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# Perks: Contractual Arrangements to Restrain Moral Hazard

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#### Abstract

*Perks* are a commodity bundle offered by an employer to an employee. They are used to directly control an employee's consumption. Consuming certain goods increases the marginal disutility of non-contractible effort. Lower consumption of such goods will make it less costly to induce an employee to put in high effort. To compensate for the decrease in such goods, an employer gives *luxurious* perks. By "luxurious" I mean that per-dollar marginal utilities of these perks are lower than those of other goods. This model explains the existence of perks such as box seat tickets and club memberships, which neither save tax nor enter the production function. Also, perks can be more luxurious at an unsuccessful outcome than at a successful outcome, and an employee with a more successful history receives more perks.

JEL Classification:

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# 1 Introduction

Why do perks exist? Why not just pay an employee in cash and let the employee purchase these products? A possible clue lies in the fact that many such perks are products that the employee seems unlikely to purchase, even if he were given the money. In other words, perks are usually luxurious. This begs the question: "Why does an employer want her employees to consume these luxuries?"

I study perks in the framework of the principal-agent model. For simplicity, suppose that there are three commodities: *money* (a numeraire good), a *condominium*, and *effort*. An agent puts in effort to produce money. The production process is stochastic, and its outcome distribution depends on effort. Effort is non-contractible. Therefore, there is a typical moral hazard problem.

Ex-post efficient allocation equates the per-dollar marginal utility of money to the perdollar marginal utility of the quality of the condominium. Assume that the marginal disutility of effort is invariant to the quality of the condominium, but decreases in the amount of money.<sup>1</sup> Suppose that the employer increases the quality of the condominium above the expost efficient allocation, and correspondingly decreases the cash salary to hold the employee's utility level constant. The implementation of higher effort becomes easier due to the lowered marginal disutility of effort. Thus, the change improves ex-ante efficiency even though expost efficiency is not satisfied. Therefore, an optimal contract would award a condominium of better quality than what an employee would have chosen, i.e. the condominium is a luxurious perk. My finding suggest that luxurious perks mitigate agency problems.

<sup>&</sup>lt;sup>1</sup>Under the interpretation that effort is reciprocal of leisure, empirical evidence demonstrates the increasing marginal disutility of effort in money. See Grossman and Hart (1983), Browning and Meghir (1991), and Bennardo and Chiappori (2003) for detailed discussion. The assumption is a simplification that any two commodities have different effects on the marginal disutility of effort. A generalized model is also presented in Appendix A.2.

I also show that perks are *more luxurious*<sup>2</sup> when the moral hazard problem is more severe. Moreover, perks can be more luxurious at an unsuccessful outcome than at a successful outcome. The media criticize lavish perks more when a firm is performing badly or is on the verge of bankruptcy. However, ex-ante efficiency requires such "excessive perks" at an unsuccessful outcome.

Alternatively, I could interpret money as tomorrow's consumption, and the condominium as today's consumption. In this interpretation, a principal devises an optimal wage scheme over two periods, and the principal wants the agent to consume more today than tomorrow.<sup>3</sup>

A dynamic model similar to Rogerson (1985) with a sequence of moral hazard problems confirms the aforementioned interpretation. I also find that a principal is more likely to encourage an agent's current period consumption when the agent's outside option – to opt out of the principal-agent relationship – is less favorable. If the principal suppresses the following period's consumption too much (by encouraging the current period's consumption), the agent might choose to sign up with another principal after the current period's consumption. Finally, I find that under a certain condition, the more senior an employee with a successful history becomes, the more perks he is awarded.

#### 1.1 Comparison with Previous Works on Perks

Some economists argue that perks lead to a moral hazard problem: an employer cannot monitor whether an employee abuses them or not. In this view, perks are "non-productive goods." Other economists, following Alchian and Demsetz (1972), consider perks as a consequence of a moral hazard problem. When the members of a profit-sharing firm have to purchase input factors personally, there is an under-investment problem (or, equivalently, a

<sup>&</sup>lt;sup>2</sup>When the difference between the per-dollar marginal utilities of money and the quality of a condominium is larger, I say that perks are more luxurious.

<sup>&</sup>lt;sup>3</sup>Henderson and Spindler (2005) also rationalize addictive perks in a dynamic setting. However, I do not assume the addictive property of perks, and give a formal mathematical modeling.

*free-rider problem*) since each does not fully appropriate the profit from these investments. If the problem is severe, it could be efficient to give the input factor as a perk, in spite of the possible abuse. This second view considers perks as "productive goods." <sup>4</sup>

Both of these views share the idea that the employer cannot observe the usage of perks. However, many expensive perks can easily be monitored. For example, it is not difficult to check whether a private jet is used for business or for personal reasons. The cost of monitoring this will be insignificant compared to the cost of the jet. In fact, it is often a legal requirement to report such expensive perks to the public. <sup>5</sup> If the use of perks is observable, it is explicitly contractible. Thus, I consider perks as a contingent payment.<sup>6</sup> Instead of devising a universal theory of perks,<sup>7</sup> I restrict my interest to this type of perks.

I do not assume that perks provide intrinsic motivation (no consumption complementarities between perks and effort), nor do I assume perks have a productive use, as in most of the literature. Those assumptions automatically justify the existence of perks. However, there are many perks that do not seem to help production or reduce the agent's cost of effort. For examples, corporate retreats involving horse back riding in Santa Fe, volleyball in Bari, or sailing in Greece, may be useful for "team building", but more frugal destinations might be equally useful. Other examples include fancy company cars, a "training program"

<sup>4</sup>Yermack (2006) uses this term for the consumption of non-productive goods and services. Jensen and Meckling (1976) and Rajan and Wulf (2006) distinguish productive and non-productive perks. Marinoa and Zábojník (2006) mainly consider perks as the consumption of productive goods. Over (2007) considers perks that have complementarities with effort and production.

<sup>5</sup>For example, new SEC rules since 2006 require public companies to list all perks over \$10,000. For top rankers in receiving perks, see

 $http://www.bayareanewsgroup.com/multimedia/mn/biz/specialreport/wtbm_underceosorts.pdf$ 

<sup>6</sup>Perks are clearly a part of compensation scheme, and contracts are renewed over time (the agent is promoted, demoted, or laid-off). Therefore, perks are also renewed. This reflects the repeated contractual relationship between a principal and an agent.

<sup>7</sup>Because of the elusiveness of the term, the SEC even refuses to define the term (SEC, 2006, p. 6553).

 $http://www.bayareanewsgroup.com/multimedia/mn/biz/specialreport/wtbm\_ceosorts.pdf$ 

on a Mediterranean island, a car service home in a Lincoln town car, and a lavish corporate holiday party.

I also do not assume that a principal and an agent can save on tax by having perks. Hypothetically, a principal could report perks as a cost of the production, get a tax deduction, and thus provide the perks at a lower cost than the agent would pay privately. However, the tax advantage explanation fails to explain why we do not often see perks in lower paid jobs. For example, there is typically no commuting subsidy for general office workers, while executives often receive corporate cars with a chauffeur. Furthermore, many perks are now fully subject to tax.<sup>8</sup>

Section 2 builds a static model, and presents the results. Section 3 develops a dynamic model, and presents the results. Section 4 concludes.

# 2 Static Model

There are two goods, money  $(m \in \mathbb{R})$  and a condominium.  $q \in \mathbb{R}$  is the quality of the condominium. The price of money is unity, and the expenditure on a condominium of quality q is pq, i.e. the expenditure increases linearly in quality q.<sup>9</sup> The principal owns a technology which produces money. The outcome of the production is stochastic. The probability of output  $s \in S$  is denoted by  $\varphi(s; e)$ , which is a function of effort  $e \in \{e_H, e_L\}$  chosen by the agent.<sup>10</sup> Utility function of the agent is quasi-separable in effort, i.e.

$$U(m,q;e) := v(m;e) + u(q).$$

<sup>&</sup>lt;sup>8</sup>Since many perks are listed to the public, they could be taxed. For example, Meg Whitman (eBay) was invited to use corporate planes for up to 200 hours of personal travel annually. That added up to more than \$773,000, plus nearly \$231,000 more to cover her tax bills for the perk.

<sup>&</sup>lt;sup>9</sup>I assume that the cost of supplying quality is linear; my results are robust to alternative assumptions, such as a convex cost function.

<sup>&</sup>lt;sup>10</sup>The assumption of two effort levels is for the simplicity of analysis. Extension to a continuum support of effort is straightforward.

A generalized model without the quasi-separability is presented in Appendix A.2. The utility function satisfies: u' > 0, u'' < 0,  $v'(m; e) := \frac{\partial v(m; e)}{\partial m} > 0$ , and  $v''(m; e) := \frac{\partial^2 v(m; e)}{\partial m^2} < 0$ .

I assume that the marginal disutility of effort,  $v(m; e_L) - v(m; e_H)$ , increases in money unless mentioned otherwise.

#### Assumption 1 (Income Effect of Money)

$$v'(m; e_H) < v'(m; e_L)$$

On the other hand, the marginal disutility of effort is constant in the quality of the condominium as  $u(\cdot)$  does not have argument e.

The principal is risk neutral. The principal's expected revenue is  $R(e) := \sum_{s \in S} s\varphi(s; e)$ . The principal designs an optimal contract  $(m(s), q(s))_{s \in S}$ : m(s) and q(s) are the two goods awarded at output s. The principal uses the two goods to enforce an optimal effort level of the agent. The principal's expected profit is

$$R(e) - \sum_{s \in S} [m(s) + pq(s)]\varphi(s; e).$$

A contract implementing  $e_L$  would employ a fixed wage, which is not interesting. Therefore, I assume that  $e_H$  is implemented. The typical incentive compatibility constraint and the participation constraint are

$$(\lambda): \sum_{s \in S} \left[ v(m(s); e_H) + u(q(s)) \right] \varphi(s; e_H) \ge \sum_{s \in S} \left[ v(m(s); e_L) + u(q(s)) \right] \varphi(s; e_L), \forall e' \quad [IC]$$

$$(\rho): \sum_{s \in S} \left[ v(m(s); e_H) + u(q(s)) \right] \varphi(s; e_H) \ge 0$$
[IR]

 $\lambda$  and  $\rho$  are the shadow value of the corresponding constraints.

The principal's problem is:

$$\max_{m(s),q(s)} \qquad R(e_H) - \sum_{s \in S} \left[ m(s) + pq(s) \right] \varphi(s; e_H) \quad s.t. \quad [IC] \text{ and } [IR]$$

I derive the following main theorem and a lemma.

Theorem 1

$$v'(m(s); e_H) = \frac{u'(q(s))}{p} \left[ 1 - \lambda(v'(m(s); e_H) - v'(m(s); e_L)) \frac{\varphi(s; e_L)}{\varphi(s; e_H)} \right]$$
(1)

*Proof.* See Appendix A.1

#### Lemma 1 $\rho > 0$ and $\lambda > 0$ .

*Proof.* See Appendix A.3

Note that the monetary value of the agent's consumption bundle at state s is

$$M(s) := m(s) + pq(s).$$

If the agent could have accessed the spot market of the goods, the agent's *ex-post* maximization would be

$$\max_{\tilde{m}(s),\tilde{q}(s)} \left[ v(\tilde{m}(s);e) + u(\tilde{q}(s)) \right] \quad \text{subject to} \quad M(s) := \tilde{m}(s) + p\tilde{q}(s).$$

The first order condition is

$$v'(\tilde{m}(s); e) = \frac{u'(\tilde{q}(s))}{p},$$
[SMC]

which I call the *spot market constraint*, [SMC]. This conditions states that per-dollar marginal utilities of the two goods are the same.

Notice that Equation (1) in Theorem 1 is not consistent with [SMC] unless  $\lambda(v'(m(s); e_H) - v'(m(s); e_L)) \frac{\varphi(s; e_L)}{\varphi(s; e_H)}$  is zero. From this observation, I derive a corollary about the relationship among the moral hazard problem, spot market access, and the income effect.

Corollary 1 [A] Equation (1) is consistent with [SMC] if there is no moral hazard problem.
[B] Without the income effect of money, Equation (1) is consistent with [SMC].

*Proof.* [A] Substituting  $\lambda$  with zero in Equation (1), I derive [SMC]. [B] From Equation (1), I derive [SMC] if  $v'(m(s); e_H) = v'(m(s); e_L)$ .

The following corollary is trivial.

**Corollary 2** In the presence of the income effect and the moral hazard problem, the profit to principal when she can use perks is larger than that when she could not use perks.

I establish three observations from Theorem 1.

Firstly, a principal implements an allocation such that per-dollar marginal utilities of the two commodities are different. (Or, equivalently, the marginal rate of substitution is not the same as the price ratio.) More specifically,  $\frac{u'(q(s))}{p} < v'(m(s); e_H)$ . The deviation from [SMC] is measured by

$$\lambda \left[ v'(m(s); e_H) - v'(m(s); e_L) \right] \frac{\varphi(s; e_L)}{\varphi(s; e_H)}.$$

For any s, the principal is giving too little m(s) (equivalently, too much q(s)) to equate perdollar marginal utilities of money and the quality of the condominium. This means that the condominium is *luxurious*: the agent would not have purchased (or rented) the condominium of the same quality, if he were given money with the right to access the spot market. By lowering the consumption of money, the principal can lower the marginal disutility of effort; It is cheaper to control the incentive problem of the agent with the lowered marginal disuitility, than with higher marginal disutility. In other words, the principal wants to give a better condominium (equivalently, less money) for *ex-ante* efficiency (implementation of higher effort with cheaper expected cost), rather than to equalize per-dollar marginal utilities of the two goods for *ex-post* efficiency ( $v'(m(s); e_H) = \frac{u'(q(s))}{p}$ ).

Secondly, suppose v(m; e) = U(m) - kme - C(e), and that the functional form satisfies the typical second order conditions. Then  $\Delta_e v'(m) := v'(m; e_H) - v'(m; e_L)$  is constant in m. For s and s' such that  $\frac{\varphi(s; e_L)}{\varphi(s; e_H)} > \frac{\varphi(s'; e_L)}{\varphi(s'; e_H)}$ ,  $\lambda \Delta_e v'(m(s)) \frac{\varphi(s; e_L)}{\varphi(s; e_H)} > \lambda \Delta_e v'(m(s')) \frac{\varphi(s'; e_L)}{\varphi(s'; e_H)}$ . Notice that s seems more likely to have come from effort  $e_L$  than s' does. This means that the deviation from  $v'(m(s); e_H) = \frac{u'(q(s))}{p}$  would be larger at an unsuccessful outcome than at a successful outcome. Media criticize lavish perks more when a firm is performing badly or on the verge of bankruptcy. However, my model rationalizes excessive perks at an unsuccessful outcome for ex-ante efficiency. Lastly, the deviation is larger for larger  $\lambda$ . More severe moral hazard problem implies more perks.

# 3 Dynamic Model

I interpret money and a condominium in the previous section as tomorrow's and today's consumption. Under this interpretation, Assumption 1 means that the marginal disutility of effort increases in tomorrow's consumption, so that a principal wants to encourage today's consumption over tomorrow's. A two period model following Rogerson (1985) confirms this interpretation under certain conditions. Then I develop a three period model to see how perks evolve over time.

#### 3.1 Two Periods Model

Time frame is as follows.

- **Period 1** : The agent chooses  $e_1$ , state  $s_1$  is realized, and consumption  $q_1(s_1)$  is awarded to the agent.
- **Period 2** : The agent chooses  $e_2(s_1)$ , state  $s_2$  is realized, and consumption  $q_2(s_1, s_2)$  is awarded.

Note that a principal can implement different effort in period 2,  $e_2(s_1)$ , depending on  $s_1$ . Also the principal can commit to the schedule  $e_2(s_1)$ . In addition, the principal can design a wage schedule to be a function of  $s_1$  even for  $q_2$ . These two characteristics of the contract could further relax the incentive compatibility constraint even when  $s_1$  and  $s_2$  are known to be independent.

For a given realization  $(s_1, s_2)$ , the utility function for the agent is

$$u(q_1(s_1); e_1) + \beta v(q_2(s_1, s_2); e_1, e_2(s_1)).$$

Note that there is income effect from consumption  $q_1(\cdot)$  and  $q_2(\cdot)$  to effort  $e_1$ , and consumption  $q_2(\cdot)$  to effort  $e_2$ .<sup>11</sup>

 $\varphi(s_1; e_1)$  is the probability that  $s_1$  is realized for given effort  $e_1$ . I allow correlation between  $s_1$  and  $s_2$ ;  $\varphi(s_1, s_2; e_1, e_2)$  is the probability that  $(s_1, s_2)$  is realized for given  $(e_1, e_2)$ . Therefore, the probability that  $s_2$  is realized for given  $s_1$  and  $(e_1, e_2)$  is  $\psi(s_2; s_1, e_1, e_2) := \frac{\varphi(s_1, s_2; e_1, e_2)}{\varphi(s_1; e_1)}$ .

For given effort schedule  $(e_1, e_2(s_1))$ , the *ex-ante* utility is

$$\sum_{s_1} u(q_1(s_1); e_1)\varphi(s_1; e_1) + \beta \sum_{s_1, s_2} v(q_2(s_1, s_2); e_1, e_2(s_1))\phi(s_1, s_2; e_1, e_2(s_1))$$

I also assume that  $e_H$  is implemented in period 1. Without that assumption, the principal's problem is to solve separate optimization problems for each realized  $s_1$  with  $e_1 = e_L$ , which is not interesting.

<sup>11</sup> The formulation of utility function here puts  $e_1$  in function v to reflect that  $q_2$  influences the marginal disutility of  $e_1$ . Alternatively, I could put  $q_2$  into function  $u(\cdot)$  to reflect the same. However, those two ways of formulating utility function are not much different. For example, suppose there is linear adverse effect of consumption to effort. Then the followings two set-ups are identical.

$$u(q_1, q_2; e_1) := U(q_1) - k_1^u q_1 e_1 - k_1^v q_2 e_1 - c(e_1), \ v(q_2; e_2) := U(q_2) - k_2^v q_2 e_2 - c(e_2)$$
$$u(q_1; e_1) := U(q_1) - k_1^u q_1 e_1 - c(e_1), \ v(q_2; e_1, e_2) := U(q_2) - \frac{k_1^v}{\beta} q_2 e_1 - k_2^v q_2 e_2 - c(e_2)$$

where  $k_1^u$  ( $k_1^v$  or  $k_2^v$ ) measures the adverse effect of consumption in period 1 (period 2) to the effort in period 1 (period 1 or 2). Also discount factor  $\beta$  is added for the second formulation. The formulation here could be called *consumption separable utility representation*, while  $u(q_1, q_2; e_1)$  and  $v(q_2; e_2)$  could be called *effort separable utility representation*.

The agent could not commit to a sequence  $(e_1, e_2(s_1))$ . There are two incentive compatibility constraints for period 1 and 2.<sup>12</sup>

$$\sum_{s_1} u(q_1(s_1); e_H)\varphi(s_1; e_H) + \beta \sum_{s_1, s_2} v(q_2(s_1, s_2); e_H, e_2(s_1))\phi(s_1, s_2; e_H, e_2(s_1))$$

$$\geq \sum_{s_1} u(q_1(s_1); e_L)\varphi(s_1; e_L) + \beta \sum_{s_1, s_2} v(q_2(s_1, s_2); e_L, e_2(s_1))\phi(s_1, s_2; e_L, e_2(s_1)) \quad [IC_1]$$

$$\beta \sum_{s_2} v(q_2(s_1, s_2); e_H, e_2(s_1))\phi(s_1, s_2; e_H, e_2(s_1))$$

$$\geq \beta \sum_{s_2} v(q_2(s_1, s_2); e_H, e'_2)\phi(s_1, s_2; e_H, e'_2) \quad \text{for all} \quad s_1, e'_2 \quad [IC_2(s_1)]$$

Note that I added  $\beta$ , and used unconditional probability  $(\phi(s_1, s_2; e_1, e_2(s_1)))$  instead of conditional probability  $(\frac{\phi(s_1, s_2; e_1 = e_H, e_2(s_1))}{\varphi(s_1; e_1 = e_H)})$  for  $[IC_2(s_1)]$ . These are for the simplicity of later notations. Also note that the deviation for  $[IC_1]$  is only with respect to  $e_L$  since  $e_2(s_1)$  is implemented by  $[IC_2(s_1)]$ , and the agent cannot commit to  $e'_2$  in period 1.

An agent can dissolve the principal agent relationship at any period to receive the outside option of value 0. There are two participation constraints for each of period 1 and 2.

$$\sum_{s_1} u(q_1(s_1); e_H)\varphi(s_1; e_H) + \beta \sum_{s_1, s_2} v(q_2(s_1, s_2); e_H, e_2(s_1))\phi(s_1, s_2; e_H, e_2(s_1)) \ge 0 \quad [\text{IR}_1]$$

$$\beta \sum_{s_2} v(q_2(s_1, s_2); e_H, e_2(s_1)) \phi(s_1, s_2; e_H, e_2(s_1)) \ge 0$$
 [IR<sub>2</sub>(s<sub>1</sub>)]

Note that I again used unconditional probability for  $[IR_2(s_1)]$ , and added  $\beta$ .

Let r to be interest rate between period 1 and period 2. The revenue to the principal at state  $(s_1, s_2)$  is  $R_1(s_1) + \frac{1}{1+r}R_2(s_1, s_2)$ , where the revenue in period 2 potentially depends on

<sup>&</sup>lt;sup>12</sup> The agent might think of the following deviation scheme that is not captured by the two incentive compatibility constraints: the agent deviates to effort  $e'_1$  influencing  $v(q_2(s_1, s_2); \cdot, e_2(s_1))$ , then deviates to  $e'_2$ . However, the effort  $e'_1$  is sunken in period 1. Therefore,  $e'_1$  should not affect the decision in period 2. In other words, even though  $v(\cdot)$  has argument of  $e_1$ , the effort in period 1 influence only the ex-ante utility in period 1 through  $v(\cdot)$ , but not the expected utility in period 2. If I employ the alternative utility formulation in the previous footnote to incorporate income effect, the point is clearer.

state  $s_1$ . The expected revenue is

$$R(e_1, e_2(s_1)) := \sum_{s_1} R_1(s_1)\varphi(s_1; e_1) + \frac{1}{1+r} \sum_{s_1, s_2} R_2(s_1, s_2)\phi(s_1, s_2; e_1, e_2(s_1))$$

The *ex-post* cost of the principal is  $q_1(s_1) + \frac{1}{1+r}q_2(s_1, s_2)$ . Therefore, the expected cost is

$$\sum_{s_1} q_1(s_1)\varphi(s_1;e_1) + \frac{1}{1+r} \sum_{s_1,s_2} q_2(s_1,s_2)\phi(s_1,s_2;e_1,e_2(s_1)).$$

The price of the good is normalized to be unity.

In sum, the principal's maximization problem for given effort schedule  $(e_H, e_2(s_1))$  is

$$\max_{q_1(\cdot),q_2(\cdot)} R(e_H, e_2(s_1)) - \left[ \sum_{s_1} q_1(s_1)\varphi(s_1; e_H) + \frac{1}{1+r} \sum_{s_1,s_2} q_2(s_1, s_2)\phi(s_1, s_2; e_H, e_2(s_1)) \right]$$
  
s.t.  $[IC_1], [IC_2(s_1)], [IR_1], [IR_2(s_1)]$ 

Let the shadow value of the constraints be  $\lambda_1$ ,  $\lambda_2(s_1)$ ,  $\rho_1$ , and  $\rho_2(s_1)$ . Also let

$$\mathbf{E} \left[ \frac{1}{v'(q_2(s_1, \cdot); e_H, e_2(s_1))} \middle| s_1 \right] = \sum_{s_2} \frac{1}{v'(q_2(s_1, s_2); e_H, e_2(s_1))} \psi(s_2; s_1, e_H, e_2(s_1)) \\ \mathbf{E} \left[ \frac{v'(q_2(s_1, \cdot); e_L, e_2(s_1))}{v'(q_2(s_1, \cdot); e_H, e_2(s_1))} \middle| s_1; e_L, e_2(s_1) \right] = \sum_{s_2} \frac{v'(q_2(s_1, s_2); e_L, e_2(s_1))}{v'(q_2(s_1, s_2); e_H, e_2(s_1))} \psi(s_2; s_1, e_L, e_2(s_1)), \\ \mathbf{E} \left[ \frac{v'(q_2(s_1, \cdot); e_H, e_2')}{v'(q_2(s_1, \cdot); e_H, e_2(s_1))} \middle| s_1; e_H, e_2' \right] = \sum_{s_2} \frac{v'(q_2(s_1, s_2); e_H, e_2(s_1))}{v'(q_2(s_1, s_2); e_H, e_2(s_1))} \psi(s_2; s_1, e_H, e_2')$$

I derive my second theorem.

Theorem 2

$$\frac{1}{\beta(1+r)} \mathbf{E} \left[ \frac{1}{v'(q_2(s_1, \cdot); e_H, e_2(s_1))} \middle| s_1 \right] \\
= \frac{1}{u'(q_1(s_1); e_H)} + \rho_2(s_1) + \lambda_2(s_1) \left( 1 - \mathbf{E} \left[ \frac{v'(q_2(s_1, \cdot); e_H, e'_2)}{v'(q_2(s_1, \cdot); e_H, e_2(s_1))} \middle| s_1; e_H, e'_2 \right] \right) \\
+ \lambda_1 \left( \frac{u'(q_1(s_1); e_L)}{u'(q_1(s_1); e_H)} - \mathbf{E} \left[ \frac{v'(q_2(s_1, \cdot); e_L, e_2(s_1))}{v'(q_2(s_1, \cdot); e_H, e_2(s_1))} \middle| s_1; e_L, e_2(s_1) \right] \right) \frac{\varphi(s_1; e_L)}{\varphi} \quad (2)$$

*Proof.* See Appendix A.4

For simple analysis of Equation (2), I assume the following.<sup>13</sup>

#### Assumption 2

$$\beta = \frac{1}{1+r}, \ u(q;e_1) = A(q) - k_1^u q e_1 - c(e_1), \ v(q;e_1,e_2) = A(q) - \frac{k_1^v}{\beta} q e_1 - k_2^v q e_2 - c(e_2)$$

 $k_1^u$  measures the income effect of  $q_1$  to  $e_1$ , and  $k_1^v$  and  $k_2^v$  measure the income effect of  $q_2$  to  $e_1$  and  $e_2$ . I derive the following from Theorem 2.

Corollary 3 With Assumption 2,

$$\mathbf{E}\left[\frac{1}{v'(q_{2}(s_{1},\cdot);e_{H},e_{2}(s_{1}))}\middle|s_{1}\right] < \frac{1}{u'(q_{1}(s_{1});e_{H})} \Leftrightarrow \underbrace{\rho_{2}(s_{1})u'(q_{1}(s_{1});e_{H})}_{(i)} - \underbrace{\lambda_{2}(s_{1})k_{2}^{v}}_{(ii)} + \underbrace{\lambda_{1}\left(k_{1}^{u} - \frac{k_{1}^{v}}{\beta}\right)\frac{\varphi(s_{1};e_{L})}{\varphi}}_{(iii)} < 0$$
(3)

*Proof.* See Appendix A.5

Term (i) comes from the participation constraint in period 2 at state  $s_1$ , term (ii) comes from the incentive compatibility constraint in period 2 at state  $s_1$ , and term (iii) comes from the incentive compatibility constraint in period 1. Terms (ii) and (iii) are also dependent on the income effect.

Without an income effect and the participation constraint in period 2, the following holds from Theorem 2. (Rogerson (1985) showed it first.)

$$\frac{1}{u'(q_1(s_1); e_H)} = \mathbf{E}\left[\frac{1}{v'(q_2(s_1, \cdot); e_H, e_2(s_1))} \middle| s_1\right].$$
 [BE(s\_1)]

By Jensen's Inequality,

$$\frac{1}{u'(q_1(s_1); e_H)} = \mathbf{E}\left[\frac{1}{v'(q_2(s_1, \cdot); e_H, e_2(s_1))} \middle| s_1\right] > \frac{1}{\mathbf{E}[v'(q_2(s_1, \cdot); e_H, e_2(s_1))|s_1]}$$
$$\Rightarrow u'(q_1(s_1); e_H) < \mathbf{E}[v'(q_2(s_1, \cdot); e_H, e_2(s_1))|s_1]. \quad (*)$$

<sup>&</sup>lt;sup>13</sup>However, note that I could give almost identical interpretation without Assumption 2.

The intuition is that an agent wants to save resource in the first period (equivalently, to increase resource in the second period) to decrease the variation of the utilities in the second period.<sup>14</sup> Therefore, the incentive compatibility constraint alone shows that the marginal utility in the first period must be smaller than the expected marginal utility in the second period. However, that is not always true with the participation constraint in period 2. For example, suppose that there is no income effect (i.e.  $k_1^u = k_1^v = k_2^v = 0$ ), but there is participation constraint in period 1. Then,  $\mathbf{E}[1/v'(q_2(s_1, \cdot); e_H, e_2(s_1))|s_1] = 1/u'(q_1(s_1); e_H) + \rho_2(s_1)$  from Theorem 2. For large  $\rho_2(s_1) > 0$ , I cannot derive the result of (\*) in general.

Corollary 3 does not answer exactly when  $\mathbf{E}[v'(q_2(s_1, \cdot); e_H, e_2(s_1))|s_1] > u'(q_1(s_1); e_H)$ holds, i.e. when the principal encourages the consumption in period 1. However, it suggests that if the right-hand side of (3) is not a large positive number, it is more likely that the principal will want to encourage the consumption in period 1. The remaining analyses interpret Corollary 3 as such.

I interpret small  $\rho_2(s_1)$  to represent the difficulty for the agent to opt out of a Principal-Agent relationship. For example, if a principal has the entire monopoly power over the employment of an agent, there would be no participation constraint. That is equivalent to  $\rho_2(s_1) = 0$ . Corollary 3 suggests that there would be more perks for the employees in thin labor market than for those in thick labor market.

Suppose that the shadow value of the moral hazard problem is large and/or the income effect  $k_2^v$  is large. Term (*ii*) indicates that Inequality (\*) is likely to hold when  $e_2(s_1) = e_H$ . On the other hand, if  $e_2(s_1) = e_L$ , it is cheaper for the principal to give full insurance to the agent to have binding participation constraint in period 2 at state  $s_1$ . In other words, the existence of perks is an evidence of the implementation of high effort.

Suppose that  $\beta$ -multiplied adverse effect of the future consumption on current effort is smaller than that of the next immediate future consumption ( $\beta k_1^u < k_1^v$ ). Then term (*iii*) is always negative. Therefore, the principal is likely to encourage the consumption in period

<sup>&</sup>lt;sup>14</sup>Because the utility function is concave, the utility variation decreases if the agent saves.

1. On the other hand, if  $\beta k_1^u > k_1^v$ , the magnitude of  $\lambda_1$  and  $\frac{\varphi(s_1;e_L)}{\varphi(s_1;e_H)}$  matters. The more likely state  $s_1$  seems to have come from effort  $e_L$  (large  $\frac{\varphi(s_1;e_L)}{\varphi(s_1;e_H)}$ ), the less likely the principal encourages the consumption in period 1. Intuitively, the principal has motive to punish the agent at such state, so that she cannot encourage the consumption in period 1 too much. The more severe the moral hazard problem in period 1 is (larger  $\lambda_1$ ), the heavier the punishment would be. Thus, the principal is less likely to encourage the consumption in period 1.

#### 3.2 Three Periods Model

I analyze how the encouragement of consumption evolves over time in this section. A three period model is enough to see the intution. The time frame is as follows.

- **Period 1** : The agent chooses  $e_1$ , state  $s_1$  is realized, and consumption  $q_1(s_1)$  is awarded.
- **Period 2** : The agent chooses  $e_2$ , state  $s_2$  is realized, and consumption  $q_2(s_1, s_2)$  is awarded.
- **Period 3** : The agent chooses  $e_3$ , state  $s_3$  is realized, and consumption  $q_3(s_1, s_2, s_3)$  is awarded.

For given realization  $(s_1, s_2, s_3)$ , the utility for the agent is

$$u(q_1(s_1); e_1) + \beta v(q_2(s_1, s_2); e_1, e_2) + \beta^2 w(q_3(s_1, s_2, s_3); e_1, e_2, e_3).$$

 $\varphi(s_1; e_1)$  is the probability that  $s_1$  is realized for given effort  $e_1$ .  $\phi(s_1, s_2; e_1, e_2)$  is the probability that  $(s_1, s_2)$  is realized for given  $(e_1, e_2)$ .  $\chi(s_1, s_2, s_3; e_1, e_2, e_3)$  is the probability that  $(s_1, s_2, s_3)$  is realized for given  $(e_1, e_2, e_3)$ .

Then the ex-ante utility, incentive compatibility constraints and participation constraints in period 1, 2, and 3, expected revenue, and expected cost are similarly defined. Details are in Appendix A.6

Let the shadow value of the incentive compatibility constraints and the participation constraints to be  $\lambda_1$ ,  $\lambda_2(s_1)$ ,  $\lambda_3(s_1, s_2)$ ,  $\rho_1$ ,  $\rho_2(s_1)$ , and  $\rho_3(s_1, s_2)$ . To save space, let w' :=  $w'(q_3(s_1, s_2, s_3); e_H, e_2(s_1), e_3(s_1, s_2))$ . Also let

$$\begin{split} \mathbf{E} \left[ \frac{1}{w'} \middle| s_1, s_2 \right] &= \sum_{s_3} \frac{1}{w'} \frac{\chi(s_1, s_2, s_3; e_H, e_2(s_1), e_3(s_1, s_2))}{\phi(s_1, s_2; e_H, e_2(s_1))}, \\ \mathbf{E} \left[ \frac{w'(q_3(\cdot); e_L, \cdot, \cdot)}{w'} \middle| s_1, s_2; e_L \right] \\ &= \sum_{s_3} \frac{w'(q_3(s_1, s_2, s_3); e_L, e_2(s_1), e_3(s_1, s_2))}{w'} \frac{\chi(s_1, s_2, s_3; e_L, e_2(s_1), e_3(s_1, s_2))}{\phi(s_1, s_2; e_L, e_2(s_1))}, \\ \mathbf{E} \left[ \frac{w'(q_3(\cdot); \cdot, e'_2, \cdot)}{w'} \middle| s_1, s_2; e'_2 \right] \\ &= \sum_{s_3} \frac{w'(q_3(s_1, s_2, s_3); e_H, e'_2, e_3(s_1, s_2))}{w'} \frac{\chi(s_1, s_2, s_3; e_H, e'_2, e_3(s_1, s_2))}{\phi(s_1, s_2; e_H, e'_2)}, \\ \mathbf{E} \left[ \frac{w'(q_3(\cdot); \cdot, \cdot, e'_3)}{w'} \middle| s_1, s_2; e'_3 \right] \\ &= \sum_{s_3} \frac{w'(q_3(s_1, s_2, s_3); e_H, e_2(s_1), e'_3)}{w'} \frac{\chi(s_1, s_2, s_3; e_H, e_2(s_1), e'_3)}{\phi(s_1, s_2; e_H, e_2(s_1))}. \end{split}$$

 $(e_H, e_2(s_1), e_3(s_1, s_2))$  is the effort schedule chosen by the principal as in the last section.

The Principal-Agent problem can be similarly formulated, and I get the following result.

#### Theorem 3

$$\begin{aligned} &\frac{1}{\beta(1+r)} \mathbf{E} \left[ \frac{1}{w'} \middle| s_1, s_2; e_H, e_2(s_1), e_3(s_1, s_2) \right] \\ &= \frac{1}{v'} + \rho_3(s_1, s_2) \beta(1+r) + \lambda_3(s_1, s_2) \beta(1+r) \left[ 1 - \mathbf{E} \left[ \frac{w'(q_3(\cdot); \cdot, \cdot, e'_3)}{w'} \middle| s_1, s_2; e'_3 \right] \right] \\ &+ \lambda_1 \beta(1+r) \left[ \frac{v'(q_2(s_1, s_2); e_L, e_2(s_1))}{v'} - \mathbf{E} \left[ \frac{w'(q_3(\cdot); e_L, \cdot, \cdot)}{w'} \middle| s_1, s_2; e_L \right] \right] \frac{\phi(s_1, e_3; e_L, e_2(s_1))}{\phi(s_1, s_2; e_H, e_2(s_1))} \\ &+ \lambda_2(s_1) \beta(1+r) \left[ \frac{v'(q_2(s_1, s_2); e_H, e'_2)}{v'} - \mathbf{E} \left[ \frac{w'(q_3(\cdot); \cdot, e'_2, \cdot)}{w'} \middle| s_1, s_2; e'_2 \right] \right] \frac{\phi(s_1, e_3; e_H, e'_2)}{\phi(s_1, s_2; e_H, e_2(s_1))} \end{aligned}$$

Note that the qualitative difference of the equation from Equation (2) is an extra shadow value,  $\lambda_1$  from two periods past. Similarly to Corollary 3, I derive the following corollary.

Corollary 4 Under Assumption 2 and

$$w(q; e_1, e_2, e_3) = A(q) - \frac{k_1^w}{\beta^2} q e_1 - \frac{k_2^w}{\beta} q e_2 - k_3^w q e_3 - c(e_3),$$

Identity (3) and the following identity are derived.

$$\mathbf{E}\left[\frac{1}{w'}\middle|s_1, s_2\right] < \frac{1}{v'} \Leftrightarrow \ \rho_3(s_1, s_2)v' - \lambda_3(s_1, s_2)k_3^w \\ + \lambda_1 \left[\frac{k_1^v}{\beta} - \frac{k_1^w}{\beta^2}\right] \frac{\phi(s_1, s_2; e_L, e_2(s_1))}{\phi} + \lambda_2(s_1) \left[k_2^v - \frac{k_2^w}{\beta}\right] \frac{\phi(s_1, s_2; e_H, e_2')}{\phi} < 0$$

Under the assumption of  $\beta k_1^v < k_1^w$  and  $\beta k_2^v < k_2^w$ , the accumulated shadow value makes the right hand side of the equation more likely to be smaller than 0.

This property of accumulated shadow value is present in N-periods model too. If Nperiods model were formulated, there are t - 1 accumulated shadow value in the inequality between periods t and t - 1. t could be interpreted as seniority. But, just because one is more senior, it does not mean that more perks are provided; the multipliers  $(\lambda_1, \lambda_2(s_1), \dots, \lambda_{t-1}(s_1, \dots, s_{t-1}))$  have to be positive for the accumulation to be meaningful. For the multiplier at a period to be positive, the implemented effort must be  $e_H$  at the period. In other words, a principal gives more perks to the agent who had more success in the past with high effort implemented. A senior staff in a firm could be understood as such an agent.

# 4 Conclusion

I have shown that (i) a principal can use perks to mitigate the moral hazard problem, (ii) perks are luxurious, (iii) perks are more luxurious when the moral hazard problem is more severe, and (iv) perks can be more luxurious at an unsuccessful outcome than at a successful outcome under a certain condition.

The principal uses perks to encourage the consumption of the goods that do not increase the marginal disutility of effort. Therefore, it becomes cheaper to implement a higher effort since the marginal disutility of effort is lowered. It is possible to interpret perks as encouragement of current consumption over future consumption. A dynamic model confirms this interpretation. Under certain conditions, the encouragement of the current period's consumption is more likely when the labor market is thin and/or when an employee is a senior.

Perks are not the only contractual agreement that could increase the consumption of commodities that do not increase the marginal disutility of effort. For example, (partially) reimbursing expenditure on personal use of company facilities, or offering company discounts are other ways to encourage such consumption.

It is well known that exclusiveness of contract is required for the efficient solution of the moral hazard problem: an agent is prohibited to privately access insurance (or contingent claims) markets. (See Fisher [1992], Tommasi and Weinschelbaum [2004], and Park [2004]) Exclusiveness clauses in insurance policies and the prohibition of insider trading also can be explained by these motives. However, the effect of restriction in private access to spot market has not been studied. My work suggests that the restrictions over access to spot markets may improve efficiency in other environments.

# A Appendix

#### A.1 Proof of Theorem 1

To save space, I use the following notational conventions.

$$v' := v'(m(s); e_H), u' := u'(q(s)), \varphi := \varphi(s; e_H)$$

The FOCs are

$$-\varphi + \lambda[v'\varphi - v'(m(s);e_L)\varphi(s;e_L)] + \rho v'\varphi = 0, -p\varphi + \lambda[u'\varphi - u'\varphi(s;e_L)] + \rho u'\varphi = 0$$

which are equivalent to

$$\frac{1}{v'} = \rho + \lambda \left[ 1 - \frac{v'(m(s); e_L)}{v'} \frac{\varphi(s; e_L)}{\varphi} \right], \frac{p}{u'} = \rho + \lambda \left[ 1 - \frac{\varphi(s; e_L)}{\varphi} \right]$$
(4)

Combining them, I get the following.

$$\frac{p/u'}{1/v'} = \frac{\rho + \lambda \left(1 - \frac{\varphi(s;e_L)}{\varphi}\right)}{\rho + \lambda \left(1 - \frac{v'(m(s);e_L)}{v'} \frac{\varphi(s;e_L)}{\varphi}\right)}$$

$$= \frac{\rho + \lambda \left(1 - \frac{v'(m(s);e_L)}{v'} \frac{\varphi(s;e_L)}{\varphi}\right) - \lambda \left(1 - \frac{v'(m(s);e_L)}{v'}\right) \frac{\varphi(s;e_L)}{\varphi}}{\rho + \lambda \left(1 - \frac{v'(m(s);e_L)}{v'} \frac{\varphi(s;e_L)}{\varphi}\right)}$$

$$= \frac{\frac{1}{v'} - \lambda \left(1 - \frac{v'(m(s);e_L)}{v'}\right) \frac{\varphi(s;e_L)}{\varphi}}{\frac{1}{v'}}$$

$$\Rightarrow pv' = u' \left[1 - \lambda (v' - v'(m(s);e_L)) \frac{\varphi(s;e_L)}{\varphi}\right] \text{ by Equation (4)}.$$

Therefore, the main theorem is proved.

### A.2 Generalized Static Model

Let the utility function of the agent to be u(m,q;e). The two first order conditions are

$$-\varphi + \lambda \left( u_m \cdot \varphi - u_m(m,q;e_L) \cdot \varphi(s;e_L) \right) + \rho u_m \varphi = 0$$
  
$$-p\varphi + \lambda \left( u_q \cdot \varphi - u_q(m,q;e_L) \cdot \varphi(s;e_L) \right) + \rho u_q \varphi = 0$$

They change into

$$\frac{1}{u_m} = \rho + \lambda \left( 1 - \frac{u_m(m,q;e_L)\varphi(s;e_L)}{u_m\varphi} \right), \frac{p}{u_q} = \rho + \lambda \left( 1 - \frac{u_q(m,q;e_L)\varphi(s;e_L)}{u_q\varphi} \right)$$

Combining them, I get the following resembling Theorem 1.

$$p \cdot u_m = u_q \left[ 1 + \lambda u_m \frac{\varphi(s; e_L)}{\varphi} \left( \frac{u_m(m, q; e_L)}{u_m} - \frac{u_q(m, q; e_L)}{u_q} \right) \right]$$

# A.3 Proof of Lemma 1

*Proof.* Note the following equation.

$$\frac{p}{u'} = \rho + \lambda \left[ 1 - \frac{\varphi(s; e_L)}{\varphi} \right].$$

Suppose  $\rho = 0$ . Then, p/u' is negative when  $\frac{\varphi(s;e_L)}{\varphi} > 1$ . Contradiction. Hence,  $\rho > 0$ . Suppose  $\lambda = 0$ , then p/u' and 1/v' are constant in s from Equation (4). Therefore, q(s) and m(s) are constant. Contradiction. Hence,  $\lambda > 0$ .

## A.4 Proof of Theorem 2

The first order conditions are:

$$-\varphi + \lambda_1 \left[ u'\varphi - u'(q_1(s_1); e_L)\varphi(s_1; e_L) \right] + \rho_1 u'\varphi = 0,$$
  
$$-\frac{1}{1+r}\phi + \beta\lambda_1 \left[ v'\phi - v'(q_2(s_1, s_2); e_L, e_2(s_1))\phi(s_1, s_2; e_L, e_2(s_1)) \right]$$
  
$$+\beta\lambda_2(s_1) \left[ v'\phi - v'(q_2(s_1, s_2); e_H, e'_2)\phi(s_1, s_2; e_H, e'_2) \right] + \beta\rho_1 v'\phi + \beta\rho_2(s_1)v'\phi = 0$$

which are

$$\frac{1}{u'} = \rho_1 + \lambda_1 \left[ 1 - \frac{u'(q_1(s_1); e_L)\varphi(s_1; e_L)}{u'\varphi} \right],$$

$$\frac{1}{\beta(1+r)v'} = \rho_1 + \rho_2(s_1) + \lambda_1 \left[ 1 - \frac{v'(q_2(s_1, s_2); e_L, e_2(s_1))\phi(s_1, s_2; e_L, e_2(s_1))}{v'\phi} \right] + \lambda_2(s_1) \left[ 1 - \frac{v'(q_2(s_1, s_2); e_H, e'_2)\phi(s_1, s_2; e_H, e'_2)}{v'\phi} \right]$$
(5)

Combining them, I get

$$\frac{1}{\beta(1+r)v'} = \frac{1}{u'} + \rho_2(s_1) + \lambda_2(s_1) \left[ 1 - \frac{v'(q_2(s_1, s_2); e_H, e'_2)\phi(s_1, s_2; e_H, e'_2)}{v'\phi} \right] \\ - \lambda_1 \left[ \frac{v'(q_2(s_1, s_2); e_L, e_2(s_1))\phi(s_1, s_2; e_L, e_2(s_1))}{v'\phi} - \frac{u'(q_1(s_1); e_L)\varphi(s_1; e_L)}{u'\varphi} \right]$$
by using (5).

That is

$$\Leftrightarrow \frac{1}{\beta(1+r)v'} \frac{\phi}{\varphi} = \frac{1}{u'} \frac{\phi}{\varphi} + \rho_2(s_1) \frac{\phi}{\varphi} + \lambda_2(s_1) \left[ 1 - \frac{v'(q_2(s_1, s_2); e_H, e'_2)\phi(s_1, e_3; e_H, e'_2)}{v'\phi} \right] \frac{\phi}{\varphi} - \lambda_1 \left[ \frac{v'(q_2(s_1, s_2); e_L, e_2(s_1))}{v'} \frac{\phi(s_1, s_2; e_L, e_2(s_1))}{\varphi} - \frac{u'(q_1(s_1); e_L)\varphi(s_1; e_L)}{u'\varphi} \frac{\phi}{\varphi} \right]$$

Summing the previous equation over  $s_2$ , I get,

$$\frac{1}{\beta(1+r)} \mathbf{E} \left[ \frac{1}{v'} \middle| s_1; e_H, e_2(s_1) \right] = \frac{1}{u'} + \rho_2(s_1) + \lambda_2(s_1) \left( 1 - \mathbf{E} \left[ \frac{v'(q_2(s_1, \cdot); e_H, e'_2)}{v'} \middle| s_1; e_H, e'_2 \right] \right) \\ - \lambda_1 \left[ \mathbf{E} \left[ \frac{v'(q_2(s_1, \cdot); e_L, e_2(s_1))}{v'} \middle| s_1; e_L, e_2(s_1) \right] - \frac{u'(q_1(s_1); e_L)}{u'} \right] \frac{\varphi(s_1; e_L)}{\varphi}$$

Rearranging, I derive the theorem.

# A.5 Proof for Corollary 3

From the below,

$$u(q;e_1) = A(q) - k_1^u q e_1 - c(e_1), v(q;e_1,e_2) = A(q) - k_1^v q e_1 - k_2^v q e_2 - c(e_2),$$

I derive

$$\mathbf{E}\left[\left.\frac{1}{v'}\right|s_1\right] = \frac{1}{u'} + \rho_2(s_1) - \lambda_2(s_1)k_2^v \mathbf{E}\left[\left.\frac{1}{v'}\right|s_1\right] + \lambda_1\left(k_1^u \frac{1}{u'} - \frac{k_1^v}{\beta} \mathbf{E}\left[\left.\frac{1}{v'}\right|s_1\right]\right)\frac{\varphi(s_1;e_L)}{\varphi}$$

By rearranging the equation, I get the result.

# A.6 Three Periods Model

The *ex-ante* utility function is

$$\begin{split} \sum_{s_1} u(q_1(s_1); e_1) \varphi(s_1; e_1) + \beta \sum_{s_1, s_2} v(q_2(s_1, s_2); e_1, e_2) \phi(s_1, s_2; e_1, e_2) \\ + \beta^2 \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1, e_2, e_3) \chi(s_1, s_2, s_3; e_1, e_2, e_3) \end{split}$$

I have three incentive compatibility constraints.

$$\sum_{s_3} w(q_3(s_1, s_2, s_2); e_1, e_2, e_3) \frac{\chi(s_1, s_2, s_3; e_1, e_2, e_3)}{\phi(s_1, s_2; e_1, e_2)}$$

$$\geq \sum_{s_3} w(q_3(s_1, s_2, s_2); e_1, e_2, e'_3) \frac{\chi(s_1, s_2, s_3; e_1, e_2, e'_3)}{\phi(s_1, s_2; e_1, e_2)} \text{ for all } s_1, s_2, e'_3 \qquad [\text{IC}_3(s_1, s_2)]$$

$$\begin{split} \sum_{s_2} v(q_2(s_1, s_2); e_1, e_2(s_1)) \frac{\phi(s_1, s_2; e_1, e_2(s_1))}{\varphi(s_1; e_1)} + \beta \sum_{s_2, s_3} w(q_3(s_1, s_2, s_2); e_1, e_2, e_3) \frac{\chi(s_1, s_2, s_3; e_1, e_2, e_3)}{\varphi(s_1; e_1)} \\ &\geq \sum_{s_2} v(q_2(s_1, s_2); e_1, e_2') \frac{\phi(s_1, s_2; e_1, e_2')}{\varphi(s_1; e_1)} \\ &+ \beta \sum_{s_2, s_3} w(q_3(s_1, s_2, s_2); e_1, e_2', e_3) \frac{\chi(s_1, s_2, s_3; e_1, e_2', e_3)}{\varphi(s_1; e_1)} \text{ for all } s_1, e_2' \qquad [IC_2(s_1)] \\ &\sum_{s_1} u(q_1(s_1); e_1)\varphi(s_1; e_1) + \beta \sum_{s_1, s_2} v(q_2(s_1, s_2); e_1, e_2(s_1))\phi(s_1, s_2; e_1, e_2(s_1)) \\ &+ \beta^2 \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1, e_2, e_3)\chi(s_1, s_2, s_3; e_1, e_2, e_3) \\ &\geq \sum_{s_1} u(q_1(s_1); e_1')\varphi(s_1; e_1') + \beta \sum_{s_1, s_2} v(q_2(s_1, s_2); e_1', e_2(s_1))\phi(s_1, s_2; e_1', e_2(s_1)) \text{ for all } e_1' \\ &+ \beta^2 \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2, e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &\geq \sum_{s_1} u(q_1(s_1); e_1')\varphi(s_1; e_1') + \beta \sum_{s_1, s_2} v(q_2(s_1, s_2); e_1', e_2(s_1))\phi(s_1, s_2; e_1', e_2(s_1)) \text{ for all } e_1' \\ &+ \beta^2 \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2, e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &\geq \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2, e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &= \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2, e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &= \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2, e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &= \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2, e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &= \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2, e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &= \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2, e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &= \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2, e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &= \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2, e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &= \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2, e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &= \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1', e_2', e_3)\chi(s_1, s_2, s_3; e_1', e_2, e_3) \\ &= \sum_{s_1, s_2, s_3}$$

Similarly, I have three participation constraints.

$$\sum_{s_1} u(q_1(s_1); e_1)\varphi(s_1; e_1) + \beta \sum_{s_1, s_2} v(q_2(s_1, s_2); e_1, e_2(s_1))\phi(s_1, s_2; e_1, e_2(s_1))$$
  
+  $\beta^2 \sum_{s_1, s_2, s_3} w(q_3(s_1, s_2, s_2); e_1, e_2, e_3)\chi(s_1, s_2, s_3; e_1, e_2, e_3) \ge 0$  [IR<sub>1</sub>]

$$\sum_{s_2} v(q_2(s_1, s_2); e_1, e_2(s_1)) \frac{\phi(s_1, s_2; e_1, e_2(s_1))}{\varphi(s_1; e_1)} + \beta \sum_{s_2, s_3} w(q_3(s_1, s_2, s_2); e_1, e_2, e_3) \frac{\chi(s_1, s_2, s_3; e_1, e_2, e_3)}{\varphi(s_1; e_1)} \ge 0$$
[IR<sub>2</sub>(s<sub>1</sub>)]

$$\sum_{s_3} w(q_3(s_1, s_2, s_2); e_1, e_2, e_3) \frac{\chi(s_1, s_2, s_3; e_1, e_2, e_3)}{\phi(s_1, s_2; e_1, e_2)} \ge 0$$
 [IR<sub>1</sub>]

The expected revenue is

$$\begin{aligned} R(e_1, e_2, e_3) &:= \sum_{s_1} R_1(s_1) \varphi(s_1; e_1) + \frac{1}{1+r} \sum_{s_1, s_2} R_2(s_1, s_2) \phi(s_1, s_2; e_1, e_2) \\ &+ \frac{1}{(1+r)^2} \sum_{s_1, s_2, s_3} R_3(s_1, s_2, s_3) \chi(s_1, s_2, s_3; e_1, e_2, e_3) \end{aligned}$$

The expected cost is

$$\sum_{s_1} q_1(s_1)\varphi(s_1;e_1) + \frac{1}{1+r} \sum_{s_1,s_2} q_2(s_1,s_2)\phi(s_1,s_2;e_1,e_2) + \frac{1}{(1+r)^2} \sum_{s_1,s_2,s_3} q_3(s_1,s_2,s_3)\chi(s_1,s_2,s_3;e_1,e_2,e_3)$$

By deriving the first order conditions of the principal's problem, and by arranging by a similar way as in Appendix A.4, I could get the result.

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