Comparison of numerical, empirical and local partition methods in ADCB specimens

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Abstract: It is well-known from the scientific literature that the asymmetric double cantilever beam (ADCB) specimen is subjected to mixed-mode I/II load at the crack tip. In these samples, the crack plane lies outside the laminate midplane. In this work, the energy release rate in modes I and II (G_I and G_{II}) are obtained by different approaches. The analytical determination of G_I and G_{II} in ADCB samples is not simple or straightforward and is usually based on partition methods. Numerical results obtained from finite element analysis (FE) are compared with the analytical local partition method (LP) for a carbon fibre epoxy AS4/3501-6 laminate. Both results are also compared with an empirical formulation obtained in previous works. Results obtained from all three methods are in good agreement.

Keywords: asymmetric double cantilever beam; ADCB; mixed mode; fracture toughness; delamination; virtual crack closure technique; VCCT.

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Jaime Viña graduated in Industrial Engineering at University of Oviedo, received his PhD in Materials Engineering from the same university. In 1988, he joined to University of Oviedo as an Assistant Professor and now he is Full Professor of Materials Science and Non-metallic Materials. His research area is the fracture and fatigue of composite materials. He is the author of more than

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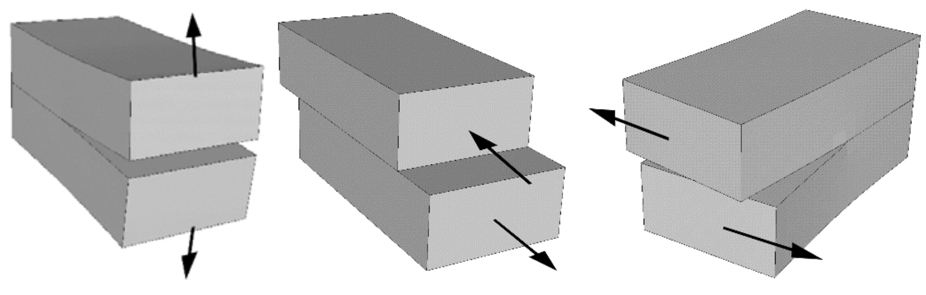
1 Introduction

The reduction of weight in automotive and aerospace structures is an important issue in order to reduce environmental impact and fuel consumption. Material selection plays an important role in this field. Aluminium alloys and composite materials are commonly used in the automotive industry but they are not so widely used as in the aerospace industry. The substitution of steel structures by composite materials leads to lighter and more efficient structures. Nevertheless, processing cost still plays an important role in the economic viability of composite materials at high production volumes, mainly for long fibre composites that exhibit the best mechanical properties to weight ratio.

The study of composite materials and the joints between dissimilar materials as steel, aluminium and composites is an important research field in the automotive industry.

Figure 1 Delamination modes, (a) mode I (b) mode II (c) mode III



Regarding the mechanical behaviour of composite materials, delamination failure is frequently found in composite structures. This fracture mode is produced by high interlaminar stresses due to material and geometric discontinuities in laminates subjected to static and dynamic loads. The delamination process can be conducted in modes I, II and III (Figure 1) and by means of the different combination between these modes. The delamination process has been studied not only for cracks inside the laminates, but also between dissimilar materials (Ning et al., 2014) where surface treatments influences the resistance to delamination.

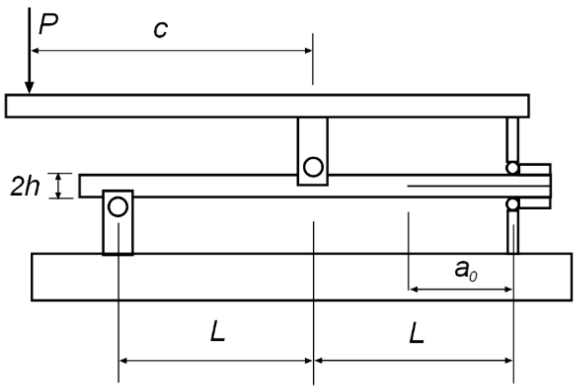
There are different procedures described in the scientific literature in order to determine pure modes I, II and III. Mode I is usually determined by means of the double cantilever beam (DCB) test. This test has been elevated to international standard (ISO 15024, 2001; ASTM D 5528, 2013). Mode II is typically determined by means of the end notched flexural (ENF) test although this test remains controversial due to the unstable crack growth and the influence of friction on the results (Brunner et al., 2008). Presently, a Japanese mode II standard test (JIS K7086, 1997), a testing protocol delivered by European Structural Integrity Society (ESIS) (Moore et al., 2001), a European aerospace testing procedure draft (Fpr EN 6034, 1995) and an ASTM standard (ASTM D7905, 2014) have been published.

Mode III is still being investigated. There are some test methods published in the scientific literature as the split cantilever beam (SCB) (Donaldson, 1988) or the edge cracked torsion (ECT) (Lee, 1993) but none of them has been elevated to international standards so far.

Nevertheless, composite failure usually involves a combination of damage mechanism instead of pure damage modes. In this sense, mixed mode tests have attracted the interest of many researches. Mixed mode I/II has been the most studied mode among the different combinations of modes I, II and III.

The mixed mode bending (MMB) test for I/II mixed mode has been widely covered by the scientific literature (Crews and Reeder, 1988; Reeder and Crews, 1990; 1992) and have been elevated to international standards (ASTM D 6671, 2013) (Figure 2). The test procedure and analytical formulation to calculate mode I and mode II are well-established for the MMB test.

Figure 2 MMB tests



Notes: *P*: applied load, a_0: initial crack length, *L*: midspan, *c*: lever length and *2h*: specimen thickness

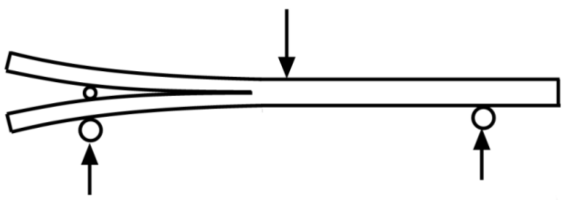
Blanco et al. (2006) found that the resultant mode mixture in the MMB test could differ considerably from the expected mode mixture depending on the equation used to calculate the distance of the lever arm. They developed an exact solution to calculate this distance.

The asymmetric double cantilever beam (ADCB) test is an alternative to the MMB test to produce a mixed mode load state at the crack tip. In these samples, the crack plane lies outside the laminate midplane. ADCB test specimens and fittings are as simple as in pure mode I tests. As a disadvantage, the mode mixity ratio at the crack tip in ADCB samples cannot be controlled by means of the test fixtures as in MMB tests. In ADCB samples, the position of the crack plane controls the mode mixity ratio. Therefore, in order to test a given mixed mode I/II a specific specimen is prepared in order to obtain the desired mode mixity ratio.

Regarding the ADCB test, the analytical determination of G_I and G_{II} is not simple or straightforward and is usually based on partition methods. Some approaches to calculate G_I and G_{II} can be found in the scientific literature (Bradley and Cohen, 1985; Charalambides et al., 1992; Hashemi et al., 1991; Hutchinson and Suo, 1991; Mangalgiri et al., 1986). Most of these methods establish an energy partition based on the local singular field ahead of the crack tip or on a global method. There are other approaches in the scientific literature as that developed by Bennati et al. (2009) based on the Timoshenko's beam theory applied to an idealised continuous distribution of elastic-brittle springs between the sublaminates.

There are other tests developed in the scientific literature in order to obtain I/II mixed mode at the crack tip as that proposed by Szekrényes (2006). This test configuration consist on a prestressed end-notched flexure (PENF) configuration (Figure 3). Mode II is produced by specimen flexure and mode I is introduced by means of a rod inserted between the sublaminates.

Figure 3 PENF test



This test configuration, as can be seen in Figure 3, combines the DCB and ENF test (for mode I and II respectively). The global configuration is an ENF test, but the rod inserted between both sublaminates generates mode I at the crack tip. This test is very simple to perform and do not require to bond hinges in the sample. Boyano et al. (2011) have also studied this test. They studied the influence of different rod positions and diameters on the mixed mode at the crack tip.

Another interesting test to analyse mixed mode is the mixed-mode end load split (MMELS) test. This test has been studied by different researchers (Hashemi et al., 1990; Kinloch et al., 1993). Blanco et al. (2006) have also analysed this test configuration comparing different approaches and proposing a more accurate alternative analysis based on the virtual crack closure technique (VCCT).

In this work, G_I and G_{II} were calculated for a unidirectional carbon fibre epoxy matrix AS4/3501-6 laminate by means of three different approaches for an ADCB test configuration. Numerical results obtained from finite element (FE) analysis were compared with an analytical local partition (LP) method (Ducept et al., 1999). Both results were also compared with an empirical formulation obtained in previous works (Mollón et al., 2010).

2 Materials and methods

2.1 Materials

The material used to perform the numerical and analytical calculations was the Hexcel AS4/3501-6 unidirectional carbon fibre reinforced epoxy laminate. The mechanical properties of this laminate are shown in Table 1.

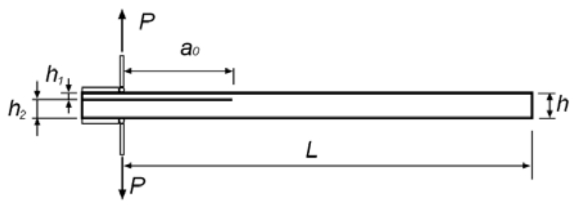
Table 1 Mechanical properties of the Hexcel AS4/3501-6 unidirectional laminate

Property		*MPa*
E_{11}	Longitudinal elastic modulus	131,000
E_{22}	Transversal elastic modulus	8,900
G_{12}	Shear elastic modulus	5,090
σ_{11}	Longitudinal tensile strength	1,954

2.2 Samples

Figure 4 shows the ADCB specimen configuration.

Figure 4 ADCB specimen



Notes: h_1: upper sublaminate thickness, h_2: bottom sublaminate thickness, h: total thickness, L: sample length, a_0: initial crack length, P: critical load.

In this work the following parameters has been set: a_0 = 50 mm, h = 5 mm, L = 150 mm and B = 25 mm.

Samples with different h_1/h_2 rates ranging from 0.25 to 1.0 were prepared in order to study different asymmetry grades.

2.3 Finite element method

An Ansys package was used to perform the numerical calculations. In order to calculate G_I and G_{II} energy release rates the two step procedure was followed (or two-step crack closure technique). In the two step method, the crack path is modelled using pairs of coincident nodes. The forces at the crack tip are calculated in a first step when the load reaches a critical value (Figure 5). The imposed displacement in the sample is then held and the coupled degrees of freedom (DOFs) of the nodes at the crack tip are released in a second step (Figure 6). Displacements are then calculated in this second step.

Figure 5 Step 1: DOFs are coupled at coincident nodes (see online version for colours)

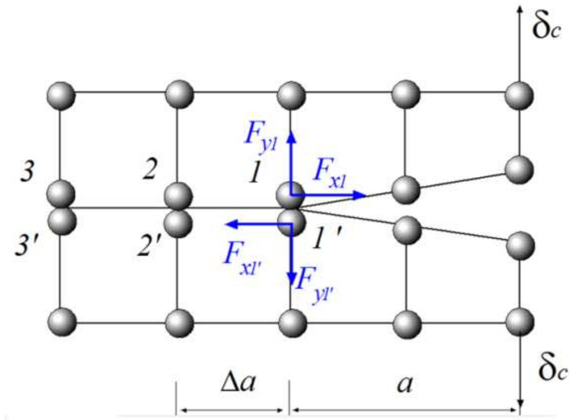
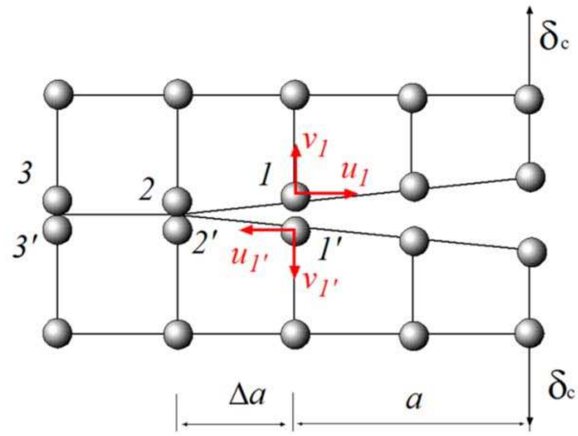


Figure 6 Step 2: DOFs at 1-1′ nodes are released (see online version for colours)



This procedure can be analytically described as follows:

$$G_I = \frac{1}{2B\Delta a}\sum_{i=1}^{n} F_{y1i}\left(v_{1i} - v_{1'i}\right)$$
$$G_{II} = \frac{1}{2B\Delta a}\sum_{i=1}^{n} F_{x1i}\left(u_{1i} - u_{1'i}\right) \tag{1}$$

where

- *B:* sample width
- *Δa:* crack length increment

- v_{1i}: vertical displacement of nodes at the crack tip
- u_{1i}: horizontal displacement of nodes at the crack tip
- F_{y1i}: vertical nodal force at the crack tip
- F_{x1i}: horizontal nodal force at the crack tip x-axe

This method is similar to the VCCT except for the nodes where the forces are calculated. In the VCCT method, forces are calculated in nodes 2-2′ with the assumption that being Δa small enough, the stress state at the crack tip does not change significantly. With this simplification, the VCCT method calculates the energy release rate in only one step.

In this case, the following expressions are used to calculate the energy release rate components:

$$G_I = \frac{1}{2B\Delta a}\sum_{i=1}^{n} F_{y2i}\left(v_{1i} - v_{1'i}\right)$$
$$G_{II} = \frac{1}{2B\Delta a}\sum_{i=1}^{n} F_{x2i}\left(u_{1i} - u_{1'i}\right) \quad (2)$$

where,

- F_{y2i}: vertical nodal force at node 2
- F_{x2i}: horizontal nodal force at node 2

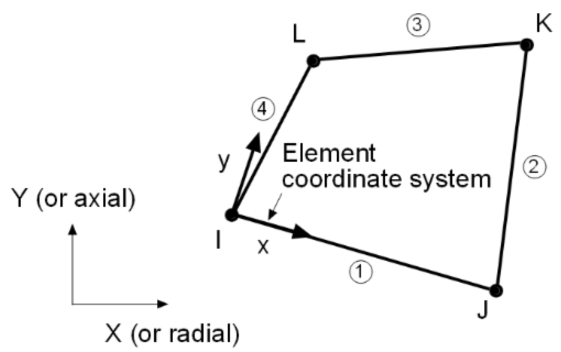
In order to perform the calculations, the following critical values where implemented in the model: G_{Ic} = 90.6 J/m^2, G_{IIc} = 943.4 J/m^2. These values where obtained experimentally in a previous work (Mollón et al., 2012).

In order to determine the critical load for each h_1/h_2 rate, a Benzeggagh-Kenane (1996) law was used:

$$\frac{G}{G_{Ic} + \left(G_{Ic} + G_{IIc}\right)\left(\frac{G_{II}}{G}\right)^{\eta}} \geq 1 \quad (3)$$

where η was set to 1.85

Figure 7 PLANE 42 element

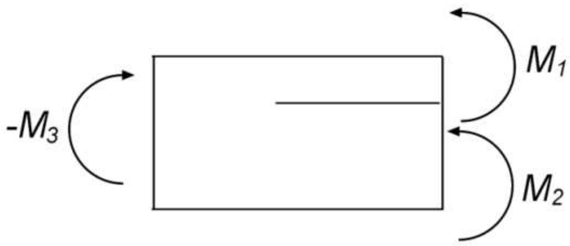


2D models with different h_1/h_2 ratios ranging from 0.25 to 1.0 were prepared. Four node 2D solid elements with two degrees of freedom at each node (translations in the nodal x and y directions) were used to build the models (Figure 7). The element length was set to 0.1 mm near the crack tip, so the ratio of the crack increment length over the initial crack length was $\Delta a/a_0 = 0.002$.

2.4 Partition method

The LP method is an analytical approach based on a stress intensity factor calculation defined by $K = K_I + iK_{II}$ (being K_I and K_{II} the mode I and mode II stress intensity factors) (Ducept et al., 1999; Hutchinson and Suo, 1991). Figure 8 shows the moments near the crack tip in a cracked specimen for a general loading state.

Figure 8 Moments near the crack tip for a general loading state



Source: Ducept et al. (1999)

The resulting expressions of the local method to calculate G_I and G_{II} are as follows (Ducept et al., 1999):

$$G_I = \frac{1}{2B^2E}\left[\frac{F\cdot\cos(\omega)}{\sqrt{Ah_1}} + \frac{M\cdot\sin(\omega+\gamma)}{\sqrt{Ih_1^3}}\right] \tag{4}$$

$$G_{II} = \frac{1}{2B^2E}\left[\frac{F\cdot\sin(\omega)}{\sqrt{Ah_1}} + \frac{M\cdot\cos(\omega+\gamma)}{\sqrt{Ih_1^3}}\right] \tag{5}$$

where

$$F = -\frac{6h_1h_2}{(h_1+h_2)^3}M_3 \tag{6}$$

$$M = M_1 - \frac{h_1^3}{(h_1+h_2)^3}M_3 \tag{7}$$

$$A = \frac{1}{1+4\left(\frac{h_1}{h_2}\right)+6\left(\frac{h_1}{h_2}\right)^2+3\left(\frac{h_1}{h_2}\right)^3} \tag{8}$$

$$I = \frac{h_2^3}{12\left(h_1^3 + h_2^3\right)} \tag{9}$$

$$\sin(\gamma) = \sqrt{A \cdot I} \cdot 6\left(\frac{h_1}{h_2}\right)^2 \cdot \left(1 + \frac{h_1}{h_2}\right) \tag{10}$$

being

- *B:* sample width
- *E:* longitudinal elastic modulus
- h_1: upper sublaminate thickness
- h_2: bottom sublaminate thickness
- M_1, M_2, M_3: moments as shown in Figure 8

ω can be determined by solving a problem involving a semi-infinite crack using integral equation methods. Hutchinson et al. (1992) and Ducept et al. (1997) have found the following expression:

$$\omega = 52.1° - 3°\left(\frac{h_1}{h_2}\right) \tag{11}$$

Equations (4) and (5) allow the determination of G_I and G_{II} for the general case showed in figure 8. In the particular case of DCB specimens, G_I and G_{II} can be obtained by substituting $M_1 = -M_2 = Pa_0$ and $M_3 = 0$ in the above equations.

2.5 Empirical equation

An empirical formulation developed by Mollón et al. (2010) to calculate the mode mixity ratio was also used to compare results. In this work, samples with different h_1/h_2 ratios were analysed and it was observed that the plot of G_I/G and G_{II}/G versus a given combination of $(h_1/h_2)^3$ can be fitted by the equation of an ellipse.

This formulation can be written as follows:

$$G_{II}/G = -\beta\sqrt{1-\alpha^2} + \beta \tag{12}$$

being

- $\alpha = \dfrac{1-\left(h_1/h_2\right)^3}{1+\left(h_1/h_2\right)^3}$ with $h_1 < h_2$
- $\beta = 0.41$ (adjusting parameter)
- $G = G_I + G_{II}$ (total energy release rate)
- G_I: mode I energy release rate
- G_{II}: mode II energy release rate
- h_1: upper sublaminate thickness

- h_2: bottom sublaminate thickness.

This is a simple and useful empirical equation that allows the determination of the mode mixity ratio G_{II}/G and G_I/G (being $G_I/G = 1 - G_{II}/G$). Nevertheless, in order to evaluate G_I and G_{II}, G must be determined by another procedure as this equation only allows the determination of the partition ratios G_{II}/G and G_I/G.

3 Results

Models with h_1/h_2 rates ranging from 0.25 to 1.0 were modelled and analysed by means of FE, LP and empirical procedures. The obtained results are shown in Tables 2 and 3.

Table 2 G_I, G_{II} and G results

h_1/h_2	G_I (J/m²)			G_{II} (J/m²)			G (J/m²)		
	FE	*LP*	*Error*	*FEM*	*LP*	*Error*	*FEM*	*LP*	*Error*
1.00	90.6	77.1	–15%	0.0	0.0	0%	90.6	77.1	–15%
0.67	90.3	76.6	–15%	5.7	5.6	–2%	96.0	82.2	–14%
0.50	96.8	85.0	–12%	16.1	15.8	–2%	112.9	100.7	–11%
0.43	105.7	94.2	–11%	24.5	23.7	–3%	130.1	117.9	–9%
0.33	119.6	110.5	–8%	41.3	38.9	–6%	160.9	149.4	–7%
0.25	135.3	131.8	–3%	66.9	58.6	–12%	202.3	190.5	–6%

Notes: FE: finite element method, LP: local partition method.

Table 3 G_I/G and G_{II}/G results

h_1/h_2	G_I/G					G_{II}/G				
	FE	*LP*	*Emp*	*Error LP-FE*	*Error Emp-FE*	*FE*	*LP*	*Emp*	*Error LP-FE*	*Error Emp-FE*
1.00	100%	100%	100%	0%	0%	0%	0%	0%	0%	0%
0.67	94%	93%	93%	–1%	–1%	6%	7%	7%	14%	10%
0.50	86%	84%	85%	–2%	–1%	14%	16%	15%	10%	7%
0.43	81%	80%	80%	–2%	–1%	19%	20%	20%	7%	5%
0.33	74%	74%	74%	–1%	0%	26%	26%	26%	2%	1%
0.25	67%	69%	69%	3%	3%	33%	31%	31%	–7%	–7%

Notes: FE: finite element method, LP: local partition method, Emp: empirical formulation

As can be seen in these tables, there is a good agreement between FE and LP methods, especially for G_I/G and G_{II}/G. Errors are in the order of 3% in the determination of G_I/G and usually below 10% in the determination of G_{II}/G. The partition mode is accurately predicted by both the LP and Emp methods.

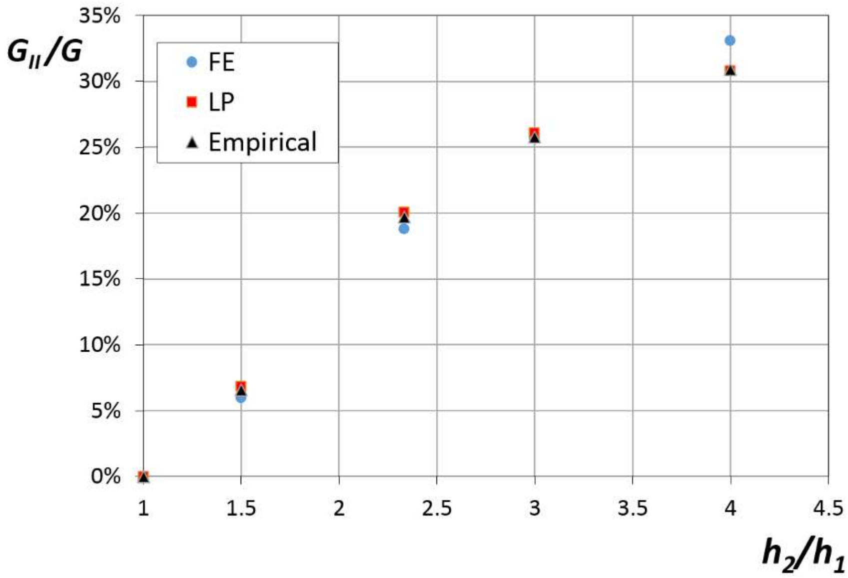
Taking FE results as a reference, the accuracy of the LP method in the determination of G_I and G_{II} vary with h_1/h_2. Errors in the calculation of G_I and G_{II} are between 3%–15% and 0%–12% respectively for the studied crack plane positions. It can be observed that the error in the determination of G_I decreases as the crack plane moves away from the midplane.

On the other hand, for G_{II} the reverse behaviour is observed. For G_{II} the best fit is obtained near the midplane and the difference between FE and LP methods increases when the crack plane moves away from the midplane.

It is also remarkable how the empirical formulation fits the LP results (Table 3). Both methods furnish very close results. The empirical formulation is very simple and effective in order to determine G_I/G and G_{II}/G mode mixity ratios. Although, as it was stated above, G must be determined by means of an alternative approach in order to obtain G_I and G_{II}. In any case, as can be seen in Table 3, the mode mixity ratios G_I/G and G_{II}/G predicted by all three methods are very close.

Figure 9 shows graphically G_{II}/G results obtained by means of FE, LP and empirical methods.

Figure 9 G_{II}/G as a function of h_2/h_1 (see online version for colours)



4 Conclusions

The ADCB test configuration is a simple and valid method to obtain mixed mode load state at the crack tip. The calculation of G_I and G_{II} at the crack onset was performed by means of FE analysis and LP method. The critical values for pure modes G_{Ic} and G_{IIc} were obtained experimentally in previous works. The critical load for each configuration was determined by FE analysis together with the Benzeggagh-Kenane law.

FE results were in good agreement with the analytical local partition method and the empirical formulation for the determination of G_I/G and G_{II}/G rates. The empirical formulation is very simple and effective in order to determine G_I/G and G_{II}/G mode mixity ratios, although G must be determined by another method in order to obtain G_I and G_{II}.

Comparing FE and LP methods, the accuracy in the determination of G_I and G_{II} vary with h_1/h_2. The errors found in the calculation of G_I and G_{II} are between 3%–15% and 0%–12% respectively for the studied h_1/h_2 rates.

Taking FE method as a reference, the maximum error in the determination of G_I is obtained in the midplane ($h_1/h_2 = 1$) while the error decreases as the crack plane moves away from the midplane. For G_{II} the opposite behaviour is observed with the best fit obtained in the midplane.

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