How does CPPI perform against the simplest guarantee strategies?

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Abstract

Capital protected structured products are popular with both investors and investment banks. A number of strategies ranging in complexity and cost exist that provide a minimum guaranteed payoff at maturity. In this paper the performance of Constant Proportion Portfolio Insurance (CPPI), a major strategy in the market, is evaluated against two simple strategies: a risk-free and a gapless investment. The CPPI strategy is general, allowing for discrete monitoring of trading ranges, ratchet features and leverage constraints. The risky asset is modelled as an asymmetric GARCH process. The CPPI's performance against these simple strategies is found to be inferior in the majority of cases and deteriorates further with the inclusion of management fees and costs. Moreover, under various risk adjusted performance ratios the CPPI is dominated by the simple gapless (buy-and-hold) strategy.

Keywords: CPPI, Constant Proportion Portfolio Insurance, management fees, discrete trading, GARCH, gap risk, return guarantees, capital guarantees.

JEL: C15, G11

1 Introduction

Risk averse investors are demanding greater security for their investments given the recent turbulence in financial markets. Portfolio insurance strategies (both static and dynamic) provide investors with the certainty of a guaranteed amount at maturity.[1] For the issuer dynamic strategies introduce the risk that the portfolio value will be below the guaranteed amount at maturity. This is known as gap risk. Under the general conditions that the investor is assumed to be risk averse and demands that a proportion (usually 100%) of their initial capital be returned at maturity, there are a number of strategies that may be considered. Simple buy-and-hold strategies that invest only the difference between the initial capital and the discounted guarantee value in the risky asset have no gap risk, either from stock price uncertainty or interest rate risk. Of course a pure risk-free

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[1] It is assumed that there is no counterparty risk for the investor i.e. no risk that the issuer will default on their obligation to pay the investor.

investment by its definition carries no risk, but also has no market participation. Constant Proportion Portfolio Insurance (CPPI) on the other hand (see Perold (1986), Black and Jones (1987) and Perold and Sharpe (1988)) is a dynamic strategy that has the potential to produce greater payoffs, but at the expense of introducing some gap risk. Although by no means a recent development, CPPI's popularity in the current structured funds market is significant, as noted by Pain and Rand (2008).

Option Based Portfolio Insurance (OBPI) is another popular strategy that exposes the seller to no gap risk. Introduced by Leland and Rubinstein (1976), the OBPI works by investing enough capital in a bond so that it grows to equal the guaranteed amount at maturity, with the remaining capital invested in options. The performance of the OBPI may also be replicated using only the underlying risky asset and bonds in a strategy known as a synthetic put. However, unlike the CPPI which gives the investor control over the exposure and risk through the multiplier value, the synthetic put's composition is entirely defined through delta hedging according to the Black-Scholes model.

The original CPPI model assumes that the risky asset follows a geometric Brownian motion (GBM), but stylized facts state the existence of fat tails, jumps and volatility clustering. The implications of such effects have been investigated in the literature. Bertrand and Prigent (2002) apply extreme value theory to determine an appropriate multiplier. Cont and Tankov (2007) find that although jumps in the asset price contribute greatly to gap risk, when using parameters estimated from actual stock returns the risk is low. Applications of CPPI using empirical data can be found in Herold et al. (2007) and Do and Faff (2004).

The managing of the gap risk of the CPPI is particularly important to the issuer since they must cover any shortfall at maturity. Gap risk arises when the risky asset value falls by more than a certain amount of its value before the portfolio can be rebalanced. Such conditions are apparent when there are discontinuities in price movements and/or when trading is restricted to discrete points in time. Balder et al. (2009) provide a comprehensive evaluation of the CPPI under discrete trading assuming the risky asset follows a GBM. This paper also adopts a discrete trading framework, but through the application of rebalancing triggers which are generally employed in practice.

Typically under discrete trading, rebalancing of the portfolio is performed at fixed predetermined intervals e.g. daily, weekly, monthly etc. Although this approach can be useful, particularly in comparing discrete models as approximations to their continuous time counterparts, in reality such rigid application of trading on fixed calendar days is not generally practiced (unless the risky asset has trading restrictions placed on it). Subject to transaction costs, it is desirable to rebalance as little as possible (incur as little costs as possible) while keeping the portfolio composition within a reasonable proximity of the model. Models restricting rebalancing of the CPPI to price movement triggers are considered in Do and Faff (2004) and Hamidi et al. (2009). Beyond the reduction in transaction costs, the implications of rebalancing triggers to risk and performance are also explored in this paper.

Under the standard assumptions, the floor of the CPPI grows at a constant risk-free rate to equal the guarantee at maturity. Even if the assumption of a constant risk-free rate is kept, a stochastic floor may be introduced whereby the floor value at anytime is partially dependent on the performance of the portfolio (and ultimately the risky asset). The use of such mechanisms are common in practice and often referred to as *ratchets* since they involve increasing the value of the floor in reaction to strong performance, but never reducing it. Stochastic floors have been introduced in the literature in Boulier and Kanniganti (1995) and extended by Mkaouar and Prigent (2007), with the former citing an improvement in performance using ratchets when leverage constraints are imposed.

This paper analyses for the first time the performance of a general CPPI strategy which has rebalancing triggers, ratchet effects and constraints on borrowing (leverage) against a gapless and risk-free investment, where the risky asset follows a ARMA-GJR-GARCH process. By considering these factors, together with transaction costs and management fees, this paper is able to provide a more accurate appraisal of the CPPI with respect to how it is actually implemented in practice. Additionally, from the perspective of the buyer, the benchmarking of the CPPI against the simplest strategies provides a valuable assessment of the CPPI against the more transparent and cost effective alternatives. The paper is organised as follows: Section 2 describes the models used, Section 3 presents the results and Section 4 concludes.

2 Models

2.1 Conditional Volatility Asset Price Model

Fat tails and volatility clustering are two stylized facts that are apparent in stock returns that cannot be captured by IID processes such as geometric Brownian Motion. A standard GARCH process is able to capture time varying volatility and excess kurtosis. In particular, if a Student-t distribution is used to model the returns distribution then higher levels of kurtosis can be achieved.

Empirical evidence suggests that negative returns are more commonly followed by periods of high volatility than equal sized positive returns. This asymmetry has been captured by a number of GARCH-based models, most notably the GJR and Exponential GARCH models. The GJR model (see Glosten et al. (1993)) is essentially the same as that of the standard GARCH, except for an additional term that captures and assigns extra weight to negative returns. In addition to the volatility of the process, the conditional mean is also important and typically captured using an ARMA model (see e.g. Rachev et al. (2007)). An ARMA(k,v)-GJR-GARCH(p,q) model is defined as

$$y_t = \mu + \sum_{i=1}^{k} a_i y_{t-i} + \sum_{j=1}^{v} b_j \epsilon_{t-j} \tag{1a}$$

$$\epsilon_t = \sigma_t \eta_t \tag{1b}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2 + \sum_{j=1}^{q} \gamma_j \epsilon_{t-j}^2 I_{t-j}(\epsilon_{t-j} < 0) \tag{1c}$$

under the conditions: $\alpha_0 \geq 0,\ \alpha_j \geq 0,\ \beta_i \geq 0,\ \alpha_j + \gamma_j \geq 0$

$$\sum_{i=1}^{p} \beta_i + \sum_{j=1}^{q} \alpha_j + \frac{1}{2} \sum_{j=1}^{q} \gamma_j < 1,$$

where y_t is the daily log-return, ϵ_t is the residual at time t, σ_t^2 is the conditional variance and α_i and β_j are the weightings given to the last period's realised volatility and forecast volatility respectively. The conditional mean and variance constants are μ and α_0 respectively. The binary indicator function is given by I and γ_j is the weighting assigned to the asymmetric term, with η_t a sequence of t-distributed random variables with zero mean and unit variance.

2.2 Discrete CPPI

In this paper it is assumed that the portfolio is monitored in discrete time. The following convention is adopted: a horizon T and $n+1$ equidistant points: $\{0 = t_0 < t_1 \ldots < t_{n-1} < t_n = T\}$, such that $t_{k+1} - t_k = \frac{T}{n}$ for $k = 0, \ldots, n-1$. Where n is the number of times the price of the risky asset is observed after the initial construction of the portfolio.

CPPI rebalances capital between a risky asset S_{t_k} and a risk-free asset. A floor F_{t_k} is calculated by discounting back from maturity T the guarantee amount G_0V_0 at the risk-free rate r, with V_0 being the initial capital and G_0 the percentage of that amount initially guaranteed at maturity. The time subscript on G is required in cases where ratchets are applied and the guarantee level may increase over time. The risk-free asset follows the same dynamics as the floor i.e. it also grows at the constant rate r. The cushion C_{t_k} is defined as the difference between the portfolio value V_{t_k} and the floor. The exposure to the risky asset E_{t_k} is defined as a multiple m of the cushion, whereby a higher m results in a greater exposure. The remainder of the capital B_{t_k} is invested in the risk-free asset. These basic relationships in the CPPI can be described by

$$V_{t_k} = F_{t_k} + C_{t_k} \tag{2}$$
$$F_{t_k} = F_0 e^{rt_k} \tag{3}$$
$$E_{t_k} = mC_{t_k} \tag{4}$$
$$B_{t_k} = V_{t_k} - E_{t_k}. \tag{5}$$

The progression of the CPPI is driven by changes in the risky asset price S_{t_k}. When S_{t_k} increases at a rate greater than r, then more capital is invested in the risky asset by selling some of the risk-free asset. If S_{t_k} grows at a rate less than r then some of the risky asset is sold and the proceeds invested in the risk-free asset. Thus V_{t_k} can be expressed in terms of S_{t_k} as (see Balder et al. (2009) for derivation)

$$V_{t_k} = \begin{cases} F_{t_k} + (V_{t_{k-1}} - F_{t_{k-1}})\left(m\frac{S_{t_k}}{S_{t_{k-1}}} - (m-1)e^{r\frac{T}{n}}\right) & \text{if} \quad V_{t_{k-1}} > F_{t_{k-1}} \\ V_{t_{k-1}} e^{r\frac{T}{n}} & \text{if} \quad V_{t_{k-1}} \leq F_{t_{k-1}}. \end{cases} \tag{6}$$

Note that Equation (6) exhibits a clear path dependence for the value of the CPPI in discrete time provided the floor has not been hit.

In the standard CPPI implementation there are no restrictions on either shorting the risky asset or on borrowing additional funds at the risk free rate. In practice however, there are restrictions in place. Constraints are imposed on the CPPI to prevent shorting of the risky asset. This is achieved by ensuring that the exposure never becomes negative:

$$C_{t_k} = \max[(V_{t_k} - F_{t_k}), 0]. \tag{7}$$

Additionally, constraints are placed on the amount of leverage (borrowing) that may be used. Defining h as a multiple limiting the maximum exposure allowed, the value of the exposure is restricted as follows

$$E_{t_k} = \min[mC_{t_k}, hV_{t_k}]. \tag{8}$$

Therefore $h = 1$ implies that the portfolio is not levered whilst $h = 2$ allows a maximum leverage of 100% i.e. up to 100% of the portfolio's current value may be borrowed at the risk-free rate r.

Although the CPPI performs well in periods of strong growth (see e.g. Perold and Sharpe (1988)) it is possible that during the lifetime of the investment any gains previously made may be lost if the risky underlying subsequently falls in value. To protect previous gains, a ratcheting mechanism may be applied that raises the guarantee by a certain percentage in reaction to a certain percentage rise in the value of the portfolio. Essentially, the exposure is reduced by an amount which is then invested into the risk-free asset. [2] Such a ratcheted floor can be described as

$$\lambda_{t_k} = \max\left(\text{floor}\left\{\frac{\ln(V_{t_k}/V_0)}{\ln(1+\nu)}\right\}, \lambda_{t_{k-1}}\right) \tag{9a}$$

$$G_{t_k} = G_0 + \lambda_{t_k}\xi \tag{9b}$$

$$F_{t_k} = V_0 G_{t_k} e^{-r(T-t_k)}, \tag{9c}$$

where ν is the percentage increase in the portfolio value that triggers the ratchet and ξ is the percentage increase applied to the guarantee. λ is the number of *clicks* of the ratchet applied up to time t_k and initially $\lambda_0 = 0$.

2.3 Rebalancing Triggers and Costs

Rebalancing the portfolio at a set number of equidistant points e.g. daily, weekly or monthly, may be inefficient when transaction costs are involved. Trading excessively wastes capital via transaction costs while trading too infrequently increases gap risk. Therefore monitoring S and rebalancing the portfolio only when it has deviated by a certain amount could be a better approach. The affect that changes in S have on the portfolio composition and risk depends on the model specification and m. Hence it is appropriate to define bounds of tolerance around the value of m that is implied by the portfolio composition at the current time. The implied m value is defined as

$$m_{t_k}^{\text{imp}} = \frac{E_{t_k^-}}{C_{t_k}} = \frac{m}{m-(m-1)\Xi_{t_k}}, \tag{10}$$

where

$$\Xi_{t_k} = \frac{S_{t_{k-x}} e^{r\frac{T}{n}x}}{S_{t_k}}$$

and $E_{t_k^-}$ is the value of the exposure at t_k after it has realised the return for that period, but before the portfolio has been rebalanced according to Equation (4). The number of

[2] Alternatively, the guarantee can be left unchanged and the holder may receive a coupon payment instead.

periods since the portfolio was last rebalanced according to m is denoted by x. The upper m^u and lower m^l rebalancing trigger points are thus defined by

$$E_{t_k} = \begin{cases} E_{t_k^-} & \text{if} \qquad m^l \leq m_{t_k}^{imp} \leq m^u \\ mC_{t_k} & \text{otherwise,} \end{cases} \tag{11}$$

ensuring rebalancing of the portfolio only occurs when the trigger points are hit. Note that if $m_{t_k}^{imp}$ increases, this means the value of the risky asset has fallen and vice versa. Furthermore, if $m_{t_k}^{imp} < 0$ then the floor has been hit and the portfolio becomes fully invested in the risk-free asset till maturity. Figure 1 illustrates the effect on m^{imp} when the value of the risky asset changes. For example if $m^{imp} = m = 4$, then a rise in the risky asset of ∼12.5% reduces m^{imp} to 3, while a fall in the risky asset of ∼6.25% increases m^{imp} to 5. To give an idea of the likelihood of such price movements occurring, the maximum daily gain and loss on FTSE 100 returns for the period considered is 5.9% and 5.6% respectively. The complete summary statistics for the period is presented in Table 9 in the Appendix.

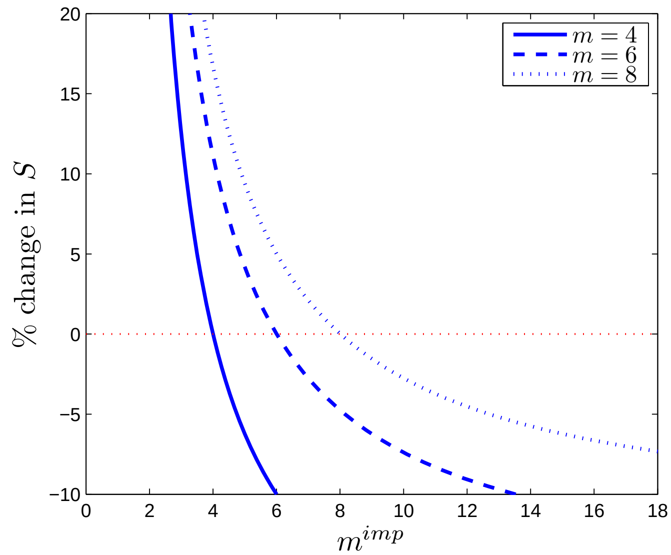

Figure 1: Implied m values

Proportional transaction costs are applied to trades resulting from rebalancing the CPPI strategy. These costs are calculated by

$$TC_{t_k} = |E_{t_k^-} - E_{t_k}|\kappa \qquad 0 < \kappa < 1, \tag{12}$$

where κ is the percentage transaction cost applied to the trade. Transaction costs are deducted from the exposure amount only when the portfolio is actually rebalanced i.e. when a rebalancing trigger is hit. In the case that the floor is breached, then all of the risky asset is sold and no further transactions take place and hence no further costs are incurred.

Investors of CPPI structured notes are charged management fees which partly cover the issuer for gap risk. These management fees are calculated and deducted on the portfolio as follows

$$\phi_{t_k} = \begin{cases} \Phi\frac{T}{n}V_{t_k} & \text{if} \quad V_{t_k}(1-\Phi\frac{T}{n}) \geq F_{t_k} \\ 0 & \text{if} \quad V_{t_k}(1-\Phi\frac{T}{n}) < F_{t_k}, \end{cases} \tag{13}$$

where Φ is the annualised percentage fee. It can be seen that a fee is only deducted every period if it will not result in the floor being violated. The fee is deducted only from the risky asset exposure amount.

2.4 Riskfree / Gapless Strategies

There are two simple strategies that guarantee a minimum payout at maturity without any risk. The first is the riskless investment which requires that all capital is invested in a risk-free bond. The second is the gapless portfolio which is a buy-and-hold strategy that is the same as a CPPI with a multiplier value of 1 i.e. the discounted value of the guarantee is invested in a risk-free bond and the remainder in the risky asset. The simplicity and transparency of these strategies means that they are subject to very little (if any) transaction and management costs. The terminal values of the risk-free V_T^{rf} and gapless $V_T^{m=1}$ portfolios are therefore

$$V_T^{rf} = V_0 e^{rT} \tag{14}$$

$$V_T^{m=1} = G_0 V_0 + (V_0 - F_0)\frac{S_T}{S_0}. \tag{15}$$

It is against these two simple strategies that the CPPI is compared.

3 Results

3.1 Experiment Design

The price path of the risky asset has been simulated using an ARMA(1,1)-GJR-GARCH(1,1) process over 1260 innovations, representing daily prices over a 5 year maturity (i.e. $T = 5$). Two separate price series, A and B, have been created with A being generated from parameters fitted from 8 years of FTSE 100 data and B being modified from A to have twice the expected return and volatility as A. A risk-free rate $r = 1.5\%$ is associated with Price Series A and $r = 3\%$ with B. Further details about the simulations can be found in the Appendix. For use in subsequent sections, the following are defined:

$$E[L_T] = E[G_T - V_T | V_T < G_T] \tag{16}$$

$$Pr[L_T] = Pr[V_T < G_T] \tag{17}$$

$$V_T^b = \max(V_T, G_T), \tag{18}$$

where $E[L_T]$ and $Pr[L_T]$ are the expected loss and percentage of losses observed respectively. V_T^b is the terminal portfolio value as obtained by the buyer i.e. with the guaranteed minimum payout in place. Throughout the tables in this results section $M[\cdot]$ represents the median value. This paper places an emphasis on the comparison of the CPPI with the gapless and riskless portfolios for each possible realisation of a price path. In all of the numerical simulations it is assumed that $V_0 = 1$ and $G_0 = 100\%$.

To gain an additional insight into the performance of the CPPI, especially in relation to the gapless portfolio, a number of performance ratios have been considered. The first is the Sharpe ratio (see e.g. Sharpe (1994)), which is the volatility adjusted excess return and defined as

$$Sharpe\ Ratio = \frac{E[R_T^b - rT]}{\sqrt{\mathrm{var}[R_T^b]}}, \tag{19}$$

where $R_T^b = \ln(V_T^b/V_0)$ i.e. the log-return on the terminal portfolio value from the buyer's perspective. The second measure employed is the Omega ratio (see e.g. Shadwick and Keating (2002)), defined as

$$Omega\ Ratio = \frac{E[\max(R_T^b - rT, 0)]}{E[\max(rT - R_T^b, 0)]}. \tag{20}$$

This is the expected gain above the threshold value rT divided by the expected loss below the threshold. The third measure is the Sortino ratio (see e.g. Sortino and Price (1994)). This measures the excess expected return over some minimum acceptable return (rT) and penalises for deviations below it:

$$Sortino\ Ratio = \frac{E[R_T^b - rT]}{DR}, \tag{21}$$

where DR is the downside risk of the target semideviation and is defined as

$$DR = \sqrt{E[\max(rT - R_T^b, 0)^2]}. \tag{22}$$

The final performance measure used is the Upside Potential Ratio (see e.g. Sortino et al. (1999)), which measures the expected excess return over rT while penalising for deviations below it. It is defined as

$$UPR = \frac{E[\max(R_T^b - rT, 0)]}{DR}. \tag{23}$$

The threshold level in all of the previously defined performance measures has been chosen as the terminal riskless investment value rT to give a point of comparison that is applicable to both the CPPI and gapless portfolios. Since these two strategies have a guaranteed return of 0, rT is the logical return to use for to compare them against.

3.2 Rebalancing Triggers and Multiplier Value

In this section the effect of trading bounds ([m^l,m,m^u]) on performance and risk is investigated. Table 1 gives the results of applying various trading bounds to the standard CPPI for price series A and B. It is shown that suitable values for the triggers can produce an enhanced performance when compared to daily trading ([4,4,4]). In particular in price series A, [2,4,6] produces higher mean and median values than daily trading, although at the cost of a greater number of floor violations. However, considering that on average the portfolio is rebalanced 2.6 times in 5 years against the daily number of 1259.9, the reduction in trading is very significant. For price series B, once again the [2,4,6] outperforms the mean and median values of both gapless and riskless portfolios. In fact its payoff is significantly better than daily trading while the number of trades at 51.4 is relatively modest. As with price series A, the number of floor violations is approximately

Table 1: Standard CPPI under various trading bounds for price series A and B.

	m^l	m	m^u	$E\left[\frac{V_T^b}{V_T^{m=1}}\right]$	$M\left[\frac{V_T^b}{V_T^{m=1}}\right]$	$E\left[\frac{V_T^b}{V_T^{rf}}\right]$	$M\left[\frac{V_T^b}{V_T^{rf}}\right]$	$Pr[L_T]$ (%)	$E[L_T]$ (bp)	Trades
	4	4	4	1.017	0.984	1.028	0.992	0.013	5.85	1259.9
	3	4	5	1.026	0.995	1.038	1.003	0.014	6.68	13.7
A	2	4	6	1.022	1.004	1.033	1.013	0.018	4.39	2.6
	3	4	4	1.023	0.996	1.035	1.004	0.011	7.67	85.5
	4	4	5	1.027	0.986	1.038	0.995	0.015	5.42	218.2
	4	4	4	1.046	0.903	1.124	0.902	0.260	9.17	1258.3
	3	4	5	1.080	0.920	1.161	0.926	0.279	10.70	210.2
B	2	4	6	1.080	0.941	1.159	0.961	0.341	12.44	51.4
	3	4	4	1.071	0.922	1.151	0.932	0.229	10.08	314.0
	4	4	5	1.067	0.905	1.148	0.901	0.310	10.32	529.5

33% higher than with daily trading. The asymmetric bound [3,4,4] can be seen to offer a significant reduction in the number of floor violations in both series A and B, while still outperforming daily trading. This follows intuitively since setting $m^u = m$ implies intolerance to any increase in risk of hitting the floor arising from a decline the risky asset.

The rebalancing triggers give significant improvement in the Sharpe ratio over daily trading, particular for the wider bounds of [2,4,6], as shown in Table 2. A comparison to the Sharpe ratio of the gapless portfolio, shown in Table 3, highlights a considerable difference in the values of the two strategies. This indicates that although the CPPI has a substantially better mean payoff, when compensated for volatility risk, it is considerably worse. Additionally, while the CPPI's Sharpe ratio declines significantly from price series A to B, the gapless portfolio's Sharpe ratio increases, furthering its case as a preferred alternative to the CPPI. Table 3 also illustrates that the gapless portfolio performs better than the riskless strategy. Note that the Sharpe ratio on S under both price series A and B remains constant at 0.127.

Table 2 gives the Sharpe, Omega, Sortino and Upside Potential (UP) ratios for the CPPI, with rT as the reference level. Once again this is from the buyer's perspective i.e. the minimum return is zero. The results show that the use of rebalancing triggers can have a strong positive effect on the CPPI, increasing the value of all of the performance ratios. With the exception of the Upside Potential ratio (and the Sortino ratio for [2,4,6]) which increases, the trend is that the values of the performance ratios are worse under price series B than A. This is due to the increase in volatility, which has a detrimental effect on the performance of the strategy. Comparing Table 2 with the gapless portfolio in Table 3, it can be seen that in the majority of cases the performance ratios of the gapless portfolio dominate that of the CPPI under price series A. Under price series B the results are more dramatic with the gapless portfolio significantly outperforming the CPPI under all performance ratios.

3.3 Ratchets

The results for the ratcheted CPPI are presented in Table 4 for both price series A and B and parameter values $\nu = 10\%$ and $\xi = 3\%$ i.e. a 10% rise in the portfolio value cause an increase in the guarantee of 3%. Comparing these results to the standard CPPI in

Table 2: Performance ratios of the CPPI for price series A and B.

	m^l	m	m^u	*Sharpe*	*Omega*	*Sortino*	*UPR*
	4	4	4	0.233	2.048	0.686	1.340
	3	4	5	0.325	2.718	1.051	1.664
A	2	4	6	0.361	2.771	1.018	1.593
	3	4	4	0.325	2.703	1.028	1.632
	4	4	5	0.285	2.446	0.934	1.580
	4	4	4	0.183	1.718	0.563	1.347
	3	4	5	0.272	2.253	0.929	1.670
B	2	4	6	0.316	2.592	1.109	1.805
	3	4	4	0.264	2.221	0.897	1.633
	4	4	5	0.221	1.916	0.712	1.488

Table 3: Performance of gapless portfolio.

	$E\left[\frac{V_T^{m=1}}{V_T^{rf}}\right]$	$M\left[\frac{V_T^{m=1}}{V_T^{rf}}\right]$	*Sharpe*	*Omega*	*Sortino*	*UPR*
A	1.010	1.009	0.464	3.308	1.037	1.486
B	1.051	1.036	0.530	6.931	2.561	2.993

Table 1 it can be seen that that the ratchets sacrifice mean performance for an increase in median performance. From the buyer's perspective under price series A, the ratcheted CPPI can be considered a desirable strategy since it outperforms the riskless and gapless strategies for both the mean and the median ratios for the rebalancing triggers [3,4,5] and [2,4,6]. Under the more volatile price series B however, although the median performance is somewhat improved over the unratcheted CPPI, it still significantly lags the riskless and gapless alternatives. Note that the ratios include the minimum guarantee payout, the expected value of which is given in the final column of Table 4.

From the seller's perspective it can be seen that impact on gap risk is negligible with the number of losses and the expected loss the same as for the unratcheted strategy. This is as expected since ratcheting the floor should not effect the risk profile of the portfolio in terms of floor violations.

Table 4: Ratcheted CPPI with $\nu = 10\%$ and $\xi = 3\%$ for price series A and B.

	m^l	m	m^u	$E\left[\frac{V_T^b}{V_T^{m=1}}\right]$	$M\left[\frac{V_T^b}{V_T^{m=1}}\right]$	$E\left[\frac{V_T^b}{V_T^{rf}}\right]$	$M\left[\frac{V_T^b}{V_T^{rf}}\right]$	$Pr[L_T]$ (%)	$E[L_T]$ (bp)	$E[G_T]$
	4	4	4	1.014	0.992	1.025	1.000	0.013	5.51	102.45%
A	3	4	5	1.022	1.002	1.034	1.010	0.014	4.62	102.65%
	2	4	6	1.020	1.005	1.030	1.014	0.018	5.03	102.35%
	4	4	4	1.024	0.928	1.095	0.940	0.267	7.48	108.11%
B	3	4	5	1.059	0.950	1.134	0.966	0.284	7.91	109.15%
	2	4	6	1.060	0.971	1.132	0.993	0.344	10.04	108.86%

Table 5: Fixed interval rebalancing under price series A.

Rebalancing Frequency	$E\left[\frac{V_T^b}{V_T^{m=1}}\right]$	$M\left[\frac{V_T^b}{V_T^{m=1}}\right]$	$E\left[\frac{V_T^b}{V_T^{rf}}\right]$	$M\left[\frac{V_T^b}{V_T^{rf}}\right]$	$Pr[L_T]$ (%)	$E[L_T]$ (bp)	Trades
Daily	1.017	0.984	1.028	0.992	0.013	5.85	1259.9
Weekly	1.025	0.990	1.036	0.998	0.162	21.19	251.8
Monthly	1.031	0.994	1.042	1.002	1.122	84.67	59.7
Quarterly	1.033	0.996	1.045	1.004	3.787	177.47	19.7
Yearly	1.035	1.005	1.047	1.013	8.095	293.73	4.9

3.4 Rebalancing Triggers vs. Fixed Trading

The performance and risk from applying fixed trading intervals is compared against that achieved through the use of rebalancing triggers in this section. From Table 5 it is evident that as the trading frequency is reduced, the mean and median performance of the portfolio increases along with the risk. In comparison to the price series A section of Table 1 it is clear that although the trading rule cannot match infrequent fixed interval trading in terms of upside performance, it can in the number of trades. Crucially however, the trading rule is drastically more effective in managing risk. These results indicate that the buyer would favour a portfolio that was rebalanced infrequently, while for the seller this would introduce additional gap risk.

3.5 Costs and Leverage

The impact of the various costs associated with a realistic application of the CPPI is explored in this section. The first part of Table 6 gives the performance and risk when a typical annual management fee of 1.5% is applied. It is clear that the impact of the fee causes the portfolio to encounter many floor violations, although the magnitude of these violations is more than 100 times smaller than usual. This lower expected loss results because the application of the fee causes the portfolio value to decline and be very close to the floor. The exposure amount is then very small and when a large drop in the risky asset price occurs, the resulting breach beyond the floor is small. Considering that there is a risk-free rate of 1.5% and expected return of 3.3% a year it is not surprising that deducting the fee heavily erodes performance.

The effects of the application of a 0.5% proportional transaction cost are displayed in the second section of Table 6. As expected the transaction costs have more of a negative effect when the portfolio is rebalanced more frequently.

In the final section of Table 6 the CPPI has an increased maximum leverage of 100% over the portfolio value. This is shown under price series B to give the strategy a greater opportunity to exploit larger increases in the risky asset. Comparing these results to those in Table 1, it can be seen that leverage has the effect of increasing the mean payoff at the expense of the median. This is because the additional leverage is utilised on those price path realisations where there is strong growth. The gap risk is largely unaffected by the use of leverage.

Table 6: Management fees, transaction costs and leverage applied to the CPPI individually under: price series A for $\Phi = 1.5\%$ and $\kappa = 0.5\%$; price series B for $h = 2$.

	m^l	m	m^u	$E\left[\frac{V_T^b}{V_T^{m=1}}\right]$	$M\left[\frac{V_T^b}{V_T^{m=1}}\right]$	$E\left[\frac{V_T^b}{V_T^{rf}}\right]$	$M\left[\frac{V_T^b}{V_T^{rf}}\right]$	$Pr[L_T]$ (%)	$E[L_T]$ (bp)
	4	4	4	0.945	0.934	0.955	0.928	5.554	0.02
$\Phi = 1.5\%$	3	4	5	0.950	0.937	0.960	0.928	4.895	0.03
	2	4	6	0.950	0.938	0.959	0.928	4.665	0.02
	4	4	4	0.981	0.960	0.991	0.967	0.016	4.97
$\kappa = 0.5\%$	3	4	5	1.021	0.990	1.032	0.998	0.016	5.89
	2	4	6	1.019	1.002	1.030	1.010	0.024	3.30
	4	4	4	1.035	0.895	1.120	0.896	0.262	9.92
$h = 2$	3	4	5	1.093	0.913	1.188	0.919	0.280	8.86
	2	4	6	1.091	0.936	1.178	0.954	0.341	12.13

4 Conclusion

Structured notes that guarantee a proportion of the initial invested capital whilst allowing participation in the market are ubiquitous, with CPPI a significant strategy. This paper compares a general CPPI strategy against two simple strategies: the risk-free and gapless investments. In the majority of cases the CPPI does not outperform the simpler gapless and risk-free strategies even before the application of fees and costs, as viewed from the buyer's perspective with a minimum guaranteed payout. Even though the expected value of the CPPI is usually significantly greater than that of the gapless portfolio, once these excess returns have been adjusted for risk and volatility the CPPI loses its attractiveness in that regard. Furthermore this paper did not consider other risk factors such as interest rate and liquidity risk, both which would impact the CPPI, but not the riskless and gapless portfolios.

The application of rebalancing triggers does much to help manage the gap risk and transaction costs of the CPPI. Yet it still fails to recapture enough performance to outperform the simple strategies under most scenarios. Ratcheting shifts the performance of the CPPI from the mean to the median and under lower volatility conditions provides a superior investment to the riskless and gapless strategies in the absence of costs. However, when volatility is higher it loses this advantage. The introduction of management fees has a negative effect on what is already generally a poorly performing strategy.

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Table 7: Fitted parameter values, standard errors and t-statistics.

Parameter	Value	Standard Error	t-Statistic
μ	5.017E-05	5.750E-05	0.873
a_1	0.624	0.158	3.966
b_1	-0.688	0.146	-4.717
α_0	1.541E-06	3.100E-07	4.971
β_1	0.906	1.397E-02	64.856
α_1	0.000	1.683E-02	0.000
γ_1	0.150	2.093E-02	7.172
DoF	27.484	1.526E-04	1.801E+05

Table 8: Ljung-Box and Kolmogorov-Smirnov statistics.

	Ljung-Box (30 lags)	KS
p-value	0.239	0.113
Critical value	43.773	0.030
Q-statistic	35.096	-
KS Statistic	-	0.027

6 Appendix

6.1 Time Series Price Simulations

Two sets of artificial simulated time series have been used in this paper. Both represent daily prices for 5 years, giving 1260 innovations across 10^6 individual realisations. The first artificial time series was fitted to daily log-returns from the FTSE 100 index from 31st January 2000 to 6th February 2008 inclusively. The summary statistics for this series are given in Table 9. An ARMA(1,1)-GJR-GARCH(1,1) model with Student-t distributed innovations was used to fit the data and subsequently provide the simulations. Resources from the Matlab Garch Toolbox were used to perform these tasks. Table 7 shows the parameters and statistics obtained from fitting the series.

The Kolmogorov-Smirnov test on the standardised residuals did not reject the null hypothesis that the data is Student-t distributed with 27.682 degrees of freedom. The results for the Ljung-Box test using 30 lags and the Kolmogorov-Smirnov is shown in Table 8.

Table 9: Summary statistics for FTSE 100 daily log-returns.

Min	-0.056	Percentiles:	
Max	0.059	1%	-0.033
Mean	-3.197e-005	5%	-0.019
Median	3.941e-004	95%	0.018
Std. dev.	0.011	99%	0.030
Skewness	-0.192		
Kurtosis	6.073		

The period selected to fit the model has a relatively low mean and volatility, resulting in the simulated series having an annualised mean and volatility of 3.3% and 14.1% respectively. A second time series was created by multiplying the μ and α_0 parameters by a factor of 2 and 4 respectively, to achieve a mean and volatility double of that of the first.