Axiomatising Questions

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Abstract

Accounts of the formal semantics of natural language often adopt a pre-existing framework. Such formalisations rely upon informal narrative to explain the intended interpretation of an expression — an expression that may have different interpretations in different circumstances, and may supports patterns of behaviour that exceed what is intended. This ought to make us question the sense in which such formalisations capture our intuitions about semantic behaviour. In the case of theories of questions and answers, a question might be interpreted as a set (of possible propositional answers), or as a function (that yields a proposition given a term that is intended to be interpreted as a phrasal answer), but the formal theory itself provides no means of distinguishing such sets and functions from other cases where they are not intended to represent questions, or their answers. Here we sketch an alternative approach to formalisation a theory of questions and answers that aims to be sensitive to such ontological considerations.

Keywords: questions, answers, ontology, methodology

1 Introduction

We first introduce some issues and concerns relating to the semantic analysis of questions and answers that are raised, or alluded to in the literature. We then summarise, in broad terms, existing formal analyses. Here we focus on semantic concerns relating to the analysis of direct questions and their answers, rather than pragmatic issues, indirect questions, or the analysis of discourse.

1.1 Questions

There are various types of questions that can be posed, even discounting implicit or indirect questions. These can be broadly classified as "wh-questions" — "Who went to London?", "What was his name?", "How did you do that?" — "polarity" questions — "Do you like cheese?", and "choice" questions — "Do you want tea or coffee?". The latter might be seen as a variant of a polarity question where there is a forced choice.

Questions can also be embedded in other propositions, for example in certain kinds of propositional attitudes — *"He knows who ate all the cheese"*, *"He wonders whether John likes Mary"*.

Questions may be refined by an additional question — "Who ate the cheese, was it John?", "Who ate the cheese, or was it the pickle?".¹

1.2 Answers

When questions are answered directly, the answer can be presented as a constituent — Q: "Who went to London?" A: "Peter and Mary."/"Nobody." — or a proposition — A: "Peter and Mary went to London."/"Nobody went to London." Answers may also be indirect, requiring some reasoning to deduce the intended answer.

Polarity questions can be answer with "yes" or "no", an adverb (of an appropriate nature) or a modal expression — Q: "Do you like cheese?", A: "Yes"/"Sometimes". The related proposition may also be spelt out in full — A: "Yes, I like cheese."/"I like cheese.".

Choice offering questions are answer with the respondent's choice being identified — Q: "Do you want tea or coffee?", A: "Tea."/"I would like tea, please.".

Answers are typically incomplete and non-exhaustive. The *appropriate* answers depend upon the context. What counts as a *relevant* answer may also depend upon what is assumed to be known.

A question may also be answered with a question — Q: "Who ate the cheese?", A: "Who do you think?"/"How should I know?".²

¹This could be viewed as a simple example of extended discourse relating to a "question under discussion" (Stalnaker, 1978; Roberts, 1996; Ginzburg & Sag, 2000).

²As when clarifying a question with an additional question, this may be an example where a more general analysis of the pragmatics of discourse is appropriate.

1.3 Relationship between focus and answers

The topic–focus contrast (see Jackendoff, 1972; Stechow, 1981, for example) is relevant when considering the interpretations of questions and answers (Stechow, 1991; Krifka, 2001). The *focus* of an utterance may be marked by some form of prosodic emphasis in speech. It typically draws attention to new information that is being introduced, or is in question. This is in contrast to the *topic*, which is taken to be understood, or presupposed. In the case of written language, we may use other cues to deduce what is in focus.

Questions provide a diagnostic test for focus: we can determine what is in focus in a proposition by considering what question the proposition might answer (Paul, 1880; Rooth, 1992) — "John likes cheese." (Q: Who likes cheese?), "John likes cheese." (Q: What does John like?). Given the nature of these diagnostic tests, it is tempting to argue that wh-terms in questions correspond to the expected focus in their answers, and that it may be appropriate to consider them as either in focus, or at least a place-holder for focus.

Focus, or emphasis, in a question may also clarify what information is sought in the case of polarity questions. In particular, knowledge of what is in focus may assist the addressee in identifying helpful information in the event of a "no" answer — Q: "*Did John eat the cheese?*", A: "No, it was Peter." (cf. "No, it was the banana.").³

For wh-questions, focus may emphasise exactly what is in question (and thus what would constitute an appropriate answer) — "Who ate the cheese?", "Who ate the cheese?", "Who ate the cheese?".

1.4 Semantic Theories of Questions

Very broadly, the various formal semantic analyses of questions and their answers can be consider to fall broadly into two camps, namely *questions as sets of answers* and *questions as structured meanings.*⁴ There are some common issues for both approaches, such as whether or not questions, answers, and propositions should all be considered to be essentially the same kind of thing.

³This is rather like a wh-question with a follow-up question (Section 1.1, "Who ate the cheese, was it John?"). Such similarities merit attention in any comprehensive account of questions and answers in dialogue.

⁴Some seek to bridge this gap (for example, Aloni & Rooy, 2002).

1.4.1 Questions as sets of answers

On the first approach, questions are conceived as sets of (possible) answers (Hamblin, 1958, 1973; Karttunen, 1977, and others). Within the the possible worlds framework, this can be formulated in terms of partitions of worlds (Groenendijk & Stokhof, 1984, 1997), where each partition represents a proposition/answer, and true answers are those partitions that contain the current world.

This approach directly models full, propositional answers. It also allows for some forms of "indirect" answer. Modelling constituent answers require a bit more work, for example by combining the constituent answer with an appropriate abstract derived from the question to produce a full propositional answer. The analysis of questions and answerhood is in effect given in the same terms as the notion of truth for propositions.

Arguably, issues concerning topic–focus, and how they relate to answers–questions, are not so easily accounted for. There are also other difficulties concerning "goodness of fit" with the data, including the the handling of choice-offering questions (Krifka, 2001).

1.4.2 Questions as structured meanings

The structured-meanings account of questions, and answers, can be motivated by observing that the topic–focus may be analysed in terms of such structured meanings (Halliday, 1967; Hull, 1975; Tichý, 1978; Stechow, 1982; Stechow & Zimmermann, 1984; Stechow, 1991; Ginzburg, 1992; Ginzburg & Sag, 2000; Krifka, 1991, 2001; Vallduví, 1992, 1993) in which "old" information is distinguished from "new" information. If wh-terms are considered to be focus-like, then we can take a similar approach to questions. The focus of the answer is corresponds to the "new" information being sought by the question. Constituent answers are answers in which the topic has been elided.

Structured meanings can be considered as "pairs" for both questions — where a wh-term is paired with the body of the question — and for topic-focus structures — where the focus terms is paired with the topic (cf. Krifka, 2001). The "focus" of an answer can be thought of as providing a "filler" for the wh-term. There is a sense in which the wh-term is "abstracted" out of the body of the question. In some accounts, a question is overtly represented by a (typed) λ -abstract, $\lambda x_T.p$, where x_T corresponds to the wh-term that is abstracted from the body of the question, p (cf. Ginzburg & Sag, 2000; Hausser, 1983; Hausser & Zaefferer, 1979; Stechow & Zimmermann, 1984). Constituent answers (or focus-terms) can then be "applied" to the question (or fill in the "missing" focus) to give a propositional answer.

Such an approach can give a straight-forward account of constituent answers, although dealing with propositional answers requires some additional formal machinery. The structured-meanings account goes someway towards relating the notion of topic–focus with that of question–answer. It can also provide accounts of choice offering questions (Krifka, 2001). Arguments can also be made that it is not as reductive as the "sets of answers" account.

2 Ontological Issues

There are a number of grounds for evaluating the pros and cons of different approaches⁵, including coverage of the data, and sensitivity to ontological issues. It is the latter that we focus on here.

In the case of set-theoretic accounts (Section 1.4.1), is it right to reduce all ontological notions to those of sets? Or possible worlds and relations over possible worlds, construed as sets?

We might consider arguments from Benacerraf (1965) on numbers, where the critical point is that numbers have structural properties whose status is independent of any particular set-theoretic characterisation. And different set-theoretic characterisations give rise to different unintended consequences — consequences that are not in accord with our understanding of the notion of number.⁶ We may wonder whether semantic notions, such as *questions* and *answers* should also be considered to have structural properties that are independent of a set-theoretic characterisation (Fox & Turner, 2012).

It seems odd for questions to *be* their (possible) answers. If that were the case, then any set of propositions/worlds would be a question. This gives rise to a methodological conundrum: how can we explore the relationship between the meaning of a question and its possible

⁵For example, see Krifka (2001) for criticism of sets of answers approach.

⁶Some set-theoretic representations allow us to express seemingly incoherent statements such as " $2 \in 3$ ". Whether such a statement is true or false does not reflect any intuitions about numbers themselves but is merely a contingent artifact of the chosen representation.

answers if a question actually is its answers, as assumed by those that follow Hamblin (1973)? It seems natural to argue that we can consider questions and their formal properties without being obliged to engage in some form of ontological reduction, just as we do with numbers. But a set-theoretic reduction appears to rule this out, at least in the formalisation.⁷

Similar arguments apply to accounts that use structured meanings to represent questions. For example, if structured meanings are pairs, how are such pairs to be distinguished from other pairs? And if questions are abstracts, how are such abstracts distinguished from other abstracts/functions?

For some, such reductions may be considered desirable. For example, Tichý (1978); Hamblin (1973); Karttunen (1977), and others, argue that questions and propositions should be the same type and that any distinction resides in our relationship to them. But this does not necessarily avoid the problem. Even if the notion of being a question is a relational one, then it can still be characterised. And presumably we wish to characterise it in a way that does not allow it to be conflated with some other, fundamentally distinct notion.

To summarise: with set-based accounts, any intuitive, ontological distinction between questions and answers, and arbitrary sets of propositions, is lost; and with conventional structured-meaning accounts, the intuitive, ontological distinction between questions (and answers), pairs, or propositional abstracts, is also lost.⁸

3 Towards a Non-reductive Analysis

Here we demonstrate a framework in which a non-reductive theory of questions can be developed. Questions will be treated somewhat like "specifications" in computer science. Answers will be those things that may "satisfy" a specification. Full propositional answers are taken to

⁷It may be possible to defend a set-theoretic reduction, and say that to entertain a question and its possible answers is to reflect on a membership relationship, or the extensional identity of a question. But then we risk stumbling into a version of the paradox of analysis (Black, 1944). We surely want to say that "considering a question and its answers" means something different to "considering the membership of a set".

⁸We could parody this reduction by observing that there appear to be no questions or answers in these formal theories of questions and answers.

have a focus that satisfies the specification. Polarity questions will be treated as questions that are answered by a propositional operator of some kind (modal or adverbial).

For our current purpose, it is not essential for us to spell out all the details here; the key objective is to demonstrate that no ontological reduction is required in order for us to develop a formal semantic analysis. We will illustrate this approach using Typed Predicate Logic (TPL), a generic framework of types and predicate logic (Turner, 2008, 2009), described below (Section 3.1). TPL frees us from the formal constraints and ontological commitments of other more rigid frameworks. In particular, it allows us to incorporate aspects of the intended interpretation into the formalisation itself, in the form of judgements and types. Some of the details of the formalisation follows the spirit of the structured-meaning approach to questions (Section 1.4.2).⁹

3.1 Typed Predicate Logic (TPL)

Typed Predicate Logic is a framework in which various kinds of theories can be formulated, both their "syntax" and proof theory. There are four basic judgements.

t Type	t is a type
s:t	s belongs to type t
t Prop	t is a proposition
$t \ (\text{or} \ t \ True)$	t is true

Theories can be formulated in this system using sequent style rules. We use a context (Γ) to simplify the presentation of rules that involve discharged assumptions. Formation rules are used to specify the grammar of a theory. For example, we can give the formation rules for conjunction and negation in a propositional logic.

$$\frac{\Gamma \vdash s \operatorname{Prop} \ \Gamma \vdash t \operatorname{Prop}}{\Gamma \vdash (s \land t) \operatorname{Prop}} \land^{F} \qquad \frac{\Gamma \vdash t \operatorname{Prop}}{\Gamma \vdash \neg t \operatorname{Prop}} \neg^{F}$$

⁹The structured-meanings perspective seems better suited to our aim of avoiding ontological reductions. It may be possible to give a non-reductive account based on some of the insights of the set-theoretic approach. We could seek to define an answerhood relation that correspond to an answer "belonging to" a question. We might hope that this would turn out to be consistent with a non-reductive theory that takes structured meanings as a starting point.

Rules governing the "logical" behaviour of such expressions can be given in terms of judgements of truth. These are formulated with constraints that ensure they only apply to the appropriate kinds of entities. We can exemplify this with introduction and elimination rules for conjunction and negation of propositions.

$$\frac{\Gamma \vdash s \operatorname{Prop} \Gamma \vdash t \operatorname{Prop} \Gamma \vdash s \Gamma \vdash t}{\Gamma \vdash s \wedge t} \wedge^{+} \frac{\Gamma \vdash s \operatorname{Prop} \Gamma, s \vdash \bot}{\Gamma \vdash \neg s} \neg^{+} \frac{\Gamma \vdash s \operatorname{Prop} \Gamma, s \vdash \bot}{\Gamma \vdash s} \neg^{-}$$

For many systems, the formation rules — which generate the wellformed expressions, including propositions — are *independent* of the rules governing other kinds of judgements. Such systems include those for which meaning-independent notion of syntax can be specified. But TPL also allows us to express formation rules that depend upon truth judgements, rather than on purely "syntactic" notions. We can exemplify this with a weak form of material implication, that only forms a proposition if the antecedent is true.

$$\frac{\Gamma \vdash s \ \mathsf{Prop} \ \ \Gamma, s \ \mathsf{True} \vdash t \ \mathsf{Prop}}{\Gamma, s \ \mathsf{True} \vdash (s \to t) \ \mathsf{Prop}} \to'^F$$

TPL allows us to define various type systems. As an example, we can present a version of Simple Type Theory (Church, 1940), with entities e, propositions p, and functions $\langle S, T \rangle$ from S to T.

$$\label{eq:relation} \begin{array}{c} \overline{\Gamma \vdash e \ \mathsf{Type}} & \overline{\Gamma \vdash p \ \mathsf{Type}} \\ \\ \overline{\Gamma \vdash S \ \mathsf{Type}} & \overline{\Gamma \vdash T \ \mathsf{Type}} \\ \\ \overline{\Gamma \vdash \langle S, T \rangle \ \mathsf{Type}} & \langle \cdot, \cdot \rangle^F & \frac{\Gamma \vdash f : \langle S, T \rangle \ \ \Gamma \vdash a : S}{\Gamma \vdash fa : T} \end{array} \circ$$

More complex types (and data structures) can be defined.

3.2 Questions and Answers in Typed Predicate Logic

We can represent questions using expressions of the form $[x : T | \phi]$. This is reminiscent of the notation of "schema" in computer science. The type T indicates what type of term answer is appropriate. The proposition ϕ must be satisfied by any constituent answer that satisfies the specification. This can be a distinct representation for questions — no ontological reduction is required.¹⁰

¹⁰Polymorphic typing can be used to account for the systematic behaviour of embedded interrogatives of distinct types (cf. Fox & Lappin, 2005, chapter 5).

$$\frac{\Gamma \vdash T \text{ Type}}{\mathsf{Quest}(T) \text{ Type }} \ \operatorname{\mathsf{Quest}}(T)^F \quad \frac{\Gamma \vdash T \text{ Type } \quad \Gamma, x: T \vdash \phi \text{ Prop}}{\Gamma \vdash [x:T \mid \phi]: \operatorname{\mathsf{Quest}}(T)} \ \left[\cdot \mid \cdot \right]^F$$

For answers, we adopt a form of structured proposition. We can represent structured propositions as $\langle f | t \rangle$, with a topic t and focus f. This can be kept distinct from the notion of a pair.

$$\frac{\Gamma, x: T \vdash t(x) \operatorname{Prop} \quad \Gamma \vdash f: T}{\Gamma \vdash \langle f \mid t \rangle \operatorname{Prop}} \quad \langle \cdot \mid \cdot \rangle^{F}$$

$$\frac{\Gamma, f: T \vdash t(f) \operatorname{Prop} \quad \Gamma \vdash a: T \quad \Gamma \vdash t(f)}{\Gamma \vdash \langle f \mid t \rangle} \quad \langle \cdot \mid \cdot \rangle^{+}$$

$$\frac{\Gamma, f: T \vdash t(f) \operatorname{Prop} \quad \Gamma \vdash f: T \quad \Gamma \vdash \langle f \mid t \rangle}{\Gamma \vdash t(f)} \quad \langle \cdot \mid \cdot \rangle^{-}$$

We can now introduce a relation, ans, that makes a well formed proposition between question and putative answer if they are of the appropriate nature. The judgement that a is a *potential* answer to a question q is then captured by the well-formedness judgement of a ans q — that is (a ans q) Prop.

$$\frac{\Gamma, x: T \vdash t \ \operatorname{Prop} \ \ \Gamma \vdash a: T}{\Gamma \vdash (\langle a \mid \lambda x.t \rangle \ \operatorname{ans} [x:T \mid t]) \ \operatorname{Prop} \ } \ {}^{\operatorname{ans} F}$$

This is the canonical propositional case. It would need to be generalised to include cases where the propositional part of the answer is "congruent" with the question, or is left unstated. If we have no analysis of ellipsis, this latter case could be approximated by allowing the topic in the answer to be optional.

If a is a *potential* answer to a question q, then we can derive wellformedness judgement, (a ans q) Prop. For it to be a *true* (or *correct*) answer, we need to be able to derive the judgement (a ans q) True (cf. Karttunen, 1977).

$$\frac{\Gamma, x: T \vdash t \ \operatorname{Prop} \quad \Gamma \vdash a: T \quad \Gamma \vdash t[x/a]}{\Gamma \vdash (\langle a \mid [\lambda x.t] \rangle \text{ ans } [x:T \mid t]) \ \operatorname{True}} \ \operatorname{ans}^+ \\ \frac{\Gamma, x: T \vdash t \ \operatorname{Prop} \quad \Gamma \vdash a: T \quad \Gamma \vdash (\langle a \mid [\lambda x.t] \rangle \text{ ans } [x:T \mid t]) \ \operatorname{True}}{\Gamma \vdash t[x/a]} \ \operatorname{ans}^-$$

It is worth observing that this framework also allows us capture fine-grained intentionality without possible worlds. Questions that have the same (possible) answers need not be equated. This is in part because we maintain an ontological distinction between questions and their (possible) answers.¹¹

4 Summary

The claim being made here is not that it is possible or appropriate to dispense with all of the meta-theoretic narrative that accompanies any well-constructed formal semantic analysis. Rather, the argument is that those aspects of the narrative that seek to apply ontological classifications and distinctions ought to feature in the formal analysis, and that they can do so given an appropriate formalism. This avoids conflating the interpretations and patterns of behaviour of expressions that can result from reducing all formal meaning to the language of set theory and simple types.

In particular, the formal theory sketched here (Section 3.2) illustrates that the notions of "being a question", and of "being an answer to a question", *can* be captured as first-class judgements *within* a formal analysis. It does not *prohibit* us from making ontological reductions if we choose. For example, (structured) propositions and questions could be given the same term representation, or the topic–focus structure could be expressed in terms of a type-membership judgement.

We can avoid conflating the interpretations and patterns of behaviour of expressions that can result from reducing all formal meaning to the language of set theory and simple types. If ontological reductions are to be made, it should through choice, not because they are imposed by the limitations of a particular semantic framework, or methodology.

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 $^{^{11}\}mathrm{TPL}$ similarly allows us to maintain fine-grained intensionality for propositions.

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