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Aggregation of endogenous information in large elections[1]

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Abstract

We study aggregation of information when voters can collect information of different precision, with increased precision entailing an increasing marginal cost. In order to properly understand the incentives to collect information we introduce another dimension of heterogeneity: on top of the ideological dimension we allow for different levels of intensity in preferences. Contrary to traditional models of endogenous information, in equilibrium, there are voters collecting information of different qualities. After characterizing all symmetric Bayesian equilibria in pure strategies for arbitrary rules of election and fairly general distribution of types. We study information aggregation in symmetric electorates and show that information aggregates even when voters collect information of different qualities.

Keywords: Endogenous Information, Aggregation of Information, Heterogeneity.
JEL Codes: D71, D72, D82.

1 Introduction

The *Condorcet Jury Theorem* has attracted a lot of attention from political scientists and economists. In its original form it states that if voters have common values and report truthfully their preferences (or signals) large democracies using the simple majority rule will elect the right candidate[1]. Most of the early work has been devoted to study the case in which there are two states of nature, two candidates and voters receive an exogenous signal that is correlated with the true state of nature (Young (1988), Mueller (2003) and Berend and Paroush (1998)).

Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997) show that the assumption that voters report truthfully is only rational under very particular circumstances. If voters behave strategically and consider that their vote is only relevant when it is pivotal, they should use more information to decide how to vote besides their private signal. Nonetheless, Feddersen and Pesendorfer (1997) sow that under very general circumstances the *Condorcet Jury Theorem* holds when voters properly condition on pivotal events.

One relevant question that is seldom addressed is how voters get the information they use to vote. The larger the electorate, the smaller the probability a vote will actually affect the outcome of the election; a rational voter will then have less incentives to acquire information if this information is costly and, in the limit, every voter should be rationally ignorant. Yariv (2004) uses this intuition and assume that, when the electorate grows the signal that a voter receives is less precise. When a voter's information worsens with the size of the electorate, the speed at which the precision of the signal a voter receives decreases is crucial for information aggregation. Martinelli (2006) provides microfoundations for the results in Yariv (2004). He allows each voter to select the quality of information they use to vote assuming an increasing marginal cost for the precision of this information. His paper is the first one to study the rational ignorance hypothesis when voters can select the quality of information from a continuum of different qualities. He shows that information aggregation depends crucially on the shape of the cost function. In particular the second and third derivative when very little information is collected determines whether there will be or not information aggregation. The results go in line with the results in Yariv (2004) regarding the speed at which a voter's quality of information decreases with the size of the electorate.[2]

Martinelli (2006) provides sufficient conditions under simple majority rule that, among

[1] We focus here in non costly voting models. There is a vast literature that assumes that voting is costly for the voter. This literature discusses what is known as the "Paradox of non-voting". See Borgers (2004) for private values analysis, Krishna and Morgan (2005) for common values analysis, and Feddersen (2004) for a survey.

[2] Martinelli (2007) assumes that voters differ on the cost of information acquisition and shows that full information aggregation is possible if there is a positive probability of information acquisition being free.

other things, imply that in equilibrium every voter that decides to collect information, collects the same quality of information. As Martinelli (2006) we study information aggregation in a two state-two candidate election when voters are allowed to select the quality of information they use before deciding how to vote but we extend Martinelli (2006) in one crucial dimension.

Instead of modelling preferences as it is traditional in voting (Martinelli (2006), Yariv (2004), Feddersen and Pesendorfer (1997) and Austen-Smith and Banks (1996)) we model voter's preferences assuming preferences with two dimensions of heterogeneity. In one dimension, voters *differ on the ideological axis*: there is "right" and "left"; on the other dimension, voters *differ on the level of concern*: there are irresponsible and responsible members. These more general preferences generate voters that endogenously collect different qualities of information which is not the case in Martinelli (2006) since all informed voters are equally informed. Our main result of the paper is that, even when voters are heterogeneously informed, information aggregation under simple majority rule is possible.

The rest of the Paper is organized as follows. We present our model in the next section. Section 3 solves the model for arbitrary rules and a fairly arbitrary composition of the electorate. We discuss the incentives to collect information and vote separately before presenting the characterization and existence result. The main results of the paper are presented in Section 4 and we provide a more technical discussion of Martinelli (2006) results there. Section 5 concludes. All proofs are provided in an Appendix.

2 The model

There are n potential voters that must decide between two options A and Q. There are two states of nature ($\omega \in \{a, q\}$) and the probability of state a is given by $\phi = \Pr(a)$ which is common knowledge. Let $\mathbf{X}= \{Q, A\}$ stand for the set of available voting actions. We refer to a generic voting rule as $\mathbf{R}$.

There are three classes of voters: **responsive, partisan for** A and **partisan for** Q. Partisan voters for A (Q) always vote for candidate A (Q), while responsive voters have contingent preferences described by $\theta = \{\theta_q, \theta_a\} \in [0, 1]^2$. Let $U(d \mid \omega)$ be the utility derived from candidate $d \in \{A, Q\}$ winning in state $\omega \in \{a, q\}$. The utility that a responsive voter with preferences θ derives for different outcomes (winning candidates) is contingent in the state and can be described by the four terms $U(A \mid a) = U(Q \mid q) = 0$, $U(A \mid q) = -\theta_q$ and $U(Q \mid a) = -\theta_a$. We refer to responsive voter i's preferences (the pair θ_q and θ_a) as her type, and to a "responsive voter type θ" simply as a "type θ".

In behavioral terms using two dimensions of heterogeneity enriches the interpretation of voters' preferences. For example, when preferences are restricted to one dimension a voter who suffers a high utility loss for selecting the wrong candidate in one state suffers only a small utility loss for a mistake in the other state. By introducing the extra dimension (concern or intensity) we are able to generate voters that actually differ on the overall level of care for any type of mistakes. On the other hand, for the quality of information to significantly differ across voters for any voting rule and any level of asymmetry in the electorate it is not enough to use only ideological differences. Imagine a voter that suffers the same utility losses for electing the wrong candidate (say for example x) in any state and another one that suffers y for any mistake. If $x < y$ the first voter has little incentives to acquire information, while the more concerned voter (second one) will be more willing to invest in order to receive a highly precise signal.

Preferences are private information. A voter is responsive with probability $(1-\alpha)$ and partisan for A (Q) with probability $\alpha\xi_A$ $(\alpha(1-\xi_A))$, where $\alpha \in (0,1)$ and $\xi_A \in (0,1)$. If the voter is responsive, her preferences are drawn independently from a distribution with cumulative distribution function F on $[0,1]^2$ with no mass points. We assume that F, α and ξ_A are common knowledge.

Once nature selects a profile of types and preferences are assigned, a voter can invest in collecting information. Each responsive voter i can select $p_i \in \left[\frac{1}{2},1\right]$ where p_i is the parameter of a Bernoulli random variable S that takes values on the set $\{s_q, s_a\}$. We assume that the probability of signal $s = s_\omega$ in state $\omega \in \{a,q\}$ is equal to the parameter p_i selected by voter i:

$$\Pr(s_\omega \mid p, \omega) = p_i \text{ for } \omega \in \{a,q\}$$

The precision cost is given by $C : \left[\frac{1}{2},1\right] \rightarrow \mathcal{R}_+$ where we assume that:

Assumption 1 *The cost function C is twice continuously differentiable everywhere in $\left[\frac{1}{2},1\right]$ and satisfies 1) $C'(p) > 0$ and $C''(p) > 0$ for all $p > \frac{1}{2}$, 2) $C\left(\frac{1}{2}\right) = C'\left(\frac{1}{2}\right) = 0$, 3) $C''\left(\frac{1}{2}\right) \geq 0$, and 4) $\lim_{p\to 1} C'(p) \rightarrow \infty$.*

The last part of assumption (1) implies that there are no fully informed voters in equilibrium. $C'\left(\frac{1}{2}\right) = 0$ simplifies the analysis but allowing for $C'\left(\frac{1}{2}\right) > 0$ would essentially introduce a fixed cost of information acquisition (see Martinelli (2006) for a discussion).

Definition 1 *A **regular committee** of size n is a committee with mandatory voting and n members in which preferences are described by the parameters $(\alpha, \xi_A) \in (0,1)^2$ and the cdf F over $[0,1]^2$ with no mass points, and continuous pdf f, the prior probability of state a is $\phi \in (0,1)$, and the cost technology of information acquisition verifies assumption (1).*

Definition 2 *A pure strategy for voter i is an investment function $P_i : [0,1]^2 \rightarrow \left[\frac{1}{2},1\right]$ and a voting function $V_i : [0,1]^2 \times \{s_q, s_a\} \rightarrow \boldsymbol{X}$, such that $P_i(\theta)$ is the investment level of responsive* *voter i type θ, and $V_i(\theta, S) = (V_i(\theta, s_q), V_i(\theta, s_a))$ is the contingent vote of responsive voter i type θ who receives the signal $S \in \{s_q, s_a\}$.*[3]

When we refer to a generic voting function, investment function or strategy, we omit the superscript indicating types. We refer to the voter's behavior (strategy) as $V(\theta, S)$ and to an arbitrary pair of votes as $(v_q, v_a) \in \mathbf{X}^2$. $V(\theta, S)$ is part of an strategy while (v_q, v_a) is basically notation. When we want to refer to a particular vote we use just v.

We will refer to a profile of strategies as $\left(\widetilde{P}, \widetilde{V}\right)$ where $\widetilde{P} = (P_1, ...P_n)$ and $\widetilde{V} = (V_1, ...V_n)$ are the profile of investment functions and voting functions for the whole committee. Analogously $\left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)$ is the profile of strategies for all players but player i. We will say that, if $V(\theta, s) = v$ for all $s \in \{s_q, s_a\}$ type θ uses an **uninformed** voting function, and if $V(\theta, s_q) \neq V(\theta, s_a)$ type θ uses an **informed** voting function. We therefore identify strategies by the voting function they employ.

Conditional on the profile of strategies of all voters but i, $\left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)$, we define the probability that the winner is $x \in \{Q, A\}$ in state $\omega \in \{q, a\}$, when voter i casts vote $v \in \mathbf{X}$, as

$$\Pr\left(x \mid \omega, v, \left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)\right) \tag{1}$$

Since a voter selects the quality of information after observing her type but before observing the signal, while the vote is decided after observing the signal, we need to define payoffs in different stages of the game. The expected utility of player i of type $\theta \in [0,1]^2$ when she votes $v \in \mathbf{X}$, and the state is $\omega \in \{q, a\}$, is

$$u^i(v \mid \theta, \omega) \equiv -\theta_\omega \Pr\left((-\omega) \mid \omega, v, \left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)\right) \tag{2}$$

where we let $(-\omega) = Q$ if $\omega = a$ and $(-\omega) = A$ if $\omega = q$. This expression is just the product of the disutility of a mistake $(-\theta_\omega)$ and the probability of a mistake in the state $\omega \in \{q, a\}$, given player i's vote v. We define the expected utility of player i of type $\theta \in [0,1]^2$ and investment choice $p \in \left[\frac{1}{2}, 1\right]$, when she votes $v \in \mathbf{X}$ after receiving the signal $s \in \{s_q, s_a\}$ as

$$U^i(p, v \mid \theta, s) \equiv \sum_{\omega \in \{q,a\}} u^i(v \mid \theta, \omega) \Pr(\omega \mid s, p) \tag{3}$$

Using (3), the expected utility of player i of type $\theta \in [0,1]^2$ and investment choice

[3] The reader may argue that voting rules should be contingent in the level of investment performed by each voter so $V : [0,1]^2 \times \left[\frac{1}{2}, 1\right]^2 \times \{s_q, s_a\} \to \mathbf{X}$. This approach substantially complicates the model without affecting any of the results. That results are unaffected follows by the fact that between the investment decision and voting decision no other public information is revealed to the voters.

$p \in \left[\frac{1}{2}, 1\right]$, for a voting function (v_q, v_a) is

$$\mathcal{U}^i\left(p, (v_q, v_a)\right) \mid \theta) \equiv \sum_{x \in \{q,a\}} U^i\left(p, v_x \mid \theta, s_x\right) \Pr\left(s_x \mid p\right) \tag{4}$$

We study symmetric Bayesian equilibria in pure strategies.

Definition 3 *A symmetric Bayesian equilibrium for the voting game in a regular committee with voting rule $\boldsymbol{R}$ and voting alternatives $\boldsymbol{X}$ is a strategy $\left(P^*(\theta), V^*(\theta, S)\right)$ such that: **1)** all voters use $\left(V^*(\theta, S), P^*(\theta)\right)$, **2)** for every $\theta \in [0,1]^2$, and for any other feasible (v_q, v_a) and p, the strategy $\left(P^*(\theta), V^*(\theta, S)\right)$ satisfies*

$$\mathcal{U}^i\left(P^*(\theta), V^*(\theta, S) \mid \theta\right) - C\left(P^*(\theta)\right) \geq \mathcal{U}^i\left(p, (v_q, v_a) \mid \theta\right) - C(p) \tag{5}$$

From now on, we omit the strategy profile of all other players as an argument of endogenous variables. Therefore, the probability of a particular outcome of the decision $x \in \{Q, A\}$, in state ω, after player i cast a vote $v \in \mathbf{X}$, is written as $\Pr(x \mid \omega, v)$. Using the assumption that every voter uses the same strategy in equilibrium, the probability that an arbitrary voter $j \neq i$ votes for $v \in \mathbf{X}$, in state ω, when all other players but i are using the strategy $\left(P(\theta), V(\theta, S)\right)$ is

$$\Pr(v \mid \omega) = (1-\alpha) \int_{\theta \in [0,1]^2} \sum_{s \in \{s_q, s_a\}} I\left(V(\theta, s) = v\right) \Pr\left(s \mid P(\theta), \omega\right) dF(\theta) + \alpha \xi_v \tag{6}$$

The first part of the right side is just the probability that a voter is responsive multiplied by the probability that a responsive voter votes for $v \in \mathbf{X}$. The second part is the probability that a voter is partisan, multiplied by the probability that a partisan member votes for $v \in \mathbf{X}$. This expression aggregates over the two sources of private information present in the model: the type of player and the signal received after investment.[4]

Recalling the expression (1) and fixing all players' strategies but i's, we also define the change in the probability of A winning when voter i switches her vote from Q to A in state ω as,

$$\Delta \Pr(\omega, Q) \equiv \Pr(A \mid \omega, A) - \Pr(A \mid \omega, Q) \tag{7}$$

Again, we must recall that $\Delta \Pr(\omega, Q)$ for $\omega \in \{q, a\}$ are conditioned on the actual profile of strategies $\left(\widetilde{P}^{-i}, \widetilde{V}^{-i}\right)$ so they both depend on the behavior of all other players. These new variables are crucial to tackle the existence problem and characterization results in a manageable way.

[4]As the reader suspects $\Pr(x \mid \omega, v)$ is a combination of $\Pr(v \mid \omega)$, for $v \in \mathbf{X}$, $x \in \{Q, A\}$ and $\omega \in \{q, a\}$.

3 Solving the model

Let T_k stand for the total number of votes for A, when there are k voters. The voting rule is defined as a pair $\mathbf{R}=(N, r)$ with $n \geq N \geq \frac{n}{2}$[5] and $r \in [0,1]$, such that A wins if $T_n > N$ and Q wins if $T_n < N$; if $T_n = N$, A wins with probability $1-r$ and Q wins with probability r.[6]

3.1 Voting Incentives

Responsive voters can use four possible voting functions: (Q, A), (A, Q), (A, A), and (Q, Q). It is straightforward to see that the voting functions (A, A) and (Q, Q) can not induce positive investment in information in equilibrium. Only (Q, A) and (A, Q) can induce positive investment in equilibrium. As the reader suspects, (A, Q) can not be optimal. The next lemma provides conditions for a vote $v \in \{Q, A\}$ to be optimal after receiving the signal $s \in \{s_q, s_a\}$ when the investment is p

Lemma 1 *In any regular committee, a necessary condition for a responsive voter to vote for A after receiving the signal $s \in \{s_q, s_a\}$, when she is type θ and the investment is p, is*

$$\theta_q \Delta \Pr(q, Q) \Pr(q \mid p, s) \leq \theta_a \Delta \Pr(a, Q) \Pr(a \mid p, s) \tag{8}$$

A necessary condition for a responsive voter type θ with investment p to vote for Q after receiving the signal $s \in \{s_q, s_a\}$ is obtained by reversing the sign of (8). Strict inequalities give sufficient conditions.

$\theta_q \Delta \Pr(q, Q) \Pr(q \mid p, s)$ is the expected cost of making a mistake (making A the winner in state q) when switching the vote from Q to A after signal s, while $\theta_a \Delta \Pr(a, Q) \Pr(a \mid p, s)$ is the expected benefit of avoiding a mistake (Q winning in state a) when switching the vote from Q to A after signal s. Therefore, (8) only states that the voter will vote in favor of A, when the expected benefit of avoiding a mistake is higher than the expected loss of making one when changing the vote from Q to A.

Responsive voters will consider how they affect the outcome of the election to decide how they vote. Since there are always partisan voters every possible configuration of votes is probable and it is always true that a voter might affect the outcome of the election. The next lemma states this formally so changing the vote always has an impact in the election and $\Delta \Pr(\omega, Q) > 0$ for any $\omega \in \{q, a\}$.

[5] The case with $N < \frac{n}{2}$ can be studied by reversing the roles of Q and A.

[6] The results are valid for all q-majority rules, such that A is the winner if the percentage of votes in favor of A is at least $N > qn$.

Lemma 2 *In any regular committee, there is some* $\zeta(\omega) > 0$ *for each* $\omega \in \{q, a\}$*, such that* $\Delta \Pr(\omega, Q) \in [\zeta(\omega), 1 - \zeta(\omega)]$*. If* $n = 1$*, then* $\Delta \Pr(\omega, Q) = 1$ *for* $\omega \in \{q, a\}$*.*

Once we know that $\Delta \Pr(\omega, Q) > 0$ for $\omega \in \{q, a\}$ it is easy to see that there are no equilibria in which all responsive voters vote for a particular candidate independently of their preferences. That is, there exist a pair of types θ_1 and θ_2 such that $V(\theta_1, S) \neq V(\theta_2, S)$. With Lemma (2) at hand, we can manipulate the expression (8) to show that A is optimal after signal $s \in \{s_q, s_a\}$ if

$$\frac{\theta_q}{\theta_a} \frac{\Pr(q \mid p, s)}{\Pr(a \mid p, s)} \leq \frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)} \tag{9}$$

Obviously, Q is optimal if the sign is reversed in expression (9). $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)}$ is determined in equilibrium, while $\frac{\theta_q}{\theta_a} \frac{\Pr(q|p,s)}{\Pr(a|p,s)}$ is the voter's private information information. Condition (9) will allow us to construct functions that separate types that prefer $v = A$ over $v = Q$ conditional on the signal and the investment.

We proceed now to determine the responsive voters' optimal voting function. There are basically two informed strategies: the strategy with the voting function (A, Q), and the strategy with the voting function (Q, A). If the signal is informative, it is not optimal for a responsive voter to vote against the information that she receives in all circumstances. A player using a strategy with the informed voting function (A, Q) is doing just that. Only uninformed voters that are indifferent between option A and Q may use (A, Q). Therefore, (A, Q) is not used in equilibrium with positive probability.

Lemma 3 *In any regular committee, the voting function* (A, Q) *may be used in equilibrium only by types that satisfy* $\frac{\theta_a}{\theta_q} = \frac{\Delta \Pr(q,Q)}{\Delta \Pr(a,Q)} \frac{(1-\phi)}{\phi}$*, and if they use it, they do not collect information; for all other types it is not optimal. The set of types who do not acquire information and use an informative strategy has measure* 0 *in equilibrium.*

If a responsive voter uses an uninformed strategy, this voter cannot be collecting information. If a voter type θ invests, it must be the case that she is following an informed strategy. Now, we can separate the types in those that always vote for A, types that always vote for Q, and types that collect information and change the vote according to the signal received. We will refer to types that always vote for A (or Q) without performing any investment as **supporters** for A (or Q), and types that invest and change their vote according to the signal received as **independents**.

3.2 Information acquisition

In this section we derive the optimal investment function for independents. Using (4), the optimal investment function of players that use the informed strategy with (Q, A) is defined implicitly by[7]:

$$C'\left(P^{*}\left(\theta\right)\right)=\theta_{q}\Delta\Pr\left(q,Q\right)\left(1-\phi\right)+\theta_{a}\Delta\Pr\left(a,Q\right)\phi \tag{10}$$

When $C'\left(\frac{1}{2}\right)=0$,[8] the fact that $C''\left(p\right)>0$ for all $p>\frac{1}{2}$, and the implicit function theorem imply that $P^{*}\left(\theta\right)$ exists, is continuously differentiable, strictly increasing and strictly concave for all $\theta\neq\left(0,0\right)$. Recalling that there exists some $\zeta\left(\omega\right)$ such that $\Delta\Pr\left(\omega,Q\right)\leq1-\zeta\left(\omega\right)$ (see Lemma (2)) we conclude that $C'\left(P^{*}\left(\theta\right)\right)\leq\max\limits_{\omega\in\{q,a\}}\left(1-\zeta\left(\omega\right)\right)$, and since $\lim\limits_{x\rightarrow1}C'\left(x\right)\rightarrow\infty$ we know that $P^{*}\left(\theta\right)\leq\eta$ for some $\eta<1$.

The informed strategy $\left(Q,A\right)$ is used whenever its expected utility, net of investment costs, is higher than the expected utility derived from using any uninformed strategy (see condition (5)). The next lemma introduces an expression to compare the informed strategy with $\left(Q,A\right)$ with the uninformed strategies.

Lemma 4 *In any regular committee, a necessary condition for voter type θ, to use a strategy with voting function $V\left(\theta,S\right)=\left(Q,A\right)$, and investment function $P^{*}\left(\theta\right)>\frac{1}{2}$ that satisfies (10), is that*

$$C'\left(P^{*}\left(\theta\right)\right)P^{*}\left(\theta\right)-C\left(P^{*}\left(\theta\right)\right)-\theta_{\omega}\Delta\Pr\left(\omega,Q\right)\Pr\left(\omega\right)\geq0 \tag{11}$$

for all $\omega\in\{q,a\}$. A sufficient condition is obtained if (11) holds with strict inequality.

The informed strategy $\left(Q,A\right)$ is preferred to the uninformed strategy $\left(A,A\right)$ if we let $\omega=a$ in condition (11), and the informed strategy $\left(Q,A\right)$ is preferred to the uninformed strategy $\left(Q,Q\right)$ if we let $\omega=q$ in condition (11). In order to determine which type prefers which strategy, we define implicitly the functions $g^{\omega}:\mathcal{R}\rightarrow\mathcal{R}$ such that the pair $\left(g^{\omega}\left(\theta_{a}\right),\theta_{a}\right)$ satisfies (11) with equality:

$$\begin{aligned}C'\left(P^{*}\left(g^{a}\left(\theta_{a}\right),\theta_{a}\right)\right)P^{*}\left(g^{a}\left(\theta_{a}\right),\theta_{a}\right)-C\left(P^{*}\left(g^{a}\left(\theta_{a}\right),\theta_{a}\right)\right) &= \theta_{a}\Delta\Pr\left(a,Q\right)\phi\\ C'\left(P^{*}\left(g^{q}\left(\theta_{a}\right),\theta_{a}\right)\right)P^{*}\left(g^{q}\left(\theta_{a}\right),\theta_{a}\right)-C\left(P^{*}\left(g^{q}\left(\theta_{a}\right),\theta_{a}\right)\right) &= g^{q}\left(\theta_{a}\right)\Delta\Pr\left(q,Q\right)\left(1-\phi\right)\end{aligned}$$

where $P^{*}\left(g^{\omega}\left(\theta_{a}\right),\theta_{a}\right)$, $\omega\in\{q,a\}$ satisfies (10). Each function g^{ω}, $\omega\in\{q,a\}$ partitions the space of types $\left[0,1\right]^{2}$ in two regions: g^{q} separates types that prefer the informed strategy $\left(Q,A\right)$ to the uninformed strategy $\left(Q,Q\right)$, and g^{a} separates types that prefer the informed strategy $\left(Q,A\right)$ to the uninformed strategy $\left(A,A\right)$.

Let $\omega=q$ in condition (11), and note that the left side of (11) is decreasing in θ_{q}. Therefore, any $\theta\in\left[0,1\right]^{2}$ such that $\theta_{q}>g^{q}\left(\theta_{a}\right)$, prefers the uninformed strategy with $\left(Q,Q\right)$ to the informed strategy with $\left(Q,A\right)$. On the other hand, if $\theta_{q}<g^{q}\left(\theta_{a}\right)$ the informed

[7] Second order conditions follow directly by convexity of C.

[8] If $C'\left(\frac{1}{2}\right)>0$, the set of types that may use the informed voting rule $\left(Q,A\right)$ must satisfy $\min\{\theta_{q},\theta_{a}\}>0$.

strategy is preferred. If $\omega = a$ in condition (11), any pair $\theta \in [0,1]^2$ such that $\theta_q > g^a(\theta_a)$ prefers the informed strategy with (Q, A) to the uninformed strategy with (A, A).

Using the implicit function theorem, each $g^\omega(\theta_a)$ for $\omega \in \{q, a\}$ exists, is continuously differentiable and strictly increasing for all $\theta \neq (0,0)$. Moreover, $g^q(\theta_a)$ is strictly convex and $g^a(\theta_a)$ is strictly concave for all $\theta \neq (0,0)$, and $g^q(\theta_a) - g^a(\theta_a)$ is strictly increasing for all $\theta_a > 0$, with $\lim_{x \to 0} g^\omega(x) = 0$, for all $\omega \in \{q, a\}$.[9]

3.3 Characterization and existence of equilibrium

3.3.1 Characterization

The functions $g^q(\theta_a)$ and $g^a(\theta_a)$ separate the space of types in three groups that use different strategies. All responsive voters type $\theta \in [0,1]^2$ with $\theta_q < g^a(\theta_a)$ use a simple strategy described by constant functions: $P(\theta) = \frac{1}{2}$ and $V(\theta, s) = A$, for $s \in \{s_q, s_a\}$. The same can be said for responsive voters with type $\theta_q > g^q(\theta_a)$ where $V(\theta, s) = Q$, for $s \in \{s_q, s_a\}$. The interesting group is the set of responsive voters $\theta \in [0,1]^2$ that satisfy $g^a(\theta_a) \leq \theta_q \leq g^q(\theta_a)$ (independents), since both the investment function and the voting function change with the type and the signal.

The functions $g^q(\theta_a)$ and $g^a(\theta_a)$ ensure the strategies are optimal, and the backward induction process ensures that the voting function is optimal when conditional on the optimal investment level. This is formally stated in the next proposition

Proposition 1 *In any regular committee of size $n \geq 1$ in which the voting rule is $\boldsymbol{R} = (N, r)$, the strategy $(P^*(\theta), V^*(\theta, S))$ that prescribes*

1. *the investment function $P^*(\theta)$ as defined in (10) for every θ that satisfies $g^a(\theta_a) \leq \theta_q \leq g^q(\theta_a)$, and $P^*(\theta) = \frac{1}{2}$ otherwise,*

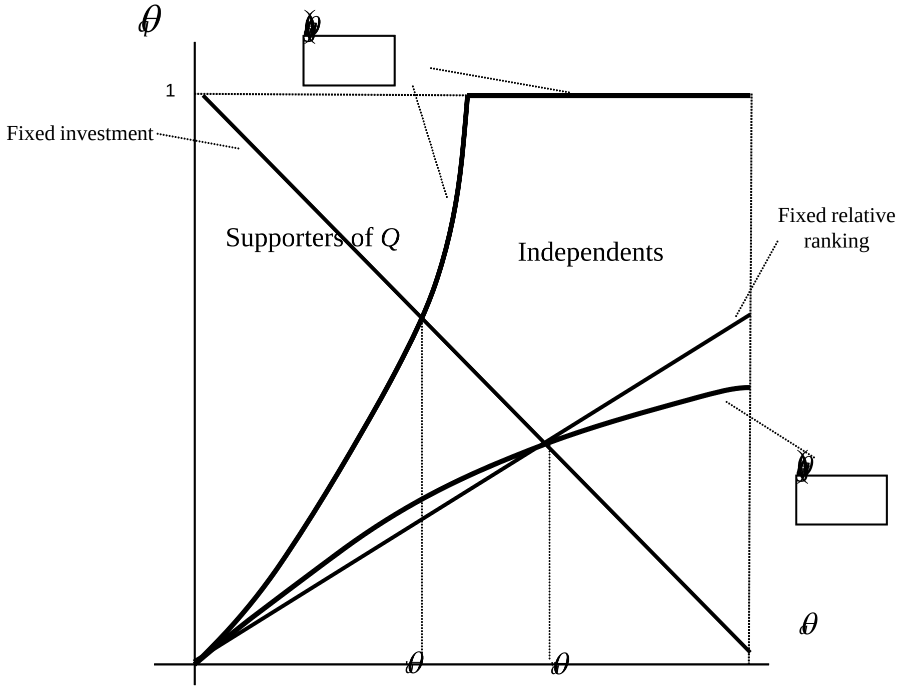
2. *the voting function $V^*(\theta, S) = (A, A)$ if $\theta_q < g^a(\theta_a)$, $V^*(\theta, S) = (Q, Q)$ if $\theta_q > g^q(\theta_a)$, and $V^*(\theta, S) = (Q, A)$ if $g^a(\theta_a) \leq \theta_q \leq g^q(\theta_a)$,*

is a symmetric Bayesian equilibrium.

[9] It is worth noting two features of these results used later for existence. First, P^* may be defined for $\theta \notin [0,1]^2$. Indeed, as long as θ satisfies $C'(1) > \sum_{\omega \in \{a,q\}} \theta_\omega \Delta \Pr(\omega, Q) \Pr(\omega)$, $P^*(\theta)$ is well defined. Second, $g^q(\theta_a)$ and $g^a(\theta_a)$ are also properly defined for all θ that satisfy $C'(1) > \sum_{\omega \in \{a,q\}} \theta_\omega \Delta \Pr(\omega, Q) \Pr(\omega)$, even if $\theta \notin [0,1]^2$.

In Figure (1) we illustrate the equilibrium in an election where the simple majority rule is in place for n odd, $\phi = \frac{1}{2}$, F symmetric around the 45^o degree line[10], and $\xi_A = \frac{1}{2}$. In this case the equilibrium is fully symmetric around the 45^o degree line: $g^a(y) = x$ iff $g^q(x) = y$ and $P^*(\theta) = P^*(\theta')$, for every $\theta = (x, y)$ and $\theta' = (y, x)$.

1



1 .pdf

Figure 1: Equilibrium under the plurality rule and n is odd. Supporters of A (Q) always vote for A (Q) and do not collect information. Independents collect information according to equation (10), and vote according to the signal received: if $S = s_a$ then $v = A$ and if $S = s_q$ then $v = Q$.

3.3.2 Existence

The fact that the equilibrium strategy is composed of an investment function that is only C^0 almost everywhere,[11] complicates the direct use of standard fixed point theorems on the space of best responses. In order to show existence we create a transformation that uses the optimal investment function and the optimal voting strategies as arguments. In this manner,

[10] That is $F(x, y) = F(y, x)$ for all $(x, y) \in [0, 1]^2$.

[11] See the equicontinuity requirement for Schauder's Fixed Point Theorem in Rudin (1973). In turns some assumption abut the differentiability of $P^*(\theta)$ is required which can be translated into $C''\left(\frac{1}{2}\right) > 0$. This condition rules out any possible aggregation of information in the limit as shown by Martinelli (2006).

the equilibrium can be described by the functions $g^{\omega}(\theta_a)$, $\omega \in \{q, a\}$ and the investment function $P^*(\theta_a)$ that are uniquely defined by $\Delta \Pr(\omega, Q)^*$, $\omega \in \{q, a\}$.

Proposition 2 *In any regular committee of size $n \geq 1$ in which the voting rule is $\boldsymbol{R}= (N, r)$, there exists a symmetric Bayesian equilibrium. Moreover, this equilibrium is characterized by the strategy $(P^*(\theta), V^*(\theta, S))$ in Proposition (1).*

Although we can not prove that for each set of primitives (ϕ, n, (N, r), $F(\cdot)$, α, ξ_A, $C(\cdot)$) there is a unique equilibrium (a unique set of $g^q(\theta_a)$, $g^a(\theta_a)$ and $P^*(\theta)$), we know that every symmetric Bayesian equilibrium is described by a set of cut off functions $g^q(\theta_a)$ and $g^a(\theta_a)$ and a investment function $P^*(\theta)$. Therefore, our characterization is valid for *all* symmetric Bayesian equilibria.

3.3.3 Intuitive characterization

Supporters for A are characterized by a high relative ratio of losses $\frac{\theta_a}{\theta_q}$ while supporters for Q are characterized by a low relative ratio of losses $\frac{\theta_a}{\theta_q}$. Independents have more balanced preferences and invest to collect information and follow the signal accordingly.

There are two main forces that drive a voter's behavior when information is endogenous: the relative ranking of alternatives ($\frac{\theta_a}{\theta_q}$) and the actual level of utility losses ($\min\{\theta_a, \theta_q\}$). When $\frac{\theta_a}{\theta_q}$ is high (biased towards A) a vote for Q is only possible if the evidence in favor of state q is overwhelming. When information is endogenous, this information depends on the level of losses through the function $\sum_{\omega \in \{a,q\}} \theta_\omega \Delta \Pr(\omega, Q) \Pr(\omega)$, so the information level increases as we move away from the origin. For a fixed level of $\frac{\theta_a}{\theta_q}$, the higher θ_a, the higher the precision selected if the informed strategy is used in equilibrium. For example, along the "fixed relative ranking" line in Figure (1), $\frac{\theta_a}{\theta_q}$ is fixed; when $\theta_a < \theta_a^{''}$ the information collected if (Q, A) were used is not too strong. Because of this imprecise information, a responsive voter cannot be too sure that the true state is q when the signal received is s_q; therefore, she prefers to save on the cost of information than buying reassurance that the true state is q. When $\theta_a > \theta_a^{''}$, the precision of the information collected if (Q, A) were used is high enough to induce a responsive voter to select Q when the signal is s_q.

What would happen if the information were free and its precision were exactly $P^*(\theta)$ for each type θ? For types (θ_q, θ_a) such that θ_q is much smaller than $g^a(\theta_a)$, the free signal does not alter their behavior: they would still be supporters for A. These voters decide strategically to ignore their information. But for types (θ_q, θ_a) such that θ_q is close but smaller than $g^a(\theta_a)$, if the information were free, they would vote in favor of Q instead of A. In essence, the reason why some (θ_q, θ_a) with θ_q close but smaller than $g^a(\theta_a)$ behave as a supporter is due to the cost of information: a signal with the precision she would have selected if it were forced to collect information would have made her change her vote, but that signal is too costly.

Alternatively, fix the precision of the information collected by informed voters along the "fixed investment" line in Figure (1). When the type θ satisfies $\theta_a < \theta_a^{'}$, the precision of the information collected when (Q, A) is used is not high enough to make the player vote in favor of A. Then there is no reason to collect information. When θ_a is close to 0, any free information would be disregarded, and if θ_a is close to $\theta_a^{'}$, free information is welcome. When $\theta_a \in \left(\theta_a^{'}, \theta_a^{''}\right)$, preferences are balanced enough for (Q, A) to be preferred given the optimal p. In that case information is collected and the signal guides the voting function. When $\theta_a > \theta_a^{''}$, the problem is that the signal in favor of Q is not strong enough, and the responsive voter becomes a supporter of A.

Besides the assume existence of partisan voters, there are supporters for A, supporters for Q and independents in any equilibrium. This is basically driven by the fact that $\frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)}$ is bounded by $\left[\frac{\zeta(a)}{1-\zeta(q)}, \frac{1-\zeta(q)}{\zeta(a)}\right]$ and $\frac{p}{1-p}$ is bounded by $\left[1, \frac{\eta}{1-\eta}\right]$. Using (9), there are always responsive voters with extreme types ($\frac{\theta_a}{\theta_q}$ big enough or small enough): the precision of the signal that would have been collected if (Q, A) were used could not have overturned the relative bias.

4 Information Aggregation

Martinelli (2006) is the first model jointly studying both the *Condorcet Jury Theorem* and the rational ignorance hypothesis in the same set up by allowing voters to collect information of the same quality. He gives individual rationality to the exogenous process that Yariv (2004) assumes, and he shows that the form of the cost function for information acquisition determines whether there is aggregation of information or not. On a first section he assumes homogeneity ($\theta_a^i = \theta_a$ and $\theta_q^i = \theta_q$ for all i) and introduces heterogeneity as an extension (θ_a^i and θ_q^i might differ for different voters). He also assumes preferences to be single dimensional so, in our terms, $\theta_q + \theta_a = 1$ and focuses on the simple majority rule.

When preferences are heterogeneous Martinelli (2006) assumes that both θ_a and θ_q are uniform and $\phi = (1 - \phi) = \frac{1}{2}$ which directly leads to $\Delta \Pr(q, Q) = \Delta \Pr(a, Q)$. Note that the first order conditions for investment (10) together with $\phi = (1 - \phi) = \frac{1}{2}$ and $\Delta \Pr(q, Q) = \Delta \Pr(a, Q)$ give that $C'(P^*(\theta)) = (\theta_q + \theta_a) \Delta \Pr(a, Q) \phi = \frac{\Delta \Pr(a,Q)}{2}$ so every voter that collects information will collect the same quality of information if $\theta_q + \theta_a = 1$. At the same time, (19) gives that voters that verify $\frac{P^*(\theta)}{1-P^*(\theta)} \geq \frac{\theta_a}{\theta_q} \geq \frac{P^*(\theta)}{1-P^*(\theta)}$ will collect information and an equilibrium with information acquisition exists for any n as long as

$C'\left(\frac{1}{2}\right) = 0$. This summarizes Theorem 5 in Martinelli (2006).[12] He finds that full aggregation of information is only possible in large electorates if $C'\left(\frac{1}{2}\right) = C''\left(\frac{1}{2}\right) = C'''\left(\frac{1}{2}\right) = 0$.[13]

Martinelli's existence and characterization results crucially depend on the fact that preferences are restricted to $\theta_a + \theta_q = 1$, the simple majority rule is in place and $\phi = \frac{1}{2}$. This is because under these conditions every informed voter selects the same quality of information. Martinelli (2006) looks for equilibria that are described by the cutoffs that separates independents from supporters and the quality of information that every informed voter collects.

A natural question arises: how general is the aggregation result given that all informed voters collect the same quality of information? In Figure (1) Martinelli's informed voters are independents along the "fixed Investment" line ($\theta_q + \theta_a = 1$). Independents with $\theta_q + \theta_a < 1$ collect less information and independents with $\theta_q + \theta_a > 1$ collect more information. It seems that the average level of information should be higher with flexible preferences (θ_q and θ_a are not perfectly correlated) and it should be easier to aggregate information in the limit. But it is not clear that the average voter is more informed under flexible preferences than under restricted preferences. If the committee is actually making better decisions under flexible preferences it might be that the average voter is collecting poor information since his information is less valuable. Then, the requirements for aggregation of information under flexible (two dimensional) preferences might be stronger than under restricted (one dimensional) preferences.

Below we show that, even when voters collect information of different qualities, information aggregation is possible if the electorate is symmetric. We define symmetric in the following way:

Definition 4 *A symmetric committee of size n is a regular committee of size n in which both states are equally likely ($\phi = \frac{1}{2}$), partisans vote for each candidate with equal probability ($\xi_A = \frac{1}{2}$), and F is symmetric around the $45°$.*

In order to study information aggregation, first we need a characterization of the equilibrium when the majority rule is in place and the number of members is odd.

Proposition 3 *In any symmetric committee of size $n = 2N + 1$ with simple majority rule, for every $N \geq 0$, there is a symmetric Bayesian equilibrium characterized by the pair*

[12] The results of the previous section can be easily extended to the case of $\theta_a + \theta_q = 1$ but neither this assumption or the assumption that $\phi = \frac{1}{2}$ are necessary to show existence or characterize the equilibria with information acquisition. It is easy to see that if $C'\left(\frac{1}{2}\right) > 0$ there is no information acquisition for sufficiently large n.

[13] If $C'\left(\frac{1}{2}\right) = C''\left(\frac{1}{2}\right) = 0$ and $C'''\left(\frac{1}{2}\right) > 0$, Martinelli (2006) obtains a specific limit (when the electorate grows) on the probability of making the right choice. This limit is decreasing in $C'''\left(\frac{1}{2}\right)$ and approaches 1 when $C'''\left(\frac{1}{2}\right) \to 0$. If $C''\left(\frac{1}{2}\right) > C'\left(\frac{1}{2}\right) = 0$ both candidates have equal chances of winning in any state of nature when the electorate is sufficiently large.

$(x_N, y_N) \in (0,1)^2$, *where* ***1)*** $\Delta \Pr(\omega, Q) = x_N$ *for* $\omega \in \{q, a\}$, ***2)*** $\Pr(A \mid a) = \frac{1}{2} + y_N(1-\alpha)$, ***3)*** $\Pr(A \mid a) = \Pr(Q \mid q)$, *and* ***4)*** $P(\theta)$ *is such that* $C'(P^*(\theta)) = \frac{(\theta_q+\theta_a)x_N}{2}$.

Along any path of equilibria indexed by N, the probability of selecting A when the state is a is equal to the probability of selecting Q when the state is q. From now on, when we refer to A being selected in state a, it should be understood that we refer to the probability that the committee selects the candidate that would have won had the true state of nature been common knowledge.

What is the effect of increasing the number of voters? Intuition suggests that the probability of being pivotal decreases when the number of voters increases. This is straightforward only if the level of information in the electorate is constant. Unfortunately, when the information collected by each voter decreases (but the number of voters remains constant) the outcome of the election becomes more random. This extra randomness translates into a higher probability of being pivotal. Nevertheless, we can prove that under the simple majority rule the effect of more voters is dominant, and investment decreases when the size of the electorate increases. Indeed,

Proposition 4 *In any symmetric committee with the simple majority rule,* $P^*(\theta)$ *and* $\Delta \Pr(\omega, Q)$ *decrease with* N.

This proves that the rational ignorance hypothesis holds in our model.[14] This does not imply that information aggregation is not possible under any circumstances. The probability that a large electorate makes the right choice depends on the speed at which information acquisition decreases in the electorate when the number of voters increases (Yariv (2004)). Indeed

Proposition 5 *In any symmetric committee of size n (odd) with the simple majority rule, if the cost function is three times differentiable with* $C''' \geq 0$ *and* $F(\theta_q, \theta_a) = \theta_q\theta_a$,[15] *then*

1. *if* $C'\left(\frac{1}{2}\right) = 0$ *and* $C''\left(\frac{1}{2}\right) > 0$, *the result of the election approaches a random variable that makes* A *the winner with probability* $\frac{1}{2}$ *in any state of nature when* N *grows arbitrarily large.*

2. *if* $C'\left(\frac{1}{2}\right) = C''\left(\frac{1}{2}\right) = 0$ *and* $C'''\left(\frac{1}{2}\right) > 0$, *when* N *grows arbitrarily large, the probability of making the right choice is bounded away from* $\frac{1}{2}$, *and the bound is decreasing on the value of* $C'''\left(\frac{1}{2}\right)$.

[14] Benz and Stutzer (2004) find empirical support for the probability of being pivotal being positively correlated with the quality of information.

[15] The uniform distribution of types and independence across parameters is a simplification: all results hold if F is symmetric arounf the 45^o degree line.

3. if $C'\left(\frac{1}{2}\right) = C''\left(\frac{1}{2}\right) = C'''\left(\frac{1}{2}\right) = 0$, the probability of making the right choice converges to 1 when N grows arbitrarily large.

Note that the aggregation result depends on the value of $C'''\left(\frac{1}{2}\right)$. Using that $C'\left(P^*\left(\theta\right)\right) = \frac{(\theta_q+\theta_a)\Delta\Pr(a,Q)}{2}$ we have that the direct effect of $\Delta\Pr\left(a,Q\right)$ on $P^*\left(\theta\right)$ is determined by the second derivative of the cost function while the change in this change $\left(\frac{\partial^2 P^*(\theta)}{\partial(\Delta\Pr(a,Q))^2}\right)$ is affected by the third derivative of the cost function. In our model increasing the number of voters increases the chances of a vote being informative but reduces the incentives to collect information of all voters. When the speed at which the average information decreases because new voters are added is slow enough, aggregation of information is possible in the limit. The cost function for information acquisition determines whether adding another member is desirable or not.

5 Conclusions

Allowing for endogenous information creates serious problems to well known established results.[16] For example, Stiglitz and Grossman (1980) show that efficiency and endogenous information are not as easily paired in competitive markets as Hayek (1945) suggested.[17] Even the existence of endogenous information equilibria is problematic[18]. Moreover, the non-existence problem appears in much simpler set ups as the demand for information is not well behaved.[19] Given these results and the fact that most of the literature on committees focuses on models with exogenous information we ask: are the exogenous information results in committees robust to the introduction of endogenous information?

We develop a model of voting where voters endogenously select the quality of the information they will use to vote.[20] Voters who receive reports or memos need to expend time and effort to understand the information. This decision is endogenous so there is no reason

[16] See Stiglitz (2002) for a broader survey of information economics discussing problems arising with endogenous information and incentives to collect this information.

[17] For example, in Prat (2002) allowing for endogenous information in the electorate will kill voters' incentives to collect private signals in the separating equilibrium (in Prat (2002) terms: z will not convey any information). Voters will rely solely on campaign advertisement to decide the candidates' valence and interest groups are indifferent between contributing or not to campaigns (see point 1, page 1007 in Prat (2002)).

[18] See Green (1977); see Dubey et al. (1987) for further developments departing from competitve markets.

[19] See Stiglitz and Radner (1984) for a seminal exposition in simple environments and Chade and Schlee (2002) for an extension to continuous set ups and generalizations.

[20] For any rule besides the unanimity rule, if there are no partisans, there is always an equilibrium where all voters vote for A (Q) and do not collect any information. Although we assume the existence of partisans all our results hold when we assume away these voters and let all voters be responsive and use non-weakly dominated strategies.

to expect that different voters will be equally informed. Modelling information acquisition in elections as a choice over a set of signals with different precision is more accurate than assuming a common source of information for all voters. In line with this observation, we allow voters to *select the correlation between the signal they will receive and the true state of nature.*

We assume the level of conflict among committee members to be richer than it is usually assumed in the literature. When information is exogenous, all the relevant interaction between the committee members can be represented by simple structures. Indeed, preferences modelled as a relative ranking of alternatives capture all the proper incentives to study voting decisions: ideological heterogeneity is enough. This restriction on preferences imposes correlation between the disutilities that a member derives from mistaken decisions. When information is endogenous, this restriction does not capture all the relevant strategic interaction: voters with the same ranking of alternatives may have different incentives to collect information. We assume that committee members' preferences are *flexible* and introduce *another dimension of heterogeneity:* committee members not only differ on their ideological position *but they also differ on the level of concern* about the outcome of the election.

We provide existence and characterization results for arbitrary rules of election, arbitrary but continuous distribution of types and heterogeneity in preferences. We give a natural and intuitive representation of the equilibrium behavior of committee members. This geometric representation of equilibrium is important in order to derive the existence result. In equilibrium, our model predicts that informed voters endogenously select different levels of information. Contrary to previous results in the literature, heterogenous preferences translate into heterogenous informed voters. This is directly related to the assumption about preferences and, in particular, to the second dimension of heterogeneity.

Aggregation results for symmetric electorates and priors are derived without imposing that every informed member must collect the same quality of information in equilibrium. Therefore, the aggregation of information derived in Martinelli (2006) does not depend on the assumed preferences and the homogeneity of information among informed voters. The "speed" at which information is lost due to the reduction in the probability of being decisive is the key ingredient in the aggregation of information (Yariv (2004)).

A crucial technical difference between our paper and Martinelli (2006) is the role played by symmetry in the aggregation results. While in Martinelli (2006) symmetry assures that every voter selects the same level of information and the probability of being pivotal in both states is the same ($\Delta \Pr(q, Q) = \Delta \Pr(a, Q)$ for every n), in our paper only the second part is used. What's the specific role of $\Delta \Pr(q, Q) = \Delta \Pr(a, Q)$? If along the equilibrium path, $\Delta \Pr(q, Q) = \Delta \Pr(a, Q)$ for every n, the cutoff functions $g^a(\theta_q)$ and $g^q(\theta_q)$ only depend on the level of information that the cutoff type selects since the slope of $g^a(\theta_q)$ and $g^q(\theta_q)$ depend on $\frac{\Delta\Pr(q,Q)}{\Delta\Pr(a,Q)}$. Hence, the effect of an extra voter on the equilibrium behavior is all determined by the probability of being pivotal and not on the relative ratio of these probabilities in each state. If $\Delta\Pr(q,Q) \neq \Delta\Pr(a,Q)$ aggregation results will depend also on how the ratio $\frac{\Delta\Pr(q,Q)}{\Delta\Pr(a,Q)}$ changes with n. In turns, this requires a full characterization of equilibrium which is, at this point, not possible unless we assume a particular cost of information function C.

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A Proofs

Proof of Lemma (1). Using the definition of expected utility in (2) and (3), we have that a voter will only follow the signal s' if

$$\mathcal{U}^i\left(P^*\left(\theta\right), V^*\left(\theta, s'\right) \mid \theta\right) \geq \mathcal{U}^i\left(P^*\left(\theta\right), v \mid \theta\right)$$

for any $v \neq V^*\left(\theta, s'\right)$. Bayes' rule gives the result. ■

Proof of Lemma (2). Assume that all players but i are using the strategy (P, V) and player i uses (P^i, V^i). Let $\Pr\left(T_m = k \mid \omega\right)$ be the probability that there are k votes for A out of the m voters when everybody uses the voting function V and the investment function P. Using the distribution function of a binomial random variable

$$\Pr\left(T_{n-1} = k \mid \omega\right) = \frac{(n-1)!}{(n-1-k)!k!}\left(\Pr\left(A \mid \omega\right)\right)^k\left(1 - \Pr\left(A \mid \omega\right)\right)^{n-1-k} \tag{12}$$

where $\Pr\left(A \mid \omega\right)$ is defined as in (6). The probability of candidate A being selected when member i votes for $x \in \{Q, A\}$ is just

$$\Pr\left(A \mid \omega, A\right) = \Pr\left(T_{n-1} = N - 1 \mid \omega\right)(1 - r) + \sum_{k=N}^{n-1} \Pr\left(T_{n-1} = k \mid \omega\right) \tag{13}$$

and

$$\Pr\left(A \mid \omega, Q\right) = \Pr\left(T_{n-1} = N \mid \omega\right)(1 - r) + \sum_{k=N+1}^{n-1} \Pr\left(T_{n-1} = k \mid \omega\right) \tag{14}$$

Therefore, using definition (7)

$$\Delta \Pr\left(\omega, Q\right) = \Pr\left(T_{n-1} = N - 1 \mid \omega\right)(1 - r) + \Pr\left(T_{n-1} = N \mid \omega\right) r \tag{15}$$

Since $\Pr\left(A \mid \omega\right) \in \left[\alpha\xi_A, 1 - \alpha\xi_Q\right]$, $\Pr\left(A \mid \omega, v\right) < 1$ for $v \in \{Q, A\}$, then $\Delta \Pr\left(\omega, Q\right) \leq 1 - \zeta^1\left(\omega\right)$ for some $\zeta^1\left(\omega\right) > 0$ small enough. On the other hand, using (12), we conclude that $\Delta \Pr\left(\omega, Q\right)$ is bigger than

$$\min\left\{\Pr\left(T_{n-1} = N - 1 \mid \omega\right), \Pr\left(T_{n-1} = N \mid \omega\right)\right\}$$

Again, using the fact that $\Pr\left(A \mid \omega\right) \in \left[\alpha\xi_A, 1 - \alpha\xi_Q\right]$, there is some $\zeta^2\left(\omega\right) > 0$ such that $\Delta \Pr\left(\omega, Q\right) \geq \zeta^2\left(\omega\right)$. Finally, the result for $n = 1$ is straightforward. ■

Proof of Lemma (3). First note that Bayes' rule gives that $\frac{\Pr(q|s_a)}{\Pr(a|s_a)} = \frac{(1-p)}{p}\frac{(1-\phi)}{\phi}$ and

$\frac{\Pr(q|s_q)}{\Pr(a|s_q)} = \frac{p}{(1-p)} \frac{(1-\phi)}{\phi}$. Using (8), the informed strategy with (A, Q) must satisfy

$$\begin{aligned} \theta_q \Delta \Pr(q, Q) \frac{p}{(1-p)} \frac{(1-\phi)}{\phi} &\leq \theta_a \Delta \Pr(a, Q) \\ \theta_q \Delta \Pr(q, Q) \frac{(1-p)}{p} \frac{(1-\phi)}{\phi} &\geq \theta_a \Delta \Pr(a, Q) \end{aligned} \tag{16}$$

so we must have that $\frac{\theta_q}{\theta_a} \frac{p}{(1-p)} \leq \frac{\theta_q}{\theta_a} \frac{(1-p)}{p}$. If $p > \frac{1}{2}$, we reach a contradiction. If $p = \frac{1}{2}$, it is necessary for (A, Q) that both inequalities in (16) hold which imply the conditions stated in the hypothesis: $\frac{\theta_a}{\theta_q} = \frac{\Delta \Pr(q,Q)}{\Delta \Pr(a,Q)} \frac{(1-\phi)}{\phi}$. ■

Proof of Lemma (4). Using condition (5), the informed strategy with voting function (Q, A), is as good as an uninformed strategy $(v_q, v_a) = (X, X)$ for $X \in \{Q, A\}$, iff

$$C(P^*(\theta)) \leq \mathcal{U}^i(P^*(\theta), (Q, A) \mid \theta) - \mathcal{U}^i\left(\frac{1}{2}, (X, X) \mid \theta\right) \tag{17}$$

where $\mathcal{U}^i\left(\frac{1}{2}, (v_q, v_a) \mid \theta\right)$ is defined by (4). Using (4) and recalling $\Delta \Pr(a, Q)$ and $\Delta \Pr(q, Q)$, (17) reduces to (11), and necessity follows. Sufficiency of (11) with strict inequality is straightforward. ■

Proof of Proposition (1). By construction of the functions $g^q(\theta_a)$ and $g^a(\theta_a)$, all types satisfy the optimal condition (5) when using the strategies defined in the proposition. It remains to show that it is actually optimal to follow that voting function after the signal is realized.

For supporters for A, condition (8) is just

$$\theta_q \leq \theta_a \frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)} \frac{\phi}{1-\phi} \tag{18}$$

for both signals. In the case of supporters for Q, we must reverse the sign of (18). In the case of the informed strategy with (Q, A) we must satisfy that

$$\begin{aligned} \theta_q &\geq \theta_a \frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)} \frac{\phi}{1-\phi} \frac{1-P^*(\theta)}{P^*(\theta)} \\ \theta_q &\leq \theta_a \frac{\Delta \Pr(a, Q)}{\Delta \Pr(q, Q)} \frac{\phi}{1-\phi} \frac{P^*(\theta)}{1-P^*(\theta)} \end{aligned} \tag{19}$$

Using the fact that $g^q(\theta_a)$ is convex and $g^a(\theta_a)$ is concave

$$\begin{aligned}
\frac{\partial g^q(0)}{\partial\theta_a}\theta_a &\leq g^q(\theta_a) \leq \frac{\partial g^q(\theta_a)}{\partial\theta_a}\theta_a \\
\frac{\partial g^a(0)}{\partial\theta_a}\theta_a &\geq g^a(\theta_a) \geq \frac{\partial g^a(\theta_a)}{\partial\theta_a}\theta_a
\end{aligned}$$

where we used that $g^\omega(0) = 0$ for $\omega \in \{a, q\}$. An application of the implicit function theorem gives that $\frac{\partial g^q(\theta_a)}{\partial\theta_a} = \frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}\frac{\phi}{1-\phi}\frac{P^*(g^q(\theta_a),\theta_a)}{1-P^*(g^q(\theta_a),\theta_a)}$ and $\frac{\partial g^a(\theta_a)}{\partial\theta_a} = \frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}\frac{\phi}{1-\phi}\frac{1-P^*(g^a(\theta_a),\theta_a)}{P^*(g^a(\theta_a),\theta_a)}$ which implies

$$\begin{aligned}
1 &\leq \frac{g^q(\theta_a)}{\theta_a}\frac{\Delta\Pr(q,Q)}{\Delta\Pr(a,Q)}\frac{1-\phi}{\phi} \leq \frac{P^*(g^q(\theta_a),\theta_a)}{1-P^*(g^q(\theta_a),\theta_a)} \\
1 &\geq \frac{g^a(\theta_a)}{\theta_a}\frac{\Delta\Pr(q,Q)}{\Delta\Pr(a,Q)}\frac{1-\phi}{\phi} \geq \frac{1-P^*(g^a(\theta_a),\theta_a)}{P^*(g^a(\theta_a),\theta_a)}
\end{aligned} \tag{20}$$

Since supporters for A satisfy $g^a(\theta_a) > \theta_q$, by the second equation in (20), condition (18) holds for these voters. Using the first equation in (20) and the fact that supporters for Q satisfy $g^q(\theta_a) < \theta_q$, condition (18) does not hold for these voters. Therefore, both uninformed strategies are consistent.

Using the right hand side of the second inequality in (20) and the fact that $\theta_q \geq g^a(\theta_a)$ for independents gives the first equation in (19). Because $g^q(\theta_a)$ is monotone it is invertible and $\theta_q \leq g^q(\theta_a)$ is equal to $\theta_a \geq (g^q)^{-1}(\theta_q)$. For any $\theta_a \geq (g^q)^{-1}(\theta_q)$ we must have that $\frac{P^*\left(\theta_q,(g^q)^{-1}(\theta_q)\right)}{1-P^*\left(\theta_q,(g^q)^{-1}(\theta_q)\right)} \leq \frac{P^*(\theta_q,\theta_a)}{1-P^*(\theta_q,\theta_a)}$; using the first equation in (20) we have that $\frac{\theta_q}{(g^q)^{-1}(\theta_q)}\frac{\Delta\Pr(q,Q)}{\Delta\Pr(a,Q)} \leq \frac{P^*(\theta_q,\theta_a)}{1-P^*(\theta_q,\theta_a)}$. Using that $\theta_a \geq (g^q)^{-1}(\theta_q)$ we have that $\frac{\theta_q}{(g^q)^{-1}(\theta_q)} \geq \frac{\theta_q}{\theta_a}$ which gives the second equation in (19).

To show the characterization is complete, we need to show that no type $\theta \in [0,1]^2$ belongs to two different groups, and the union of independents and supporters covers all $[0,1]^2$. But this is obvious, since $g^a(\theta_a)$ and $g^q(\theta_a)$ cross each other only at $(0,0)$ for $\theta_a \geq 0$. ■

Proof of Proposition (2). $P^*(\theta)$ changes smoothly with $\Delta\Pr(\omega, Q)$, $\omega \in \{q, a\}$ for all $\theta \neq (0,0)$; this is a direct application of the implicit function theorem to (10). Using the definitions of $g^\omega(\theta_a)$, $\omega \in \{q, a\}$,

$$\begin{aligned}
\frac{\partial g^a(\theta_a)}{\partial\Delta\Pr(a,Q)} &= \frac{\theta_a}{\Delta\Pr(q,Q)}\frac{1-P^*(g^a(\theta_a),\theta_a)}{P^*(g^a(\theta_a),\theta_a)}\frac{\phi}{(1-\phi)} \\
\frac{\partial g^a(\theta_a)}{\partial\Delta\Pr(q,Q)} &= -\frac{g^a(\theta_a)}{\Delta\Pr(q,Q)}
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial g^q(\theta_a)}{\partial \Delta \Pr(a,Q)} &= \frac{\theta_a}{\Delta \Pr(q,Q)} \frac{P^*(g^q(\theta_a),\theta_a)}{1-P^*(g^q(\theta_a),\theta_a)} \frac{\phi}{(1-\phi)} \\
\frac{\partial g^q(\theta_a)}{\partial \Delta \Pr(q,Q)} &= -\frac{g^q(\theta_a)}{\Delta \Pr(q,Q)}
\end{aligned}
$$

which implies that both $g^a(\theta_a)$ and $g^q(\theta_a)$ are continuous in $\Delta \Pr(\omega,Q)$, $\omega \in \{q,a\}$. Therefore $\Pr(A \mid \omega)$, $\omega \in \{q,a\}$ are continuous in $\Delta \Pr(\omega,Q)$, $\omega \in \{q,a\}$ for all θ.

Let $X = \left[\alpha\xi_A, 1-\alpha\xi_Q\right]^2$ and $Y = [\zeta(a), 1-\zeta(a)] \times [\zeta(q), 1-\zeta(q)]$. Trivially $X \times Y$ is compact and convex subset of an euclidean space. Let $(x_1,x_2) \in X$ and $(y_1,y_2) \in Y$ be generic elements of these spaces.

In (10) replace $\Delta \Pr(a,Q)$ for y_1 and $\Delta \Pr(q,Q)$ for y_2 and define $P^*(\theta)$ implicitly for all $\theta \neq (0,0)$ in terms of y_1 and y_2 as $P^*(\theta \mid y_1,y_2)$. Now define first the cut off functions $g^a(\theta_a)$ and $g^q(\theta_a)$ by replacing $\Delta \Pr(a,Q)$ for y_1 and $\Delta \Pr(q,Q)$ for y_2 in the corresponding conditions (11) and using the function $P^*(\theta \mid y_1,y_2)$: $g^a(\theta_a \mid y_1,y_2)$ and $g^q(\theta_a \mid y_1,y_2)$.

Let $K_i : Y \to \left[\alpha\xi_A, 1-\alpha\xi_Q\right]$ for $i=1,2$ be such that

$$
\begin{aligned}
\frac{K_1(y_1,y_2)-\alpha\xi_A}{1-\alpha} &\equiv \int_0^1 \int_0^{\min\{1,g^a(\theta_a|y_1,y_2)\}} dF(\theta) + \int_0^1 \int_{\min\{1,g^a(\theta_a|y_1,y_2)\}}^{\min\{1,g^q(\theta_a|y_1,y_2)\}} P^*(\theta \mid y_1,y_2)\, dF(\theta) \\
\frac{K_2(y_1,y_2)-\alpha\xi_A}{1-\alpha} &\equiv \int_0^1 \int_0^{\min\{1,g^q(\theta_a|y_1,y_2)\}} dF(\theta) - \int_0^1 \int_{\min\{1,g^a(\theta_a|y_1,y_2)\}}^{\min\{1,g^q(\theta_a|y_1,y_2)\}} P^*(\theta \mid y_1,y_2)\, dF(\theta)
\end{aligned}
$$

$K_i, i=1,2$ are continuous in (y_1,y_2). Here K_1 plays the role of $\Pr(A \mid a)$ and K_2 plays the role of $\Pr(A \mid q)$

Let $K_3 : \left[\alpha\xi_A, 1-\alpha\xi_Q\right] \to [\zeta(a), 1-\zeta(a)]$ and $K_4 : \left[\alpha\xi_A, 1-\alpha\xi_Q\right] \to [\zeta(q), 1-\zeta(q)]$ be defined such that

$$
\begin{aligned}
K_3(x_1) &\equiv \frac{(n-1)!}{(n-1-N)!\,(N-1)!} (x_1)^{N-1} (1-x_1)^{n-1-N} \chi(x_1) \\
K_4(x_2) &\equiv \frac{(n-1)!}{(n-1-N)!\,(N-1)!} (x_2)^{N-1} (1-x_2)^{n-1-N} \chi(x_2)
\end{aligned}
$$

where $\chi(x) = \left(\frac{(1-x)(1-r)}{(n-N)} + \frac{xr}{N}\right)$. Trivially, $K_i, i=3,4$ are continuous in x_1 and x_2 respectively. Note that $K_3(x_1)$ plays the role of $\Delta \Pr(a,Q)$ and $K_4(x_2)$ plays the role of $\Delta \Pr(q,Q)$ in (15).

Let $\Gamma : X \times Y \to X \times Y$ be defined as $\Gamma \equiv (K_1,K_2,K_3,K_4)$ which is continuous. Therefore, applying Brouwer fixed point theorem (see Border (1985)), there is some $(x_1^*,x_2^*,y_1^*,y_2^*) \in X \times Y$ such that $\Gamma(x_1^*,x_2^*,y_1^*,y_2^*) = (x_1^*,x_2^*,y_1^*,y_2^*)$.

The fact that $(x_1^*, x_2^*, y_1^*, y_2^*)$ is an equilibrium follows trivially. Let $x_1^* = \Pr(A \mid a)$, $x_2^* = \Pr(A \mid q)$, $y_1^* = \Delta\Pr(a, Q)$ and $y_2^* = \Delta\Pr(q, Q)$. Since Γ has embedded the description of the best response functions $(g^a(\theta_a), g^q(\theta_a))$ and $P^*(\theta)$, for any pair $\Delta\Pr(\omega, Q)$, $\omega \in \{q, a\}$, the transformation Γ gives the optimal probabilities of voting for A in each state $\omega \in \{q, a\}$. (x_1^*, x_2^*) only ensures that actually we have a fixed point in the probabilities of voting that are constructed using $\Delta\Pr(\omega, Q)$, $\omega \in \{q, a\}$. ■

Proof of Proposition (3). First we are going to prove that $\Pr(Q \mid q) = \Pr(A \mid a)$ iff $\Delta\Pr(a, Q) = \Delta\Pr(q, Q)$. Using (7) and (12),

$$\Delta\Pr(\omega, Q) = \frac{2N!}{N!N!}\Pr(Q \mid \omega)^N (1 - \Pr(Q \mid \omega))^N \tag{21}$$

If $\Pr(Q \mid q) = \Pr(A \mid a)$ it is trivial to see that $\Delta\Pr(a, Q) = \Delta\Pr(q, Q)$.

Now assume that $\Delta\Pr(a, Q) = \Delta\Pr(q, Q)$. The first order condition for investment is just $C'(P^*(\theta)) = \frac{(\theta_q + \theta_a)\Delta\Pr(a,Q)}{2}$ so $P^*(\theta_1, \theta_2) = P^*(\theta_2, \theta_1)$. By definition,

$$\begin{aligned} C(P^*(g^a(\theta_a), \theta_a)) &= \frac{((\theta_a + g^a(\theta_a)) P^*(g^a(\theta_a), \theta_a) - \theta_a)\Delta\Pr(a, Q)}{2} \\ C(P^*(g^q(\theta_a), \theta_a)) &= \frac{((\theta_a + g^q(\theta_a)) P^*(g^q(\theta_a), \theta_a) - g^q(\theta_a))\Delta\Pr(a, Q)}{2} \end{aligned} \tag{22}$$

Let $\theta_a = \theta_1$ and $g^q(\theta_1) = \theta_2$ in the second equation of (22) to get

$$C(P^*(\theta_2, \theta_1)) = \frac{((\theta_1 + \theta_2) P^*(\theta_2, \theta_1) - \theta_2)\Delta\Pr(a, Q)}{2} \tag{23}$$

Using that $P^*(\theta_1, \theta_2) = P^*(\theta_2, \theta_1)$ on (23), it follows that

$$C(P^*(\theta_1, \theta_2)) = \frac{((\theta_1 + \theta_2) P(\theta_1, \theta_2) - \theta_2)\Delta\Pr(a, Q)}{2} \tag{24}$$

Let $\theta_2 = \theta_a$ in (24) and comparing with the first equation of (22) it follows that $g^a(\theta_2) = \theta_1$. It is easy to see the inverse also follows which implies that $g^q(\theta_a) = \theta_q \iff g^a(\theta_q) = \theta_a$ or $g^a = (g^q)^{-1}$ (geometric symmetry around the 45° line).

Now we have to calculate $\Pr(Q \mid q)$ and $\Pr(A \mid a)$. This expressions are:

$$\begin{aligned} \frac{\Pr(Q \mid q) - \frac{\alpha}{2}}{1 - \alpha} &= \int_0^1 \int_{\min\{g^q(\theta_a),1\}}^1 dF(\theta) + \int_0^1 \int_{\min\{g^a(\theta_a),1\}}^{\min\{g^q(\theta_a),1\}} P^*(\theta)\, dF(\theta) \\ \frac{\Pr(A \mid a) - \frac{\alpha}{2}}{1 - \alpha} &= \int_0^1 \int_0^{\min\{g^a(\theta_a),1\}} dF(\theta) + \int_0^1 \int_{\min\{g^a(\theta_a),1\}}^{\min\{g^q(\theta_a),1\}} P^*(\theta)\, dF(\theta) \end{aligned} \tag{25}$$

Recall that $\frac{\partial g^a(\theta_a)}{\partial \theta_a} \leq \frac{\Delta \Pr(a,Q)}{\Delta \Pr(q,Q)} \leq \frac{\partial g^q(\theta_a)}{\partial \theta_a}$, which implies that there is some $\theta_a^{**} \leq 1$ such that $g^q\left(\theta_a^{**}\right) = 1$, and the previous expressions are

$$\begin{aligned} \frac{\Pr\left(Q \mid q\right) - \frac{\alpha}{2}}{1-\alpha} &= \int_0^{\theta_a^{**}} \int_{g^q(\theta_a)}^{1} dF\left(\theta\right) + \mathcal{T} \\ \frac{\Pr\left(A \mid a\right) - \frac{\alpha}{2}}{1-\alpha} &= \int_0^{1} \int_{0}^{g^a(\theta_a)} dF\left(\theta\right) + \mathcal{T} \end{aligned} \tag{26}$$

where

$$\begin{aligned} \mathcal{T} &\equiv \int_0^1 \int_{\min\{g^a(\theta_a),1\}}^{\min\{g^q(\theta_a),1\}} P^*\left(\theta\right) dF\left(\theta\right) \\ &\equiv \int_0^1 \int_{g^a(\theta_a)}^{\theta_a} P^*\left(\theta\right) dF\left(\theta\right) + \int_0^{\theta_a^{**}} \int_{\theta_a}^{g^q(\theta_a)} P^*\left(\theta\right) dF\left(\theta\right) \end{aligned}$$

Reversing the order of integration, using that $g^a = \left(g^q\right)^{-1}$ and F symmetric we have

$$\begin{aligned} \int_0^{\theta_a^{**}} \left(\int_{\theta_a}^{g^q(\theta_a)} P^*\left(\theta\right) f\left(\theta\right) d\theta_q \right) d\theta_a &= \int_0^1 \left(\int_{(g^q)^{-1}(\theta_q)}^{\theta_q} P^*\left(\theta\right) f\left(\theta\right) d\theta_a \right) d\theta_q \\ &= \int_0^1 \left(\int_{g^a(\theta_q)}^{\theta_q} P^*\left(\theta\right) f\left(\theta\right) d\theta_a \right) d\theta_q \end{aligned}$$

so $\mathcal{T} = 2\int_0^1 \int_{g^a(\theta_a)}^{\theta_a} P^*\left(\theta\right) dF\left(\theta\right)$. Using the same argument we have

$$\int_0^1 \int_0^{g^a(\theta_a)} f\left(\theta\right) d\theta_a d\theta_q = \int_0^{\theta_a^{**}} \int_{g^q(\theta_q)}^{1} f\left(\theta\right) d\theta_q d\theta_a$$

which in turns implies that $\Pr\left(A \mid a\right) = \Pr\left(Q \mid q\right)$. Now we are ready to show that there is such equilibrium.

Let $X' = \{(x_1, x_2) \in X : x_1 = x_2\}$ and $Y' = \{(y_1, y_2) \in Y : y_1 = y_2\}$ where where $X = \left[\alpha\xi_A, 1 - \alpha\xi_Q\right]^2$ and $Y = \left[\zeta\left(a\right), 1 - \zeta\left(a\right)\right] \times \left[\zeta\left(q\right), 1 - \zeta\left(q\right)\right]$ as in the Proof of Proposition

(2). Since X' is closed and convex in X and Y' is closed and convex in Y, the argument used in that proof with the transformation $\Gamma : X \times Y \to X \times Y$ is valid and the Brouwer fixed point theorem gives that there is $(x_1, x_2, y_1, y_2) \in X' \times Y'$ such that $\Gamma(x_1, x_2, y_1, y_2) = (x_1, x_2, y_1, y_2)$ and $x_1 = x_2$, $y_1 = y_2$.

To complete the proof we define $y_N = 2\int_0^1 \int_{g^a(\theta_a)}^{\theta_a} \left(P^*(\theta) - \frac{1}{2}\right) dF(\theta)$ and $x_N = \frac{2N!}{N!N!} \left(\frac{1}{4} - (y_N)^2 (1-\alpha)^2\right)^N$

■

Proof of Proposition (4). We first prove the following Lemma:

Lemma 5 *In any symmetric committee with the simple majority rule* $\Pr(A \mid a)$ *is decreasing in* N*.*

Proof. *We prove this by contradiction. Let* $\varphi(x) = \left(\frac{1}{2}\right)^2 - x^2(1-\alpha)^2$*. Assume that* $\Pr(A \mid a) = P_N$ *increases with* N*; therefore, we must have that* $(\varphi(P_N))^N \geq (\varphi(P_{N+1}))^N$*. Using that* $\frac{2N!}{N!N!} > \frac{1}{4}\frac{2N+2!}{N+1!N+1!}$ *and* $\varphi(P_{N+1}) \leq \frac{1}{4}$*,*

$$\frac{2N!}{N!N!}(\varphi(P_N))^N \geq \frac{2N+2!}{N+1!N+1!}(\varphi(P_{N+1}))^{N+1}$$

and, $\Delta \Pr(\omega, Q)$ *in (21) decreases with* N*, which in turns imply that* $P^*(\theta)$ *must decrease with* N *as well. Therefore* $\frac{1-P^*(\theta)}{P^*(\theta)}$ *increases with* N *and the slope of the function* $g^a(\theta_a)$ *increases while the slope of the function* $g^q(\theta_a)$ *decreases. Then, the functions* $g^a(\theta_a)$ *and* $g^q(\theta_a)$ *get closer when* N *increases.*

Recalling the expression for $\frac{\Pr(A|a) - \frac{\alpha}{2}}{1-\alpha}$ *and using the symmetry of the equilibrium, and some algebra gives,*

$$\frac{\Pr(A \mid a) - \frac{\alpha}{2}}{1-\alpha} = \frac{1}{2} + 2\int_0^1 \int_{g^a(\theta_a)}^{\theta_a} \left(P^*(\theta) - \frac{1}{2}\right) dF(\theta) \tag{27}$$

and because the slope of $g^a(\theta_a)$ *increases and* $P^*(\theta)$ *decreases, we must have that* $\Pr(A \mid a)$ *is also decreasing with* N*.* ■

Once we know that $\Pr(A \mid a)$ decreases with N, we must also have that investment decreases and therefore, $\Delta \Pr(\omega, Q)$ must also be decreasing in N. Since $P^*(\theta)$ changes monotonically with $\Delta \Pr(\omega, Q)$, it must be the case that $P^*(\theta)$ changes in the same way for all types $(\theta_q, \theta_a) \in [0,1]^2$. Using equation (27), the fact that $\frac{\partial g^a(\theta_a)}{\partial \theta_a} = \frac{1-P^*(g^a(\theta_a),\theta_a)}{P^*(g^a(\theta_a),\theta_a)}$ we have that if $P^*(\theta)$ increases with N, it must be the case that $\Pr(A \mid a)$ also increases. A contradiction.

Using the investment function the result on $\Delta \Pr(\omega, Q)$ follows. ■

Proof of Proposition (5). We follow Martinelli (2006) although we must consider a continuum of types of voters instead of an homogeneously informed voter. First we are going to construct a random variable that describes the difference between the probability of voting for one candidate and the other. Then we are going to apply Berry-Esseen Theorem (see Bickel and Doksum (2000)).

Let $y_N = 2\int_0^1 \int_{g^a(\theta_a)}^{\theta_a} \left(P^*(\theta) - \frac{1}{2}\right) d\theta_q d\theta_a$ on (27). Define the random variable M_i^N such that $M_i^N \equiv \frac{1}{2} - y_N(1-\alpha)$ if $v = A$ and $M_i^N \equiv -\frac{1}{2} - y_N(1-\alpha)$ if $v = Q$. It is easy to see that $E\left(M_i^N\right) = 0$, $E\left(\left(M_i^N\right)^2\right) = \frac{1}{4} - (y_N(1-\alpha))^2$ and $E\left(\left|M_i^N\right|^3\right) = 2\left(\left(\frac{1}{2}\right)^4 - (y_N(1-\alpha))^4\right)$.

Therefore, $\overline{M}_i^N \equiv \frac{M_i^N}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}$ is a random variable with zero mean, variance equal to 1 and $E\left(\left|\overline{M}_i^N\right|^3\right) = \frac{2\left(\left(\frac{1}{2}\right)^2+(y_N(1-\alpha))^2\right)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}$. Define $M^N \equiv \sum_{i=1}^{2N+1} \overline{M}_i^N = \frac{T_{2N+1}(A)-\frac{2N+1}{2}-(2N+1)y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}$ and recall that $T_{2N+1} \equiv \sum_{i=1}^{2N+1} I(v_i = A)$ is the number of votes for A out of $2N+1$ voters. We know that A is the winner if $T_{2N+1} > N$ and Q is the winner if $T_{2N+1} \leq N$; therefore we require that $M^N = \frac{T_{2N+1}(A)-\frac{2N+1}{2}-(2N+1)y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}} > \frac{-\frac{1}{2}-(2N+1)y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}$ for $T_{2N+1} > N$. Let $\mathcal{M}^N \equiv \frac{M^N}{\sqrt[2]{(2N+1)}}$ and $\mathcal{F}^N$ be its distribution. The probability of A being the winner is just the probability that $\mathcal{M}^N > -\frac{\frac{\frac{1}{2}+(2N+1)y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}}{\sqrt[2]{(2N+1)}}$; if we let $J^N(y_N) \equiv -\frac{\frac{\frac{1}{2}+(2N+1)y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}}{\sqrt[2]{(2N+1)}}$ the probability of A being the winner is just $1 - \mathcal{F}^N\left(J^N(y_N)\right)$. Replacing, we get

$$J^N(y_N) = -\frac{1}{2\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)(2N+1)}} - \frac{\sqrt[2]{(2N+1)}y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}.$$

Let Φ be the cdf of a $(0,1)$ normal random variable, so we can apply the Berry-Esseen Theorem[21] to get that $\lim_{N\to\infty} \mathcal{F}^N\left(J^N(y_N)\right) \to \Phi\left(J^N(y_N)\right)$ if $E\left(\left|\overline{M}_i^N\right|^3\right)$ is finite, which is the case for some N big enough so y_N is close to 0 since $\Delta \Pr(\omega, Q)$ approaches 0 and hence $P^*(\theta) \to \frac{1}{2}$.[22] Now using that Φ is continuous we must have that $\lim_{N\to\infty} \Phi\left(J^N(y_N)\right) \to$

[21] Let $X_1, ...X_n$ be $i.i.d$ with mean $\mu = 0$ and $\sigma^2 = 1$. Then, for all n
$\sup\left|\Pr\left(\frac{\sum_{i=1}^n X_i}{\sqrt[2]{n}} \leq t\right) - \Phi(t)\right| \leq \frac{33}{4}\frac{E|X_1|^3}{\sqrt[2]{n}}$. See Bickel and Doksum (2000).

[22] Paradoxically, the fact that adding a new voter decreases the average informativeness of each vote is helpful in order to prove aggregation.

$\Phi\left(\lim_{N\to\infty} J^N(y_N)\right)$, which implies that $\lim_{N\to\infty} \mathcal{F}^N\left(J^N(y_N)\right) \to \Phi\left(\lim_{N\to\infty} J^N(y_N)\right)$.

The problem is now the limit of $J^N(y_N)$ or $\lim_{N\to\infty} \widehat{J}^N(y_N)$ where $\widehat{J}^N(y_N) = -\frac{\sqrt[2]{(2N+1)}y_N(1-\alpha)}{\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)}}$ since $\frac{1}{2\sqrt[2]{\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)(2N+1)}} \to 0$ as N grows. If $\sqrt[2]{(2N+1)}y_N(1-\alpha) \to \infty$, it follows that $\widehat{J}^N(y_N) \to -\infty$, and $1-\mathcal{F}^N\left(J^N(y_N)\right) \to 1$ which makes A the winner almost surely in state a.

Recall that $\Delta \Pr(a,Q) = \frac{(2N)!}{N!N!}\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)^N$ therefore

$$C'(P^*(x)) = \frac{x}{2}\frac{(2N)!}{N!N!}\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)^N \tag{28}$$

$$y_N = 2\int_0^1\left(\int_{\theta_a+g^a(\theta_a)}^{2\theta_a}\left(P(x)-\frac{1}{2}\right)dx\right)d\theta_a$$

Assume that $C''\left(\frac{1}{2}\right) = l > 0$; since $P^*(x)$ is concave we must have $P^*(x) \leq P^*(0) + \left.\frac{\partial P^*(x)}{\partial x}\right|_{x=0} x$ so

$$z_N \leq \frac{(1-\alpha)}{l}\frac{(2N)!}{N!N!}\frac{\sqrt[2]{N}}{2^{2N}}\left(1-\frac{(2z_N)^2}{N}\right)^N\int_0^1\left(\int_{\theta_a+g^a(\theta_a)}^{2\theta_a} x\,dx\right)d\theta_a \tag{29}$$

where we used that $\frac{\partial P^*(x)}{\partial x} = \frac{1}{2C''(P^*(x))}\frac{(2N)!}{N!N!}\left(\frac{1}{4}-(y_N(1-\alpha))^2\right)^N$ and define $z_N \equiv y_N(1-\alpha)\sqrt[2]{N}$. Let $\frac{(2N)!}{N!N!}\frac{\sqrt[2]{N}}{2^{2N}}\left(1-\frac{(2z_N)^2}{N}\right)^N \equiv h(z_N,N)$ and note that $e^{-4z_N} = \lim_{N\to\infty}\left(1-\frac{(2z_N)^2}{N}\right)^N$ and $\lim_{N\to\infty}\frac{(2N)!}{N!N!}\frac{\sqrt[2]{N}}{2^{2N}} = \pi^{-1}$ so $h(z_N,N) \to e^{-4z_N}\pi^{-1}$. Therefore, since $\int_0^1\int_{\theta_a+g^a(\theta_a)}^{2\theta_a} x\,dx \to 0$ we must have that $\lim_{N\to\infty} z_N = 0$. This proves the first part of the proposition.

Now , for the case that $C''\left(\frac{1}{2}\right) = 0$ and $C'''\left(\frac{1}{2}\right) \geq 0$ we have that

$$\begin{aligned} y_N &\geq 2\int_0^1\left(\left(P^*(\theta_a+g^a(\theta_a))-\frac{1}{2}\right)\int_{\theta_a+g^a(\theta_a)}^{2\theta_a} dx\right)d\theta_a \\ &\geq 2\int_0^1\left(P^*(\theta_a+g^a(\theta_a))-\frac{1}{2}\right)(\theta_a-g^a(\theta_a))\,d\theta_a \end{aligned}$$

Using concavity of P^*: $P^*(\theta_a + g^a(\theta_a)) \geq P^*(0) + \frac{\partial P^*(x)}{\partial x}_{x=\theta_a+g^a(\theta_a)} (\theta_a + g^a(\theta_a))$ it follows

$$\begin{aligned} z_N &\geq (1-\alpha) h(z_N, N) \int_0^1 \frac{\left((\theta_a)^2 - (g^a(\theta_a))^2\right)}{C''(P^*(g^a(\theta_a) + \theta_a))} d\theta_a \\ &\geq (1-\alpha) h(z_N, N) \int_0^1 \mathcal{H}_1(\theta_a) d\theta_a \end{aligned}$$

where $\mathcal{H}_1(\theta_a) \equiv \frac{\left((\theta_a)^2 - (g^a(\theta_a))^2\right)}{C''(P^*(2\theta_a))}$ and we used $C''' \geq 0$. Using L'Hopital's rule we have that

$$\lim_{\Delta \Pr(a,Q)\to 0} \mathcal{H}_1(\theta_a) = \lim_{\Delta \Pr(a,Q)\to 0} \frac{-2(g^a(\theta_a)) \frac{\partial g^a(\theta_a)}{\partial \Delta \Pr(a,Q)}}{C'''(P^*(2\theta_a)) \frac{\partial P^*(2\theta_a)}{\partial \Delta \Pr(a,Q)}}$$

Using the system of equations for $P^*(g^a(\theta_a) + \theta_a)$ and $g^a(\theta_a)$

$$\begin{aligned} C(P^*(g^a(\theta_a) + \theta_a)) &= \frac{((\theta_a + g^a(\theta_a)) P^*(g^a(\theta_a) + \theta_a) - \theta_a) \Delta \Pr(a,Q)}{2} \\ C'(P^*(g^a(\theta_a) + \theta_a)) &= \frac{(\theta_a + g^a(\theta_a)) \Delta \Pr(a,Q)}{2} \end{aligned}$$

we have

$$\begin{aligned} \frac{\partial g^a(\theta_a)}{\partial \Delta \Pr(a,Q)} &= -\frac{\mathcal{H}_2(\theta_a)}{P^*(g^a(\theta_a) + \theta_a) \Delta \Pr(a,Q)} \\ \frac{\partial P^*(g^a(\theta_a) + \theta_a)}{\partial \Delta \Pr(a,Q)} &= \frac{\theta_a}{2P^*(g^a(\theta_a) + \theta_a) C''(P^*(g^a(\theta_a) + \theta_a))} \\ \frac{\partial P^*(2\theta_a)}{\partial \Delta \Pr(a,Q)} &= \frac{\theta_a}{C''(P^*(2\theta_a))} \end{aligned} \tag{30}$$

where we define $\mathcal{H}_2(\theta_a) = (\theta_a + g^a(\theta_a)) P^*(g^a(\theta_a) + \theta_a) - \theta_a$. So

$$\begin{aligned} \lim_{\Delta \Pr(a,Q)\to 0} \mathcal{H}_1(\theta_a) &= \lim_{\Delta \Pr(a,Q)\to 0} \frac{2(g^a(\theta_a)) \frac{\mathcal{H}_2(\theta_a)}{P^*(g^a(\theta_a)+\theta_a)\Delta \Pr(a,Q)}}{C'''(P^*(2\theta_a)) \frac{\theta_a}{C''(P^*(2\theta_a))}} \\ &= \lim_{\Delta \Pr(a,Q)\to 0} \frac{\frac{2}{P^*(g^a(\theta_a)+\theta_a)} \frac{(g^a(\theta_a))}{\theta_a} \frac{\mathcal{H}_2(\theta_a)}{\Delta \Pr(a,Q)}}{\frac{C'''(P^*(2\theta_a))}{C''(P^*(2\theta_a))}} \\ &= 4 \lim_{\Delta \Pr(a,Q)\to 0} \frac{\mathcal{H}_2(\theta_a) C''(P^*(2\theta_a))}{C'''(P^*(2\theta_a)) \Delta \Pr(a,Q)} \end{aligned}$$

L'Hopital again gives that $\lim\limits_{\Delta \Pr(a,Q)\to 0} \frac{\mathcal{H}_2(\theta_a)C''(P^*(2\theta_a))}{\Delta \Pr(a,Q)}$ is equal to

$$\lim_{\Delta \Pr(a,Q)\to 0}\left(\mathcal{H}_2\left(\theta_a\right)C'''\left(P^*\left(2\theta_a\right)\right)\frac{\partial P^*\left(2\theta_a\right)}{\partial \Delta \Pr\left(a,Q\right)}+\frac{\partial \mathcal{H}_2\left(\theta_a\right)}{\partial \Delta \Pr\left(a,Q\right)}C''\left(P^*\left(2\theta_a\right)\right)\right)$$

Using the expressions for $\frac{\partial P^*(g^a(\theta_a)+\theta_a)}{\partial \Delta \Pr(a,Q)}$ and $\frac{\partial g^a(\theta_a)}{\partial \Delta \Pr(a,Q)}$ to get $\frac{\partial \mathcal{H}_2(\theta_a)}{\partial \Delta \Pr(a,Q)}$ in (30) we have that $\lim\limits_{\Delta \Pr(a,Q)\to 0} \frac{\mathcal{H}_2(\theta_a)C''(P^*(2\theta_a))}{\Delta \Pr(a,Q)}$ is equal to

$$\begin{aligned}&\lim_{\Delta \Pr(a,Q)\to 0}\frac{\theta_a\left(g^a\left(\theta_a\right)+\theta_a\right)C''\left(P^*\left(2\theta_a\right)\right)}{2P^*\left(g^a\left(\theta_a\right)+\theta_a\right)C''\left(P^*\left(g^a\left(\theta_a\right)+\theta_a\right)\right)}\\&+\lim_{\Delta \Pr(a,Q)\to 0}\left(\frac{\mathcal{H}_2\left(\theta_a\right)C'''\left(P^*\left(2\theta_a\right)\right)\theta_a}{C''\left(P^*\left(2\theta_a\right)\right)}-\frac{\mathcal{H}_2\left(\theta_a\right)C''\left(P^*\left(2\theta_a\right)\right)}{\Delta \Pr\left(a,Q\right)}\right)\end{aligned} \tag{31}$$

and some algebra gives that $2\lim\limits_{\Delta \Pr(a,Q)\to 0} \frac{\mathcal{H}_2(\theta_a)C''(P^*(2\theta_a))}{\Delta \Pr(a,Q)}$ is equal to

$$\begin{aligned}&\lim_{\Delta \Pr(a,Q)\to 0}\mathcal{H}_2\left(\theta_a\right)\theta_a\frac{C'''\left(P^*\left(2\theta_a\right)\right)}{C''\left(P^*\left(2\theta_a\right)\right)}\\&+\lim_{\Delta \Pr(a,Q)\to 0}\frac{\theta_a\left(g^a\left(\theta_a\right)+\theta_a\right)C''\left(P^*\left(2\theta_a\right)\right)}{2P^*\left(g^a\left(\theta_a\right)+\theta_a\right)C''\left(P^*\left(g^a\left(\theta_a\right)+\theta_a\right)\right)}\end{aligned}$$

Since $\mathcal{H}_2\left(\theta_a\right)\geq 0$ (see (16)) and $\lim\limits_{\Delta \Pr(a,Q)\to 0}\frac{\theta_a(g^a(\theta_a)+\theta_a)}{2P^*(g^a(\theta_a)+\theta_a)}=2\left(\theta_a\right)^2$

$$2\lim_{\Delta \Pr(a,Q)\to 0}\frac{\mathcal{H}_2\left(\theta_a\right)C''\left(P^*\left(2\theta_a\right)\right)}{\Delta \Pr\left(a,Q\right)}\geq 2\left(\theta_a\right)^2\lim_{\Delta \Pr(a,Q)\to 0}\frac{C''\left(P^*\left(2\theta_a\right)\right)}{C''\left(P^*\left(g^a\left(\theta_a\right)+\theta_a\right)\right)}$$

using that $C'''\geq 0$ and $g^a\left(\theta_a\right)<\theta_a$, it follows that

$$\lim_{\Delta \Pr(a,Q)\to 0}\frac{\mathcal{H}_2\left(\theta_a\right)C''\left(P^*\left(\theta_a+g^a\left(\theta_a\right)\right)\right)}{\Delta \Pr\left(a,Q\right)}\geq\left(\theta_a\right)^2$$

so using that $C''\left(P^*\left(2\theta_a\right)\right)\geq C''\left(P^*\left(\theta_a+g^a\left(\theta_a\right)\right)\right)$ we have

$$\lim_{\Delta \Pr(a,Q)\to 0}\mathcal{H}_1\left(\theta_a\right)\geq 4\lim_{\Delta \Pr(a,Q)\to 0}\frac{\left(\theta_a\right)^2}{C'''\left(P^*\left(\theta_a+g^a\left(\theta_a\right)\right)\right)}$$

and therefore if $\lim\limits_{\Delta \Pr(a,Q)\to 0}C'''\left(P^*\left(\theta_a+g^a\left(\theta_a\right)\right)\right)\to 0$ (as it is for $C'''\left(\frac{1}{2}\right)=0$) we have $\lim\limits_{\Delta \Pr(a,Q)\to 0}\mathcal{H}_1\left(\theta_a\right)\to\infty$, which proves that $z_N\to\infty$. If $C'''\left(\frac{1}{2}\right)=l>C''\left(\frac{1}{2}\right)=0$, a lower bound for z_N is obtained. ■