# Empirical investigation of a tradable credits scheme on travel demand: a household utility based approach incorporating travel money and travel time budgets 

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#### Abstract

: We investigate the influence of a new mobility management measure, the tradable credits scheme (TCS), on the daily travel mode choices of individuals. Generally, we assume the individuals' travel consists of different modes, e.g. private car mode and mass transit mode. In order to control the rapid increase in use of the private car mode in an area, policy makers may wish to implement a TCS basing on the VKT (vehicle kilometre travelled). The effects of the TCS are investigated in this paper based on a utility-theory travel demand model proposed by Golob et al. (1981), a household utility based model incorporating proposed travel money and travel time budgets. The empirical investigation is based on comparison studies of the short-term response and long-term effects with and without TCS. It finds that the implementation of TCS has not a clear impact to the value of time of household in the short-term, and the presence of TCS will not affect the linear relationship between travel time budget and travel money budget over long term. Numerical results demonstrate that the TCS will affect the travel distance of the available transport modes differentially, according to different levels of annual household income.


Keywords: travel demand management, tradable credits scheme, travel budget, utility, VKT

## 1. Introduction

Urban transport has an overriding role in economic activity and growth, and is of major importance for the quality of life of individuals as well as for regional productivity. However, the presence of negative externalities such as congestion, pollution and accidents brings increasing social and environmental stress, which has emphasized the urgent need for an effective, efficient and socially feasible road transport system (Grant-Muller and Xu, 2014). The future development and implementation of effective transport policies in the urban road transport sector depends primarily on how well understood they are. In the environmental management field, the terms 'permits' or 'credits', which provide a 'pull' mechanism have evolved over a relatively long period, particularly in relation to pollution control where they has been well studied and used in practice. The Kyoto Protocol proposed the use of a system of emission permits as an economic tool for climate change mitigation. However, the use of credits or permits for transport demand management, known as the tradable credits scheme (TCS), is a relatively new measure both in theory and in practice (Yang and Wang, 2011). There are no wide-scale implementations of such a scheme in the world to date. The lack of a practical application of this economic measure may be attributable to an undeveloped and incomplete theoretical foundation and particular practical issues that are still to be resolved. However, the tradable credits scheme appears to be a promising policy tool for mobility management and has received increasing attention in recent years.

If a TCS were to be implemented in an urban transport system, the budget for credits will become an additional resource considered within individuals' mode choice, i.e., the introduction of a TCS would create a new expenditure item, but it would also create a new source of income. The net effect depends on whether an individual is a net buyer or seller of credits. The use of a private car will become subject to additional monetary costs if there is a wish to use it beyond the limit of the initial credit allocation. There are various options within the scope of the scheme: people may want to fully use their credits, buy additional credits (where their use of the car exceeds the initial allocation), or save and sell them for financial gain. As a result, individuals have to not only decide on the necessity of the trips they take, but also on how they wish to manage their budget of allocated credits. Mode choice for a typical individual in each geographic area will be affected by the individuals' transport budget, the travel cost for different modes and the individuals' attitudes.

To implement a TCS, a fundamental question is how to measure the effects of a TCS quantitatively with respect to the VKT. In this study, we examine how travelers' mode choice preferences could be influenced by implementing a TCS. The study supposes that the regional authority is responsible for implementing a TCS, the initial credit allocation is free and each household receive a certain number of credits (representing VKT). In order to reduce the VKT of private car, each household (in maximizing their utility), must consider their travel mode based on the credits distributed. That is, the individual must consider the permitted number of VKT and the credit price.

To investigate the influence of a given TCS, we present a microeconomic quantitative analysis framework based on a utility theory based travel demand model developed by Golob et al. (1981). Travel patterns are compared before and after the introduction of a TCS. We further investigate the short-term response and long-term effects with and without a TCS. This is important in order that policy makers might understand how a TCS would need to be designed and how it will affect the travel demand in the future.

The organization of the paper is as follows. In Section 2, based on the utility model proposed by Golob et al. (1981), we present a utility model for the situation where a tradable credits scheme is applied. In Section 3, and in Section 4, we separately investigate the short-term response and longterm effects with and without a TCS. In Section 5, we investigate the effects of the TCS based on the empirical example given by Golob et al. (1981), and the paper is concluded in Section 6.

## 2. Methodology

Golob et al. (1981) proposed a utility-theory travel demand model which incorporated travel budget constraints including average daily time and money expenditure on travel, as will be introduced in Section 2.1. The methodology involves utility theory from microeconomics, which is based on the premise of rational choice behaviour. The rational choice behaviour asserts that a decision-maker is able to rank possible alternatives in order of personal preference and will choose the alternative that is ranked highest, subject to relevant constraints placed on the choice decision.

### 2.1 A general form

Let $U$ represent household utility, $x$ is the amount or quantity of travel, i.e. VKT in this paper. $c$ is the consumption of non-travel goods and services, and $t$ is the leisure time. A definition of the commodity groups relevant to modelling travel decisions is:

$$
\begin{equation*}
U=U(x, c, t) \tag{1}
\end{equation*}
$$

We set the price indices for travel and general consumption as $p_{x}$ and $p_{c}$, respectively, and $Y$ is the household disposable income. The household faces the following money budget constraint when allocating expenditure:

$$
\begin{equation*}
p_{x} x+p_{c} c \leq Y \tag{2}
\end{equation*}
$$

We set $t_{x}$ as the given time per unit distance travelled, $t_{c}$ is the general consumption time, and $T^{\prime}$ is the total time available to all household members. The time budget constraint is then:

$$
\begin{equation*}
t_{x} x+t+t_{c} c \leq T^{\prime} \tag{3}
\end{equation*}
$$

Assuming for the first approximation that the time spend for general consumption is relatively constant over the range of consumption levels in the short-term, $T=T^{\prime}-t_{c} c$ can be used as the time constraint. Therefore, the utility-maximum problem can be written as

$$
\begin{array}{ll}
\operatorname{Max}_{x, c, t} & \phi(x)+\varphi(c)+\xi(t) \\
\text { s.t. } & p_{x} x+p_{c} c \leq Y \\
& t_{x} x+t \leq T \tag{6}
\end{array}
$$

Where $\phi(x), \varphi(c), \xi(t)$ represent utilities due to travel, general consumption and leisure respectively. The utility maximization problem can be further written as

$$
\begin{equation*}
\underset{x}{\operatorname{Max}}\left[\phi(x)+\varphi\left(\frac{Y}{p_{c}}-\frac{p_{x}}{p_{c}} x\right)+\xi\left(T-t_{x} x\right)\right] \tag{7}
\end{equation*}
$$

Corresponding to this, the first optimality condition is

$$
\begin{equation*}
\phi^{\prime}(x)-\frac{p_{x}}{p_{c}} \varphi^{\prime}\left(\frac{Y}{p_{c}}-\frac{p_{x}}{p_{c}} x\right)-t_{x} \xi^{\prime}\left(T-t_{x} x\right)=0 \tag{8}
\end{equation*}
$$

With the assumption that each utility component $\phi(x), \varphi(c)$, and $\xi(t)$ is monotonically increasing and quasi-concave, it has the following qualitative properties according to the first order optimal condition:
(1) Travel $(x)$ can never decreases as income $(Y)$ increases;
(2) Travel $(x)$ can never decrease as the amount of available time ( $T$ ) increases;
(3) Travel decreases with increasing costs (the demand curve for travel is always downward sloping);
(4) Travel increases as speed increases.

Eqs. (4-6) can be further investigated in the multi-modal case. Supposing that there are $m$ available transport modes, $i=1, \cdots, m$, and each mode $i$ is associate with average speed $v_{i}$ and $\operatorname{cost} p_{i}$. The general utility maximization model can be rewritten as

$$
\begin{equation*}
\underset{x}{\operatorname{Max}} U=\sum_{i=1}^{m} \phi_{i}\left(x_{i}\right)+\varphi\left(Y-\sum_{i=1}^{m} p_{i} x_{i}\right)+\xi\left(T-\sum_{i=1}^{m} \frac{x_{i}}{v_{i}}\right) \tag{9}
\end{equation*}
$$

The first order optimal condition for (9) is

$$
\begin{equation*}
\phi_{i}^{\prime}\left(x_{i}\right)-p_{i} \varphi^{\prime}\left(Y-\sum_{i=1}^{m} p_{i} x_{i}\right)-\frac{1}{v_{i}} \xi^{\prime}\left(T-\sum_{i=1}^{m} \frac{x_{i}}{v_{i}}\right)=0, i=1, \cdots, m \tag{10}
\end{equation*}
$$

From (10), the travel by each mode is adjusted to the point where the marginal benefit gained is equal to the marginal cost incurred. The marginal cost $\left(\phi_{i}^{\prime}\left(x_{i}\right)\right)$ consists of two terms which are attributed to money and time, where the marginal monetary $\operatorname{cost}\left(\varphi^{\prime}\left(Y, p_{1}, \cdots, p_{m}\right)\right)$ is a function of income and travel costs on all modes, and the marginal time cost $\left(\xi^{\prime}\left(T, v_{1}, \cdots, v_{m}\right)\right.$ ) is a function of the available time and speeds on all modes.

Considering the commuters in each household who can either take mass transit or drive a private car to his/her destination, and denoting $x_{1}$ as the average distance travelled by mass transit (unit: vehicle
kilometres travelled (VKT)), whilst $x_{2}$ represents the average distance travelled by private car (unit: VKT), a logarithmic utility model with the twin money and time constraints is specified as follows:
$\underset{x_{1}, x_{2}}{\operatorname{Max}} U=a_{1} \log x_{1}+a_{2} \log x_{2}+b_{1} \log \left[Y-\left(p_{1} x_{1}+p_{2} x_{2}\right)\right]+b_{2} \log \left[T-\left(\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}}\right)\right]$
where $p_{2}$ is the per-kilometre commute cost of private car driving, $p_{1}$ is the per-kilometre commute cost for mass transit, and $Y$ is the household disposable income. The average speed of mass transit is $v_{1}$, and the average speed of private cars is $v_{2} . a_{i}$ is the attraction of mode $i$, and $b_{1}$ and $b_{2}$ are the utility weights for general consumption and leisure time, respectively.

Combining (10) and (11), we have the following optimization conditions:

$$
\begin{align*}
& \frac{a_{1}}{x_{1}}-\frac{b_{1} p_{1}}{Y-\left(p_{1} x_{1}+p_{2} x_{2}\right)}-\frac{b_{2}}{v_{1}\left[T-\left(\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}}\right)\right]}=0  \tag{12}\\
& \frac{a_{2}}{x_{2}}-\frac{b_{1} p_{2}}{Y-\left(p_{1} x_{1}+p_{2} x_{2}\right)}-\frac{b_{2}}{v_{2}\left[T-\left(\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}}\right)\right]}=0 \tag{13}
\end{align*}
$$

Eqs.(12)-(13) represent an intractable set of non-linear equations. It should be noted that Golob et al. (1981) just used an approximation to the solutions of Eqs. (12-13) under the assumption that total travel expenditure is a relatively small proportion of income, that is, $p_{1} x_{1}+p_{2} x_{2} \ll Y$, and $\frac{x_{1}}{v_{1}}+$ $\frac{x_{2}}{v_{2}} \ll T$.

### 2.2 The case of a tradable credits scheme

Suppose the regulatory authority implements a tradable credits scheme. The initial credit distribution is free and each household in the study area receives a certain number of credits that permits travel by car: $\bar{x}_{2}$. Individuals in each household then need to consider the amount of kilometres that are allowed by private car and the price of a credit $\left(p_{e}\right)$ if they wish to travel further more kilometres by car. Under the tradable credits scheme, the utility maximization problem (11) for each representative household can then be formulated as follows:

$$
\begin{array}{cc}
\underset{x_{1}, x_{2}}{\operatorname{Max}} & U=a_{1} \log x_{1}+a_{2} \log x_{2} \\
\text { s.t. } & p_{1} x_{1}+p_{2} x_{2}+p_{e}\left(x_{2}-\bar{x}_{2}\right) \leq Y \\
& \frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}} \leq T \\
& x_{1} \geq 0, x_{2} \geq 0 \tag{17}
\end{array}
$$

where $p_{e}$ is the price of tradable credits, $\bar{x}_{2}$ represents credits received per household, e.g., each credit/license entitles the holder to travel one kilometre by car.

The utility maximization problem can be further written as

$$
\begin{equation*}
\underset{x_{1}, x_{2}}{\operatorname{Max}} U=\sum_{i=1}^{2} a_{i} \log x_{i}+b_{1} \log \left[Y+p_{e} \bar{x}_{2}-\left(p_{1} x_{1}+\left(p_{2}+p_{e}\right) x_{2}\right)\right]+b_{2} \log \left[T-\left(\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}}\right)\right]( \tag{18}
\end{equation*}
$$

Similarly, combining (10) and (18), we have the following optimization conditions:

$$
\begin{equation*}
\frac{a_{1}}{x_{1}}-\frac{b_{1} p_{1}}{Y+p_{e} \bar{x}_{2}-\left(p_{1} x_{1}+\left(p_{2}+p_{e}\right) x_{2}\right)}-\frac{b_{2}}{v_{1}\left[T-\left(\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}}\right)\right]}=0 \tag{19}
\end{equation*}
$$

$$
\begin{equation*}
\frac{a_{2}}{x_{2}}-\frac{b_{1}\left(p_{2}+p_{e}\right)}{Y+p_{e} \bar{x}_{2}-\left(p_{1} x_{1}+\left(p_{2}+p_{e}\right) x_{2}\right)}-\frac{b_{2}}{v_{2}\left[T-\left(\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}}\right)\right]}=0 \tag{20}
\end{equation*}
$$

Similar to Eqs. (12)-(13), these are an intractable set of nonlinear equations for which we cannot state the analytical solution directly. Comparing Eqs. (12)-(13) and Eqs. (19)-(20), both sets state that travel by private car mode and mass transit mode per household are adjusted to the point where the marginal benefit gained is equal to the marginal cost incurred. The marginal cost for both modes consists of two terms which are attributed to money and time, where the marginal monetary cost is a function of income $(Y)$, travel cost by transit $\left(p_{1}\right)$ and private car $\left(p_{2}\right)$. The marginal time cost ( $\xi^{\prime}\left(T, v_{1}, \cdots, v_{m}\right)$ ) is a function of the available time ( $T$ ), average speed by transit $\left(v_{1}\right)$ and private car $\left(v_{2}\right)$. However, in the presence of a tradable credits scheme, the cost of using a private car will consist of the cost of private car driving $p_{2}$ and the price of credits $p_{e}$.

## 3. Short-term response with and without a tradable credits scheme

Considering we cannot present the analytical solution of Eqs. (12)-(13) and Eqs. (19)-(20) directly, we firstly consider a special case of the absence of a time constraint, that is,

$$
\begin{equation*}
\underset{x_{1}, x_{2}}{\operatorname{Max}} U=a_{1} \log x_{1}+a_{2} \log x_{2}+b_{1} \log \left[Y-\left(p_{1} x_{1}+p_{2} x_{2}\right)\right] \tag{21}
\end{equation*}
$$

In the presence of a TCS,

$$
\begin{equation*}
{\left.\underset{x_{1}, x_{2}}{\operatorname{Max}} U=\sum_{i=1}^{2} a_{i} \log x_{i}+b_{1} \log \left[Y+p_{e} \bar{x}_{2}-\left(p_{1} x_{1}+\left(p_{2}+p_{e}\right) x_{2}\right)\right] .\right] . ~}_{\text {and }} \tag{22}
\end{equation*}
$$

Corresponding to Eqs. (12)-(13), the optimal conditions can be rewritten as

$$
\begin{align*}
& \frac{a_{1}}{x_{1}}-\frac{b_{1} p_{1}}{Y-\left(p_{1} x_{1}+p_{2} x_{2}\right)}=0  \tag{23}\\
& \frac{a_{2}}{x_{2}}-\frac{b_{1} p_{2}}{Y-\left(p_{1} x_{1}+p_{2} x_{2}\right)}=0 \tag{24}
\end{align*}
$$

We have the analytical solution for transit mode and private car mode without TCS:

$$
\begin{align*}
& x_{1}=\frac{1}{p_{1}} \frac{a_{1}}{a_{1}+a_{2}+b_{1}} Y  \tag{25}\\
& x_{2}=\frac{1}{p_{2}} \frac{a_{2}}{a_{1}+a_{2}+b_{1}} Y \tag{26}
\end{align*}
$$

Corresponding to Eqs. (19)-(20), in the presence of TCS the optimal conditions are:

$$
\begin{align*}
& \frac{a_{1}}{x_{1}}-\frac{b_{1} p_{1}}{Y+p_{e} \bar{x}_{2}-\left(p_{1} x_{1}+\left(p_{2}+p_{e}\right) x_{2}\right)}=0  \tag{27}\\
& \frac{a_{2}}{x_{2}}-\frac{b_{1}\left(p_{2}+p_{e}\right)}{Y+p_{e} \bar{x}_{2}-\left(p_{1} x_{1}+\left(p_{2}+p_{e}\right) x_{2}\right)}=0 \tag{28}
\end{align*}
$$

We have the analytical solution for the transit mode and private car mode with TCS:

$$
\begin{align*}
& x_{1}=\frac{1}{p_{1}} \frac{a_{1}}{a_{1}+a_{2}+b_{1}}\left(Y+p_{e} \bar{x}_{2}\right)  \tag{29}\\
& x_{2}=\frac{1}{p_{2}+p_{e}} \frac{a_{2}}{a_{1}+a_{2}+b_{1}}\left(Y+p_{e} \bar{x}_{2}\right) \tag{30}
\end{align*}
$$

According to the solutions (25)-(26) without a TCS, the VKT on transit and private car modes are constant and proportional to income $Y$. In comparison to the solutions (29)-(30) with a TCS, the VKT for transit and private modes are proportional to the sum of income $Y$ and the potential benefit ( $p_{e} \bar{x}_{2}$ ).

The benefit $p_{e} \bar{x}_{2}$ can be treated as a transport subsidy paid to an individual if the number of credits $\bar{x}_{2}$ is not used, and the credits price $p_{e}$ will affect the VKT for transit and private car modes.

We can now investigate some short-term responses according to Section 2.1 and Section 2.2. From Eqs. (12)-(13) and Eqs. (19)-(20),the simultaneous presence of a money budget and time budget brings nonlinearities between travel distance with two modes and income and available time, whether the TCS applies or not. Comparing the single money budget constraint with the solutions (25-26) without TCS and the solutions (29-30) with TCS, the simultaneous presence of time and money constraints thus brings deviations from the travel money and time budgets, and the presence of TCS will further worsen the deviation.

In the presence of a two-budget model (11) without TCS, we can further investigate the resulting expression for the ratio of the marginal utility of time to the marginal utility of money,

$$
\begin{equation*}
\frac{\partial U}{\partial T} / \frac{\partial U}{\partial Y}=\frac{b_{2}}{b_{1}} \frac{Y-\left(p_{1} x_{1}+p_{2} x_{2}\right)}{T-\left(\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}}\right)} \tag{31}
\end{equation*}
$$

That is, as pointed out by Golob et al. (1981), the value of time given by (31) implied by the two budget logarithmic utility model is directly proportional to the money available for non-travel consumption $\left(Y-\left(p_{1} x_{1}+p_{2} x_{2}\right)\right)$ and inversely proportional to time available for non-travel discretionary purposes ( $T-\left(\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}}\right)$ ). The constant proportionality $\left(\frac{b_{2}}{b_{1}}\right)$ presences the taste of the household for which the model is calibrated. The value of time given by (31) can be also considered as a function of travel conditions, which reflected in per-kilometre commute cost ( $p_{1}$ and $p_{2}$ ) and average travel speed ( $v_{1}$ and $v_{2}$ ). Generally, these conditions might be dependent on the time of day (e.g., peak vs non-peak) as well as the land use characteristics of the transportation system or the household's location within an urban area.

In the presence of two-budget model (18) with TCS, the ratio of the marginal utility of time to the marginal utility of money can be written as

$$
\begin{equation*}
\frac{\partial U}{\partial T} / \frac{\partial U}{\partial Y}=\frac{b_{2}}{b_{1}} \frac{Y+p_{e} \bar{x}_{2}-\left[p_{1} x_{1}+\left(p_{2}+p_{e}\right) x_{2}\right]}{T-\left(\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}}\right)} \tag{32}
\end{equation*}
$$

Although the presence of a TCS will affect the VKT for transit and private car modes, expression (32) demonstrates that the value of time won't change clearly with TCS in contrast with expression (31) without TCS, considering that travel time and money budgets are relatively small proportions of total money and time. Under the assumption that total travel expenditure is a relatively small proportion of income, that is, $p_{1} x_{1}+p_{2} x_{2} \ll Y$, and $\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}} \ll T$, the value of time of household can be approached as

$$
\begin{equation*}
\frac{\partial U}{\partial T} / \frac{\partial U}{\partial Y}=\frac{b_{2}}{b_{1}} \frac{Y+p_{e} \bar{x}_{2}-\left[p_{1} x_{1}+\left(p_{2}+p_{e}\right) x_{2}\right]}{T-\left(\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}}\right)} \approx \frac{b_{2}}{b_{1}} \frac{Y-\left(p_{1} x_{1}+p_{2} x_{2}\right)}{T-\left(\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}}\right)} \approx \frac{b_{2}}{b_{1}} \frac{Y}{T} \tag{33}
\end{equation*}
$$

Furthermore, the coefficients $b_{1}, b_{2}$ defined as sociodemographic and life style variables, which differ by household. Whether TCS applies or not, the value of time concept takes into account both supply side and demand side variables and requires careful data analysis.

## 4. Long-term effects with and without a tradable credits scheme

Following on from the consideration of short-term responses with and without a TCS, another important issue is how travel time and travel money budgets per household are determined over the long-term with and without TCS. That is, whether a TCS is applied or not, travel budgets might be
considered fixed (or approximately fixed per household) in the short run. These budgets can be expected to change in the long run, e.g., according to changes in household incomes, available times, or firms and residential location.

By applying a TCS, there are undoubtedly long-term effects on travel budgets. If we assume that the transit mode and private car mode are available, then the fixed travel money budget is given by,

$$
\begin{equation*}
T M^{*}=p_{1} x_{1}+\left(p_{2}+p_{e}\right) x_{2} \tag{34}
\end{equation*}
$$

and the fixed travel time budget is given by,

$$
\begin{equation*}
T T^{*}=\frac{x_{1}}{v_{1}}+\frac{x_{2}}{v_{2}} \tag{35}
\end{equation*}
$$

The decision problem faced by a household over the long run can be specified in utility terms as

$$
\begin{equation*}
\underset{T M^{*}, T T^{*}}{\operatorname{Max}} U=\operatorname{alog}\left(x_{1}+x_{2}\right)+b_{1} \log \left(Y+p_{e} \bar{x}_{2}-T M^{*}\right)+b_{2} \log \left(T-T T^{*}\right) \tag{36}
\end{equation*}
$$

Here it is proposed that the household faces choices involving trading off the utility from total travel $\left(a \log \left(x_{1}+x_{2}\right)\right)$, against consumption and leisure utilities in determining travel budgets at the longterm stage of the travel decision process. The optimal conditions for an optimum in (36) are

$$
\begin{align*}
& \frac{a}{x_{1}+x_{2}}-\frac{b_{1} p_{1}}{Y+p_{e} \bar{x}_{2}-T M^{*}}-\frac{b_{2}}{v_{1}\left(T-T T^{*}\right)}=0  \tag{37}\\
& \frac{a}{x_{1}+x_{2}}-\frac{b_{1}\left(p_{2}+p_{e}\right)}{Y+p_{e} \bar{x}_{2}-T M^{*}}-\frac{b_{2}}{v_{2}\left(T-T T^{*}\right)}=0 \tag{38}
\end{align*}
$$

Joining the conditions (37-38), we have

$$
\begin{equation*}
\frac{b_{1} p_{1}}{Y+p_{e} \bar{x}_{2}-T M^{*}}+\frac{b_{2}}{v_{1}\left(T-T T^{*}\right)}=\frac{b_{1}\left(p_{2}+p_{e}\right)}{Y+p_{e} \bar{x}_{2}-T M^{*}}+\frac{b_{2}}{v_{2}\left(T-T T^{*}\right)} \tag{39}
\end{equation*}
$$

Setting

$$
\begin{equation*}
\beta=\frac{b_{2}\left(\frac{1}{v_{1}}-\frac{1}{v_{2}}\right)}{b_{1}\left(p_{2}+p_{e}-p_{1}\right)} \tag{40}
\end{equation*}
$$

We have

$$
\begin{equation*}
T-T T^{*}=\beta\left(Y+p_{e} \bar{x}_{2}-T M^{*}\right) \tag{41}
\end{equation*}
$$

From Eq. (40), the coefficient $\beta$ is relative to time for leisure, money for residual consumption and the price of credits. It is positive if the transit mode and private car mode do not dominate each other with the TCS, i.e., if $p_{2}+p_{e}>p_{1}$ for $v_{1}<v_{2}$, or $p_{2}+p_{e}<p_{1}$ for $v_{1}>v_{2}$. This condition is expected to be satisfied in the case that the private car mode is faster than the transit mode $v_{1}<v_{2}$, however with the implementation of a TCS the cost of private car mode is more expensive $p_{2}+p_{e}>p_{1}$.
From Eq. (41), we have

$$
\begin{equation*}
T T^{*}=\beta T M^{*}+\left[T-\beta\left(Y+p_{e} \bar{x}_{2}\right)\right] \tag{42}
\end{equation*}
$$

Eq. (42) represents that, under the application of a TCS, the travel time budget and travel money budget are linearly related over the long-term.
Without a TCS, setting $T M^{\prime *}=p_{1} x_{1}+p_{2} x_{2}$, and $\beta^{\prime}=\frac{b_{2}\left(\frac{1}{v_{1}}-\frac{1}{v_{2}}\right)}{b_{1}\left(p_{2}-p_{1}\right)}$, we have

$$
\begin{equation*}
T T^{*}=\beta^{\prime} T M^{\prime *}+\left(T-\beta^{\prime} Y\right) \tag{43}
\end{equation*}
$$

Comparing Eq. (42) and Eq.(43), we conclude that the travel time budget and travel money budget are independent of the TCS. That is, the presence of TCS or not does not affect the linear relationship between travel time budget and travel money budget over long-term. However, the presence of a TCS will adjust the travel time budget and travel money budget (from $\beta^{\prime}$ to $\beta$ ) and affect the travel demand for transit mode and private car mode. Therefore, Eq. (42) explains how the long-term travel time budget and travel money budget changes with a TCS.

The slope of the derived linear relationships is independent of income and available time, but is dependent on the travel costs (including the price of credits if a TCS applies) and speed. The intercept is dependent on income and the available time as well as travel times and costs. It is positive for $T>\beta^{\prime} Y$, which means that time is relatively more abundant in relation to money. In the presence of a TCS, the price of credits, as part of the travel costs, will also affect the intercept, and the intercept is positive for $T>\beta\left(Y+p_{e} \bar{x}_{2}\right)$.

## 5. Numerical example

According to Zahavi (1979) and Golob et al. (1981), applying the simplified version of the model to aggregate district-level travel data for Washington, D.C. in 1968 resulted in an estimated daily travel distance by car and transit mode (per representative household by income level) within the range of $\$ 4,000-\$ 11,000$. The details are summarized in Table 1. The total travel time expenditure per household is the door-to-door travel time as reported by the respondents, which increases with household income. Travel money expenditure was derived from the reported travel distance by mode. Table 1 also details the unit costs of travel by private car and by bus.

According to Table 1, without the tradable credits scheme, the total travel distance is 298.566 km with difference income levels, where the total car travel distance is 218.808 km , and the total bus travel distance is 79.758 km . We assume that a TCS will be implemented in this area. Assuming the case where the authority decides to reduce the demand for private car travel. According to the current total car travel distance $(218.808 \mathrm{~km})$, the authority will set the total number of credits as 200 ( 1 credits equals 1 km ). It is supposed that the initial credits are distributed equally, i.e., we can set $\overline{\mathrm{x}}_{2}=25$ per household for the eight income levels ( $\$ 4,000-\$ 11,000)(200 / 8=25)$.

Since we cannot derive the equilibrium price directly from the proposed model, we firstly assume the unit credits price is below the unit price of bus, e.g., setting $p_{e}=0.025$, we have the car travel distance and bus travel distance with this TCS, as shown in Table 1. Comparing the car travel distance and bus travel distance without a TCS, we find that the car travelling per household for different income levels decreases, and the bus travelling per household for different income levels increases. The total travel distance with different income levels decreases to 261.577 km in contrast with the 298.566 km without TCS, the total car travel distance with different income levels decrease to 144.866 km in contrast with the 218.808 km before TCS, however, the total bus travel distance with different income levels increase to 116.711 km in contrast with the 79.758 km before TCS.

Further, the decreased ratio of the car travel and the increased ratio of the bus travel are different for different income groups. We define the ratio of car travel ( $R_{c a r}$ ) and the ratio of bus travel ( $R_{b u s}$ ) as Eq.(44)

$$
\begin{equation*}
R_{c a r}=\frac{\left|x_{2}-x_{2}^{\prime}\right|}{x_{2}}, R_{b u s}=\frac{\left|x_{1}-x_{1}^{\prime}\right|}{x_{1}} \tag{44}
\end{equation*}
$$

where the $x_{2}^{\prime}$ and the $x_{1}^{\prime}$ represent the car travel distance and the bus travel distance under the TCS with $p_{e}=0.025$. As shown in Fig. 1, under the presence of a TCS, the car use per household for the high income group decreases more, in contrast with the increased bus use per household.

Table 1. Summary of estimated travel distance per household by income, averaged by district, Washington D.C., 1968

| Annual Income, $\$$ | 4,000 | 5,000 | 6,000 | 7,000 | 8,000 | 9,000 | 10,000 | 11,000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Cars/Household | - | 0.1 | 0.35 | 0.71 | 1.02 | 1.29 | 1.54 | 1.76 |
| Time Budget, min. | 121.2 | 121.2 | 125.4 | 132 | 137.4 | 144.6 | 151.8 | 157.8 |

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| Money Budget, \$ |  | 0.51 | 0.75 | 1.24 | 2.01 | 2,82 | 3.17 | 3.53 | 3.88 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Car <br> Travel <br> Without <br> TCS | Unit time, $\mathrm{min} / \mathrm{km}$ | 4.44 | 4.00 | 3.75 | 3.16 | 2. 86 | 2.50 | 2.31 | 2.14 |
|  | Unit cost, \$/km | 0.104 | 0.096 | 0.092 | 0.081 | 0.075 | 0.068 | 0.064 | 0.060 |
|  | $\begin{aligned} & \text { Distance, } \\ & \text { km } \end{aligned}$ | 0.018 | 2.445 | 8.454 | 19.795 | 34.173 | 42.423 | 50.863 | 60.636 |
| Bus <br> Travel Without TCS | Unit time, $\mathrm{min} / \mathrm{km}$ | 8.82 | 8.00 | 7.50 | 6.32 | 5.71 | 5.00 | 4.62 | 4.29 |
|  | Unit cost, \$/km | 0.037 | 0.037 | 0.037 | 0.037 | 0.037 | 0.037 | 0.037 | 0.037 |
|  | $\begin{aligned} & \text { Distance, } \\ & \text { km } \\ & \hline \end{aligned}$ | 13.732 | 13.928 | 12.493 | 10.988 | 6.947 | 7.709 | 7.425 | 6.536 |
| Total Distance without TCS, km/Household |  | 13.751 | 16.372 | 20.947 | 30.784 | 41.120 | 50.132 | 58.289 | 67.172 |
| TCS price |  | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 | 0.025 |
| Car Travel Distance with TCS, km |  | 0.014 | 1.848 | 6.308 | 14.140 | 23.686 | 28.187 | 32.827 | 37.856 |
| Bus Travel Distance with TCS, km |  | 13.734 | 14.226 | 13.566 | 13.816 | 12.199 | 14.826 | 16.444 | 17.900 |
| Total Distance with TCS, km/Household |  | 13.748 | 16.074 | 19.874 | 27.960 | 35.886 | 43.014 | 49.271 | 55.755 |

The effect of TCS can be further illustrated in Fig. 2, which shows the changes of total travel distance, car travel distance and bus travel distance before and after the implementation of TCS. As investigated in Golob et al. (1981), the estimated travel distances (total travel distance, car travel distance and bus travel distance) per household by mode appear as a continuouss curve (the curves before the TCS), which correspond to observed values well. This is especially the case in the light of the fact that the estimated values were not calibrated to observed proportions of travelers using the two modes, but were derived from the observed travel budgets and theoretical relationships. Therefore, this modelling approach provide us to continue investigate the impact of TCS.

Fig. 2 shows that the application of TCS will bring different effects with respect to income levels. Generally, the changes of total travel distance are not so big with a household annual income within the range of $\$ 4000-\$ 6000$, in contrast with bigger changes with a household annual income within the range of $\$ 7000-\$ 11,000$. The changes of total travel distance with different income level attribute the corresponding changes of car travel distance and bus travel distance, with the increase of bus travel distance and the decrease of car travel distance of the implementation of TCS, e.g., for the household travel at the annual income $\$ 4000$, the total travel distance, car travel distance and bus travel distance, before and after the implementation of TCS, is $13.751 \mathrm{~km}(13.748 \mathrm{~km}), 0.018 \mathrm{~km}(0.014 \mathrm{~km})$, and $13.732 \mathrm{~km}(13.734 \mathrm{~km})$, separately. The change of the total travel distance is minor (only 0.003 km ). However, for the household travel at the annual income \$11,000, the total travel distance, car travel distance and bus travel distance, before and after the implementation of TCS, is 67.172 km ( 55.755 km ), $60.636 \mathrm{~km}(37.856 \mathrm{~km})$, and 6.536 km ( 17.9 km ), separately. The change of the total distance is 11.417 km .


Fig. 1 Comparison of the ratio of car travel $\left(R_{c a r}\right)$ and the ratio of bus travel $\left(R_{b u s}\right)$ with and without TCS


Fig. 2 Daily travel distance per household by mode and by income with and without TCS

## 6. A case study for the city of Milan

Further studies and investigations are necessary. Besides the theoretical investigations with different dimensions, an empirical study is necessary with recent data. The city of Milan and its surrounds constitute a metropolitan area positioned in the center of the Po valley, Northern Italy (Mussone, et al., 2014). In the exact centre of the city there is an area called "Cerchia dei Bastioni" (Bastioni for brevity) that was the subject of a charging policy called "Area C", started since16th January 2012, to mitigate congestion and then reduce pollution. Really, the boundary of "Area C" is contained in the "Bastioni" and is slightly smaller as the ring roads surrounding Area C are not included. This network structure has been calibrated with real traffic data obtained over many years, and provides a realistic 'supply' for the scenario studies. The Area C might provide an importance case for us to carry out simulation studies and investigate the effects of TCS.

## 7. Conclusion

Investigations the potential effects of TCS to travel demand have been carried out based on existing studies. A household utility based model incorporating travel money and travel time budgets proposed. Existing concepts and mathematical models have been proved to be effective in modelling consumer behaviour in microeconomics, the proposed approach attempts to investigate the potential impacts of TCS and their potential role in travel demand management under reasonable theoretical assumptions. We investigate the short-term response and long-term effects with and without TCS. We find that the implementation of TCS has not a clear impact to the value of time of household in the short-term, and the presence of TCS does not affect the linear relationship of travel time budget and travel money budget over long-term. However, the presence of TCS will adjust the travel time budget and travel money budget and affect the travel demand for transit mode and private car mode. Numerical results demonstrate that TCS will affect the travel distance for available transport modes (e.g., bus and car). The effects of TCS to different household are different with respect to the annual income.

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