SAPIENZA
UNIVERSITÀ DI ROMA

Ph.d. Dissertation in Systems Engineering

DYNAMICAL AREA COVERAGE BY MOBILE SENSOR NETWORKS
Analysis, Modeling and Control

Simone Gabriele

ADVISOR
Prof. Paolo Di Giamberardino

COORDINATOR
Prof. Carlo Bruni

CICLE XXI
2005-2008

CONTENTS

CHAPTER 1

INTRODUCTION

In the last years,under the boost of the recent technological advances in wireless networking and miniaturizing of electro-mechanical systems, multiagent systems are receiving a great and great attention. Such systems could find numerous and various fields of applications like, for example, environmental monitoring of lands, seas or cities, cleaning of parks, squares or lakes, mine clearance, critical structures surveillance, and so on.

In this context, it is growing the number of application based on the development and the use of large groups of autonomous vehicles, able to coordinate their actions by exchanging data over had hoc communication networks. The control of the behavior of such complex systems introduces interesting problems related with control theory, data fusion, distributed computation, networking and so on. This Thesis faces the problem of covering a given field of interest with a mobile sensor network. In this case, the mobility is used to access all the points of the field within a prefixed time, so that the coverage is guaranteed not at any time instant, like for static networks, but within any prefixed time interval. The meaning of coverage obviously depends from the considered sensing model; however, it is possible to claim that a given set of interest is covered when all the significant information, distributed over the field as a function of the coordinates, has been collected. Sensing ranges of instrumentation are obviously limited and, if sensors have a fixed location, their number and the covered area are strongly related by a direct proportional relationship. On the other hand, once the coverage is accepted over a time interval, then mobility can be used to expand the sensor network range in the same same spirit of the sentries that walk coming and going on the castles' walls. However, mobility introduces challenging coordination problems. In fact, in order to satisfy performance requirements like, for example, the collisions avoidance, it is necessary to coordinate sensors motion introducing suitable constraints.

1.1 Wireless Sensor Networks

Sensor networks consist of individual nodes that are able to interact with their environment by sensing or controlling physical parameters; these nodes have to collaborate to fulfill their tasks as, usually, a single node is incapable of doing so; they use wireless communication to enable this collaboration. In essence, the nodes without such a network contain at least some computation, wireless communication, and sensing or control functionalities. Despite the fact that these networks also often include actuators, the term wireless sensor network has become the commonly accepted name. Sometimes, other names like *wireless sensor and actuator networks* are also found.
Wireless sensor networks are powerful in that they are amenable to support a lot of very different real-world applications as shown in Garca-Hernndez *et al.* (2007); Glaser (2004); Lewis (2004); Mainwaring *et al.* (2002); Porter *et al.* (2005); they are also a challenging research and engineering problem because of this very flexibility. In fact there is no single set of requirements that clearly classifies all Wireless Sensor Networks, and there is also not a single technical solution that encompasses the entire design space as shown by the specialized literature (Akyildiz *et al.* (2002); Holger Karl (2005); Santi (2005); Stojmenovic (2005)). Moreover, the number, price, and potentially low accuracy of individual nodes is relevant when comparing a distributed system of many sensor nodes to a more centralized version with fewer, more expensive nodes of higher accuracy. Simpler but numerous sensors that are close to the phenomenon under study can make the architecture of a system both simpler and more efficient.

1.1.1 Mobile Sensor Networks

Endowing nodes in a sensor network with mobility drastically expands the spectrum of the network's capabilities. Moreover, assuming that each mobile node possesses a certain amount of decision making autonomy gives rise to a dynamic system with a considerable amount of flexibility, depending on the extent to which the nodes can cooperate in order to perform a *mission*. This flexibility, for example, allows us to handle a large number of data source targets with a much smaller number of nodes that can move and visit the targets over time to perform various tasks. Naturally, mobility also implies an additional layer of complexity. For example, if communication connectivity is to be maintained, we must ensure that each node remains within range of at least some other nodes. We must also take into account that mobility consumes a considerable amount of energy, which amplifies the need for various forms of power control. Another interesting aspect of mobility is that the exact location of nodes is not always available to other nodes or to a base station. This is especially true in settings where GPS tracking is not applicable, such as locating people or important equipment in a building. The location detection problem is a particularly challenging one, although we do not discuss it in this paper. Taking a system and control theory perspective, mobile networks provide the opportunity to exercise real-time cooperative control involving their nodes Cassandras & Li (2005). The goal of cooperative control is to coordinate the actions of the nodes so as to achieve a common objective. Its most popular application to date has been in networks of autonomous vehicles Fiorelli *et al.* (2004); Grocholsky *et al.* (2006); King *et al.* (2004); Stubbs *et al.* (2006); Tang & Ozguner (2005); Wang & Hussein (2007).

1.2 Area Coverage

Coverage represents a key measure of the quality of service provided by a sensor network. Area Coverage is always referred to a set, named *set of interest*, and to an action, then, covering means acting on all the physical locations of the set of interest.

Within the several actions that can be considered, such as manipulating, cleaning, watering and so on, sensing is certainly one of the most considered in literature.

1.2.1 Static Sensors

Considering static sensors, the coverage problem, as been addressed several point of view.

Coverage Evaluation Several studies addressed the problem of evaluating how well an area is monitored or tracked by sensors deployed in deterministic or stochastic way. In Meguerdichian *et al.* (2001) a polynomial time algorithm for evaluation of coverage obtained by a network of sensors with known positions is presented. The algorithm is obtained by combination of computational geometry (Voronoy diagrams) and graph theoretic techniques (graph search algorithms) is presented. Sensors with uniform sensing ability are considered.

In Li *et al.* (2003) for the same problem a distributed algorithm is proposed. In addition a more general sensing model is considered in which sensing ability diminishes as the distance increases.

In Huang & Tseng (2005) the k-coverage evaluation problem is formulated as a decision problem, whose goal is to determine whether every point in the service area of the sensor network is covered by at least k sensors, where k is a predefined value. The sensing ranges of sensors can be unit disks or non-unit disks. Moreover irregular sensing regions are considered.

These approaches are deterministic, such as sensors positions are known. A different idea is considering sensors that are deployed on the field of interest according with a given probability distribution. The problem is then evaluating the coverage of a field of interest achieved deploying a given number of sensors with a given distribution.

In Liu & Towsley (2004) area coverage for a large-scale randomly placed sensor network under the Boolean sensing model (a point in the space is covered if it is sufficiently near to one sensor) is obtained using results in stochastic geometry. The required sensor density to achieve a target area coverage level is derived.

The problem of coverage evaluation of heterogeneous sensor networks is considered in Lazos & Poovendran (2006). The coverage problem is formulated as a set intersection problem and faced with tools of integral geometry. Sensors are deployed according to an arbitrary stochastic distribution. Sensing areas of sensors can have any arbitrary shape, moreover sensors need not have an identical sensing capability.

Energy Optimization Considering overdeployed sensor networks, the coverage problem can be formulated as choosing an optimal set of sensors needed to achieve coverage of the field of interest. In this sense a centralized approximation algorithm that delivers a suboptimal solution is presented in Zhou *et al.* (2004). Moreover a distributed version of the algorithm is considered.
A fully distributed algorithm, that can be implemented with local information and low message complexity is proposed in Hefeeda & Bagheri (2007). Moreover the algorithm does not require that sensors know their locations.
For many kind of sensors it is possible to control the sensing range. Anyway a wide sensing radius entails consumption of a lot of energy. For this sensors the coverage problem can be formulated in terms to optimize energy consumption need to achieve full coverage using a given set of pre-deployed sensors.
In Wang & Medidi (2007) two local sensing radii optimization schemes based on one-hop approximation of Delaunay Triangulation are proposed in order to minimize the energy consumption and extend the lifetime of networks .

Optimal Sensor Deployment If sensors need to be deployed, an interesting problem could be finding the deployment strategy that maximize the achieved coverage. If a deterministic deployment is considered the problem became finding optimal positions for sensors.
In Chakrabarty *et al.* (2002) grid coverage strategies for effective surveillance and target location in distributed sensor networks are presented. The sensor field is represented as a grid (two or three-dimensional) of points (coordinates). An integer linear programming (ILP) solution for minimizing the cost of sensors for complete coverage of the sensor field is proposed.
A polynomial time algorithm for the problem of deployment of sensor nodes to optimize the coverage improvement is proposed in Hou *et al.* (2006).
An important result came from computational geometry Du *et al.* (1999). The so called centroidal Voronoy configurations are proposed as optimal solution to the problem of optimizing location for sensors with infinite ranges in which sensing ability diminishes as the distance increases.

1.2.2 Mobile Sensors

The introduction of mobile sensors allow developing networks in which sensors, starting from an initial random deployment, evaluate and move trough optimal locations.
In Li & Cassandras (2005) maximizing coverage using sensors with limited range, while minimizing communications cost, is formulated as an optimization problem. A gradient algorithm is used to drive sensors from initial positions to suboptimal locations.
In Howard (2002) an incremental deployment algorithm is presented. Nodes are deployed one-at-at time into an unknown complex environment, with each node making use of information gathered by previously deployed nodes. The algorithm is designed to maximize network coverage while ensuring line-of-sight between nodes.
A stable, distributed, feedback control law, to drive sensors to centroidal Voronoy configurations, that are, as said, critical points of the sensors locations optimization problem (Du *et al.* (1999)) , is presented in Cortes *et al.* (2004).
In Sameera & Gaurav S. (2004) a distributed, scalable algorithm based on artificial potential fields is presented. The constraint that each of the nodes has at least K neighbors (sensors within its communication ranges), where K is a user-specified parameter, is considered.
Obstacles avoidance is considered in Zou & Chakrabarty (2003) and Wong *et al.* (2004).
In Tsai *et al.* (2004) the maximization of visibility information for mobile observers when obstacles to vision are present in the environment has been studied in the level set framework. Suboptimal solutions are proposed.
Moreover mobile nodes can be used to handle faults as proposed in Wang *et al.* (2006) and in Sekhar *et al.* (2005)

Dynamic Coverage The natural evolution of these kind of approaches moves in the direction of giving a greater motion capabilities to the network. And once the sensors can move autonomously in the environment, the measurements can be performed also during the motion (*dynamic coverage*). Then,under the assumption, reasonable in many applications, that synchronous or asynchronous discrete time measures are acceptable instead of continuous ones, the number of sensors can be strongly reduced. Moreover, faults or critical situations can be faced and solved more efficiently, simply changing the paths of the working sensors. Clearly, coordinated motion of such *dynamic sensors network*, imposes additional requirements, such as avoiding collisions or preserving communication links between sensors. In order to better motivate why and when a mobile sensor network can be a more successful choice than a static one, some considerations are reported, even in an approximated way. If a dynamic network is considered, the area covered by sensors is a time function and, clearly, it not decreases as time passes. A simplified discrete time model of the evolution of the area still uncovered, at (discrete) time $t = k + 1$, by a dynamic sensors network, can be given by the following differences equation

$$A_u(k+1) = \left(1 - \frac{\dot{A}_N}{A_{tot}}\right) A_u(k) \tag{1.1}$$

where

$$\dot{A}_N = \frac{v_{max}}{2\rho_S} A_{tot} \left(1 - \left(1 - \frac{\pi \rho_S^2}{A_{tot}}\right)^N\right)$$

represents the area covered in the time unit by a number N of mobile sensors subject to the maximum motion velocity v_{max}. Measurements are then modeled as obtained deploying randomly N static sensors on the workspace every $\frac{2\rho_S}{v_{max}}$ seconds. Denoting by

$$A_u(0) = A_{tot} \left(1 - \frac{\pi \rho_S^2}{A_{tot}}\right)^N$$

the initial condition for area to be covered, at each discrete time $t = k$ the fraction of area covered is given by

$$A_{\%}(k) = 1 - \frac{A_u(k)}{A_{tot}} = 1 - \left[\frac{A_u(0)}{A_{tot}} \left(1 - \frac{\dot{A}_N}{A_{tot}}\right)^k\right] \tag{1.2}$$

The evolution computed using (1.2) with $N = 5$, $N = 10$ and $N = 15$ has been compared with the results of simulations where the approach described in Chapter 5 of this Thesis is applied. In Fig. 1.1 this comparison is reported, showing that (1.2) is a good model for describing the relationship between the area covered and the time using a dynamic solution.

Then, referring to surveillance tasks, the model defined in 1.1 can be used to evaluate the minimum number of sensors (with given ρ_S and v_{max}) required to cover a given fraction $\tilde{A}_{\%}$ of the area of interest according to a given measurement rate. In fact, it

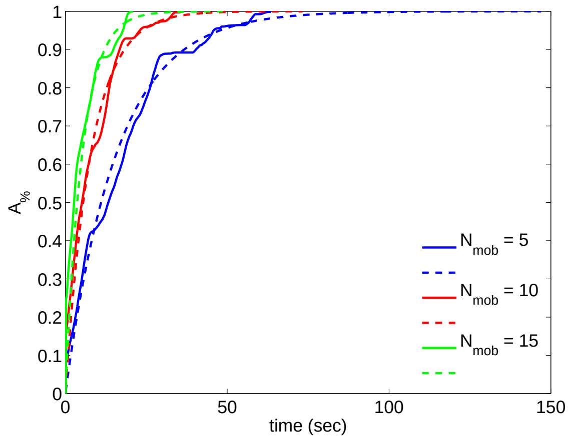


Figure 1.1: Comparison between coverage evolution obtained by the model (1.1) (*dashed*) and simulations of the proposed coverage strategy (*solid*) for different numbers of moving sensors

is possible to write the relation between the maximum rate at witch the network can cover the $\tilde{A}_{\%}$ fraction of A_{tot} and the number of moving sensors as

$$f = \frac{\log\left(1 - \frac{\dot{A}_N}{A_{tot}}\right)}{\log\left(1 - \tilde{A}_{\%}\right) - N\log\left(1 - \frac{\pi\rho_S^2}{A_{tot}}\right)} \tag{1.3}$$

Such a relationship between N and f is depicted in Fig. 1.2, showing, as intuitively expected, almost a proportionality between number of sensors and frequency of measurement at each point of the area A_{tot}.

The motivation and the support of the dynamic solution is evidenced by Fig. (1.1): lower is the refresh frequency of the measurements at each point (that is higher are the time intervals between measurements) and lower is the number of sensors required, once sensors motion is introduced.

1.3 Thesis Outline

This work is organized in two parts.

Part I The fist part concerns optimal dynamic coverage, such as the problem of offline evaluating controls for a mobile sensor network in order to maximize the area

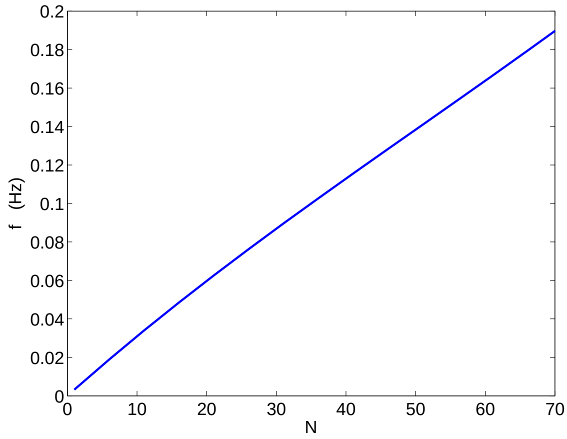


Figure 1.2: Maximum measure rate f in function of number of moving sensors.

covered during a time interval.
In Chapter 2, the optimal coverage problem is formulated in general. General formulations of *Optimal Control* and *Nonlinear Programming* problems are recalled in order to introduce notations useful for understanding following chapters.
In Chapter 3, the optimal overage control is considered for a single mobile sensor. At first, after the introduction of a continuous time, the problem is formulated as an optimal control problem. Because of the difficulty of solving that problem, an approximation is performed introducing a discrete time model. Using this model optimal coverage can be formulated as a nonlinear programming model. This formulation allows to evaluate suboptimal solutions.
In Chapter 4, the approach adopted in Chapter 3 is extended to a mobile sensor network. the usage of multiple mobile sensors introduce motion coordination problems that are solved introducing distance constraints to the optimization problem. Particular attention is given to *connectivity maintenance*. Moreover heterogeneous sensors are considered. Node fault robustness is also studied as a particular case.

Results presented in the first part of the thesis has been published as authors original work in Gabriele & Di Giamberardino (2007a,b, 2008a,c,e,f); Gabriele & Giamberardino (2008)

Part II The second part of the thesis concerns approaches to dynamic coverage that are suitable for online applications, such as, approaches based on the development of feedback law driving the sensor network to cover the set of interest.
In Chapter 5, an artificial potential based control law that guarantees to drive a mobile sensor network to totally cover the set of interest is proposed.
In Chapter 6, motion coordination is considered in order to satisfy collisions avoidance and connectivity maintenance constraints. In particular a distributed strategy for connectivity maintenance is proposed.

Results presented in the first part of the thesis has been published as authors original work in Gabriele & Di Giamberardino (2008b,d,e)

In Chapter 7, conclusions and avenues for future research are given.

Part I

Optimal Dynamic Coverage

CHAPTER 2

OPTIMAL COVERAGE

This chapter introduces the first part of this thesis, concerning the centralized approach to *dynamic coverage* of given set of interest within a time interval. The point of view is similar to the one considered in robot motion planning, where the objective is to find admissible path between a starting and final configurations. In LaValle (2006) a complete description of robot motion planning is given, coverage planning and optimal planning problems are also mentioned. In Choset (2001) a survey on coverage planning for mobile robots is given. The basic idea of the reviewed works is to exactly or approximately decompose the set of interest, in which there are obstacles, into convex cells that can be covered via simple back-and-forth motions. With the same approach in Acar *et al.* (2006) a coverage algorithm based on the Morse cell decomposition is proposed.

In Cecil & Marthler (2004) the coverage path planning problem for multiple sensors is studied, with a variational approach, in the level set framework. Obstacles occlusions are considered, suboptimal solutions are proposed also in three dimensional environments (Cecil & Marthler (2006)).

In Wang (2003) various problems associated with optimal path planning for mobile observers are considered. The existence of solutions is discussed first. Then, optimality conditions are derived by considering local path perturbations. Numerical algorithms for solving the corresponding approximate problems are proposed.

These works consider the problem under a geometric point of view, doing elaborations on a map of the environment, without considering the dynamics of the mobile objects. In Hokayem *et al.* (2007a) the problem of dynamic coverage control of a convex polygonal region in the plane using N agents with bounded velocities is addressed. Trajectories are planned by the agents through a distributed algorithm and then executed. Upper bound on the completion time as well as the number of messages that need to be exchanged by the agents are provided. With the same approach periodic coverage is considered in Hokayem *et al.* (2007b).
A path planning algorithm for cooperative coverage for nonholonomic vehicles is presented in Ahmadzadeh *et al.* (2007). Constraints such as collision avoidance and specifications on initial and final positions are considered. An approximation of the trajectories of vehicles using sequence of waypoints is performed. Each way point is treated as a moving particle in the space. Interaction forces between the particles are defined. Trajectories generated by the waypoints in the equilibria of the multi particle system satisfy the constraints generating a suboptimal solution to the coverage problem.
Moreover, optimal coverage is strongly related with is one of the most widely known combinatorial optimization problems, such as the *traveling salesperson problem* (TSP) and in particular with its *stochastic* version. It concerns with finding a minimum length closed path through a finite set of points randomly generated on a given set.
Stochastic TSP has been addressed for the Dubin vehicle in Savla *et al.* (2005), for a double integrator in Savla *et al.* (2006), for the reeds-shepp car and the differential drive robot in Enright & Frazzoli (2006). The main idea is tiling the set of interest with geometric objects, named *beads*, constructed on the base of the mobile object motion model. The mobile sensor sweep the whole set of interest visiting all the beads rows in sequence top-to-down, alternating between left-to-right and right-to-left passes.

In this thesis an approach to *Optimal Coverage Control* is proposed. So, given a generic mobile sensor (that can be view as composed by one or more mobile objects), the objective is to evaluate how to optimally use its motion capabilities, in a given time interval, in order to enlarge its range of measure.

A general definition of the optimal coverage problem in given in 2.1. Considering continuous time dynamics for the mobile sensor, the problem can be formulated as an

finite-time optimal control problem. Section 2.2contain a basic definition of the generic optimal control problem.
Analytical solutions of the optimal control formulation of the optimal coverage problem are in general very hard to compute, so, it is possible to approximate the formulation considering discrete time dynamics. In this case a more tractable nonlinear programming formulation can be given. For this reason, in section 2.3 the basic nonlinear programming problem is introduced .

2.1 Optimal Coverage Problem

Let W be a compact subset of the real Euclidean space called the *set of interest*. A *generic mobile sensor* can be represented by:

- A configuration space $\mathcal{C}$, that is the space of possible positions that the sensor may attain.
- A dynamic model that describe the evolution of sensor configuration in time, according to a control input u and that can be express by:

$$f(\ddot{q}(t), \dot{q}(t), q(t), u(t)) = 0$$

- A *visibility set* $M = M(q(t)) \subseteq W$, that is the subset of W on witch the sensor in configuration $q(t)$ can do measures.

Let's consider a time interval $\Theta = [t_i, t_f]$ and the evolution of the sensor configuration during such time interval time interval $(q(t),\ t \in \Theta)$.

The the subset of $\mathcal{W}$ on which the sensor do measures during Θ can be represented as:

$$M_\Theta(q) = \bigcup_{t \in \Theta} M(q(t)) \tag{2.1}$$

The area covered by the sensor set during Θ is then the measure of $M_\Theta(q)$:

$$A_\Theta(q) = \mu(M_\Theta(q)) \tag{2.2}$$

The general *optimal coverage* problem consists of evaluating the sensor trajectory, and the correspondent input function, that maximize the area covered during the given time interval.

$$\max_{(u,q) \in D} A_\Theta(q) \tag{2.3}$$

The admissible set D is represented by:

$$D = \{(u, q) : f(\ddot{q}(t), \dot{q}(t), q(t), u(t)) = 0;\ g(\ddot{q}(t), \dot{q}(t), q(t), u(t)) \leq 0\}$$

were g represents additional constraints that can be defined on the sensor input or configuration. In the following chapters the optimal coverage problem will be formulated, for specific senor models, at first as an optimal control problem ,then, after discretization, as a non linear programming one. For this reason sections 2.2 and 2.3 will be dedicated to introduce very basic definitions on this kind of problems.

2.2 Optimal Control

Let's consider a dynamic system described by the following differential equation

$$\dot{z}(t) = f(z(t), u(t), t) \tag{2.4}$$

where $z(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the input vector and f is a function of class C^2 with respect to its arguments.

Let's define some cost functional

$$J(z, u)$$

that depends from state and input trajectories in a given time interval $\Theta = [t_i, t_f]$. Let's assume that starting and final state are constrained to belong to some open sets:

$$z(t_i) \in D_i z(t_f) \in D_f$$

and that additional constraints are defined on state and input trajectories:

$$g[z(t), u(t), t] \leq 0\ \forall t \in [t_i, t_f]$$

The general optimal control problem,that concerns the evaluation of admissible state and input trajectories that minimize the cost functional, can be defined as follows:

$$\min_{(\mathbf{z},\mathbf{u}) \in \mathcal{D}} J(\mathbf{z}, \mathbf{u}) \tag{2.5}$$

Where the admissible set D is represented by:

$$\begin{aligned} D = &\{(z, u) \in \bar{C}^1[t_i, t_f] \times \bar{C}^0[t_i, t_f] :\ z(t_i) \in D_i;\ z(t_f) \in D_f; \\ &h[z(t), \dot{z}(t), u(t), t] = \dot{z}(t) - f(z(t), u(t), t)\ \forall t \in [t_i, t_f]; \\ &g[z(t), \dot{z}(t), u(t), t] \leq 0\ \forall t \in [t_i, t_f]\} \end{aligned}$$

Deep analysis of optimal control theory can be found in Bruni & di Pillo (1993); Sontag (1998).

2.3 Nonlinear Programming

Let's consider a scalar function $J(v)$ defined as follows:

$$J : \mathbb{R}^n \longrightarrow \mathbb{R}$$

Let's define a set $D \subseteq \mathbb{R}^n$ defined by the following equality constraints:

$$h(v) = 0 \tag{2.6}$$

and inequality constraints:

$$g(v) \leq 0 \tag{2.7}$$

with h and g vectorial functions with dimensions $\mu < n$ and σ respectively.

Definition 2.3.1. *A point $v^* \in D$ is said to be a constrained local minimum point of J in D if exists a neighborhood $S(v^*, \epsilon)$ such that:*

$$J(v^*) \leq J(v)\ \forall v \in S(v^*, \epsilon) : h(v) = 0,\ g(v) \leq 0$$

Definition 2.3.2. *A point $v^0 \in D$ is said to be a constrained global minimum point of J in D if :*

$$J(v^0) \leq J(v)\ \forall v \in \mathbb{R}^n : h(v) = 0,\ g(v) \leq 0$$

Obviously, every global minimum is also a local minimum, the converse, in general is not true.

The general optimal control problem,that concerns the determination of existence and the evaluation of a global minimum point of the cost function in the admissible set D, can be defined as follows:

$$\min_{\mathbf{v} \in \mathcal{D}} J(\mathbf{v}) \tag{2.8}$$

Where the admissible set D is represented by:

$$D = \{v \in \mathbb{R}^n :\ h(v) = 0,\ g(v) \leq 0\}$$

Deep analysis of nonlinear programming can be found in Bruni & di Pillo (1993); Nocedal & Wright (1999).

CHAPTER 3

OPTIMAL COVERAGE FOR A SINGLE MOBILE SENSOR

In this chapter the optimal coverage problem for a single mobile sensor is considered. At first a continuous time model and an optimal control formulation of the coverage problem are proposed, then, after discretization, the problem is reformulated and solved as a nonlinear programming one.

3.1 Continuous Time Formulation

3.1.1 Motion Model

Sensor is modeled, from a dynamic point of view, as a material point moving on $\mathbb{R}^2$. Planar motion is considered only for sake of simplicity, all results can be immediately extended to the 3D motion case.

The classical simple equations of motion are the satisfied:

$$\ddot{\mathbf{q}} = \mathbf{u} \tag{3.1}$$

where $\mathbf{q} = (q_1, q_2)^T$ indicates the sensor configuration and $\mathbf{u} = (u_1, u_2)^T$ the input force. For sake of simplicity mass is assumed to be unitary.
Linearity of 3.1 allow to write dynamics in the form:

$$\begin{aligned}\dot{\mathbf{z}}(t) &= A\mathbf{z}(t) + B\mathbf{u}(t) \\ \mathbf{q}(t) &= C\mathbf{z}(t)\end{aligned} \tag{3.2}$$

where

$$\mathbf{z} = (\dot{q}_1, q_1, \dot{q}_2, q_2)^T$$

and

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The evolutions of state and output are related with the input one by the following, well known equations:

$$\mathbf{z}(t) = \phi(\mathbf{z}(0), \mathbf{u}(t)) = e^{At}\mathbf{z}(0) + \int_0^t e^{A(t-\tau)} B\mathbf{u}(\tau) d\tau \tag{3.3}$$

and

$$\mathbf{q}(t) = \psi(\phi(\mathbf{z}(0), \mathbf{u}(t))) = C\phi(\mathbf{z}(0), \mathbf{u}(t)) \tag{3.4}$$

3.1.2 Sensing Model

Proximity sensing model is considered, so the mobile sensor is assumed to take measures, at time t, within a circular area of radius ρ around its current position $\mathbf{q}(t)$. The sensor field of measure is then a sphere of center $\mathbf{q}(t)$ and radius ρ:

$$M(\mathbf{q}(t)) = \{\mathbf{p} \in W : \|\mathbf{q}(t) - \mathbf{p}\| \leq \rho\} \tag{3.5}$$

3.1.3 Coverage Problem Formulation

3.1.3.1 Objective Functional

In 2.1 the area covered by a mobile sensor during a time interval Θ is defined as the union of the measure sets over Θ.

This quantity is very hard to compute, also for the simple measure set model introduced in 3.1.2.

An alternative coverage measure is here introduced that is based on the concept of distance between a sensor trajectory and a point. Let's define the distance between a point $\mathbf{p} \in \mathcal{W}$ and a sensor trajectory $\mathbf{q}$, within a time interval $\Theta = [0, t_f]$, as

$$d_\Theta(\mathbf{q}, \mathbf{p}) = \min_{t \in \Theta} \|\mathbf{q}(t) - \mathbf{p}\| \tag{3.6}$$

It is easy to see that d_Θ is a continuous functional of the sensor trajectory $\mathbf{q}$. In general d_Θ is not convex in $\mathbf{q}$.

Considering the proximity sensing model defined in 3.1.2, we can say that a point $\mathbf{p} \in \mathcal{W}$ is covered by the sensor during Θ if and only if:

$$d_\Theta(\mathbf{q}, \mathbf{p}) \leq \rho$$

So, making use of the function

$$\mathrm{pos}(\chi) = \begin{cases} \chi & \text{if} \quad \chi > 0 \\ 0 & \text{if} \quad \chi \leq 0 \end{cases} \tag{3.7}$$

that fixes to zero any non positive value, it is possible to define the following non negative functional:

$$\hat{d}_\Theta(\mathbf{q}, \mathbf{p}, \rho) = \mathrm{pos}\left(d_\Theta(\mathbf{q}, \mathbf{p}) - \rho\right) \tag{3.8}$$

A measure of coverage achieved by a trajectory $\mathbf{q}$ can be computed integrating functional $\hat{d}$ over the whole set of interest:

$$J(\mathbf{q}) = \int_{\mathcal{W}} \hat{d}_\Theta(\mathbf{q}, \mathbf{p}, \rho) \tag{3.9}$$

According with equation (3.4) the dependence of functional J from sensor state $\mathbf{z}$ and the input functions $\mathbf{u}$ can be explicated:

$$J(\mathbf{z}, \mathbf{u}) = \int_{\mathcal{W}} \hat{d}_\Theta(C\mathbf{z}, \mathbf{p}, \rho) = \int_{\mathcal{W}} \hat{d}_\Theta(C\phi(\mathbf{z}(0), \mathbf{u}), \mathbf{p}, \rho) \tag{3.10}$$

Functional J is continuous in its arguments. In general J is not convex.

3.1.3.2 Constraints

Physical limits on the actuators (for the motion) and/or on the sensors (in terms of velocity in the measure acquisition) suggest to consider bounded inputs and velocities.

$$\|\dot{\mathbf{q}}(t)\| \leq \mathbf{v}_{max}$$
$$\|\mathbf{u}(t)\| \leq \mathbf{u}_{max}$$

Expressions above can be rewritten as inequality constraints, on generalized inputs and state:

$$g_1(\mathbf{z}(t)) = \|B^T\mathbf{z}(t)\| - \mathbf{v}_{max} \leq \mathbf{0} \tag{3.11}$$

$$g_2(\mathbf{u}(t)) = \|\mathbf{u}(t)\| - \mathbf{u}_{max} \leq \mathbf{0} \tag{3.12}$$

According with missions it is possible to introduce several kind of geometric constraints. Box constraints can be introduced on the sensor configuration allowing the sensor to move only within a box subset of $\mathbb{R}^2$:

$$\mathbf{q}_{min} \leq \mathbf{q}(t) \leq \mathbf{q}_{max}$$

Also this constraints can be expressed as inequality constraints on the sensor network state.

$$g_3(\mathbf{z}(t)) = C\mathbf{z}(t) - \mathbf{z}_{max} \leq \mathbf{0} \tag{3.13}$$

$$g_4(\mathbf{z}(t)) = \mathbf{z}_{min} - C\mathbf{z}(t) \leq 0 \tag{3.14}$$

Moreover staring and/or final positions can be fixed:

$$\mathbf{q}(0) = \mathbf{q}_{start}$$
$$\mathbf{q}(t_f) = \mathbf{q}_{end}$$

An other interesting example is the closed trajectory constraint:

$$\mathbf{z}(0) = \mathbf{z}(t_f)$$

Closed trajectories can be repeated in times, obtaining periodic coverage of the same space, that can be very useful in surveillance missions. That constraints can be expressed as equality constraints on the starting and/or final state:

$$\chi(\mathbf{z}(0), \mathbf{z}(t_f)) = 0 \tag{3.15}$$

3.1.3.3 Optimal Control Problem

Summarizing, the optimal coverage problem be formulated as an optimal control problem.

$$\min_{\mathbf{z},\mathbf{u}} J(\mathbf{z},\mathbf{u}) = \int_{\mathcal{W}} \hat{d}_{\Theta}(\mathbf{z},\mathbf{u}),\mathbf{p},\rho)$$
$$(\mathbf{z},\mathbf{u}) \in \mathcal{D} \tag{3.16}$$

where the admissible set $\mathcal{D}$ is defined as:

$$\begin{aligned}
\mathcal{D} = \{(\mathbf{z},\mathbf{u}) \in \mathcal{C}^1 : & \chi(\mathbf{z}(t_f),\mathbf{z}(t_f)) = 0 \\
& h(\mathbf{z},\dot{\mathbf{z}},\mathbf{u}) = A\mathbf{z} + B\mathbf{u} - \dot{\mathbf{z}} = 0 \\
& g_1(\mathbf{z}(t)) = \|B^T\mathbf{z}(t)\| - \mathbf{v}_{max} \leq \mathbf{0} \\
& g_2(\mathbf{u}(t)) = \|\mathbf{u}(t)\| - \mathbf{u}_{max} \leq \mathbf{0} \\
& g_3(\mathbf{z}(t)) = C\mathbf{z}(t) - \mathbf{z}_{max} \leq \mathbf{0} \\
& g_4(\mathbf{z}(t)) = \mathbf{z}_{min} - C\mathbf{z}(t) \leq 0\}
\end{aligned}$$

The non convexity of the cost functional makes this problem not convex, so, several *suboptimal* solutions may exist.
Anyway, in general, it is very hard to evaluate analytically one of this solutions. For this reason in the next section discretization is introduced that allow to make the problem tractable.

3.2 Discrete Time Formulation

3.2.1 Discrete Time Motion Model

If discrete time is considered the sensor motion model assumes the following expression:

$$\begin{aligned}
\mathbf{z}((n+1)T_s) &= A_d\mathbf{z}(nT_s) + B_d\mathbf{u}(nT_s) \\
\mathbf{q}(nT_s) &= C\mathbf{z}(nT_s)
\end{aligned}$$

where

$$A_d = e^{AT_s} \quad B_d = \int_0^{T_s} e^{A\tau} B d\tau$$

According with the introduced model it is possible to write state and output values at time nT_s as:

$$\mathbf{z}(nT_s) = {A_d}^n\mathbf{z}(0) + \sum_{k=0}^{n-1} {A_d}^k B_d\mathbf{u}((n-1)T_s - kT_s) \tag{3.17}$$

and

$$\mathbf{q}(nT_s) = C{A_d}^n\mathbf{z}(0) + C\sum_{k=0}^{n-1} {A_d}^k B_d\mathbf{u}((n-1)T_s - kT_s) \tag{3.18}$$

Representing the sensor input sequence from time $t = 0$ to time $t_f = (n_f - 1)T_s$ as:

$$\mathbf{u}_{n_f} = \begin{pmatrix} \mathbf{u}(0) \\ \mathbf{u}(T_s) \\ \vdots \\ \mathbf{u}((n_f-1)T_s) \end{pmatrix}$$

and defining the following vectors

$$\mathbf{v}_{n_f} = \begin{pmatrix} \mathbf{z}(0) \\ \mathbf{u}_{n_f} \end{pmatrix} \qquad H(n) = \begin{pmatrix} A_d^n & A_d^{n-1}B_d & \dots & B_d & 0 & \dots & 0 \end{pmatrix}$$

it is possible to rewrite equations 3.17 and 3.18 in the following compact form:

$$\mathbf{z}(nT_s) = H(n)\mathbf{v}_{n_f} \tag{3.19}$$

and

$$\mathbf{q}(nT_s) = CH(n)\mathbf{v}_{n_f} \tag{3.20}$$

Let's represent state and output sequences with the following vectors:

$$\mathbf{z}_{n_f} = \begin{pmatrix} \mathbf{z}(0) \\ \mathbf{z}(T_s) \\ \vdots \\ \mathbf{z}(n_fT_s) \end{pmatrix} \qquad \mathbf{q}_{n_f} = \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{q}(T_s) \\ \vdots \\ \mathbf{q}(n_fT_s) \end{pmatrix}$$

According with 3.19 and 3.20, vectors $\mathbf{q}_{n_f}$ and $\mathbf{z}_{n_f}$ can be written as linear functions of vector $\mathbf{v}_{n_f}$:

$$\mathbf{z}_{n_f} = H_{n_f}\mathbf{v}_{n_f} \tag{3.21}$$

and

$$\mathbf{q}_{n_f} = C_{n_f}H_{n_f}\mathbf{v}_{n_f} \tag{3.22}$$

where

$$H_{n_f} = \begin{pmatrix} H(0) \\ H(1) \\ \vdots \\ H(n_f) \end{pmatrix} \qquad C_{n_f} = \begin{pmatrix} C & 0 & \dots & 0 \\ 0 & C & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C \end{pmatrix}$$

3.2.2 Sensing Model

Space discretization is also performed. More precisely the set of interest $\mathcal{W}$ is divided into square cells c_k. Every cell is represented by its centroid $\mathbf{p}_k$. The sensor is assumed to cover the cell c_k at time nT_s if its distance between its position $\mathbf{q}(nT_s)$ and the cell centroid il less than ρ. The sensor field of measure at time nT_s is then the union of the cells that have their centroids within a sphere of center $\mathbf{q}(nT_s)$ and radius ρ:

$$M(\mathbf{q}(nT_s)) = \{\bigcup_k c_k : \|\mathbf{q}(nT_s) - \mathbf{p}_k\| \leq \rho\} \tag{3.23}$$

3.2.3 Coverage Problem Formulation

3.2.3.1 Objective Function

Let's define the distance between a cell c_k and a sensor discrete trajectory $\mathbf{q}$ as

$$d_\Theta(\mathbf{q}, \mathbf{p}_k) = \min_{n \in [0, n_f]} \|\mathbf{q}(nT_s) - \mathbf{p}_k\| \tag{3.24}$$

It is important to observe that considering the discrete time sensor dynamic, d_Θ become a scalar function of the sensor configurations sequence. This function is again, in general, not convex.
As done for the continuous time formulation let's define:

$$\hat{d}_\Theta(\mathbf{q}, \mathbf{p}, \rho) = \mathrm{pos}\left(d_\Theta(\mathbf{q}, \mathbf{p}) - \rho\right) \tag{3.25}$$

A measure of coverage performance achieved by a discrete time trajectory $\mathbf{q}$ on can be computed summing function $\hat{d}$ over all the cells:

$$J(\mathbf{q}) = \sum_k \hat{d}_\Theta(\mathbf{q}, \mathbf{p}_k, \rho) \tag{3.26}$$

According with equation (3.22) the dependence of function J from vector $\mathbf{v}_{n_f}$ can be explicated:

$$J(\mathbf{v}_{n_f}) = \sum_k \hat{d}_\Theta(\mathbf{q}_{n_f}, \mathbf{p}_k, \rho) = \sum_k \hat{d}_\Theta(C_{n_f} H_{n_f} \mathbf{v}_{n_f}, \mathbf{p}_k, \rho) \tag{3.27}$$

Function J is again continuous in its arguments, but, in general, not convex.

3.2.3.2 Nonlinear Programming Problem

After discretization the optimal coverage problem can be formulated as a nonlinear programming problem. The objective is to find, into an admissible set $\mathcal{D}$, the vector $\mathbf{v}^*_{n_f}$ that minimize the cost function:

$$\mathbf{v}^*_{n_f} = \min_{\mathbf{v}_{n_f}} J(\mathbf{v}_{n_f}) = \sum_k \hat{d}_\Theta(C_{n_f} H_{n_f} \mathbf{v}_{n_f}, \mathbf{p}_k, \rho)$$

$$\mathbf{v}^*_{n_f} \in \mathcal{D}$$

Considering discrete time versions of constraints defined in 3.1.3.2 the admissible set $\mathcal{D}$ can be defined as:

$$\begin{aligned}
\mathcal{D} = \{\mathbf{v}_{n_f} \in \mathbb{R}^{m(4+2n_f)} :& \chi(\mathbf{z}(0), \mathbf{z}(n_f T_s)) = 0 \\
& g_{1,0}(\mathbf{z}(0)) = \|B^T \mathbf{z}(0)\| - \mathbf{v}_{max} \leq \mathbf{0} \\
& \vdots \\
& g_{1,n_f}(\mathbf{z}(n_f T_s)) = \|B^T \mathbf{z}(n_f T_s)\| - \mathbf{v}_{max} \leq \mathbf{0} \\
& g_{2,0}(\mathbf{u}(0)) = \|\mathbf{u}(0)\| - \mathbf{u}_{max} \leq \mathbf{0} \\
& \vdots \\
& g_{2,n_f}(\mathbf{u}(n_f T_s)) = \|\mathbf{u}(n_f T_s)\| - \mathbf{u}_{max} \leq \mathbf{0} \\
& g_{3,0}(\mathbf{z}(0)) = C\mathbf{z}(0) - \mathbf{z}_{max} \leq \mathbf{0} \\
& \vdots \\
& g_{3,n_f}(\mathbf{z}(n_f T_s)) = C\mathbf{z}(n_f T_s) - \mathbf{z}_{max} \leq \mathbf{0} \\
& g_{0,4}(\mathbf{z}(0)) = \mathbf{z}_{min} - C\mathbf{z}(0) \leq 0 \\
& \vdots \\
& g_{4,n_f}(\mathbf{z}(n_f T_s)) = \mathbf{z}_{min} - C\mathbf{z}(n_f T_s) \leq 0\}
\end{aligned}$$

Suboptimal solutions can be computed using numerical methods. In the simulations performed, the SQP (Sequential Quadratic Programming) (Appendix A) method has been applied.

3.3 Simulations

In this section simulation results are presented that show the effectiveness of the proposed methodology.

In all the simulations the following values are assumed for the mobile sensor parameters :

$$u_{max} = 0.5 \qquad v_{max} = 1.5$$
$$\rho = 1$$

For the first three cases a time interval $\Theta = 15sec$ is considered. In the first simulation the problem of optimal coverage of box shaped set of interest is faced. Both starting and final state are free. The mobile sensor trajectory is displayed in subfigure 3.1(c). On the background the coverage status of the set of interest is displayed, darker zones represent the uncovered cells. In subfigures 3.1(a) and 3.1(b) evolutions of controls and speeds are respectively displayed.

In the second simulations starting state is fixed to:

$$\mathbf{z}(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Results are displayed in figure 3.2.
In figure 3.3 the same scenario is considered on a disk shaped set of interest. In the last simulation a time interval $\Theta = 25sec$ is considered.
the closed trajectory constraint is introduced, so:

$$\mathbf{z}(0) = \mathbf{z}(t_f)$$

Results are displayed in figure 3.4.

As said before, the optimal control problem has been formulated as a non convex optimization problem. For this problem the S.Q.P. method allow to evaluate solutions that only locally optimal, as the ones displayed above. Anyway, intuitively, it seems that the presented solutions are good even if suboptimal.

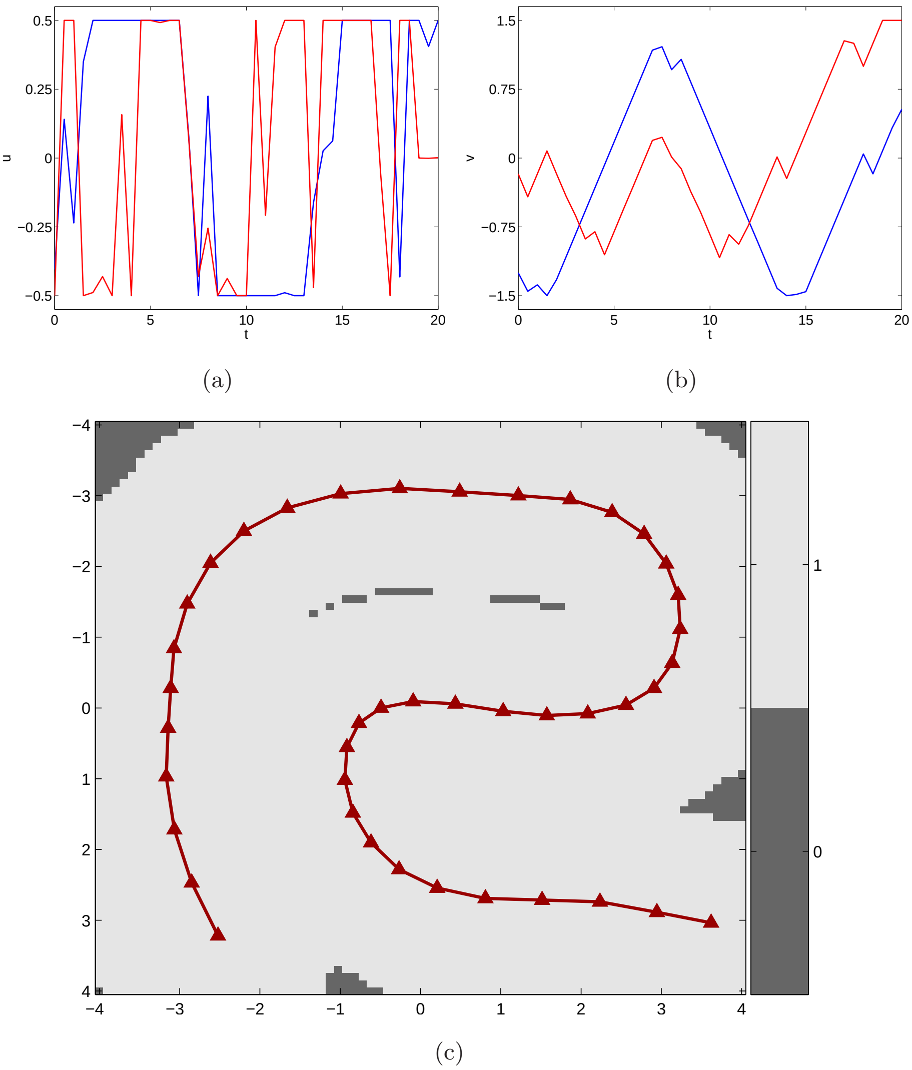


Figure 3.1: One sensor covering a boxed area. (a) Control components evolution. (b) Speed components evolution. (c) Sensor trajectory and coverage status of the set of interest

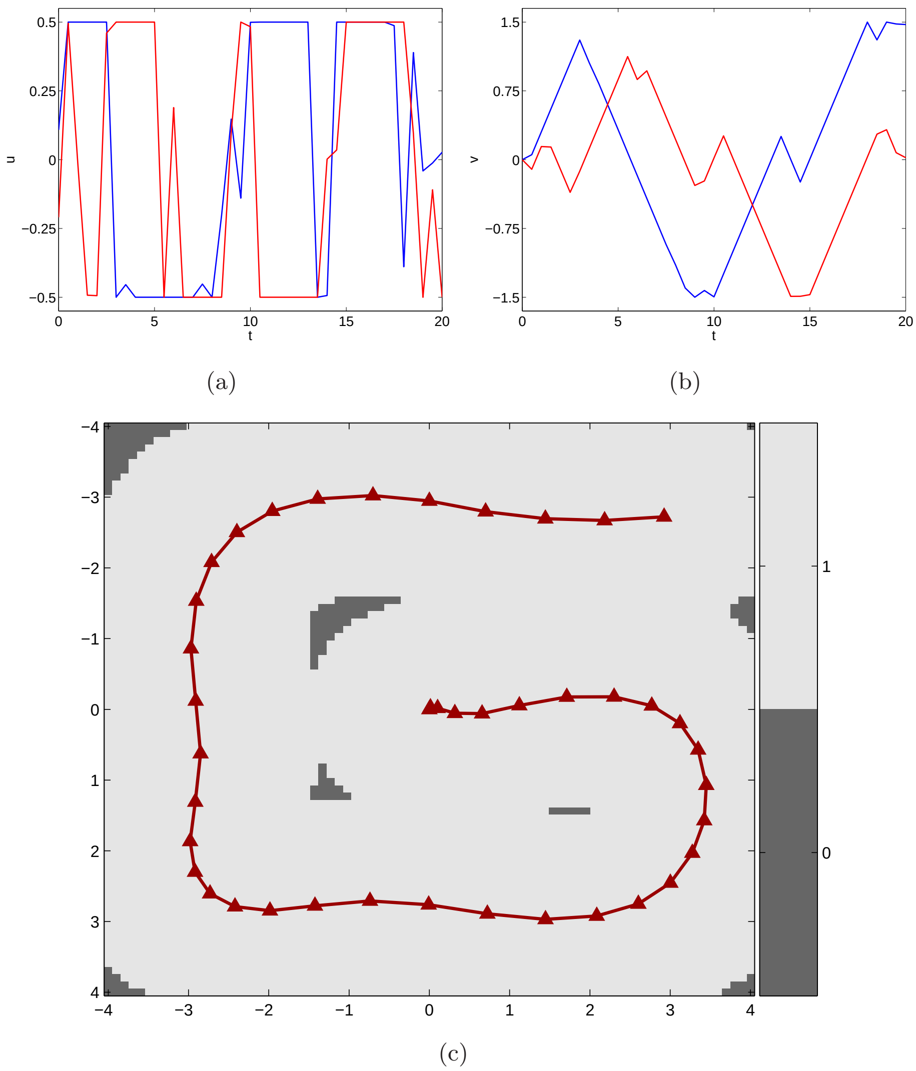
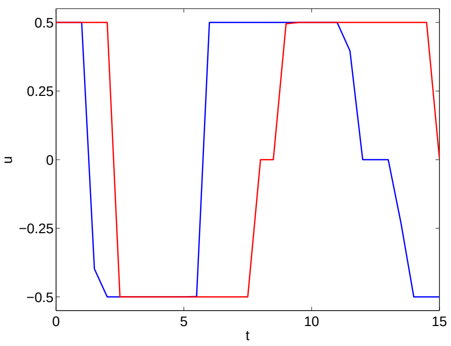
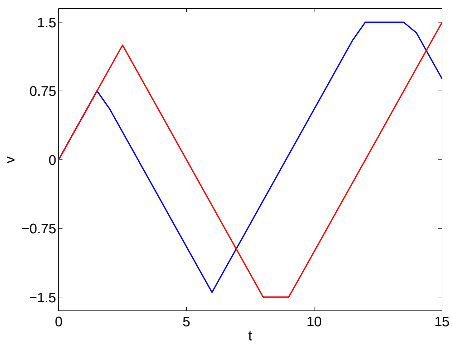


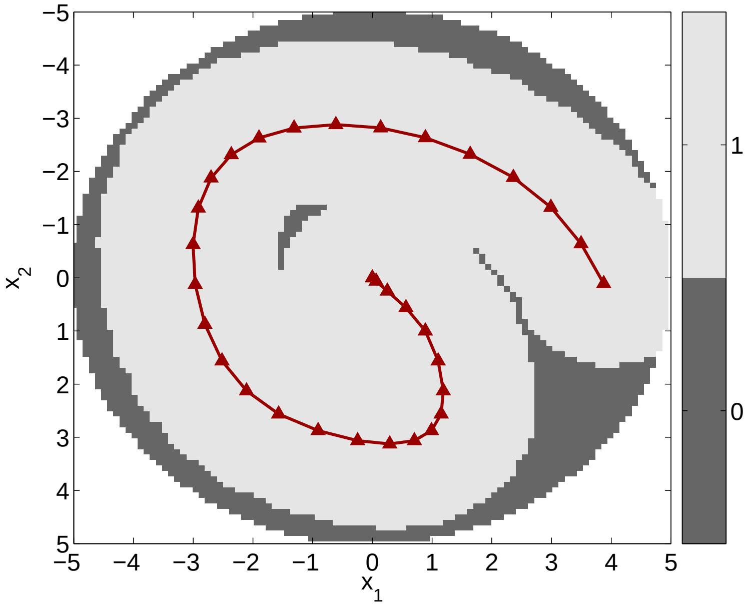
Figure 3.2: One sensor covering a box shaped area with fixed starting point. (a) Control components evolution. (b) Speed components evolution. (c) Sensor trajectory and coverage status of the set of interest



(a)



(b)



(c)

Figure 3.3: One sensor covering a circular area with fixed starting point. (a) Control components evolution. (b) Speed components evolution. (c) Sensor trajectory and coverage status of the set of interest

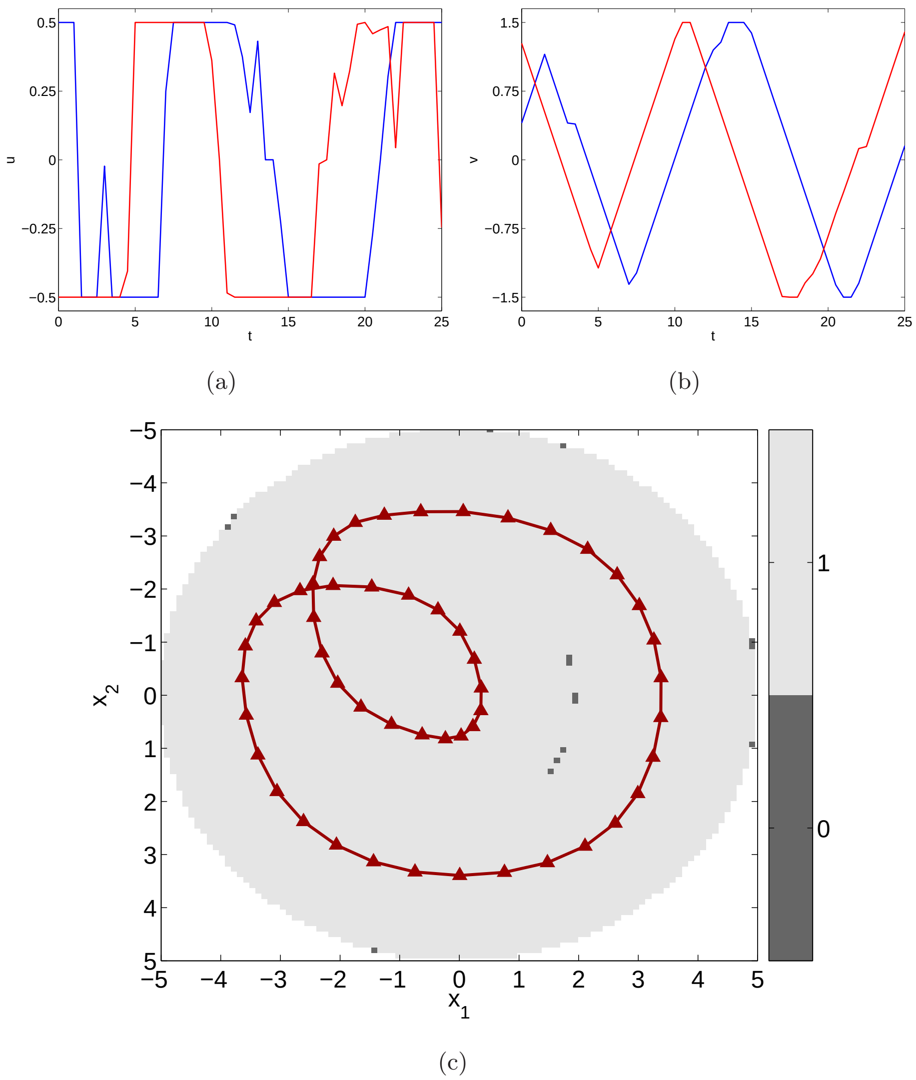


Figure 3.4: One sensor covering a circular area making a cyclic trajectory. (a) Control components evolution. (b) Speed components evolution. (c) Sensor trajectory and coverage status of the set of interest

CHAPTER 4

OPTIMAL COVERAGE FOR A MOBILE SENSOR NETWORK

In this chapter the optimal coverage problem is formulated for a mobile sensor network, such as, a set of m mobile sensors connected by an ad hoc communication network. The use of multiple sensors introduces challenging motion coordination problems. Coordination is needed to guarantee performances as, for example, avoidance of collisions or maintenance of communication quality features. Particular attention is given to the problem of maintaining connectivity of the communication network. In a mobile sensor network the communication network must be modeled as a configuration dependent dynamic graph in witch topology changes continuously while sensors moves. In order to maintain connectivity it is then necessary to introduce constrains on the relative positions of sensors. The simplest way to do that is maintaining the edges of the starting communication graph that's always assumed to be connected. An alternative way is imposing flocking behavior (Olfati-Saber (2006)) to sensors as in Hussein & Stipanovic (2007b). Anyway, this approaches affect strongly the motion capability of sensors. It is then more desirable to allow topology to change over time, even though that introduce challenging dynamic graph control problems.

In Mesbahi (2004), starting from a class of problems associated with control of distributed dynamic systems, a controllability framework for state-dependent dynamic

graphs is considered.
In Kim & Mesbahi (2005) the positions of a dynamic state-dependent graph vertexes are controlled in order to maximize the second smallest eigenvalue of the Laplacian matrix, also named *algebraic connectivity* and that has emerged as a critical parameter that influences the stability and robustness properties of dynamic systems that operate over an information network.
K-hop connectivity preservation is considered, in Zavlanos (2005), for a network with dynamic nodes. A centralized control framework that guarantees maintenance of this property is developed. Connectivity is modeled as an invariance problem and transformed into a set of constraints on the control variable.
In this Chapter a centralized approach to connectivity maintenance is proposed, that is based on preservation of the edges of a *Spanning Tree* of the communication graph .

In the framework introduced in Chapter 3 coordination can be implemented introducing constraints in the optimal coverage problem.
The approach is similar to the one proposed in Schouwenaars *et al.* (2001) for the problem fuel-optimal path planning of multiple vehicles. Here the basic problem is to have the vehicles move from an initial dynamic state to a final state without colliding with each other, while at the same time avoiding other stationary and moving obstacles. the problem is formulated as a linear program with mixed integer/linear constraints that account for the collisions avoidance.

Moreover the case of heterogeneous sensors is considered. Sensor network nodes were called *heterogeneous* with respect to different aspects.
In ling Lam & hui Liu (2007), the problem of deploying a set of mobile sensor nodes, with heterogeneous sensing ranges, to give coverage is addressed.
In Lazos & Poovendran (2006), evaluating coverage of a set of sensors, with arbitrary different shapes, deployed according to an arbitrary stochastic distribution is formulated as a set intersection problem.
In Hussein *et al.* (2007) the use of two classes of vehicles are used to dynamically cover a given domain of interest. The first class is composed of vehicles, whose main responsibility is to dynamically cover the domain of interest. The second class is composed of coordination vehicles, whose main responsibility is to effectively communicate coverage information across the network.

The problem of deploying nodes, equipped with different sets of sensing units, is studied in Shih *et al.* (2007) in order to cover a sensing field in which multiple attributes are required to be sensed.
In this Chapter the case of different magnitudes to be measured on a given set of interest is considered. In many applications,it is required to measure several feature of the field of interest, then, mobile agents are often equipped with a multiple sensor system. Heterogeneity between the network nodes is then considered, like in Shih *et al.* (2007), with respect to the set of sensing units with witch they are equipped. Moreover different sensors can have different sensing ranges.
The mathematical model needed to introduce heterogeneous sensors can be used to face an other important problem for sensors networks such as node fault robustness.

In section 4.1 the dynamic sensor network is defined as a generic mobile sensor. The optimal coverage problem is, at first, formulated in continuous time as an optimal control problem (section 4.2), then, after discretization as a nonlinear programming problem (section 4.3), as done for the single sensor case. Simulations results are presented in section 4.4.

4.1 General Formulation

Let W be the set of interest. Let $\Xi = \{\xi_1, \xi_2, \dots\}$ be the set of magnitudes of interest defined on W. A mobile sensor network is composed by agents, called sensors or nodes, able to move, to do measures on W and to communicate with each other. More formally, each *sensor* can be represented by:

- A configuration space $\mathcal{C}^{(i)}$, that is the space of possible positions ($q^{(i)}$) that the sensor may attain.
- A dynamic model that describe the evolution of sensor configuration in time, according to a control input $u^{(i)}$ and that can be express by:

$$f^{(i)}(\ddot{q}^{(i)}, \dot{q}^{(i)}, q^{(i)}, u^{(i)}) = 0$$

- A set $\Xi^{(i)} \subseteq \Xi$, that is the subset of magnitudes that the sensor can measure

- For every magnitude $\xi_j \in \Xi^{(i)}$, a *visibility* set $M_j^{(i)} = M_j^{(i)}(q^{(i)}(t)) \subseteq W$, that is the subset of W within the sensor in configuration $q^{(i)}(t)$ can measure magnitude ξ_j.

Looking at the whole network is possible to define generalized configuration $q = \{q^{(1)}, q^{(2)}, \ldots, q^{(m)}\}$ and generalized input $u = \{u^{(1)}, u^{(2)}, \ldots, u^{(m)}\}$.
Generalized dynamic model can written as:

$$f(\ddot{q}, \dot{q}, q, u) = 0$$

The subset of W on which the sensor network in configuration $q(t)$ can measure magnitude ξ_j is given by

$$M_j(q(t)) = \bigcup_{i:\xi_j \in \Xi^{(i)}} M_j^{(i)}(q^{(i)}(t))$$

Considering a time interval Θ and a generalized trajectory of the sensor network ($q(t)$ $t \in \Theta$) Let's denote with $q(\Theta)$ the evolution of the generalized network configuration during a given time interval Θ. It is possible to define the the subset of W on witch the magnitude ξ has been measured as:

$$M_{j,\Theta}(q) = \bigcup_{t \in \Theta} M_j(q(t)) \tag{4.1}$$

Looking at the whole magnitudes set, the subset of W on witch all magnitudes $\xi \in \Xi$ have been measured by the network can be defined as:

$$M_{\Theta}(q) = \bigcap_{\xi \in \Xi} M_{j,\Theta}(q) \tag{4.2}$$

The area covered by the sensor network with respect to the whole magnitudes set during Θ is then the measure of $M_{\Theta}(q)$:

$$A_{\Theta}(q) = \mu(M_{\Theta}(q)) \tag{4.3}$$

Node Fault Robustness Robustness w.r.t node faults is, obviously, a very desirable characteristic for a sensor network (Hazon & Kaminka (2005)). For dynamic sensor networks robustness can be achieved online, by dynamically changing sensors trajectories when a node fault happen, or outline, by oversizing the sensor network and planning sensors trajectories in order to guarantee coverage performances in case of faults. The second approach is the one considered here. In particular is possible to see how robust trajectories planning can be view as a particular heterogeneous sensor network trajectory planning. Let's consider a magnitude ξ defined on the sensor network workspace W, that can be measured within a radius ρ_ξ. Let's assume that m sensors allow to reach the desired coverage performances w.r.t. the magnitude ξ. As said, it is possible to reach robustness oversizing the sensor network, such as augmenting the number of sensors.

Let's want to make the network robust to the fault of h sensors, then, at least $m + h$ sensors must be used. To plan sensors trajectories customize the coverage problem adding $\binom{m+h}{m} - 1$ auxiliary magnitudes, measurable within the same radius of ξ. Let's call $\hat{\xi}_j \quad j = 1, 2, ..., \binom{m+h}{m}$ the new set of magnitudes of interest. Let's consider all the combination of m sensors and let's call them $\{\sigma\}_j \quad j = 1, 2, ..., \binom{m+h}{m}$. Let's associate magnitudes to sensors with the following law:

$$\hat{\xi}_j \in \Xi^{(i)} \Longleftrightarrow i \in \{\sigma\}_j$$

The multiple magnitudes coverage problem , so customized, is equivalent to the robust coverage problem w.r.t the magnitude ξ. In case of multiple magnitudes of interest the same operation must be done with respect to every magnitude.

4.1.1 Communication

Sensors can be view as nodes of communication network that con be represented by the graph

$$\mathcal{G}(t) =< V_\mathcal{G}, E_\mathcal{G}(t) >$$

where $V_\mathcal{G} = \{1, 2, \ldots, m\}$ indicate the vertexes set and $E_\mathcal{G}(q)$ indicates the edges set. Edges set depends from the network generalized configuration. While sensors moves network configuration changes in time, so the communication graph, and in particular its edges set, is time varying.

4.2 Continuous Time Formulation

4.2.1 Motion Model

Every sensor is modeled, from a dynamic point of view, as described in section 3.1.1. So, adding apexes and pedexes to the notation introduced in for the single section case, we can say that the generic $i-th$ sensor moves under the following motion model

$$\begin{aligned}\dot{\mathbf{z}}^{(i)}(t) &= A^{(i)}\mathbf{z}^{(i)}(t) + B^{(i)}\mathbf{u}^{(i)}(t)\\ \mathbf{q}^{(i)}(t) &= C^{(i)}\mathbf{z}^{(i)}(t)\end{aligned}$$

Considering the whole network the vector

$$\mathbf{z}(t) = \begin{pmatrix}\mathbf{z}^{(1)}(t) & \mathbf{z}^{(2)}(t) & \cdots & \mathbf{z}^{(m)}(t)\end{pmatrix}^T$$

can be defined to denote the generalized configuration, and the vector

$$\mathbf{q}(t) = \begin{pmatrix}\mathbf{q}^{(1)}(t) & \mathbf{q}^{(2)}(t) & \cdots & \mathbf{q}^{(m)}(t)\end{pmatrix}^T$$

to denote the generalized position that is represented, for each t, by m points in the Euclidean space.

At the same manner the generalized input is defined as as:

$$\mathbf{u}(t) = \begin{pmatrix}\mathbf{u}^{(1)}(t) & \mathbf{u}^{(2)}(t) & \cdots & u^{(m)}(t)\end{pmatrix}^T$$

Generalized dynamics for the whole network can, then, be written as:

$$\begin{aligned}\dot{\mathbf{z}}(t) = A\mathbf{z}(t) + B\mathbf{u}(t)\\ \mathbf{q}(t) = C\mathbf{z}(t)\end{aligned}$$

where:

$$A = \begin{pmatrix} A^{(1)} & 0 & \dots & 0\\ 0 & A^{(2)} & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & A^{(m)}\end{pmatrix} \qquad B = \begin{pmatrix} B^{(1)} & 0 & \dots & 0\\ 0 & B^{(2)} & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & B^{(m)}\end{pmatrix}$$

$$C = \begin{pmatrix} C^{(1)} & 0 & \dots & 0\\ 0 & C^{(2)} & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & C^{(m)}\end{pmatrix}$$

According with 3.3 and 3.4, generalized configuration evolution and network generalized trajectory are related with generalized input by:

$$\mathbf{z}(t) = \Phi(\mathbf{z}(0), \mathbf{u}(t)) = \begin{pmatrix} \phi_1(\mathbf{z}^{(1)}(0), \mathbf{u}^{(1)}(t)) \\ \phi_2(\mathbf{z}^{(2)}(0), \mathbf{u}^{(2)}(t)) \\ \vdots \\ \phi_m(\mathbf{z}^{(m)}(0), \mathbf{u}^{(m)}(t)) \end{pmatrix} \tag{4.4}$$

and

$$\mathbf{q}(t) = \Psi(\Phi(\mathbf{z}(0), \mathbf{u}(t))) = \begin{pmatrix} \psi_1(\phi_1(\mathbf{z}^{(1)}(0), \mathbf{u}^{(1)}(t))) \\ \psi_2(\phi_2(\mathbf{z}^{(2)}(0), \mathbf{u}^{(2)}(t))) \\ \vdots \\ \psi_m(\phi_m(\mathbf{z}^{(m)}(0), \mathbf{u}^{(m)}(t))) \end{pmatrix} \tag{4.5}$$

.

4.2.2 Sensing Model

As done in section 3.1.2 proximity sensing model is assumed for every sensor.

It is assumed that at every time t the generic i-th sensor can measure magnitude $\xi_j \in \Xi^{(i)}$ in a circular area of radius ρ_j around its current position $\mathbf{q}^{(i)}(t)$. The sensor field of measure respect to ξ_j is than a disk of center $\mathbf{q}^{(i)}(t)$ and radius $\rho_j^{(i)}$:

$$M_j^{(i)}(\mathbf{q}^{(i)}(t)) = \{p \in W : \|\mathbf{q}^{(i)} - \mathbf{p}\| \leq \rho_j^{(i)} \quad \xi_j \in \Xi^{(i)}\} \tag{4.6}$$

4.2.3 Communication Model

For communication between sensors the well known proximity model is assumed, such as, two sensors communicate directly if they are enough *near*. The communication network can then be modeled as an Euclidean graph.

$$\mathcal{G}(t) =< V_{\mathcal{G}}, E_{\mathcal{G}}(\mathbf{q}(t)) >$$

where

- $V_{\mathcal{G}} = \{1, \ldots, m\}$ represents the vertexes set.
- $E_{\mathcal{G}}(\mathbf{q}(t)) = \{(i,j) \ : \ \|\mathbf{q}^{(i)}(t) - \mathbf{q}^{(j)}(t)\| \leq \rho_C\}$ represents the edges set. It depends from the sensor network configuration and then it is time varying.

There is an edge between two sensors if the distance between them is smaller than a given communication radius ρ_C.

In case of homogeneous communication radia, that is the one considered in this paper, the communication network graph $\mathcal{G}$ is undirected in fact:

$$(i, j) \in E_{\mathcal{G}}(\mathbf{q}(t)) \Longleftrightarrow (j, i) \in E_{\mathcal{G}}(\mathbf{q}(t))$$

4.2.4 Coverage Problem Formulation

4.2.4.1 Objective Functional

With respect to every magnitude of interest ξ_j, it is possible to define the distance between a point $\mathbf{p} \in \mathcal{W}$ and a sensor network generalized trajectory $\mathbf{q}$, within a time interval $\Theta = [0, t_f]$, as

$$d_{j,\Theta}(\mathbf{q}, \mathbf{p}) = \min_{t \in \Theta;\ i : \xi_j \in \Xi^{(i)}} \|\mathbf{q}^{(i)}(t) - \mathbf{p}\| \tag{4.7}$$

Considering the proximity sensing model defined in section 4.2.2, we can say that magnitude ξ_j can be measured on point $\mathbf{p} \in \mathcal{W}$ is covered by the sensor during Θ if and only if:

$$d_{\Theta}(\mathbf{q}, \mathbf{p}) \leq \rho_j$$

So, making use of the function

$$\text{pos}(\chi) = \begin{cases} \chi & \text{if} \quad \chi > 0 \\ 0 & \text{if} \quad \chi \leq 0 \end{cases} \tag{4.8}$$

that fixes to zero any non positive value, it is possible to define, for every magnitude, the following non negative functional:

$$\hat{d}_{j,\Theta}(\mathbf{q}, \mathbf{p}, \rho_j) = \text{pos}\left(d_{j,\theta}(\mathbf{q}, \mathbf{p}) - \rho_j\right) \tag{4.9}$$

A measure of the coverage achieved by a trajectory $\mathbf{q}$, with respect to a magnitude ξ_j, can be computed integrating functional $\hat{d}_{j,\Theta}$ over the whole set of interest:

$$J_j(\mathbf{q}) = \int_{\mathcal{W}} \hat{d}_{j,\Theta}(\mathbf{q}, \mathbf{p}, \rho_j) \tag{4.10}$$

Looking at the whole magnitudes of interest set Ξ, a possible evaluation how a given generalized trajectory $\mathbf{q}(t)t \in \Theta$ cover the set of interest W, can be doune using the following functional:

$$J(\mathbf{q}) = \sum_j J_j(\mathbf{q}) \tag{4.11}$$

According to equation (4.4) the dependence of functional J from sensor state $\mathbf{z}$ and the input functions $\mathbf{u}$ can be explicated:

$$J(\mathbf{z}, \mathbf{u}) = \sum_j \int_{\mathcal{W}} \hat{d}_{j,\Theta}(C\mathbf{z}, \mathbf{p}, \rho_j) = \sum_j \int_{\mathcal{W}} \hat{d}_{j,\Theta}(C\phi(\mathbf{z}(0), \mathbf{u}), \mathbf{p}, \rho_j) \tag{4.12}$$

Functional J is continuous in its arguments. In general J is not convex.

4.2.4.2 Constraints

As done in section 3.1.3.2 physical limits on the actuators (for the motion) and/or on the sensors (in terms of velocity in the measure acquisition) must be considered.

$$\|\dot{\mathbf{q}}^{(i)}(t)\| \leq \mathbf{v}_{max}^{(i)}$$
$$\|\mathbf{u}^{(i)}(t)\| \leq \mathbf{u}_{max}^{(i)}$$

Looking at the whole network,

$$\|\dot{\mathbf{q}}(t)\| \leq \mathbf{v}_{max}$$
$$\|\mathbf{u}(t)\| \leq \mathbf{u}_{max}$$

where

$$\mathbf{v}_{max} = \begin{pmatrix} \mathbf{v}_{max}^{(1)} & \mathbf{v}_{max}^{(2)} & \cdots & v_{max}^{(m)} \end{pmatrix}^T$$

and

$$\mathbf{u}_{max} = \begin{pmatrix} \mathbf{u}_{max}^{(1)} & \mathbf{u}_{max}^{(2)} & \cdots & u_{max}^{(m)} \end{pmatrix}^T$$

Expressions above can be rewritten as inequality constraints, on generalized inputs and state:

$$g_1(\mathbf{z}(t)) = \|B^T\mathbf{z}(t)\| - \mathbf{v}_{max} \leq \mathbf{0} \tag{4.13}$$

$$g_2(\mathbf{u}(t)) = \|\mathbf{u}(t)\| - \mathbf{u}_{max} \leq \mathbf{0} \tag{4.14}$$

At the same manner for box constraints on sensors positions are expressed, for every sensor, by:

$$\mathbf{q}^{(i)}_{min} \leq \mathbf{q}^{(i)}(t) \leq \mathbf{q}^{(i)}_{max}$$

Looking at the whole network, constraints on the generalized configuration have the following structure:

$$\mathbf{q}_{min} \leq \mathbf{q}(t) \leq \mathbf{q}_{max}$$

where

$$\mathbf{q}_{max} = \begin{pmatrix} \mathbf{q}^{(1)}_{max} & \mathbf{q}^{(2)}_{max} & \cdots & q^{(m)}_{max} \end{pmatrix}^T$$

and

$$\mathbf{q}_{min} = \begin{pmatrix} \mathbf{q}^{(1)}_{min} & \mathbf{q}^{(2)}_{min} & \cdots & q^{(m)}_{min} \end{pmatrix}^T$$

Also this constraints can be expressed as inequality constraints on the sensor network state.

$$g_3(\mathbf{z}(t)) = C\mathbf{z}(t) - \mathbf{z}_{max} \leq \mathbf{0} \tag{4.15}$$

$$g_4(\mathbf{z}(t)) = \mathbf{z}_{min} - C\mathbf{z}(t) \leq 0 \tag{4.16}$$

Moreover equality constraints on starting and/or final sate of the sensor network can be introduced in order to instroduce box constraints on sensors positions as in or to impose closed trajectories.

$$\chi(\mathbf{z}(0), \mathbf{z}(t_f)) = 0 \tag{4.17}$$

Motion Coordination While using multiple sensors it became necessary to coordinate motion in order to avoid conflicts and to guarantee sensor network performances.

One of the basic requirements can be avoiding collisions between moving sensors. So, distance between sensors must be lower bounded. For every couple of sensors the following distance constraints must be introduced

$$\|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\| \geq \rho_B$$

Those distance constraints can be rewritten as inequality constraints on the sensor network state:

$$g_5^{(i,j)}(\mathbf{z}(t)) = \rho_B - \|C(\mathbf{z}^{(i)} - \mathbf{z}^{(j)})\| \leq 0 \tag{4.18}$$

Several requirements can be done on the structure of the communication network. For example, it is possible to impose a fixed network topology. That can be useful, for example, to fix the level of redundancy on the communication link an than to reach node fault tolerance. To maintain a fixed network topology every sensor must maintain direct communication with a subset of its starting neighbors that is fixed in time. Indicating with $\mathcal{G}_d =< V_{\mathcal{G}}, E_{\mathcal{G}_d}(t) >$ the graph that represents desired topology, where $E_{\mathcal{G}_d}(t) \subseteq E_{\mathcal{G}}(t)\ \forall t \in \Theta$.

According to section 4.2.3, for every edge of $\mathcal{G}_d$ a distance constrain between a couple of sensors must be introduced, so maintaining a desired topology $\mathcal{G}_d$ means to satisfy the following constrains set $\forall t \in \Theta$:

$$\|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\| \leq \rho_C \quad \forall\, (i,j) \in E_{\mathcal{G}_d}(t) \tag{4.19}$$

An other important requirement on the communication network is connectivity. Communication network connectivity is necessary for data exchange and transmission, but also for sensor localization, coordination and commands communication. Fixed topology maintenance is, obviously, a particular case of connectivity maintenance if the desired topology is connected. Anyway, this approach introduces strong constrains on sensors motion. These constraints can be relaxed if only connectivity is needed, allowing network topology to change over time.
As said before, the communication model introduced in section 4.2.3 makes the communication graph $\mathcal{G}(t)$ to be undirected. A undirected graph is connected if and only if it contain a spanning tree. So it is possible to maintain network connectivity constraining every sensor just to maintain direct communication links that corresponds to the edges of a spanning tree of the communication tree.

Assigning a weight at every edge of $E_{\mathcal{G}}$ is possible to define the *Minimum Spanning Tree* of $\mathcal{G}$ as the spanning tree with minimum weight (Figure 4.1). In particular being $\mathcal{G}$ an Euclidean graph it come natural to define the edges weights as:

$$w(i,j) = \|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\|$$

in this case the minimum spanning tree is said *Euclidean* (EMST). The EMST can be easily and efficiently computed by standard algorithms (such as Prim's algorithm or Kruskal's algorithm). Indicating the EMST with $\mathcal{T}(t) =< V_{\mathcal{T}}, E_{\mathcal{T}}(t) >$, where$V_{\mathcal{T}} =$

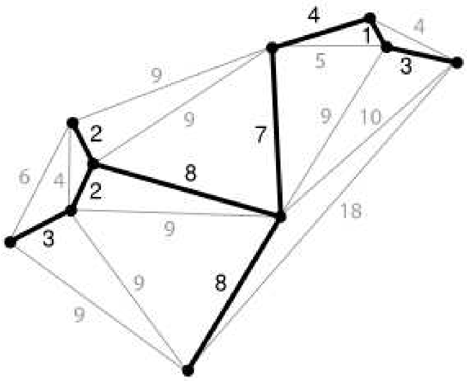


Figure 4.1: Minimum Spanning Tree for a planar weighted undirected graph

$V_{\mathcal{G}}$ and $E_{\mathcal{T}}(t) \subseteq E_{\mathcal{G}}(t)$, maintaining the communication network connectivity means satisfying the following constrains $\forall t \in \Theta$:

$$\|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\| \leq \rho_C \quad \forall\, (i,j) \in E_{\mathcal{T}}(t) \tag{4.20}$$

The minimum spanning tree of the communication network graph changes while sensors moves, so the neighbors set of every node change over time making the network topology dynamic.

Both fixed topology and connectivity maintenance can be rewritten as inequality constraints on the sensor network state

$$g_6^{(i,j)}(\mathbf{z}(t)) = \|C(\mathbf{z}^{(i)} - \mathbf{z}^{(j)})\| - \rho_C \leq 0 \tag{4.21}$$

4.2.4.3 Optimal Control Problem

Summarizing, the optimal coverage problem be formulated as an optimal control problem.

$$\min_{(\mathbf{z},\mathbf{u}) \in \mathcal{D}} J(\mathbf{z}, \mathbf{u}) \tag{4.22}$$

where the admissible set $\mathcal{D}$ is defined as:

$$\begin{aligned}\mathcal{D} = \{(\mathbf{z},\mathbf{u}) \in \mathcal{C}^1 : &\chi(\mathbf{z}(t_f),\mathbf{z}(t_f)) = 0\\ &h(\mathbf{z},\dot{\mathbf{z}},\mathbf{u}) = A\mathbf{z} + B\mathbf{u} - \dot{\mathbf{z}} = 0\\ &g_1(\mathbf{z}(t)) = \|B^T\mathbf{z}(t)\| - \mathbf{v}_{max} \leq \mathbf{0}\\ &g_2(\mathbf{u}(t)) = \|\mathbf{u}(t)\| - \mathbf{u}_{max} \leq \mathbf{0}\\ &g_3(\mathbf{z}(t)) = C\mathbf{z}(t) - \mathbf{z}_{max} \leq \mathbf{0}\\ &g_4(\mathbf{z}(t)) = \mathbf{z}_{min} - C\mathbf{z}(t) \leq 0\\ &g_5^{(i,j)}(\mathbf{z}(t)) = \rho_B - \|C(\mathbf{z}^{(i)} - \mathbf{z}^{(j)})\| \leq 0\\ &g_6^{(i,j)}(\mathbf{z}(t)) = \|C(\mathbf{z}^{(i)} - \mathbf{z}^{(j)})\| - \rho_C \leq 0\}\end{aligned}$$

The non convexity of the cost functional makes this problem not convex, so, several *suboptimal* solutions may exist.
Anyway, in general, it is very hard to evaluate analytically one of this solutions. For this reason in the next section discretization is introduced that allow to make the problem tractable.

4.3 Discrete Time Formulation

4.3.1 Sensors Discretized Dynamics

The discrete time motion model of the generic $i-th$ sensor is well described by the following equations:

$$\begin{aligned}\mathbf{z}^{(i)}((n+1)T_s) &= A_d^{(i)}\mathbf{z}^{(i)}(nT_s) + B_d^{(i)}\mathbf{u}^{(i)}(nT_s)\\ \mathbf{q}^{(i)}(nT_s) &= C^{(i)}\mathbf{z}^{(i)}(nT_s)\end{aligned}$$

Looking at the whole network generalized dynamics can be written as:

$$\begin{aligned}\mathbf{z}((n+1)T_s) &= A_d\mathbf{z}(nT_s) + B_d\mathbf{u}(nT_s)\\ \mathbf{q}(nT_s) &= C\mathbf{z}(nT_s)\end{aligned}$$

where:

$$A_d = \begin{pmatrix} A_d^{(1)} & 0 & \dots & 0 \\ 0 & A_d^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & A_d^{(m)} \end{pmatrix} = e^{AT_s}$$

$$B_d = \begin{pmatrix} B_d^{(1)} & 0 & \dots & 0 \\ 0 & B_d^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & B_d^{(m)} \end{pmatrix} = \int_0^{T_s} e^{A\tau} B d\tau$$

According with the introduced dynamic model it is possible to write generalized state and output values at time nT_s as:

$$\mathbf{z}(nT_s) = {A_d}^n \mathbf{z}(0) + \sum_{k=0}^{n-1} {A_d}^k B_d \mathbf{u}((n-1)T_s - kT_s) \tag{4.23}$$

and

$$\mathbf{q}(nT_s) = C{A_d}^n \mathbf{z}(0) + C \sum_{k=0}^{n-1} {A_d}^k B_d \mathbf{u}((n-1)T_s - kT_s) \tag{4.24}$$

Representing the generalized sensor network input sequence from time $t = 0$ to time $t_f = (n_f - 1)T_s$ as:

$$\mathbf{u}_{n_f} = \begin{pmatrix} \mathbf{u}(0) \\ \mathbf{u}(T_s) \\ \vdots \\ \mathbf{u}((n_f - 1)T_s) \end{pmatrix}$$

and defining the following vectors

$$\mathbf{v}_{n_f} = \begin{pmatrix} \mathbf{z}(0) \\ \mathbf{u}_{n_f} \end{pmatrix} \qquad H(n) = \begin{pmatrix} A_d^n & A_d^{n-1} B_d & \dots & B_d & 0 & \dots & 0 \end{pmatrix}$$

it is possible to rewrite equations 4.23 and 4.24 in the following compact form:

$$\mathbf{z}(nT_s) = H(n)\mathbf{v}_{n_f} \tag{4.25}$$

and

$$\mathbf{q}(nT_s) = CH(n)\mathbf{v}_{n_f} \tag{4.26}$$

Let's represent generalized state and output sequences with the following vectors:

$$\mathbf{z}_{n_f} = \begin{pmatrix} \mathbf{z}(0) \\ \mathbf{z}(T_s) \\ \vdots \\ \mathbf{z}(n_f T_s) \end{pmatrix} \qquad \mathbf{q}_{n_f} = \begin{pmatrix} \mathbf{q}(0) \\ \mathbf{q}(T_s) \\ \vdots \\ \mathbf{q}(n_f T_s) \end{pmatrix}$$

According with 4.25 and 4.26, vectors $\mathbf{q}_{n_f}$ and $\mathbf{z}_{n_f}$ can be written as linear functions of vector $\mathbf{v}_{n_f}$:

$$\mathbf{z}_{n_f} = H_{n_f} \mathbf{v}_{n_f} = \tag{4.27}$$

and

$$\mathbf{q}_{n_f} = C_{n_f} H_{n_f} \mathbf{v}_{n_f} \tag{4.28}$$

where

$$H_{n_f} = \begin{pmatrix} H(0) \\ H(1) \\ \vdots \\ H(n_f) \end{pmatrix} \qquad C_{n_f} = \begin{pmatrix} C & 0 & \dots & 0 \\ 0 & C & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C \end{pmatrix}$$

4.3.2 Sensing Model

As done for a single sensor the workspace is divideded into square cells c_k identified by thei centroids $\mathbf{p}_k$. The generic $i - th$ sensor is assumed measure magnitude ξ_j on cell c_k at time nT_s if its distance between its position $\mathbf{q}^{(i)}(nT_s)$ and the cell centroid il less than $\rho_j^{(i)}$. The i-th sensor field of measure, with respect to magnitude ξ_j, at time nT_s is then the union of the cells that have their centroids within a sphere of center $\mathbf{q}^{(i)}(nT_s)$ and radius $\rho_j^{(i)}$:

$$M_j^{(i)}(\mathbf{q}^{(i)}(nT_s)) = \{\bigcup_k c_k : \|\mathbf{q}^{(i)}(nT_s) - \mathbf{p}_k\| \leq \rho_j^{(i)}\} \tag{4.29}$$

4.3.3 Coverage Problem Formulation

4.3.3.1 Objective Function

With respect to every magnitude of interest ξ_j it is possible to define the distance between a cell c_k and a generalized sensor network discrete trajectory $\mathbf{q}$ as

$$d_{j,\Theta}(\mathbf{q}_{n_f}, \mathbf{p}_k) = \min_{n \in [0, n_f];\ i : \xi_j \in \Xi^{(i)}} \|\mathbf{q}(nT_s) - \mathbf{p}_k\| \tag{4.30}$$

Considering discrete time dynamic, $d_{j,\Theta}$ become a scalar function of the generalized sensor network configurations sequence. This function is again, in general, not convex. As done for the continuous time formulation let define:

$$\hat{d}_{j,\Theta}(\mathbf{q}_{n_f}, \mathbf{p}, \rho_j) = \text{pos}\left(d_{j,\Theta}(\mathbf{q}_{n_f}, \mathbf{p}) - \rho_j\right) \tag{4.31}$$

A measure of coverage performance achieved by a discrete time generalized trajectory $\mathbf{q}$, with respect to magnitude ξ_j, can be computed summing function $\hat{d}_{j,\Theta}$ over all the cells:

$$J_j(\mathbf{q}_{n_f}) = \sum_k \hat{d}_{j,\Theta}(\mathbf{q}_{n_f}, \mathbf{p}_k, \rho_j) \tag{4.32}$$

Looking at the whole magnitudes of interest set Ξ, a possible evaluation how a given generalized trajectory $\mathbf{q}(t) t \in \Theta$ cover the set of interest W, can be doune using the following function:

$$J(\mathbf{q}_{n_f}) = \sum_j J_j(\mathbf{q}_{n_f}) \tag{4.33}$$

According with equation (4.28) the dependence of function J from vector $\mathbf{v}_{n_f}$ can be explicated:

$$J(\mathbf{v}_{n_f}) = \sum_j \sum_k \hat{d}_{j,\Theta}(\mathbf{q}_{n_f}, \mathbf{p}_k, \rho_j) = \sum_j \sum_k \hat{d}_{j,\Theta}(C_{n_f} H_{n_f} \mathbf{v}_{n_f}, \mathbf{p}_k, \rho_j) \tag{4.34}$$

Function J is again continuous in its arguments, but, in general, not convex.

4.3.3.2 Nonlinear Programming Problem

After discretization the optimal coverage problem can be formulated as a nonlinear programming problem. The objective is to find, into an admissible set $\mathcal{D}$, the vector $\mathbf{v}^*_{n_f}$ that minimize the cost function:

$$\mathbf{v}^*_{n_f} = \min_{v_{n_f}} J(\mathbf{v}_{n_f})$$
$$\mathbf{v}_{n_f} \in \mathcal{D}$$

where the admissible set $\mathcal{D}$ is defined as:

$$
\begin{aligned}
\mathcal{D} = \{\mathbf{v}_{n_f} \in \mathbb{R}^{m(4+2n_f)} : & \chi(\mathbf{z}(0), \mathbf{z}(n_f T_s)) = 0 \\
& g_{1,0}(\mathbf{z}(0)) = \|B^T \mathbf{z}(0)\| - \mathbf{v}_{max} \leq \mathbf{0} \\
& \vdots \\
& g_{1,n_f}(\mathbf{z}(n_f T_s)) = \|B^T \mathbf{z}(n_f T_s)\| - \mathbf{v}_{max} \leq \mathbf{0} \\
& g_{2,0}(\mathbf{u}(0)) = \|\mathbf{u}(0)\| - \mathbf{u}_{max} \leq \mathbf{0} \\
& \vdots \\
& g_{2,n_f}(\mathbf{u}(n_f T_s)) = \|\mathbf{u}(n_f T_s)\| - \mathbf{u}_{max} \leq \mathbf{0} \\
& g_{3,0}(\mathbf{z}(0)) = C\mathbf{z}(0) - \mathbf{z}_{max} \leq \mathbf{0} \\
& \vdots \\
& g_{3,n_f}(\mathbf{z}(n_f T_s)) = C\mathbf{z}(n_f T_s) - \mathbf{z}_{max} \leq \mathbf{0} \\
& g_{0,4}(\mathbf{z}(0)) = \mathbf{z}_{min} - C\mathbf{z}(0) \leq 0 \\
& \vdots \\
& g_{4,n_f}(\mathbf{z}(n_f T_s)) = \mathbf{z}_{min} - C\mathbf{z}(n_f T_s) \leq 0 \\
& g_{5,0}^{(i,j)}(\mathbf{z}(0)) = \rho_B - \|C(\mathbf{z}^{(i)}(0) - \mathbf{z}^{(j)}(0))\| \leq 0 \\
& \vdots \\
& g_{5,n_f}^{(i,j)}(\mathbf{z}(n_f T_s)) = \rho_B - \|C(\mathbf{z}^{(i)}(n_f T_s) - \mathbf{z}^{(j)}(n_f T_s))\| \leq 0 \\
& g_{6,0}^{(i,j)}(\mathbf{z}(0)) = \|C(\mathbf{z}^{(i)}(0) - \mathbf{z}^{(j)}(0))\| - \rho_C \leq 0\} \\
& \vdots \\
& g_{6,n_f}^{(i,j)}(\mathbf{z}(n_f T_s)) = \|C(\mathbf{z}^{(i)}(n_f T_s) - \mathbf{z}^{(j)}(n_f T_s))\| - \rho_C \leq 0\}
\end{aligned}
$$

The non convexity of the cost functional makes this problem not convex, so, several *suboptimal* solutions may exist.
In the simulations performed, the SQP (Sequential Quadratic Programming) (Appendix A) method has been applied to find some of those solutions.

4.4 Simulations

In this section simulation results for different cases are displayed to show the effectiveness and the generality of the proposed method. The first case concerns the coverage of a box shaped workspace, within a time interval $\Theta = 15$ *sec*, with a sensor network, with three homogeneous nodes. A single magnitude in then considered. The following values are assumed for sensors parameters:

$$u_{max} = 1.5 \quad v_{max} = 1.5 \quad \rho_B = 0.5$$

Sensors range is assumed to be unitary. Communication between two nodes is assumed to be reliable within a maximum range of

$$\rho_c = 5.5$$

Both collisions avoidance and connectivity maintenance constraints are considered. The mobile sensors trajectories are displayed in subfigure 4.2(d). On the background the coverage status of the set of interest is displayed, darker zones represent the uncovered cells. In subfigures 4.2(a) and 4.2(b) evolutions of controls and speeds are respectively displayed. In subfigure 4.2(c) evolutions of the relative distances between sensors are plotted. As is visible, collisions are avoided.

In the second simulation an heterogeneous sensor network covering a box shaped workspace within a time interval $\Theta = 15$ *sec* is considered. Three magnitudes of interest are defined,

$$\Xi = \{\xi_1, \xi_2, \xi_3\}$$

Sensors can measure the three measure within the following radia, so $\forall i$

$$\rho_{\xi_1}^{(i)} = 2 \quad \rho_{\xi_2}^{(i)} = 1 \quad \rho_{\xi_3}^{(i)} = 3$$

The following values are assumed for sensors parameters:

$$u_{max} = 1.5 \quad v_{max} = 1.5 \quad \rho_B = 0.5$$

Communication between two nodes is assumed to be reliable within a maximum range of

$$\rho_c = 5.5$$

The sensor network is composed by 4 nodes, with different sensing capabilities.

$$\Xi_1 = \{\xi_1, \xi_2\} \quad \Xi_2 = \{\xi_2, \xi_3\}$$

$$\Xi_3 = \{\xi_1, \xi_2\} \quad \Xi_4 = \{\xi_2, \xi_3\}$$

Both collisions avoidance and connectivity maintenance constraints are considered. Results are displayed in figure 4.3. Subfigures 4.3(d),4.3(e) and 4.3(f) show dynamic coverage with respect to each magnitude of interest. So, for each ξ_j trajectories of sensors able to measure it ($\mathbf{q}^{(i)} \,:\, \xi_j \in \Xi^{(i)}$) are plotted. In subfigure 4.3(g) the whole magnitude set is considered. The coverage status of the set of interest is displayed on the background. Coverage with respect to the magnitude set is represented by a color code displayed in the side color bar.

In the third simulation a similar scenario but with a generic shaped workspace is considered. Results are displayed in figure 4.4

In the fourth and last simulation the case of node fault robustness requirement is considered. A three nodes sensor network measuring a single magnitude on a box shaped workspace is considered. The following values are assumed for sensors parameters:

$$u_{max} = 1.5 \quad v_{max} = 1.5 \quad \rho_B = 0.5$$

Sensors range is assumed to be unitary. Results are displayed in figure 4.5 Subfigures 4.5(d),4.5(e) and 4.5(f) show the case of fault of one of the three sensors

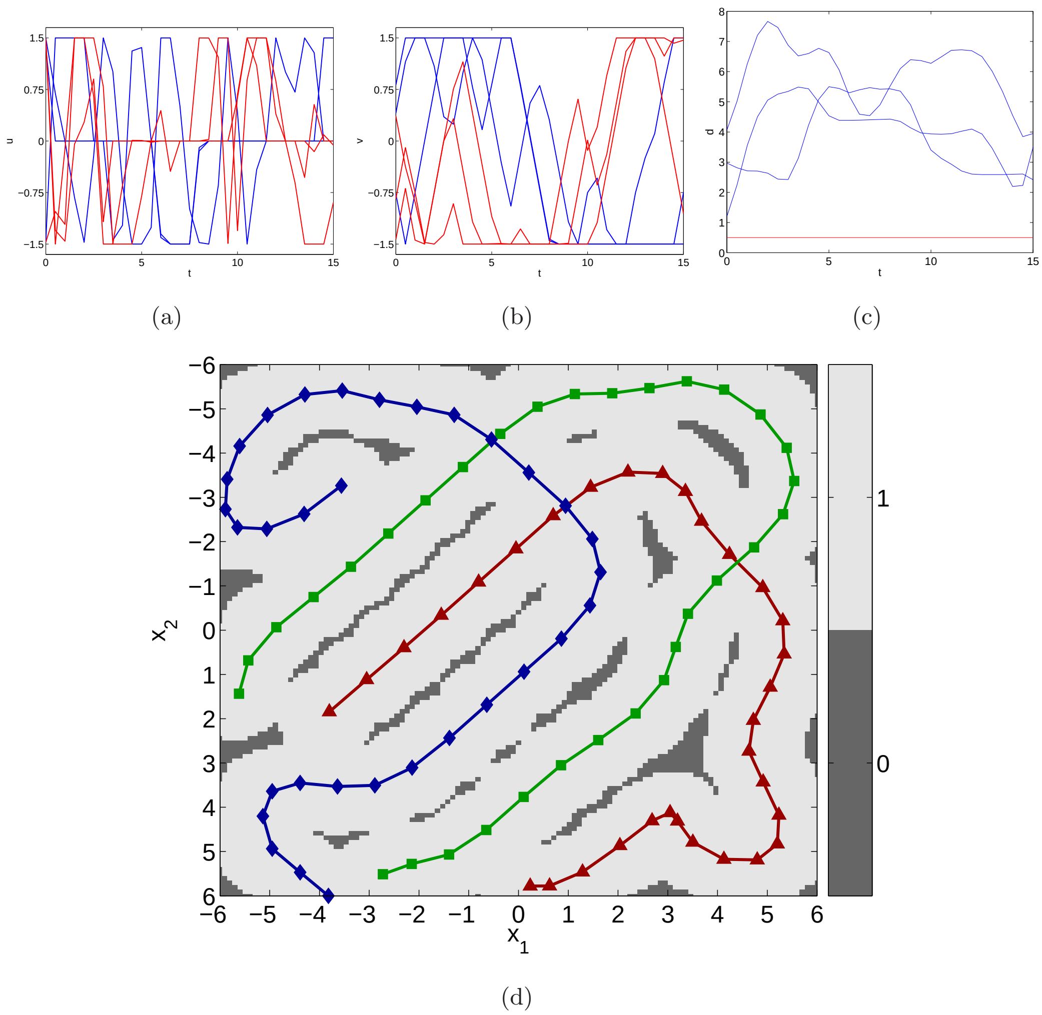
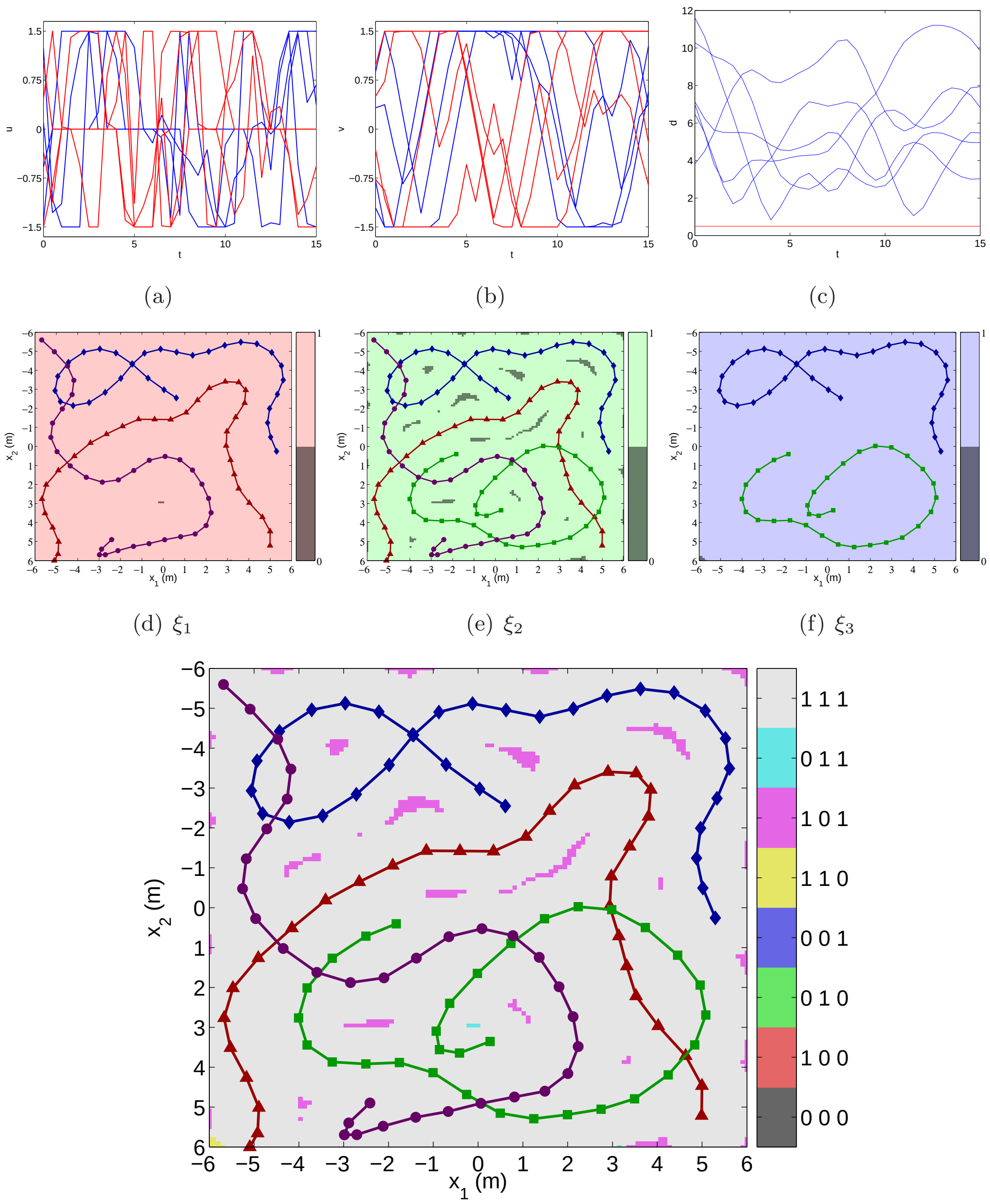


Figure 4.2: Coverage of a box shaped workspace with a dynamic sensor network with three homogeneous nodes. (a) Control components evolution. (b) Relative distances between all vehicles, the red line represents minimum distance for collisions avoidance (rho_B). (c) Sensors trajectories and coverage status of the set of interest.

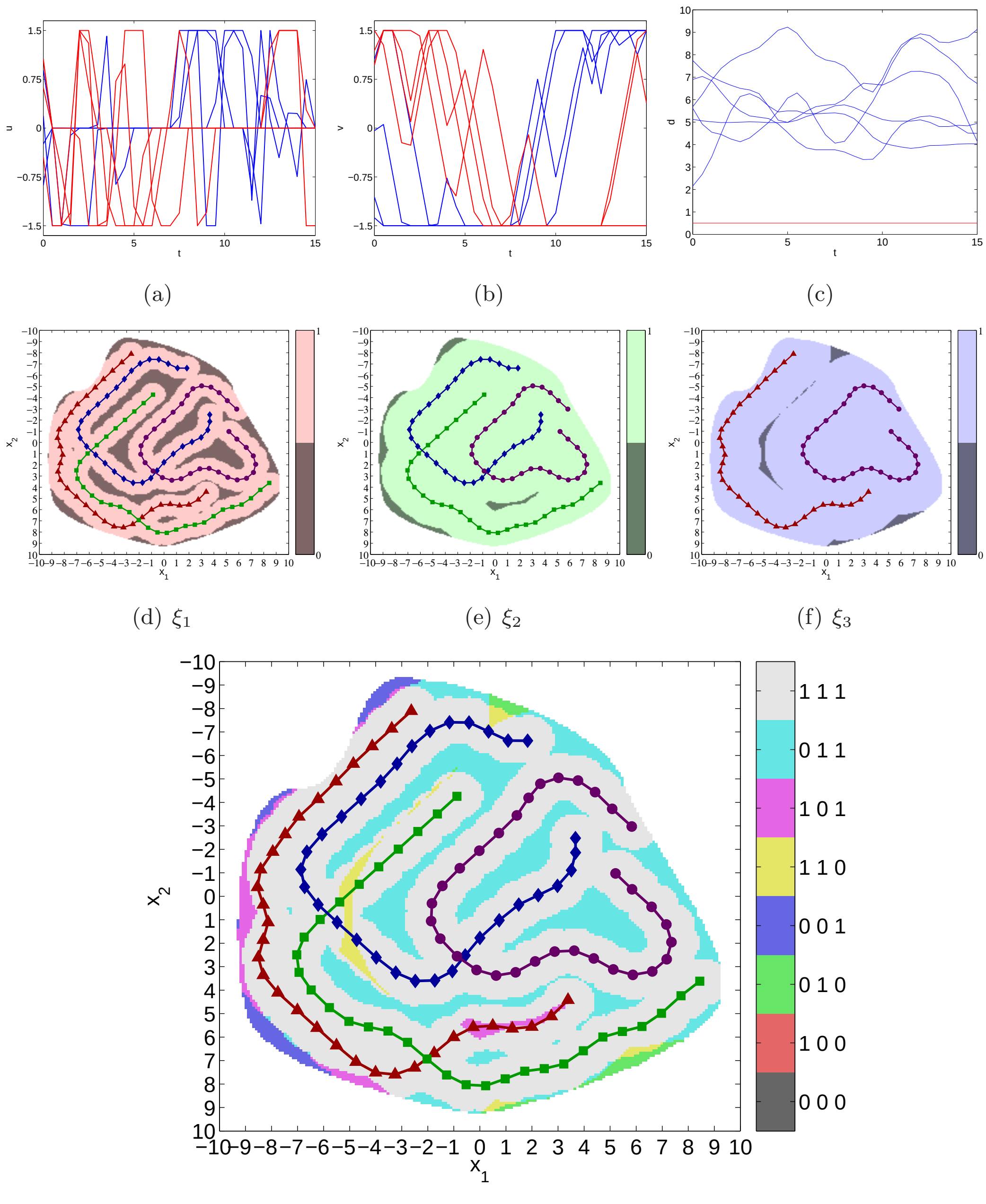


(a) (b) (c)

(d) ξ_1 (e) ξ_2 (f) ξ_3

(g) ξ_1 ξ_2 ξ_3

Figure 4.3: Coverage of a box shaped workspace with an heterogeneous dynamic sensors network. (a) Control components evolutions. (b) Relative distances between all vehicles, the red line represents minimum distance for collisions avoidance (rho_B). (c)-(d)-(e) Dynamic coverage w.r.t. each magnitude. (f) Nodes trajectories and coverage status w.r.t the whole magnitudes set

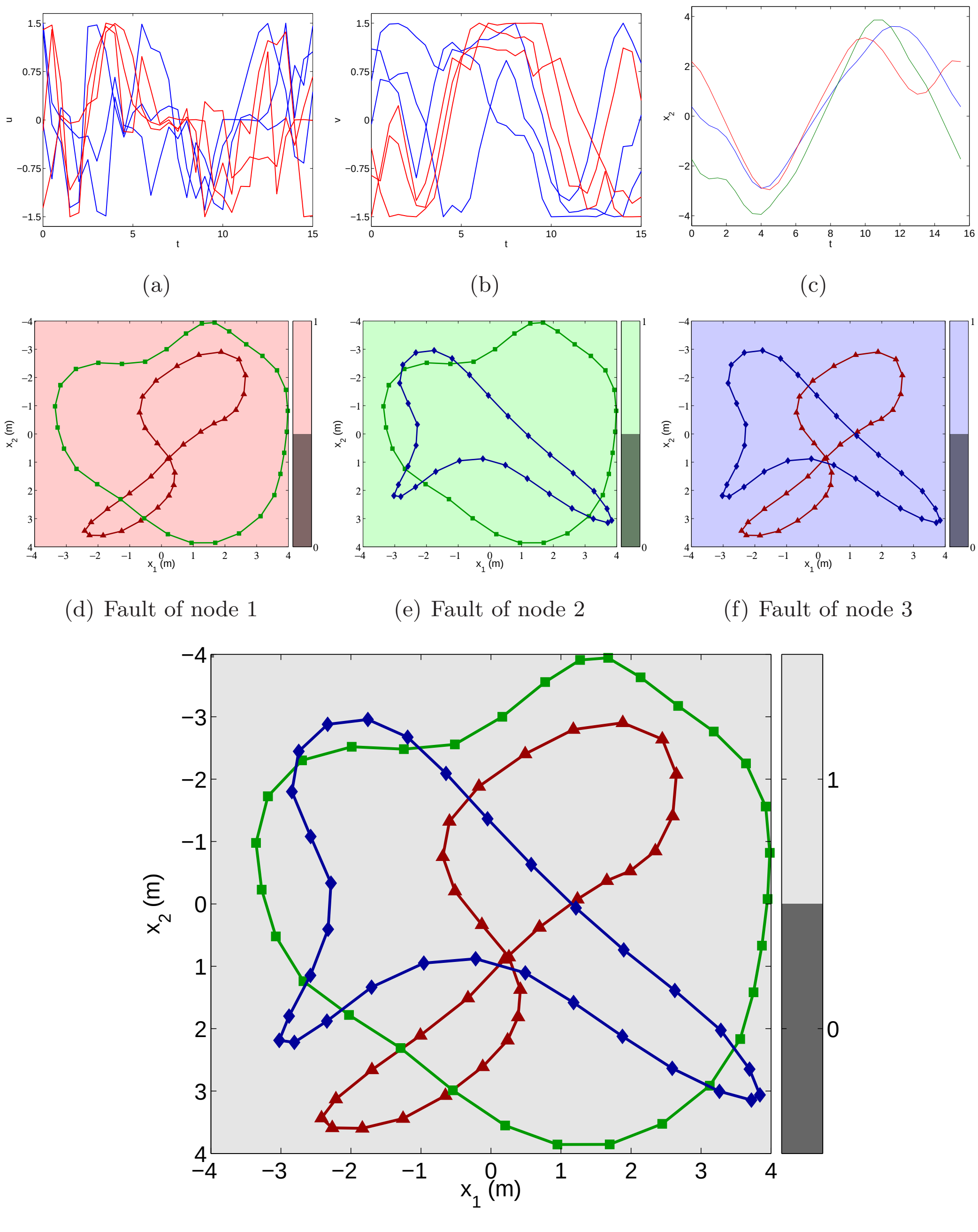


(a) (b) (c)

(d) ξ_1 (e) ξ_2 (f) ξ_3

(g) ξ_1 ξ_2 ξ_3

Figure 4.4: Coverage of a generic shaped workspace with an heterogeneous dynamic sensors network. (a) Control components evolutions. (b) Relative distances between all vehicles, the red line represents minimum distance for collisions avoidance (rho_B). (c)-(d)-(e) Dynamic coverage w.r.t. each magnitude. (f) Nodes trajectories and coverage status w.r.t the whole magnitudes set



(a) (b) (c)

(d) Fault of node 1 (e) Fault of node 2 (f) Fault of node 3

(g) No faults

Figure 4.5: Robust area surveillance with a mobile sensors network

Part II

Distributed Dynamic Coverage

CHAPTER 5

DISTRIBUTED COVERAGE CONTROL

THE second part of the Thesis concerns approaches to dynamic coverage that are suitable for online applications, such as, approaches based on the development of feedback laws. At every time step the sensor network must evaluate how to move according to its configuration, such as the positions of its nodes, and to the coverage status of the set of interest (sensors will move toward uncovered areas).

In Burgard *et al.* (2005) the problem of choosing appropriate target points for a team of mobile robots so that they simultaneously explore different regions of the environment is considered. A centralized approach for the coordination of multiple robots, which takes into account the cost of reaching a target point and its utility, is proposed. Whenever a target point is assigned to a specific robot, the utility of the unexplored area visible from this target position is reduced. In this way, different target locations are assigned to the individual robots. Global knowledge of both positions of all the robots and of the coverage status of the workspace, represented by an occupancy grid, is assumed.

In Franchi *et al.* (2007) a randomized approach to multirobot exploration is considered. Each robot of the team explore the workspace autonomously. When two robots meet,

such as they are so close to communicate, they exchange all informations about their past configurations. In that every robot can take into account the presence of the others and plan its motion toward areas which appear to be unexplored by itself as well as the rest of the team.
Dynamic coverage by mobile sensor networks is considered also in Hussein & Stipanovic (2006). Agents move in order to increase the area covered with time until every point in the given area has been covered with a prescribed coverage level termed *effective coverage*. The agents try to collectively minimize an error function that indicate how well the area has been covered up to time t by moving along the negative gradient of that function. However, this gradient-based approach might possibly drive the agents to a local minimum of the error function where they get stuck. A strategy for escaping those trapping situations is proposed.
The same author, taking inspiration from the study of active sensing performed in Grocholsky (2002), presented in Hussein (2007) a control strategy based on the discrete Kalman filter. Here, the problem of estimating a spatially-decoupled scalar field using a network of finite-range sensor vehicles is considered. The approach relies on using the Kalman filter to estimate the field and, on the filter's prediction step to plan the vehicles next move to maximize the quality of the field estimate. A strategy for avoiding converging to local minima of the coverage cost is proposed.
A common feature of the cited works is the fact that global information on at least the coverage status of the workspace are required. Communication is, in general used to exchange those informations.
There exist particular cases in which these global informations are not necessary as shown in Pavone & Frazzoli (2007). Here coverage of a circular and obstacle-free environment is achieved using a control policy that is static (i.e., memoryless), decentralized, and does not rely on coverage maps or other global informations.

In this Chapter an artificial potential based control law, that guarantees to drive a mobile sensor network to totally cover the set of interest is proposed. Very weak assumptions are needed on the sensing model, so increasing the applicability of the proposed solution.
The proposed approach is suitable for distributed implementation. In a distributed

control architecture each sensor evaluates its inputs with only locally available informations. For networked agents, the concept of locally available informations must be related not only to space but also to time in the sense that also those informations, that are intrinsically not local in the space, can flow trough the communication network becoming available to all the sensors under a certain time delay, once that the connectivity of the communication network is assumed and assured. For this reason, in the next Chapter, particular attention will be given to communication connectivity maintenance.

In section 5.1 a simplified motion model for mobile sensors is defined. In section 5.2 a general sensing model is defined, it is then shown how it can it include well known coverage models. The coverage control law is described in section 5.3 that is shown to drive the sensor network to totally cover the set of interest. Simulations results are presented in 5.4.

5.1 Sensors Motion Model

Sensors are modeled, from the dynamic point of view, as material points moving on on the Euclidean plane ($Q = \mathbb{R}^2$). Planar motion is considered only for sake of simplicity, all results can be immediately extended to the 3D motion case. Every sensor is assumed to satisfy the following simple discrete-time kinematic equation of motion:

$$\mathbf{q}^{(i)}((n+1)T_S) = \mathbf{q}^{(i)}(nT_S) - T_S\mathbf{u}^{(i)}(nT_S) \tag{5.1}$$

with

$$i = 1, \ldots, m$$

where $\mathbf{q}^{(i)}$ indicate the position of sensor i. The control velocity of sensor i is denoted with $u^{(i)}$, it is assumed to be bounded

$$\|\mathbf{u}^{(i)}\| \leq u_{max} \tag{5.2}$$

Bounds on control velocity represent the actuators limits. For simplicity they are assumed to be equals for all the sensors, in general different bounds ($u_{max}^{(i)}$) can be considered to describe the case of sensors equipped with different actuators. Looking at the whole network a generalized configuration and generalized input can be defined as:

$$\mathbf{q} = \begin{pmatrix} \mathbf{q}^{(1)} \\ \vdots \\ \mathbf{q}^{(m)} \end{pmatrix} \qquad \mathbf{u} = \begin{pmatrix} \mathbf{u}^{(1)} \\ \vdots \\ \mathbf{u}^{(m)} \end{pmatrix}$$

5.2 Sensing Model

Let's indicate with $\mathcal{W} \subset \mathbb{R}^2$ the *set of interest*, that must be covered with sensors measures and that is discretized into a set of cells. Every cell c_k is identified by its centroid $\mathbf{p}_k$

Proximity measure model is considered, so every mobile sensor is assumed to take measures within a circular set of radius ρ around its current position $\mathbf{q}^{(i)}$. Such a set under sensor *visibility* will be denoted as

$$M^{(i)}(n) = \{\mathbf{p}_k \ : \|\mathbf{q}^{(i)}(nT_S) - \mathbf{p}_k\| \leq \rho\} \tag{5.3}$$

The measure set of the whole network is given by the union of the measure sets of every sensor

$$M(n) = \bigcup_i M^{(i)}(n) \tag{5.4}$$

5.2.1 Residual Information

On every cell centroid $\mathbf{p}_k$ a variable $\gamma_k(n)$ is defined that indicate the *residual information* contained into the cell c_k at time nT_S. Measurement on cell c_k are modeled as reductions of the corresponding residual information γ_k.

In order to more precisely define the residual information γ_k, let's do the following assumptions:

1. $\gamma_k(n)$ is positive and not decreasing in time

$$\gamma_k(n) \in \mathbb{R}^+ \qquad \gamma_k(n+1) - \gamma_k(n) \leq 0$$

2. If $\mathbf{p}_k$ is in the visibility set of the sensor network, $\gamma_k(n)$ decrease and became zero within a finite number N of time steps.

$$\gamma_k(n) > 0 \wedge \mathbf{p}_k \in M(\mathbf{q}(n)) \rightarrow \gamma_k(n+1) - \gamma_k(n) < 0$$

$$\mathbf{p}_k \in M(\mathbf{q}(n+\nu)) \ \nu = 0, \ldots, N \rightarrow \gamma_k(n+N) = 0$$

As said γ_k represents the residual information contained in cell c_k and then it is a measure of how much cell c_k is *uncovered.* A measure of the residual information of the whole set of interest is, then, given by:

$$\Gamma(n) = \sum_k \gamma_k(n)$$

Full coverage is achieved if and only if $\Gamma = 0$.
Coverage definition can be done choosing opportunely γ_k and its evolution law. In the following paragraphs two examples are proposed for well known coverage definitions.

Effective Coverage The definition of effective coverage was introduced in Hussein & Stipanovic (2006). For every sensor an instantaneous coverage function $A_i(\mathbf{q}_i, \mathbf{p}_k)$ is defined, with the following properties:

$$A_i(\mathbf{q}_i, \mathbf{p}_k) \begin{cases} > 0 \text{ if } \mathbf{p}_k \in M^{(i)} \\ = 0 \text{ otherwise} \end{cases}$$

The *effective coverage* achieved by sensor σ_i on point $\mathbf{p}_k$ is given by:

$$\mathcal{T}_i(\mathbf{p}_k, n) = \sum_{\nu=1,2,\ldots,n} A_i(\mathbf{q}_i(\nu), \mathbf{p}_k)$$

For this definition of coverage γ_k assumes following structure

$$\gamma_k(n) = pos(\mathcal{C}_p^* - \sum_i \mathcal{T}_i(\mathbf{p}_k, n))$$

where C_p^* is a positive constant and

$$pos(\xi) = \begin{cases} \xi \text{ if } \xi \geq 0 \\ 0 \text{ if } \xi < 0 \end{cases} \tag{5.5}$$

A particular case of effective coverage is K-coverage in which it is required that every point is measured K at least times. In this case:

$$A_i(\mathbf{q}_i, \mathbf{p}_k) = \begin{cases} 1 \text{ if } \mathbf{p}_k \in M^{(i)} \\ 0 \text{ otherwise} \end{cases}$$

$$\gamma_k(n) = pos\left(K - \sum_i \mathcal{T}_i(\mathbf{p}_k, n)\right)$$

Is easy to see that that assumptions on γ_k are verified.

Stochastic Coverage Let's assume that the sensor network goal is to estimate a spatially decoupled scalar field x defined on $\mathcal{W}$. The field is assumed to be static but measurements are corrupted by noise. On every point $\mathbf{p}_k$ the value of $x_k = x(\mathbf{p}_k)$ can be view as a random variable. The network is assumed to estimate the value of such variables using a Kalman filter. The evolution model is described by the following equation

$$x_k(n+1) = x_k(n) \tag{5.6}$$

Every sensor that make measures on $x_k(n)$ with the following observation model:

$$z_k^{(i)}(n) = H_k^{(i)}(n) * x_k(n) + v_k^{(i)}(n) \tag{5.7}$$

where $v_k^{(i)}(n)$ is a zero mean Gaussian observation noise with variance $R_k^{(i)}(n) = E[{v_k^{(i)}}^2(n)]$, and

$$H_k^{(i)}(n) = \begin{cases} 1 \text{ if } \mathbf{p}_k \in M^{(i)}(n) \\ 0 \text{ otherwise} \end{cases}$$

The whole network observation model have the following structure:

$$\mathbf{z}_k(n) = \mathbf{H}_k(n) * x_k(n) + \mathbf{v}_k(n) \tag{5.8}$$

At every time step the estimation of $x_k(n)$ is updated following the well known law:

$$\hat{x}_k(n+1) = \hat{x}_k(n) + \mathbf{K}_k(n)\left(\mathbf{z}_k(n) - \mathbf{H}_k(n) * \hat{x}_k(n)\right)$$

Where $\mathbf{K}_k(n)$ is the Kalman gain at time nT_S.
The covariance of the estimation assumes the following expression:

$$P_k(n+1) = (1 - \mathbf{K}_k(n)\mathbf{H}_k(n))P_k(n)$$

Estimation covariance decrease asymptotically to 0. Defining a maximum acceptable value P_{max} for estimation covariance the residual information γ_k can be defined as:

$$\gamma_k(n) = pos(P_k(n) - P_{max})$$

Is easy to see that that assumptions on γ_k are verified.

5.2.1.1 Distributed Computation

It is important to remark how the knowledge of the residual information evolution allow each sensor to take into account the behavior of the others and then, to implicitly cooperate with them. Anyway, in order to evaluate $\gamma_k(n)$ it is necessary, in general to know the evolution of the generalized configuration of the whole sensor network from time 0 to time nT_S. In a distributed control architecture the assumption that every sensor knows the full network configuration $\mathbf{q}(nT_S)$ at time nT_S can not be used.
A more realistic case is the one in which every sensor can evaluate the sensor network configuration $\mathbf{q}((n - \delta_i)T_S)$with a certain delay δ_i. The delay depends from the communication network topology that, necessarily, must be always connected.
Every sensor can, then, evaluate, with only locally available informations, a *delayed* residual information:

$$\hat{\gamma}_k^{(i)}(nT_S) = \gamma_k((n - \delta_i)T_S)$$

Obviously, $\hat{\gamma}_k^{(i)}$ satisfies all the assumptions done in 5.2.1. Moreover it is easy to see that $\hat{\gamma}_k^{(i)}$ overestimates γ_k, so:

$$\hat{\Gamma}^{(i)} = 0 \Longrightarrow \Gamma = 0$$

where $\hat{\Gamma}^{(i)} = \sum_k \hat{\gamma}_k^{(i)}$.
Once the delayed residual information is zero, full coverage is achieved.

5.3 Coverage Control

According with the considered proximity based sensor model, every dynamic sensor is assumed to be subject, for every cell c_k, to a virtual potential that depends from $\|\mathbf{q}^{(i)} - \mathbf{p}_k\|$, other than, as is reasonable from the residual information γ_k.
Anyway, knowledge of exact residual information it is not required. So, in the rest

of this section we will consider an overestimating approximation ,as, for example the *delayed* residual information defined in 5.2.1.1.

The coverage potential function, that can be evaluated with only local communication, has the following structure:

$$f_k^{(i)} = \hat{\gamma}_k^{(i)} pos \left(\frac{1}{\beta} - \frac{1}{\|\mathbf{q}^{(i)} - \mathbf{p}_k\|^{\alpha}} \right) \tag{5.9}$$

where $\alpha > 1$, and $\beta \in \mathbb{R}^+$ is positive constant, necessary to avoid singularity points and small as needed.

Considering the whole workspace every dynamic node is subject to the following potential:

$$f^{(i)} = \sum_k f_k^{(i)}(\mathbf{q}^{(i)}) \tag{5.10}$$

The control strategy proposed, as in the classical potential based approaches, is to drive sensors in the direction of the negative gradient of $f^{(i)}$.

$$\mathbf{u}_{cov}^{(i)} = -\frac{\partial f^{(i)}}{\partial \mathbf{q}^{(i)}} \tag{5.11}$$

where

$$\frac{\partial f^{(i)}}{\partial \mathbf{q}^{(i)}} = \begin{cases} \sum_k \hat{\gamma}_k^{(i)}(n) \frac{\mathbf{q}^{(i)} - \mathbf{p}_k}{\|\mathbf{q}^{(i)} - \mathbf{p}\|^{\alpha+1}} \text{ if } \|\mathbf{q}^{(i)} - \mathbf{p}_k\| \geq \beta \\ \\ 0 \text{ otherwise} \end{cases} \tag{5.12}$$

The motion model of the generic i-th sensor can then be written as

$$\mathbf{q}^{(i)}((n+1)T_S) = \mathbf{q}^{(i)}(nT_S) - T_S \left. \frac{\partial f^{(i)}}{\partial \mathbf{q}^{(i)}} \right|_{nT_S} \tag{5.13}$$

A global potential function for the sensor network can then be defined as:

$$f = \sum_i f^{(i)} \tag{5.14}$$

Function f is positive and vanish only when the whole workspace is *covered*, such as, when $\hat{\Gamma} = 0$.

The gradient of f respect to the generalized configuration $\mathbf{q}$ is given by

$$\frac{\partial f}{\partial \mathbf{q}} = \begin{pmatrix} \frac{\partial f^{(1)}}{\partial \mathbf{q}^{(1)}} \\ \frac{\partial f^{(2)}}{\partial \mathbf{q}^{(2)}} \\ \vdots \\ \frac{\partial f^{(m)}}{\partial \mathbf{q}^{(m)}} \end{pmatrix} \tag{5.15}$$

the coverage control law can then be written for the whole network as:

$$\mathbf{q}((n+1)T_S) = \mathbf{q}(nT_S) - K\frac{\partial f}{\partial \mathbf{q}}T_S \tag{5.16}$$

In order to show that, under the proposed control strategy, the sensor network is driven to totally cover the set of interest, it is sufficient to show that:

$$\Delta f(nT_S) = f((n+1)T_S) - f(nT_S) < 0$$

According with 5.14 and 5.10 the variation of function f within a time step can be written as:

$$\begin{aligned}\Delta f(nT_S) &= f((n+1)T_S) - f(nT_S)\\ &= \sum_i \sum_k \Delta f_k^{(i)}(nT_S)\\ &= \sum_i \sum_k [f_k^{(i)}((n+1)T_S) - f_k^{(i)}(nT_S)]\end{aligned}$$

As shown in 5.9 functions $f_k^{(i)}$ can be written as:

$$f_k^{(i)}(nT_S) = f_k^{(i)}(\hat{\gamma}_k^{(i)}(n), \mathbf{q}^{(i)}(nT_S))$$

Then, according with the well known expression of Taylor series for functions of several variables, it is possible to write the increment of function $f_k^{(i)}$ as:

$$\begin{aligned}\Delta f_k^{(i)}(nT_S) = &\left.\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}}\right|_{nT_S} \Delta\hat{\gamma}_k^{(i)}(n) +\\ &\left.\frac{\partial f_k^{(i)}}{\partial \mathbf{q}^{(i)}}^T\right|_{nT_S} \Delta\mathbf{q}^{(i)}(nT_S) + o({T_S}^2)\end{aligned}$$

From equation 5.13 follows:

$$\begin{aligned}\Delta f_k^{(i)}(nT_S) = &\left.\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}}\right|_{nT_S} \Delta\hat{\gamma}_k^{(i)}(n) +\\ &\left.\frac{\partial f_k^{(i)}}{\partial \mathbf{q}^{(i)}}^T\right|_{T_S} \left.\frac{\partial f^{(i)}}{\partial \mathbf{q}^{(i)}}\right|_{nT_S} + o({T_S}^2)\end{aligned}$$

According with the expression above the variation of function f can be rewritten as:

$$\Delta f(nT_S) = \sum_i \sum_k \left.\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}}\right|_{nT_S} \Delta\hat{\gamma}_k^{(i)}(n) - \sum_i \left[\sum_k \left.\frac{\partial f_k^{(i)}}{\partial \mathbf{q}^{(i)}}\right|_{nT_S} \right]^T \left.\frac{\partial f^{(i)}}{\partial \mathbf{q}^{(i)}}\right|_{nT_S} + o({T_S}^2)$$

From equation 5.10 follows:

$$\Delta f(nT_S) = \sum_i \sum_k \left.\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}}\right|_{nT_S} \Delta\hat{\gamma}_k^{(i)}(n) - \sum_i \left.\left\| \frac{\partial f^{(i)}}{\partial \mathbf{q}^{(i)}} \right\|^2\right|_{nT_S} + o({T_S}^2)$$

From equation 5.14 follows:

$$\Delta f(nT_S) = \sum_i \sum_k \left.\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}}\right|_{nT_S} \Delta\hat{\gamma}_k^{(i)}(n) - \left.\left\| \frac{\partial f}{\partial \mathbf{q}} \right\|^2\right|_{nT_S} + o({T_S}^2)$$

From the equations above follows that:

$$\Delta f(nT_S) < 0 \Longleftrightarrow \sum_i \sum_k \left.\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}}\right|_{nT_S} \Delta\hat{\gamma}_k^{(i)}(n) - \left.\left\| \frac{\partial f}{\partial \mathbf{q}} \right\|^2\right|_{nT_S} < 0$$

Observing that, from equation 5.9,

$$\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}} = pos\left(\frac{1}{\beta} - \frac{1}{\|\mathbf{q}^{(i)} - \mathbf{p}_k\|^\alpha}\right) \geq 0$$

and, as assumed in 5.2.1,

$$\Delta\hat{\gamma}_k^{(i)}(n) = \hat{\gamma}_k^{(i)}(n+1) - \hat{\gamma}_k^{(i)}(n) \leq 0$$

it is possible to see that:

$$\sum_i \sum_k \left.\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}}\right|_{nT_S} \Delta\hat{\gamma}_k^{(i)}(n) + \left.\left\| \frac{\partial f}{\partial \mathbf{q}} \right\|^2 \right|_{nT_S} \leq 0$$

Then, in order to exclude the possibility of local minima, it is necessary to study stationary points, such as points in wich:

$$\sum_i \sum_k \left.\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}}\right|_{nT_S} \Delta\hat{\gamma}_k^{(i)}(n) + \left.\left\| \frac{\partial f}{\partial \mathbf{q}} \right\|^2 \right|_{nT_S} = 0$$

Obviously the condition above is verified when the whole workspace is covered and then $f = 0$. Excluding the trivial case mentioned above, we can say that, to be verified it is necessary that :

$$\frac{\partial f}{\partial \mathbf{q}} = \mathbf{0}$$

Lemma 5.3.1. *Points* $\mathbf{q}_e$: $\left.\frac{\partial f}{\partial \mathbf{q}}\right|_{\mathbf{q}_e} = \mathbf{0}$ *are not local minima points of* $f(\mathbf{q})$*.*

Proof. Let us consider a stationary point $\mathbf{q}_e$: $\left.\frac{\partial f}{\partial \mathbf{q}}\right|_{\mathbf{q}_e} = \mathbf{0}$ and consider a perturbation $\mathbf{\Delta q}$ such that $\mathbf{\Delta q} = \mathbf{q} - \mathbf{q}_e$. Assuming, without loss of generality that $\hat{\gamma}_k^{(i)}$ is constant, the value of $f(\mathbf{q})$ for $\mathbf{q} = \mathbf{q}_e$ can be written as

$$f(\mathbf{q}_e) = f(\mathbf{q}) + \frac{\partial f}{\partial \mathbf{q}}^T (\mathbf{q} - \mathbf{q}_e) + o(\|\mathbf{q} - \mathbf{q_e}\|^2)$$

Introducing the following notation

$$\mathbf{r}_k = \mathbf{q} - \mathbf{p}_k \qquad \mathbf{r}_{e,k} = \mathbf{q}_e - \mathbf{p}_k$$

the variation of value of f from $\mathbf{q}_e$ to $\mathbf{q}$ can be written as:

$$
\begin{aligned}
\Delta f &= f(\mathbf{q}_e) - f(\mathbf{q}) \\
&= -\frac{\partial f}{\partial \mathbf{q}}^T \mathbf{\Delta q} + o(\|\mathbf{\Delta q}\|^2) \\
&= -\sum_i \sum_k \hat{\gamma}_k^{(i)} \frac{\mathbf{r}_k^{(i)T} \mathbf{\Delta q}^{(i)}}{\|\mathbf{r}_k^{(i)}\|^{\alpha+1}} + o(\|\mathbf{\Delta q}\|^2) \\
&= -\sum_i \sum_k \frac{\hat{\gamma}_k^{(i)} \left(\mathbf{r}_{k,e}^{(i)} + \mathbf{\Delta q}^{(\mathbf{i})}\right)^T \mathbf{\Delta q}^{(\mathbf{i})}}{[(\mathbf{r}_{k,e}^{(i)} + \mathbf{\Delta q}^{(\mathbf{i})})^T (\mathbf{r}_{k,e}^{(i)} + \mathbf{\Delta q}^{(\mathbf{i})})]^{\frac{\alpha+1}{2}}} \\
&\quad + o(\|\mathbf{\Delta q}\|^2) \\
&= -\sum_i \sum_k \frac{\hat{\gamma}_k^{(i)} \left(\mathbf{r}_{k,e}^{(i)T} \mathbf{\Delta q}^{(\mathbf{i})} + \|\mathbf{\Delta q}^{(\mathbf{i})}\|^2\right)}{\left[\|\mathbf{r}_{k,e}^{(i)}\|^2 + \|\mathbf{\Delta q}^{(\mathbf{i})}\|^2 + 2\mathbf{r}_{k,e}^{(i)T} \mathbf{\Delta q}^{(\mathbf{i})}\right]^{\frac{\alpha+1}{2}}} \\
&\quad + o(\|\mathbf{\Delta q}\|^2)
\end{aligned}
$$

if $\|\mathbf{\Delta q}^{(\mathbf{i})}\|$ is sufficiently small

$$
\begin{aligned}
&\approx -\sum_i \sum_k \hat{\gamma}_k^{(i)} \frac{\mathbf{r}_{k,e}^{(i)T} \mathbf{\Delta q}^{(\mathbf{i})}}{\left[\|\mathbf{r}_{k,e}^{(i)}\|^2 + 2\mathbf{r}_{k,e}^{(i)T} \mathbf{\Delta q}^{(\mathbf{i})}\right]^{\frac{\alpha+1}{2}}} \\
&\geq -\sum_i \sum_k \hat{\gamma}_k^{(i)} \frac{\mathbf{r}_{k,e}^{(i)T} \mathbf{\Delta q}^{(\mathbf{i})}}{\|\mathbf{r}_{k,e}^{(i)}\|^{\alpha+1}} \\
&= \left.\frac{\partial f}{\partial \mathbf{q}}\right|_{\mathbf{q}_e}^T \mathbf{\Delta q} = 0
\end{aligned}
$$

So there exist points $\mathbf{q}$ in a neighborhood of $\mathbf{q}_e$ such that $f(\mathbf{q}_e) \geq f(\mathbf{q})$ and then points $\mathbf{q}_e$ cannot be local minima of f. □

To avoid stationary situations, and achieve monotonic decreasing of f and then guaranteed full coverage of the set of interest, it is sufficient to perturb sensors configurations. It can be done, for example, adding a random signal $\mathbf{r}(nT_S)$ to the control input.

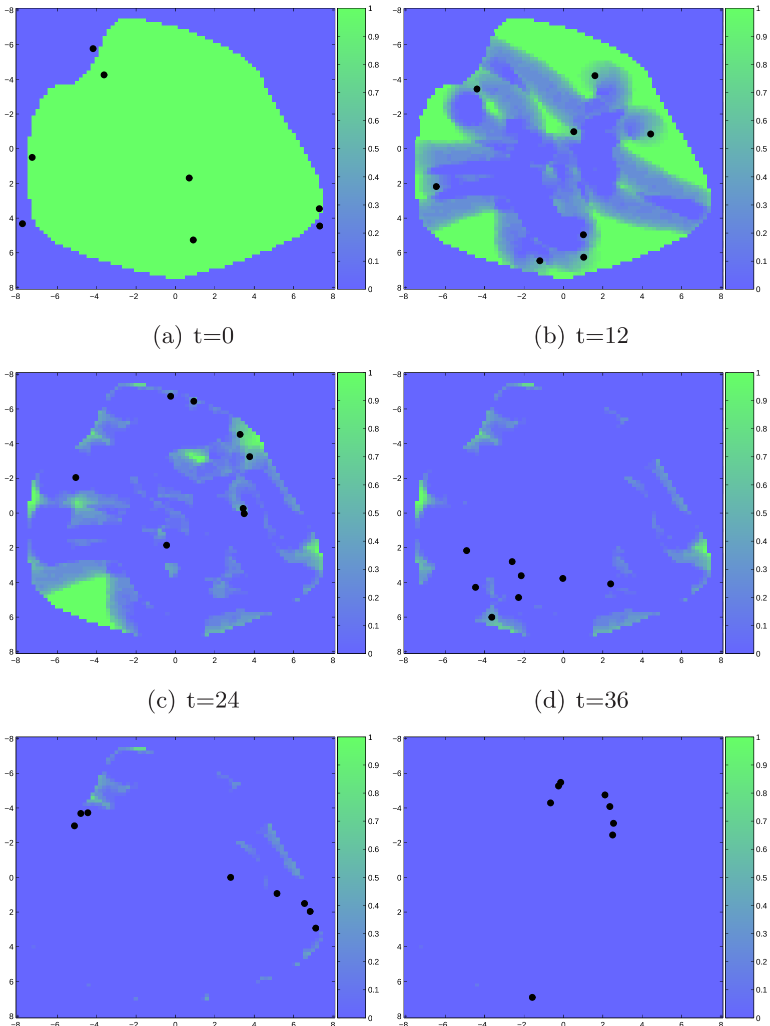
$$
\mathbf{q}((n+1)T_S) = \mathbf{q}(nT_S) - \nabla f_{\mathbf{q}} T_S + \mathbf{r}(nT_S) \tag{5.17}
$$

5.4 Simulations

In this section simulations are discussed in order to put in evidence the effectiveness of the proposed methodology. The sensor network is assumed to be composed of eight nodes having unitary sensing radius ($\rho_S = 1$) and unitary bounded inputs ($u_{max} = 1$). Moreover nodes are assumed to share the residual information map γ_k $\forall k$. As said in 5.2.1.1 this hypothesis is very strong, anyway, the aim of this section is only to show how the proposed control law drives sensors to totally cover the set of interest. As said in 5.3, that does not depend from the particular choice of γ_k.
The global knowledge assumption will be relaxed in the next Chapter where, once that communication connectivity will be assured by motion coordination, an approximation of γ_k, that can be computed in distributed way, will be used.
In the first simulation K-coverage is considered with $K = 15$, so the residual information γ_k is defined as in 5.2.1. In figure 5.1 the evolution of the sensor network configuration is displayed together with the residual information map.
Figure 5.2 shows the evolution of the percentage residual information.

In the second simulation stochastic coverage is considered. The sensor network is asked to estimate the static scalar field x displayed in figure 5.3(a).

In figure 5.4 the evolution of the sensor network configuration is displayed together with the estimations of the residual information and of the field x.
Figure 5.3(b) shows the evolution of the percentage residual information.



(a) t=0 (b) t=12

(c) t=24 (d) t=36

(e) t=48 (f) t=60

Figure 5.1: K-Coverage. Evolution of the sensor network configuration and residual information map (K=15).

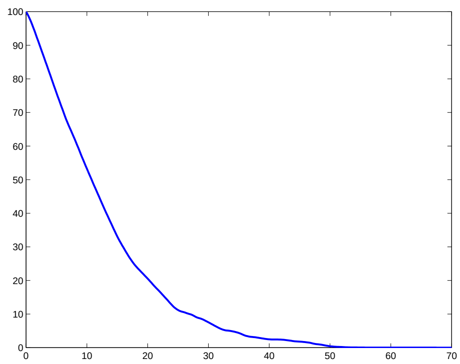
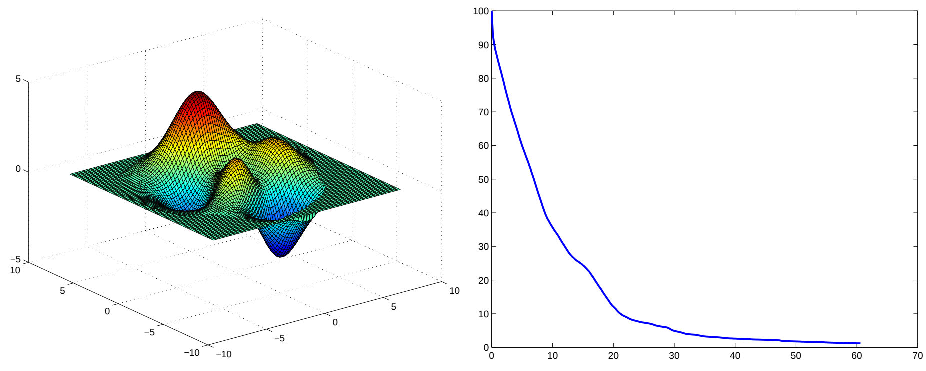


Figure 5.2: K-Coverage. Evolution of the percentage residual information of the set of interest.



(a) (b)

Figure 5.3: Stochastic Coverage. Static scalar field x to be estimated. Evolution of the percentage residual information of the set of interest.

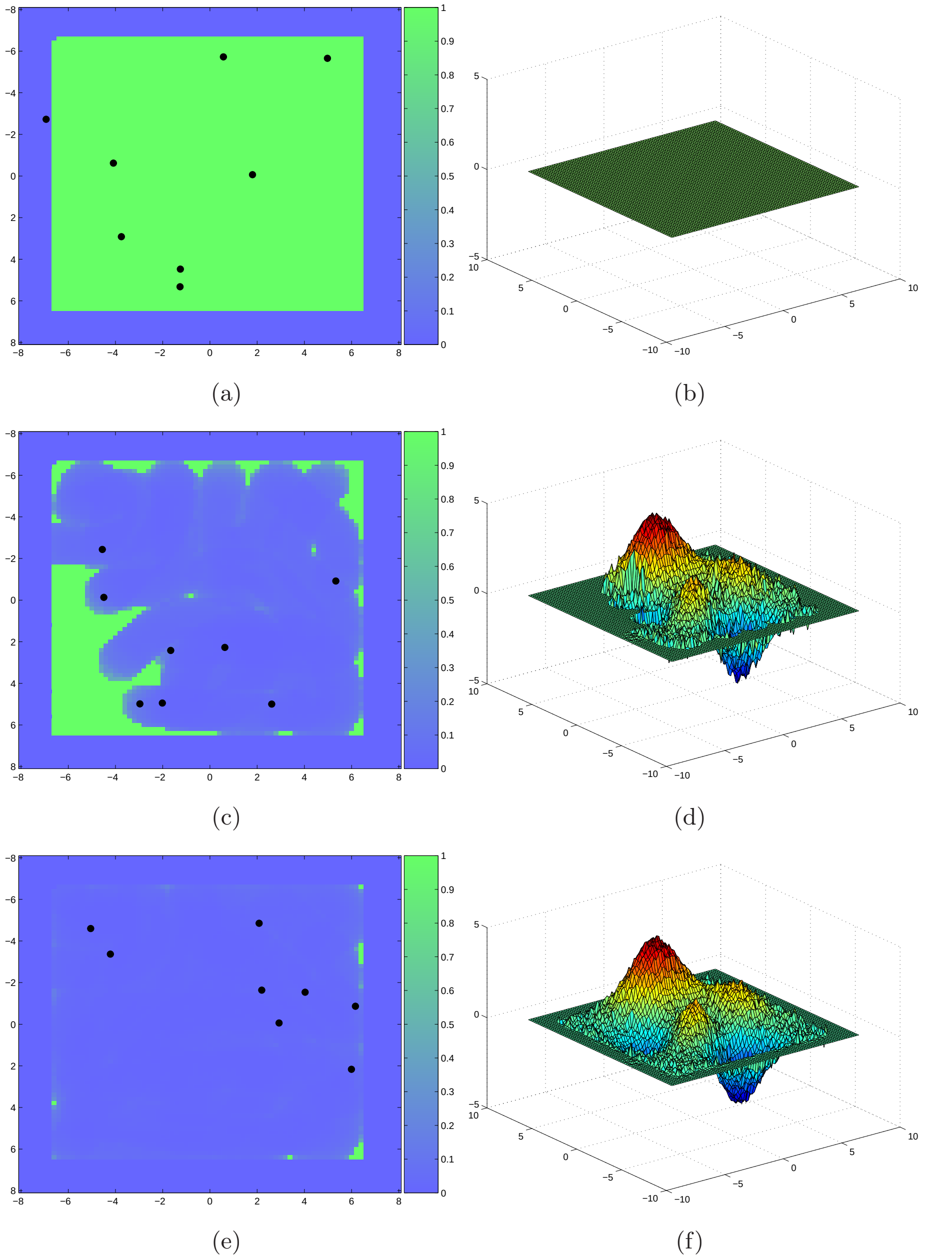


Figure 5.4: Stochastic Coverage. Evolution of the sensor network configuration, estimation of the residual information map (left column), and estimation of the scalar field x (right column).

CHAPTER 6

DISTRIBUTED MOTION COORDINATION

In this chapter motion coordination is considered. As previously said the usage of multiple mobile sensors introduce challenging coordination problems. In Chapter 4 coordination was implemented introducing constraints to the optimal coverage problem. In a distributed control architecture, as the one proposed in the second part of this thesis, coordination must be achieved using online feedback.

Distributed motion coordination is a popular topic for multi-agent systems.

In Leonard & Fiorelli (2001) a framework for coordinated and distributed control of multiple autonomous vehicles using artificial potentials and virtual leaders is introduced. Artificial potentials define interaction control forces between neighboring vehicles and are designed to enforce a desired inter-vehicle spacing. A virtual leader is a moving reference point that influences vehicles in its neighborhood by means of additional artificial potentials. Virtual leaders can be used to manipulate group geometry and direct the motion of the group. The approach provides a construction for a Lyapunov function to prove closed-loop stability using the system kinetic energy and the artificial potential energy. Dissipative control terms are included to achieve asymptotic stability. This approach is then applied in gradient climbing tasks for mobile sensor networks in Ogren *et al.* (2004).

In Olfati-Saber & Murray (2002a,b) distributed stabilization of formations of multiple vehicles using structural potential functions is addressed. The key idea in formation stabilization is using natural potential functions obtained from structural constraints of a desired formation in a way that leads to a collision-free, distributed, and bounded state feedback law for each vehicle obtained from the desired formations graphs.
In these approaches the *interaction topology* is maintained fixed.
In Jadbabaie *et al.* (2003), starting from the study done in Vicsek *et al.* (1995) on the motion of interacting particles, an approach to motion coordination of autonomous vehicles based on the nearest neighbor rules is proposed.Possible changes in nearest neighbors over time and then in interaction topology are explicitly taken into account.
In Olfati-Saber (2006) a deep study of the problem of flocking of multi-agent dynamic systems is presented. Here, a systematic method for construction of cost functions (or collective potentials) for flocking is provided. It is shown that migration of flocks can be performed using a peer-to-peer network of agents, i.e. *flocks need no leaders*. A *universal* definition of flocking for particle systems with similarities to Lyapunov stability is given.
A survey on collective motion problems and algorithms for sensor networks is presented in Ganguli *et al.* (2005).
Artificial potential based strategies for motion coordination of mobile sensor networks while dynamically covering a given field of interest are proposed in Hussein & Stipanovic (2007a), in order to guarantee collisions avoidance, and in Hussein & Stipanovic (2007b) in order to guarantee flocking behavior.

In this Chapter a motion coordination strategy is proposed in order to assure some desired features of the sensor network. Very weak assumptions are needed on the constraint model, so increasing the applicability of the proposed solution. Particular attention is given to collisions avoidance and communication connectivity maintenance. In particular a distributed approach to connectivity maintenance is presented. Every sensors evaluate constraints on its position using, only, information on its neighbors. To do that a subgraph of the communication network graph is computed in distributed way at every time. This graph is showed to be connected, because it contains a minimum spanning tree. Maintenance of edges of this graph entails communication network connectivity. This approach introduces less motion constrains respect to flocking or fixed topology maintenance.

6.1 Motion Constraints

While performing coverage tasks a sensor network is often asked to satisfy many kind of constraints on the generalized configuration $\mathbf{q}$, and then on sensors positions.

$$\mathbf{q} \in \Omega \subset \mathbb{R}^{2m}$$

The following assumptions are done on the functional structure of constraints:

1. Function g explode if the network configuration approaches to a not admissible configuration:
$$\forall \tilde{\mathbf{q}} \notin \Omega \quad \|\mathbf{q} - \tilde{\mathbf{q}}\| \to 0 \Longrightarrow g(\mathbf{q}) \to \infty$$

2. The sum of the partial gradients of $g(\mathbf{q})$ with respect to every sensor configuration $\mathbf{q}^{(i)}$ is null:
$$\sum_i \frac{\partial g}{\partial \mathbf{q}^{(i)}} = \mathbf{0}$$

Examples of such constraints are given in the following subsections.

6.1.1 Collisions Avoidance

In order to avoid collisions between sensors it is necessary to introduce minimum distance constraints

$$\|\mathbf{q}^{(i)}(nT_S) - \mathbf{q}^{(i)}(nT_S)\| \geq \rho_B$$

Taking inspiration from Stipanovic *et al.* (2007), for every couple of sensors the following potential function is defined:

$$g_{coll}^{(i,j)}(\mathbf{q}^{(i)}, \mathbf{q}^{(j)}) = \frac{pos(R_B - \|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\|)}{pos(\|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\| - \rho_B)} \tag{6.1}$$

where $R_B > \rho_B$ can be view as an activation radius, so the collisions avoidance potential has effect only if

$$\rho_B \leq \|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\| \leq R_B$$

If $R_B \leq \rho_C$, such as, the activation radius of avoidance potential is less than the communication radius of sensors, as is reasonable, avoidance potential can be evaluated with only local informations.
The derivative of the potential function is given by:

$$\frac{\partial g_{coll}^{(i,j)}}{\partial \mathbf{q}^{(i)}} = \frac{\rho_B - R_B}{(\|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\| - \rho_B)^2} \frac{\mathbf{q}^{(i)} - \mathbf{q}^{(j)}}{\|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\|}$$

Every sensor is subject to the following collisions avoidance potential

$$g_{coll}^{(i)}(\mathbf{q}) = \sum_{j \neq i} g_{coll}^{(i,j)}(\mathbf{q}^{(i)}, \mathbf{q}^{(j)}) \tag{6.2}$$

Looking at the whole network the collisions avoidance potential is given by:

$$g_{coll}(\mathbf{q}) = \sum_{i} g_{coll}^{(i)}(\mathbf{q}) \tag{6.3}$$

observing that

$$\frac{\partial g_{coll}^{(i,j)}}{\partial \mathbf{q}^{(i)}} = -\frac{\partial g_{coll}^{(i,j)}}{\partial \mathbf{q}^{(j)}}$$

it is easy to see that $g_{coll}(\mathbf{q})$ satisfy assumptions done in 6.1.

6.1.2 Connectivity Maintenance

The proximity model is again assumed for the communication between mobile sensors. Let's recall the definition of the communication graph:

$$\mathcal{G} =< V_{\mathcal{G}}, E_{\mathcal{G}}(\mathbf{q}) >$$

where

- $V_{\mathcal{G}} = \{1, \ldots, m\}$ represents the vertexes set.
- $E_{\mathcal{G}}(\mathbf{q}) = \{(i,j) \ : \ \|\mathbf{q}_i - \mathbf{q}_j\| \leq \rho_C\}$ represents the edges set. Edges are weighted with the Euclidean distance between nodes:

$$w(i,j) = \|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\|$$

Being $\mathcal{G}$ an undirected graph that's assumed to be connected at time $t = 0$, it is possible to maintain network connectivity just maintaining links that belong to a minimum spanning tree. For exactly evaluating the minimum spanning three the knowledge of the configuration of the whole network, and then of the positions of all the sensors, is needed. This approach can be used only in a centralized control architecture, in which a central computer, that has global informations, evaluates inputs for all the network nodes. In a distributed architecture, every sensor must evaluate constraints on its position using only locally available informations. As done for the distributed evaluation of the residual information, it would be possible to use communication in order to collect all the needed global informations. In this sense, many strategies has been proposed in past years, as for example in Awerbuch (1987); Gallagher *et al.* (1983); Garay *et al.* (1996). However, this approach is not suitable for connectivity maintenance of state dependent networks because of delays. The distributed evaluation of the minimum spanning tree, in fact, require that sensors exchange a certain number of messages that depends from the size of the network. During the information flow, the sensor network configuration must not change and that can strongly affect the motion capabilities of sensors. In order to overcome this difficulties an alternative solution is proposed.

Let's introduce some useful notations (6.1):

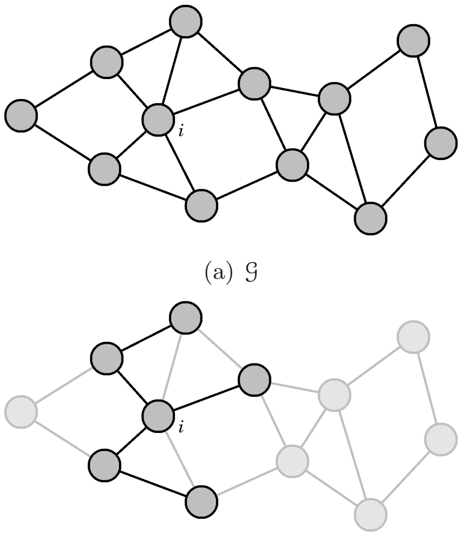
- $\mathcal{G}_i$ is the the Euclidean graph that has the node i and its neighbors as vertexes (obviously $\mathcal{G}_i \subset \mathcal{G}$) .
- $\mathcal{T}_{\mathcal{G}_i}$ is a MST of $\mathcal{G}_i$.
- $E^i_{\mathcal{G}} \subset E_{\mathcal{G}_i} \subset E_{\mathcal{G}}$ is the set of edges of $\mathcal{T}_{\mathcal{G}_i}$ connected with q_i.

The graph $\mathcal{T}_{\mathcal{G}}{}^*$ is defined as:

$$\mathcal{T}_{\mathcal{G}}{}^* =< V_{\mathcal{T}_{\mathcal{G}}{}^*}, E_{\mathcal{T}_{\mathcal{G}}{}^*} >$$

where:

- $V_{\mathcal{T}_{\mathcal{G}}{}^*} = V_{\mathcal{G}}$
- $E_{\mathcal{T}_{\mathcal{G}}{}^*} = \left\{ e_{i,j} \in E^i_{\mathcal{G}} \quad | \quad e_{i,j} \in \mathcal{T}_{\mathcal{G}_i} \right\}$



(a) $\mathcal{G}$ (b) $\mathcal{G}_i^*$

(c) $\mathcal{T}_{\mathcal{G}_i}$ (d) $E_{\mathcal{G}}^i$

Figure 6.1: Construction of the approximated MST $\mathcal{T}_{\mathcal{G}}{}^*$

It's well known that a spanning tree $\mathcal{T}$ must satisfy the following property named *cycle property*:

Property 6.1.1. *For any cycle $\mathcal{C}$ in $\mathcal{G}$, if the weight of an edge e of $\mathcal{C}$ is larger than the weights of other edges of $\mathcal{C}$, then this edge cannot belong to $\mathcal{T}$.*

Using the cycle property is possible to prove the following theorem.

Theorem 6.1.1. *$\mathcal{T}_{\mathcal{G}}{}^*$ contain a minimum spanning tree $\mathcal{T}_{\mathcal{G}}$ of $\mathcal{G}$.*

Proof. Let $(i,j) \in E_{\mathcal{G}}^i$ be an edge $\mathcal{T}_{\mathcal{G}}$ that is not contained in $\mathcal{T}_{\mathcal{G}}{}^*$.
It must exist a cycle $\mathcal{C}_i \subset \mathcal{G}_i$ such that, it is composed by (i,j) and by a subset of the edges of the local MST $\mathcal{T}_{\mathcal{G}_i}$, with the corresponding nodes.
Being $\mathcal{T}_{\mathcal{G}_i}$ an MST, for the cycle property, (i,j) must be the edge of $\mathcal{C}_i$ with largest weight. Then, because $\mathcal{T}_{\mathcal{G}}$ is an MST too, and must satisfy the cycle property, it can't contain edge (i,j). That contradict the starting assumption, so if an edge is contained in $\mathcal{T}_{\mathcal{G}}$, it must also be contained in $\mathcal{T}_{\mathcal{G}}{}^*$. □

Every sensor can then compute constraints necessary to maintain, at every time, edges of $\mathcal{T}_{\mathcal{G}}{}^*$ and then communication network connectivity, just knowing the position of its neighbors

.

$$\|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\| \leq \rho_C \qquad (i,j) \in E^i_{\mathcal{G}} \tag{6.4}$$

6.1.2.1 Potential Function

As done in 6.1.1 a potential function is defined that describe the connectivity maintenance constraint. For every constrained couple of sensors the following potential function is defined:

$$g^{(i,j)}_{conn}(\mathbf{q}^{(i)}, \mathbf{q}^{(j)}) = \frac{pos(\|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\| - R_C)}{pos(\rho_C - \| \mathbf{q}^{(i)} - \mathbf{q}^{(j)}\|)} \tag{6.5}$$

where $R_C < \rho_C$ can be view as an activation radius, so the collisions avoidance potential has effect only if

$$R_C \leq \|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\| \leq \rho_C$$

The derivative of the potential function in given by:

$$\frac{\partial g^{(i,j)}_{conn}}{\partial \mathbf{q}^{(i)}} = \frac{\rho_C - R_C}{(\rho_C - \|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\|)^2} \frac{\mathbf{q}^{(i)} - \mathbf{q}^{(j)}}{\|\mathbf{q}^{(i)} - \mathbf{q}^{(j)}\|}$$

every sensor is, then, subject to the following connectivity maintenance potential

$$g^{(i)}_{conn}(\mathbf{q}) = \sum_{j\,:\,(i,j)\in E^i_{\mathcal{G}}} g^{(i,j)}_{conn}(\mathbf{q}^{(i)}, \mathbf{q}^{(j)}) \tag{6.6}$$

Looking at the whole network the connectivity maintenance potential is given by:

$$g_{conn}(\mathbf{q}) = \sum_i g^{(i)}_{conn}(\mathbf{q}) \tag{6.7}$$

observing that, because communication links to be maintained are undirected,

$$\frac{\partial g^{(i,j)}_{conn}}{\partial \mathbf{q}^{(i)}} = -\frac{\partial g^{(i,j)}_{conn}}{\partial \mathbf{q}^{(j)}}$$

it is easy to see that $g_{conn}(\mathbf{q})$ satisfy assumptions done in 6.1.

6.2 Constrained Coverage Control

Motion coordination potentials defined int the previous section can be combined with the dynamic coverage strategy proposed in Chapter 5. To do that a potential function that take into account both coverage and constraints is defined. For each network node this new potential function assume the following structure

$$F(\mathbf{q}) = f(\mathbf{q}) + g(\mathbf{q}) \tag{6.8}$$

where f represents the coverage potential function defined in 5.3. The control strategy proposed in order to both achieve coverage and satisfy constraints is, again, to drive sensors in the direction of the negative gradient of the potential function F.

$$\mathbf{q}((n+1)T_S) = \mathbf{q}(nT_S) - \nabla F_{\mathbf{q}} T_S$$

The control input for every sensor is then given by:

$$\mathbf{u}^{(i)} = -\frac{\partial F}{\partial \mathbf{q}^{(i)}} = -\nabla f^{(i)}_{\mathbf{q}^{(i)}} - \frac{\partial g}{\partial \mathbf{q}^{(i)}} \tag{6.9}$$

as done in 5.3, to show that, under the proposed control strategy, the sensor network is driven to totally cover the set of interest, it is sufficient to show that:

$$\Delta F(nT_S) = F((n+1)T_S) - F(nT_S) < 0$$

Moreover if the condition above holds and if at time 0 the sensor network configuration $\mathbf{q}(0)$ is admissible, for the assumptions made on the g function structure, constraints will be satisfied for every time.

$$\Delta F(nT_S) = \sum_i \sum_k \left.\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}}\right|_{T_S} \Delta \hat{\gamma}_k^{(i)}(n) +$$

$$\sum_i \left[\left.\left(\frac{\partial f^{(i)}}{\partial \mathbf{q}^{(i)}} + \frac{\partial g}{\partial \mathbf{q}^{(i)}} \right)\right|_{T_S} \right]^T \Delta \mathbf{q}^{(i)}(nT_S) + o({T_S}^2)$$

from equation 6.9 follows

$$= \sum_i \sum_k \left.\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}}\right|_{T_S} \Delta \hat{\gamma}_k^{(i)}(n) -$$

$$\sum_i \left\| \left.\left(\frac{\partial f^{(i)}}{\partial \mathbf{q}^{(i)}} + \frac{\partial g}{\partial \mathbf{q}^{(i)}} \right)\right|_{T_S} \right\|^2 + o({T_S}^2)$$

From equations above it follows that

$$\Delta F(nT_S) < 0 \Longleftrightarrow \sum_i \sum_k \left.\frac{\partial f_k^{(i)}}{\partial \hat{\gamma}_k^{(i)}}\right|_{T_S} \Delta \hat{\gamma}_k^{(i)}(n) - \left\| \frac{\partial F}{\partial \mathbf{q}} \right\|_{T_S}^2 < 0$$

It is, then, necessary to study only stationary points, such as

$$\mathbf{q}_e \; : \; \nabla F_{\mathbf{q}=\mathbf{q}_e} = \mathbf{0}$$

Lemma 6.2.1. *Points* $\mathbf{q}_e \; : \; \nabla F_{\mathbf{q}=\mathbf{q}_e} = \mathbf{0}$ *are not local minima of* $F(\mathbf{q})$.

Proof. Let's consider a stationary point $\mathbf{q}_e : \; \nabla F_{\mathbf{q}=\mathbf{q}_e} = \mathbf{0}$ and consider a perturbation $\boldsymbol{\Delta}\mathbf{q} = \mathbf{q} - \mathbf{q}_e$ such that

$$\boldsymbol{\Delta}\mathbf{q}^{(\mathbf{1})} = \boldsymbol{\Delta}\mathbf{q}^{(\mathbf{2})} = \cdots = \boldsymbol{\Delta}\mathbf{q}^{(\mathbf{m})} = \boldsymbol{\Delta}$$

The value of $F(\mathbf{q})$ for $\mathbf{q} = \mathbf{q}_e$ can be written as

$$F(\mathbf{q}_e) = F(\mathbf{q}) + \nabla F_{\mathbf{q}}^T \, (\mathbf{q} - \mathbf{q}_e) + o(\|\mathbf{q} - \mathbf{q}_\mathbf{e}\|^2)$$

Introducing the following notation

$$\mathbf{r}_k = \mathbf{q} - \mathbf{p}_k \qquad \mathbf{r}_{e,k} = \mathbf{q}_e - \mathbf{p}_k$$

the variation of value of F from $\mathbf{q}_e$ to $\mathbf{q}$ can be written as:

$$
\begin{aligned}
\Delta F &= F(\mathbf{q}_e) - F(\mathbf{q}) \\
&= -\nabla F_{\mathbf{q}}^T \boldsymbol{\Delta}\mathbf{q} + o(\|\boldsymbol{\Delta}\mathbf{q}\|^2) \\
&= -\sum_i \sum_k \hat{\gamma}_k^{(i)} \frac{\mathbf{r}_k^{(i)T} \boldsymbol{\Delta}\mathbf{q}^{(i)}}{\|\mathbf{r}_k^{(i)}\|^{\alpha+1}} \\
&\quad - \sum_i \frac{\partial g}{\partial \mathbf{q}^{(i)}}^T \boldsymbol{\Delta}\mathbf{q}^{(i)} + o(\|\boldsymbol{\Delta}\mathbf{q}\|^2) \\
&= -\sum_i \sum_k \hat{\gamma}_k^{(i)} \frac{\mathbf{r}_k^{(i)T} \boldsymbol{\Delta}\mathbf{q}^{(i)}}{\|\mathbf{r}_k^{(i)}\|^{\alpha+1}} \\
&\quad - \left[\sum_i \frac{\partial g}{\partial \mathbf{q}^{(i)}} \right]^T \boldsymbol{\Delta} + o(\|\boldsymbol{\Delta}\mathbf{q}\|^2)
\end{aligned}
$$

for property 2

$$
\begin{aligned}
&= -\sum_i \sum_k \hat{\gamma}_k^{(i)} \frac{\mathbf{r}_k^{(i)T} \boldsymbol{\Delta}\mathbf{q}^{(i)}}{\|\mathbf{r}_k^{(i)}\|^{\alpha+1}} + o(\|\boldsymbol{\Delta}\mathbf{q}\|^2) \\
&= -\sum_i \sum_k \frac{\hat{\gamma}_k^{(i)} \left(\mathbf{r}_{k,e}^{(i)T} \boldsymbol{\Delta}\mathbf{q}^{(\mathbf{i})} + \|\boldsymbol{\Delta}\mathbf{q}^{(\mathbf{i})}\|^2 \right)}{\left[\|\mathbf{r}_{k,e}^{(i)}\|^2 + \|\boldsymbol{\Delta}\mathbf{q}^{(\mathbf{i})}\|^2 + 2\mathbf{r}_{k,e}^{(i)T} \boldsymbol{\Delta}\mathbf{q}^{(\mathbf{i})} \right]^{\frac{\alpha+1}{2}}} \\
&\quad + o(\|\boldsymbol{\Delta}\mathbf{q}\|^2)
\end{aligned}
$$

if $\|\boldsymbol{\Delta}\mathbf{q}^{(\mathbf{i})}\|$ is sufficiently small

$$
\begin{aligned}
&\approx -\sum_i \sum_k \hat{\gamma}_k^{(i)} \frac{\mathbf{r}_{p,e}^{(i)T} \boldsymbol{\Delta}\mathbf{q}^{(\mathbf{i})}}{\left[\|\mathbf{r}_{p,e}^{(i)}\|^2 + 2\mathbf{r}_{p,e}^{(i)T} \boldsymbol{\Delta}\mathbf{q}^{(\mathbf{i})} \right]^{\frac{\alpha+1}{2}}} \\
&\geq -\sum_i \sum_k \hat{\gamma}_k^{(i)} \frac{\mathbf{r}_{p,e}^{(i)T} \boldsymbol{\Delta}\mathbf{q}^{(\mathbf{i})}}{\|\mathbf{r}_{p,e}^{(i)}\|^{\alpha+1}} \\
&= \nabla f_{\mathbf{q}=\mathbf{q}_e}^T \boldsymbol{\Delta}\mathbf{q} = 0
\end{aligned}
$$

So there exist points $\mathbf{q}$ in a neighborhood of $\mathbf{q}_e$ such that $F(\mathbf{q}_e) \geq F(\mathbf{q})$ and then points $\mathbf{q}_e$ cannot be local minima of f. $\square$

To avoid stationary situations it is, then, sufficient to perturb sensors configurations.

6.3 Simulations

This section presents simulation results for constrained coverage control.
Motion coordination is introduced in the same cases considered in the previous chapter. Once connectivity maintenance of the communication network is assured it is possible to relax the hypothesis of global knowledge. So every sensor configuration evolves with the following law:

$$\begin{aligned}\mathbf{q}^{(i)}((n+1)T_S) =& \mathbf{q}^{(i)}(nT_S) + \mathbf{u}^{(i)}_{cov}(nT_S) \\ &+ \sum_{j \neq i} \frac{\partial g^{(i,j)}_{coll}}{\partial \mathbf{q}^{(i)}} + \sum_{j\,:\,(i,j)\in E^i_{\mathcal{G}}} \frac{\partial g^{(i,j)}_{conn}}{\partial \mathbf{q}^{(i)}}\end{aligned}$$

Being $R_B < \rho_C$ evaluating collisions avoidance constraints require the knowledge of only a subset of the neighbors of the generic i-th sensor.
The following values are assumed for the parameters of connectivity maintenance and collisions avoidance constraints:

$$\rho_B = 0.5 \qquad R_B = 1$$
$$\rho_C = 5.5 \qquad R_C = 4$$

In the first simulation the K-Coverage sensing model is considered (K=15). Under the hypothesis of communication network connectivity the global configuration of the sensor network can be *locally available* with a certain delay δ. Every sensor can then evaluate an approximation of the residual information of the workspace. In particular the generic i-th sensor is assumed to know the following approximated sensor network configuration:

$$\hat{\mathbf{q}}_i(nT_S) = \begin{pmatrix} \mathbf{q}^{(1)}(n_\delta T_S) \\ \vdots \\ \mathbf{q}^{(i)}(nT_S) \\ \vdots \\ \mathbf{q}^{(m)}(n_\delta T_S) \end{pmatrix}$$

with:

$$n_\delta = \begin{cases} n - \delta & \text{if } n > \delta \\ 0 & \text{if } n \leq \delta \end{cases}$$

The value chosen for the delay is $\delta = 8$ that is the worst case if unitary delay is assumed for the communication channel.
Every sensor can evaluate $\hat{\gamma}_k^{(i)}$, for every cell c_k, with only locally available informations. In figure 6.2 the evolution of the sensor network configuration is displayed together with the estimation of the residual information made by one of the sensors, that's represented with a bigger dot. Communication network constraints are also shown, in particular the edges that must maintained constraining distances between sensors are displayed as black lines. The communication network topology changes but connectivity is preserved.

In figure 6.3(a) distances between sensors are displayed, the constant indicates the collision radius. As shown collisions between sensors are avoided.

Relaxing the hypothesis of global knowledge obviously affect the coverage dynamic, in figure 6.3(b) the evolution of the percentage residual information of the whole interest set is displayed and compared with the case of global knowledge.

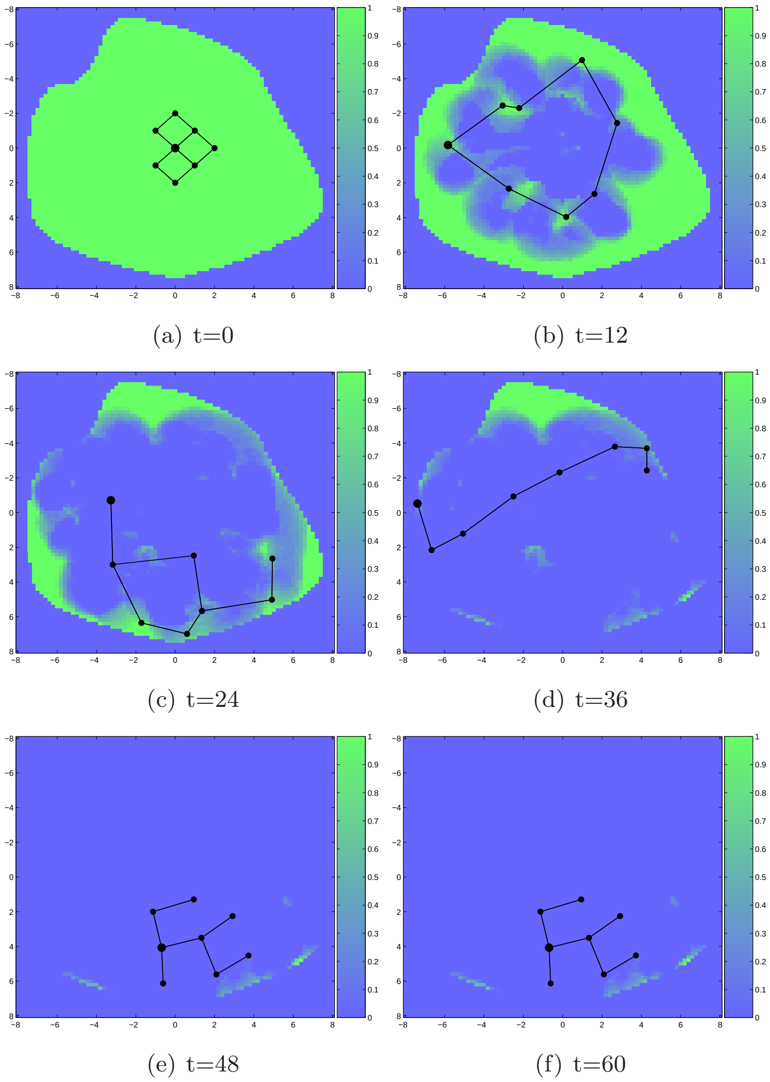
In the second simulation stochastic coverage is considered. The sensor network is asked to estimate the static scalar field x displayed in figure 5.3(a).

As said in 5.2.1, the residual information of a generic cell is given by the covariance P_k of the estimation of the value of x_k.
The generic i-th sensor is assumed to know the delayed sensor network configuration $\hat{\mathbf{q}}_i(nT_S)$ and the corresponding measures, so:

$$\hat{\mathbf{z}}_k(n) = \begin{pmatrix} z_k^{(1)}(n_\delta) \\ \vdots \\ z_k^{(i)}(n) \\ \vdots \\ z_k^{(m)}(n_\delta) \end{pmatrix} \qquad \hat{\mathbf{H}}_k(k) = \begin{pmatrix} H_k^{(1)}(n_\delta) \\ \vdots \\ H_k^{(i)}(n) \\ \vdots \\ H_k^{(m)}(n_\delta) \end{pmatrix}$$

where

$$H_k^j(n_\delta) = \begin{cases} H_k^j(n-\delta) & \text{if } n > \delta \\ 0 & \text{otherwise} \end{cases}$$

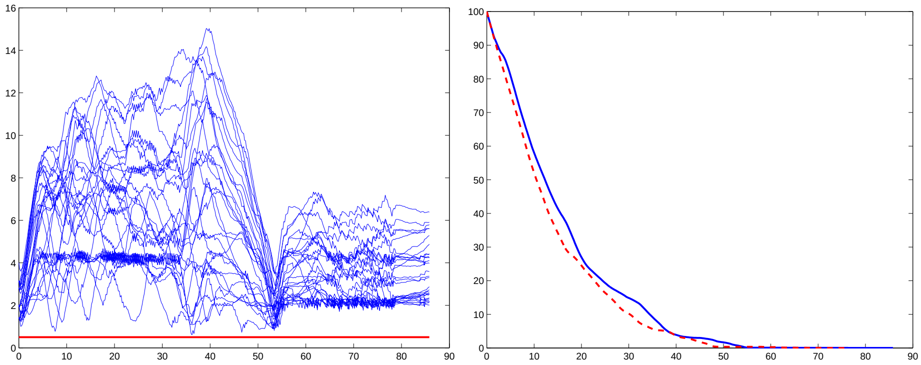


(a) t=0 (b) t=12

(c) t=24 (d) t=36

(e) t=48 (f) t=60

Figure 6.2: K-Coverage with collisions avoidance and connectivity maintenance constraints. Evolution of the sensor network configuration and estimation of the residual information made by one sensors (bigger dot). The black lines indicate the edges of the communication network that must maintained constraining distances between sensors in order to assure global connectivity.

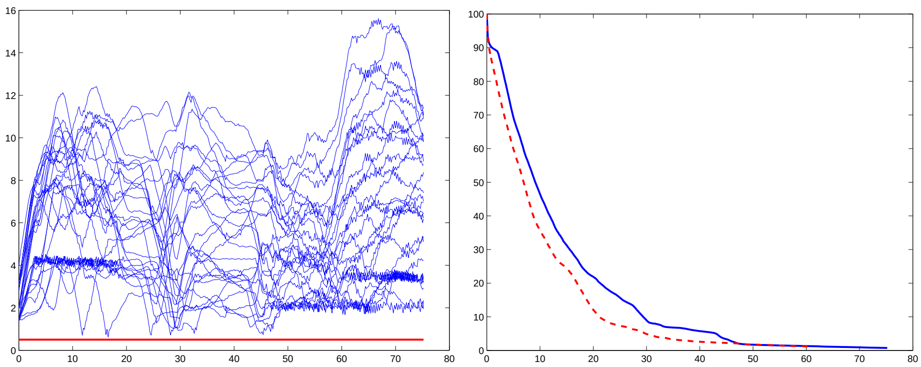


(a) (b)

Figure 6.3: Stochastic Coverage. (a) Evolution of the percentage estimated residual information of the whole set of interest (solid). Comparison with the case of global knowledge (dashed). (b) Relative distances between sensors.

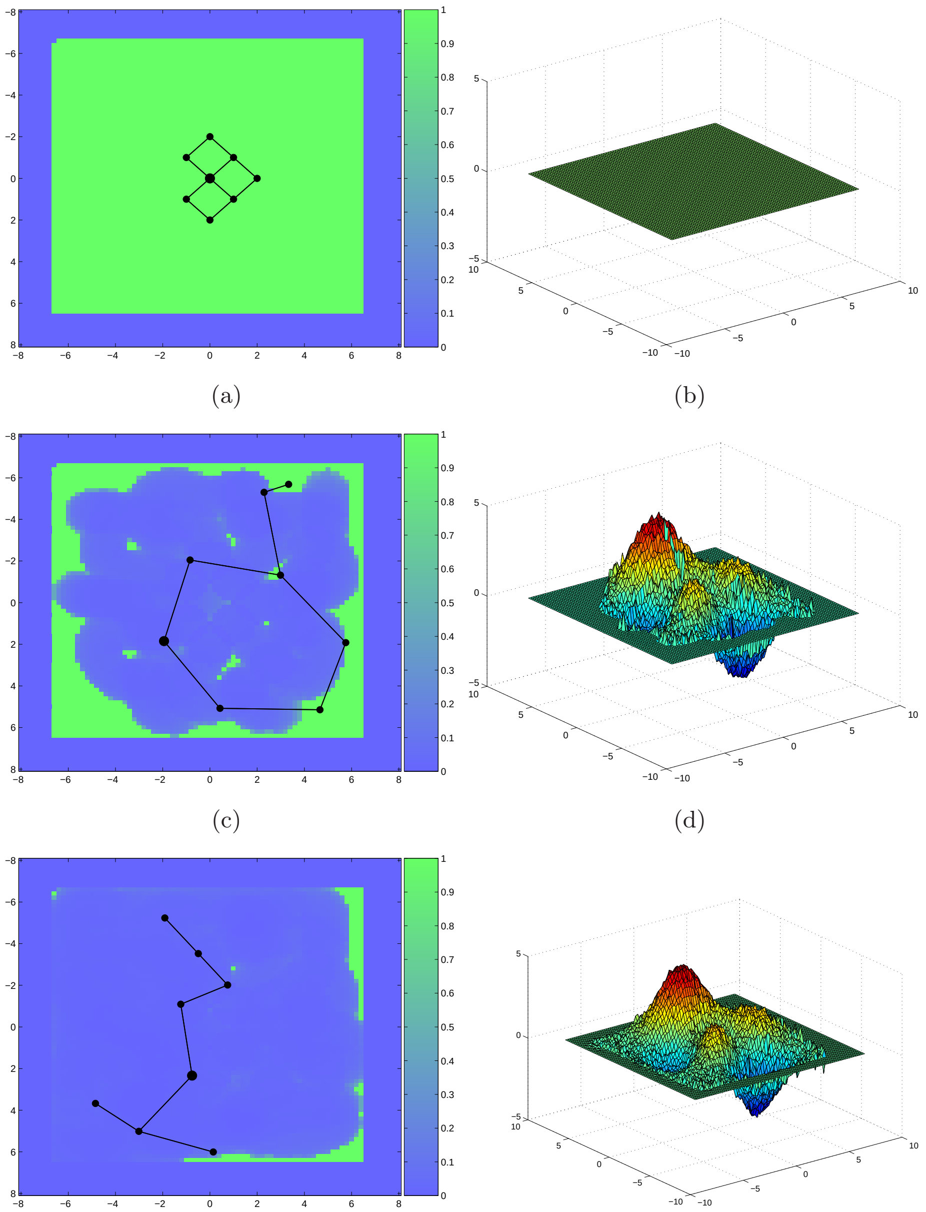
In figure 6.5 the evolution of the sensor network configuration is displayed together with the estimations of the residual information and of the field x made by one of the sensors that is represented with a bigger dot.

In figure 6.4(a) distances between sensors are displayed to show the avoidance of collisions. In figure 6.4(b) the evolution of the percentage residual information of the whole interest set is displayed and compared with the case of global knowledge. As in the previous case there is a loss of performances with respect to the case of global coverage.



(a) (b)

Figure 6.4: Stochastic Coverage. (a) Evolution of the percentage estimated residual information of the whole set of interest (solid). Comparison with the case of global knowledge (dashed). (b) Relative distances between sensors.



(a) (b) (c) (d) (e) (f)

Figure 6.5: Stochastic Coverage with collisions avoidance and connectivity maintenance constraints. Evolution of the sensor network configuration, estimation of the residual information map (left column), and estimation of the scalar field x (right column) made by one sensors (bigger dot). The black lines indicate the edges of the communication network that must maintained constraining distances between sensors in order to assure global connectivity.

CHAPTER 7

CONCLUSIONS

This chapter summarizes the main results of the thesis, and indicates possible future extensions.

Summary

This thesis concerns the problem of dynamic coverage with mobile sensor networks. Once the loss of continuous measurements is acceptable, as happen in many applications, mobility can be used to expand the sensor network range. Anyway controlling multiple mobile agents introduces challenging coordination problems. In this thesis motion coordination is considered especially in terms of collisions avoidance and connectivity maintenance.
In the first part optimal dynamical coverage is considered. Here the objective is evaluating optimal controls for the mobile senors in order to maximize the area covered by the network in given time interval. The problem is formulated at first an an optimal control problem, then, after discretization, as a nonlinear programming problem. The proposed formulation allow to handle many cases including, for example, closed trajectories, suitable for periodic coverage and surveillance. Moreover the case of mobile nodes with non homogeneous sensing capabilities is considered. As a particular case redundant coverage for achieving robustness to node faults is addressed. In this context

motion coordination is implemented introducing constraint to the resulting optimization problems. Because of its computational complexity, this approach can be used only offline and only for small networks.

In the second part a distributed solution to dynamic area coverage is presented. A feedback control law is proposed that that can be evaluated by each sensor with only locally available informations and that is shown working with also very general sensing and constraint models.
In a distributed control architecture connectivity of the communication network is a crucial aspect. For this reason the problem of distributed connectivity maintenance for is considered. A solution based on the evaluation of an approximated MST of the communication network is proposed. The approximated graph contain the MST and then it is connected. Maintaining its edges entails, then, connectivity maintenance. While the performances are lower than a centralized approach (but still effective) both computation and data transfer (since only local data are required) are considerably reduced, making possible the implementation of such a result for large senor networks and online real time applications.

Future Improvements

A major challenge posed to the forthcoming research consists in relaxing the simplifying assumptions done in this work.

- The *motion model* assumed, especially in the developing of the distributed control strategy is very simple. A more complicated model could be considered including, for example, nonholonomic constraints.
- The *sensing model* based on proximity, is not suitable for describing several real sensors as, for example, cameras. It could be generalized including non symmetric models.
- Assumptions on the *communication model* can be relaxed allowing unidirectional communication links.

- Coverage is considered only with respect to static magnitudes. When considering time varying magnitudes the coverage measure of a given point must decrease in time while the point is not in the sensing range of the network. Including this case could be an important improvement.

- The *set of interest* is always assumed to be free. Including obstacles could be an interesting topic for future research.

Moreover sensors cooperation could be improved. In the proposed approach the cooperation between sensors is implicit. Excluding coordination tasks as collisions avoidance and connectivity maintenance the behavior of a sensor is not directly conditioned by the ones of the others.
In fact, for evaluating coverage control every sensor consider only its position and the coverage status of the set of interest. The evolution of the coverage status (or of the residual information) of the set of interest depending from the behavior of all the sensors implement that implicit cooperation. Probably explicit cooperation could improve the senor network performances.

APPENDIX A

SEQUENTIAL QUADRATIC PROGRAMMING

This appendix presents basic notion on sequential quadratic programming methods. A detailed description can be found in Nocedal & Wright (1999). In A.1 the general constrained optimization problem has been defined as:

$$\min_{v \in \mathcal{D}} J(v) \tag{A.1}$$

Where the admissibile set D is represented by:

$$D = \{v \in \mathbb{R}^n : \; h(v) = 0, \; g(v) \leq 0\}$$

Referring to the problem A.1 the **Lagrange function** is defined as the following linear combination of the objective function and of the constraints functions:

$$L(v, \lambda, \eta) = L(v) + \lambda^T h(v) + \eta^T g(v)$$

where $\lambda \in \mathbb{R}^\mu$ and $\eta \in \mathbb{R}^\sigma$ are called multipliers.

As the name implies, sequential quadratic programming (SQP) methods are iterative methods which solve at each iteration a quadratic programming problem (QP). For the problem A.1 at the $k-th$ iteration (starting from the current iterate v_k) the following problem must be solved for the next search direction:

$$\min_{\mathbf{p}} \frac{1}{2}\mathbf{p}^T W_k \mathbf{p} + \nabla J_k^T \mathbf{p}$$
$$s.t. \nabla h(v_k)^T \mathbf{p} + h(v_k) = 0$$
$$\nabla h(v_k)^T \mathbf{p} + h(v_k) = 0$$

Here W_k is usually a positive semi-definite approximation of $\left.\frac{\partial^2 L}{\partial \mathbf{v}^2}\right|_k$.
The subproblems above can be efficiently solved using the well known QP techniques.

REFERENCES

Acar, Choset & Lee, J.Y. (2006). Sensor-based coverage with extended range detectors. *IEEE Transactions on Robotics and Automation*, **22**, 189–198. 12

Ahmadzadeh, A., Jadbabaie, A., Kumar, V. & Pappas, G.J. (2007). Stable multi-particle systems and application in multi-vehicle path planning and coverage. In *Decision and Control, 2007 46th IEEE Conference on*, 1467–1472. 13

Akyildiz, I., Su, W., Sankarasubramaniam, Y. & Cayirci, E. (2002). A survey on sensor networks. *IEEE Communications Magazine*, **40**, 102–114. 2

Awerbuch, B. (1987). Optimal distributed algorithms for minimum weight spanning tree. In *Proceedings of 19th Symposium on Theory of Computing*. 75

Bruni, C. & di Pillo, G. (1993). *Metodi variazionali per il controllo ottimo*. 16

Burgard, W., Moors, M., Stachniss, C. & Schneider, F. (2005). Coordinated multi-robot exploration. *IEEE Transactions on Robotics*, **21**, 376–386. 55

Cassandras, G., C. & Li, W. (2005). Sensor networks and cooperative control. *European Journal of Control*, **11**, 436–463. 3

Cecil & Marthler (2004). A variational approach to search and path planning using level set methods. Tech. rep., UCLA CAM. 12

CECIL & MARTHLER (2006). A variational approach to path planning in three dimensions using level set methods. *Journal of Computational Physics*, **221**, 179–197. 12

CHAKRABARTY, K., IYENGAR, S., QI, H. & CHO, E. (2002). Grid coverage for surveillance and target location in distributed sensor networks. *IEEE Transactions on Computers*, **51**, 1448–1453. 5

CHOSET (2001). Coverage for robotics - a survey of recent results. *Annals of Mathematics and Artificial Intelligence*, **31**, 113–126. 12

CORTES, J., MARTINEZ, S., KARATAS, T. & BULLO, F. (2004). Coverage control for mobile sensing networks. *IEEE Transactions on Robotics and Automation*, **20**, 243–255. 6

DU, Q., FABER, V. & GUNZBURGER, M. (1999). Centroidal voronoi tessellations: applications and algorithms. *SIAM Review*, **41**, 637–676. 5, 6

ENRIGHT, J. & FRAZZOLI, E. (2006). The traveling salesman problem for the reeds-shepp car and the differential drive robot. In *Proceedings of the 45th IEEE Conference on Decision and Control (CDC2006)*, 3058–3064. 13

FIORELLI, E., LEONARD, N., BHATTA, P., PALEY, D., BACHMAYER, R. & FRATANTONI, D. (2004). Multi-auv control and adaptive sampling in monterey bay. In *Proceddings of the IEEE Autonomous Underwater Vehicles : Workshop on Multiple AUV Operations (AUV04)*. 3

FRANCHI, A., FREDA, L., ORIOLO, G. & VENDITTELLI, M. (2007). A randomized strategy for cooperative robot exploration. In *Proceedings of the 2007 IEEE International Conference on Robotics and Automation (ICRA2007)*. 55

GABRIELE, S. & DI GIAMBERARDINO, P. (2007a). Communication constraints for mobile sensors networks. In *Proceedings of the 11th WSEAS International Conference on Systems*. 9

GABRIELE, S. & DI GIAMBERARDINO, P. (2007b). Dynamic sensor networks. an approach to optimal dynamic field coverage. In *ICINCO 2007, Proceedings of the Fourth*

International Conference on Informatics in Control, Automation and Robotics, Intelligent Control Systems and Optimization. 9

Gabriele, S. & Di Giamberardino, P. (2008a). *Developments in Robotics, Automation and Control*, chap. 1. I-Tech Education and Publishing. 9

Gabriele, S. & Di Giamberardino, P. (2008b). Distributed connectivity maintenance of dynamic sensor networks. In *Proceedings of the IASTED International Conference on Sensor Networks*. 10

Gabriele, S. & Di Giamberardino, P. (2008c). Measurement noise reduction in dynamic sensor networks. In *Proceedings of the 12th WSEAS International Conference on Systems*. 9

Gabriele, S. & Di Giamberardino, P. (2008d). Mobile sensors networks: a distributed solution to the area coverage problem. In *Proceedings of IEEE Mediterranean Conference on Control (MED)*. 10

Gabriele, S. & Di Giamberardino, P. (2008e). Mobile sensors networks under communication constraints. *WSEAS Transactions on Systems*. 9, 10

Gabriele, S. & Di Giamberardino, P. (2008f). Node fault robustness for heterogeneous dynamic sensor networks. In *Proceedings of the 7th WSEAS Int. Conf. on Instrumentation, Measures, Circuits and Systems (IMCAS'08)*. 9

Gabriele, S. & Giamberardino, P.D. (2008). A unified approach for heterogeneity and node fault robustness in dynamic sensor networks. *WSEAS TRANSACTIONS on COMMUNICATIONS*, **7**, 685–694. 9

Gallagher, R., Humblet, P. & Spira, P. (1983). A distributed algorithm for minimum-weight spanning trees. *ACM Transactions on Programming Languages and Systems*, **5(1)**, 66–77. 75

Ganguli, A., Susca, S., Martinez, S., Bullo, F. & Cortes, J. (2005). On collective motion in sensor networks: sample problems and distributed algorithms. In *Proceedings of the 44th IEEE Conference on Decision and Control (CDC2005) and the European Control Conference (ECC2005)*. 72

Garay, A., Juan, Kutten, S. & Peleg, D. (1996). A sub-linear time distributed algorithm for minimum-weight spanning trees. In *Proceedings of IEEE Symposium on Foundations of Computer Science*. 75

Garca-Hernndez, C.F., Ibargengoytia-Gonzlez, P.H., Garca-Hernndez, J. & Prez-Daz, J.A. (2007). Wireless sensor networks and applications: a survey. *IJCSNS International Journal of Computer Science and Network Security*, **3**, 264–273. 2

Glaser, S.D. (2004). Some real-world applications of wireless sensor nodes. In *Proceedings of SPIE Symposium on Smart Structures & Materials/ NDE*. 2

Grocholsky, B. (2002). *Information-Theoretic Control of Multiple Sensor Platforms*. Ph.D. thesis, Department of Aerospace, Mechatronic and Mechanical Engineering of theUniversity of Sydney. 56

Grocholsky, B., Keller, J., Kumar, V. & Pappas, G. (2006). Cooperative air and ground surveillance. *IEEE Robotics & Automation Magazine*, 16–26. 3

Hazon, N. & Kaminka, G.A. (2005). Redundancy, efficiency and robustness in multi-robot coverage. In *Proceedings of the 2005 IEEE International Conference on Robotics and Automation (ICRA2005)*. 34

Hefeeda, M. & Bagheri, M. (2007). Randomized k-coverage algorithms for dense sensor networks. In *26th IEEE International Conference on Computer Communications(INFOCOM2007)*. 5

Hokayem, P., Stipanovic, D. & Spong, M. (2007a). Dynamic coverage control with limited communication. In *Proceedings of American Control Conference (ACC07)*, 4878–4883. 13

Hokayem, P., Stipanovic, D. & Spong, M. (2007b). On persistent coverage control. In *46th IEEE Conference on Decision and Control (CDC07)*, 6130–6135. 13

Holger Karl, A.W. (2005). *Protocols and Architectures for Wireless Sensor Networks*. Wiley. 2

Hou, Y.T., Lee, T.C., Chen, C.M. & Jeng, B. (2006). Node placement for optimal coverage in sensor networks. In *Proceedings of IEEE International Conference on Sensor Networks, Ubiquitous, and Trustworthy Computing*. 5

Howard, S., Mataric (2002). An incremental self-deployment for mobile sensor networks. *Autonomus Robots*. 6

Huang, C.F. & Tseng, Y.C. (2005). The coverage problem in a wireless sensor network. *Mobile Networks and Applications*, **10**, 519–528. 4

Hussein, I. & Stipanovic, D. (2006). Effective coverage control using dynamic sensor networks. In *Decision and Control, 2006 45th IEEE Conference on*, 2747–2752. 56, 59

Hussein, I.I. (2007). A kalman filter-based control strategy for dynamic coverage control. In *Proceedings of the 2007 American Control Conference (ACC07)*. 56

Hussein, I.I. & Stipanovic, D.M. (2007a). Effective coverage control for mobile sensor networks with guaranteed collision avoidance. *IEEE Transactions on Control Systems Technology*, **15**, 642–657. 72

Hussein, I.I. & Stipanovic, D.M. (2007b). Effective coverage control using dynamic sensor networks with flocking and guaranteed collision avoidance. In *American Control Conference, 2007. ACC '07*, 3420–3425. 31, 72

Hussein, I.I., Stipanovic, D.M. & Wang, Y. (2007). Reliable coverage control using heterogeneous vehicles. In *Decision and Control, 2007 46th IEEE Conference on*, 6142–6147. 32

Jadbabaie, A., Lin, J. & Morse, A.S. (2003). Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions On Automatic Control*, **48**, 988–1000. 72

Kim, Y. & Mesbahi, M. (2005). On maximizing the second smallest eigenvalue of a state-dependent graph laplacian. In *Proceedings of American Control Conference*. 32

King, E., Kuwata, Y., Alighanbari, M., Bertuccelli, L. & How, J. (2004). Coordination and control experiments on a multi-vehicle testbed. In *Proceeding of the American Control Conference*. 3

LaValle, S.M. (2006). *Planning Algorithms*. Cambridge University Press. 12

Lazos, L. & Poovendran, R. (2006). Stochastic coverage in heterogeneous sensor networks. *ACM Transactions on Sensor Networks (TOSN)*, **2**, 325 – 358. 4, 32

Leonard, N.E. & Fiorelli, E. (2001). Virtual leaders, artificial potentials and coordinated control of groups. In *Proceedings of the 40th IEEE Conference on Decision and Control (CDC2001)*. 71

Lewis, F.L. (2004). *Smart Environments: Technologies, Protocols and Applications*, chap. 2. D.J. Cook and S. K. Das. 2

Li, W. & Cassandras, C. (2005). Distributed cooperative coverage control of sensor networks. In *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC '05. 44th IEEE Conference on*, 2542–2547. 6

Li, X.Y., Wan, P.J. & Frieder, O. (2003). Coverage in wireless ad hoc sensor networks. *IEEE Transactions on Computers*, **52**, 753–763. 4

ling Lam, M. & hui Liu, Y. (2007). Heterogeneous sensor network deployment using circle packings. In *Robotics and Automation, 2007 IEEE International Conference on*, 4442–4447. 32

Liu, B. & Towsley, D. (2004). A study of the coverage of large-scale sensor networks. In *Proceeding of IEEE Internationa Conference on Mobile Ad Hoc Sensor Systems (MASS)*. 4

Mainwaring, A., Polastre, J., Szewczyk, R., Culler, D. & Anderson, J. (2002). Wireless sensor networks for habitat monitoring. In *Proceedings of the 1st ACM international workshop on Wireless sensor networks and applications*. 2

Meguerdichian, S., Koushanfar, F., Potkonjak, M. & Srivastava, M. (2001). Coverage problems in wireless ad-hoc sensor networks. In *INFOCOM 2001. Twentieth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 3, 1380–1387vol.3. 4

Mesbahi, M. (2004). On state-dependent dynamic graphs and their controllability properties. In *Proceedings of 43rd IEEE Conference on Decision and Control*. 31

Nocedal, J. & Wright, S. (1999). *Numerical optimization*. Springer. 16, 90

Ogren, P., Fiorelli, E. & Leonard, N. (2004). Cooperative control of mobile sensor networks:adaptive gradient climbing in a distributed environment. *IEEE Transactions on Automatic Control*, **49**, 1292–1302. 71

Olfati-Saber, R. (2006). Flocking for multi-agent dynamic systems: algorithms and theory. *Automatic Control, IEEE Transactions on*, **51**, 401–420. 31, 72

Olfati-Saber, R. & Murray, R. (2002a). Distributed structural stabilization and tracking for formations of dynamic multi-agents. In *Decision and Control, 2002, Proceedings of the 41st IEEE Conference on*, vol. 1, 209–215vol.1. 72

Olfati-Saber, R. & Murray, R.M. (2002b). Distributed cooperative control of multiple vehicle formations using structural potential functions. In *in IFAC World Congress*. 72

Pavone, M. & Frazzoli, E. (2007). Decentralized policies for geometric pattern formation. In *American Control Conference, 2007. ACC '07*, 3949–3954. 56

Porter, J., Arzberger, P., Braun, H.W., Bryant, P. & Gage, S. (2005). Wireless sensor networks for ecology. *BioScience*, **55**, 561–572. 2

Sameera, P. & Gaurav S., S. (2004). Constrained coverage for mobile sensor networks. In *IEEE International Conference on Robotics and Automation*. 6

Santi, P. (2005). *Topology Control in Wireless Ad Hoc and Sensor Networks*. Wiley. 2

Savla, K., Bullo, F. & Frazzoli, E. (2005). On traveling salesperson problems for dubins&vehicle: stochastic and dynamic environments. In *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC '05. 44th IEEE Conference on*, 4530–4535. 13

Savla, K., Bullo, F. & Frazzoli, E. (2006). On traveling salesperson problems for a double integrator. In *Decision and Control, 2006 45th IEEE Conference on*, 5305–5310. 13

SCHOUWENAARS, T., MOOR, B.D., FERON, E. & HOW, J. (2001). Mixed integer programming for multi-vehicle path planning. In *In Proceedings of the European Control Conference (ECC2001)*, 2603–2608. 32

SEKHAR, A., MANOJ, B. & MURTHY, C.S.R. (2005). Dynamic coverage maintenance algorithms for sensor networks with limited mobility. In *Proceedings of the 3rd IEEE International Conference on Pervasive Computing and Communications (PerCom 2005)*. 6

SHIH, K.P., CHEN, H.C. & LIU, B.J. (2007). Integrating target coverage and connectivity for wireless heterogeneous sensor networks with multiple sensing units. In *Networks, 2007. ICON 2007. 15th IEEE International Conference on*, 419–424. 33

SONTAG, E.D. (1998). *Mathematical Control Theory*. 16

STIPANOVIC, D.M., HOKAYEM, P.F. & SILJAK, D.D. (2007). Cooperative avoidance control for multiagent systems. *Journal of Dynamic Systems, Measurement, and Control*, **129**, 699. 73

STOJMENOVIC, I. (2005). *Handbook of Sensor Networks Algorithms and Architecture*. Wiley. 2

STUBBS, A., VLADIMEROU, V., FULFORD, A.T., KING, D., STRICK, J. & DULLERUD, G.E. (2006). Multivehicle systems control over networks. *IEEE Control Systems Magazine*. 3

TANG, Z. & OZGUNER, U. (2005). Motion planning for multitarget surveillance with mobile sensor agents. *Robotics, IEEE Transactions on [see also Robotics and Automation, IEEE Transactions on]*, **21**, 898–908. 3

TSAI, CHENG, OSHER, BURCHARD & SAPIRO (2004). Visibility and its dynamics in a pde based implicit framework. *Journal of Computational Physics*, **199**, 260–290. 6

VICSEK, T., CZIROK, A., BEN-JACOB, E., COHEN, I. & SOCHET, O. (1995). Novel type of phase transition in a system of self-driven particles. *PHYS REV LETT.*, **75**, 1226. 72

WANG, J. & MEDIDI, S. (2007). Energy efficient coverage with variable sensing radii in wireless sensor networks. In *Proceedings of Third IEEE International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob 2007)*. 5

WANG, P.C., HOU, T.W. & YAN, R.H. (2006). Maintaining coverage by progressive crystal-lattice permutation in mobile wireless sensor networks. In *International Conference on Systems and Networks Communication (ICSNC06)*, 42–42. 6

WANG, P.K.C. (2003). Optimal path planing based on visibility. *Journal of Optimization Theory and Applications*, **117**, 157–181. 12

WANG, Y. & HUSSEIN, I.I. (2007). Cooperative vision-based multi-vehicle dynamic coverage control for underwater applications. In *Proceedings of 16th IEEE International Conference on Control Applications*. 3

WONG, T., TSUCHIYA, T. & KIKUNO, T. (2004). A self-organizing technique for sensor placement in wireless micro-sensor networks. In *Proceedings of 18th International Conference on Advanced Information Networking and Applications (AINA 2004)*. 6

ZAVLANOS, G., M.M. PAPPAS (2005). Controlling connectivity of dynamic graphs. In *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC '05. 44th IEEE Conference on*. 32

ZHOU, Z., DAS, S. & GUPTA, H. (2004). Connected k-coverage problem in sensor networks. In *Computer Communications and Networks, 2004. ICCCN 2004. Proceedings. 13th International Conference on*, 373–378. 5

ZOU, Y. & CHAKRABARTY, K. (2003). Sensor deployment and target localization based on virtual forces. In *Proceedings of IEEE INFOCOM*. 6