Extended linear-homothetic preferences and the Cournot-Bertrand profit differential*

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Abstract

This paper introduces the 'extended linear-homothetic' preferences to model consumer choice. Specifically, we extend Datta and Dixon's (2000) 'standard linear-homothetic' preferences by adding an additional term to the unit cost function. This term captures the relative importance of price interactions within sectors on the unit cost of utility. In an economy composed of a large number of sectors (K) with a sufficiently large number of firms (n) in each, the 'extended linear-homothetic' preferences yield (perceived) linear demands in own strategy *and* competitors' strategies - where goods are characterized as substitutes. Thus, the linearity and homotheticity properties of the preferences open the possibility to develop a tractable model of oligopoly in general equilibrium. An additional novelty introduced by the 'extended linear-homothetic' preferences is the presence of a sectoral-specific price index in product demand. For n small, this implies that firms internalize the sectoral price effects of their individual pricing strategies. The latter, we argue, may provide us with a link between nonatomistic price and wage setters and the monetary authority.

Keywords: Duality; Homotheticity; Oligopolistic Competition; General Equilibrium.

JEL classification: D11; D21; D43; L1.

1. Introduction

We present an extended version of the linear-homothetic (LH) preferences developed by Datta and Dixon (2000). In an economy composed of a large number of sectors (K) with a sufficiently large number of firms (n) in each, the extended LH preferences result in a direct product demand function for each good that is linear in own price *and* competitors' prices within the oligopolistic sector - where goods are characterized as substitutes. Furthermore, direct product demands depend inversely upon a sectoral-specific price index, indicating that consumers' demand for each good is responsive to a sectoral price indicator.

The extended LH preferences produce (perceived) linear demand systems that exhibit similar properties to the ones we find in, for example, Singh and Vives (1984). A particular advantage of using the extended LH preferences to model consumer choice is that they can be easily integrated into a general equilibrium model. This would allow us both, to explore the general equilibrium outcomes delivered by (a)symmetric differentiated oligopolies (competing strategically in either prices or quantities), and to formalize the macroeconomic outcomes of asymmetric right-to-manage wage bargaining under alternative degrees of centralization. Furthermore, we argue that, under certain conditions, the product demand functions derived from the extended LH preferences may provide us with a link between nonatomistic price and wage setters and the monetary authority.

We follow Datta and Dixon (2000) in adopting the dual approach, and we extend their unit cost function by adding an additional term which captures the relative importance of price interactions within sectors on the unit cost of utility. Hence, the unit cost of utility has two parameters, γ and δ, which determine (direct) product demand elasticity at symmetric equilibrium, that is when all individual prices are identical due to symmetry across sectors. Intuitively, given γ, parameter δ captures the intensity of competition in the economy: from the monopolistically competitive economy ($\delta = 0$) to the oligopolistically competitive one ($\delta > 0$).

As an application of the extended LH preferences to partial equilibrium analysis, we derive inverse demand functions and we compare equilibrium profits obtained in the Cournot and Bertrand games. The analysis concludes that the extended LH preferences produce one of the standard results in oligopolistic theory: quantity-setting competition yields higher profits than price-setting competition when goods are substitutes and marginal costs are exogenous, as in Singh and Vives's (1984) seminal paper.

Finally, we briefly explore some of the outcomes that would be potentially delivered by integrating the extended LH preferences into a general equilibrium framework. We consider two alternative product market structures: (i) K and n large - where firms are aggregate *and* sectoral price takers; and (ii) K large and n small - where each firm internalizes the sectoral price effect of its individual pricing strategy. The model predicts that, in general equilibrium, the Bertrand markup is lower than the Cournot one. This result is consistent with partial equilibrium analysis, which provides stronger support for adopting the extended LH preferences in order to model oligopoly in general equilibrium.

The paper is organized as follows. Section 2 presents the extended LH preferences and derives the system of product demand functions (direct and inverse). Section 3 applies the preferences to a standard partial equilibrium problem - whether Cournot competition delivers higher profits than Bertrand competition in the presence of substitutes. Section 4 anticipates and discusses some of the results the model would produce in general equilibrium. Finally, section 5 closes with a conclusion and further remarks.

2. The extended LH preferences

The economy consists of K sectors with n ($n \geq 2$) firms in each, where F_{ik} denotes firm i of sector k producing good x_{ik}. Following Datta and Dixon (2000) we define the expenditure function as follows:

$$E(p, u) = b(p)\, u, \tag{1}$$

where $p \in \Re_{+}^{nK}$ is the price vector of the nK goods and u represents alternative positive utility levels. The unit cost function $b(p) : \Re_{+}^{nK} \longrightarrow \Re_{+}$ takes the form:

$$b(p) = \mu + \delta\Psi + \gamma[\mu - \pi], \tag{2}$$

where $\delta > 0$, $\gamma > 0$ and the following price indices are defined:

$$\mu = \frac{\sum_{k=1}^{K}\sum_{i=1}^{n} p_{ik}}{nK}\quad ; \Psi = \frac{\sum_{k=1}^{K}\psi_k}{K}; \psi_k = \left(\frac{2\sum_{i=1}^{n}\sum_{\substack{j=1\\ i<j}}^{n} p_{ik}\, p_{jk}}{n(n-1)} \right)^{\frac{1}{2}} ; \pi = \left(\frac{\sum_{k=1}^{K}\sum_{i=1}^{n} p_{ik}^{2}}{nK} \right)^{\frac{1}{2}}. \tag{3}$$

The unit cost of utility is composed of several aggregate price indices: μ is the arithmetic average of individual prices, Ψ is the arithmetic average of sectoral-specific price indices, where ψ_k captures the interaction of prices within sector k or within-sector effects, and π is the variance of prices from zero. The novelty to Datta and Dixon (2000) is parameter δ in expression (2), where δ captures the relative importance of within-sector effects. Hence, for $\delta = 0$ we have the standard LH preferences, a larger δ implies more important within-sector effects. Notice that for analytical simplicity δ is assumed identical across sectors, and that $b(p)$ would produce Leontieff preferences when $\delta = 0$ and $\gamma = 0$. In the *symmetric* solution, where all individual prices are identical: $p_{ik} = P \; \forall i, k$, the unit cost function amounts to $b(p) = (1+\delta)P$ since $\mu = \psi_1 = \psi_2 = ... = \psi_K = \Psi = \pi = P$; out of the symmetric solution $\mu < \pi$. Validity of the unit cost function defined by (2) and (3) is proven in appendix A.

Applying Shephard's lemma to (2) it follows that:

$$\frac{p_{ik} x_{ik}}{Y} = \frac{\partial b}{\partial p_{ik}} \frac{p_{ik}}{b} \equiv \alpha_{ik}, \tag{4}$$

hence, the share of aggregate nominal expenditure Y going to good x_{ik} equals the elasticity of the unit cost function with respect to p_{ik}. Evaluating (4) from (2) and (3) and re-arranging yields the Marshallian demand function for *representative* good x_{ik} as given by:

$$x_{ik} = \frac{Y}{bnK} \left[1 + \gamma + \frac{\delta}{(n-1)} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{p_{jk}}{\psi_k} - \gamma \frac{p_{ik}}{\pi} \right], \tag{5}$$

such that, in an identical fashion, we obtain the Marshallian demand function for every good produced in the economy. Given $K > 0$, $n \geq 2$, $\gamma > 0$ and $\delta > 0$, it follows that consumer's demand of x_{ik} depends inversely upon its price (p_{ik}), a price index of the sector where the good belongs to (ψ_k) and a cost-of-living index (b, as interpreted from Datta and Dixon (2001)). Correspondingly, it depends positively upon aggregate nominal expenditure (Y), an aggregate price index capturing the distance of prices from a baseline price vector set to zero (π) and the individual prices of the goods produced by other firms in the sector ($\sum_{j\neq i}^{n} p_{jk}$).

Assuming K and n large, (5) is perceived linear in p_{ik} and $\sum_{j\neq i}^{n} p_{jk}$ by F_{ik}, that is, each firm takes b, π and its corresponding sectoral index ψ as given when making optimal production decisions[1]. Additionally, given the parametric assumptions of the model - where $\gamma > 0$, $\delta > 0$ and K and n are large positive numbers - it follows from (5) that goods across sectors are independent, *in the sense* that cross-price elasticities are zero, and goods within a sector are substitutes, *in the sense* that cross-price elasticities are positive[2]. Thus, this theoretical setting characterizes goods within sectors as substitutes. Further properties of the product demand function given by (5) are similar to the ones we find in Datta and Dixon (2000). Specifically, it is straightforward to

[1] Thus, aggregate price taking follows Dixit and Stiglitz's (1977) monopolistic competition model. A novel aspect introduced by the extended LH preferences is the presence of a sectoral-specific price index in firm's direct demand. In order to justify that an individual firm, say F_{ik}, takes the sectoral price index ψ_k as given, we need to assume that n is not too small. In the extreme case where $n = 2$, it is reasonable to think that an individual firm would take into account the effect of its strategy on the sectoral price index ψ_k. In other words, x_{ik} would not be linear in p_{ik} and $\sum_{j\neq i}^{n} p_{jk}$. For simplicity, we address here the case where n is relatively large, such that an individual firm takes ψ_k as given. The case of n small is addressed in Section 4.

[2] From (5) it follows that $\phi_{ik}^{j} = (\partial x_{ik}/\partial p_{jk})\left(p_{jk}/x_{ik}\right) = (\delta(\frac{p_{jk}}{\psi_k}))/((n-1)(1+\gamma+\frac{\delta}{(n-1)}\sum_{j\neq i}^{n}\frac{p_{jk}}{\psi_k} - \gamma\frac{p_{ik}}{\pi})) > 0$.

check that own-price elasticity and markup vary along the linear product demand schedule, where (the absolute value of) elasticity is increasing in its own price. Additionally, the product demand function given by (5) is bounded by the following choke-off price[3]:

$$\overline{p}_{ik} = \frac{\pi}{\gamma}\left[1 + \gamma + \frac{\delta}{(n-1)}\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{p_{jk}}{\psi_k}\right], \tag{6}$$

and the maximum quantity at zero-price:

$$\overline{x}_{ik} = \frac{Y}{bnK}\left[1 + \gamma + \frac{\delta}{(n-1)}\sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{p_{jk}}{\psi_k}\right]. \tag{7}$$

A closer analysis of the role of parameters γ and δ in the model follows from the *symmetric* solution, where $p_{ik} = P \; \forall i, k$ hence $\mu = \psi_1 = \psi_2 = ... = \psi_K = \Psi = \pi = P$ and $b = (1+\delta)P$. From (5), it is straightforward to show that parameters $\{\gamma, \delta\}$ determine the *symmetric* own-price elasticity of direct product demand, specifically $|\widehat{\varepsilon}_{ik}| = (\partial x_{ik}/\partial p_{ik})\,(p_{ik}/x_{ik}) = \gamma/(1+\delta)$ and $|\widehat{\varepsilon}_{ik}| > 1 \leftrightarrow \gamma > 1 + \delta$. Therefore, $\{\gamma, \delta\}$ parameterize product demand elasticity, such that $\gamma \in (1+\delta, \infty)$ and $(1+\delta)$ sets its lower-bound. Notice that having $\gamma > \delta$ from the elasticity condition ensures that, in absolute terms, the own-price effect on product demand is greater than the sum of the cross-price effects[4].

[3] Let us emphasize that, unlike the Marshallian product demand functions derived in Datta and Dixon (2000) and in this section, the Constant Elasticity of Substitution/Cobb-Douglas product demand functions exhibit constant elasticity (and markup) along product demand, strict convexity and uncut axes.

[4] Out of the symmetric solution, we assume $\gamma/\pi > \delta/\psi_k$ for the own-price effect to dominate

Intuitively, given γ, we interpret δ as a parameter indicating the intensity of competition in the economy - recall that in (2) δ is assumed identical across sectors. For $\delta = 0$, we have a very large number (nK) of identical firms operating in the economy and competing over a share of aggregate nominal expenditure (α_{ik}). This is the monopolistically competitive scenario, where goods are independent *in the sense* that competitors' individual prices do not explicitly figure in product demand. For $\delta > 0$, an increase in δ leads the firm to perceive that a one percent increase in its price reduces its share of aggregate expenditure by a smaller percentage[5]. As a result, a higher price strategy is forthcoming. Therefore, as δ increases the competitive-enhancing effect that competing over Y brings about is reduced, such that a less competitive outcome follows in the economy. In summary, the monopolistically competitive economy $(\delta = 0)$ would yield a more competitive outcome than the oligopolistically competitive one $(\delta > 0)$.

Finally, the inverse demand function for the *representative* good is derived from (5) as follows:

$$p_{ik} = \frac{\psi_k \, \pi}{(\gamma\psi_k - \delta\pi)Z_1} \left[(1+\gamma)Z_1 - bnK\,\delta\,\pi \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{x_{jk}}{Y} - bnK\, Z_2 \, \frac{x_{ik}}{Y} \right], \qquad (8)$$

the sum of the cross-price effects. Finally, note that, in symmetric equilibrium, the own-price elasticity effect of a change in δ, where $\partial \left|\widehat{\varepsilon}_{ik}\right| / \partial\delta = -\gamma/(1+\delta)^2 < 0$, strictly dominates the sum of the cross-price elasticity effects, where $\partial\widehat{\phi}^{j}_{ik}/\partial\delta = 1/((n-1)(1+\delta)^2) > 0$. Specifically, $\left|\partial \left|\widehat{\varepsilon}_{ik}\right| / \partial\delta\right| > (n-1)\partial\widehat{\phi}^{i}_{jk}/\partial\delta \leftrightarrow \gamma > 1$, which holds under the assumptions of the model.

[5] From (2), (3) and (4) it is straightforward to evaluate that: $\xi_{ik} = (\partial\alpha_{ik}/\partial p_{ik})(p_{ik}/\alpha_{ik}) = 1 - ((\gamma(\frac{p_{ik}}{\pi}))/(1+\gamma+\frac{\delta}{(n-1)}\sum_{j\neq i}^{n}\frac{p_{jk}}{\psi_k} - \gamma\frac{p_{ik}}{\pi}))$, such that $\xi_{ik} < 0$ and $\partial\xi_{ik}/\partial\delta > 0$. At the symmetric solution, $\widehat{\xi}_{ik} = 1 - (\gamma/(1+\delta)) < 0$ and $\partial\widehat{\xi}_{ik}/\partial\delta = \gamma/(1+\delta)^2 > 0$.

where $Z_1 \equiv (\gamma\psi_k(n-1)+\delta\pi)$ and $Z_2 \equiv ((\gamma\psi_k-\delta\pi)(n-1)+\delta\pi) > 0$ since $\gamma\psi_k > \delta\pi$. Accordingly, we obtain the inverse demand function of every good produced in the economy.

Next, we derive some partial equilibrium outcomes obtained by modelling consumer choice following the extended LH preferences. Specifically, we evaluate whether the derived (direct and inverse) demand functions produce one of the standard results in oligopoly theory in the context of exogenous labor costs - namely, that quantity-setting competition yields higher profits than price-setting competition in the presence of imperfect substitutes.

3. An application to the Cournot-Bertrand profit differential

There is a continuum of consumer-workers consuming goods from each of the K sectors the economy is composed of. As a result, an individual firm cannot significantly influence the income of its consumer-workers, hence firms take income (Y) as given. Consider one of the n-size differentiated sectors. Firms exhibit constant and identical marginal labor costs (w) according to the following short-run technology: $x = l$, where l stands for labor units. Each firm maximizes nominal profits in the knowledge of the product demand function it faces, where competitors' strategies are taken as given. For K and n large, firms are aggregate and sectoral price takers, that is, they treat as parameters the aggregate price indices $\{b, \pi\}$ and the sectoral-specific price index ψ in product demand. We consider two types of product market competition: firms in the oligopoly cooperatively choose to compete in either prices, à la Bertrand, or quantities, à la Cournot.

3.1. Price-setting competition

The representative firm F_{ik} maximizes $\Omega_{ik} = (p_{ik} - w)x_{ik}$ in order to choose p_{ik} subject to the demand function given by (5). There are n simultaneous and symmetric optimizations in sector k. Hence, the unique Bertrand-Nash equilibrium price is the simultaneous solution to the n-size vector of Bertrand-Nash best-reply functions in prices as given by:

$$p^{B*} = \frac{\psi_k \left((1+\gamma)\pi + \gamma w\right)}{2\gamma\psi_k - \delta\pi}. \tag{9}$$

Introducing (9) in (5) yields equilibrium quantity under Bertrand competition as follows:

$$x^{B*} = \frac{Y\,\gamma \left((1+\gamma)\pi\psi_k - (\gamma\psi_k - \delta\pi)w\right)}{bnK\pi \left(2\gamma\psi_k - \delta\pi\right)}. \tag{10}$$

Finally, from (9) and (10) we obtain the following expression for Bertrand-Nash equilibrium profits of every firm in sector k:

$$\Omega^{B*} = \frac{Y\,\gamma \left((1+\gamma)\pi\psi_k - (\gamma\psi_k - \delta\pi)w\right)^2}{bnK\pi \left(2\gamma\psi_k - \delta\pi\right)^2}. \tag{11}$$

3.2. Quantity-setting competition

In the quantity-setting game, the representative firm maximizes profits in order to choose x_{ik} subject to the inverse demand function given by (8). Once again, the unique Cournot-Nash equilibrium quantity is the simultaneous solution to the n-size vector of Cournot-Nash best-reply functions in quantities. This is given by:

$$x^{C*} = \frac{Y\,Z_1 \left((1+\gamma)\pi\psi_k - (\gamma\psi_k - \delta\pi)w\right)}{bnK\pi\psi_k \left(2\gamma\psi_k(n-1) - \delta\pi(n-3)\right)}. \tag{12}$$

Substituting (12) into (8) yields equilibrium price under Cournot competition as follows:

$$p^{C*} = \frac{\pi\psi_k(1+\gamma)(\gamma\psi_k(n-1) - \delta\pi(n-2)) + Z_1(\gamma\psi_k - \delta\pi)w}{(\gamma\psi_k - \delta\pi)\,(2\gamma\psi_k(n-1) - \delta\pi(n-3))}. \tag{13}$$

Equilibrium profits in the Cournot-Nash solution follow from (12) and (13) as given by:

$$\Omega^{C*} = \frac{YZ_1(\gamma\psi_k(n-1) - \delta\pi(n-2))((1+\gamma)\pi\psi_k - (\gamma\psi_k - \delta\pi)w)^2}{bnK\pi\psi_k(\gamma\psi_k - \delta\pi)(2\gamma\psi_k(n-1) - \delta\pi(n-3))^2}. \tag{14}$$

3.3. The Cournot-Bertrand profit differential

Consider two symmetric sectors of the economy $\{S_1, S_2\}$ where competition takes place à la Cournot and à la Bertrand, respectively. We assume that each firm in each sector takes as given *identical* aggregate and sectoral price indices when making its optimal choice in the product market. Hence, from expressions (11) and (14), we derive the Cournot-Bertrand profit differential as follows:

$$D = \Omega_1^{C*} - \Omega_2^{B*} \equiv \tag{15}$$

$$\frac{Y\,\delta^3\pi^2(2\gamma\psi(n-1) - \delta\pi(n-2))((1+\gamma)\pi\psi - (\gamma\psi - \delta\pi)w)^2}{bnK\psi(\gamma\psi - \delta\pi)(2\gamma\psi - \delta\pi)^2(2\gamma\psi(n-1) - \delta\pi(n-3))^2}.$$

Given the assumptions of the model - where n and K are large positive numbers, $\gamma > 0$, $\delta > 0$ and $\gamma\psi > \delta\pi$ - it is straightforward to conclude that the denominator and the numerator of expression (15) are strictly positive. Hence, the profit differential is positive: Cournot profits are above Bertrand profits when goods are imperfect substitutes. In other words, the partial equilibrium profits obtained by the representative firm in S_1 are greater than the partial equilibrium profits obtained by the representative firm in S_2.

Under identical aggregate and sectoral price taking, we have shown that the extended LH preferences produce one of the results established by Singh and Vives (1984): firm's preference of Cournot competition over Bertrand when goods are perceived as substitutes by consumers, in the sense of positive cross-price elasticities, and marginal costs are exogenous[6]. For $\delta = 0$, we find that the profit differential collapses to zero as profits fall to the monopolistically competitive solution.

5. General equilibrium: Initial results under alternative product market structures

This section investigates some of the outcomes that would be delivered by embedding the extended LH preferences in general equilibrium. In particular, we are interested in inspecting the specifications for equilibrium real wage and markup[7]. We focus on the symmetric outcome, hence firms anticipate symmetric equilibrium in aggregate (and sectoral, when applicable) price indices. The following product market structures are considered:

Structure (i): K large and n large

By assuming K and n large, we justify that each firm takes b, π and its corresponding sectoral index ψ as given when making its sectoral strategic de-

[6] Notice that if we solve the model anticipating symmetry in aggregate price indices we obtain simpler partial equilibrium outcomes.

[7] We do not embed the extended LH preferences in a general equilibrium setting. We simply investigate some of the results and applications that such general equilibrium framework could produce.

cisions. From (5), it follows that the direct product demand function for the representative good simplifies to:

$$x_{ik} = \frac{y}{(1+\delta)nK}\left[1+\gamma+\frac{\delta}{(n-1)}\sum_{\substack{j=1\\j\neq i}}^{n}\frac{p_{jk}}{P}-\gamma\frac{p_{ik}}{P}\right], \tag{16}$$

where symmetry in price indices is anticipated, that is $\mu = \psi_1 = \psi_2 = ... = \psi_K = \Psi = \pi = P$ and $b = (1+\delta)P$, and y is real aggregate expenditure, that is $y = Y/P$. Accordingly, from (8) we re-write the inverse demand function for good x_{ik} as follows:

$$p_{ik} = \frac{P}{\gamma-\delta}\left[1+\gamma-\frac{nK\,\delta(1+\delta)}{\gamma(n-1)+\delta}\sum_{\substack{j=1\\j\neq i}}^{n}\frac{x_{jk}}{y}-\frac{nK(1+\delta)((\gamma-\delta)(n-1)+\delta)}{\gamma(n-1)+\delta}\frac{x_{ik}}{y}\right]. \tag{17}$$

From the first order condition of profit optimization: $p_{ik}\left(1-(1/\left|\varepsilon_{ik}\right|)\right) = w$, it follows that the equilibrium real wage under price-setting competition in each oligopoly is given by:

$$\left(\frac{w}{P}\right)^{B*} = \frac{\gamma-(1+\delta)}{\gamma}, \tag{18}$$

where $(w/P)^{B*} < 1$, and the markup under Bertrand competition takes the following functional form[8]: $\widehat{\lambda}_{ik}^{B} = 1/\left|\widehat{\varepsilon}_{ik}^{B}\right| = (1+\delta)/\gamma$. Correspondingly, the equilibrium real wage delivered by quantity-setting competition is given by:

$$\left(\frac{w}{P}\right)^{C*} = \frac{(\gamma-\delta)(\gamma(n-1)+\delta)-(1+\delta)((\gamma-\delta)(n-1)+\delta)}{(\gamma-\delta)(\gamma(n-1)+\delta)}, \tag{19}$$

such that $(w/P)^{C*} < 1$, and the Cournot markup in general equilibrium under

[8] Recall that $\left|\widehat{\varepsilon}_{ik}^{B}\right| > 1 \leftrightarrow \gamma > 1+\delta$.

symmetry is given by[9]: $\widehat{\lambda}_{ik}^{C} = 1/\left|\widehat{\varepsilon}_{ik}^{C}\right| = ((1+\delta)((\gamma-\delta)(n-1)+\delta))/((\gamma-\delta)(\gamma(n-1)+\delta))$.

From (18) and (19), we conclude that quantity-setting competition produces a lower real wage, hence a higher markup, in general equilibrium[10]. Notice that the equilibrium markup under Bertrand competition is invariant to the number of firms in the oligopoly whereas the equilibrium markup under Cournot decreases with n [11]. In other words, in the model, as it stands, an increase in n would simply replicate the Bertrand economy.

Finally, let us point out that from (16), (17) and the optimizing behavior of firms, we can derive *linear* labor demand functions that would depend upon aggregate variables $\{y, P\}$. In a general equilibrium framework, this would allow us to formalize the macroeconomic outcomes of asymmetric right-to-manage Nash bargaining under alternative degrees of centralization and types of product market competition[12].

Structure (ii): K large and n small

By assuming n small, we imply that the effect of an individual price on the corresponding sectoral price index cannot be ignored. The bottom line would

[9] Note that $\left|\widehat{\varepsilon}_{ik}^{C}\right| > 1 \leftrightarrow \gamma > \underline{\gamma}^{C}$ where $\underline{\gamma}^{C} = ((n-1)+\delta(2n-3)+((n-1)^2+\delta(2(n-1)+\delta(4n-3)))^{1/2})/(2(n-1))$. Hence, $\gamma > \underline{\gamma}^{C}$ is the implicit assumption on γ for inverse demand to be elastic.

[10] The comparison yields that $(w/P)^{B*} > (w/P)^{C*} \leftrightarrow \delta^2(1+\delta) > 0$, which holds $\forall\delta > 0$.

[11] Specifically, $\partial(w/P)^{C*}/\partial n = \delta^2(1+\delta)/((\gamma-\delta)(\gamma(n-1)+\delta)^2) > 0$ for the assumed range of parameter values.

[12] A general equilibrium model of unionized oligopoly embedding the extended LH preferences can be found in Correa López (2003).

be a duopoly in each sector or $n = 2$. As a result, the representative firm F_{ik} would take into account the effect of its price strategy p_{ik} on its sectoral price index ψ_k [13]. Accordingly, the assumption of a large number of oligopolies still implies aggregate price taking, such that the effect of an individual price on aggregate indices can be ignored. Under this structure, the direct product demand function for good x_{ik} is given by:

$$x_{ik} = \frac{y}{nK(1+\delta)}\left[1 + \gamma + \frac{\delta}{(n-1)}\sum_{\substack{j=1 \\ j\neq i}}^{n}\frac{p_{jk}}{\psi_k} - \gamma\frac{p_{ik}}{P}\right], \qquad (20)$$

where symmetry in aggregate price indices is anticipated, i.e. $\mu = \Psi = \pi = P$ and $b = (1+\delta)P$. The price elasticity of demand at symmetric equilibrium is now given by: $\left|\widehat{\varepsilon}_{ik}^{B}\right| = (\gamma/(1+\delta)) + (\delta/(n(1+\delta)))$. From the latter it follows that, if firm F_{ik} takes into account the effect of its strategy on ψ_k it will perceive a more elastic product demand function. Elasticity depends upon n to the extent that a larger n implies a smaller impact of firm F_{ik}'s price on the sectoral price index, thus a reduced incentive of the individual firm to follow a low price strategy.

The presence of a sectoral-specific price index in product demand opens an interesting possibility for modelling the strategic interactions that may occur between the monetary authority and price and wage setters[14]. We might anticipate a new transmission mechanism of monetary policy-making. The basic

[13] Or, in other words, x_{ik} would not be linear in p_{ik} and $\sum_{j\neq i}^{n} p_{jk}$.

[14] Furthermore, notice that the model would not produce feedback effects since the assumption of K large implies that economic agents take as given aggregate price indices at partial equilibrium.

message conveyed in this transmission mechanism is straightforward: as long as firms (and unions) care about the sectoral price effects of their individual pricing strategies *and* as long as the central bank monitors sectoral price indices when designing monetary policy, real effects might be expected from: (i) the specific institutional environment under which monetary policy is conducted, and (ii) the monetary policy rule adopted by the monetary authority.

4. Conclusion

The 'extended linear-homothetic' preferences developed in this paper produce (perceived) linear product demand functions that exhibit similar properties to the ones more frequently used in the analysis of differentiated oligopolies (see, for example, Singh and Vives (1984)). A particular advantage of using the 'extended linear-homothetic' preferences to model consumer choice, lies in its potential application to the study of oligopoly in a tractable general equilibrium framework. In addition, an important novelty introduced by the 'extended linear-homothetic' preferences is the presence of a sectoral-specific price index in firm's direct demand. For n small, an individual firm would internalize the sectoral price effects of its product (and labor) market strategies. In a general equilibrium environment, this could provide us with a link between nonatomistic price and wage setters and, for example, the monetary authority. Investigating these issues is left for further work.

Appendix A. Validity of the expenditure and unit cost functions

The domain of function $b(p)$ is defined by $S \equiv \{p \in \Re^{nk}_{+} : p_{ik} > 0, i = 1, ..., n, k = 1, ..., K\}$. We check that $b(p)$ exhibits the sufficient properties: (i) Non-negative and non-decreasing in prices; (ii) Homogeneity of degree one and concavity in p; (iii) Continuous differentiability. As Datta and Dixon (2000) emphasize property (iii) is not necessary for validity but for the application of Shephard's lemma. Given our assumptions on parameter values, where $n \geq 2$, $\delta > 0$ and $\gamma > 0$, it is straightforward to conclude that $b(p)$ is continuously differentiable and homogeneous of degree one.

Concavity in p is proven by checking that $b_1(p)$ and $b_2(p)$ are concave, where $b_1(p) = \mu + \gamma[\mu - \pi]$ and $b_2(p) = \delta\Psi$. Specifically, concavity of $b_1(p)$ implies that $\varphi b_1(\widetilde{p}) + (1-\varphi)b_1(\overline{p}) \leqslant b_1(\varphi\widetilde{p} + (1-\varphi)\overline{p})$ where $0 < \varphi < 1$ and $\{\widetilde{p}, \overline{p}\} \in S$. In order to assess the concavity of $b_2(p)$ we start by checking the concavity of the *representative* sectoral price index ψ_k, whose domain $s_k : s_k \subset S$ is defined by $s_k \equiv \{p_k \in \Re^{n}_{+} : p_{ik} > 0, i = 1, ..., n\}$. Denote H^{ψ_k} as the Hessian matrix associated to ψ_k. Hence, given a n-size sector k, where $n \geq 2$, it is straightforward to check that *all* principal minors of H^{ψ_k} exhibit the following signs: $\left|H^{\psi_k}_m\right| < 0$ for m odd and $m < n$, $\left|H^{\psi_k}_m\right| > 0$ for m even and $m < n$ and $\left|H^{\psi_k}_m\right| = 0$ for $m = n$ (i.e. when $\left|H^{\psi_k}_m\right| = \left|H^{\psi_k}\right|$). More specifically, for $m < n$, the *leading* principal minors can be expressed as follows:

$$\left|H^{\psi_k}_m\right| = \frac{(-1)^m}{(n-1)^{m+1}\, n^{m+1}\, \psi_k^{m+2}}\left[-n(m-1)(n-1)\psi_k^2 - \right.$$

$$(m-2)\left(\left(\sum_{\substack{j=1\\j\neq 1}}^{n} p_{jk}\right)^2 + \left(\sum_{\substack{j=1\\j\neq 2}}^{n} p_{jk}\right)^2 + ... + \left(\sum_{\substack{j=1\\j\neq m}}^{n} p_{jk}\right)^2\right) +$$

$$2\,(\sum_{\substack{j=1\\j\neq 1}}^{n} p_{jk}\sum_{\substack{j=1\\j\neq 2}}^{n} p_{jk} + \sum_{\substack{j=1\\j\neq 1}}^{n} p_{jk}\sum_{\substack{j=1\\j\neq 3}}^{n} p_{jk} + ... + \sum_{\substack{j=1\\j\neq 1}}^{n} p_{jk}\sum_{\substack{j=1\\j\neq m}}^{n} p_{jk} + \sum_{\substack{j=1\\j\neq 2}}^{n} p_{jk}\sum_{\substack{j=1\\j\neq 3}}^{n} p_{jk} +$$

$$\sum_{\substack{j=1\\j\neq 2}}^{n} p_{jk}\sum_{\substack{j=1\\j\neq 4}}^{n} p_{jk} + ... + \sum_{\substack{j=1\\j\neq 2}}^{n} p_{jk}\sum_{\substack{j=1\\j\neq m}}^{n} p_{jk} + ... + \sum_{\substack{j=1\\j\neq (m-1)}}^{n} p_{jk}\sum_{\substack{j=1\\j\neq m}}^{n} p_{jk})\,],$$

which is re-written as:

$$\left|H_m^{\psi_k}\right| = \frac{(-1)^m}{(n-1)^{m+1}\,n^{m+1}\,\psi_k^{m+2}} \tag{21}$$

$$\left[-n(m-1)(n-1)\psi_k^2 - (m-2)\sum_{i=1}^{m}\left(\sum_{\substack{j=1\\j\neq i}}^{n} p_{jk}\right)^2 + 2\sum_{i=1}^{m}\sum_{\substack{q=1\\i<q}}^{m}\left(\sum_{\substack{j=1\\j\neq i}}^{n} p_{jk}\sum_{\substack{j=1\\j\neq q}}^{n} p_{jk}\right)\right],$$

such that the long term in brackets in (21) equals zero for $m = n$. Overall, we conclude that the Hessian matrix associated to ψ_k is negative semidefinite, hence, ψ_k is concave. Note that the sectoral price indices $\{\psi_1, \psi_2, ..., \psi_K\}$ have the same functional form as ψ_k in their corresponding subset of S. Thus, they are also characterized by negative semidefinite Hessian matrices whose principal minors exhibit the pattern of signs described above.

Finally, given $\Psi = (\sum_{k=1}^{K}\psi_k)/K$ the Hessian matrix associated to Ψ is given by:

$$H^{\Psi} = \begin{pmatrix} H^{\psi_1} & 0 & 0 & ... & 0 \\ 0 & H^{\psi_2} & 0 & ... & 0 \\ 0 & 0 & H^{\psi_3} & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & H^{\psi_K} \end{pmatrix}, \tag{22}$$

where the positive constant term $(1/K)$ is omitted for simplicity. The Hessian matrix H^{Ψ} is $nK \times nK$, where 0 stands for a $n \times n$ null matrix. From the structure of (22) and the analysis of H^{ψ_k} it can be shown that H^{Ψ} is negative semidefinite, more particularly, all its principal minors exhibit the following signs: $\left|H_m^{\Psi}\right| \leq 0$ for m odd and $m < nk$, $\left|H_m^{\Psi}\right| \geq 0$ for m even and $m < nk$ and $\left|H_m^{\Psi}\right| = 0$ for $m = nk$ (i.e. where $\left|H_m^{\Psi}\right| = \left|H^{\Psi}\right|$). Overall, we can conclude that Ψ and, hence, $b_2(p)$ are concave.

Given the domain defined by S property (i) is re-written such that $b(p)$ has to be positive and non-decreasing in prices. Property (i) implies that *demands are non-negative*; it also implies that an additional unit of utility is costly. As Datta and Dixon (2000) emphasize if prices are so dispersed that the higher surpass the choke-off price, then, some "raw" demands will become negative. Datta and Dixon (2000) consider two alternatives to formally get around this possibility. The first one is to set an upper-bound limit on γ, however, this alternative is not favoured since it would explicitly set an upper limit (that may be overly restrictive) on product demand elasticity. The second alternative is to develop what they call the restricted Linear Homothetic cost function $B(p)$. This function meets all the properties of a valid cost function and, especially, it meets property (i) since it is designed to "cap" those prices that exceed the choke-off price.

After developing the restricted LH cost function, Datta and Dixon (2000, 2001) support the use of the unrestricted $b(p)$ without imposing a restriction on γ. This is justified by the nature of firms operating in imperfectly competitive markets. Profit maximizing firms will not, in general, set prices so high to make demands negative. In parallel, oligopolistic firms competing in imperfect substitutes will not find profitable to set a price above the choke-off price. This argument allows us to use the unrestricted unit cost function $b(p)$ in (2) comfortably. Finally, from the analysis derived above we note that the expenditure function in (1) is homothetic.

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