Investigating the Rate of Company Compulsory Liquidation in the UK via Bayesian Inference and Frequentist Statistical Methods

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Abstract

In this paper, we investigate the rate of company compulsory liquidation (insolvency) (CCL) via Bayesian inference applied with the use of 'OpenBUGS' and 'R' software mediums. This study follows on from a previous 'frequentist' statistical based study of one of the authors and introduces the usefulness of Bayesian inference as the statistical tool. CCL occurs when a company creditor successfully petitions the courts for a winding up order. There are a finite number of variables that can affect the possibility of company creditors instigating CCL. The model in this study included those variables found to be statistically significant within the data range contained in the previous 'frequentist' study. We commence by presenting the model from the previous 'frequentist' study along with an analysis of the results of that study. We then introduce the concept of Bayesian inference to the study and apply the inference to the 'frequentist' model, and analyse the results of the new model. Finally, we then compare the two sets of findings and report our conclusions, and the implications for the modeling of insolvencies.

I. Introduction

This paper explores the use of Bayesian Statistical inference when developing models of UK incorporated company insolvency by compulsory liquidation. We compare this approach with the more conventional 'frequentist' approach that is usually undertaken when building statistical models.

The main aim of this paper seeks to explore the usefulness of Bayesian Statistical inference as a tool to explain UK incorporated company insolvency by compulsory liquidation. The paper commences with a 'frequentist' based time series statistical model of company insolvency across the time period 1972 to 1992. The 'frequentist' model used in this paper was developed by Evans (2002), based upon the company insolvency models as devised by Cuthbertson and Hudson (1989, 1991 and 1996). We then introduce the application of Bayesian inference to the same model to determine whether or not the results of the extrapolation (i.e. the projection (or in Bayesian terms, the posterior)), based upon prior and likelihood information, are an efficient predictor of the rate of compulsory insolvency (liquidation) amongst UK incorporated companies

Compulsory liquidation occurs when a creditor successfully petitions the courts and since 1986, current UK legislation permits the courts to appoint an official receiver whose role it is to minimise the losses associated with the insolvent company. In the UK, there are two other types of company insolvency i.e. voluntary liquidation (where the receiver is immediately appointed by the creditors following a creditor's meeting) and members liquidation, (i.e. usually the alternative choice of a re-organisation of the company if the members believe the company is heading towards insolvency). The data set used in this study encompasses the Insolvency Act (1986) and we expect that the appointment of an administrator should have some inverse effect upon the rate of compulsory liquidations towards the latter end of the data range of the time series.

Bayesian inference has been in existence as long as the laws of probability; it was Thomas Bayes (circa 1701 to 1761) who addressed the relationship of conditional probability theory but, numerical techniques at that time permitted only the solutions to relatively simple examples. The introduction of specific computer software such as 'BUGS' has simplified the techniques of solving multiple integrals when dealing with complex models and this has recently permitted statisticians an opportunity to assess the usefulness of Bayes' theorem. 'Frequentists', in contrast, are using statistical techniques recently developed and evolved by the two main 'schools' of statistical thought i.e. the basis of the methodologies as developed by the 'Neyman-Pearson-Wald' school (hypothesis testing) (Neyman & Pearson (1928), Wald (1943), (1944)) and the 'Fisher' school (likely to be everything else) (Fisher (1925), (1935)).

There are general advantages attributed to the use of Bayesian inference methods (Samaniego (2010), Koop (2003), West & Harrison (1997), Lancaster (2005), Ibrahim, Chen & Sinha (2010)) over the use of 'frequentist' methods, for which company insolvency will benefit from.

Relationships such as the use of prior data (if it is available) and the determination of a 'likelihood' to assist with a posterior result suggests that the evolvement of economic circumstances over time can be captured within the estimation process by a continual updating of the process and this lends itself to computation with information technology. With 'frequentist' statistics, such evolvement is not so easily captured in the estimation process.

If different prior values are used on the same data set, then it is possible that different posterior results may occur and while this may be true especially in the case of the use of incorrect prior values, it is also true to emphasise the use of 'vague' or 'non-informative' priors that will permit Bayesian inference (i.e. seemingly without a prior) to be applied to an estimation process similar to that of the application of 'frequentist' statistics where no priors are used. Bayesian statistics therefore gains an advantage over 'frequentist' statistics since Bayesian statistics can be used with or without a prior value where-as 'frequentist' statistics cannot.

We used PC-GIVE, 'R', 'MS Excel' and 'OpenBugs' software. 'OpenBugs and 'R' software are available on the internet and (currently) free to download and use.

This paper is organised as follows: section II considers the theory, section III considers data and estimation issues, section IV considers the 'frequentist' model and the application of Bayesian inference via the 'BUGS' software and section V draws conclusions.

II. Theoretical Considerations

There are a finite number of variables that will exercise a degree of uncertainty for the company and thus create pressure for the company to survive in the long run. For example, changes in interest rates, foreign exchange rates, unemployment, the real wage are some of the variables expected to create uncertainty for the company. An increase in the number of companies ('births') (Hudson (1987)) will create competitive pressure while company failure ('deaths') may achieve the reverse and decrease competitiveness but, it may also imply a drop in demand for certain commodities thus creating further pressure on those surviving companies. Some companies however, will fail due to misfortune and/or bad management policies and practice such as a failure to control cash flow, costs and/or an inability to change to current market conditions, all of which will create pressure on company profits. We include the effects of company 'births' and 'deaths' into our model.

The probability of insolvency is likely to be intensified to those incorporated companies whose ability to borrow is limited (traditionally 'smaller' companies). A company will be at risk of compulsory insolvency if it returns a series of net negative profits (losses) and creditors are unable to satisfy themselves that the trend of net negative profits will decrease. Additionally, if the creditors' assessment of the current liquidation value of the company is greater than their assessment of the discounted value of expected future net negative profits and the company's expected future liquidation value then, the probability of compulsory insolvency is likely to be high. Hence, profit margins are likely to be a major determinant of compulsory insolvency and we include this variable into our model. Wadwhani (1986) provides an excellent example of the 'Fisher' effect on the role of interest rates when debt is not indexed and thus creates additional pressure on the company to fund its interest repayments. We include this effect within our income gearing variable.

We model the number of compulsory liquidations as a proportion of the number of economically active companies and we commence with an autoregressive distributed lagged model and follow the general to specific methodology (GTSM) developed by Hendry (1983) to determine the 'best parsimonious equation' (bpe).

(The bpe is the optimal (or economical) equation based upon the rejection of other explanatory variables due to their respective coefficients failing the necessary 't' tests and/or collectively, failing other statistical tests of model stability and/or validity (Hendry (1983)). Hendry's (1983) technique identifies the valid components of a model by repeated statistical testing and by eliminating those variables found not to be statistically significant, until the 'frequentist' parsimonious equation was determined. For the long run analysis, we adopted the pioneering work undertaken by Pesaran, Shin and Smith (1996).

Similar to the empirical study of Cuthbertson and Hudson (1996), we found the point estimates of the coefficients on the three quarterly dummy variables to be negative and increasing in value over time and to address this increase, we included a 'shift' dummy with a value of zero everywhere except for 1988(Q1) to 1992(Q2) time period.

The 'best parsimonious equation' insolvency model (bpe) was selected from Evans (2002). The composition of the model was set up as follows:

$$PCL_t = \alpha_{11} + \alpha_1 \sum_{j=1}^{3} \left(\frac{PCL_{t-j}}{3} \right)_t + \alpha_2 \Delta IG_{t-1} + \alpha_3 IG_{t-7} - \alpha_4 \Pi_{t-9} + \alpha_5 \sum_{j=8}^{11} \left(\frac{PB_{t-j}}{4} \right)_t + \alpha_6 Q_{t-1} + \alpha_7 Q_{t-2}$$
$$+ \alpha_8 Q_{t-3} + \alpha_9 SH88 - 89_t \qquad (1.0)$$

where:

PCL is the Proportion of Compulsory Liquidations, (α_1)

IG is Income Gearing, (Δ represents changes in), (α_2), (α_3) Π is the profit margin, (α_4)
PB is the proportion of company 'births' (averaged across t-8 to t-11 time periods), (α_5)
Q1, Q2 and Q3 are quarterly dummies (for quarterly data), (α_6), (α_7), (α_8) and
SH88-89 is a shift dummy and represents a structural change in the data of the model, (α_9).

III. Data and Estimation Issues

The frequentist model made use of the data set from 1972(Q1) to 1992(Q2) inclusive, with the estimation across 1972(Q1) to 1989(Q4) with the remaining 10 sets of data used for extrapolation. Quarterly (seasonally unadjusted) data was used in the empirical model and was selected from various issues of Financial Statistics, Economic Trends Annual Supplements, Department of Trade and Industry (DTI) - Companies in Active register and (DTI) New Incorporations. Government data collection services for the number of actual companies, the number of company births, the number of compulsory liquidations and the number of other company 'deaths' were subsequently rationalised and re-organised by governmental departments and hence, it proved difficult to locate some of the necessary data beyond the year 2000 (to update the data set). It was thought likely to require other data sources to proxy one or more of these variables as and when the data set was to be extended beyond the year 2000. Manipulation of the data (to create lags, ratios etc.) were undertaken in PC-GIVE, Microsoft Excel and/or 'R'. The Proportion of Compulsory Liquidations (PCL) and the 'births' of new incorporations (PB) were measured as a percentage of the number of economically active companies. Income Gearing (IG) was measured by Industrial and Commercial Companies (ICC) net interest rate payments as a percentage of ICC net profits where net profits are gross profits minus non trading income. The profit margin (PM) was measured by ICC pre-tax profit as a percentage of ICC value added. The quarterly dummies were found to be statistically significant as was the shift dummy (SH88-89).

It was assumed that data values in the secondary sources were accurate and complete although it was noted that there may have been at least one data observation (due to its outlying 'effect' in relation to the remainder of the data) recorded inaccurately. This 'inaccuracy' was catered for by interpolation. It should be further noted that when there are missing (or incomplete/inaccurate) data observations, the 'BUGS' software (in this case 'OpenBUGS') will estimate the model by simulating missing values based upon other values in the distribution of the data.

IV. The 'frequentist' Model (Equation 1.0)

$$PCL_t = 0.0465 + 0.7108 \sum_{t-1}^{3} \left(PCL_{t-j}/3 \right) + 0.0017 \Delta IG_{t-1} + 0.0017 IG_{t-7} - 0.0082 \Pi_{t-9} +$$

$$\quad (2.823)\ (8.1123) \qquad\qquad (2.4797) \qquad (4.087) \qquad (-2.863)$$

$$0.0214 \sum_{t-1}^{4} \left(PB_{t-j}/4 \right) - 0.0327 Q_1 - 0.0502 Q_2 - 0.1159 Q_3 - 0.0345 SH88 - 89, \qquad (1.1)$$

$$\quad (3.878) \qquad\qquad (-4.916) \quad (-7.888) \quad (-16.357) \quad (-3.465) \qquad (..)\ \text{'t' values}$$

For purposes of clarity, equation 1.1 is set into table 1.0

Dependent Variable	$(PCL)_t$		
Regressors	Coef.	Std. Err.	t'-values
$\sum(PCL_{t-j}/3)_t$	0.7107954	0.08762	8.11229
ΔIG_{t-1}	0.0016538	0.00067	2.47975
IG_{t-7}	0.0016583	0.00041	4.08704
Π_{t-9}	-0.0081647	0.00285	-2.86269
$\sum(PB_{t-j}/4)_t$	0.0214060	0.00552	3.87777
Q_1	-0.0327333	0.00666	-4.91621
Q_2	-0.0502264	0.00637	-7.8879
Q_3	-0.1159811	0.00709	-16.35681
$SH88\text{-}89_t$	-0.0345039	0.00996	-3.46464
Constant	0.0464655	0.01646	2.82289
$R^2 =$	0.98		

Table 1.0: The Proportion of Compulsory Liquidations – the 'Frequentist' Empirical Model

Key:

PCL_t — The Proportion of Compulsory Liquidations (i.e. the number of compulsory insolvencies divided by the number of economically active companies (* 100%))

$\sum(PCL_{t-j}/3)_t$ — (lagged) moving average of PCLs

IG_{t-7} — Income Gearing : ICC net interest rate payments divided by ICC net profits (* 100).   (Net profits are gross profits minus non trading income).

Π_{t-9} — Profit Margin : ICC pre-tax profit divided by ICC value added (* 100).

$\sum(PB_{t-j}/4)_t$ — (lagged) moving average of the profit margin.   (Profit Margin is ICC pre-tax profit divided by ICC value added (* 100)).

Q — quarterly (seasonal) dummy variable(s)

$SH88\text{-}89$ — 'shift' dummy variable to take into account the effects of the Insolvency Act of 1986 and the increasing absolute values of the associated 't' tests of the dummy variables.

Diagnostic checking of the model was based upon the interpretations of the statistical tests associated with the 'frequentist' method and these were as follows:

$R^2 = 0.98$; Rbar$^2 = 0.96$; $\sigma = 0.0019$; $F(10, 72) = 393.39\ [0.0000]\ \{1.99\}$; DW = 2.164; (LM(4) $F(4, 57) = 1.4)\ [0.2453]\ \{2.53\}$; RSS = 0.02661; ARCH (4, 53) = 0.88 [0.4811] {2.53}; BJ(2) (χ^2)= 1.78 {5.9}; HET (15,45) = 2.88 [0.0031] {1.84}; Ram (RESET) $F(1, 60) = 38.421\ [0.0000]\ \{4\}$

([.] = marginal significance level, {.} = 5% critical value).

For the purposes of interpretation, all signs on the parameter coefficient estimates appeared to be correct in terms of economic theory (discussed below) and statistically significant. The R^2 term returned a value of 0.98 suggesting that approx. 98% of the changes in the dependent variable could be explained by changes in the explanatory variables. DW = 2.164 indicating the existence of serial correlation, but due to the inclusion of the dependent variable as a lagged, explanatory variable then the Lagrange Multiplier (LM) test for serial correlation was preferred to DW. In this case, the Lagrange Multiplier test for serial correlation $F(4, 57)$ (4 lags) = 1.4 (critical value = 2.53) hence the null hypothesis was not rejected (i.e. there was no evidence of serial correlation). The 'F' test for an overall significance of the regression was not rejected ($F(10,72)$ =393.39 {1.99}).

The Bera & Jarque Test for normality BJ(2) (x^2)= 1.78 {5.9} was not rejected and this indicated the equation possessed normally distributed, white noise errors. The ARCH test for fourth order conditional heteroscedasticity ARCH $(4, 53)$ = 0.88 [0.4811] {2.53} did not reject the null hypothesis although the failure of the model to satisfy the tests for HET and Ram (RESET), indicated the model may be mis-specified in terms of functional form.

Failure on functional form is indicative of either a mis-specified model, or mis-specification in the functional form of the model. There were two patterns of thought at this point:

- The original model before Hendry's (1983) GTSM could be subjected to the Ram (RESET) test to see if the missing variable(s) was (were) rejected during the GTSM procedure.
- The missing variable(s) could be considered (or re-considered if rejected as above) and inserted (re-inserted) into the model for re-estimation.

Both approaches were adopted. In the first part above, the general model (before Hendry's (1983) GTSM) was subjected to the Ram (RESET) test and the subsequent test results for 38 independent variables (including lags of the explanatory variables) = 17.5 which was still indicative of a rejection of the null hypothesis. It should be noted that due to the failure of the statistical tests on parameter coefficients, other explanatory variables thought to be economically theoretical were rejected following Hendry's (1983) GTSM procedure.

Despite the failure of the model due to functional form, we believed the model to be reasonably sound and useful to use for comparison purposes with Bayesian inference techniques.

Income Gearing (*IG*) was defined as the ratio of net interest payments of ICCs as a percentage of net profits. The 'frequentist' model (table 1) included the *change* in, and also the *level* of income gearing as statistical significant explanatory variables. The positive signed coefficient on both *IG* variables of the model, suggested the higher the level of income gearing is, the higher will be the level of compulsory liquidations. So either an increase (decrease) in net interest payments or a decrease (increase) in net profits (or both), should imply an increase (decrease) in compulsory liquidations. Net interest payments will increase if (for example) debt is not indexed, thus lending some support to the theory of Wadhwani (1986). Net profits are equal to the excess of revenue over costs and if revenue is higher than costs, then net profits will be healthy, and the firm is expected to survive. However, a cyclical downturn in demand may reduce the difference between a company's revenue and cost curves and then the firm may enter financial distress and may possibly find difficulty in its ability to survive into the longer term.

The change in income gearing appeared to have a significant effect on compulsory liquidations after one time period, which suggested compulsory liquidations were almost immediately affected by movements in changes of income gearing.

A profit definition (i.e. revenue ≥ costs) also appears in the numerator for the profit margin (Π), and any increases (decreases) in profits (via the negatively signed profit margin coefficient), will determine decreases (increases) in compulsory liquidations. So the possibility of linear correlation between income gearing and the profit margin may exist, but examination of these variables within the correlation matrix of the software results (see Evans, (2002)), suggested a correlation coefficient of (approx.) 0.3, which was indicative of very little linear association between the two variables.

For the profit margin, the 'frequentist' model suggested profits had a significant effect on compulsory liquidations after nine time periods (approx. two years). The rationale for this phenomenon appears to suggest that the level of profits have a 'sluggish' passage through a company's accounts. Since the sign of the profit margin coefficient is (and was theoretically expected to be) negative, then increases in profits will create a downward push on compulsory liquidations and vice versa.

During a recession, profits are expected to fall and would thus apply pressure to the company's future financial viability and ultimately, its prospects for survival. Since incorporated companies usually enjoy the benefits of limited (financial) liability, then directors free of such financial liabilities, will invariably attempt to keep the company in productive operation for as long as possible in the expectation that the economy will move out of the recession. Hence, a noticeable reduction in profits would therefore slowly filter through the system but, if the reduction trend continued, then creditors expectations of the company survival chances would change accordingly.

The graph of the results across this time period (superimposed with the actual values) is available in *figure 1*.
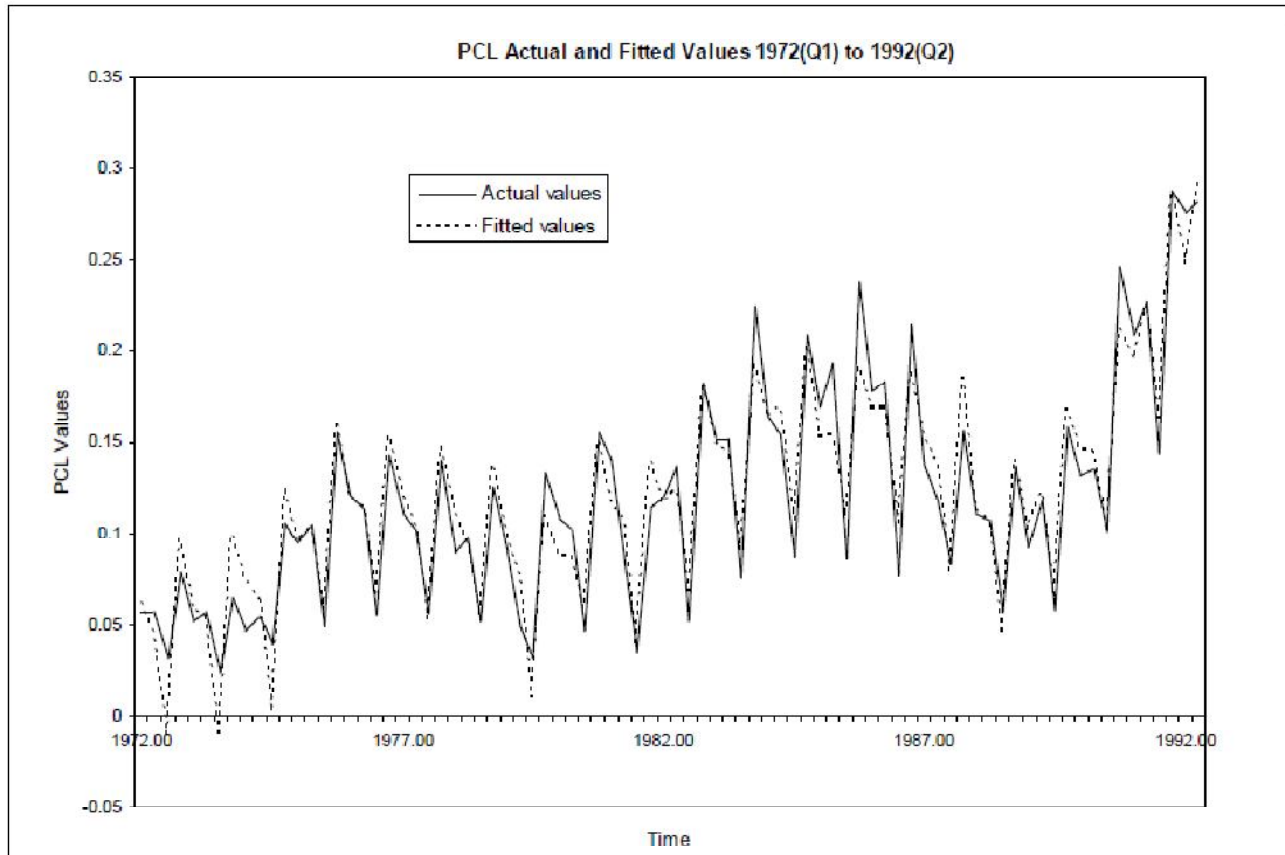

fig. 1

The 'frequentist' model was deemed to fit the data reasonably successfully and the long run analysis of the 'frequentist' model was based upon the long run techniques following the work of Pesaran, Shin and Smith (1996) where knowledge of the order of integration of the variables is not a pre-requisite to determine the existence of a long run relationship. Pesaran, Shin and Smith's (1996) approach for testing for the existence of a long-run relationship between two (or more) economic time series variables, is a 'bounds' test procedure rather than a cointegration test procedure, and is based upon the 'F' statistic for testing of the joint null hypothesis of H_0 : $\phi = 0 = \mathfrak{z}$, (or in the case of more than one independent (x) variable, $\phi = 0 = \mathfrak{z}1 = \mathfrak{z}2 =$ etc.), against the alternative hypothesis of $H_1 : \phi \neq 0$ in an unrestricted error correction equation.

A property of an integrated time series is that it possesses an unbounded variance (Banjeree et al 1993), but the dependent variable of equation 1.0 is expressed as a 'proportion' i.e. the proportion of compulsory liquidations (PCL) and the series is therefore confined to lie between (and including) 0 and 1 (or 0 and 100%). Hence, the PCL series must possess the property of a bounded variance which suggests that it is not integrated (more specifically, the PCL series is not I(1)) and thus cannot be cointegrated. It is possible of course, to get round the problem of the bounded variance series by analysis of the integration properties of the natural log of compulsory liquidations, or by assuming the series behaves 'as if' it is an integrated series. However, using the argument that the series cannot be I(1), then the lower bound of 2.163 as suggested by Pesaran Shin and Smith (1996) (i.e. the I(0) bound), is taken to approximate to the true 5% critical value in this test.

Estimation of the restricted model of equation 1.0 produced an 'F' test statistic of 2.9135 and hence, is greater than Pesaran Shin and Smith's (1996) lower bound of 2.163, so there is evidence to reject the null hypothesis in favour of a long run relationship.

Detailed analysis of the theory behind the application of a long run relationship analysis using Pesaran, Shin and Smith's (1996) approach, is contained in Evans (2002).

The 'Bayesian' model

The Bayesian model was estimated in 'OpenBUGS' with all variables set with a normal(0.1, 0.1) prior (N(0.1, Var = 10)) and the (Bayesian) parameter estimates for the model in equation 1.0 are presented as follows in table 2.0 below:

Dependent Variable	$(PCL)_t$	BAYES
Regressors		Coef.
$\sum(PCL_{t}/3)_t$	$\alpha[1]$	0.7139
ΔIG_{t-1}	$\alpha[2]$	0.001652
IG_{t-7}	$\alpha[3]$	0.001639
Π_{t-9}	$\alpha[4]$	-0.00809
$\sum(PB_{t}/4)_t$	$\alpha[5]$	0.02127
Q_1	$\alpha[6]$	-0.03291
Q_2	$\alpha[7]$	-0.05051
Q_3	$\alpha[8]$	-0.1162
$SH88\text{-}89_t$	$\alpha[9]$	-0.03441
Constant	$\alpha[11]$	0.02331

Table 2.0: PCLs – The Parameter Estimates for the Frequentist 'bpe' empirical model as estimated by 'Open BUGS'

Table 3.0 includes and incorporates the parameter estimates of the coefficient values from the frequentist 'best parsimonious equation' alongside the Bayesian parameter estimates as depicted in table 2.0:

Dependent Variable	$(PCL)_t$	BAYES	FREQ
Regressors		Coef.	Coef.
$\sum(PCL_{t}/3)_t$	$\alpha[1]$	0.7139	0.7107954
ΔIG_{t-1}	$\alpha[2]$	0.001652	0.0016538
IG_{t-7}	$\alpha[3]$	0.001639	0.0016583
Π_{t-9}	$\alpha[4]$	-0.00809	-0.0081647
$\sum(PB_{t}/4)_t$	$\alpha[5]$	0.02127	0.021406
Q_1	$\alpha[6]$	-0.03291	-0.0327333
Q_2	$\alpha[7]$	-0.05051	-0.0502264
Q_3	$\alpha[8]$	-0.1162	-0.1159811
$SH88\text{-}89_t$	$\alpha[9]$	-0.03441	-0.0345039
Constant	$\alpha[11]$	0.02331	0.0464655

Table 3.0: PCLs – A Comparison of the Parameter Estimates for the Bayesian and frequentist estimation methods

As a recap, the frequentist estimation method provided a model that successfully 'passed' all of the statistical tests except for HET and Ram (RESET), indicating the model was mis-specified in terms of functional form. Otherwise, the frequentist estimation method for the 'best parsimonious equation' indicated all of the signs on the parameter coefficient estimates appeared to be correct in terms of economic theory and statistically significant. The R^2 term returned a value of 0.98 suggesting that approx. 98% of the changes in the dependent variable could be explained by changes in the explanatory variables. As a result, the frequentist 'best parsimonious equation' appears to be the 'norm' to compare other models to.

For the Bayesian estimation of the same empirical model, the parameter coefficient estimates have the same 'signs' and (except for the constant term), are almost identical in magnitude to those of the frequentist estimation method. Hence, it can be assumed that the Bayesian method of estimation provides an efficient estimate of the parameter coefficients.

Diagnostic Checking of the Bayesian Model

With Bayesian inference, diagnostic checking is not quite the same as with the frequentist method. Diagnostic checking is based more upon beliefs of uncertainty and can take up considerable time since there are little (if any) statistical tests to undertake (although the traditional frequentist statistical tests can be used). In frequentist statistics, diagnostic checking traditionally commences with and examination of the parameter estimates of the model and whether or not they have 'passed' the 't' tests and if their 'signs' make logical sense when compared with the dependent variable (or in the case of econometrics, economic sense). The Bayesian equivalent to this is whether or not the marginal posterior distributions look 'right' i.e. are the parameter estimates consistent with logical (or economic) theory. As an example, changes in compulsory liquidation are assumed to be inversely (negatively) related to profits since, increases in profits should indicate a drop in compulsory liquidations (company finances are strong). This indicates a 'belief' in the 'signs' of the parameters of the explanatory variables. Additionally, consider the 95% 'credible interval' (the Bayesian equivalent of a 'confidence interval'), - does a change of sign occur within this interval? – if so, caution must be exercised with the interpretations of the parameter estimates. (This is the equivalent of the frequentist 't' test for the value of the parameter (coefficient) to be no different to zero).

Traditionally in Bayesian inference, there are two methods to adopt for model checking i.e. informal and formal model checking procedures.

With informal model checking procedures, the same frequentist CLRM assumptions hold and consideration needs to be applied to the error term and the distribution of the errors associated with the posterior results, to check for consistency with the prior beliefs. The posterior errors are traditionally expected to be 'independent and identically distributed' (iid) and follow a N(0, tau1) distribution (to satisfy the assumptions of the model). The basic method used to examine the distribution of the errors, is by a residual 'QQ' plot.

Formal model checking is based upon predictive distributions since checking a model is the process of comparing the predictions to actual evidence. i.e. how accurate are the predictions? There are two fundamental types of predictive distributions viz. the prior predictive distribution and the posterior predictive distribution
The prior predictive distribution (also known as the marginal likelihood) is defined as:

$$p(y) = \int p(y \mid \theta) p(\theta) d\theta \qquad (2.0)$$

(where $p(y \mid \theta)$ is the likelihood and $p(\theta)$ is the prior).

There are differing types of prior viz. informative (when much prior information is known), non-informative (when there is little prior information known) and improper (where the sum or the integral of the prior values do not need to be finite (i.e. they do not need to sum or integrate to unity)). As an example for an informative prior, the prior predictive distribution would compare features of the data with what the model predicts but, this is somewhat subjective. For example, suppose 'y' (from equation 2.0) is a (posterior) predictive time series set of observations (formulated by the prior and the likelihood), then the predictive distribution of the residuals associated with 'y' can be calculated. The next step would be to collect the actual data associated to the time period of the prediction and calculate their auto-correlation and compare them to the auto-correlation attributed to the prediction. A simple method to assist the comparison would be to plot the residuals on a graph. The prior predictive distribution indicates how the data should appear so, if there is any element of doubt between the appearance of the predictive distribution of the residuals and the distribution of the residuals from the actual data associated to the prediction time period, then the interpretation would be to conclude that the model is not set up correctly.

The above section (the prior predictive distribution) discussed the appearance of the predicted data before the actual data was viewed. The posterior predictive distribution considers an updated equation of (2.0) to include the actual data observations (y^{obs}):

$$p(\bar{y}\,|\,y^{obs}) = \int p(\bar{y}\,|\,y^{obs},\theta)p(\theta\,|\,y^{obs})d\theta \qquad (3.0)$$

(where $\bar{y}$ represents the estimates for the data)

There is a simple algorithm associated with the posterior prediction i.e.

- Sample θ from the posterior distribution,
- Insert the sample of θ into the conditional distribution (equation (3.0)),
- Given θ and yobs, sample y from the conditional distribution,
- Repeat the above steps until the results from the sample y of the conditional distribution converge.

There is also the possibility of using the Posterior odds and model choice ratio (not covered here) when there are two or more similar models (but with competing theories) that are capable of explaining changes in the same dependent variable.

Estimation of the Bayesian model

Following the estimation of the 'best parsimonious equation' by the Bayesian model, the first diagnostic analysis to undertake was to investigate the parameter estimates of the model and see if the respective 'signs' make economic sense. In this case, an examination of table 3.0 indicates the 'signs' of the parameter estimates following the Bayesian inference, are the same as the 'signs' of the parameter estimates following the frequentist inference.

The next step was to consider the 95% 'credible interval' to identify whether or not there was a change in the sign of each of the parameter coefficient estimates. The Bayesian model parameter estimates (from table 2.0) and the 'credible intervals' for the respective parameter estimates are displayed in table 4.0 below:

Dependent Variable	(PCL)ᵢ	BAYES		
Regressors		Coef.	Val2.5pc	Val97.5pc
$\Sigma(PCL_{t-i}/3)_t$	$\alpha[1]$	0.7139	0.2059	1.223
ΔIG_{t-1}	$\alpha[2]$	0.001652	-0.002266	0.005548
IG_{t-7}	$\alpha[3]$	0.001639	-7.546E-4	0.004031
Π_{t-8}	$\alpha[4]$	-0.00809	-0.02507	0.008536
$\Sigma(PB_{t-i}/4)_t$	$\alpha[5]$	0.02127	-0.01123	0.05345
Q_1	$\alpha[6]$	-0.03291	-0.07191	0.005901
Q_2	$\alpha[7]$	-0.05051	-0.08757	-0.01353
Q_3	$\alpha[8]$	-0.1162	-0.1575	-0.07486
$SH88-89_t$	$\alpha[9]$	-0.03441	-0.09226	0.02365
Constant	$\alpha[11]$	0.02331	-0.02413	0.07275

Table 4.0: PCLs – the Bayesian Parameter Estimates and their 'Credible Intervals'

Examination of table 4.0 indicates there were six of the nine explanatory variables plus the constant term, where a change of sign occurred across the credible interval. This indicates a possible value for the parameter coefficient value to be equal to zero and hence, care must be taken with the interpretation.

For the three explanatory variables where a change of sign across the credible interval did not occur, two of the parameter coefficient estimates were 'quarterly' dummy variables. The only explanatory variable whose parameter coefficient demonstrated a definitive value not equal to zero, was the (lagged) moving average of PCLs. Hence, this model (although 'passing' all of the 't' tests for parameter estimate validity and thus, indicating a reasonably sound prediction model under the frequentist method), 'fails' the second of the Bayesian model checking procedures. As a result, the Bayesian model really needs to be re-addressed.

V. Conclusion

The frequentist method provided a model that 'passed' all of the statistical tests except for functional form and we believed this model to be reasonably sound and useful to use for extrapolation and for comparison purposes with Bayesian inference techniques.

The Bayesian method provided a model that was similar in many respects to the frequentist model (in terms of parameter coefficient estimates and 'signs') but analysis of the 95% 'credible intervals' (and the consequent change in sign of the parameter estimates) indicated that the value of most of the parameter estimates could possibly be equal to zero. The Bayesian model needs to be re-addressed – and a possible solution to resolving the change of signs of 95% 'credible intervals' may be by re-addressing the beliefs relating to the prior information.

Our conclusion for the analysis of this compulsory liquidation model by the use of Bayesian inference is that it is a useful tool (especially with the use of prior information as additional information is probably better than no information), but care needs to be applied with the interpretation of the estimates.

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[1] This text introduced the phrase of 'the null hypothesis'.

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