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Consistent circuit technique for zero-sequence currents evaluation in interconnected single/three-phase power networks

This paper deals with a rigorous and mathematically consistent technique for circuit analysis of modern electrical power systems consisting in the interconnection of three-phase components and single-phase active loads. Indeed, it is well known that the standard technique based on the symmetrical components transformation is commonly used in the analysis of symmetrical three-phase systems. Nowadays, however, the evolution of power systems towards the custom power conditioning (e.g., active filtering) and the smart grid model requires the inclusion into the analytical tool of single-phase active loads. Starting from the symmetrical components transformation in its rational form instead of its classical form, a rigorous circuit representation of the interconnection of a three-phase system with single-phase active loads is derived in the paper. The proposed circuit representation allows the analysis of complex power systems by means of basic circuit techniques. In particular, the paper focuses on the evaluation of the zero-sequence component of the currents in any branch of the power system. The application of the proposed circuit technique is demonstrated through an example consisting in the analysis of an active filter designed to force to zero the current in the fourth wire of the mains.

Keywords: Symmetrical components transformation; sequence circuits; single-phase active loads; zero sequence current; active filtering.

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1. Introduction

It is well known that modern power systems for electrical energy distribution are moving towards the new concept of Smart Grid (SG) (e.g., see [1]) and custom power conditioning (e.g., active filtering for load balancing and harmonic filtering [2]-[3]). Roughly speaking, the traditional model for electrical systems consisting in the production of electrical energy in large power plants, move energy into high-voltage transmission systems, and then into lower-voltage networks reaching the end users, is nowadays about to be overcome by the SG model. Indeed, while the traditional model is characterized by reduced flexibility due to its inherent unidirectional and passive features, the SG model allows to integrate the energy coming from large power plants with that generated from renewable sources, usually dispersed and intermittent. A key feature of the SG and power conditioning models is the capacity to dynamically manage generators and both active and passive loads to cope with peak power, compensate sudden voltage reductions, etc. [1], [4].

Within the framework depicted above, the standard symmetrical components transformation used for the circuit analysis of electrical power systems [5]-[6] needs to be extended and generalized [7], in order to take into account new network topologies where loads are typically active loads, and where single-phase active networks are connected to the grid [8]. Indeed, to the Authors' knowledge, in the existing technical literature related to the symmetrical components transformation, a rigorous circuit approach allowing the

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analysis under general topological and source conditions is still lacking. In this work, a slightly changed symmetrical components transformation will be used with respect to the classical approach, i.e., a self-adjoint transformation matrix will be used in order to preserve conservation of power [9]-[12]. This approach leads to a straightforward derivation of an equivalent circuit in order to obtain a rigorous representation of the interconnection of single-phase active networks to a three-phase network [13]-[14]. This result allows the analysis of complex networks with general source and topology conditions by means of basic circuit techniques. In particular, in this paper the analysis is aimed at evaluating the zero-sequence component of currents circulating in the network branches. The use of the analytical circuit results derived in the paper will be shown through the application to an example consisting in the analysis of an active filtering system with the particular feature of forcing to zero the current in the fourth wire of the mains [15].

2. Symmetrical components transformation in rational form

Symmetrical three-phase power systems under sinusoidal steady-state can be conveniently analyzed by resorting to the well-known symmetrical components transformation. Indeed, even in the case of mutual coupling between the phases, the assumption of symmetrical system results in three uncoupled sequence circuits. Solving each sequence circuit is much simpler than solving the system as a whole.

The transformation matrix, in its rational form, is defined as

$$\mathbf{S} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \tag{1}$$

where

$$\alpha = e^{j\frac{2}{3}\pi} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \tag{2}$$

and $\alpha^2 = \alpha^*$. The transformation matrix is a Hermitian (or self-adjoint) matrix, i.e., $\mathbf{S}^{-1} = \mathbf{S}^{T*}$. This is a crucial property within the approach presented in this paper since the conservation of power is guaranteed and therefore consistent equivalent circuits can be derived. On the contrary, in its classical form the transformation (1) does not hold the power conservation property [5].

The symmetrical components transformation when applied to phasor voltages provides

$$\begin{bmatrix} V_+ \\ V_- \\ V_\mathbf{0} \end{bmatrix} = \mathbf{S} \cdot \begin{bmatrix} V_\mathrm{a} \\ V_\mathrm{b} \\ V_\mathrm{c} \end{bmatrix} \tag{3}$$

where V_+, V_- and V_0 are the positive, negative, and zero sequence voltages. Of course, the same transformation applies to phasor currents.

Symmetrical three-phase passive components (i.e., lines and loads) can be described in terms of an impedance matrix with the following structure

$$\mathbf{Z} = \begin{bmatrix} Z & Z_m & Z_m \\ Z_m & Z & Z_m \\ Z_m & Z_m & Z \end{bmatrix} \tag{4}$$

By defining the column vectors

$$\mathbf{V}_s = \begin{bmatrix} V_+ \\ V_- \\ V_0 \end{bmatrix}, \ \mathbf{I}_s = \begin{bmatrix} I_+ \\ I_- \\ I_0 \end{bmatrix}, \ \mathbf{V} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}, \ \mathbf{I} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{5}$$

the transformed current/voltage relationship for a symmetrical passive component can be written

$$\mathbf{V}_s = \mathbf{S}\mathbf{Z}\mathbf{S}^{-1}\mathbf{I}_s = \mathbf{Z}_s\mathbf{I}_s \tag{6}$$

where

$$\mathbf{Z}_s = \begin{bmatrix} Z_+ & 0 & 0 \\ 0 & Z_- & 0 \\ 0 & 0 & Z_0 \end{bmatrix} \tag{7}$$

and

$$Z_+ = Z_- = Z - Z_m \tag{8}$$

$$Z_0 = Z + 2Z_m \tag{9}$$

The diagonal form of the sequence impedance matrix (7) leads to the above-mentioned uncoupled sequence circuits when the transformation is applied to the whole three-phase system.

3. Connection with single-phase networks

A key point for developing an analytical tool able to include a single-phase network in the general three-phase framework outlined in Section 2 is taking into account the constraints imposed by the connection between the two kinds of networks.

The most common connection between a three-phase and a single-phase network is the symmetrical connection, consisting in the star connection represented in Fig. 1, where G is the reference terminal for voltages.

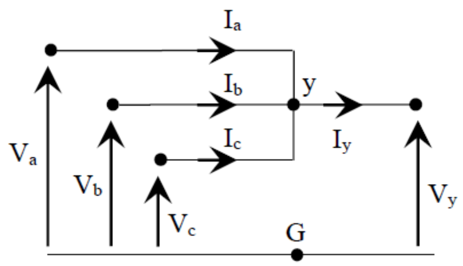

Fig. 1. Star connection between a three-phase and a single-phase network.

The constraints given by the connection are

$$V_a = V_b = V_c = V_y \tag{10}$$

$$I_a + I_b + I_c = I_y \tag{11}$$

By taking into account such constraints into the symmetrical components transformation it can be readily obtained

$$V_+ = 0,\ V_- = 0 \tag{12}$$

$$V_0 = \sqrt{3}V_y,\ I_0 = \frac{1}{\sqrt{3}} I_y \tag{13}$$

It follows that positive and negative sequences appear as short-circuit connection, whereas, as far as the zero sequence component is concerned, the symmetrical connection acts as an ideal transformer, with transfer ratio $\sqrt{3}$, between the three-phase and the single-phase sides (see Fig. 2 where the arrow below the transfer ratio is directed from the primary to the secondary side of the ideal transformer) [13]. It is worth noticing that this achievement was possible because the rational form for the transformation matrix was adopted instead of the more commonly used classical form.

The above result allows the analysis of the whole zero-sequence circuit, including the single-phase network that can be easily taken into account by exploiting the well known properties of the ideal transformer concerning the transformation of voltages, currents, and impedances. More specifically, a single-phase network connected at the right side in Fig. 2 is equivalent to a single-phase network at the left-side having: a) the same topology; b) impedances multiplied by 3; c) voltages and voltage sources multiplied by $\sqrt{3}$; d) currents and current sources divided by $\sqrt{3}$. Of course, inverse transformations should be considered to perform the analysis on the single-phase side.

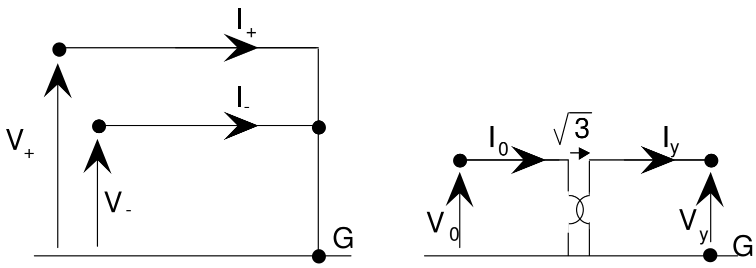

Fig. 2. Symmetrical-components equivalent circuits of a star connection: positive and negative sequence circuits on the left, and zero sequence circuit on the right.

4. Example

The network transformation outlined in Section 3 is here applied to the network shown in Fig. 3 for illustration purpose. The network consists in three three-phase sources with series (possibly coupled) impedances connected to a single-phase network with arbitrary topology and possibly including voltage/current sources and impedances (symbolically represented by V_k, I_k, and Z_k in Fig. 3).

Positive and negative sequence circuits (here not represented) can be readily derived from (12) leading to short-circuited common points of the three why connections. Thus, as a well-known result, single-phase networks connected to star common points do not affect network topology of positive and negative sequence circuits.

Fig. 4 represents the zero sequence circuit related to the three-phase system, coupled to the single-phase system through ideal transformers according to the results reported in Section 3. It can be seen the fictitious reference point G which is common to all the ideal transformers.

Fig. 5 represents the same circuit as in Fig. 4, with the single-phase network directly connected with the zero-sequence network taking into account the ratio of the ideal transformers. Such a transformation holds the network topology, but the voltage/current sources and the impedances values are transformed as mentioned above.

The whole network represented in Fig. 5 can be solved by resorting to standard circuit techniques. In particular, the zero sequence currents can be evaluated in any network branch.

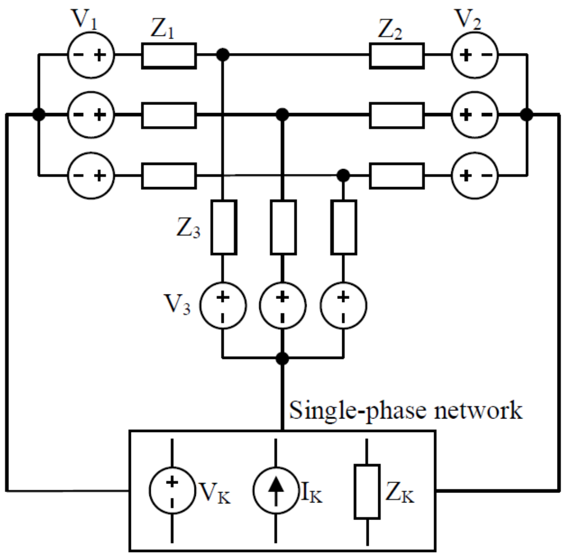

Fig. 3. Sample network used to illustrate the circuit transformations derived in Section 3.

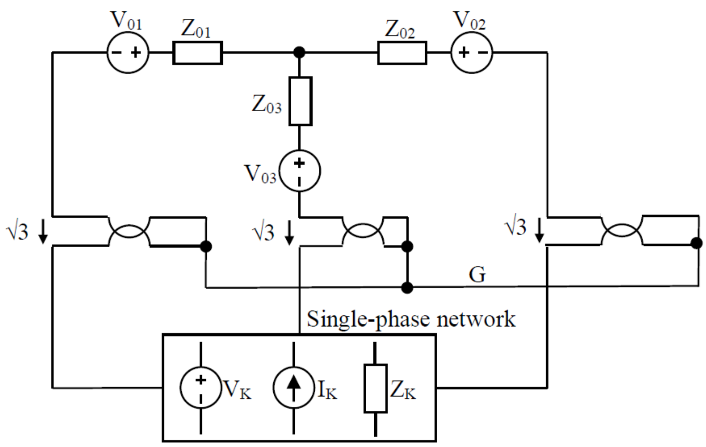

Fig. 4. Zero sequence circuit for the sample network shown in Fig. 3.

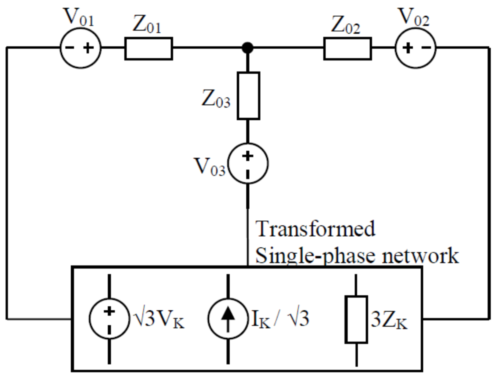

Fig. 5. Zero sequence circuit of Fig. 4 with the single-phase network reported through the ideal transformers.

5. Application to Active Filter Analysis

An interesting application of the theory developed in Section 3 is the analysis of the system represented in Fig. 6 (reduced form of [15]), including an active power filter consisting in a three-phase voltage source inverter (VSI) and a half-bridge inverter (CH) connected between the mains and a four-wire distorting load. The variables referred to the three phases *a, b, c* are indicated in bold as a single three-element vector.

Three-phase load capacitors filter the VSI current ripple and cooperate with the active filter in the reactive power compensation. The central point of d.c. link capacitors is connected to the fourth wire such that high voltage spikes towards the ground, as well as disturbances and capacitive ground currents at the switching frequency, are avoided. The job of VSI is to maintain the line current $\boldsymbol{i}_s$ sinusoidal and balanced. Under these operating conditions the active filter at the line side acts as an ideal resistive and balanced load. It performs unit power factor, load balancing and harmonic filtering. The mains zero-sequence current i_{sn} is forced to zero provided that the current $\boldsymbol{i}_s$ is balanced. In order to avoid voltage oscillation of central point of d.c. link capacitors, single-phase inverter CH is driven to adsorb the load neutral current, so as to keep $\boldsymbol{i}_N$=0. In Fig. 7 the equivalent circuit under sinusoidal steady state is shown. Notice that the four modulated voltages of VSI and half-bridge inverter CH are represented as single voltage sources $\mathbf{V}_F$ and V_H connected to the central point of d.c. link capacitors (not indicated).

In Fig. 8 positive/negative sequence circuit is represented. As it was expected, the single-phase network is not included in such representation.

In Fig. 9, however, where the zero sequence circuit is represented, the single-phase network corresponding to the half-bridge inverter CH must be included, according to the results derived in Section 3. In particular, notice that the impedance is multiplied by the factor 3, whereas voltages and currents are multiplied and divided by $\sqrt{3}$, respectively. Analysis of the circuit in Fig. 9 allows a proper design of the active filter with respect, in particular, to its capability of forcing to zero both the current I_{sn} in the fourth wire of the mains and d.c. central-point current I_N. Note that, as a result, there is no zero-sequence current in three-phase capacitors, and the no-load zero-sequence line voltage (if any) is applied to the load. Finally, it is worth noticing that the specific application shown in Figs. 6-9 was considered only to the aim of illustrating the methodological use of the analytical results derived in Section 3. In fact, the numerical simulation of the system in Fig. 6 was already presented in [15], and therefore it is beyond the scope of this paper.

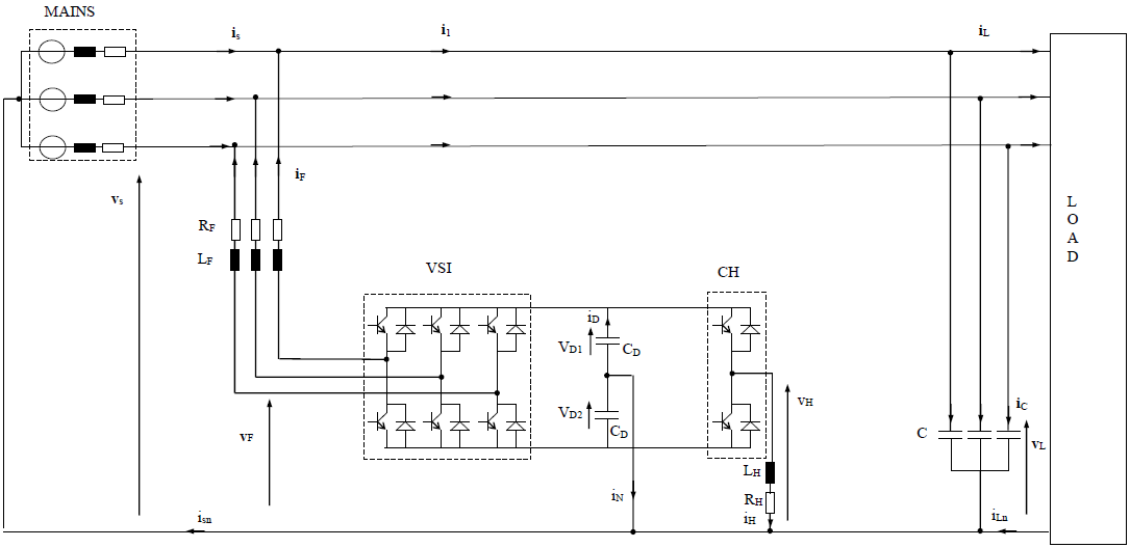

Fig. 6. Application example used to illustrate the theoretical results derived in Section 3. The system includes a three-phase voltage source inverter connected to a half-bridge inverter (i.e., a single-phase network).

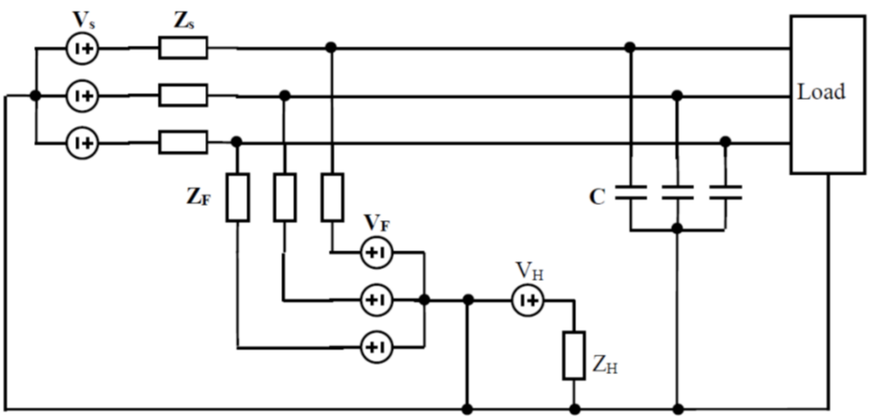

Fig. 7. Equivalent circuit of the system in Fig. 6 under sinusoidal steady state.

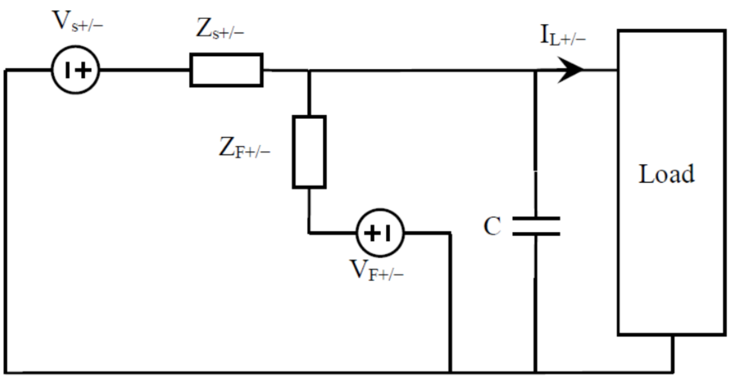

Fig. 8. Positive/negative sequence circuit for the circuit shown in Fig. 7.

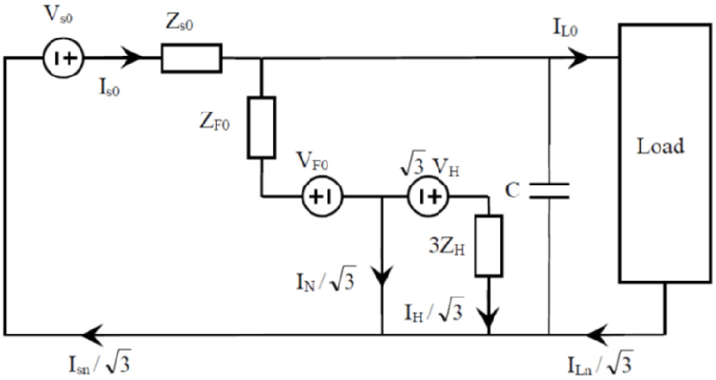

Fig. 9. Zero sequence circuit of the circuit in Fig. 7. The single-phase network and the related variables are represented according to the analytical results derived in Section 3.

6. Conclusion

The proposed circuit technique provides a straightforward approach for the evaluation of zero-sequence currents in power systems including three-phase components and active single-phase loads. The approach is consistent with the standard symmetrical components transformation provided that its rational form is used. The proposed methodology includes active loads, therefore it is well suited to the analysis of modern power systems based on the smart grid model and custom power conditioning. Future work will be devoted to the comparison of the proposed circuit technique with the numerical methods commonly used in the analysis of complex power systems.

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