Contracting with Endogenous Entry*

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December 2016

Abstract

We analyze entry in markets where a principal contracts with a privately informed agent. Before learning his production cost, the agent knows his probability of having a low cost — his ex ante "type" — and decides whether to pay an entry fee to contract with the principal. There are two cut-off equilibria that determine the possible types of an agent who actually enters the market, and neither equilibrium can be discarded by standard selection criteria. Contrasting with standard intuition, in the equilibrium with the highest cut-off an increase in the entry fee reduces the marginal type of the agent who enters, thus increasing entry and the expected cost of an entrant. This equilibrium is selected by a criterion based on "robustness to equilibrium risk," even though the equilibrium with the lowest cut-off is Pareto dominant for the agent. Public policies that increase entry barriers may be welfare improving.

JEL Classification: D43, D82, L13, L51.

Keywords: Entry, Vertical Contracting, Asymmetric Information.

*We would like to thank Alessandro Bonatti, Piero Tedeschi, Antonio Rosato, the Editor and the referees for extremely helpful comments.

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1 Introduction

Entry costs affect entry and the characteristics of firms in a market. But although barriers to entry (like minimum capital or advertising intensity, and concession or license fees) are high in many industries, there is also evidence of high entry rates in markets with high barriers to entry (see, e.g., Geroski, 1995, and Kessides, 1986, for evidence on decentralized markets). Even in markets based on contracting relations (like labour markets, procurement and retailing), participation by agents is not costless, since they often pay substantial entry or search costs in order to be able to contract with a principal. This suggests that the characteristics of agents who contract in a market may not be exogenous, but may be determined by their entry decisions.

In order to analyze how entry costs and barriers to entry affect entry decisions and the characteristics of the agents and firms that enter a market, we study a simple model in which a privately informed agent (e.g., a downstream firm) first chooses whether to enter, and then contracts with a principal in the market (e.g., an upstream firm). Before entering and learning his actual production cost, the agent privately observes his *ex ante type*, which is the probability of being efficient and having a low production cost once hired by the principal. We assume that there is a fixed entry cost that determines the mass of agents (i.e., the ex ante types) that enters the market in equilibrium, because they expect to obtain positive net profit. After entry, the principal offers a direct revelation mechanism to the agent to maximize interim expected profit, given his expectation of the agent's ex ante type.

We show that the entry game has two different cut-off equilibria, in which the agent enters if and only if his type is sufficiently high. In one equilibrium the mass of agents entering the market is decreasing in the entry cost, a feature that is common to many standard IO models (see, e.g., Mankiw and Whinston, 1986). In the equilibrium with the highest cut-off for the agent's types, by contrast, the mass of agents entering the market is lower and an increase in the entry cost reduces the marginal type that enters and increases the mass of agents in the market.

The intuition for this counterintuitive result is that, if the entry cost increases, the information rent of the marginal agent type that enters the market has to increase in order to make entry profitable. In the equilibrium with the highest cut-off, this extra information rent can arise through a reduction in the expected probability that an agent who enters has a low cost, which reduces the distortion that the principal imposes on the quantity produced by a high-cost agent.[1] In other words, an expansion in the set of ex ante types of agents who enter the market generates a positive externality on agents in the market, since it relaxes the rent-extraction/efficiency trade-off faced by the principal and allows him to increase production. Hence, contrasting with standard intuition, in the equilibrium with the highest cut-off an increase in the entry cost, or higher barriers to entry, induces neither a reduction in the mass of agents who enter, nor an increase in the quality (i.e., in the expected probability of being efficient) of those agents who enter.

We show that neither of the two cut-off equilibria is always Pareto dominant,[2] and that neither of the two equilibria is risk dominant in a simultaneous coordination game in which players choose among equilibria. Hence, neither of the two equilibria of our model can always be discarded by standard selection criteria.

Moreover, we also consider equilibrium selection from the point of view of the agent alone, since he is the player who initiates the game by choosing whether to enter in our environment. From this perspective, while the equilibrium with the lowest cut-off is Pareto dominant for the agent, the equilibrium with the highest cut-off is the only one that is "robust to equilibrium risk" for the agent, according to a selection criterion that we introduce. Specifically, we show that while a coordination failure between the principal and the agent in the equilibrium with the highest cut-off does not induce losses for the agent, a coordination failure in the equilibrium with the lowest cut-off induces some of the agent's types to earn negative profits in the market. This suggests that the equilibrium with the highest cut-off may actually prevail in the presence of agents that are not willing to face the risk created by uncertainty with respect to the principal's equilibrium choice.

Finally, we analyze the impact of higher barriers to entry on welfare and players' profits, and we show that while in one equilibrium of our model all players, including a regulator, have an incentive to minimize the entry cost, in the other equilibrium public policies that reduce entry costs do not necessarily increase welfare and profits. The reason is that, in the equilibrium with the highest cut-off, a marginal increase in the entry cost increases the set of the agent's types that enter the market and hence increases efficiency by reducing the quantity distortion due to the presence of asymmetric information.

Our assumptions that agents pay the entry cost before observing the contract offered by the principal, and obtain additional information about their characteristics after entry,

[1] By contrast, in the equilibrium with the lowest cut-off the extra information rent that the marginal agent type obtains when the entry cost increases arises through an increase in the probability that this agent has a low cost.

[2] More precisely, the equilibrium with the highest cut-off is never Pareto dominant, while the equilibrium with the lowest cut off is Pareto dominant for some distributions of the agent's type and values of the entry cost, but not for all distributions and entry costs.

capture relevant elements of many economic environments. For example, workers often pay non-negligible search costs to find potential employers, before learning the precise terms of the labour contract that they will be offered by an employer and discovering their productivity for the specific job that they will be required to do. Similarly, consumers interested in acquiring non-standardized products often pay substantial search costs to locate a seller, before observing the precise sale mechanism used by the seller and the specific characteristics of the actual product on sale. Moreover, in auctions for spectrum licenses governments often select the actual allocation mechanism only after a long consultation process that includes potential bidders,[3] who devote substantial internal resources and hire consultants to participate in the process and prepare for the auction, before learning the exact mechanism that will eventually be selected by the seller and obtaining a precise estimate of the profitability of the licenses on sale.[4] Our analysis suggests that, in all these environments, a principal may not have an incentive to reduce entry or search costs that privately-informed agents have to pay to interact with him and be offered a contract, since a reduction of these costs may actually reduce entry. We discuss various specific applications of our simple model in Section 6.

Related Literature. Courty and Li (2000) first considered agents who are privately informed about the distribution of their ex-post types. In a model of price discrimination where the buyer is privately informed about the distribution of his valuation, before learning the actual valuation, they show that the optimal sequential screening policy is a menu of contracts consisting of an advance payment and a refund payment in case of no consumption. In contrast to Courty and Li (2000), in our model the principal can only contract with the agent after he enters and learns his cost, which is arguably a reasonable assumption in the presence of frictions that prevent ex ante contracting, like costly experimentation and specialization, or when the principal lacks commitment power. An intermediate approach is taken by Deb and Said (2015) and the literature on dynamic mechanism design analyzing a principal who has only partial, or limited, commitment power.

Entry in an adverse selection framework has also been analyzed in the auction literature — see, e.g., Samuelson (1985), McAfee and McMillan (1987), Engelbrecht-Wiggans (1993), Levin and Smith (1994), and Menezes and Monteiro (2000). In these models,

[3]Our model can be directly interpreted as an approximation of a procurement auction, where competition among suppliers is expected to be low.

[4]By contrast, in the literature analyzing entry in auctions, it is commonly assumed that bidders observe the mechanism before choosing whether to participate.

however, entry only affects the number of bidders, while the distribution of the agents' types is exogenously fixed, because the mechanism is fixed by the seller before bidders decide whether to participate in the auction. Closer to our model is Chakraborty and Kosmopoulou (2001) who analyze an auction in which bidders observe a private signal on the common value of the object on sale and pay an entry fee to participate in the auction. There are two main differences between their model and our paper. First, while entry in an auction depends on the intensity of competition that bidders expect to face, in our model there is a single agent. Second, in our model with production the rent/efficiency trade-off bites both on the extensive and on the intensive margin: the profitability of entry is determined by the agent's expected information rent, which depends on the quantity distortion for an inefficient agent.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3, we characterize the equilibria, present the main results, and analyze the effects of changing the entry cost. In Section 4 we compare equilibria. In Section 5 we analyze the optimal choice of entry costs by the different players. Finally, Section 6 discusses applications of our model and Section 7 concludes. All proofs are in the Appendix.

2 The Model

Players and Environment. We consider a market where a principal delegates to an agent the production of q units of a good, which yields a gross surplus $S(q)$ to the principal. $S(\cdot)$ is continuously differentiable, strictly increasing and concave, with $S'(0) = +\infty$ and $S'(+\infty) = 0$. Both players are risk neutral.

The agent privately learns his marginal cost of production $\theta \in \Theta \equiv \{\underline{\theta}, \overline{\theta}\}$ when he enters the market, and the probability of the agent having a low cost is $\nu = \Pr[\theta = \underline{\theta}]$. We depart from the standard environment analyzed in the literature by assuming that, before entering the market, the agent is privately informed about ν — i.e., his *ex ante type* — which is distributed on $[0, 1]$ with cumulative distribution function $G(\nu)$. This assumption allows us to make the agent's entry problem non-trivial: without private information on ν, the agent always enters the market (provided that the entry cost is not to high), and entry provides no information on the agent's characteristics to the principal. For example, before being hired by a firm and discovering his actual marginal cost of production, which will depend on the specific characteristics of his job, a manager may be better informed than his potential employer about the likelihood of having a low cost, which also depends on his ability and on his current experience and qualification, and this information may affect his decision to look for a job.

The agent has to pay a fixed sunk cost $F > 0$ in order to enter the market and be able to interact with the principal. This may be interpreted as a search cost required to find a potential employer, or as a cost of entry into the market which represents a barrier to entry.

Contracts. We assume without loss of generality that the contract offered by the principal to the agent in the market is deterministic and consists in a menu $\{q(m), t(m)\}_{m \in \Theta}$, which specifies the quantity $q(\cdot) : \Theta \to \mathbb{R}^+$ produced by the agent and the transfer $t(\cdot) : \Theta \to \mathbb{R}^+$ paid to the agent by the principal, both contingent on the reported cost $m \in \Theta$. A standard version of the Revelation Principle holds in our environment because, as we will show in Lemma 1, the principal has no incentive to use a wider set of contracts that include messages on ν by the agent.

Therefore, the principal's utility is equal to $S(q) - t$, and the agent's utility is

$$U(\cdot) \equiv t - \theta q.$$

The agent's ex ante outside option is normalized to zero (while his outside option after entry is $-F$). We discuss various applications of this simple model in Section 6.

Timing. The timing of the game is as follows.

1. The agent is privately informed about the ex-ante type ν and chooses whether to pay the entry cost F to enter the market and learn his marginal cost θ.
2. If the agent enters, the principal offers a contract.
3. If the agent accepts the contract, he reports his cost, produces and receives the transfer.

The assumption that the agent chooses whether to enter before learning his cost is necessary to make our analysis interesting: an agent who knows his cost never enters the market. The reason is that an agent with a high cost has no incentive to enter because he obtains no rent in the market. And, as a consequence, a standard hold up logic implies that an agent with a low cost does not enter either.[5]

[5] Notice that the same unravelling argument also holds in a model with a continuum of possible costs for the agent: given any candidate equilibrium with entry, since the principal knows the possible costs of an agent in the market, the agent with the highest cost who should enter has an incentive to deviate and stay out.

With an alternative timing in which the agent learns θ after the principal offers a contract, all our results hold if we impose a "strong" limited liability constraint that ensures that the agent obtains non-negative utility for each cost — i.e., that $t - \theta q \geq 0$, $\forall \theta$.[6]

Equilibrium Concept. A Perfect Bayesian Equilibrium consists of a set $M^* \subseteq [0,1]$ of ex ante agent's types that enter the market, and of a contract $\{q^*(\theta), t^*(\theta)\}_{\theta \in \Theta}$ offered by the principal and accepted by the agent. M^* can also be interpreted as the quantity of potential agents who enter the market, with the principal randomly selecting an agent among them with whom to contract.

We consider truthful equilibria in which: (i) the principal offers the incentive compatible contract that maximizes his expected profit contingent on the correct expectation about the set M^*; (ii) an agent enters into the market if, and only if, his expected utility from interacting with the principal (contingent on the principal holding the correct expectation about the set M^*) is higher than F.

3 Equilibrium Analysis

3.1 Preliminaries

We start by showing that we can restrict the analysis to contracts that only depend on the agent's cost and to cut-off entry strategies for the agent.

Lemma 1 *There is no loss of generality in considering contracts that only depend on the agent's costs, and not on the agent's ex ante type.*

The reason for this result is simple: after the agent enters and learns his cost, the agent's utility does not depend on his ex ante type — i.e., the 'Spence-Mirrlees' condition does not hold with respect to ν because players contract after entry has occurred.

Let $\Delta\theta \equiv \overline{\theta} - \underline{\theta}$. Given a contract $\{q(m), t(m)\}_{m \in \Theta}$, the incentive compatibility constraints for a low-cost and a high-cost agent are

$$U(\underline{\theta}) \equiv t(\underline{\theta}) - \underline{\theta} q(\underline{\theta}) \geq t(\overline{\theta}) - \underline{\theta} q(\overline{\theta}) = U(\overline{\theta}) + \Delta\theta q(\overline{\theta}), \tag{1}$$

[6] When the principal offers a contract before the agent learns his cost, contracts must only satisfy an *ex ante* participation constraint for the agent. Without limited liability, this constraint is binding (see, e.g., Laffont and Martimort, 2002, Ch. 2, p. 57), the agent obtains no rent (in expectation), and the principal implements the efficient outcome. Limited liability prevents full surplus extraction by the principal, since it is equivalent to imposing an interim participation constraint.

and

$$U\left(\overline{\theta}\right) \equiv t\left(\overline{\theta}\right) - \overline{\theta} q\left(\overline{\theta}\right) \geq t\left(\underline{\theta}\right) - \overline{\theta} q\left(\underline{\theta}\right) = U\left(\underline{\theta}\right) - \Delta\theta q\left(\underline{\theta}\right);$$

while the participation constraints require that the contract yields non-negative utility to the agent, for each realization of his cost — i.e., $U(\theta) \geq 0$, $\theta = \underline{\theta}, \overline{\theta}$.

Standard arguments allow to show that, in equilibrium, only the incentive compatibility constraint of the low-cost agent binds — i.e., condition 1 holds with equality — and that only the participation constraint of the high-cost agent binds — i.e., $U\left(\overline{\theta}\right) = 0$.[7] This implies that an agent who has a low cost obtains an information rent equal to

$$U\left(\underline{\theta}\right) = \Delta\theta q\left(\overline{\theta}\right),$$

while an agent who has a high cost obtains no rent in the market. As usual, the information rent is increasing in the quantity produced by a high-cost agent. The agent pays F and enters if and only if the expected information rent that he obtains from interacting with the principal is higher than the entry cost — i.e.,

$$\Pr\left[\theta = \underline{\theta}\right] \times U\left(\underline{\theta}\right) = \nu\Delta\theta q\left(\overline{\theta}\right) > F.$$

Lemma 2 *In any equilibrium, the agent adopts a cut-off entry strategy — i.e., there is a unique value ν^* such that the agent enters if and only his ex ante type is greater than ν^*.*

The intuition for this result is straightforward. Given a contract offered by the principal, the agent's information rent is increasing in his probability of having a low cost. Therefore, if entry is profitable for an agent with a given ex ante type, then it is also profitable for any agent with a higher type.

Lemma 2 implies that, in any equilibrium, the set $M^* \equiv [\nu^*, 1]$ is uniquely characterized by ν^*, the lowest agent's type who chooses to enter.

3.2 Equilibria

Suppose that there is a cut-off x such that the principal believes that the agent enters in the market if and only if $\nu \geq x$, so that the principal assigns probability

$$\mathbb{E}\left[\nu | \nu \geq x\right] = \int_{\nu \geq x} \nu dG\left(\nu | \nu \geq x\right)$$

[7] See Laffont and Martimort (2002).

to the agent having a low cost in the market. Conditional on entry by the agent, the principal offers the contract $\{q(m,x), t(m,x)\}_{m\in\Theta}$ that maximizes his expected profit. Substituting the binding incentive and participation constraints for the agent, the principal's expected profit is

$$\mathbb{E}[\nu|\nu \geq x]\left[S(q(\underline{\theta},x)) - \underline{\theta}q(\underline{\theta},x) - \Delta\theta q(\overline{\theta},x)\right] + (1-\mathbb{E}[\nu|\nu \geq x])\left[S\left(q\left(\overline{\theta},x\right)\right) - \overline{\theta}q\left(\overline{\theta},x\right)\right].$$

The standard solution to this problem yields 'no distortion at the top' — i.e., $q(\underline{\theta},x) \equiv q(\underline{\theta})$ is such that $S'(q(\underline{\theta})) = \underline{\theta}$ — and 'downward distortion for the inefficient type' — i.e., $q\left(\overline{\theta},x\right)$ is such that

$$S'\left(q\left(\overline{\theta},x\right)\right) = \overline{\theta} + \frac{\mathbb{E}[\nu|\nu \geq x]}{1-\mathbb{E}[\nu|\nu \geq x]}\Delta\theta. \tag{2}$$

Hence, the principal induces a high-cost agent to produce a quantity that is lower than the efficient one, in order to make it less atractive for a low-cost agent to misreport his marginal cost.

It follows that, given an entry cut-off x, the expected information rent of an agent with type ν is

$$\nu\Delta\theta \times S'^{-1}\left(\overline{\theta} + \frac{\mathbb{E}[\nu|\nu \geq x]}{1-\mathbb{E}[\nu|\nu \geq x]}\Delta\theta\right).$$

Let

$$\Gamma(x) \equiv x\Delta\theta \times S'^{-1}\left(\overline{\theta} + \frac{\mathbb{E}[\nu|\nu \geq x]}{1-\mathbb{E}[\nu|\nu \geq x]}\Delta\theta\right)$$

be the expected information rent of an agent of type x, when the principal expects an agent to enter if and only if $\nu \geq x$. In other words, $\Gamma(\cdot)$ represents the expected information rent of the agent with the lowest ex-ante type who enters the market in an equilibrium. Notice that: (i) the function $\Gamma(x)$ is continuous and non-negative for $x \in [0,1]$; (ii) $\Gamma(0) = 0$ because an agent with type $\nu = 0$ is certain to have a high cost an hence obtains no utility in the market; and (iii) $\Gamma(1) = 0$ because, when the marginal type who enters is $\nu = 1$, the principal knows that an agent in the market has a low cost and hence the agent obtains no information rent.[8] To simplify the exposition, we assume that the function $\Gamma(\cdot)$ is single peaked.[9]

[8]See the Appendix for details.

[9]A sufficient condition for this is that $\left|\frac{\partial^2\mathbb{E}[\nu|\nu\geq x]}{\partial^2 x}\right|$ is not too large and that $S'''(\cdot) \leq 0$. In footnote 12 we explain why our qualitative results do not hinge on this assumption.

The equilibrium cut-off $x = \nu^*$ is defined by the 'zero profit' condition

$$\nu^* \Delta\theta \times S'^{-1}\left(\overline{\theta} + \frac{\mathbb{E}\left[\nu|\nu \geq \nu^*\right]}{1-\mathbb{E}\left[\nu|\nu \geq \nu^*\right]}\Delta\theta\right) - F = 0 \quad \Leftrightarrow \quad \Gamma\left(\nu^*\right) = F. \tag{3}$$

In fact, condition 3 identifies the *marginal agent's type in equilibrium* in the market: an agent with ex-ante type ν^* is indifferent between paying F to interact with the principal or staying out of the market, given the equilibrium contract that he expects the principal to offer, when the principal correctly believes that the agent enters if and only if his type is weakly higher than ν^*.

Theorem 1 *If $F < \max_{x\in[0,1]} \Gamma(x)$, there are two equilibria of the game: one in which the agent adopts a cut-off entry strategy $\underline{\nu}^*$ and one in which the agent adopts a cut-off entry strategy $\overline{\nu}^*$, where $0 < \underline{\nu}^* < \overline{\nu}^* < 1$. If $F = \max_{x\in[0,1]} \Gamma(x)$, there is a unique equilibrium with cut-off entry strategy $\underline{\nu}^* = \overline{\nu}^*$.*

For any entry strategy $\nu^* \in \{\underline{\nu}^*; \overline{\nu}^*\}$, the equilibrium contract features the equilibrium quantities $q^*(\underline{\theta}) = S'^{-1}(\underline{\theta})$ and

$$q^*\left(\overline{\theta}, \nu^*\right) = S'^{-1}\left(\overline{\theta} + \frac{\mathbb{E}\left[\nu|\nu \geq \nu^*\right]}{1-\mathbb{E}\left[\nu|\nu \geq \nu^*\right]}\Delta\theta\right);$$

and the equilibrium transfers $t^*\left(\overline{\theta}\right) = \overline{\theta} q^*\left(\overline{\theta}, \nu^*\right)$ and

$$t^*\left(\underline{\theta}\right) = \Delta\theta q^*\left(\overline{\theta}, \nu^*\right) + \underline{\theta} q^*\left(\underline{\theta}\right),$$

where $q^*\left(\overline{\theta}, \nu^*\right)$ is decreasing in ν^*.

If $F > \max_{x\in[0,1]} \Gamma(x)$, the agent does not enter into the market.

Figure 1 shows the function $\Gamma(\cdot)$ and the two cut-off equilibria.[10] The shape of the function $\Gamma(x)$ depends on two contrasting effects of a higher cut-off x on the equilibrium information rent of the marginal agent's type. On the one hand, a higher x implies a higher probability of obtaining an information rent for the marginal agent's type. On the other hand, however, a higher x also increases the principal's expectation of the probability that the agent has a low cost conditional entry — i.e., $E[\nu|\nu \geq x]$ — and, hence, the distortion of the quantity produced by a high-cost agent, which reduces the agent's rent (other things being equal). The function $\Gamma(x)$ is concave because the effect of an increase in x on $q\left(\overline{\theta}, x\right)$ is stronger when x is larger, since in this case the principal contracts with

[10]Of course, the function $\Gamma(\cdot)$ should not be interpreted as representing the expected utility that different ex ante types of the agent obtain by entering the market.

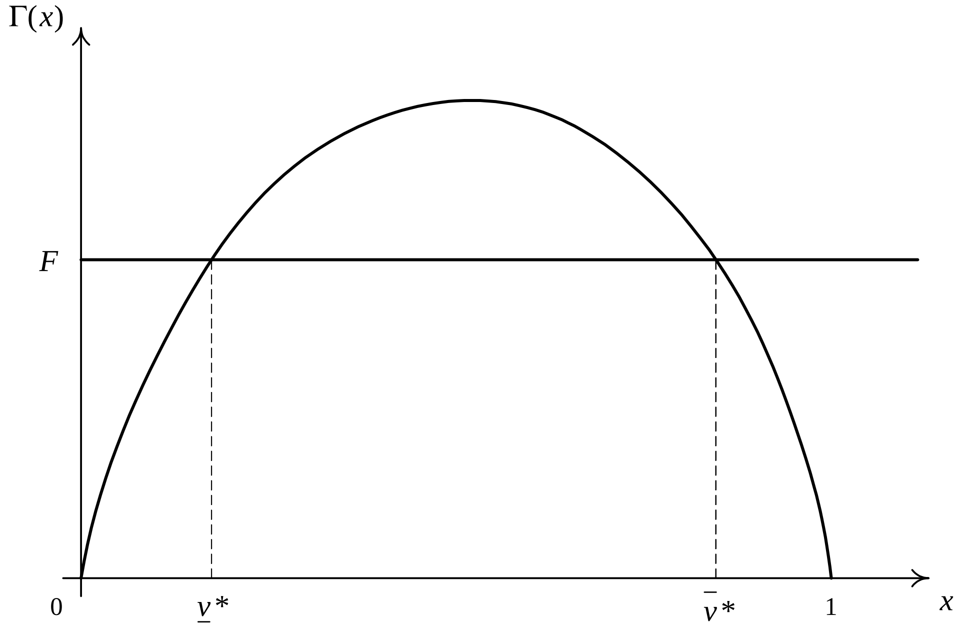

Figure 1: Entry cut-off equilibria

an agent who is relatively more likely to have a low-cost, and hence he is more willing to distort the quantity produced by a high-cost agent. In fact, at the extreme when $x = 1$, the principal shuts down production of a high-cost agent, so that the agent's expected information rent must necessarily decrease for x sufficiently large.[11]

Since $\Gamma(\cdot)$ is single peaked by assumption, there are exactly two values of ν^* that satisfy condition 3, provided F is not too high: the first effect dominates when $\Gamma(x)$ is increasing and pins down $\underline{\nu}^*$, the second effect dominates when $\Gamma(x)$ is decreasing and pins down $\overline{\nu}^*$.[12]

Compared to the equilibrium with cut-off $\underline{\nu}^*$, in the equilibrium with cut-off $\overline{\nu}^*$: (i) a larger mass of the agent's types $(1-\underline{\nu}^*)$ enters the market (see Figure 1) and (ii) after entry the principal contracts with an agent with a lower expected ν and, hence, he reduces the distortion of the quantity produced when the agent actually has a high cost (since this is relatively more likely in expectation).

Remark. Notice that allowing the agent to send a cheap talk message to the principal about ν before the principal offers the contract does not affect our results. The reason is that the agent cannot credibly communicate any information with a cheap talk message,

[11] The properties of the function $\Gamma(\cdot)$ are discussed in more details in the proof of Theorem 1. See also Laffont and Martimort (2002, p. 71).

[12] If $\Gamma(\cdot)$ is not single peaked, there may be more than two equilibria, but the equilibrium with the highest cut-off always has the same properties of the equilibrium with cut-off $\overline{\nu}^*$ in our model.

since all agent's type would want the principal to believe that their type is $\nu = 0$ in order to obtain the highest possible information rent. In other words, in contrast to standard cheap talk games where senders have different ideal points, in our model every ex ante type of the agent has the same "ideal point," which is represented by the situation in which the principal does not distort the quantity of the high-cost agent. Of course, the principal would like to elicit information on ν and contract on it before the agent learns its cost — see Courty and Li (2000) who analyze a sequential screening model where the principal has the commitment power to contract with the agent ex ante.

3.3 Comparative Statics

We now analyze the effect of varying the fixed cost of entry in the two equilibria of our model characterized in Theorem 1.

Proposition 1 *The cut-off $\underline{\nu}^*$ is increasing in F and the cut-off $\overline{\nu}^*$ is decreasing in F.*

The comparative statics results for the equilibrium with cut-off $\underline{\nu}^*$ is consistent with the results of many standard IO models: as entry becomes more costly, only ex ante types who expect to be relatively more efficient and have a low cost with higher probability choose to enter.

By contrast, in the equilibrium with cut-off entry strategy $\overline{\nu}^*$, an increase in the fixed cost of entry reduces the marginal agent's type in the market, which yields a higher probability that the agent enters the market — i.e., a larger set of ex ante types of the agent enter into the market — and lower expected probability that the agent has a low cost when he enters. Therefore, when F increases and the equilibrium has cut-off $\overline{\nu}^*$, the principal interacts with a less efficient agent on average, conditional on entry.

The intuition for this result is as follows: when F increases, the information rent obtained by the marginal agent's type $\overline{\nu}^*$ has to increase in order to induce him to enter the market. This increase is achieved through a reduction in $\mathbb{E}[\nu|\nu \geq \overline{\nu}^*]$, which reduces the distortion imposed by the principal on the quantity produced by a high-cost agent. Hence, contrary to a standard intuition, in the equilibrium of our model with the highest cut-off an increase in the cost of entry induces neither a reduction in the mass of the agent's ex-ante types who enter the market, nor an increase in the expected efficiency of an agent who enters.

4 Comparing Equilibria

In this Section we compare the two equilibria characterized in Theorem 1, in order to analyze whether there are reasons to expect that any of them may not be played in our model. We first show that no equilibrium always Pareto dominates the other and that both equilibria survive the risk dominance selection criterion (Harsanyi and Selten, 1988) applied to a coordination game where players choose among equilibria. We then focus on the agent, the player who chooses first in our environment. While the equilibrium with the lowest cut-off yields a higher utility for the agent, the equilibrium with the highest cut-off is the only one that is "robust to equilibrium risk" (in a sense that is formalized below) from his point of view.

4.1 Pareto Dominance

The equilibrium with cut-off $\underline{\nu}^*$ yields a higher utility for every agent's ex ante type than the equilibrium with cut-off $\overline{\nu}^*$. First, all types $\nu \geq \overline{\nu}^*$ obtain a strictly higher information rent in this equilibrium than in the equilibrium with cut-off $\overline{\nu}^*$, because $q^*\left(\overline{\theta}, \nu^*\right)$ is decreasing in ν^* and hence

$$\nu \Delta\theta q^*\left(\overline{\theta}, \underline{\nu}^*\right) > \nu \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right).$$

Second, types between $\underline{\nu}^*$ and $\overline{\nu}^*$ also strictly prefer the equilibrium with cut-off $\underline{\nu}^*$, because they do not enter the market in the other equilibrium. Third, all types $\nu \leq \underline{\nu}^*$ are indifferent between the two equilibria, because they enter in neither of the equilibria.

From the point of view of the principal, however, no equilibrium is Pareto dominant. In order to show this, it is useful to notice that, conditional on entry, the principal is better off in the equilibrium with cut-off $\overline{\nu}^*$, since in this equilibrium he contracts with an agent who is more efficient on average.

Lemma 3 *The principal's interim expected profit is higher in the equilibrium with cut-off $\overline{\nu}^*$ than in the equilibrium with cut-off $\underline{\nu}^*$.*

For any equilibrium with cut-off $\nu^* \in \{\underline{\nu}^*, \overline{\nu}^*\}$, the principal's expected profit can be written as

$$V\left(\nu^*\right) = \left(1 - G\left(\nu^*\right)\right) \widetilde{V}\left(\nu^*\right),$$

where $\widetilde{V}\left(\nu^*\right)$ denotes the principal's interim profit and $1 - G\left(\nu^*\right)$ is the probability that the agent enters. Hence, the difference between the principal's expected profits in the two equilibria is

$$V(\underline{\nu}^*) - V(\overline{\nu}^*) \equiv \widetilde{V}(\overline{\nu}^*)\left[G(\overline{\nu}^*) - G(\underline{\nu}^*)\right] - (1 - G(\underline{\nu}^*))\left[\widetilde{V}(\overline{\nu}^*) - \widetilde{V}(\underline{\nu}^*)\right], \qquad (4)$$

which depends on the differences between the probabilities of entry and between the principal's interim profits in the two equilibria.

The first term of this expression is positive since $G(\overline{\nu}^*) > G(\underline{\nu}^*)$: there is more entry in the equilibrium with cut-off $\underline{\nu}^*$ than in the equilibrium with cut-off $\overline{\nu}^*$, which increases the principal's expected profit (other things being equal). By Lemma (3), however, the second term of the expression is negative: conditional on entry, the principal's interim profit is higher in the equilibrium with cut-off $\overline{\nu}^*$. In general, which of these two contrasting effects dominates depends on their relative strengths.

When F is sufficiently small, the effect of the probability of entry dominates and the principal always obtains a higher profit in the equilibrium with cut-off $\underline{\nu}^*$. The reason is that $\underline{\nu}^* \to 0$ and $\overline{\nu}^* \to 1$ as $F \to 0$ (see Figure 1), so that the agent almost never enters in the equilibrium with cut-off $\overline{\nu}^*$ and $V(\overline{\nu}^*) \to 0$, while he almost always enters in the equilibrium with cut-off $\underline{\nu}^*$. By contrast, when F increases, the distance between the equilibrium cut-offs $\overline{\nu}^*$ and $\underline{\nu}^*$ shrinks, which reduces the difference between the probabilities of entry in the two equilibria. In the Appendix we show that, for F close to $\max_x \Gamma(x)$, the principal obtains a higher profit in the equilibrium with cut-off $\overline{\nu}^*$ when the c.d.f. $G(\nu)$ is concentrated around $\nu = 1$.[13] In this case, the probability of entry is almost 1 in both equilibria, and the effect of the principal's interim profit in expression (4) dominates.

Summing up, depending on F and $G(\cdot)$ either the equilibrium with cut-off $\underline{\nu}^*$ Pareto dominates the equilibrium with cut-off $\underline{\nu}^*$ for both the principal and the agent, or no equilibrium Pareto dominates the other (while the equilibrium with cut-off $\underline{\nu}^*$ never Pareto dominates the equilibrium with cut-off $\overline{\nu}^*$ for both the principal and the agent).

Proposition 2 *No equilibrium is Pareto dominant for all F and $G(\cdot)$.*

Therefore, neither of the two equilibria of our model can always be discarded by a Pareto dominance criterion.

[13] Formally, this requires the hazard rate of $G(\cdot)$ to be sufficiently small.

4.2 Risk Dominance

In order to investigate the robustness of our equilibria to the logic of risk dominance, we analyze a coordination game between the principal and an agent of type ν, where players simultaneously choose between the two equilibria of our model, and apply the standard definition by Harsanyi and Selten (1988) of a risk dominant equilibrium for two-player, two-strategy game.[14]

Suppose that the agent plays according to the equilibrium with cut-off $x \in \{\underline{\nu}^*; \overline{\nu}^*\}$, while the principal plays according to the equilibrium with cut-off $y \in \{\underline{\nu}^*; \overline{\nu}^*\}$. In this case, the agent enters if and only if $\nu \geq x$, and the principal offers a contract with quantity $q^*\left(\overline{\theta}, y\right)$. Abusing notation, let

$$U(x, y, \nu) \equiv \mathbb{I}_x(\nu)\left[\nu \Delta\theta q^*\left(\overline{\theta}, y\right) - F\right]$$

be the utility of an agent of type ν, where the indicator function $\mathbb{I}_x(\nu)$ is equal to 1 if the agent enters and zero otherwise; and let

$$\begin{aligned} V(y, x) \equiv & \int_x^1 \nu\left(S\left(q^*\left(\underline{\theta}\right)\right) - \underline{\theta} q^*\left(\underline{\theta}\right) - \Delta\theta q^*\left(\overline{\theta}, y\right)\right) dG(\nu) \\ & + \int_x^1 (1-\nu)\left(S\left(q^*\left(\overline{\theta}, y\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, y\right)\right) dG(\nu), \end{aligned}$$

be the expected utility of the principal (since he knows that he contracts with an agent whose type is weakly higher than x).

Consider the following coordination game, where players simultaneously choose among the two equilibria of our model.

		Agent of type ν: $\underline{\nu}^*$		Agent of type ν: $\overline{\nu}^*$	
Principal	$\underline{\nu}^*$	$V(\underline{\nu}^*, \underline{\nu}^*)$	$U(\underline{\nu}^*, \underline{\nu}^*, \nu)$	$V(\underline{\nu}^*, \overline{\nu}^*)$	$U(\overline{\nu}^*, \underline{\nu}^*, \nu)$
	$\overline{\nu}^*$	$V(\overline{\nu}^*, \underline{\nu}^*)$	$U(\underline{\nu}^*, \overline{\nu}^*, \nu)$	$V(\overline{\nu}^*, \overline{\nu}^*)$	$U(\overline{\nu}^*, \overline{\nu}^*, \nu)$

This game has two pure strategy Nash equilibria: one in which both players choose $\underline{\nu}^*$, and one in which both players choose $\overline{\nu}^*$. An equilibrium risk dominates the other if and

[14]Risk dominance cannot be applied directly to our model because the principal's strategy space is not binary and there is incomplete information. Notice however that, although we have defined our model as a sequential game, the environment can also be interpreted as a game in which the agent and the principal choose simultaneously, since the agent reveals his type to the principal only after being offered a contract.

only if the product of the deviation losses is highest in the former equilibrium (Harsanyi and Selten, 1988).

Proposition 3 *In the simultaneous coordination game, neither the equilibrium* $(\underline{\nu}^*, \underline{\nu}^*)$ *nor the equilibrium* $(\overline{\nu}^*, \overline{\nu}^*)$ *is risk dominant.*

Therefore, neither of the two equilibria of our model can be discarded by a risk dominance criterion.

4.3 Robustness to Equilibrium Risk

We now consider equilibrium selection from the point of view of the agent alone, since he is the player who moves first in our environment and chooses whether to initiate interaction with the principal by entering the market.[15] While the equilibrium with cut-off $\underline{\nu}^*$ yields a higher utility for the agent than the equilibrium with cut-off $\overline{\nu}^*$ (as we have shown above), the equilibrium with cut-off $\overline{\nu}^*$ is the only one that is "robust to equilibrium risk" for the agent, and may be preferred by an agent who is uncertain about the choice of equilibrium by the principal.

To see why the equilibrium with cut-off $\overline{\nu}^*$ may be preferred by the agent, notice that an agent with ex ante type higher than $\overline{\nu}^*$ obtains a strictly positive utility from entering the market in any of the two equilibria, while an agent with ex ante type between $\underline{\nu}^*$ and $\overline{\nu}^*$ obtains a positive utility from entering in the equilibrium with cut-off $\underline{\nu}^*$, but not in the equilibrium with cut-off $\overline{\nu}^*$. Therefore, the equilibrium with cut-off $\underline{\nu}^*$ may be perceived as more risky by the agent, if he is uncertain about the equilibrium that will be played by the principal.

More precisely, suppose that, when choosing whether to enter, the agent expects the principal to offer with equal probability each of the two equilibrium contracts — i.e., a contract with quantity $q^*\left(\overline{\theta}, \underline{\nu}^*\right)$ or a contract with quantity $q^*\left(\overline{\theta}, \overline{\nu}^*\right)$ for a high-cost agent. In this case, agents with ex-ante types greater than $\overline{\nu}^*$ enter since they obtain a strictly higher profit from entering than from staying out, with both equilibrium contracts. By contrast, agents with ex-ante types lower than $\underline{\nu}^*$ do not enter since they always obtain a negative profit from entering. For agents with ex-ante types in the interval $[\underline{\nu}^*, \overline{\nu}^*]$, there exists a threshold $k \in (\underline{\nu}^*, \overline{\nu}^*)$ such that agents with ex-ante types $\in (k, \overline{\nu}^*]$ enter, while agents with ex-ante types $\in [\underline{\nu}^*, k)$ stay out. Therefore, some agent's type that should enter in the equilibrium with cut-off $\underline{\nu}^*$ do not enter in the presence of equilibrium uncertainty.

[15] This is consistent with the logic of equilibrium refinements for signalling games, that focus on senders.

To formalize this idea, we introduce the following definition that allows us to select among the equilibria of our model, when the agent faces some uncertainty about which of the two equilibrium contracts will be offered by the principal in the market.

Definition 1 *An equilibrium with cut-off $\nu^* \in \{\underline{\nu}^*; \overline{\nu}^*\}$ is robust to equilibrium risk for the agent if all agent's types $\nu > \nu^*$ who enter in equilibrium would also enter in the presence of a positive probability, which may be arbitrarily small, that the principal plays the other equilibrium.*

Based on the discussion above, the only equilibrium that is robust to equilibrium risk for the agent is the one with cut-off $\overline{\nu}^*$. In fact, if the agent assigns a positive probability to the principal playing the equilibrium with cut-off $\overline{\nu}^*$, then an agent with ex ante type sufficiently close to $\underline{\nu}^*$ strictly prefers not to enter, since his expected information rent is lower than F.[16]

Proposition 4 *The equilibrium with cut-off $\overline{\nu}^*$ is robust to equilibrium risk for the agent, while the equilibrium with cut-off $\underline{\nu}^*$ is not robust to equilibrium risk for the agent.*

Therefore, an agent who is uncertain about the equilibrium that will actually be played by the principal is likely to select the equilibrium with cut-off $\overline{\nu}^*$ rather than the equilibrium with cut-off $\underline{\nu}^*$, although he obtains a lower utility in the former equilibrium.

Finally, following the "Max-Min" logic considered in Bassetto and Phelan (2008), we analyze which equilibrium is selected by a player who maximizes the lowest possible payoff that he may obtain by choosing a particular equilibrium, when the other player chooses the other equilibrium. In other words, we compare the off-equilibrium payoffs in the coordination game where players choose among the two equilibria of our model. In the Appendix, we show that $V(\overline{\nu}^*, \underline{\nu}^*) > V(\underline{\nu}^*, \overline{\nu}^*)$ and $U(\overline{\nu}^*, \underline{\nu}^*, \nu) \geq U(\underline{\nu}^*, \overline{\nu}^*, \nu)$ — i.e., both the principal and the agent obtain a higher payoff by choosing the equilibrium with cut-off $\overline{\nu}^*$, when the other player chooses a different equilibrium from the one they choose. Therefore, a principal and an agent who prefer the best worst-case scenario select the equilibrium with the highest cut-off in our model.

[16] If the function $\Gamma(\cdot)$ is not single peaked and there are more than two equilibria, then the only equilibrium that is robust to equilibrium risk is the one with the highest cut-off.

5 Players' Choice of Entry Cost

In this section, we analyze how changes in the entry cost affects social welfare, the agent's expected utility and the principal's expected profit. We assume that while players may be able to change F, they cannot directly influence entry or the contract offered by the principal. Moreover, we restrict the analysis to $F \in \left[\underline{F}, \max_{x\in[0,1]} \Gamma(x)\right]$, with $\underline{F} > 0$ and arbitrarily small — i.e., we assume that players cannot completely eliminate the entry cost.[17] We first consider the equilibrium with cut-off $\overline{\nu}^*$, and then discuss how the results change in the other equilibrium characterized in Theorem 1.

Proposition 5 *In the equilibrium with cut-off $\overline{\nu}^*$, no player wants to minimize the entry cost F.*

The intuition, as well as the proof, of this result is straightforward. In the equilibrium with cut-off $\overline{\nu}^*$, choosing F arbitrarily close to zero reduces to zero the probability of entry and, hence, players' profits and social welfare, which is never optimal for any of the players.

In reality, players may only be able to modify entry costs marginally, and costs may be sticky in the short run. For example, entry costs may depend on some exogenous technological factors on which players only have a limited influence. To analyze this context, we conduct a comparative statics analysis starting from a positive and discrete entry cost and ask whether players have an incentive to slightly reduce or increase this cost around its initial level. We show that the incentive to change F depends on the identity of the player who has the ability to marginally affect it.

First consider a social planner. Ex-ante total expected welfare is

$$\int_{\overline{\nu}^*}^{1} \mathrm{W}(\nu)\, dG(\nu), \tag{5}$$

where

$$\mathrm{W}(\nu) \equiv \nu \left[S\left(q^*\left(\underline{\theta}\right)\right) - \underline{\theta} q^*\left(\underline{\theta}\right)\right] + (1-\nu)\left[S\left(q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right] - F$$

is the (equilibrium) total welfare when an agent of type ν enters the market.

[17] Of course, if $F > \max_{x\in[0,1]} \Gamma(x)$ no agent enters and total welfare is zero.

Differentiating (5) with respect to F and rearranging

$$\underbrace{\frac{g\left(\overline{\nu}^*\right)}{1-G(\overline{\nu}^*)}\left|\frac{\partial\overline{\nu}^*}{\partial F}\right|\mathrm{W}\left(\overline{\nu}^*\right)}_{\text{Entry effect }(+)}+\underbrace{\Delta\theta\frac{\partial q^*\left(\overline{\theta},\overline{\nu}^*\right)}{\partial\overline{\nu}^*}\frac{\partial\overline{\nu}^*}{\partial F}\mathbb{E}\left[\nu|\nu\geq\overline{\nu}^*\right]}_{\text{Production effect }(+)}-\underbrace{1}_{\text{Entry cost effect }(-)}.$$

A change in the entry cost around the initial level has three effects on welfare. First, an increase in F reduces $\overline{\nu}^*$ and increases the mass of agents who enter, thus increasing welfare: the entry effect.[18] This effect is proportional to $\mathrm{W}(\overline{\nu}^*)$, the welfare created in equilibrium by the marginal type, and to $\left|\frac{\partial\overline{\nu}^*}{\partial F}\right|$, a measure of how responsive is $\overline{\nu}^*$ to changes in the entry cost. Second, an increase in F reduces the distortion of the quantity produced by a high-cost agent, which increases welfare: the production effect.[19] This effect is proportional to $\Delta\theta$, a measure of the severity of the adverse selection problem: a larger $\Delta\theta$ induces the principal to distort the production of the inefficient type more (other things being equal), thus enhancing the mitigating effect of an increase on the quantity distortion. Third, an increase in F increases the entry cost paid by an agent who enters: the entry cost effect.

In sum, the net effect on social welfare of a marginal increase in F is positive when the first two effects dominate the third one. In this case, a social planner can increase welfare in the market by increasing the entry cost.

Consider now the agent. The ex-ante expected information rent of an agent, net of the entry cost, is

$$\int_{\overline{\nu}^*}^{1}\left[\nu\Delta\theta q^*\left(\overline{\theta},\overline{\nu}^*\right)-F\right]dG\left(\nu\right).$$

Differentiating with respect to F and rearranging

$$-\underbrace{\left[\overline{\nu}^*\Delta\theta q^*\left(\overline{\theta},\overline{\nu}^*\right)-F\right]}_{=0}+\int_{\overline{\nu}^*}^{1}\left[\nu\Delta\theta\frac{\partial q^*\left(\overline{\theta},\overline{\nu}^*\right)}{\partial\overline{\nu}^*}\frac{\partial\overline{\nu}^*}{\partial F}-1\right]dG\left(\nu\right)=$$

$$=\underbrace{\Delta\theta\frac{\partial q^*\left(\overline{\theta},\overline{\nu}^*\right)}{\partial\overline{\nu}^*}\frac{\partial\overline{\nu}^*}{\partial F}\mathbb{E}\left[\nu|\nu\geq\overline{\nu}^*\right]}_{\text{Rent effect }(+)}-\underbrace{1}_{\text{Entry cost effect }(-)}.$$

A change in the entry cost has two contrasting effects on on the agent's expected utility. A higher F increases the agent's expected rent because it reduces the quantity distortion

[18] Note that, by definition, $\mathrm{W}(\overline{\nu}^*)>0$ because if $\mathrm{W}(\overline{\nu}^*)<0$ then also the agent's rent would be negative when his ex-ante type is $\overline{\nu}^*$.

[19] Recall that $\frac{\partial q^*(\overline{\theta},\overline{\nu}^*)}{\partial\overline{\nu}^*}<0$ because when, the expected probability of the agent having a low cost increases, the principal has an incentive to distort production for the high-cost agent more.

imposed by the principal: the rent effect. But a higher F also increases the cost paid by an agent who enters.

Interestingly, the expression for the effect of a variation in F on the agent's profit is identical to the one for the planner excluding the entry effect. This is because the agent does not take into account the effect of a change in F on total welfare. Hence, when the agent prefers to increase the entry cost around its initial level to increase his utility, the social planner wants to do the same since social welfare is also increasing in F.

Finally, consider the principal. His ex-ante expected profit is

$$\int_{\overline{\nu}^*}^{1} \left\{\nu \left[S\left(q^*\left(\underline{\theta}\right)\right) - \underline{\theta} q^*\left(\underline{\theta}\right) - \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right] + (1-\nu)\left[S\left(q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right]\right\} dG\left(\nu\right).$$

Differentiating with respect to F,

$$\frac{\partial q^*\left(\overline{\theta}, \overline{\nu}^*\right)}{\partial \overline{\nu}^*} \frac{\partial \overline{\nu}^*}{\partial F} \int_{\overline{\nu}^*}^{1} \left\{-\nu \Delta\theta + (1-\nu)\left[S'\left(q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \overline{\theta}\right]\right\} dG\left(\nu\right)$$
$$-\left\{\overline{\nu}^* \left[S\left(q^*\left(\underline{\theta}\right)\right) - \underline{\theta} q^*\left(\underline{\theta}\right) - \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right] + (1-\overline{\nu}^*)\left[S\left(q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right]\right\} g\left(\overline{\nu}^*\right) \frac{\partial \overline{\nu}^*}{\partial F}.$$

Using the free entry condition $\overline{\nu}^* \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right) = F$ and dividing and multiplying the first term by $1 - G\left(\overline{\nu}^*\right)$, by the Envelope Theorem this expression can be rewritten as

$$-g\left(\overline{\nu}^*\right) \mathrm{W}\left(\overline{\nu}^*\right) \frac{\partial \overline{\nu}^*}{\partial F},$$

which is always positive. Hence, the principal always benefits from an increase in F: since quantities are optimally chosen to maximize his profit after entry, the principal simply wants to maximize entry into the market.

Finally, in the equilibrium with cut-off $\underline{\nu}^*$, all players are harmed by an increase in F around its initial value because, by Proposition 1, a reduction in F reduces the equilibrium cut-off $\underline{\nu}^*$ and all the effects that we have identified are negative in this case.

6 Applications

Our results can be framed in several different market applications to analyze entry.

Regulation and procurement. In Baron and Myerson (1982), the principal is a regulator who maximizes a weighted average of consumers' surplus $CS\left(q\right)$ and a regulated monopoly's profit $U\left(q\right) = t - \theta q$, where θ is the marginal cost of production and q the quantity produced. Letting α be the weight assigned to the monopolist's profit, the principal's profit can be written as

$$V(\cdot) = CS(q) - \theta q - \alpha U(q).$$

In this environment, $\nu = \Pr[\theta = \underline{\theta}]$ can be interpreted as an ex-ante idiosyncratic characteristic of a firm that captures, for example, its technological standard or its ability to successfully perform R&D or other cost-reducing activities; while F can be interpreted either as a start-up fee or, in the procurement case, as the cost of specialization required to produce the specific product requested by the procurer.

Our results suggest that, in equilibrium, an increase in the start up fee or in the specialization cost may result in the selection of a less efficient firm on average.

Non-linear pricing. The principal can also be interpreted as the seller of a good with marginal cost of production $c(q)$ who faces a continuum of potential buyers. The principal's utility function is

$$V(\cdot) = t - c(q),$$

where t represents the price and q the quantity of the good. A buyer's utility function is

$$U(\cdot) = \theta u(q) - t,$$

where θ is a measure of his taste for the good (as in Maskin and Riley, 1984) or his preference for higher-quality products (as in Mussa and Rosen, 1978, where q represents the quality of the good). Changing notation, $\nu = \Pr[\theta = \overline{\theta}]$ can be interpreted as the ex-ante confidence of the buyer in the fit between the good's characteristics and her private tastes; while F can be interpreted as a proxy for search and experimentation costs that a consumer has to pay in order to reach the seller and discover his actual taste for the specific good on sale.

Our results suggest that, in equilibrium, higher search and experimentation costs may result in the seller interacting with buyers who expect to have a lower taste for the good.

Financial contracting. In Freixas and Laffont (1990), the principal is a lender in a financial market who provides a loan of size k, with repayment t, to a borrower. Capital costs Rk, where R is the risk-free interest rate that the lender would earn in an alternative investment. Therefore, the lender's profit is

$$V(\cdot) = t - Rk,$$

while the borrower's profit is

$$U(\cdot) = \theta f(k) - t,$$

where $f(k)$ is the return on capital that the borrower is able to generate and θ is a productivity shock. In this context, $\nu = \Pr\left[\theta = \overline{\theta}\right]$ can be interpreted as the ex-ante probability of the borrower's return being high, and F can be interpreted as a fixed fee that the borrower has to pay to submit a loan application.

Our results suggests that a higher submission fee may induce borrowers with a lower probability of having high return on capital, on average, to apply for a loan.

Labor contracts. In Green and Khan (1983) and Hart (1983), the principal is a union, or a set of workers, that provides its labor force l to a firm and has full bargaining power in determining the labor contract. The firm's profit is

$$U(\cdot) = \theta f(l) - t,$$

where $f(l)$ is the return on labor, t is the wage, and θ is a productivity shock which is observed by the firm but not by the union. The union's utility function is

$$V(\cdot) = t - \psi(l),$$

where $\psi(l)$ is the monetary disutility of labor. In this context, F can be interpreted a fixed cost that the firm has to pay to contract with the union.

Our results suggests that, when it is more costly to contract with a union, firms that expect to be less efficient and have lower productivity, on average, may choose to do so.

Distribution channels. Following the literature on vertical contracting in distribution channels (e.g., Gal-Or, 1991, 1999; Martimort, 1996; Pagnozzi *et al.*, 2016), the principal can be interpreted as an upstream supplier who sells an intermediate good to a downstream retailer who distributes the final product to final consumers and is better informed than the supplier on some relevant characteristics of the market. Assuming for simplicity that production is costless for the supplier, its utility function is

$$V(\cdot) = t,$$

where t is the price paid by the retailer and q the quantity of the good. The retailer's utility function is

$$U(\cdot) = P(\theta, q)\, q - t,$$

where $P(\theta, q)$ is the demand function and θ is a measure consumers' preferences for the final product, with $\frac{\partial P(\cdot)}{\partial \theta} > 0$. In this context, $\nu = \Pr\left[\theta = \overline{\theta}\right]$ can be interpreted as the retailer's ability to appeal to consumers' preferences, while F can be interpreted as a specialization costs that the retailer has to pay in order to be able to sell the product of a specific manufacturer.

Our results suggest that an increases in the retailer's specialization costs may induce manufacturer to contract with retailers that are less capable of appealing to consumers' preferences, on average.

7 Conclusions

We studied a simple principal-agent model in which the distribution of the agent's type is endogenous and determined by an entry condition that equalizes the agent's expected information rent to the fixed entry cost.

Under standard assumptions, our game has two cut-off equilibria that determine the mass and the characteristics of agents who enter the market. Contrasting with standard intuition, in the equilibrium with the highest cut-off an increase in the entry cost increases the mass of agents who enter and reduces the expected efficiency of the entrant. This equilibrium is selected by a criterion based on "robustness to equilibrium risk." Hence, our analysis suggests that in industries characterized by adverse selection, barriers to entry may be positively correlated with entry by privately informed agents. Moreover, increasing entry costs may not necessarily decrease welfare.

Our simple environment has multiple interpretations — like non-linear pricing, financial and labor contracting, regulation and procurement — and may help interpreting the effects of changes in entry costs on welfare and profits in a variety of applications.

A Appendix

Proof of Lemma 1. Because contracts are offered after entry decisions and the agent discovers his cost, when the agent decides whether to accept the contract in stage 3 his ex ante type does not affect his utility. Hence, the principal cannot screen the agent along the ν dimension and induce him to reveal his ex ante type. ■

Proof of Lemma 2. Fix the contract offered by the principal. If a type ν' chooses to enter, then $\nu'\Delta\theta q\left(\overline{\theta}\right) > F$. Therefore, any type $\nu > \nu'$ also obtains an expected information rent higher than F and chooses to enter. If a type ν'' chooses not to enter, then $\nu''\Delta\theta q\left(\overline{\theta}\right) < F$. Therefore, any type $\nu < \nu''$ also obtains an expected information rent lower than F and chooses not to enter. ■

Proof of Theorem 1. Notice that $\Gamma(0) = 0$ and that (since $S'(0) = +\infty$)

$$\begin{aligned}\Gamma(1) &= \Delta\theta \times S'^{-1}\left(\overline{\theta} + \frac{\mathbb{E}[\nu|\nu \geq 1]}{1-\mathbb{E}[\nu|\nu \geq 1]}\Delta\theta\right) \\ &= \Delta\theta \times S'^{-1}(+\infty) = 0.\end{aligned}$$

Moreover,

$$\Gamma'(x) = \underbrace{\Delta\theta \times S'^{-1}(\cdot)}_{>0} + \underbrace{x\frac{(\Delta\theta)^2}{S''(\cdot)}\frac{\mathbb{E}[\nu|\nu \geq x]}{(1-\mathbb{E}[\nu|\nu \geq x])^2}\frac{\partial\mathbb{E}[\nu|\nu \geq x]}{\partial x}}_{<0}, \tag{6}$$

where $\Gamma'(0) > 0$ since the second term of equation (6) is equal to zero when $x = 0$, and $\Gamma'(1) < 0$ since $q^*\left(\overline{\theta}, 1\right) = 0$. Hence, since the function $\Gamma(x)$ is single peaked by assumption, when $F < \max_{x\in[0,1]}\Gamma(x)$ equation 3 has exactly two solutions, both strictly positive and lower than 1. So there are two cut-off entry equilibrium strategies.

The expressions for the quantities chosen by the principal for the equilibrium contracts immediately follow from the discussion preceding the statement of the theorem and from equation (2). Finally, the definition of $U(\cdot)$ and the binding individual rationality and participation constraints yield the equilibrium transfers.

The quantity $q^*\left(\overline{\theta}, \nu^*\right)$ is decreasing in ν^* because $\frac{\partial\mathbb{E}[\nu|\nu\geq\nu^*]}{\partial\nu^*} \geq 0$ and $S'^{-1}(\cdot)$ is decreasing.

When $F = \max_{x\in[0,1]}\Gamma(x)$, equation 3 has a unique solution equal to $\arg\max_{x\in[0,1]}\Gamma(x)$. By contrast, when $F > \max_{x\in[0,1]}\Gamma(x)$, equation 3 is never satisfied. ■

Proof of Proposition 1. Recall that entry is determined by the condition $\Gamma(\nu^*) = F$ (see condition 3). In the equilibrium with cut-off $\overline{\nu}^*$, by the Implicit Function Theorem we have

$$\frac{\partial\overline{\nu}^*}{\partial F} = \frac{1}{\frac{\partial}{\partial x}\Gamma(\overline{\nu}^*)}.$$

This is negative since $\frac{\partial}{\partial x}\Gamma\left(\overline{\nu}^*\right) < 0$ (because the equilibrium exists if and only if $F < \max_{x\in[0,1]}\Gamma(x)$ and $\Gamma(\cdot)$ is single peaked by assumption).

In the equilibrium with cut-off $\underline{\nu}^*$, by the Implicit Function Theorem we have

$$\frac{\partial \underline{\nu}^*}{\partial F} = \frac{1}{\frac{\partial}{\partial x}\Gamma\left(\underline{\nu}^*\right)}.$$

This is positive since $\frac{\partial}{\partial x}\Gamma\left(\underline{\nu}^*\right) > 0$ (because the equilibrium exists if and only if $F < \max_{x\in[0,1]}\Gamma(x)$ and $\Gamma(\cdot)$ is single peaked). ■

Proof of Lemma 3. For any equilibrium with cut-off entry strategy $\nu^* \in \{\underline{\nu}^*, \overline{\nu}^*\}$, the principal's interim expected profit — i.e., conditional on the agent entering the market — is

$$\begin{aligned}\widetilde{V}\left(\nu^*\right) \equiv\ & \mathbb{E}\left[\nu|\nu \geq \nu^*\right]\left[S\left(q^*\left(\underline{\theta}\right)\right) - \underline{\theta}q^*\left(\underline{\theta}\right) - \Delta\theta q^*\left(\overline{\theta},\nu^*\right)\right] \\ & + \left(1 - \mathbb{E}\left[\nu|\nu \geq \nu^*\right]\right)\left[S\left(q^*\left(\overline{\theta},\nu^*\right)\right) - \overline{\theta}q^*\left(\overline{\theta},\nu^*\right)\right].\end{aligned}$$

Differentiating with respect to ν^* and using the Envelope Theorem,

$$\frac{d\widetilde{V}\left(\nu^*\right)}{d\nu^*} = \frac{\partial\mathbb{E}\left[\nu|\nu \geq \nu^*\right]}{\partial\nu^*}\left\{\begin{array}{c}\left[S\left(q^*\left(\underline{\theta}\right)\right) - \underline{\theta}q^*\left(\underline{\theta}\right) - \Delta\theta q^*\left(\overline{\theta},\nu^*\right)\right] - \\ -\left[S\left(q^*\left(\overline{\theta},\nu^*\right)\right) - \overline{\theta}q^*\left(\overline{\theta},\nu^*\right)\right]\end{array}\right\}.$$

Hence, since $\frac{\partial\mathbb{E}[\nu|\nu\geq\nu^*]}{\partial\nu^*} > 0$,

$$\frac{d\widetilde{V}\left(\nu^*\right)}{d\nu^*} > 0 \quad\Leftrightarrow\quad S\left(q^*\left(\underline{\theta}\right)\right) - \underline{\theta}q^*\left(\underline{\theta}\right) - \Delta\theta q^*\left(\overline{\theta},\nu^*\right) > S\left(q^*\left(\overline{\theta},\nu^*\right)\right) - \overline{\theta}q^*\left(\overline{\theta},\nu^*\right).$$

This inequality always holds since, by the definition of $q^*\left(\underline{\theta}\right)$ and strict concavity of $S(\cdot)$,

$$\begin{aligned}S\left(q^*\left(\underline{\theta}\right)\right) - \underline{\theta}q^*\left(\underline{\theta}\right) - \Delta\theta q^*\left(\overline{\theta},\nu^*\right) &> S\left(q^*\left(\overline{\theta},\nu^*\right)\right) - \underline{\theta}q^*\left(\overline{\theta},\nu^*\right) \\ &> S\left(q^*\left(\overline{\theta},\nu^*\right)\right) - \overline{\theta}q^*\left(\overline{\theta},\nu^*\right).\end{aligned}$$

Therefore, the principal's interim expected profit is increasing in ν^*. ■

Proof of Proposition 2. In order to prove that there is no Pareto dominant equilibrium for the principal, we show that the difference between the principal's expected profit in the equilibrium with cut-off $\underline{\nu}^*$ and the principal's expected profit in the equilibrium with cut-off $\overline{\nu}^*$ is ambiguous and depends on the model's primitives.

First, for $F \to 0$, $\underline{\nu}^* \to 0$, $\overline{\nu}^* \to 1$ and, hence, $V\left(\underline{\nu}^*\right) > V\left(\overline{\nu}^*\right)$.

Let $\overline{F} \equiv \max_{x\in[0,1]}\Gamma(x)$. The fact that $\left(\overline{\nu}^* - \underline{\nu}^*\right) \to 0$ as $F \to \overline{F}$ implies that $\lim_{F\to\overline{F}}\left[V\left(\underline{\nu}^*\right) - V\left(\overline{\nu}^*\right)\right] = 0$. In order to analyze the difference between the principal's expected profits when F is close to $\overline{F}$, we consider a first-order Taylor approximation for

$F \to \overline{F}^{-}$:

$$V\left(\underline{\nu}^*\right) - V\left(\overline{\nu}^*\right) \approx \left(F - \overline{F}\right) \lim_{F \to \overline{F}^{-}} \frac{\partial\left[V\left(\underline{\nu}^*\right) - V\left(\overline{\nu}^*\right)\right]}{\partial F}.$$

Differentiating with respect to F and using the Envelope Theorem,

$$\lim_{F \to \overline{F}^{-}} \frac{\partial\left[V\left(\underline{\nu}^*\right) - V\left(\overline{\nu}^*\right)\right]}{\partial F} = \left[g\left(\widehat{\nu}\right)\widetilde{V}\left(\widehat{\nu}\right) - \left(1 - G\left(\widehat{\nu}\right)\right)\widetilde{V}'\left(\widehat{\nu}\right)\right] \lim_{F \to \overline{F}^{-}} \left(\frac{\partial \overline{\nu}^*}{\partial F} - \frac{\partial \underline{\nu}^*}{\partial F}\right),$$

where

$$\widehat{\nu} \equiv \arg\max_{x \in [0,1]} \Gamma\left(x\right)$$

and $\widehat{\nu} \in (0, 1)$ by strict concavity of $\Gamma\left(\cdot\right)$.

Therefore, for F sufficiently close to $\overline{F}$,

$$V\left(\underline{\nu}^*\right) - V\left(\overline{\nu}^*\right) \approx \left(F - \overline{F}\right) \left[g\left(\widehat{\nu}\right)\widetilde{V}\left(\widehat{\nu}\right) - \left(1 - G\left(\widehat{\nu}\right)\right)\widetilde{V}'\left(\widehat{\nu}\right)\right] \lim_{F \to \overline{F}^{-}} \left(\frac{\partial \overline{\nu}^*}{\partial F} - \frac{\partial \underline{\nu}^*}{\partial F}\right).$$

By assumption, $F < \overline{F}$, and, by Proposition 1, $\frac{\partial \overline{\nu}^*}{\partial F} < 0$ and $\frac{\partial \underline{\nu}^*}{\partial F} > 0$. Hence, $V\left(\underline{\nu}^*\right) < V\left(\overline{\nu}^*\right)$ if and only if

$$g\left(\widehat{\nu}\right)\widetilde{V}\left(\widehat{\nu}\right) - \left(1 - G\left(\widehat{\nu}\right)\right)\widetilde{V}'\left(\widehat{\nu}\right) < 0 \quad \Leftrightarrow \quad \frac{g\left(\widehat{\nu}\right)}{1 - G\left(\widehat{\nu}\right)} < \frac{\widetilde{V}'\left(\widehat{\nu}\right)}{\widetilde{V}\left(\widehat{\nu}\right)}. \tag{7}$$

The right-hand-side of condition (7) is strictly positive since $\widetilde{V}'\left(\cdot\right) > 0$ (see the proof of Lemma 3), $\widetilde{V}\left(\underline{\nu}^*\right) > 0$, and $\underline{\nu}^* < \widehat{\nu}$ (implying that $\widetilde{V}\left(\widehat{\nu}\right) > 0$).

Consider a parametrized c.d.f. $G\left(\nu, \lambda\right)$ (with $\lambda \geq 0$, $G\left(0, \lambda\right) = 0$, and $G\left(1, \lambda\right) = 1$) such that the hazard rate $\frac{g(\nu,\lambda)}{1-G(\nu,\lambda)}$ is strictly decreasing in λ — i.e., higher values of λ reflect stochastic hazard rate dominance (which also implies first-order stochastic dominance).[20] Moreover, assume that there exists $\overline{\lambda} > 0$ such that, when $\lambda \geq \overline{\lambda}$,

$$\frac{g\left(\nu, \lambda\right)}{1 - G\left(\nu, \lambda\right)} = 0 \quad \forall \nu \in [0, 1].$$

That is, for λ sufficiently large the c.d.f. $G\left(\nu, \lambda\right)$ is degenerate around 1. In this case, for F close to $\overline{F}$ and λ sufficiently large, $V\left(\underline{\nu}^*\right) < V\left(\overline{\nu}^*\right)$. ■

Proof of Proposition 3. The equilibrium $\left(\overline{\nu}^*, \overline{\nu}^*\right)$ risk dominates the equilibrium $\left(\underline{\nu}^*, \underline{\nu}^*\right)$ if

$$\begin{aligned} &\left(U\left(\underline{\nu}^*, \overline{\nu}^*, \nu\right) - U\left(\overline{\nu}^*, \overline{\nu}^*, \nu\right)\right)\left(V\left(\underline{\nu}^*, \overline{\nu}^*\right) - V\left(\overline{\nu}^*, \overline{\nu}^*\right)\right) > \\ &> \left(U\left(\overline{\nu}^*, \underline{\nu}^*, \nu\right) - U\left(\underline{\nu}^*, \underline{\nu}^*, \nu\right)\right)\left(V\left(\overline{\nu}^*, \underline{\nu}^*\right) - V\left(\underline{\nu}^*, \underline{\nu}^*\right)\right), \quad \forall \nu \in [0, 1]. \end{aligned} \tag{8}$$

[20] The parametrized exponential and the Gamma distributions satisfy this property.

By contrast, the equilibrium $(\underline{\nu}^*, \underline{\nu}^*)$ risk dominates the equilibrium $(\overline{\nu}^*, \overline{\nu}^*)$ if the inequality sign in condition (8) is reversed. Notice that the differences $V(\underline{\nu}^*, \overline{\nu}^*) - V(\overline{\nu}^*, \overline{\nu}^*)$ and $V(\overline{\nu}^*, \underline{\nu}^*) - V(\underline{\nu}^*, \underline{\nu}^*)$ are always strictly negative and, by the Inada conditions on $S(\cdot)$, bounded from below. We show that the sign of the inequality in condition (8) depends on ν.

First, for types $\nu < \underline{\nu}^*$, $\mathbb{I}_x(\nu) = 0$ for every $x \in \{\underline{\nu}^*; \overline{\nu}^*\}$ and, hence, $U(x, y, \nu) = 0$ for every $x, y \in \{\underline{\nu}^*; \overline{\nu}^*\}$. Second, for types $\nu > \overline{\nu}^*$, $\mathbb{I}_x(\nu) = 1$ for every $x \in \{\underline{\nu}^*; \overline{\nu}^*\}$ and, hence,

$$U(\underline{\nu}^*, \overline{\nu}^*, \nu) - U(\overline{\nu}^*, \overline{\nu}^*, \nu) = U(\overline{\nu}^*, \underline{\nu}^*, \nu) - U(\underline{\nu}^*, \underline{\nu}^*, \nu) = 0.$$

Therefore, in both these cases, condition (8) holds with equality.

Third, for types $\nu \in [\underline{\nu}^*, \overline{\nu}^*]$, $\mathbb{I}_{\underline{\nu}^*}(\nu) = 1$ and $\mathbb{I}_{\overline{\nu}^*}(\nu) = 0$. Hence,

$$U(\underline{\nu}^*, \overline{\nu}^*, \nu) - U(\overline{\nu}^*, \overline{\nu}^*, \nu) = \nu \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right) - F$$

and

$$U(\overline{\nu}^*, \underline{\nu}^*, \nu) - U(\underline{\nu}^*, \underline{\nu}^*, \nu) = F - \nu \Delta\theta q^*\left(\overline{\theta}, \underline{\nu}^*\right).$$

Notice that, by definition of $\underline{\nu}^*$ and $\overline{\nu}^*$:

(*i*) For $\nu = \overline{\nu}^*$, $F - \nu\Delta\theta q^*\left(\overline{\theta}, \underline{\nu}^*\right) < 0$ and $\nu\Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right) - F = 0$;

(*ii*) For $\nu = \underline{\nu}^*$, $\nu\Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right) - F < 0$ and $F - \nu\Delta\theta q^*\left(\overline{\theta}, \underline{\nu}^*\right) = 0$.

Therefore, for ν sufficiently close to $\underline{\nu}^*$ condition (8) holds; while for ν sufficiently close to $\overline{\nu}^*$ condition (8) holds with the inequality sign reversed. ■

Proof of Proposition 4. For any probability $\varepsilon > 0$ that the principal plays the equilibrium with cut-off $\overline{\nu}^*$ (and probability $(1 - \varepsilon)$ that the principal plays the equilibrium with cut-off $\underline{\nu}^*$), the agent with type $\underline{\nu}^*$ obtains a strictly negative utility form entering and, by continuity, there always exists a positive mass of the agent's types that are sufficiently close to $\underline{\nu}^*$ who also obtain a strictly negative utility form entering. The reason is that an agent with type $\underline{\nu}^*$ obtains an utility equal to zero when he enters and the principal offers the contract associated to the equilibrium with cut-off $\underline{\nu}^*$, but obtains a strictly negative utility when he enters and the principal offers the contract associated to the equilibrium with cut-off $\overline{\nu}^*$ because $q^*\left(\overline{\theta}, \overline{\nu}^*\right) < q^*\left(\overline{\theta}, \underline{\nu}^*\right)$. Hence, the equilibrium with cut-off $\underline{\nu}^*$ is not robust to equilibrium risk for the agent.

By contrast, the equilibrium with cut-off $\overline{\nu}^*$ is robust to equilibrium risk for the agent because: (*i*) all the agent's types $\nu > \overline{\nu}^*$ who enter in this equilibrium obtain a strictly positive utility from entering regardless of the equilibrium played by the principal, and (*ii*) an agent with type $\overline{\nu}^*$ obtains an utility equal to zero when he enters and the principal offers the contract associated to the equilibrium with cut-off $\overline{\nu}^*$ and obtains a strictly positive utility when he enters and the principal offers the contract associated to the equilibrium with cut-off $\underline{\nu}^*$. Hence, regardless of the probability that the principal plays the equilibrium with cut-off $\underline{\nu}^*$, all agent's types $\nu > \overline{\nu}^*$ enter the market. ■

"Max-Min" Logic. First consider the principal. By choosing $\underline{\nu}^*$, he obtains at worst $V(\underline{\nu}^*, \overline{\nu}^*)$; while by choosing $\overline{\nu}^*$, he obtains at worst $V(\overline{\nu}^*, \underline{\nu}^*)$. The difference between these two expected profits is

$$V(\overline{\nu}^*, \underline{\nu}^*) - V(\underline{\nu}^*, \overline{\nu}^*) = (S(q^*(\underline{\theta})) - \underline{\theta} q^*(\underline{\theta})) \int_{\underline{\nu}^*}^{\overline{\nu}^*} \nu dG(\nu) +$$
$$+ \int_{\overline{\nu}^*}^{1} \left[(1-\nu)\left(S\left(q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \nu \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right] dG(\nu)$$
$$- \int_{\overline{\nu}^*}^{1} \left[(1-\nu)\left(S\left(q^*\left(\overline{\theta}, \underline{\nu}^*\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, \underline{\nu}^*\right)\right) - \nu \Delta\theta q^*\left(\overline{\theta}, \underline{\nu}^*\right)\right] dG(\nu)$$
$$+ \int_{\underline{\nu}^*}^{\overline{\nu}^*} \left((1-\nu)\left(S\left(q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \nu \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) dG(\nu).$$

This expression is always positive since:

(*i*) $S(q^*(\underline{\theta})) - \underline{\theta} q^*(\underline{\theta}) > 0$.

(*ii*) Optimality of $q^*\left(\overline{\theta}, \overline{\nu}^*\right)$ implies that

$$\int_{\overline{\nu}^*}^{1} \left[(1-\nu)\left(S\left(q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \nu \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right] dG(\nu) >$$
$$\int_{\overline{\nu}^*}^{1} \left[(1-\nu)\left(S\left(q^*\left(\overline{\theta}, \underline{\nu}^*\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, \underline{\nu}^*\right)\right) - \nu \Delta\theta q^*\left(\overline{\theta}, \underline{\nu}^*\right)\right] dG(\nu).$$

(*iii*) Finally,

$$\int_{\underline{\nu}^*}^{\overline{\nu}^*} \left((1-\nu)\left(S\left(q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \nu \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) dG \geq 0$$

because

$$(1-\overline{\nu}^*)\left(S\left(q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \overline{\nu}^* \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right) >$$
$$(1-\mathbb{E}[\nu|\nu \geq \overline{\nu}^*])\left(S\left(q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \overline{\theta} q^*\left(\overline{\theta}, \overline{\nu}^*\right)\right) - \mathbb{E}[\nu|\nu \geq \overline{\nu}^*] \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right) \geq 0$$

where the first inequality follows from $\overline{\nu}^* < \mathbb{E}[\nu|\nu \geq \overline{\nu}^*]$, and the second inequality follows from optimality of $q^*\left(\overline{\theta}, \overline{\nu}^*\right)$ and the Inada conditions.

Second, consider the agent. By choosing $\underline{\nu}^*$, he obtains, at worst $U(\underline{\nu}^*, \overline{\nu}^*, \nu)$; while by choosing $\overline{\nu}^*$ he obtains at worst $U(\overline{\nu}^*, \underline{\nu}^*, \nu)$. In this case,

$$U(\overline{\nu}^*, \underline{\nu}^*, \nu) \geq U(\underline{\nu}^*, \overline{\nu}^*, \nu) \quad \Leftrightarrow$$

$$\mathbb{I}_{\overline{\nu}^*}(\nu)\left[\nu \Delta\theta q^*\left(\overline{\theta}, \underline{\nu}^*\right) - F\right] \geq \mathbb{I}_{\underline{\nu}^*}(\nu)\left[\nu \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right) - F\right].$$

This inequality always holds since:

(i) For types $\nu < \underline{\nu}^*$, $\mathbb{I}_{\overline{\nu}^*}(\nu) = \mathbb{I}_{\underline{\nu}^*}(\nu) = 0$.

(ii) For types $\nu > \overline{\nu}^*$, $\mathbb{I}_{\overline{\nu}^*}(\nu) = \mathbb{I}_{\underline{\nu}^*}(\nu) = 1$ and $q^*\left(\overline{\theta}, \underline{\nu}^*\right) > q^*\left(\overline{\theta}, \overline{\nu}^*\right)$.

(iii) For types $\nu \in [\underline{\nu}^*, \overline{\nu}^*]$, $\mathbb{I}_{\underline{\nu}^*}(\nu) = 1$, $\mathbb{I}_{\overline{\nu}^*}(\nu) = 0$, and $\nu \Delta\theta q^*\left(\overline{\theta}, \overline{\nu}^*\right) < F$.

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