

# On the selection by MCDM methods of the optimal system for seismic retrofitting and vertical addition of existing buildings



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## ABSTRACT

In the current paper a novel procedure to select the optimal solution both for seismic retrofitting of existing RC buildings and for super-elevation of existing masonry constructions has been implemented by using three different Multi-Criteria Decision Making (MCDM) (TOPSIS, ELECTRE and VIKOR) methods.

The procedure application has been faced with reference to two case studies.

The first intervention has been studied on a real full-scale 3D RC structure retrofitted with different seismic protection devices mainly based on metal materials, whose performances were experimentally evaluated in a previous research project. All the applied MCDM methods have provided the same result, that is the dominating role exerted by aluminium shear panels for seismic retrofitting of the analysed structure.

On the other hand, different innovative and traditional constructive systems have been examined to increase the number of floors of existing masonry buildings. The effectiveness of these interventions in improving the base building behaviour has been proved on a typical building of the South Italy. The study results, achieved by using the three MCDM methods inspected, have provided as an optimal solution the cold-formed steel systems thanks to their prerequisites of lightness, economy and sustainability.

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## 1. Introduction

In the last years the Italian construction market has focused the attention on the restoration of existing buildings rather than the edification of new constructions.

This activity has been dictated from need of upgrading buildings designed to withstand vertical loads only or subjected to either a new seismic classification or a change of use, therefore requiring an increased load bearing capacity. To this purpose, new seismic analysis rules have been implemented, they making several changes to the regulatory framework for existing buildings, that is both considering their ultimate limit state design and taking into account deformations and displacements as design parameters. When these rules are related to new buildings, a specified performance of such constructions under earthquake, which should be able to attain a predicted performance level, should be guaranteed by the designer. On the contrary, this aim is more difficult to be pursued for existing buildings, where the primary target is to increase their seismic safety, ensuring, at the same time, the human life safety.

Common intervention techniques modifying the performance level of an existing building are aimed at its retrofitting or vertical addition, which are respectively carried out either to provide earthquake resistance, when a design for gravity loads is performed, or to increase its living volume.

Within this field, innovative intervention techniques are very popular and available in a large number, they being differentiated each other also for the various application difficulty. However, a macroscopic subdivision among them can be made: some systems alter the seismic demand in terms of Peak Ground Acceleration (PGA) and, therefore, reduce the horizontal seismic force, whereas other systems improve the seismic structural response.

In the current paper, which is a revised and updated version of the conference paper [1], a significant number of these techniques has been used for retrofitting some real full-scale 3D RC frames, as it will be shown in Section 2.

On the other hand, different innovative and traditional constructive systems are used to increase the number of floors of existing buildings. The effectiveness of these interventions in improving the base building behaviour, represented by a typical masonry construction of the urban tissue of a generic South Italy town, is shown in Section 5.

In both structural modification applications, the optimal solution for retrofitting and vertical addition purposes has been

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individuated by applying three different Multi-Criteria Decision Making (MCDM) methods (TOPSIS, ELECTRE and VIKOR), which have always provided the same solutions as the competition winners.

As a whole, the main contribution of the paper to the current knowledge into the field of structural modification interventions is the application of three MCDM methods, different each to other both for basis theories and results provided, to give three rankings among considered alternatives able to individuate with a greatest certainty the optimal consolidation and super-elevation solutions.

## 2. The Multi-Criteria Decision Making (MCDM) methods

The Multi-Criteria Decision Making (MCDM) methods are mathematical tools allowing to solve a decision problem through the selection of the optimal alternative meeting a given number of criteria. Therefore, a multi-criteria analysis is the formulation of the convenience opinion of an intervention according to most criteria, examined independently or interactively.

All decision problems regarding a multi-criteria evaluation are investigated by considering the following factors:

- A “goal” or a set of “goal”, which represent the general aim to be achieved.
- A Decision Maker (DM) or a group of Decision Makers (DMs) involved in the selection process, who are responsible of the evaluation procedure.
- A set of decisional alternatives, which are the fundamental elements for evaluation and selection processes.
- An evaluation set, used by the DM to evaluate the performance of alternatives.
- The preferences of DM, which are typically expressed in terms of weights assigned to the evaluation criteria.
- A set of scores, expressing the value of the alternative  $i$  with respect to the criterion  $j$ .

In particular, any MCDM method is based on two basic parameters, that is the decision matrix  $D$ , where the performance of different alternatives with respect to each criterion is reported, and the criteria weight vector, which provides the importance that the DM, or the group of DMs, gives to each selected criterion.

In the present paper three MCDM methods have been inspected and applied to two structural modification intervention case studies.

First of all, the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method [2] has been used thanks to its application easy. This method represents the various alternatives as points of a vector space having dimensions equal to the criteria number, so that the different solutions performances become the coordinates in the assumed vector space. Therefore, with this very practical method, both the better alternative is identified and an alternative ranking is defined.

Afterwards, the ELECTRE (ELimination and Choice Traslating REality) method [3], which provides relationship of dominance (outranking) among various options, and the VIKOR method [4], which delivers, analogously to the TOPSIS method, a ranking among selected alternatives, have been considered and applied to the case studies. A detailed description of the three used methods is reported as follows.

The TOPSIS method creates two additional ideal alternatives that guide the DM to choose the optimal alternative among those considered. These two ideal alternatives are the optimal solution ( $A^+$ ), having the paramount performance over all criteria, and the worst one ( $A^-$ ). So, the decision problem solution is represented

by the alternative having, at the same time, the minimum distance from  $A^+$  and the maximum distance from  $A^-$ .

The first practical step of the method requires that the decision matrix  $D$  should be made of dimensionless elements in order to compare each to other criteria with different units. This gives rise to the matrix  $R$ , which is made of parameters  $r_{ij}$  calculated in the following manner:

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{k=1}^n a_{kj}^2}} \quad (1)$$

where  $a_{ij}$  are the decision matrix elements.

In this way all matrix elements are without measurement units.

Later on, the weighted decision matrix  $V$ , composed of elements  $v_{ij}$ , is achieved by multiplying  $r_{ij}$  elements for the criteria weight vector  $\omega_j$  according to the following relationship:

$$v_{ij} = \omega_j \times r_{ij} \quad (2)$$

After this, the method requires to assess two ideal alternatives. All alternatives considered, together with the two ideal ones, are considered as virtual points in a vector space, whose coordinates are their performance against the established criteria.

Subsequently, the distances among each alternative and the ideal ones are calculated. So, the preference list among alternatives can be generated by considering the following parameters:

$$S_{i^+} = \sqrt{\sum_{j=1}^m J(v_{ij} - v_{j^+})^2} \quad \text{per } i = 1, 2, \dots, n$$

$$S_{i^-} = \sqrt{\sum_{j=1}^m J(v_{ij} - v_{j^-})^2} \quad \text{per } i = 1, 2, \dots, n \quad (3)$$

and taking as optimal solution the alternative having the minimum value of the factor  $C_{i^+}$ , calculated as follows:

$$C_{i^+} = \frac{S_{i^-}}{S_{i^-} + S_{i^+}} \quad (4)$$

In general, MCDM methods are used to help the DM or a group of DMs to make objective choices not influenced by the evaluation process responsible. In order to test the validity of the achieved results, the weight of each single criterion, taken one by one, is varied from 0 to 1, leaving all others unchanged, aiming at verifying if the ranking is changed or not. If the ranking does not change, then the stability of the solution found is confirmed.

The generic weight absolute change able to reach a solution different from the one identified with the starting weights is indicated with *Absolute Top (AT)*, where “absolute” means that there is a value absolute change and “top” indicates that this change modifies the alternative ranking top. Then, for each criterion, by dividing *AT* for the criterion weight, the *Percentage Top (PT)* is obtained, it representing the weight change altering the ranking first solution.

Finally, the stability measure of the solution is made by calculating the sensitivity parameter, achieved as reciprocal of the corresponding *PT* value. The solution will be more stable as much as more the *PT* values are high.

In this context, robust criteria are defined when the *AT* values change does not provoke a decision problem solution alteration. So, robust criteria are, of course, no sensitive to the final solution definition, since their weight variations do not change the ranking. Therefore, when both a large number of criteria are stable and *PT* values are high, it is possible to declare that the decision problem outcome is sufficiently sure, it being not influenced by the DM personal choices.

On the other hand, the ELECTRE method has the ultimate goal to build the so called outranking (domination) relationships among considered alternatives.

An alternative can be defined as dominated if there is another alternative that responds better than that towards one or more criteria.

The method involves binary comparisons among alternatives with respect to each criterion. The set of relationships can be either complete or there may be a failure, when the DM has not given any preference for an alternative over another.

The application of the method follows simple steps, similar to the TOPSIS method ones. As a first step, the decision matrix is represented with dimensionless elements according to the relationship (1). Afterwards, the weighted decision matrix is calculated on the basis of the expression (2), which provides  $v_{ij}$  values.

Then, the concordance set  $C_{kp}$  and the divergence set  $D_{kp}$  are determined. The concordance set of an alternative  $A_k$  with respect to the alternative  $A_p$  is made of the criteria where the  $v_{kj}$  parameter of the matrix  $V$  is greater than the corresponding  $v_{pj}$  value. Subsequently, the concordance index  $c_{kp}$  is calculated as sum of criteria weights contained in the concordance set:

$$c_{kp} = \sum_{j \in C_{kp}} \omega_j \quad \text{for } j = 1, 2, \dots, m \quad (5)$$

On the other hand, the divergence index  $d_{kp}$  is gotten from the following equation:

$$d_{kp} = \frac{\max_{j \in D_{kp}} |y_{kj} - y_{pj}|}{\max_j |y_{kj} - y_{pj}|} \quad (6)$$

The concordance index expresses the importance of the alternative  $A_k$  with respect to the alternative  $A_p$ , while the divergence one has the opposite meaning.

Successively, the determination of other threshold parameters used to impose the outclassed relationship is made. These parameters are the concordance threshold  $S_c$ :

$$S_c = \frac{1}{n(n-1)} \sum_{k=1}^n \sum_{p=1, p \neq k}^n c_{kp} \quad (7)$$

and the discrepancy threshold  $S_d$ :

$$S_d = \frac{1}{n(n-1)} \sum_{k=1}^n \sum_{p=1, p \neq k}^n d_{kp} \quad (8)$$

The outclassed relationship is expressed as follows:

$$A_k \text{ outclasses } A_p \text{ if and only if } c_{kp} \geq S_c \text{ and } d_{kp} \leq S_d \quad (9)$$

The latter relationship provides an additional matrix  $E$ , which individuates either the alternative dominating all over the others or to eliminate an alternative group dominated by remaining ones. This matrix is built by putting either one, if the outclassed relationship is satisfied, or zero, when the relationship is not fulfilled. From the matrix  $E$  the columns with at least one unitary element should be eliminated, since these alternatives are dominated from others. So, as a method final results, also the alternative ranking can be formulated and the optimal solution, as that with the less number of unitary elements, can be individuated.

Finally, the VIKOR (compromise ranking) method is based on definition of three scalar parameters generating the alternative ranking.

First, the optimal ( $a_{j+}$ ) and the worst ( $a_{j-}$ ) performances of each alternative against the same criteria  $j$  are identified. After these performances are known, the calculation of scalar parameters  $S_i$  and  $R_i$  is performed as follows:

$$S_i = \sum_{j=1}^m \frac{\omega_j (a_{j+} - a_{ij})}{a_{j+} - a_{j-}}; \quad R_i = \max_j \left[ \frac{\omega_j (a_{j+} - a_{ij})}{a_{j+} - a_{j-}} \right] \quad (10)$$

where  $\omega_j$  represents the criteria weight.

The above two scalar parameters provide, for each alternative, the scalar parameter  $Q_i$ , calculated as:

$$Q_i = v \frac{S_i - S^+}{S^- - S^+} + (1 - v) \frac{R_i - R^+}{R^- - R^+} \quad (11)$$

where

$$\begin{aligned} S^* &= \min_i S_i; & S^- &= \max_i S_i; \\ R^* &= \min_i R_i; & R^- &= \max_i R_i; \end{aligned} \quad v = 0.5 \quad (12)$$

Therefore, from  $Q_i$ , the alternative ranking is generated and the optimal option, called compromise solution  $A'$  and having the lowest value of  $Q_i$ , is found.

### 3. Selection of the optimum seismic retrofitting system of a real RC structure

#### 3.1. The experimental campaign ILVA-IDEM

The possibility of increasing the knowledge on the retrofitting of existing RC buildings has represented the main objective of the experimental activity performed in the period 2000–2005 by the research group headed by Federico M. Mazzolani on a real structure located within the ex steel mill of Bagnoli in Naples and destined to be demolished.

The purpose of the experimental campaign, called “ILVA-IDEM” (acronym of ILVA Intelligent DEMolition”), was to evaluate and compare each to other different retrofitting techniques of existing structures mainly based on metallic materials. The detailed contents of this wide experimental, numerical and theoretical activity are reported in [5]. The original structure was not designed to withstand any horizontal loads, since its erection was made when the area of Bagnoli was not considered as a seismic prone zone. For this reason, the structure was designed to sustain vertical loads only. The building (Fig. 1) had a rectangular lengthened plan shape (41.6 m × 6.50 m) and it developed on two floors with a first and second floor heights on the ground of 3.55 m and 6.81 m, respectively. It was composed by twenty-six 30 × 30 cm columns and beams, located along the building perimeter only, supporting RC joists-hollow tiles mixed floors. Inverse T-beams were used as building foundations.

In order to increase the potential number of structures to be tested with different upgrading solutions, slabs were cut at the first and second floor, so to achieve six separate simple structures (sub-units) to be analysed. Before the slab cutting, both the internal partitions and the external claddings of the building were removed.

Seven different retrofitting techniques, which represents the alternatives (A) for the application of MCDM methods, have been considered to retrofit the structural sub-units:



Fig. 1. The building tested within the ILVA-IDEM research project before experimentation.

1. Base isolation with rubber bearings (BI) (A1)
2. Buckling Restrained Bracings (BRB) (A2)
3. Carbon-Fibre Reinforced Polymers (C-FRP) (A3)
4. Eccentric Bracings (EB) (A4)
5. Shape Memory Alloy (SMA) bracings (A5)
6. Steel Shear Panels (SSP) (A6)
7. Aluminium Shear Panels (ASP) (A7)

Each technique was associated to a given structural sub-unit, as shown in Fig. 2, where it is noticed that in the sixth module the building staircase was located and, therefore, no seismic protection device was installed.

The base isolation system was submitted to free vibration and ambient vibration tests. Instead, static inelastic tests were carried out for all the other systems. Differently from other techniques, the SMA bracing system was tested both statically and dynamically (free vibration tests).

In the following, a summary of test results is given, also showing a final comparison among them. First of all, the base isolation system with circular cross-section rubber bearings was inserted into the largest module by cutting preliminarily the ground floor column bases and, subsequently, by connecting their ends with X-shaped steel bracings in order to create a stiff base diaphragm.

Free vibration tests were carried out on both the original (fixed) structure and the base isolated one. Fig. 3 illustrates the base-isolated structure, the maximum lateral deformation of the rubber bearing, the displacement transducer and the structure displacements measured during the release tests.

Fig. 4a illustrates the sub-structure equipped with BRBs, which were placed in number of two, but in different vertical planes, for each story. The tested BRBs belonged to the ‘only-steel’ type, with

the core made of one single steel plate (25 mm × 10 mm) and two restraining rectangular steel tubes (100 mm × 50 mm × 5 mm) [6]. Two tests were carried out, differing only for some detailing of the yielding core end portions [7,8]. The first type of BRB tested is shown in Fig. 4b, while the correspondent force–displacement diagram is shown in Fig. 4c. The second type of BRB tested, arranged to be opened for inspection and monitoring at the test end, is shown in Fig. 4d. The force displacement diagram achieved from the second test is shown in Fig. 4e.

The sub-structure reinforced with C-FRP is depicted in Fig. 5a. The strengthening system consisted of longitudinal C-FRP pultruded strips, externally bonded to the RC columns, and transverse C-FRP confining sheets. The original (bare) RC sub-structure was tested under a monotonic increasing roof displacement, while the upgraded structure was tested under load reversals. Fig. 5b illustrates the column-sway and the beam-sway mechanisms exhibited by the original RC sub-structure. Fig. 5c shows the comparison of response between the original structure and the upgraded one.

Fig. 6a illustrates the typology of the EB system adopted for seismic retrofitting purpose. Three tests were carried out by using always the same RC frame, but changing the link cross-section and some connection details [9].

In the first test the link-to-diagonal connection failure was occurred. Instead, in the second test, both the shear failure of link-to-diagonal connection bolts and a relatively small plastic deformation of links, thus indicating link over-strength larger than expected, were recorded. Finally, in the third test, as on the basis of previous studies [10], a significant plastic deformation of links was occurred, but once again failure with the (predominantly) shear rupture of link-to-diagonals connection bolts was observed. The



Fig. 2. Techniques under study: (1) BI; (2) BRB; (3) C-FRP; (4) EB; (5) SMA, SSP and ASP.

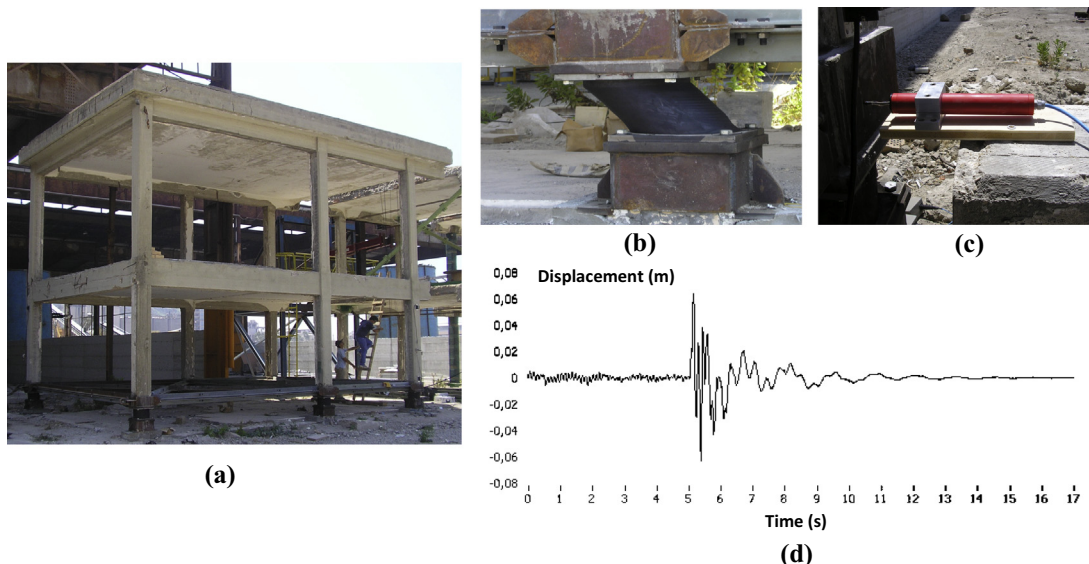


Fig. 3. The base isolated structure (a), the deformed rubber bearing at the test end (b), the displacement transducer (c) and the displacement vs. time diagram of the retrofitted structure (d).

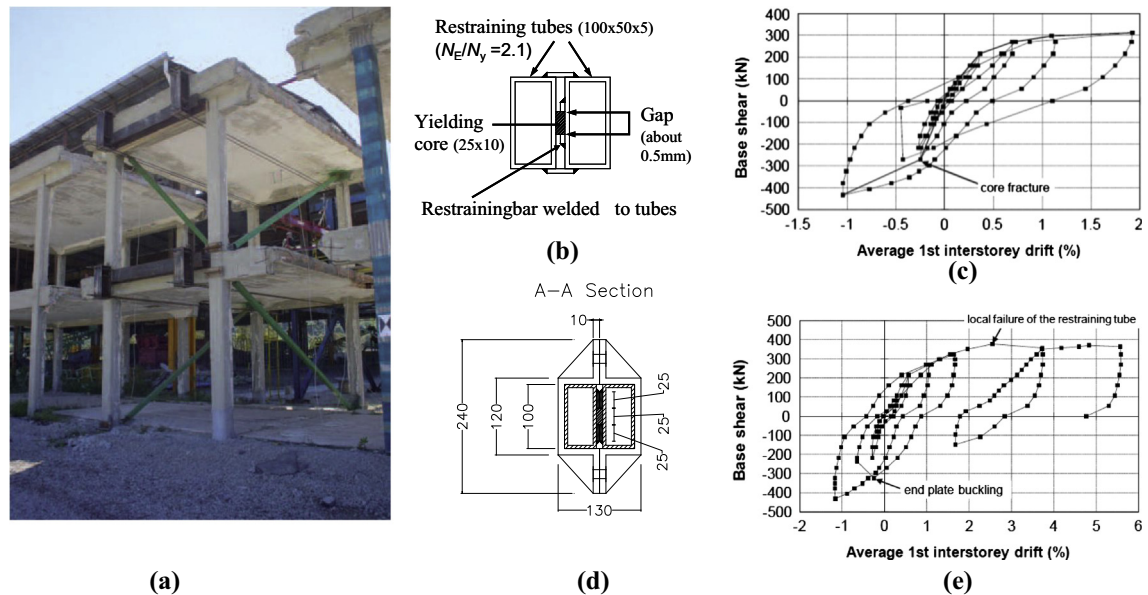


Fig. 4. The structure equipped with BRB (a): BRB type 1 (b), results of the test n. 1 (c), BRB type 2 (d) and results of the test n. 2 (e).

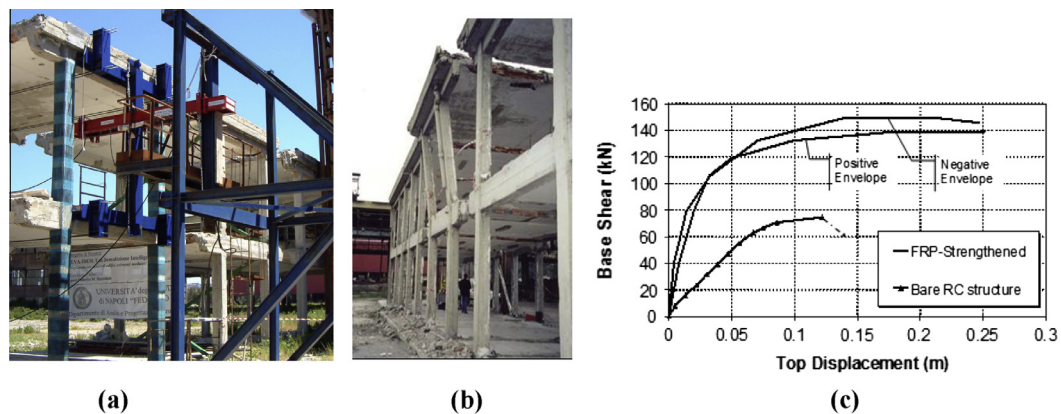


Fig. 5. The structure retrofitted with C-FRP (a), the collapse mechanism of the bare structure (b) and the summary of test results (c).

experimental responses of tests n. 1, n. 2 and n. 3 are shown in Fig. 6b, c and d, respectively.

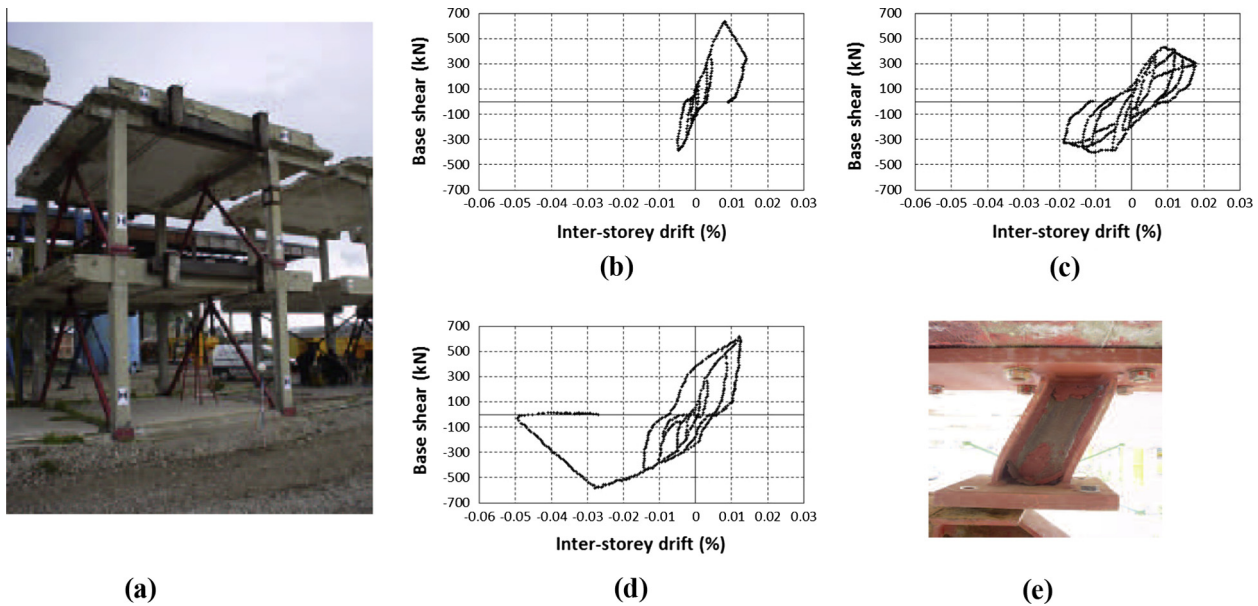
Later on, the research group of the University of Basilicata, based on the results of previous studies [11], proved the effectiveness of NiTi SMA-based bracings for retrofitting purposes. The in-situ activity consisted of push-over and cyclic tests, as well as release tests. Fig. 7a shows the SMA braces mounted on the structure. Fig. 7b gives the results in terms of the top storey displacement – time diagram of the free vibration test on the upgraded structure, which was first pulled in the bracing direction and then left free to move. In the same figure, the displacement reduction and the damping increase of the retrofitted structure as respect to the bare RC sub-structure is shown.

Shear panels (SP) made of aluminium alloy [12,13] and steel [14,15] have been used as retrofitting system of the sub-structure depicted in Fig. 8a. After an appropriate reinforcement of the bare RC structure [16,17], a couple of shear panels was inserted into the frame ground floor by means of surrounding hinged steel frames (Fig. 8b and c). The effectiveness of the proposed upgrading intervention was proved by two experimental cyclic tests (Fig. 8d) [18], whose results showed that the dissipation

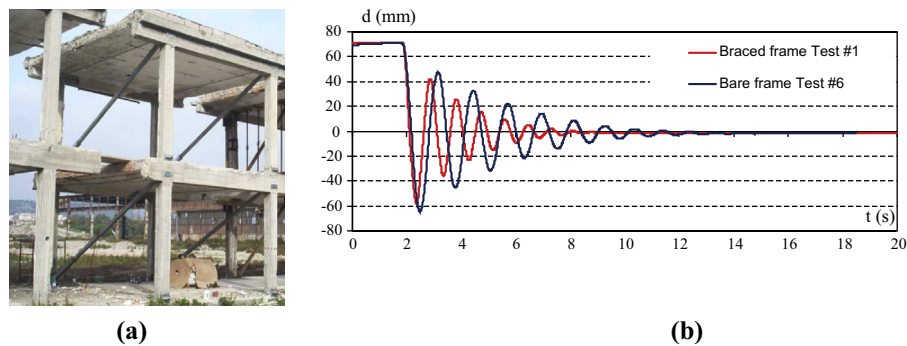
capacity of the structure retrofitted with aluminium shear panels was more satisfactory than the one endowed with steel plates, due to the excellent hysteretic characteristics of the used aluminium alloy [19,20].

Fig. 9 shows a comparison of the experimental results on the five upgrading systems (C-FRP, EB, BRB, SSP and ASP) examined by the research group of the University of Naples, also illustrating the improvement of the retrofitted structures response with respect to the bare RC structure one.

Actually, the four tested RC structures were slightly different for their in-plan dimensions, but their lateral-load response can be assumed to be fairly the same because governed by the four square-section columns, which were identically reinforced. As it can be seen and it could be expected, steel bracings, both EB and BRB, and shear panels were generally able to produce very large increase of stiffness and strength, while the C-FRP system appreciably increased the lateral displacement capacity of the structure but with a low increase in strength (about 2 times). The second type of tested BRB, however, reached a maximum displacement approximately equal to that of the C-FRP upgraded structure. In Fig. 9, the event of a brittle rupture is highlighted with a star.



**Fig. 6.** The structure reinforced with EBF (a), base shear ( $V$ ) – inter-storey drift ( $i$ ) diagrams related to test n. 1 (b), test n. 2 (c), and test n. 3 (d) and link-to-diagonal bolts failure in the test n. 3 (e).



**Fig. 7.** SMA bracings inserted into the module n. 5 (a) and time history displacement diagram comparison between the bare structure and the retrofitted one (b).

The maximum increase in load bearing capacity was reached with EB, which showed a base shear from 5.5 to 8 times greater than the bare structure one. In case of BRB the increment was averagely 4.25 times, very similar as in case of SP (about 4 times).

### 3.2. The used alternatives and criteria

In the decision problem related to the ILVA-IDEM project, the alternatives, which represent the decision matrix rows, are the different seismic protection techniques described in the previous section.

Conversely, seven performance criteria have been considered, they being distinguished into quantitative criteria, expressing the generic alternative with a number, and qualitative criteria, represented by verbal assessments converted into numbers to be included in the decision matrix  $D$ .

The quantitative criteria are:

- (1) Intervention cost, which is always considered when a retrofitting design is performed. It is a cost criterion that should be minimised as much as possible.

- (2) Vulnerability reduction, determined as the ratio between the maximum PGA sustained by the retrofitted structure and the bare structure one. It is a benefit criterion that should be maximised as much as possible.

On the other hand, the qualitative criteria are:

- (1) Intervention feasibility, which includes all those impediments that can arise for the intervention realisation, namely the availability of workmanship, materials and technologies. It is a benefit criterion and the preferences of DMs, expressed as verbal judgments, should be transformed into numbers in order to be placed in the decision matrix.
- (2) Disturbance to occupiers, which is an essential criterion if the retrofitting intervention is carried out on a building in use. In fact, depending on the intervention type, either the occupant removal or the production shift can occur or not.
- (3) Functional and aesthetic compatibility: the aesthetic component and the functionality of the intervention, which are very important topics when the structure is used especially for residential purposes, represent a benefit criterion expressed into a verbal way.

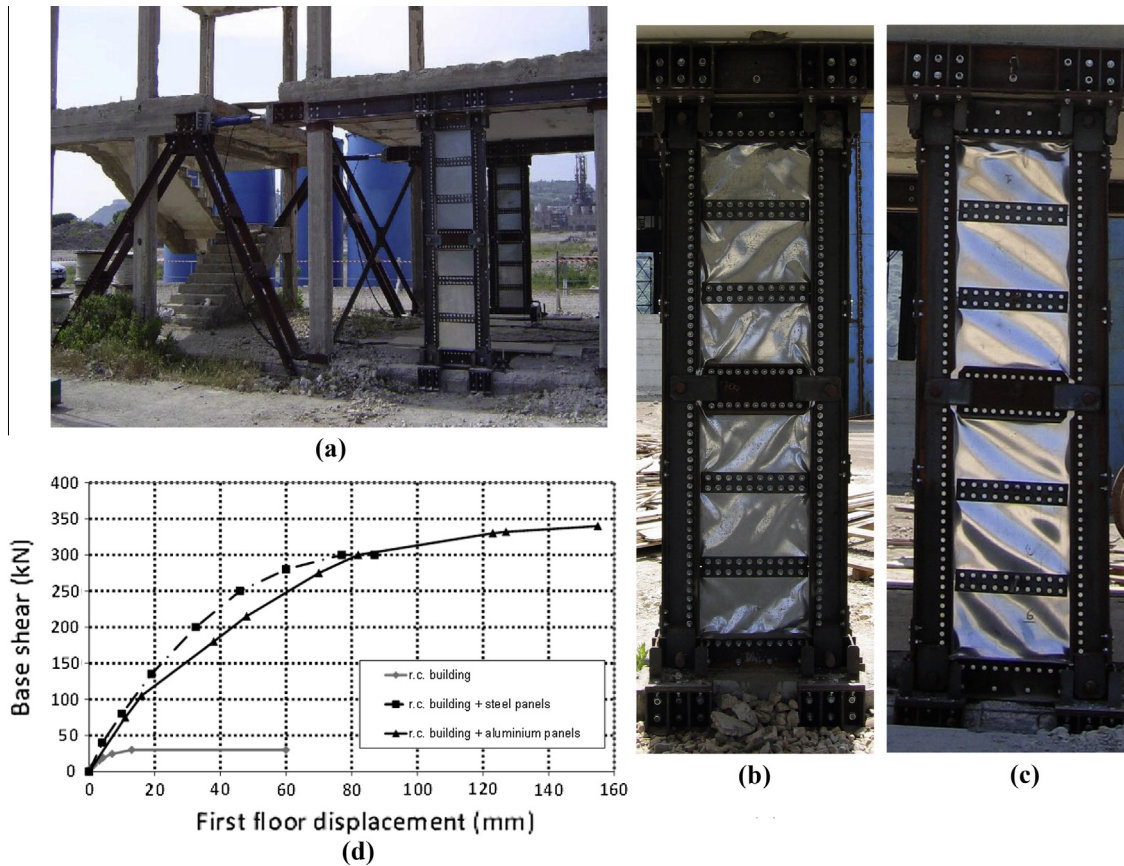


Fig. 8. The sub-structure retrofitted with metal shear panels (a), final deformed shape of tested steel (b) and aluminium (c) devices and comparison between the bare structure response and the behaviour of upgraded structures (d).

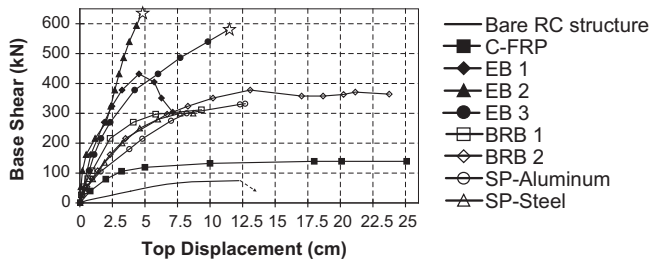


Fig. 9. Comparison among responses of retrofitted structures.

- (4) Reversibility, which is the capacity of a given alternative to be easily removed from the building when other interventions are requested. It is a benefit criterion and, therefore, the preference is devoted to techniques having this prerequisite.
- (5) Protection from damage, which is related to the need of preserving the integrity of the seismic protection devices throughout the structure life. Among the different alternatives, the one providing both the better security, and, over time, a constant capacity response, should be individuated. It is a benefit criterion.

3.3. The criteria weight vector and the decision matrix

All MCDM methods require two elements to be applied: the criteria weight vector, representing the importance that DM gives to different criteria, and the decision matrix, containing the performance of individual alternatives towards the considered criteria.

Table 1

Weights of criteria for seismic upgrading of an existing RC structure.

Criterion		Weight (%)
Cost	C1	7.00
Feasibility	C2	5.00
Disturbance to occupiers	C3	20.00
Functional and aesthetic compatibility	C4	10.00
Reversibility	C5	35.00
Vulnerability reduction	C6	20.00
Protection from damage	C7	3.00

In the current analysis case the criteria weights have been determined by using the AHP method developed by Saaty (Table 1) [21].

Criteria weights have been decided by Authors, who have assigned maximum importance to reversibility, since this prerequisite is at the basis of modern restoration charts. After this, disturbance to people (limited building interruption) and vulnerability reduction (increase of seismic resistance) represent the criteria with maximum weights. On the other hand, the intervention cost, with a weight of 7% only, is not considered as a very important criterion since major attention has been devoted to the building seismic performance improvement.

After weights have been assigned to the various criteria, the decision matrix has been implemented, as shown in Table 2.

3.4. The decisional problem solution

3.4.1. The TOPSIS method

Once defined the two above basic elements, firstly the TOPSIS method has been employed to solve the decisional problem.

**Table 2**The decision matrix  $D$  for the seismic upgrading problem of an existing RC structure.

Alternative		Criterion						
		C1	C2	C3	C4	C5	C6	C7
Base isolation	A1	€ 36,359.66	0.024	0.317	0.036	0.025	2.450	0.404
Buckling restrained bracings	A2	€ 2649.22	0.039	0.069	0.321	0.154	2.420	0.074
Carbon fibre-reinforced polymers	A3	€ 42,821.28	0.167	0.296	0.058	0.044	1.000	0.028
Eccentric bracings	A4	€ 1537.68	0.083	0.059	0.097	0.114	2.560	0.187
Shape memory alloy bracings	A5	€ 49,000.00	0.058	0.034	0.100	0.084	2.680	0.065
Steel shear panels	A6	€ 12,466.85	0.324	0.114	0.196	0.276	2.140	0.129
Aluminium shear panels	A7	€ 13,010.83	0.305	0.110	0.192	0.303	2.430	0.114

**Table 3**The matrix  $R$  (RC structure – TOPSIS method).

	C1	C2	C3	C4	C5	C6	C7
A1	0.4737	0.0495	0.6712	0.0797	0.0542	0.4023	0.8281
A2	0.0345	0.0789	0.1461	0.7136	0.3322	0.3974	0.1518
A3	0.5579	0.3430	0.6276	0.1293	0.0943	0.1642	0.0566
A4	0.0200	0.1695	0.1258	0.2158	0.2468	0.4204	0.3827
A5	0.6384	0.1186	0.0730	0.2215	0.1820	0.4401	0.1331
A6	0.1624	0.6629	0.2415	0.4349	0.5962	0.3514	0.2639
A7	0.1695	0.6256	0.2340	0.4277	0.6545	0.3991	0.2328

So, starting from the matrix  $D$ , the matrix  $R$ , having dimensionless elements, has been achieved (Table 3) and, consequently, also the matrix  $V$  has been determined (Table 4).

Therefore, the ideal alternatives have been individuated (Table 5) and the alternative ranking, based on the distance of each retrofitting technique from ideal ones, has been drawn (Table 6), it providing aluminium shear panels as optimal retrofitting option.

Finally, from the sensitivity analysis performed, the stability of the solution found has been asserted due to one robust criterion (C6) and high values of PT parameters for the other criteria (Table 7).

#### 3.4.2. The ELECTRE method

First, the application of the method has led to the determination of matrixes  $R$  and  $V$ , respectively reported in Tables 8 and 9.

The method final phase allows for the definition of the matrix  $E$ , which provides the solution of the MCDM problem (Table 10).

From the analysis results it is shown that the dominant alternative is A7, that is aluminium shear panels, as already found by the TOPSIS method. After this solution, the preference can be attributed to SSP (2nd place), BRB (3rd place), BI, C-FRP and EB (4th place) and SMA (5th place).

#### 3.4.3. The VIKOR method

The first step of the VIKOR method is the calculation of two scalar parameters, that is  $S_i$  and  $R_i$  (Table 11).

Afterwards, the definition of another scalar parameter  $Q_i$  for each alternative allows for finding both the decisional problem solution (minimum  $Q_i$  value) and the classification among different retrofitting systems considered (Table 12).

**Table 4**The matrix  $V$  (RC structure – TOPSIS method).

	C1	C2	C3	C4	C5	C6	C7
A1	0.0332	0.0025	0.1342	0.0080	0.0190	0.0805	0.0248
A2	0.0024	0.0039	0.0292	0.0714	0.1163	0.0795	0.0046
A3	0.0391	0.0171	0.1255	0.0129	0.0330	0.0328	0.0017
A4	0.0014	0.0085	0.0252	0.0216	0.0864	0.0841	0.0115
A5	0.0447	0.0059	0.0146	0.0222	0.0637	0.0880	0.0040
A6	0.0114	0.0331	0.0483	0.0435	0.2087	0.0703	0.0079
A7	0.0119	0.0313	0.0468	0.0428	0.2291	0.0798	0.0070

As in the two previous analysis cases, shear panels made of aluminium alloys represent the optimal solution for resolving the investigated MCDM problem.

#### 3.4.4. Further investigations

Even if sensitivity checks carried out with three used MCDM methods have provided encouraging results, since the decision problem resolution can be affected by the DM judgments, further analyses have been performed.

First, one by one criterion has been assumed as dominant (weight of 70%), with all the others having secondary importance (weight of 5%), and, subsequently, two criteria have been considered as equally predominant (weight of 37.5%), whereas the remaining ones have the same negligible weight (5%).

These analysis have been carried out with each of the three MCDM methods examined.

The results of the two above investigations are synthesised in Fig. 10, where it is evident that with TOPSIS and VIKOR methods aluminium shear panels are on the ranking top, whereas with the ELECTRE method the leadership is assigned to steel shear panels, immediately followed by eccentric bracings.

The final synthesis of all results achieved from this additional investigation by using the three used MCDM methods is summarised in Fig. 11, where the percentage of each solution as a ranking top is shown. In the same figure, one time again it is confirmed that the optimal retrofitting system of the RC structure under study is represented by aluminium shear panels. This confirms the reliability of results achieved in the first part of the study.

## 4. The optimum technology for vertical addition of masonry buildings

### 4.1. Super-elevation of existing buildings

The MCDM methods have been also used to select the optimal technique for vertical addition of existing masonry buildings.

A single structural masonry unit extrapolated from a linear building aggregate, representative of the heritage built-up erected in Naples at the beginning of '900, has been selected as a case study. This building, made of 50 cm thick tuff stones and covering an area of about 120 m<sup>2</sup>, is developed on two storeys having inter-storey heights of 4.20 m and 3.20 m at the ground floor and at the first one, respectively. The compression and shear resistance of masonry are  $f_k = 1$  MPa and  $f_{vk0} = 0.10$  MPa, respectively. A 3D view of the study masonry unit is depicted in Fig. 12a.

**Table 5**

The virtual alternatives (RC structure – TOPSIS method).

	C1	C2	C3	C4	C5	C6	C7
A <sup>+</sup>	0.0014	0.0331	0.0146	0.0714	0.2291	0.0880	0.0248
A <sup>-</sup>	0.0447	0.0025	0.1342	0.0080	0.0190	0.0328	0.0017



**Table 6**

Alternative ranking for RC structure seismic retrofitting according to the TOPSIS method.

Alternative				$C_i^*$
First	A	7	Aluminium shear panels	0.831
Second	A	6	Steel shear panels	0.799
Third	A	2	Buckling restrained bracings	0.586
Fourth	A	4	Eccentric bracings	0.486
Fifth	A	5	Shape memory alloy bracings	0.436
Sixth	A	1	Base isolation	0.176
Seventh	A	3	Carbon-fibre reinforced polymers	0.087

**Table 7**

Sensitivity analysis (RC structure – TOPSIS method).

Criteria	Weight	AT	PT (%)	Sensitivity
C1	0.070	0.585	836	0.001
C2	0.050	0.579	1158	0.001
C3	0.200	0.542	271	0.004
C4	0.100	0.640	640	0.002
C5	0.350	0.320	91	0.011
C6	0.200	–	–	–
C7	0.030	0.334	1113	0.001

**Table 8**The matrix  $R$  (RC structure – ELECTRE method).

	C1	C2	C3	C4	C5	C6	C7
A1	0.4737	0.0495	0.6712	0.0797	0.0542	0.4023	0.8281
A2	0.0345	0.0789	0.1461	0.7136	0.3322	0.3974	0.1518
A3	0.5579	0.3430	0.6276	0.1293	0.0943	0.1642	0.0566
A4	0.0200	0.1695	0.1258	0.2158	0.2468	0.4204	0.3827
A5	0.6384	0.1186	0.0730	0.2215	0.1820	0.4401	0.1331
A6	0.1624	0.6629	0.2415	0.4349	0.5962	0.3514	0.2639
A7	0.1695	0.6256	0.2340	0.4277	0.6545	0.3991	0.2328

**Table 9**The matrix  $Y$  (RC structure – ELECTRE method).

	C1	C2	C3	C4	C5	C6	C7
A1	0.0332	0.0025	0.1342	0.0080	0.0190	0.0805	0.0248
A2	0.0024	0.0039	0.0292	0.0714	0.1163	0.0795	0.0046
A3	0.0391	0.0171	0.1255	0.0129	0.0330	0.0328	0.0017
A4	0.0014	0.0085	0.0252	0.0216	0.0864	0.0841	0.0115
A5	0.0447	0.0059	0.0146	0.0222	0.0637	0.0880	0.0040
A6	0.0114	0.0331	0.0483	0.0435	0.2087	0.0703	0.0079
A7	0.0119	0.0313	0.0468	0.0428	0.2291	0.0798	0.0070

**Table 10**The matrix  $E$  (RC structure – ELECTRE method).

	A1	A2	A3	A4	A5	A6	A7
A1	0	0	0	0	0	0	0
A2	1	0	1	1	1	0	0
A3	0	0	0	0	0	0	0
A4	0	0	0	0	1	0	0
A5	0	0	0	0	0	0	0
A6	1	1	1	1	1	0	0
A7	1	1	1	1	1	1	0

The study structural unit is susceptible to be super-elevated due to the following reasons: (1) both 6 m spaced load-bearing shear walls (less than 7 m, which is the maximum limit allowable for new buildings) and masonry area – to – gross building area ratios in the two main directions compliant to the limits of the Italian technical code [22] are present; (2) the horizontal load-bearing system, made of steel profiles and hollow tiles, is completed with a slab anchored to the beams by appropriate connectors in order

**Table 11**The scalar parameters  $S_i$  and  $R_i$  (RC structure – VIKOR method).

Alternative		$S_i$	$R_i$
A1	Base isolation	0.7787	0.3500
A2	Buckling restrained bracings	0.3188	0.1879
A3	Carbon fibre-reinforced polymers	0.9212	0.3267
A4	Eccentric bracings	0.4057	0.2377
A5	Shape memory alloy bracings	0.4945	0.2755
A6	Steel shear panels	0.2366	0.0643
A7	Aluminium shear panels	0.1718	0.0538

**Table 12**

Alternative ranking for RC structure seismic retrofitting according to the VIKOR method.

Alternative			$Q_i$
First	A7	Aluminium shear panels	0.000
Second	A6	Steel shear panels	0.061
Third	A2	Buckling restrained bracings	0.325
Fourth	A4	Eccentric bracings	0.466
Fifth	A5	Shape memory alloy bracings	0.590
Sixth	A1	Base isolation	0.905
Seventh	A3	Carbon fibre-reinforced polymers	0.961

to have a rigid diaphragm; (3) tie beams able to both distribute the forces among shear walls and create a building box behaviour are found.

Therefore, on this masonry building, the design of a super-elevated floor has been conceived, it requiring the demolition of a significant part of the original roof slab with a reduction of the permanent loads equal to 1.11 kN/m<sup>2</sup>.

To this purpose, the following different vertical addition constructive technologies have been considered:

- glued laminated timber (Fig. 12b);
- reinforced concrete (Fig. 12c);
- hot-rolled steel (Fig. 12d);
- tuff masonry (Fig. 12e);
- cold-formed steel (Fig. 12f).

So, other than traditional construction technologies (RC, ordinary steel and masonry systems), also innovative ones, represented by cold-formed steel systems and glued laminated timber structures, have been examined.

The glued laminated timber is preferred to timber, which was used in the past for erection of floors, since it allows to obtain sections having general shape with minimal defects and high structural performance. The possibility of reducing the dimensions of the vertical addition structure members has led in some cases to the use of reinforced concrete. A solution widely used also for the retrofitting of existing buildings is the one based on hot-rolled steel elements, organised into either moment resisting frames or pinned ones. Finally, an innovative solution that combines the use of light materials with structural types distributing the vertical loads uniformly on the masonry walls, is represented by cold-formed systems.

The result of the performed study, framed within a more large research activity carried out by the first Author [23,24], is to use the same MCDM methods already applied in the previous section in order to decide which vertical addition construction system provides the optimal performance.

#### 4.2. The basic elements of MCDM methods

The alternatives of MCDM methods are the various technologies used for vertical addition purpose, which have been previously listed and synthetically illustrated.

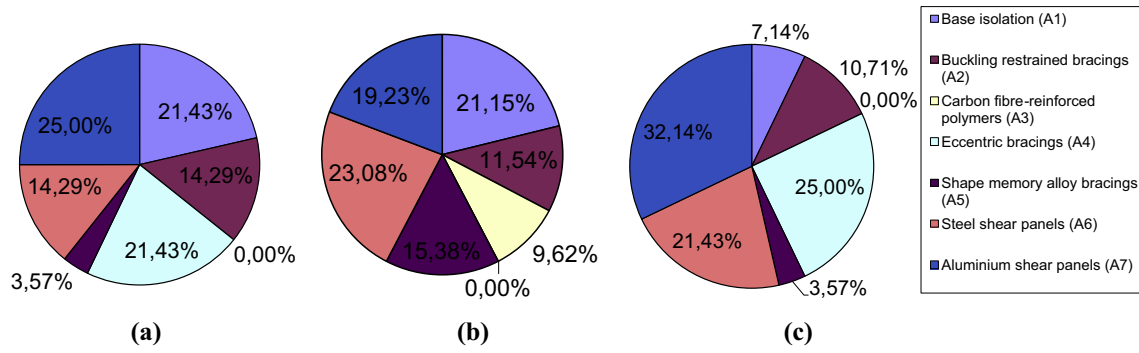


Fig. 10. Results of some further analysis carried out with TOPSIS (a), ELECTRE (b) and VIKOR (c) methods by changing the criteria weight for solving the seismic retrofitting decision problem of existing RC structures.

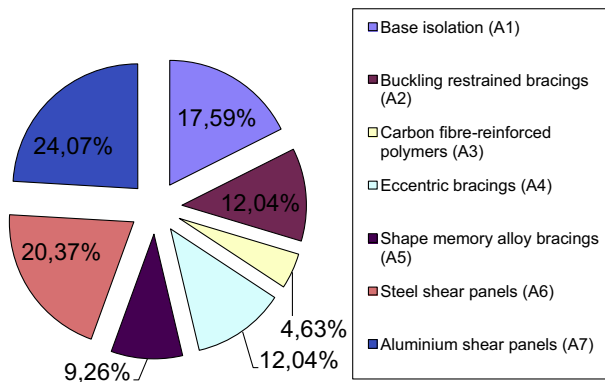


Fig. 11. Final results of some further analysis carried out with all examined MCDM methods.

Instead, about considered criteria, it is important to underline that not only economic and structural parameters, but also environmental factors related to the construction pollution reduction, have been taken into account.

In particular, in the current application, the following criteria have been examined:

- Life Cycle Assessment (LCA), which is an analysis method evaluating a set of interactions that the vertical addition structure has with the environment, considering its whole life cycle, from cradle to grave.
- Environmental Performance Index ( $EP_i$ ), which is another criterion allowing for an environmental comparison about various alternatives. More in detail, it is the amount of energy either consumed or expected to meet the different needs associated with the building standard use.

- Vertical addition system cost, assessed taking into account the updated price list of the Italian Campania Region for building systems.
- Maximum vertical load  $Q_{v\ eff}$ , representing the maximum load sustained by the base masonry piers.
- Peak Ground Acceleration (PGA), which represents the maximum acceleration reached by the vertical addition system.

In particular, the last two criteria have been determined by implementing a FEM model of the selected structural masonry unit, provided with lateral restraints to reproduce the presence of adjacent buildings (see Fig. 13), by means of the PRO\_SAP structural analysis program [25]. A linear stress-strain behaviour in compression and a negligible tensile strength have been assumed for masonry. Shell elements with 30 cm mesh side length have been used for masonry walls, whereas beam elements have been employed to model all the other vertical addition system components.

Initially, as already declared, any MCDM problem involves the definition of both the decision matrix  $D$  and the criterion weight  $w_i$ . In particular, the matrix  $D$  has been defined by considering that the matrix elements  $d_{ij}$  are the rating of the alternative  $A_i$  with respect to the criterion  $C_j$  (Table 13).

Instead, the criterion weights, expressed in percentage terms so that their sum is equal to one, have been determined through the AHP (Analytic Hierarchy Process) method developed by Saaty [14], considering the linear scale of relative importance among alternatives (Table 14).

Criterion weights have been assigned by Authors, who have conferred, contrary to the previous case, the highest importance to the intervention cost (37%) and minimum relevance to energetic parameters (LCA (8%) and  $EP_i$  (5%)).

Since the analysis results can be conditioned from the DM opinions, finally the judgment consistency has been verified in order to

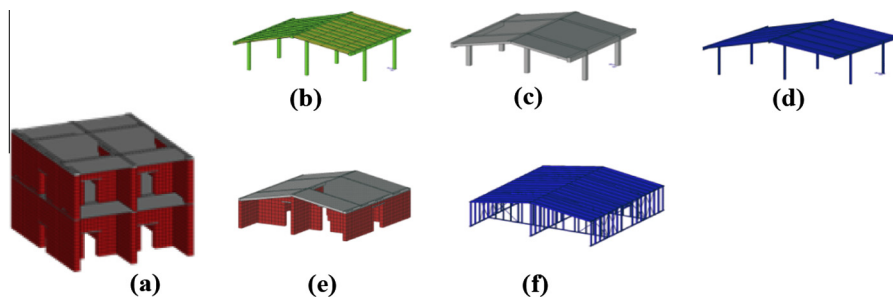


Fig. 12. The study masonry structural unit (a) and the different vertical addition structures (from b to f).



Fig. 13. Typical example of a structural unit extracted from a masonry building aggregate.

**Table 13**  
The decision matrix *D* for the vertical addition problem of an existing masonry building.

Alternative		LCA	EP <sub>i</sub>	Cost of the vertical addition system	Q <sub>v eff</sub>	PGA
		– C1	kW m/m <sup>2</sup> year C2	€/m <sup>2</sup> C3	KN/m <sup>2</sup> C4	m/s <sup>2</sup> C5
Glued laminated timber	A1	0.2096	162.80	174.00	6.16	0.1680
Reinforced concrete	A2	0.8766	149.80	85.62	2.50	0.1760
Hot-rolled steel	A3	0.5731	195.70	95.48	5.85	0.1600
Tuff masonry	A4	0.8405	168.30	103.92	10.50	0.1340
Cold formed steel	A5	0.5731	183.60	111.94	13.19	0.1800

guarantee the objectivity of the final solution. In particular, the Consistency Ratio (CR) was evaluated by checking that it did not exceed the limit specified by Saaty.

After this initial phase common to any MCDM process, the TOPSIS, ELECTRE and VIKOR methods have been applied in order to solve the decision problem.

### 4.3. Decisional problem solution

First, the TOPSIS method has been applied through the same phases already accomplished in the previous analyses. In particular, the following steps have been followed: (1) definition of the normalised decision matrix *R* (Table 15), considering that alternatives are judged on the basis of criteria involving different physical quantities; (2) calculation of the weighted normalised decision matrix *V* (Table 16), obtained by multiplying each column of the matrix *R* for the corresponding weight; (3) determination of virtual optimal and worst solutions (Table 17); (4) distance calculation

**Table 14**  
Weights of criteria for the vertical addition problem of an existing masonry building.

Criterion		Weight (%)
LCA	C1	8.00
EP <sub>i</sub>	C2	5.00
Super-structure cost	C3	37.00
Q <sub>v eff</sub>	C4	20.00
PGA	C5	30.00

**Table 15**  
The matrix *R* (masonry building – TOPSIS method).

	C1	C2	C3	C4	C5
A1	0.1421	0.4214	0.6574	0.3235	0.4570
A2	0.5943	0.3877	0.3235	0.1313	0.4787
A3	0.3885	0.5065	0.3607	0.3072	0.4352
A4	0.5698	0.4356	0.3926	0.5514	0.3645
A5	0.3885	0.4752	0.4229	0.6926	0.4896

**Table 16**  
The matrix *V* (masonry building – TOPSIS method).

	C1	C2	C3	C4	C5
A1	0.0114	0.0211	0.2432	0.0647	0.1371
A2	0.0475	0.0194	0.1197	0.0263	0.1436
A3	0.0311	0.0253	0.1335	0.0614	0.1306
A4	0.0456	0.0218	0.1453	0.1103	0.1093
A5	0.0311	0.0238	0.1565	0.1385	0.1469

**Table 17**  
The virtual alternatives (masonry building – TOPSIS method).

	C1	C2	C3	C4	C5
A*	0.0475	0.0194	0.1197	0.1385	0.1469
A-	0.0114	0.0253	0.2432	0.0263	0.1093

**Table 18**

Ranking of alternatives for vertical addition of an existing masonry building according to the TOPSIS method.

Alternative	$C_i^*$
First A5 Cold formed steel	0.785
Second A4 Tuff masonry	0.714
Third A3 Hot-rolled steel	0.592
Fourth A2 Reinforced concrete	0.543
Fifth A1 Glued laminated timber	0.242

**Table 19**

Sensitivity analysis (masonry building – TOPSIS method).

Criterion	Weight	AT	PT (%)	Sensitivity
C1	0.076	0.339	443	0.002
C2	0.048	–	0	–
C3	0.156	–	0	–
C4	0.268	0.235	88	0.011
	0.451	–	0	–

**Table 20**

The matrix  $R$  (masonry building – ELECTRE method).

	C1	C2	C3	C4	C5
A1	0.1421	0.4214	0.6574	0.3235	0.4570
A2	0.5943	0.3877	0.3235	0.1313	0.4787
A3	0.3885	0.5065	0.3607	0.3072	0.4352
A4	0.5698	0.4356	0.3926	0.5514	0.3645
A5	0.3885	0.4752	0.4229	0.6926	0.4896

**Table 21**

The matrix  $V$  (masonry building – ELECTRE method).

	C1	C2	C3	C4	C5
A1	0.0114	0.0211	0.2432	0.0647	0.1371
A2	0.0475	0.0194	0.1197	0.0263	0.1436
A3	0.0311	0.0253	0.1335	0.0614	0.1306
A4	0.0456	0.0218	0.1453	0.1103	0.1093
A5	0.0311	0.0238	0.1565	0.1385	0.1469

**Table 22**

The matrix  $E$  (masonry building – ELECTRE method).

	A1	A2	A3	A4	A5
A1	0	1	1	1	0
A2	0	0	0	0	0
A3	0	1	0	0	0
A4	0	1	0	0	0
A5	0	1	1	1	0

among each alternative and virtual ones; (5) relative distance calculation from the optimal solution ( $C_i^*$ ) and establishment of a consequent preference order ranking (Table 18).

The applied method has shown that the cold-formed steel system is the optimal super-elevation solution. The results have also been validated through a sensitivity analysis, used to verify that the final results are not influenced by the DM judgments. In fact, by assessing the absolute ( $AT$ ) and the percentage ( $PT$ ) variation parameters of the criteria weight, it has been shown that the solution is stable. In fact, as shown in Table 19,  $C_2$ ,  $C_3$  and  $C_5$  are strong criteria, since  $S = 1/PT = 0$  (that is the  $C_2$ ,  $C_3$  and  $C_5$  criteria weight change does not involve modifications in the preference order classification), while the influence of other criterion weight variation is quite modest, it producing only in few cases different preference order.

**Table 23**

The scalar parameters  $S_i$  and  $R_i$  (masonry building – VIKOR method).

Alternative	$S_i$	$R_i$
A1 Glued laminated timber	0.6156	0.3700
A2 Reinforced concrete	0.3561	0.2000
A3 Hot-rolled steel	0.3526	0.1373
A4 Tuff masonry	0.5325	0.3000
A5 Cold formed steel	0.1670	0.1102

**Table 24**

Ranking of alternatives for vertical addition of an existing masonry building according to the VIKOR method.

Rank	$Q_i$
First A5 Cold formed steel	0.000
Second A3 Hot-rolled steel	0.259
Third A2 Reinforced concrete	0.384
Fourth A4 Tuff masonry	0.773
Fifth A1 Glued laminated timber	1.000

Second, the ELECTRE method application has provided the matrixes  $R$  and  $V$ , reported respectively in Tables 20 and 21.

From the analysis of the matrix  $E$  it is shown that the dominant alternatives for vertical addition are cold-formed steel and glued laminated timber structures, followed by masonry and hot-rolled steel systems (2nd place) and the reinforced concrete one (3rd place) (Table 22).

Finally, from the VIKOR method, the scalar parameters  $S_i$  and  $R_i$  have been firstly calculated, as shown in Table 23. Afterwards, the ranking of alternatives is established by means of the parameter  $Q_i$  (Table 24). Analogously to the previous methods, the optimal solution is always represented by the cold-formed steel system, which has the lowest value of  $Q_i$ .

## 5. Conclusions

In the current paper the choice of the optimal solution for two structural modification interventions, namely seismic retrofitting and vertical addition, of existing buildings has been individuated by means of three MCDM methods, where different alternatives are judged by a Decision Maker (DM) on the basis of several comparison criteria, that is structural, economic and environmental parameters. The novelty of the work, articulated into two case studies, is to find with all investigated methods impartial MCDM problem solutions, which, therefore, do not depend on the personal choices of the DM.

The first intervention has been studied on the basis of the experimental campaign results performed on a real full-scale 3D RC structure upgraded with different seismic retrofitting devices. The MCDM analyses with all used methods have provided the same result, that is the dominating role exerted by aluminium shear panels. The same result has been confirmed also when more in depth investigations with the same three MCDM methods have been carried out. In these cases, where larger combination of criterion weights have been considered, it was shown that the optimal performances are provided by metal shear panels, which lead the final global ranking and are immediately followed by base isolation system and steel BRB and eccentric bracings. As a consequence, also the leading role occupied by metal devices as seismic retrofitting technique of existing RC frames has been proved.

On the other hand, as second intervention typology, the vertical addition of an existing masonry structural unit by means of traditional and innovative technologies has been done. The study results, achieved by using the same three methods used for seismic

retrofitting assessment of the examined RC structure, have provided as optimal solution the cold-formed steel systems thanks to their prerequisites, such as lightness, economy and sustainability. The reliability of the solution found has been validated from the performed sensitivity check, where three of five criteria are robust and the variation influence of other criterion weight is quite modest, it producing rarely the alternative position change within the ranking.

Finally, the combined application of three investigated MCDM methods will allow the practitioners, dealing with structural modification interventions, to individuate with the highest probability an objective optimal solution for retrofitting and vertical addition purposes under the economic, structural and environmental points of view.

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