Inflation Tax and the Hidden Economy.

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Abstract

Differential tax analysis is used to show how the optimal mix of inflation tax and direct taxation changes with the relative size of the hidden economy. The larger the relative size of the hidden economy, the smaller the optimal ratio of direct tax to inflation tax. Anecdotal empirical evidence supports this result.

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1 Introduction.

Raising tax revenue when the hidden economy forms a large proportion of the total economy is a pressing problem in many countries. Inflation tax (seignorage) is seen as one solution, however, this leads to price inflation if the growth of the money supply exceeds the growth of the total economy. This raises the question: How does the relative size of the hidden economy affect the optimal inflation tax?

The analysis is in the context of the optimal inflation tax model proposed by Phelps (1973). Phelps's model is modified to include a hidden economy as well as the visible economy. Agents in the hidden economy are able to avoid direct taxation[1] but unable to avoid the inflation tax because they require cash to carry out transactions. The paper proceeds as follows. In this section the motivation for the paper is continued. In section 2 the model is presented and in section 3 it is solved. In section 4 the results are discussed along with some anecdotal empirical evidence and possible extensions. A final section concludes.

1.1 What is the Hidden Economy?

The hidden, black, informal, parallel, second, shadow or underground economy represents all economic activity which is not subject to direct taxation. In countries with developed tax-gathering authorities the hidden economy is typically associated with criminal activity. In developing countries, with less developed tax-gathering authorities, the hidden economy is typically made up of a large agricultural economy which is not subject to direct taxation because the costs of levying taxes outweigh the potential revenues. In such a context the hidden economy need not be illegal.

1.2 Inflation Tax and Seignorage.

Seignorage is the difference between the face value of money and the cost of manufacturing it. Because of different manufacturing costs, the seignorage value of coins is quite low, the seignorage value of paper money is considerably higher and the seignorage value of the monetary base created through *money on call* nearly equals the full value of the monetary base increase. In this paper, the simplifying assumption is made that the seignorage value is exactly equal to the full value of the new money supply. Subject to conditions (i) to (v) outlined in section 2, the "revenue from the inflation tax is simply its contribution to the government's *seignorage* ... a tax on *liquidity*".[2]

Seignorage can be a useful source of government revenue and all governments use it to some extent. Fisher (1982) estimated the magnitude of seignorage for several industrialised countries and found that this accounted for 6.1% of government revenue during 1960-73 and 5.9% during 1973-78. The problem with raising large amounts of revenue through seignorage is that it expands the monetary base, potentially leading to high price inflation. For example, Fisher (1982) found that during 1960-73 Italy raised 9.8% of government revenue by seignorage and experienced 4.66% average inflation, during 1973-79 seignorage rose to 16% and average inflation rose to 16.38%.

[1] In this context direct taxation includes income and/or expenditure taxes.

[2] Phelps (1973), page 75.

2 The Inflation Tax: Concept and Measure.

As in Phelps (1973), assume an economy where the government raises revenue by an inflation tax and by a direct tax. In contrast to Phelps (1973), the private sector includes a hidden economy that is not subject to the direct tax. The private sector is made up of the visible and hidden economies in the proportions $1-\alpha$ and α respectively. The relative size of the hidden economy has implications for the government's choice of the optimal tax-mix between the inflation and direct tax.

The simplifying assumptions underlying this analysis are that (i) agents forecast inflation perfectly, (ii) the natural rate of output is not affected by the level of inflation, (iii) the economy is closed so that inflation is irrelevant as a stabilisation policy, (iv) there are no costs to adjusting wage and prices, and (v) no interest is paid on money balances.

2.1 The Government Sector.

The problem facing the government is one of having to maximise social welfare by co-ordinating the actions of its three administrative branches; the expenditure branch, the treasury and the central bank. The total tax yield is constant and differential tax analysis is used to allocate taxes between inflation and direct taxation in order to maximise utility in the private sector. The notation and equation numbering follows closely that of Phelps (1973) so as to facilitate any comparisons.

The expenditure branch determines government expenditure (G) and benefits (B) as a function of time and independently of any other factors.

$$G = \gamma(t) \geq 0, \quad B = \beta(t) \geq 0 \tag{1}$$

The treasury, in financing these expenditures, faces the budget constraint of matching all costs and all revenues, including payments on the existing debt,

$$T + \frac{\dot{D}}{p} = G + B + i_D \frac{D^*}{p} \tag{2}$$

where T is direct taxation on the visible economy, D is the accumulated debt which includes both the public debt (D^*) and money (M), D^* is the part of the accumulated debt held by the private sector, p is the price level and i_D is the nominal interest rate on the debt.

The central bank sets the time path of the money supply (M) independently of the treasury,

$$M = D - D^* \tag{3}$$

The money supply affects the price level, therefore, the central bank is able to determine alternative price level programs in the form,

$$p(t) = \phi(t; \pi) \tag{4}$$

where π is the target level of inflation for the central bank.

2.2 The Private Sector.

The behaviour of the private sector is determined by the following consumption demand (C) and manhour supply (H) functions. Total consumption is determined by consumption in the hidden economy (C^h) and consumption in the visible economy (C^v). Total hours are determined by hours in the hidden economy (H^h) and hours in the visible economy (H^v). The parameter α determines the proportion of the population N that is in each sector, so

$$C = C^v(\tilde{Y}^v, W; ...; (1-\alpha)N; t) \; + \; C^h(\tilde{Y}^h, W; ...; \alpha N; t) \tag{5}$$

$$H = H^v(\tilde{Y}^v, W; ...; (1-\alpha)N; t) \; + \; H^h(\tilde{Y}^h, W; ...; \alpha N; t) \tag{6}$$

where real net disposable income $\tilde{Y}$ in each sector is determined by the population proportions and by the difference between revenues and costs,

$$\tilde{Y}^v = (1-\alpha)\left[\bar{Y} + B + i_D\frac{D^*}{p} - \pi\left(\frac{M+D^*}{p}\right)\right] - T \tag{7a}$$

$$\tilde{Y}^h = \alpha\left[\bar{Y} + B + i_D\frac{D^*}{p} - \pi\left(\frac{M+D^*}{p}\right)\right] \tag{7b}$$

where real disposable wealth is given by,

$$W = K + \frac{D}{p} = K + \frac{M+D^*}{p} \tag{8}$$

and potential pre-tax income is,

$$\tilde{Y} = r_K K + wH \tag{9}$$

where K is the real value of the capital stock, r_K is the return on the capital stock and w is the wage rate. Note that homotheticity is assumed throughout, that is, the visible and hidden economies are scale values of one another. This implies that $Y^i, B^i, M^i, D^i, H^i, N^i$ and W^i for $i = v, h$ are purely functions of the population proportions in the hidden and visible economies with the exception of the tax burden T that only affects the visible economy.

2.3 Measuring Seignorage.

In this sub-section the seignorage equation (15) and the marginal rate of substitution between direct taxes and inflation (17) under the optimal policy trajectory are specified. Firstly, the tax burden is invariant to changes in the tax-mix, this is ensured by the "forcing function" (10) which causes taxes T to change in response to changes in inflation π such that the tax burden θ remains constant over time,

$$\theta(t) = T + \frac{\pi M}{p}\frac{(i_D - \pi)}{p} \tag{10}$$

where θ the "wondrous dynamic parameter"[3] is a function only of time t. Secondly, the level of wealth is also invariant to changes in the tax-mix. This ensures that the real level of wealth is a function only of time and is not affected by the level of inflation,

$$\Delta(t) = \frac{D}{p} = \frac{M+D^*}{p} \tag{11}$$

[3]Phelps (1973), page 73.

Thirdly, to avoid a wealth effect at time $t = 0$ from an increase in inflation, we require the price level to remain constant at this time,

$$p(0) = p_0 > 0 \tag{12}$$

of course, the rate of inflation may change at time $t = 0$ following a change in policy but the price level should not.

Substituting equations (3), (11) and (12) into (10) and re-arranging gives seignorage tax at time zero,[4]

$$i_D \frac{M}{p_0} = \theta(0) - T + (i_D - \pi)\Delta(0) \tag{13}$$

where increased inflation generates increased seignorage and reduces the tax burden. There may be an additional increase in seignorage revenue insofar as the real rate of interest $(i_D - \pi)$ may fall.

To further simplify the analysis, the arbitrage condition that the real rate of return on the debt (D) equals the real rate of return on capital (K) and is a constant (ς) is imposed,

$$(i_D - \pi) = r_K = \varsigma \tag{14}$$

Substituting in (14) for the arbitrage condition into (13) defines,

$$(\varsigma + \pi)\frac{M}{p_0} = \theta(0) - T + \varsigma\Delta(0) \tag{15}$$

Equation (15) specifies how much the private sector pays to hold a particular level of liquidity. A behavioural equation describing money demand by the private sector is required, thus liquidity preference is specified by,

$$\frac{M}{p_0} = L(Y, r_K + \pi, K + D/p_0) \tag{16}$$

Thus, differentiating equation (13) with respect to inflation gives equation (17), where the implicit tax rate on liquidity is $i = \varsigma + \pi$, i.e. the foregone real return on capital plus the inflation tax revenue.

$$\frac{-dT}{d\pi} = \frac{d}{d\pi}\left(\frac{iM}{p_0}\right) = \frac{M}{p_0} + iL_{r+\pi} + iL_Y\frac{dY}{d\pi} \tag{17}$$

Equation (17) describes the marginal change in tax to changes in inflation.

[4] Start by substituting (3) into the second term on the right hand side of equation (10) to give, $\theta(t) = T + \frac{\pi(D-D^*)}{p} - \frac{(i_D-\pi)D^*}{p}$. Then, cancel matching items in the second and third terms, $\theta(t) = T + \frac{\pi D}{p} - \frac{i_D D^*}{p}$. Substituting equation (11) into the second and third terms on the right hand side gives, $\theta(t) = T + \pi\Delta(t) - i_D\left(\Delta(t) - \frac{M}{p}\right)$. Collect terms with $\Delta(t)$, $\theta(t) = T + (\pi - i_D)\Delta(t) + i_D\frac{M}{p}$. Substitute in using (12) for values at time zero, $\theta(0) = T + (\pi - i_D)\Delta(0) + i_D\frac{M}{p_0}$. Re-arranging with seignorage tax $i_D\frac{M}{p}$ on the left hand side gives equation (13).

2.4 Addressing Friedman's Propositions.

Phelps (1973) addresses two propositions by Friedman (1971). The first proposition is that seignorage revenue may actually fall following an increase in inflation. Phelps shows how his own model is consistent with this result subject to particular values for the interest elasticity of liquidity preference. The second proposition is that there may be no conflict between full liquidity and inflation tax revenue maximisation but Phelps states that his own model is inconsistent with this: "If, with Friedman, we identify full liquidity ... as occurring if and only if $i \leq 0$, and if we assume, again with Friedman, that

$$L(Y, 0, K + D_0/p_0) < \infty \tag{18}$$

then the revenue from the inflation tax, $i_D M/p_0$ must be non-positive at full liquidity. At any inflation rate too large for full liquidity, but not so large that $M/p = 0$, inflation tax revenue is positive. Hence there is a conflict between acquiring revenue and achieving full liquidity."[5]

3 Optimising the Revenue from Inflation.

The government sets policy targets to maximise the private sector's welfare and in doing this takes into account the private sector's responses. To solve the model, the preferences of the private sector and the constraints facing both the private and government sectors are specified.

3.1 The Private Sector's Preferences.

The behaviour of the private sector is obviously relevant to the formulation of the tax-mix. In this section the utility maximising behaviour of the visible and hidden economies is obtained using Lagrangean functions. To begin an aggregate utility function is specified for each economy, liquidity is included in the utility functions as it provides a service to private agents,

$$U^v = U(C^v, S^v, L^v, H^v) \tag{19a}$$
$$U^h = U(C^h, S^h, L^h, H^h) \tag{19b}$$

where agents in both economies have the same preference structure over the level of consumption ($U_C^{v,h} > 0$), the level of saving ($U_S^{v,h} > 0$), the level of liquidity ($U_L^{v,h} > 0$), and manhours worked ($U_H^{v,h} < 0$). So private agents are the same in all aspects except that some happen to operate in the visible economy and others in the hidden economy.

For simplicity, Phelps assumes a short-run framework such that the capital stock is taken as pre-determined. Just two taxes are assumed, the inflation tax and a proportional wage tax. To simplify factor pricing, capital and manhours are assumed to be perfect substitutes with constant marginal returns. Government expenditure is fixed, liquidity is costless to produce and gross economic output is given by,

$$Y = \bar{w}H + (\bar{r} + \bar{\delta})K = C + G + \dot{K} + \bar{\delta}K \tag{20}$$

where $\bar{w}$, $\bar{r}$ and $\bar{\delta}$ are all fixed in time and where w is the pre-tax wage, r is the real rate of interest and δ is the rate of capital depreciation.

[5] Phelps (1973), page 76.

3.2 The Government Budget Constraint.

The government budget constraint is calculated with inflation and direct taxes changing such that the prescribed path of government debt $\theta(t) = \dot{\Delta} = (\dot{D} - \pi D)/p$ remains unchanged. The proportional income tax rate is τ and pre-tax earnings are $Z = \bar{w}H$. Using equation (1) and the two equalities,[6]

$$D/p = \Delta \tag{21}$$

$$G + B + (\bar{r} + \pi)(D/p - L) - \tau Z - \pi D/p = \dot{\Delta} \tag{22}$$

gives the budget constraint facing the government,

$$(1-\alpha)\tau Z + iL = \gamma + \beta + \bar{r}\Delta - \dot{\Delta}. \tag{23}$$

The left hand side of equation (23) represents all revenues to the government and the right hand side all expenditures. Marginal tax analysis requires that the magnitude of equation (23) remains constant and that only the tax mix of the right hand side change as policy changes.

3.3 The Private Sector Budget Constraint.

The budget constraint for the private sector, with all expenditures on the left hand side and all revenues on the right hand side is given by,

$$C + S = (1-\tau)(1-\alpha)Z + \alpha Z + (r+\delta)K + \bar{B} + i(\Delta - L) - \pi\Delta - \delta K. \tag{24}$$

Assuming all aspects of the visible and hidden economies are a function of the population proportions, with the exception of the income tax burden, equation (24) can be re-arranged and split between the two sectors,

$$(1-\alpha)[C + S + iL] = (1-\alpha)[\bar{r}W + B + (1-\tau)Z] \tag{25a}$$
$$\alpha \quad [C + S + iL] = \alpha \quad [\bar{r}W + B + Z] \tag{25b}$$

where wealth and savings are respectively given by,

$$W = K + \Delta, \quad S = \dot{W}. \tag{26}$$

3.4 The Private Sector's Optimising Behaviour.

In order to derive the behaviour of the visible and hidden economies the first order conditions for the following two Lagrangeans must be satisfied.

$$\Lambda^v = U^v(C^v, S^v, L^v, H^v) - \lambda^v(1-\alpha)([C + S + iL] - [B + \bar{r}W + (1-\tau)Z]) \tag{27a}$$
$$\Lambda^h = U^h(C^h, S^h, L^h, H^h) - \lambda^h \ \alpha \ ([C + S + iL] - [B + \bar{r}W + Z]) \tag{27b}$$

[6] At this point Phelps (1973) switches notation and uses t to denote the direct tax rate rather than time. For the sake of continuity, here t continues to denote time and τ denotes the direct tax rate.

Note that this optimisation, like that of Phelps, is in the context of a static analysis. The resulting first order conditions for the visible and hidden economies are,

$$\begin{aligned} U_C^v &= U_S^v = \lambda^v(1-\alpha) \\ U_L^v &= \lambda^v(1-\alpha)i = U_C^v i \\ U_H^v &= -\lambda^v(1-\alpha)(1-\tau)\bar{w} = -U_C^v(1-\tau)\bar{w} \\ \text{or, } U_Z^v &= -\lambda^v(1-\alpha)(1-\tau) = -U_C^v(1-\tau) \end{aligned} \tag{28a}$$

$$\begin{aligned} U_C^h &= U_S^h = \lambda^h\alpha \\ U_L^h &= \lambda^h\alpha i = U_C^h i \\ U_H^h &= -\lambda^h\alpha\bar{w} = -U_C^h\bar{w} \\ \text{or, } U_Z^h &= -\lambda^h\alpha = -U_C^h \end{aligned} \tag{28b}$$

Using these first order conditions we can specify the maximised utility (U^*) subject to the tax rates from direct taxes τ and the inflation tax rate i. Writing the value functions for this optimisation in the visible and hidden economies,

$$V^v(\tau,i) = U^{v*}[C(\tau,i),S(\tau,i),L(\tau,i),Z(\tau,i)] \tag{29a}$$

$$V^h(\tau,i) = U^{h*}[C(\tau,i),S(\tau,i),L(\tau,i),Z(\tau,i)] \tag{29b}$$

The effect of the tax rates on the optimised level of utility is given by the derivatives on equations (29a) and (29b),

$$\begin{aligned} V_\tau^v(\tau,i) &= U_C^{v*}\tfrac{\partial C}{\partial\tau} + U_S^{v*}\tfrac{\partial S}{\partial\tau} + U_L^{v*}\tfrac{\partial L}{\partial\tau} + U_Z^{v*}\tfrac{\partial Z}{\partial\tau} \\ V_i^v(\tau,i) &= U_C^{v*}\tfrac{\partial C}{\partial i} + U_S^{v*}\tfrac{\partial S}{\partial i} + U_L^{v*}\tfrac{\partial L}{\partial i} + U_Z^{v*}\tfrac{\partial Z}{\partial i} \end{aligned} \tag{30a}$$

$$\begin{aligned} V_\tau^h(\tau,i) &= U_C^{h*}\tfrac{\partial C}{\partial\tau} + U_S^{h*}\tfrac{\partial S}{\partial\tau} + U_L^{h*}\tfrac{\partial L}{\partial\tau} + U_Z^{h*}\tfrac{\partial Z}{\partial\tau} \\ V_i^h(\tau,i) &= U_C^{h*}\tfrac{\partial C}{\partial i} + U_S^{h*}\tfrac{\partial S}{\partial i} + U_L^{h*}\tfrac{\partial L}{\partial i} + U_Z^{h*}\tfrac{\partial Z}{\partial i} \end{aligned} \tag{30b}$$

Substituting the first order conditions in (28a) and (28b) into (30a) and (30b) gives,

$$\begin{aligned} V_\tau^v(\tau,i) &= U_C^{v*}\left[\tfrac{\partial C}{\partial\tau} + \tfrac{\partial S}{\partial\tau} + i\tfrac{\partial L}{\partial\tau} - (1-\tau)\tfrac{\partial Z}{\partial\tau}\right] \\ V_i^v(\tau,i) &= U_C^{v*}\left[\tfrac{\partial C}{\partial i} + \tfrac{\partial S}{\partial i} + i\tfrac{\partial L}{\partial i} - (1-\tau)\tfrac{\partial Z}{\partial i}\right] \end{aligned} \tag{31a}$$

$$\begin{aligned} V_\tau^h(\tau,i) &= U_C^{h*}\left[\tfrac{\partial C}{\partial\tau} + \tfrac{\partial S}{\partial\tau} + i\tfrac{\partial L}{\partial\tau} - \tfrac{\partial Z}{\partial\tau}\right] \\ V_i^h(\tau,i) &= U_C^{h*}\left[\tfrac{\partial C}{\partial i} + \tfrac{\partial S}{\partial i} + i\tfrac{\partial L}{\partial i} - \tfrac{\partial Z}{\partial i}\right] \end{aligned} \tag{31b}$$

Differentiation of the budget constraint represented by equations (25a) and (25b) gives,

$$\begin{aligned} (1-\alpha)Z + (1-\alpha)\left[\tfrac{\partial C}{\partial\tau} + \tfrac{\partial S}{\partial\tau} + i\tfrac{\partial L}{\partial\tau} - (1-\tau)\tfrac{\partial Z}{\partial\tau}\right] &= 0 \\ (1-\alpha)L + (1-\alpha)\left[\tfrac{\partial C}{\partial i} + \tfrac{\partial S}{\partial i} + i\tfrac{\partial L}{\partial i} - (1-\tau)\tfrac{\partial Z}{\partial i}\right] &= 0 \end{aligned} \tag{32a}$$

$$\begin{aligned} \alpha\left[\tfrac{\partial C}{\partial\tau} + \tfrac{\partial S}{\partial\tau} + i\tfrac{\partial L}{\partial\tau} - \tfrac{\partial Z}{\partial\tau}\right] &= 0 \\ \alpha L + \alpha\left[\tfrac{\partial C}{\partial i} + \tfrac{\partial S}{\partial i} + i\tfrac{\partial L}{\partial i} - \tfrac{\partial Z}{\partial i}\right] &= 0 \end{aligned} \tag{32b}$$

Substituting equations (32a) and (32b) into (31a) and (31b) generates expressions for changes in the optimal level of utility as the policy mix changes. These equations are of prime interest from the government's standpoint in determining the optimal tax-mix policy,

$$
\begin{aligned}
V_\tau^v(\tau,i) &= -U_C^{v*}Z && (33a)\\
V_i^v(\tau,i) &= -U_C^{v*}L && (33b)\\
V_\tau^h(\tau,i) &= -U_C^{h*}0 = 0 && (33c)\\
V_i^h(\tau,i) &= -U_C^{h*}L && (33d)
\end{aligned}
$$

These equations are analogous to those numbered (33) in Phelps's paper. Equation (33c) is the only one that has no analogous equivalent, it suggests that at the margin, the level of utility in the hidden economy does not diminish when the burden of direct wage taxation increases.

3.5 The Government's Optimising Behaviour.

A benevolent government would set tax policy to maximise utility as represented by equations (29a) and (29b), subject to the constraint in equation (23). To establish this optimal tax policy the following Lagrangean is specified,

$$\Psi(\tau,i) \;=\; (1-\alpha)V^v(\tau,i)+\alpha V^h(\tau,i) \;+\; \mu[\tau(1-\alpha)Z+iL-\bar{R}] \qquad (34)$$

Notice that the government weights the utility functions of the visible and hidden economies in accordance to their population shares. Setting $\bar{R}=\tau(1-\alpha)Z+iL$, the first order derivatives for this Lagrangean are $\frac{\partial\Psi}{\partial\tau}=(1-\alpha)V_\tau^v+\alpha V_\tau^h+\mu\frac{\partial R}{\partial\tau}$ and $\frac{\partial\Psi}{\partial i}=(1-\alpha)V_i^v+\alpha V_i^h+\mu\frac{\partial R}{\partial i}$. The corresponding first order conditions are,

$$
\begin{aligned}
(1-\alpha)V_\tau^v+\alpha V_\tau^h &= -\mu\frac{\partial R}{\partial\tau}\\
(1-\alpha)V_i^v+\alpha V_i^h &= -\mu\frac{\partial R}{\partial i}
\end{aligned}
\qquad (35)
$$

The government sets the tax-mix policy that maximises utility (minimises tax distortions) in both the visible and hidden economies, so the derivatives of utility with respect to inflation tax must be equal $V_i^v=V_i^h$. This implies through equations (33a,b,c,d) that $U_C^{v*}=U_C^{h*}=U_C^*$. This last condition, together with the conditions represented by (33a,b,c,d) can be substituted into (35) to give,

$$
\begin{aligned}
(1-\alpha)U_C^*Z+\alpha U_C^*0 = \mu\frac{\partial R}{\partial\tau} \quad &\Rightarrow \quad \frac{U_C^*}{\mu}=\frac{\partial R/\partial\tau}{(1-\alpha)Z}\\
(1-\alpha)U_C^*L+\alpha U_C^*L = \mu\frac{\partial R}{\partial i} \quad &\Rightarrow \quad \frac{U_C^*}{\mu}=\frac{\partial R/\partial i}{L}
\end{aligned}
$$

Equating the two expressions above gives the government policy target,

$$\frac{\partial R/\partial\tau}{(1-\alpha)Z}=\frac{\partial R/\partial i}{L}=\frac{U_C^*}{\mu}, \qquad (36)$$

this defines the tax-mix that maximises social welfare subject to a constant total tax revenue, $\bar{R}=\tau(1-\alpha)Z+iL$. The increases in overall income (I) required to compensate for any change in the direct tax rate τ or the inflation tax rate i are,

$$\left(\frac{\partial I}{\partial\tau}\right)_{\bar{V}^{v,h}}=-Z, \quad \left(\frac{\partial I}{\partial i}\right)_{\bar{V}^{v,h}}=-L, \qquad (37)$$

The subscripts in equations (37) indicate that utility is being kept constant in the respective sectors by compensating agents for changes in the tax-mix. Taking partial derivatives of the government revenue function and substituting in (37) specifies,

$$\begin{aligned}\frac{\partial R}{\partial \tau} &= (1-\alpha)\left[\tau\left[\left(\frac{\partial Z}{\partial \tau}\right)_{\bar{V}^v} - Z\frac{\partial Z}{\partial I}\right] + i\left[\left(\frac{\partial L}{\partial \tau}\right)_{\bar{V}^v} - Z\frac{\partial L}{\partial I}\right] + Z\right] \\ &\quad + \alpha\left[\tau\left[\left(\frac{\partial Z}{\partial \tau}\right)_{\bar{V}^h} - Z\frac{\partial Z}{\partial I}\right] + i\left[\left(\frac{\partial L}{\partial \tau}\right)_{\bar{V}^h} - Z\frac{\partial L}{\partial I}\right] + Z\right] \\ \frac{\partial R}{\partial i} &= (1-\alpha)\left[\tau\left[\left(\frac{\partial Z}{\partial i}\right)_{\bar{V}^v} - L\frac{\partial Z}{\partial I}\right] + i\left[\left(\frac{\partial L}{\partial i}\right)_{\bar{V}^v} - L\frac{\partial L}{\partial I}\right] + L\right] \\ &\quad + \alpha\left[\tau\left[\left(\frac{\partial Z}{\partial i}\right)_{\bar{V}^h} - L\frac{\partial Z}{\partial I}\right] + i\left[\left(\frac{\partial L}{\partial i}\right)_{\bar{V}^h} - L\frac{\partial L}{\partial I}\right] + L\right]\end{aligned} \tag{38}$$

By re-arranging, these equations can be expressed more compactly as,

$$\begin{aligned}\frac{\partial R}{\partial \tau} &= (1-\alpha)\left[\tau\left(\frac{\partial Z}{\partial \tau}\right)_{\bar{V}^v} + i\left(\frac{\partial L}{\partial \tau}\right)_{\bar{V}^v}\right] + \alpha\left[\tau\left(\frac{\partial Z}{\partial \tau}\right)_{\bar{V}^h} + i\left(\frac{\partial L}{\partial \tau}\right)_{\bar{V}^h}\right] + Z\left(1 - \tau\frac{\partial Z}{\partial I} - i\frac{\partial L}{\partial I}\right) \\ \frac{\partial R}{\partial i} &= (1-\alpha)\left[\tau\left(\frac{\partial Z}{\partial i}\right)_{\bar{V}^v} + i\left(\frac{\partial L}{\partial i}\right)_{\bar{V}^v}\right] + \alpha\left[\tau\left(\frac{\partial Z}{\partial i}\right)_{\bar{V}^h} + i\left(\frac{\partial L}{\partial i}\right)_{\bar{V}^h}\right] + L\left(1 - \tau\frac{\partial Z}{\partial I} - i\frac{\partial L}{\partial I}\right)\end{aligned} \tag{39}$$

where the substitutions $\left(\frac{\partial L}{\partial \tau}\right)_{\bar{V}^v} = \left(\frac{\partial Z}{\partial i}\right)_{\bar{V}^v}$ and $\left(\frac{\partial L}{\partial \tau}\right)_{\bar{V}^h} = \left(\frac{\partial Z}{\partial i}\right)_{\bar{V}^h}$ have been made, these hold thanks to Slutsky symmetry.

Substituting the behavioural equations (39) into the policy target equations (36) yields the following rather unwieldy equation which represents a special case of equation (3) in Ramsey (1927),

$$\begin{aligned}\frac{U_C^*}{\mu} - \left(1 - \tau\frac{\partial Z}{\partial I} - i\frac{\partial L}{\partial I}\right) &= \frac{(1-\alpha)\left[\tau\left(\frac{\partial Z}{\partial \tau}\right)_{\bar{V}^v} + i\left(\frac{\partial L}{\partial \tau}\right)_{\bar{V}^v}\right] + \alpha\left[\tau\left(\frac{\partial Z}{\partial \tau}\right)_{\bar{V}^h} + i\left(\frac{\partial L}{\partial \tau}\right)_{\bar{V}^h}\right]}{(1-\alpha)Z} \\ &= \frac{(1-\alpha)\left[\tau\left(\frac{\partial Z}{\partial i}\right)_{\bar{V}^v} + i\left(\frac{\partial L}{\partial i}\right)_{\bar{V}^v}\right] + \alpha\left[\tau\left(\frac{\partial Z}{\partial i}\right)_{\bar{V}^h} + i\left(\frac{\partial L}{\partial i}\right)_{\bar{V}^h}\right]}{L}\end{aligned} \tag{40}$$

Assuming there are no cross substitution effects in the demand for one commodity (Z, L) with respect to the price of the other commodity (i, τ), then $(\partial L/\partial \tau)_V = (\partial Z/\partial i)_V = 0$ and equation (40) can be simplified to,

$$\frac{\tau}{Z}\left[(1-\alpha)\left(\frac{\partial Z}{\partial \tau}\right)_{\bar{V}^v} + \alpha\left(\frac{\partial Z}{\partial \tau}\right)_{\bar{V}^h}\right] = \frac{(1-\alpha)i}{L}\left[(1-\alpha)\left(\frac{\partial L}{\partial i}\right)_{\bar{V}^v} + \alpha\left(\frac{\partial L}{\partial i}\right)_{\bar{V}^h}\right] \tag{41}$$

Equation (41) is the policy rule for the government, it specifies the optimal direct tax and liquidity tax rates (τ, i) as functions of the compensated marginal labour, marginal liquidity demands and the relative size of the hidden economy $(\partial Z/\partial \tau, \partial L/\partial i, \alpha)$.

Recall from equation (17) that the implicit tax rate on liquidity is $i = \varsigma + \pi$ where ς is the constant real rate of interest on capital and π is inflation. Departing from Phelps's presentation, substitute this into equation (41) and re-arrange to give,

$$\frac{\tau}{\varsigma + \pi} = (1-\alpha)\frac{Z}{L}\frac{\left[(1-\alpha)\left(\frac{\partial L}{\partial i}\right)_{\bar{V}^v} + \alpha\left(\frac{\partial L}{\partial i}\right)_{\bar{V}^h}\right]}{\left[(1-\alpha)\left(\frac{\partial Z}{\partial \tau}\right)_{\bar{V}^v} + \alpha\left(\frac{\partial Z}{\partial \tau}\right)_{\bar{V}^h}\right]} \tag{42}$$

Equation (42) suggests that the larger the relative size of the hidden economy, the higher the optimal ratio of inflation to the tax rate. Of course, this assumes that the compensated labour and liquidity demands do not change so as to counteract the effect of α. To define the exact magnitudes of these optimal tax rates one would have to specify a utility function for the private sector and the government's fiscal policy.

4 Implications for Government Policy.

As discussed above, equations (41) and (42) suggest some implications of the relative size of the hidden economy on the optimal tax-mix. Equation (41) is analogous to equation (41) in Phelps (1973), with the addition of parameters representing the presence of a hidden economy within the private sector. Equation (42) has no analogous equation in Phelps (1973) but illustrates the relation between the size of the hidden economy and the ratio of direct taxes to inflation.

4.1 The Limiting Cases, $\alpha = 0$ and $\alpha = 1$.

When there is no hidden economy ($\alpha = 0$), equation (41) above collapses to equation (41) in Phelps (1973). As in Phelps's article, an implication of equation (41) is that if the optimal level of direct taxation is positive, then so will be the level of inflation taxation. When there is no visible economy ($\alpha = 1$) the right hand side of equation (41) equals zero and the optimal direct tax rate is zero ($\tau = 0$).

4.2 The Intermediate Cases, $0 < \alpha < 1$.

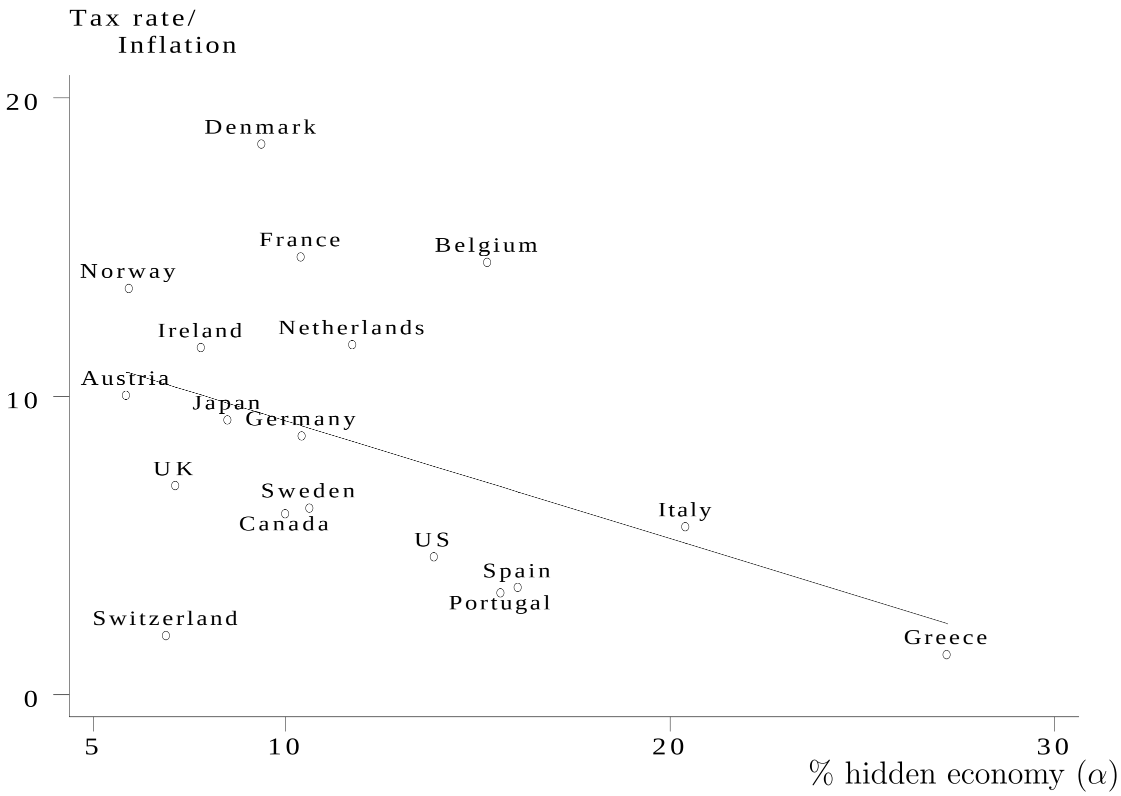
Equation (41) can also be used to consider the implications on the optimal tax-mix for intermediate cases. As the size of the hidden economy increases the right hand side of equation (41) becomes smaller, thus requiring a smaller tax rate τ relative to the inflation tax rate i to maintain the relation. These intermediate cases also imply a positive inflation tax rate for any positive direct tax rate.

4.3 Some Anecdotal Evidence.

The anecdotal evidence presented in this sub-section supports the implications of equation (42), that the ratio of direct taxes to inflation and the size of the hidden economy are inversely related. The data summarises average inflation and hidden economy size over four years for forty-one countries. Average values over time are reported because the theoretical model is static and we therefore want to abstract from any dynamics. Average values are taken over as many countries and years as the data permits. In Figures 1, 2 and 3, the data are presented by country groups that face similar economic circumstances. In Figure 4 the data for all countries are presented. These country groups include OECD member states, economies in transition and developing economies.

In Figure 1 summary data and plots for OECD member states are reported. These countries represent industrialised economies and for the most part they seem to have relatively small hidden economies and relatively high tax to inflation ratios. The most evident outlier is Greece with the largest "% hidden economy" and the lowest "Tax rate to Inflation" ratio. The fitted line is obtained by OLS with mean tax to inflation ratio a function of the hidden economy share, the negative relation between these seems clear. Two outliers off the fitted line are Belgium and Switzerland. Belgium seems to have a particularly high tax rate and low inflation rate. Switzerland, on the other hand, seems to have a particularly low tax rate.

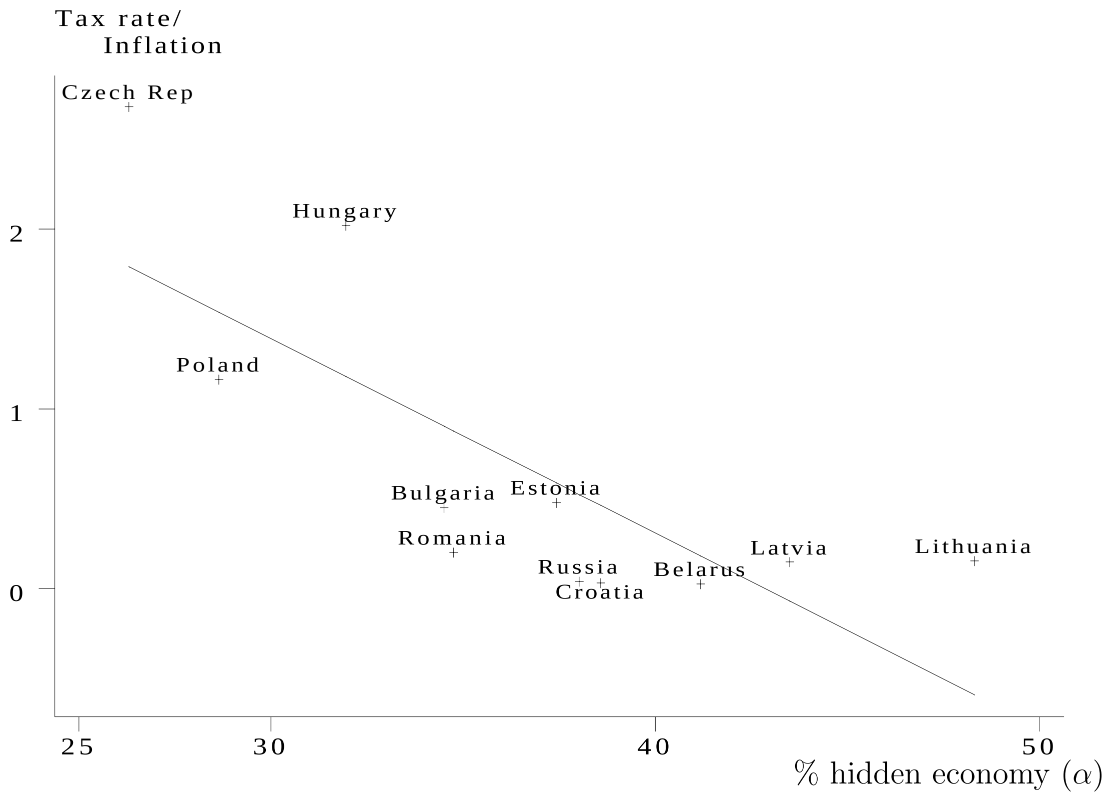
Summary data and plots for countries in economic transition are reported in Figure 2. These represent old Soviet member states and countries that were in the Soviet



Country	Hidden[a]	Tax rate[b]	Inflation[c]	$\frac{\text{Tax rate}}{\text{Inflation}}$
Austria	5.9 %	35.6 %	3.5 %	10.02
Belgium	15.2 %	43.1 %	3.0 %	14.47
Canada	10.0 %	20.7 %	3.4 %	6.04
Denmark	9.4 %	38.7 %	2.1 %	18.44
France	10.4 %	40.6 %	2.8 %	14.65
Germany	10.4 %	30.1 %	3.5 %	8.66
Greece	27.2 %	23.2 %	17.5 %	1.32
Ireland	7.8 %	32.8 %	2.8 %	11.61
Italy	20.4 %	31.4 %	5.6 %	5.61
Japan	8.5 %	22.0 %	2.4 %	9.18
Netherlands	11.8 %	33.4 %	2.8 %	11.70
Norway	5.9 %	41.1 %	3.0 %	13.60
Portugal	15.6 %	34.3 %	10.1 %	3.39
Spain	16.0 %	20.6 %	5.8 %	3.57
Sweden	10.6 %	41.6 %	6.7 %	6.23
Switzerland	6.9 %	9.1 %	4.7 %	1.96
UK	7.2 %	36.1 %	5.2 %	6.98
US	13.9 %	17.9 %	3.9 %	4.60

[a]Unofficial GDP as a proportion of total GDP (α), mean values over 1990-1993. Taken from Johnson et al. (1999) table A1, estimated using the currency demand method.
[b]Ratio of government revenue to GDP, mean values over 1990-1991. Both taken from IMF (1999), country tables, lines 81 and 99 respectively.
[c]Mean values over 1990-1991. Taken from IMF (1999), table 64 on consumer prices.

Figure 1: Tax to inflation ratio and hidden economy size, OECD member states.



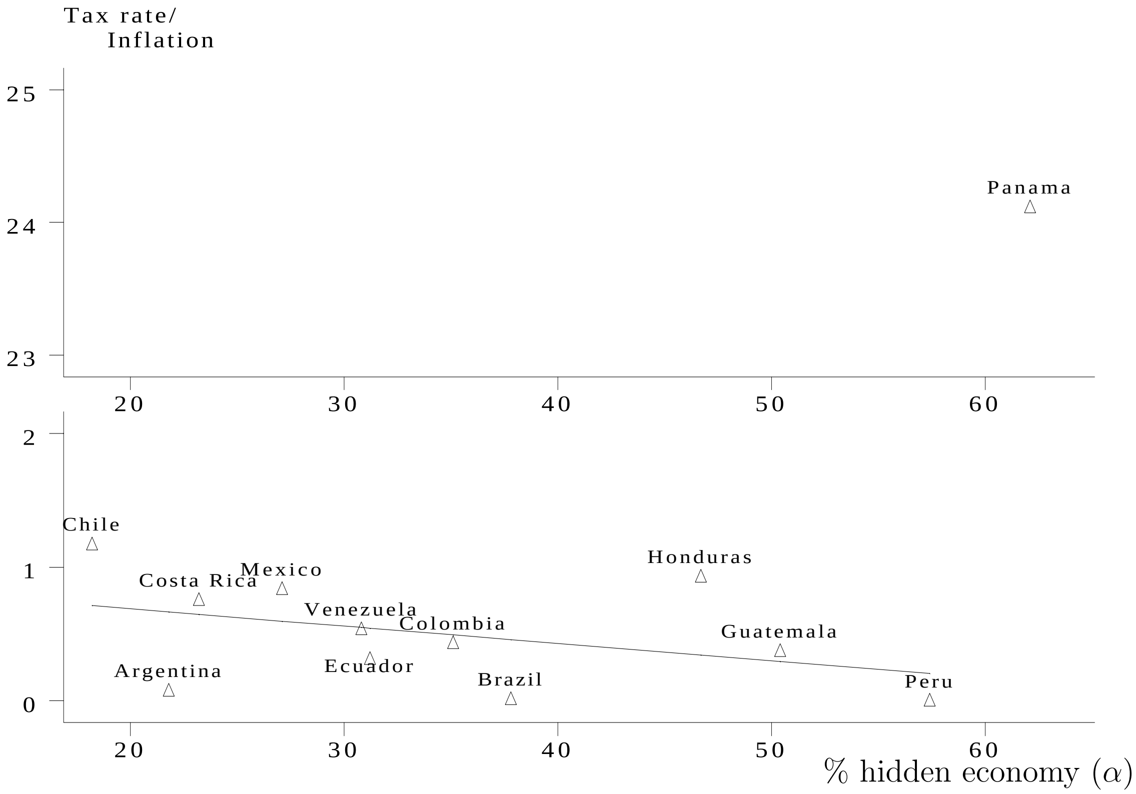
Country	Hidden[a]	Tax rate[b]	Inflation[c]	$\frac{\text{Tax rate}}{\text{Inflation}}$
Belarus	41.2 %	33.8 %	1271.8 %	0.03
Bulgaria	34.5 %	36.1 %	80.6 %	0.45
Croatia	38.6 %	12.9 %	420.3 %	0.03
Czech Republic	26.3 %	33.6 %	12.5 %	2.68
Estonia	37.4 %	30.8 %	64.4 %	0.48
Hungary	32.0 %	46.8 %	23.1 %	2.02
Latvia	43.5 %	15.2 %	103.2 %	0.15
Lithuania	48.3 %	23.5 %	153.3 %	0.15
Poland	28.6 %	41.4 %	35.6 %	1.16
Romania	34.8 %	32.0 %	158.9 %	0.20
Russia	38.0 %	14.3 %	360.8 %	0.04

[a] Unofficial GDP as a proportion of total GDP (α), mean values over 1992-1995. Taken from Lacko (1999) table 8, estimated using the household electricity demand approach.

[b] Ratio of government revenue to GDP, mean values over 1990-1991. Both taken from IMF (1999), country tables, lines 81 and 99 respectively. Mean values over 1991-3 for the Czech Republic and Lithuania, mean values over 1992-3 for Croatia and Poland, values in 1993 for Latvia and Russia.

[c] Mean values over 1990-1991. Taken from IMF (1999), table 64 on consumer prices. Any gaps are filled using data for consumer price indices from UN (1996), table 51.

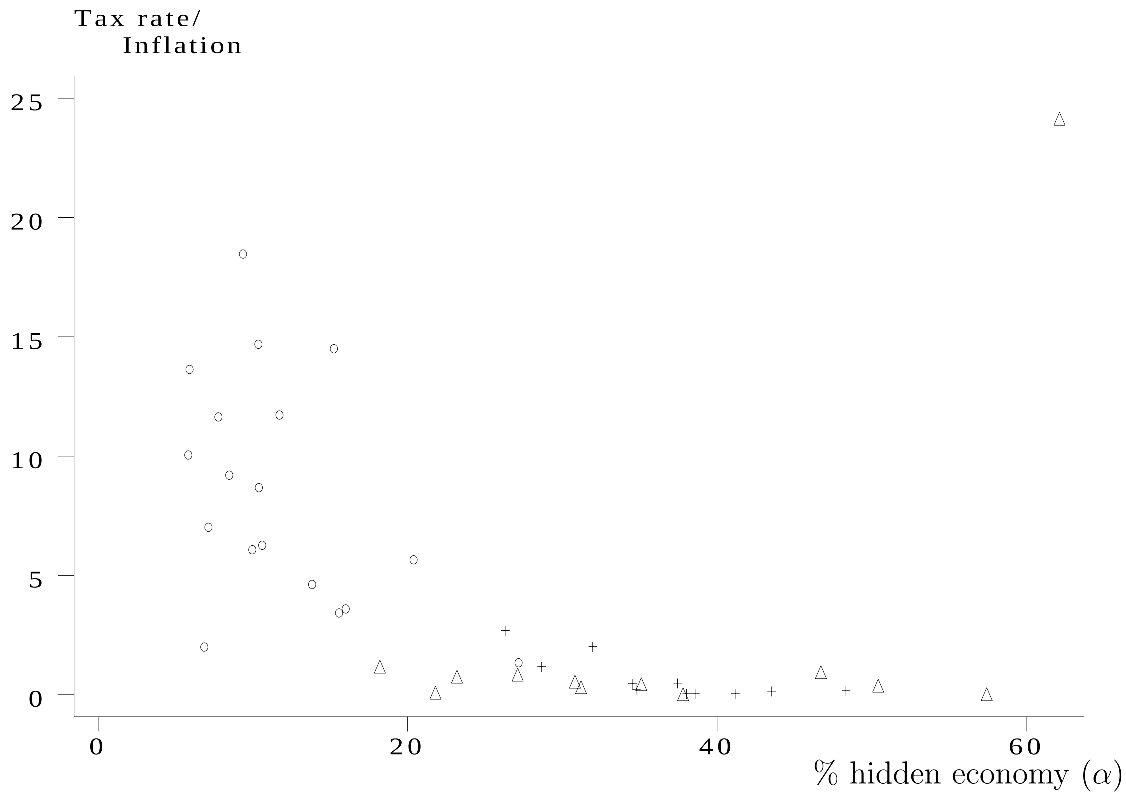
Figure 2: Tax to inflation ratio and hidden economy size, Transition economies.



Country	Hidden[a]	Tax rate[b]	Inflation[c]	$\frac{\text{Tax rate}}{\text{Inflation}}$
Argentina	21.8 %	51.5 %	630.3 %	0.08
Brazil	37.8 %	28.6 %	1565.0 %	0.02
Chile	18.2 %	22.3 %	19.0 %	1.18
Colombia	35.1 %	12.0 %	27.3 %	0.44
Costa Rica	23.2 %	15.1 %	19.8 %	0.76
Ecuador	31.2 %	15.6 %	49.2 %	0.32
Guatemala	50.4 %	9.1 %	24.1 %	0.38
Honduras	46.7 %	17.9 %	19.2 %	0.93
Mexico	27.1 %	15.7 %	18.7 %	0.84
Panama	62.1 %	26.5 %	1.1 %	24.12
Peru	57.4 %	12.0 %	2003.3 %	0.01
Venezuela	30.8 %	19.6 %	36.1 %	0.54

[a] Unofficial GDP as a proportion of total GDP (α), mean values over 1990-1993. Taken from Johnson et al. (1999) table A1, estimated using the Multiple Indicators Multiple Causes method.
[b] Ratio of government revenue to GDP, mean values over 1990-1991. Both taken from IMF (1999), country tables, lines 81 and 99 respectively.
[c] Mean values over 1990-1991. Taken from IMF (1999), table 64 on consumer prices.

Figure 3: Tax to inflation ratio and hidden economy size, Developing economies.



○ OECD member states (1990-1993).
+ Transition economies (1992-1995).
△ Developing economies (1990-1993).

Figure 4: Tax to inflation ratio and hidden economy size, all three groups.

sphere of influence. These countries have faced political and economic difficulties in recent years, this is reflected in relatively large inflation rates and relatively large hidden economy shares. Again, the positive relation between the tax to inflation ratio and the relative size of the hidden economy seems clear. No particular outliers seem evident, though it seems that a quadratic function would have provided a far better fitted line.

Summary statistics for developing countries are reported in Figure 3. This set of countries is limited to Latin America, estimates for the size of the hidden economy for countries in the African and Asian continents are very limited. Evidence of a positive correlation between the tax to inflation ratio and the relative size of the hidden economy is much weaker. Panama is the obvious outlier in this group. If the fitted line had included Panama, the relation between the "Tax rate to Inflation" ratio and "% hidden economy" would have appeared positive.

Figure 4 illustrates data points for countries in all three categories. The relation between the "Tax rate to Inflation" ratio and the "% hidden economy" appears negative, non-linear and well defined. The obvious outlier in this group is Panama, with the highest percentage hidden economy share and the highest "Tax rate to Inflation" ratio for any of the forty-one countries illustrated here. Factors that may explain why Panama is an outlier include the fixed exchange rate of one Balboa to one US $, the large proportion of government revenues that accrue from the Panama Canal and the large size of the hidden economy, source: CIA (1999).

4.4 Extensions.

Equations (41) and (42) are only suggestive of the relation between direct taxes and inflation taxes. To obtain more precise policy implications one would have to calibrate the model, specify preference functions and run simulations.

There are also important model extensions to be considered. Some of these include *heterogeneity, dollarisation, feed-back, endogeneity, dynamics* and *corruption*. The hidden and visible economies may be quite *Heterogeneus* in their access to financial instruments, hidden transactions are more likely to require cash and agents in the hidden economy may have less access to inflation-proofed financial instruments. This may imply that the inflation tax may be a good instrument for taxing the hidden economy without having too much impact on the visible economy. *Dollarisation* describes the process by which a foreign currency (most often the US $) may be used in preference to the domestic currency. This generally occurs when confidence in the domestic currency falls. Increasing the inflation tax in these circumstances may be self-defeating because it further diminishes confidence in the domestic currency and because the hidden transactions carried out in the foreign currency are not subject to the inflation tax. Dollarisation is a good example of why *Feed-Back* mechanisms are important and why the relative size of the hidden economy may be *endogenous*. *Feed-Back* mechanisms also highlight the importance of the *dynamic* implications of policy. If Fiscal and Monetary policies are used in some degree in response to economic cycles *dynamic* aspects are particularly important when undertaking any statistical analysis. Finally, one must consider the issue of *corruption* within the government sector. Throughout the analysis we have assumed that the government is benevolent and competent, if the government fails on either it may no longer be a welfare optimiser. Inadequate governments may mean that first-best policies are time-inconsistent, government institutions so inadequate as to be considered corrupt and wasteful may induce a larger proportion of the private sector into the hidden economy.

5 Conclusion.

Equations (41) and (42) suggest that the optimal tax-mix of inflation and direct taxes changes with the relative size of the hidden economy. The tax-mix is optimal in the sense that it minimises tax distortions subject to a particular government expenditure policy. Subject to marginal preferences over labour supply and liquidity preference not changing radically at the margin, a larger hidden economy implies a tax-mix with higher inflation relative to the direct tax rate. Anecdotal empirical evidence from disparate groups of countries suggests that a positive correlation between the relative size of the hidden economy and the level of price inflation is observed in practice.

Of course, price inflation is determined by a wider set of parameters than just the relative size of the hidden economy. Though the relative size of the hidden economy may only be of marginal importance, it is true that most macroeconomic analysis ignores it completely. If, as Enste and Schneider (1998, 1999) suggest, the global trend is for an increase in the relative size of the hidden economy, then inclusion of these aspects into macroeconomic analysis is becoming increasingly important.

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