Eigenvector for 2007.

Appendix B

Tables & Figures from Chapter 7

	VIX	P_t^{copper}	x_t^{IMACEC}	π_t	r_t	G_t^1	G_t^2	G_t^3	G_t^{81}	G_t^{82}	G_t^{83}
Mean	22.028	1.512	0.526	0.258	3.803	0.110	-0.053	-0.015	-0.005	0.005	-0.009
Median	21.090	0.558	0.651	0.220	3.500	0.090	-0.031	0.004	-0.003	0.006	-0.005
Maximum	59.890	35.424	7.215	1.110	8.250	1.212	0.698	0.674	0.109	0.102	0.089
Minimum	10.420	-36.471	-5.830	-1.029	0.500	-0.864	-0.770	-0.674	-0.138	-0.114	-0.151
Std. Dev.	9.139	8.954	1.587	0.383	2.004	0.325	0.210	0.182	0.046	0.045	0.038
Skewness	1.415	-0.158	-0.197	-1.029	0.185	0.082	-0.353	0.341	-0.185	-0.283	-0.519
Kurtosis	5.772	5.883	7.176	3.587	2.329	4.159	5.425	5.866	3.321	2.657	4.346
Jarque-Bera	80.447	46.275	90.184	2.863	3.010	7.021	32.687	44.472	1.233	2.243	14.795
Probability	0.000	0.000	0.000	0.238	0.222	0.030	0.000	0.000	0.540	0.326	0.001
Sum	2709	199.665	64.674	34.388	467.750	13.476	-6.499	-1.796	-0.623	0.581	-1.061
Sum Sq. Dev.	10190	10502.89	307.350	19.399	489.907	12.904	5.370	4.050	0.253	0.243	0.180
Observations	123	123	123	123	123	123	123	123	123	123	123

Table B.1: Descriptive statistics of Variables used in the VAR estimation. Note: Sample period from 2000M11 to 2011M01. Authors calculations.

1. VIX: Is the Chicago Board Options Exchange Market Volatility Index. Source: Bloomberg.

2. P_t^{Copper}: Is the seasonally adjusted monthly change of the London Metal Exchange copper price. Source: Bloomberg.

3. x_t^{IMACEC}: Is the monthly change of the IMACEC (monthly indicator of economic activity which cover 90% of goods and services included in the GDP). Source: Chilean Central Bank.

4. r_t: Is the Chilean Central Bank Monetary Policy Rate. Source: Chilean Central Bank.

Company	Business Sector
SQM A	Mining
ENERSIS	Energy
ANDINA B	Food and beverage
FALABELLA	Commerce and retail
VAPORES	Shipping
ENDESA	Energy
ORO BLANCO	Mining
CAP	Metalmechanics
CALICHERA A	Mining
ANDINA A	Food and beverage
CMPC	Forest products, cellulose
CCU	Food and beverage
COPEC	Energy and natural resources
ENTEL	Telecommunications
MADECO	Metalmechanics
BCI	Banking and Financial Services
LAN	Airline and cargo
COLBUN	Energy
SAN PEDRO	Food and beverage
QUINENCO	Conglomerate
CINTAC	Metalmechanics
ALMENDRAL	Various investments
BSANTANDER	Banking and Financial Services
CONCHATORO	Food and beverage
SMCHILE B	Banking and Financial Services
GASCO	Energy and natural resources
INFORSA	Forest products, cellulose
PARAUCO	Commerce and retail
AGUASA	Water resources
CRISTALES	Plastics and glasses
CHILE	Banking and Financial Services
CEMENTOS	Construction
IANSA	Food and beverage
CGE	Energy

(a) Largest Eigenvector

Company	Business Sector
CALICHERAA	Mining
ORO BLANCO	Mining
SQM A	Mining
NORTEGRAN	Investments and Real Estate
SQM B	Mining
ANDINA A	Food and Beverage
ANDINA B	Food and Beverage
CMPC	Forest Products related
IANSA	Agro-Industrial products
ECL	Energy
COLBUN	Energy
CAMPOS	Agro-Industrial products
CINTAC	Metalmechanics
INFORSA	Forest Products related
COPEC	Energy and Natural Resources

(b) Second Largest Eigenvector

Company	Business Sector
CAMPOS	Agro-Industrial products
IANSA	Agro-Industrial products
NORTEGRAN	Investments and Real Estate
ORO BLANCO	Mining
CALICHERAA	Mining
SM CHILE E	Mining
ECL	Energy
QUINTEC	Investments and Real Estate
NAVARINO	Transport and Logistics
EMBONOR B	Food and Beverage
PUCOBRE A	Mining
SQM B	Mining
GENER	Energy
PROVIDA	Pension Fund
SOMELA	Metalmechanics
CINTAC	Metalmechanics
ENERSIS	Energy
COPEC	Energy and Natural Resources
ENDESA	Energy
ENTEL	Telecommunications
CMPC	Forest Products related
MADECO	Mining Related
BSANTANDER	Banking and Financial Services
FALABELLA	Commerce and Retail
LAN	Airline Services
CGE	Energy
ALMENDRAL	Investments and Real Estate
CAP	Metalmechanics

(a) Third Largest Eigenvector

Company	Business Sector
CGE	Energy
ELECMETAL	Metalmechanics
KOPOLAR	Food and Beverage
ORO BLANCO	Mining
EMBONOR B	Food and Beverage
MARINSA	Investments and Real Estate
SECURITY	Banking and Financial Services
CAP	Metalmechanics
ANDINA A	Food and Beverage
CALICHERAA	Mining
ENERSIS	Energy
CHILE	Banking and Financial Services
BANMEDICA	Investments (health related)
NORTEGRAN	Investments and Real Estate
PARAUCO	Commerce and Retail
BSANTANDER	Banking and Financial Services
EMBONOR A	Food and Beverage
EMILIANA	Food and Beverage
SM CHILE E	Banking and Financial Services
ENDESA	Energy
TELCOY	Telecommunications
CINTAC	Metalmechanics
CONCHATORO	Food and Beverage
FOSFOROS	Matches producers

(b) Smallest Eigenvector

Table B.2: Significant Participating Firms in Each Eigenvector

Appendix C

Additional Analyses from Chapter 8

C.1 Proof Equation 4.1

For this result in Equation 4.1, note for any real non-negative vector U_1 in (3.31), the q power of the matrix denoted as Θ'^q, can be solved iteratively using the infinity norm of the vector $\Theta'^q U_q$ denoted as $||\Theta'^q U_q||_\infty$ to normalise the vector as in the power iteration equation:

$$U_{q+1} = \frac{\Theta' U_q}{||\Theta' U_q||_\infty} = \frac{\Theta'^q U_1}{||\Theta'^q U_1||_\infty}. \tag{C.1}$$

The iteration is said to have converged at $q+1$ when $U_{q+1} = U_q = V^L$ with an epsilon margin of error. The vector V^L is the left eigenvector of the matrix and $||\Theta'^q V^L||_\infty = \lambda_{max}(\Theta')$. Multiplying through by $\lambda_{max}(\Theta')$ in the power iteration equation, we have the eigenvalue equation $\Theta' V^L = \lambda_{max} V^L \equiv \Theta'^q U_1$. This leads to the result in Equation 4.1. That is, the potential capital loss for country i is given by $u_{i,\#} \cong \lambda_{max}(\Theta') v_{i,\infty}$.

C.2 Simulation Algorithm for LRMES

Here we describe the simulation based procedure developed by Brownlees and Engle (2012) to construct the LRMES forecasts. That is, we are interested in computing the LRMES of country i on period T at horizon h, conditional on a global market decline equal to C (which for this case we set it to 40%),

$$LRMES_{i,T} = E_T(R_{i,T+1:T+h}|R_{m,T+1:T+h} < C). \tag{C.2}$$

The authors assume parameters to be known while in practice they use estimated parameters using all of the information available up to time T.

1. Construct the GARCH-DCC standardised innovations

$$\epsilon_{m,t} = \frac{r_{m,t}}{\sigma_{m,t}} \quad and \quad \xi_{i,t} = \left(\frac{r_{i,t}}{\sigma_{i,t}} - \rho_{i,t}\frac{r_{m,t}}{\sigma_{m,t}}\right) / \sqrt{1-\rho^2_{i,t}}, \tag{C.3}$$

for each $t = 1, ..., T$. Notice that by construction $\epsilon_{m,t}$ and $\xi_{i,t}$ are $i.i.d \sim$ F(0,1).

2. Sample with replacement Sxh pairs of standardised innovations $[\epsilon_{m,t}\xi_{i,t}]'$. Use these to construct S pseudo samples of GARCH-DCC innovations from period $T+1$ to period $T+h$, that is

$$\begin{bmatrix} \epsilon^s_{m,T+t} \\ \xi^s_{i,T+t} \end{bmatrix}_{t=1,...,h} \qquad s = 1, ..., S. \tag{C.4}$$

3. Feed the pseudo samples of GARCH-DCC innovations into the DCC and GARCH filters respectively using as initial conditions the last values of the conditional correlation $\rho_{i,T}$ and variances $\sigma^2_{m,T}$ and $\sigma^2_{i,T}$. This step delivers S pseudo samples of GARCH-DCC returns from period $T+1$ to period $T+h$ conditional on the realised process up to time T, that is

$$\begin{bmatrix} r^s_{m,T+t} \\ r^s_{i,T+t} \end{bmatrix}_{t=1,...,h} |F_T \quad s = 1, ..., S. \tag{C.5}$$

4. Construct the multi-period arithmetic firm return of each pseudo sample

$$R^s_{i,T+1:T+h} = exp\left[\sum_{t=1}^{h} r^s_{i,T+t}\right] - 1, \tag{C.6}$$

and compute the multi-period arithmetic market return $R^s_{m,T+1:T+h}$ analogously.

5. Compute the LRMES as the Monte Carlo average of the simulated multi-period returns conditional on the systemic event

$$LRMES_{i,T} = \frac{\sum_{s=1}^{S} R_{i,T+1:T+h} I\{R_{m,T+1:T+h} < C\}}{\sum_{s=1}^{S} I\{R_{m,T+1:T+h} < C\}}. \tag{C.7}$$

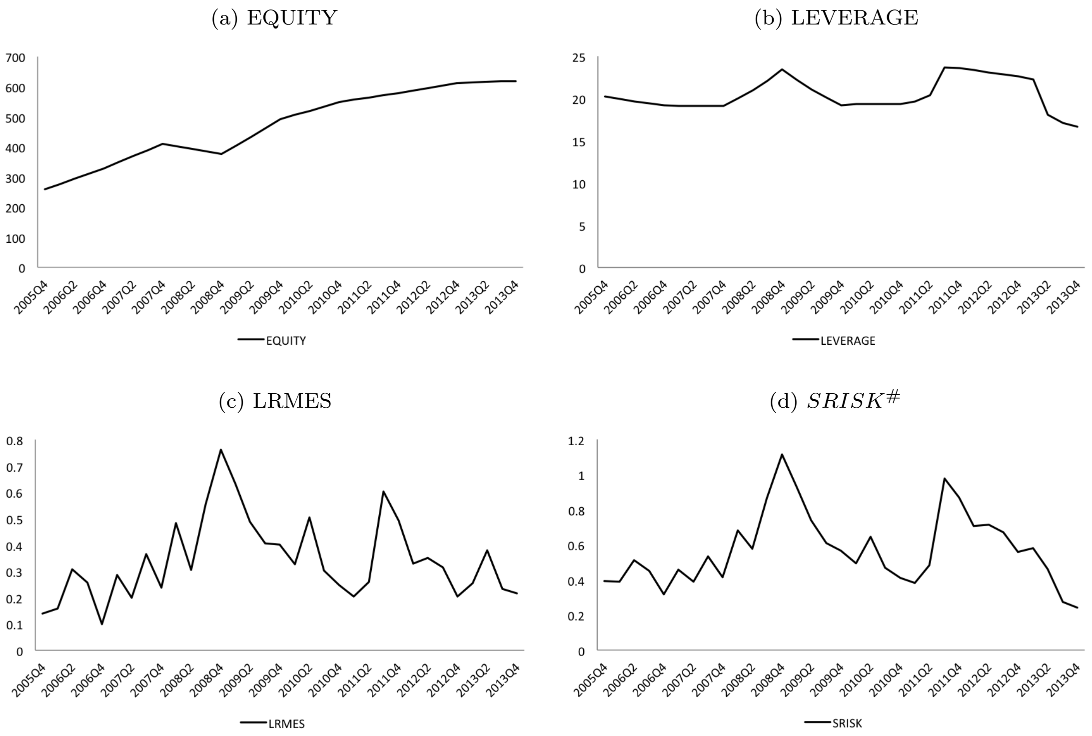
C.3 Breakdown of the Core Global Banking System: $SRISK^{\#}$

As we can see from Figure C.1, leverage was, on average, already high of the order of 20 for the 18 national banking systems during 2005Q4, peaking at 23 in 2008Q4. If we think of the minimum leverage ratio stipulated in Basel III of 3%[1], which is equivalent to a leverage of the order of 33. This would imply that these national banking systems, on average, exhibit lower leverage than what Basel III expects from them. However, in our sample, there are countries such as Ireland that exhibit a leverage well above 33, prior to the crisis, reaching a peak of 51, during 2008Q4. In contrast, there are countries such as Canada that exhibit a leverage of 6 over the entire sample. This definitively, tends to drive the results. As suggested by Engle et al. (2014), the SRISK could be weighted by the market value of equity. However, this neglects the systemic importance of many GIIPS countries, as they carry a very small weight in the overall equity and these countries were precisely the ones that had the greatest potential to trigger capital losses in the international banking system.

In addition, the LRMES remains subdued at 25% right up to 2007Q1, peaking to 76% during 2008Q4. This compensates the weight of leverage in the $SRISK^{\#}$ formula, therefore, preventing this latter from picking up in the run up to the crisis.

[1]Under Basel III, "leverage ratio" is defined as the ratio between Tier 1 capital and the bank's average total consolidated assets (sum of the exposures of all assets and non-balance sheet items).

Figure C.1: Average Equity, Leverage, LRMES, and $SRISK^{\#}$



C.4 Estimation Methodology for the Capital Threshold for Losses

In Section 3.5 we argued that the stability of the networked system is given by the condition $\lambda_{max}(\Theta) < \rho$, where ρ is the capital loss threshold. These thresholds have to be carefully specified. In this appendix we determine how the absolute capital threshold, ρ, that works as a buffer against losses relates to the regulatory Tier 1 Capital requirement.

Based on the Basel II criteria, the capital inadequacy in a bank from losses of receivables from counterparties can be defined as follows:

$$\frac{EquityCapital - LGD}{RWA} < 10\% = T_{RWA}. \tag{C.8}$$

Here LGD is loss given default, which is the magnitude of likely losses on the exposure, usually expressed as a percentage of the exposure and RWA is risk weighted assets. This condition states that equity capital minus losses should be greater than 10% of risk weighted assets. However, as the practical aspects of avoiding insolvency requires recapitalization, it is important to see the equivalence of the above Basel rule with permissible LGD as function of Equity capital, denoted as T_C. This equivalence is important as it hints at the permissible loss the country should experience before being considered in distress. If we assume that this condition is exactly met at $T_C \cdot Equitycapital = LGD$, by substituting this into C.8 when the condition in C.8 is exactly met, we have:

$$T_C = MAX\left(0, 1 - T_{RWA}\frac{RWA}{EquityCapital}\right). \tag{C.9}$$

For instance, when $\frac{EquityCapital}{RWA} = 10\% = T_{RWA}$, we have $T_C = 0$, which means that there is no room to lose anything and the bank/banking system requires recapitalization. In contrast, if we allow a relatively small number for $\frac{EquityCapital}{RWA} = 12\%$ but above $T_{RWA} = 10\%$, we have $T_C = 16.6\%$. What this implies is that when banks/banking systems have an average Equity Capital/RWA of 12%, then by C.8, the permissible loss that can be sustained before being considered in distress is 16.6% of Equity Capital. Based on this, we conclude that the Basel II criteria of capital adequacy can be highly misleading in terms of signalling distress in the system. In Table C.2 we calculated T_C for every country for all the years and we plot the minimum of T_C together with $\lambda_{max}(\Theta)$. We also provide the number of banking systems that require recapitalization as indicated by their T_C for every quarter. Here we observe that before the GFC, capital buffers were very low ranging between 14% to 9% during 2008Q4. This period also coincides with more than the half of the national banking systems requiring recapitalization. This trend is reversed as we approach 2013Q4, where banks started to build their capital buffers from historical lows to 9% to more than 35% in 2013Q4.

In general, there are growing concerns about the reliability of the denominator of capital ratios, similar to the previous loss of confidence in the numerator in the run-up to the 2007 financial crisis. Back then, market participance moved away from regulatory measures[2] and focused instead on capital measures, which better reflected true loss-absorbing capital. While Basel III attempted to correct the main deficiencies of the denominator by adopting more stringent

[2]e.g., Core Tier 1 in Europe and Tangible Common Equity or Tier 1 Common in the US.

definitions of capital, there is an increasing loss of confidence in the way that banks calculate their RWA (particularly the ones using Basel II advanced IRB approach). This is why some market participants prefer to rely on unweighted capital measures to assess solvency or to require a higher capital ratio to compensate for the possible understatement of RWAs (Leslé and Avramova, 2012). For our analysis, this suggests that the Tier 1/RWA ratio is overstated and therefore we should expect T_C to be much smaller in case RWAs were properly calculated.

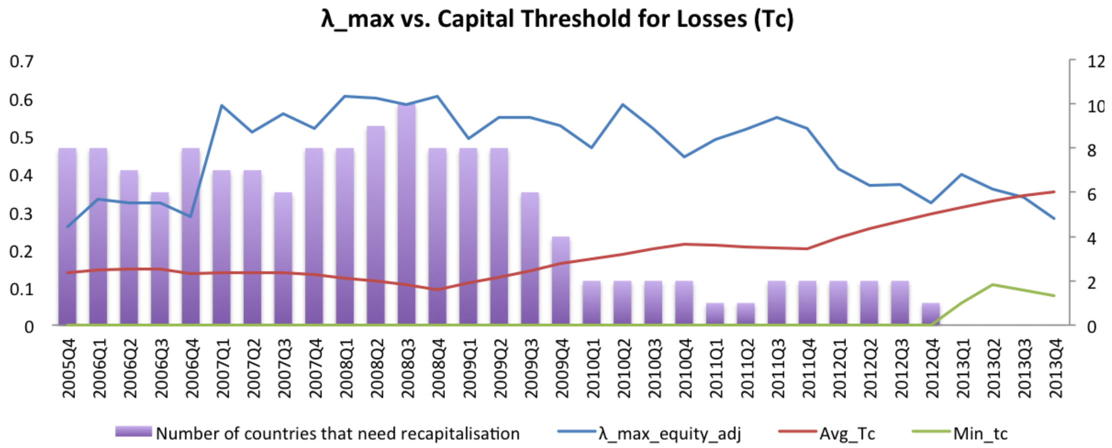


Figure C.2: Note: For quarters between 2011Q4-2012Q4 the Maximum Eigenvalue, λ_{max} (right axis), is calculated for CGBS network without Greece, when it was virtually bankrupt with negative equity capital in banking system. Nonetheless, the main source of instability is given by the GIIPS countries.

C.5 Cross-Border Flows Breakdown used to Rescale λ_{max}

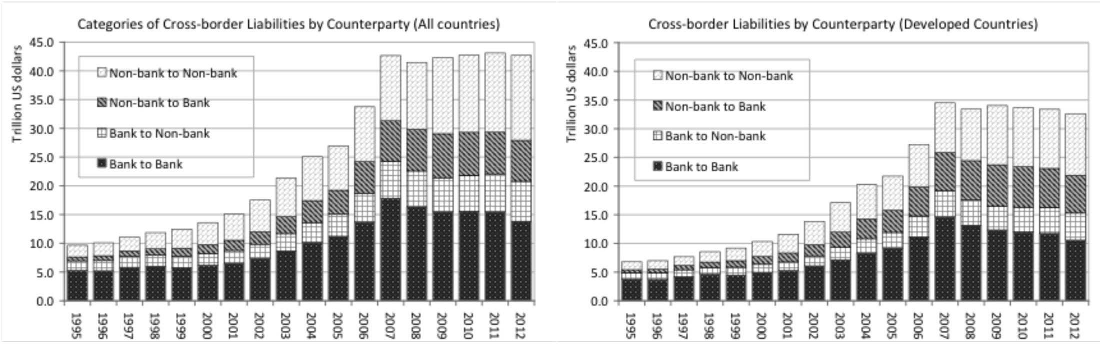


Figure C.3: Cross-border liabilities by type of counterparty. Left panel shows cross-border debt liabilities by pairwise classiffication of borrower and lender. "Bank to bank" refers to cross-border claims of banks on other banks (BIS banking statistics table 7A minus 7B). "Bank to non-bank" refers to cross-border claims of banks on non-banks (BIS table 7B). Claims of non-banks are from BIS international debt security statistics, tables 11A and 11B). The right panel shows cross-border debt liabilities of developed countries according to BIS classiffication. Source: Bruno and Shin (2015a).

	Daily Returns MSCI Financials and World				$Leverage = \frac{Liabilities+Equity}{Equity}$			
Country	Mean	Std	Max	Min	Mean	Std	Max	Min
Australia	0.00%	1.39%	9.14%	-8.87%	17.65	0.93	18.82	16.01
Austria	-0.05%	2.60%	17.13%	-18.53%	16.50	1.19	19.87	14.73
Belgium	-0.06%	2.36%	16.70%	-34.30%	22.25	2.97	29.60	17.66
Canada	0.01%	1.41%	11.70%	-13.12%	6.76	4.59	12.09	2.12
France	-0.01%	2.34%	17.05%	-11.80%	23.83	2.25	29.76	19.44
Germany	-0.01%	2.03%	16.08%	-12.50%	25.88	3.62	32.81	17.93
Greece	-0.16%	4.33%	25.74%	-34.97%	36.44	36.33	105.63	12.80
India	0.04%	2.18%	19.23%	-13.82%	14.56	0.6	16.14	13.82
Ireland	-0.17%	5.26%	38.53%	-56.65%	32.15	9.01	51.96	17.82
Italy	-0.04%	2.22%	15.60%	-10.49%	14.83	1.02	16.82	13.60
Japan	-0.02%	2.00%	14.49%	-12.40%	19.80	1.34	23.39	17.88
Netherlands	-0.02%	3.08%	21.46%	-20.31%	25.35	2.91	32.30	18.50
Portugal	-0.09%	2.44%	15.28%	-13.17%	21.35	1.71	24.60	18.46
Spain	-0.02%	2.17%	19.55%	-12.25%	18.34	1.24	21.84	15.92
Sweden	0.02%	1.98%	12.84%	-9.50%	23.01	1.77	26.53	18.57
Switzerland	-0.03%	2.11%	16.11%	-11.75%	21.22	1.78	24.82	17.94
Turkey	0.02%	2.35%	15.41%	-11.71%	9.26	1.54	16.00	8.27
United Kingdom	-0.02%	1.99%	17.61%	-13.43%	25.69	4.83	39.86	19.20
United States	-0.01%	2.34%	16.04%	-17.80%	13.43	1.32	15.60	11.64
MSCI World	0.01%	1.16%	8.90%	-7.37%	-	-	-	-

Table C.1: Source: Bloomberg for the MSCI and Bankscope for Leverage. Authors calculations

Country	Equity (in Billion USD)				RWA (in Billion USD)			
	Mean	Std	Max	Min	Mean	Std	Max	Min
Australia	165.34	66.01	247.48	32.34	1132.7	324.5	1506.3	261.0
Austria	131.19	23.06	154.06	63.54	726.5	181.1	1042.7	317.8
Belgium	145.89	17.63	194.46	110.41	811.9	215.1	1107.6	401.9
Canada	242.60	136.16	435.96	94.27	573.4	640.8	1472.4	3.0
France	822.30	189.27	1141.83	401.32	3792.7	762.1	4660.4	2146.9
Germany	759.67	245.53	1161.15	247.19	3962.2	1197.2	5909.3	902.8
Greece	24.98	12.82	48.69	5.94	272.3	77.5	460.0	121.0
India	119.16	42.44	177.35	49.86	408.2	262.2	1158.5	7.7
Ireland	59.34	11.94	77.92	37.89	446.2	107.8	627.0	308.0
Italy	395.21	59.94	470.74	223.98	2473.9	515.9	2930.5	1180.4
Japan	1211.97	372.44	1674.03	673.06	4301.7	2996.4	8356.2	810.2
Netherlands	269.06	39.57	335.77	166.61	1242.9	135.1	1511.2	1034.2
Portugal	35.33	6.35	45.89	19.39	357.1	49.4	403.8	225.7
Spain	267.24	76.14	407.98	124.90	2192.8	492.7	2798.3	1108.4
Sweden	96.01	26.41	149.44	49.16	769.4	80.2	865.0	526.8
Switzerland	266.90	45.07	366.91	179.67	814.6	253.7	1485.9	428.3
Turkey	79.84	28.94	129.63	26.13	351.7	204.8	703.0	25.7
United Kingdom	1076.48	244.07	1441.59	637.18	5985.7	749.7	7014.2	3896.0
United States	2801.10	671.71	3765.27	1774.46	13779.2	3037.9	17590.3	7611.9

Table C.2: Equity and Risk Weighted Assets (RWA, in Billion USD) from Bankscope. Authors calculations

Country	Tier 1 Capital (in Billion USD)				Tier 1 Capital /RWA (%)			
	Mean	Std	Max	Min	Mean	Std	Max	Min
Australia	104.64	41.30	157.36	22.19	8.97%	1.28%	10.96%	7.15%
Austria	88.89	28.20	120.03	26.25	11.97%	1.64%	14.13%	8.26%
Belgium	89.96	30.10	127.12	34.36	11.25%	3.37%	15.72%	6.05%
Canada	77.73	86.00	190.63	0.48	16.47%	3.59%	27.04%	12.22%
France	461.74	182.22	677.44	119.30	11.6%	3.07%	15.81%	5.55%
Germany	436.17	211.98	848.40	67.92	10.42%	2.64%	14.49%	7.29%
Greece	17.71	9.59	34.08	1.60	6.43%	2.64%	11.20%	0.6%
India	61.60	33.92	120.61	10.58	28.7%	33.6%	136.7%	10.41%
Ireland	45.75	9.97	61.43	25.52	10.47%	2.16%	16.88%	8.07%
Italy	231.46	55.55	282.98	100.93	9.34%	1.18%	11.40%	7.86%
Japan	832.05	239.20	1153.12	444.09	31.40%	19.82%	67.74%	11.40%
Netherlands	152.02	24.84	222.37	113.06	12.37%	2.29%	15.24%	8.80%
Portugal	32.83	9.63	46.85	16.31	9.06%	1.92%	12.76%	6.69%
Spain	213.70	71.80	304.70	86.88	9.50%	1.52%	12.95%	7.65%
Sweden	82.37	22.64	114.84	41.81	10.63%	2.52%	14.22%	7.47%
Switzerland	146.84	51.73	269.24	74.74	17.93%	2.04%	20.11%	13.63%
Turkey	59.86	23.63	93.65	17.10	22.58%	12.47%	66.46%	12.77%
United Kingdom	820.21	239.23	1093.89	319.62	13.42%	2.70%	16.52%	8.20%
United States	1623.80	482.37	2316.40	874.23	11.61%	1.19%	13.17%	9.30%

Table C.3: Tier 1 Capital and Risk Weighted Assets (RWA, in Billion USD) from Bankscope. Authors calculations

C.6 Alternative Measures of Systemic Instability from Network Theory

A hypothetical disconnection of the network is expected to lead to a global economic crisis, since it disconnects international markets (Joseph et al., 2014). To measure this effect in the Global Banking Network, we introduce the concept of *algebraic connectivity* of a graph, which is defined as the real-part of the smallest non-zero eigenvalue of the normalised Laplacian, which is defined as:

$$L_N = I - D_{in}^{-1} \cdot W^T, \tag{C.10}$$

where $D_{in}^{-1} = diag(1/s_1^{in}, ..., 1/s_N^{in})$ is the diagonal matrix consisting of all node's instrengths. The Laplacian matrix has exactly one zero eigenvalue for every strongly connected component. A graph is said to be *strongly connected*, if there is a directed path between any pair of nodes (i, j). If such a path does not exists for all node pairs, but the underlying undirected graph is connected, the network is said to be *weakly-connected.* For the case of a single strongly connected component, the emphalgebraic connectivity, λ_1 is the second smallest eigenvalue of L_N. It is a measure for the robustness of a network against edge/node removal. This is because a zero-value means the decomposition of the network into two disconnected components. The results of this exercise are in Figure C.4, where we plot the *algebraic connectivity*, λ_1 together with λ_{max} for the whole sample. Here we observe that whenever λ_{max} spikes up, there is a tweak in the algebraic connectivity. This means that before a financial turmoil unravels, the network becomes more disconnected. While this could be considered as an early warning, the absence of a proper threshold unables to pinpoint exactly when system stability is at stake. In contrast, $lambda_{max}$ depicts growing instability and there is an specified threshold, through the condition $\lambda_{max}(\Theta') < \rho$ above which we can consider the system to be unstable.

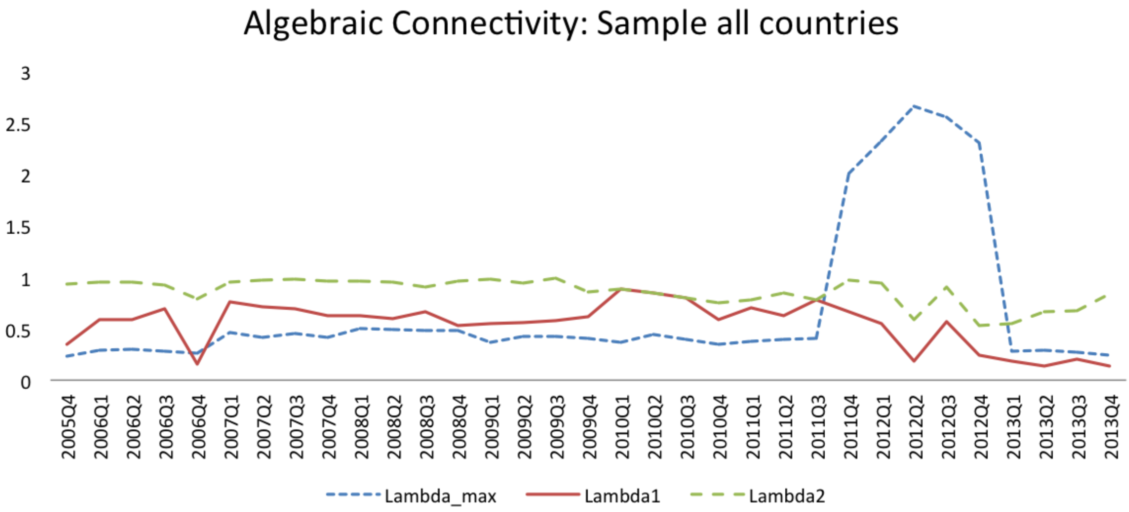


Figure C.4: Algebraic connectivity, λ_1 and λ_{max}. Author Calculations.

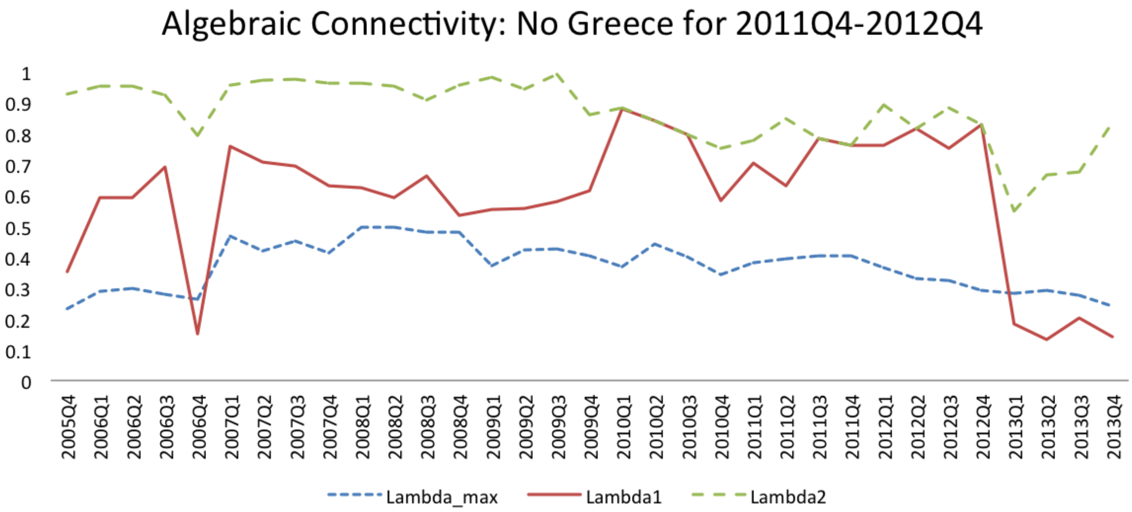


Figure C.5: Algebraic connectivity, λ_1 and λ_{max}, excluding Greece for 2011Q4-2012Q4. Author Calculations.

BIS	Bank for International Settlement
C^{SAMP}	Sample Correlation Matrix
C^{SAMP1}	Filtered Sample Correlation Matrix by keeping the Largest Eigenvalue
$C^{SAMP-RMT1}$	Filtered Sample Correlation Matrix by the method of Plerou et al (2002)
$C^{SAMP-RMT2}$	Filtered Sample Correlation Matrix by the method of Laloux et al (2000)
C^{CAPM}	Sample Correlation Matrix
$C^{CAPM-RMT1}$	Filtered CAPM Correlation Matrix by the method of Plerou et al (2002)
$C^{CAPM-RMT2}$	Filtered CAPM Correlation Matrix by the method of Laloux et al (2000)
C^{RS}	Sample Correlation Matrix
$C^{RS-RMT1}$	Filtered Regime-Switching Correlation Matrix by the method of Plerou et al (2002)
$C^{RS-RMT2}$	Filtered Regime-Switching Correlation Matrix by the method of Laloux et al (2000)
CAPM	Capital Asset Pricing Model
CCC	Constant Conditional Correlation
CDF	Cumulative Distribution Function
CGBSN	Core Global Banking System Network
DCC-MES	Dynamic Conditional Correlation Marginal Expected Shortfall
DCC-ΔCoVaR	Dynamic Conditional Correlation Delta Value-at-Risk
ΔCoVaR	Delta Conditional Value-at-Risk
EQ	Equity Capital
EWMA	Exponentially Weighted Moving Average
FPE	Final Prediction Error
GFC	Global Financial Crisis
GIIPS	Greece-Ireland-Italy-Portugal-Spain
γ_{max}	Gamma Ratio
I^k	Inverse Participation Ratio
λ	Maximum Eigenvalue of the Stability Matrix Ω
LTQ	Liquidity
LRMES	Long-run Marginal Expected Shortfall
LVG	Leverage
MA	Moving Average
MAPE	Mean Average Percentage Error
MAR	Minimum Acceptable Return
MES	Marginal Expected Shortfall
MP	Marčenko-Pastur Distribution
MPT	Markowitz Portfolio Theory
MSCO	Morgan Stanly Composite Index
MSE	Mean Square Error
MV-GARCH	Multivariate Generalized Autoregressive Conditional Heterokedasticity
N_{eff}	Effective Portfolio Diversification
N_{90}	Portfolio Diversification
Ω_{est}	Estimated Risk of the Portfolio
Ω_{real}	Realised Risk of the Portfolio
Ω	Omega Ratio
PR	Participation Ratio
Π	Cumulated Returns
RMT	Random Matrix Theory
RS-GARCH	Regime-Switching Generalised Autoregressive Conditional Heteroskedasticity Model
RWA	Risk Weighted Assets
Sortino	Sortino Ratio
SP	Sharpe Ratio
SRI	Systemic Risk Index
SRISK	Systemic Risk
V^L	Left Eigenvector Centrality
V^R	Right Eigenvector Centrality
VAR	Vector Autoregressive

Table C.4: Acronyms often used in the thesis.

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